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Multi-user Two-way Deterministic Modulo 2 Adder Channels – When Adaptation Is Useless

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***Abstract*—In two-way channels nodes are both sources and destinations of messages, allowing them to "adapt" or "interact" in the sense that their next channel input may be a function of their past received signals. This "adaptation" and how to best exploit it lies at the heart of two-way communication problems, rendering them particularly complex and challenging. It would be useful to know when adaptation is *not* beneficial from a capacity perspective. Certain examples exist: it is known that for the point-to-point two-way modulo 2 adder channel, and the point-to-point Gaussian two-way channel, adaptation does not increase capacity. In this work we show that the same is true for certain classes of deterministic multi-user two-way channels. In particular, we consider a class of multi-user two-way modulo 2 adder channels, which include the two-way modulo 2 adder MAC/BC channel, the two-way modulo 2 adder interference channel, and the two-way modulo 2 adder Z channel. For all three channel models we obtain the capacity region, which may be achieved using simple time-sharing.**

I. Introduction

Two-way communication refers to the exchange of two messages between two users. What distinguishes two-way communication from one-way communication is that nodes are both sources and destinations of messages and may both transmit and receive over the channel. This allows nodes to adapt their current channel inputs to the past received signals. Shannon first proposed and studied the point-to-point two-way channel in 1961 [1], where he provided inner and outer bounds which are not tight in general. While Shannon's bounds have been tightened [2]–[4], capacity of the two-way channel in general remains open, with the two-way (deterministic) binary multiplier channel being a notable example of a simple channel for which capacity remains elusive.

However, capacity is known for several channel models in which, essentially, the confusion between signals caused by the two-way nature of the problem, may be resolved by each receiver. For example, if we consider a two-way deterministic modulo 2 adder channel (the channel outputs $Y = Y_1 = Y_2$ are all equal to the modulo 2 sum of the binary channel inputs, $X_1 \oplus X_2$), we easily see that the capacity region is one bit per channel use per user, resulting in two parallel noise-free point-to-point channels. In essence, here one user's own signal does not affect reception of the other user's signal and both the $\rightarrow$ and the $\leftarrow$ "directions" may simultaneously carry information. In this case, allowing for adaptation at the transmitters does not effect the capacity. In a similar fashion, it was shown that the capacity of a Gaussian two-way point-to-point channel is equal to two parallel Gaussian channels, which may be achieved without the use of adaptation at the nodes [5].

Contributions. In this paper, we are interested in determining whether similar statements can be made in multi-user channels rather than point-to-point channels. To do so, we consider several multi-user deterministic modulo 2 adder channels – channels models which are of interest due to their simplicity and also bear relationship to the DOF of the multi-user channels [6]. All inputs and outputs are binary, there is no noise, and the signals are added modulo 2. Specifically, we consider the two-way deterministic binary adder Multiple Access / Broadcast channel (MAC/BC with 4 messages), the two-way deterministic modulo 2 adder interference channel (4 messages), and the two-way deterministic modulo 2 adder Z channel (6 messages). We ask whether adaptation may increase the capacity regions of these deterministic multi-user two-way modulo 2 adder channels. We will show that it does not, and that capacity may actually be achieved by simple time-sharing schemes between nodes transmitting in the same "direction", while nodes in opposite directions may simultaneously transmit as in the point-to-point modulo 2 adder and Gaussian channel models. In deriving outer bounds for these channel models, we use carefully chosen genies and Markov chain structures.

Related Work. The first of our three channel models is a deterministic modulo 2 adder MAC/BC channel. An achievable rate region and an outer bound of a similar multi-user (multiple-access and broadcast with a common message) half-duplex two-way channel is derived in [7], [8] respectively. The former [7] considers a Gaussian channel model, while the latter [8] is derived for the general discrete memoryless channel. This differs from our model in that we assume full-duplex operation, have 2 broadcast messages rather than a common one, and consider a modulo 2 adder channel model. The second of our three channel models is a two-way modulo 2 adder interference channel. The capacity region of the one-way modulo 2 adder interference channel is introduced in [9]; the capacity region of more general class of deterministic interference channels is known [10]. The two-way interference channel is first considered in [11], where an outer bound region for the general discrete memoryless model is proposed. The two-way interference channel (IC) bears some similarities with

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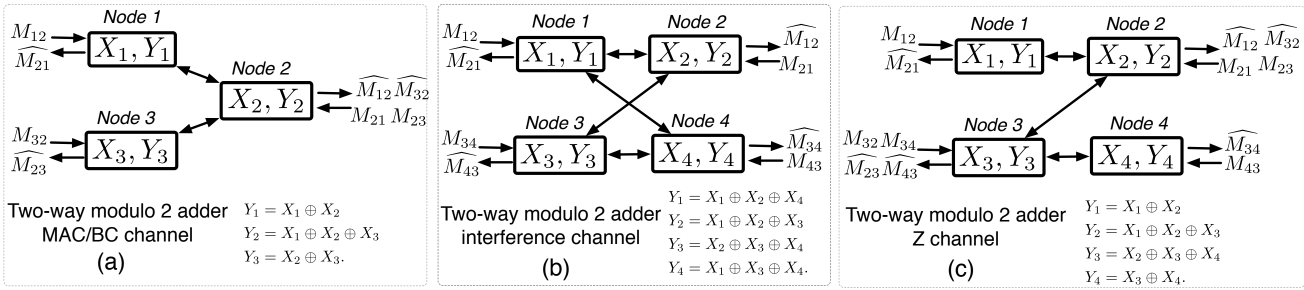

Fig. 1. Three multi-user two-way deterministic modulo 2 adder channel models. M_{jk} denotes the message from node j to node k.

interference channels with feedback and generalized feedback [12]–[14]. The third channel model we consider is the two-way deterministic modulo 2 adder Z channel. The one-way Z channel was first studied in [15], in which the capacity region of a special class of degraded Z channels and an outer bound for general Z channel are obtained. The capacity region of the one-way deterministic Z channel is found in [16]. As we see, little work has emerged on multi-user two-way channel models, and as such we first consider simple modulo 2 adder channel models. Since the submission of this work, we have extended our results to the corresponding linear deterministic channel models that model Gaussian channels at high SNR [17], where we show that adaptation is useless in the MAC/BC and Z channels, and partial adaptation (defined in [17]) is useless for the two-way IC.

Outline of Paper. We define three multi-user two-way modulo 2 adder channels in Section II and present the capacity regions of these three channel models in Section III, IV and V respectively. Section VI concludes the paper.

II. Models, Definitions and Notations

We consider three multi-user two-way binary deterministic adder channels (we drop the "deterministic" from now on) which are shown in Fig. 1, where we see that all nodes act as both transmitters (encoders) and receivers (decoders), and let M_{jk} denote the message from node j to node k:

• *the two-way modulo 2 adder MAC/BC channel:* transmitter 1 and 3 send independent messages M_{12} and M_{32} to receiver 2, respectively forming a Multiple Access Channel (MAC) in the $\rightarrow$ direction. Meanwhile, transmitter 2 sends two independent messages M_{21} and M_{23} to receiver 1 and 3, respectively, forming a Broadcast Channel (BC) in the $\leftarrow$ direction.

• *the two-way modulo 2 adder interference channel:* transmitter 1 and 3 send messages M_{12} and M_{34} to receiver 2 and 4, respectively, forming an interference channel in the $\rightarrow$ direction. Similarly, transmitter 2 and 4 send messages M_{21} and M_{43} to receiver 1 and 3 respectively, forming another interference channel in the $\leftarrow$ direction.

• *the two-way modulo 2 adder Z channel:* transmitter 1 and 4 send messages M_{12} and M_{43} to receiver 2 and 3 respectively. Transmitter 2 and 3 send messages (M_{21}, M_{23}) and (M_{32}, M_{34}) to receivers 1,3 and 2,4 respectively. We thus have two Z channels in opposite directions. This channel model contains interference, multiple-access, broadcast and two-way features.

The binary channel inputs and outputs of user $j \in \{1, 2, 3, 4\}$ at discrete time i are $X_{j,i}$ and $Y_{j,i}$, which lie in the alphabets $\mathcal{X}_j \in \{0, 1\}$ and $\mathcal{Y}_j \in \{0, 1\}$ respectively. We use "$\oplus$" to denote modulo 2 addition. Let $A_j^i = (A_{j,1}, A_{j,2}, ..., A_{j,i})$, for any given time i, and say that a node employs *adaptation* if its encoding function which yields the channel inputs at time i is a function of the previously received outputs, $X_{j,i} = f_j(M_{jk}, Y_j^{i-1})$, where f_j ($j \in \{1, 2, 3, 4\}$) are deterministic functions. If a node behaves in a *non-adaptive* fashion then its inputs are functions of its messages only, i.e. $X_{j,i} = f_j(M_{jk})$. The messages M_{jk} are uniformly distributed in $\{1, 2, \cdots 2^{nR_{jk}}\}$ for j, k in the appropriate sets depending on the channel model, where R_{jk} denotes the rate of message M_{jk}. At each time step $0 \leq i \leq n$, encoder j selects the next input $X_{j,i}(M_{jk}, Y_j^{i-1})$ (which may also be a function of 2 messages in the Z channel). Receiver k uses a decoding function $g_k : \mathcal{Y}_k^n \rightarrow \widehat{\mathcal{M}}_{jk}$ to obtain an estimate $\widehat{M}_{jk}$ of the transmitted message M_{jk}. The capacity region is the supremum over all rate tuples which simultaneously drive the probability that any of the estimated messages is not equal to the true message, to zero as $n \rightarrow \infty$. We now proceed to define the channel model for each different channel and obtain its capacity region.

III. The Capacity Region of the Two-way Modulo 2 Adder MAC/BC

The two-way modulo 2 adder MAC/BC is discrete and memoryless and at each channel use is described by

$$\begin{aligned} Y_1 &= X_1 \oplus X_2 \\ Y_2 &= X_1 \oplus X_2 \oplus X_3 \\ Y_3 &= X_2 \oplus X_3. \end{aligned}$$

If all nodes employ adaptation, then at channel use i

$$\begin{aligned} X_{1,i} &= f_1(M_{12}, Y_1^{i-1}) \\ X_{2,i} &= f_2(M_{21}, M_{23}, Y_2^{i-1}) \\ X_{3,i} &= f_3(M_{32}, Y_3^{i-1}). \end{aligned}$$

In this case, the capacity region may be stated as follows.

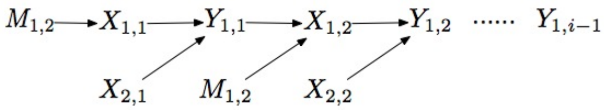

Fig. 2. The Markov chain used in the outer bound proof of Theorem 1.

Theorem 1: The capacity region of the two-way modulo 2 adder MAC/BC channel is the set of non-negative rate tuples $(R_{12}, R_{32}, R_{21}, R_{23})$ such that

$$R_{12} + R_{32} \le 1 \quad (1)$$
$$R_{23} + R_{21} \le 1. \quad (2)$$

Proof: We first prove the converse; achievability may be argued via time-sharing.

Proof of bound (1)*:* The first bound follows from the sum-rate outer bound of the multiple-access channel. By Fano's inequality,

$$\begin{aligned} n(R_{12} + R_{32} - \epsilon) &\le I(M_{12}, M_{32}; Y_2^n) \\ &= H(Y_2^n) - H(Y_2^n | M_{12}, M_{32}) \\ &\le \sum_{i=1}^{n} [H(Y_{2,i} | Y_2^{i-1})] \\ &\le \sum_{i=1}^{n} [H(Y_{2,i})] \\ &\le n \end{aligned}$$

where the last inequality follows as $Y_{2,i}$ is a binary random variable whose entropy is thus bounded by 1.

Proof of bound (2)*:*

$$\begin{aligned} & n(R_{21} + R_{23} - \epsilon) \\ & \overset{(a)}{\le} I(M_{21}; Y_1^n | M_{12}, M_{32}) + I(M_{23}; Y_3^n | M_{12}, M_{32}, M_{21}) \\ & \overset{(b)}{\le} H(Y_1^n | M_{12}, M_{32}) - H(Y_1^n | M_{21}, M_{12}, M_{32}) \\ & + H(Y_3^n | M_{12}, M_{32}, M_{21}) \\ & \overset{(c)}{\le} \sum_{i=1}^{n} [H(Y_{1,i}) - H(Y_{1,i} | M_{21}, M_{12}, M_{32}, Y_1^{i-1}) \\ & + H(Y_{3,i} | M_{12}, M_{32}, M_{21}, Y_3^{i-1})] \\ & \overset{(d)}{=} \sum_{i=1}^{n} [H(Y_{1,i}) - H(X_{1,i} \oplus X_{2,i} | M_{21}, M_{12}, M_{32}, Y_1^{i-1}, X_1^i) \\ & + H(X_{2,i} \oplus X_{3,i} | M_{12}, M_{32}, M_{21}, Y_3^{i-1}, X_3^i)] \\ & \overset{(e)}{\le} \sum_{i=1}^{n} [H(Y_{1,i}) \\ & - H(X_{2,i} | M_{21}, M_{12}, M_{32}, Y_1^{i-1}, X_1^i, X_2^{i-1}, X_3^i, Y_3^{i-1}) \\ & + H(X_{2,i} | M_{21}, M_{12}, M_{32}, Y_1^{i-1}, X_1^i, X_2^{i-1}, X_3^i, Y_3^{i-1})] \\ & = \sum_{i=1}^{n} [H(Y_{1,i})] \\ & \le n \end{aligned}$$

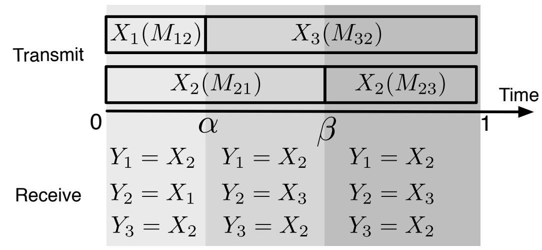

Fig. 3. Time-sharing achievability scheme for two-way modulo 2 adder MAC/BC channel.

where (a) follows from Fano's inequality, (b) drops a negative entropy term, (c) uses the chain rule and conditioning reduces entropy. In (d), $X_1^i = f_1(M_{12}, Y_1^{i-1})$ and $X_3^i = f_3(M_{32}, Y_3^{i-1})$. We cancel $X_{1,i}$ and $X_{3,i}$ in the entropy term in (e) since we know X_1^i and X_3^i respectively. In addition, we introduce genies $X_2^{i-1}, Y_3^{i-1}, X_3^i$ in the negative entropy term. For the third entropy term, X_2^{i-1} may be obtained from $Y_3^{i-1} = X_2^{i-1} \oplus X_3^{i-1}$ (bit-wise modulo 2) since we know X_3^{i-1}. We may obtain Y_1^{i-1} using the Markov chain illustrated in Fig. 2. Finally, X_1^i is given by the encoding function $X_1^i = f_1(M_{12}, Y_1^{i-1})$ (with slight abuse of notation, as vectors are denoted here). We again bound $H(Y_{1,i}) \le 1$.

We are able to achieve this outer bound using two time-sharing random variables *without adaptation*: α time-shares between channel inputs X_1 and X_3 for the MAC channel in the $\rightarrow$ direction, while β time-shares between the messages M_{21} and M_{23} (encoded as $X_2(M_{21})$ and $X_2(M_{23})$ respectively) in the BC in the $\leftarrow$ direction. Both directions ignore the received signals and use i.i.d. Bernoulli(1/2) codebooks. For clarity, we illustrate who transmits when in Figure 3, where we note the inputs and outputs in order to emphasize that at all times, both receivers may decode 1 bit per channel use after canceling their own transmitted codeword.

■

Remark 2: Without adaptation, the encoding functions would become

$$\begin{aligned} X_{1,i} &= f_1(M_{12}) \\ X_{2,i} &= f_2(M_{21}, M_{23}) \\ X_{3,i} &= f_3(M_{32}) \end{aligned}$$

This channel model thus resembles a MAC channel simultaneously transmitting with a BC channel with restricted nodes. In this case, we are still able to achieve the capacity region of Theorem 1 using time-sharing. As such, we see that allowing nodes to adapt does not increase the capacity region of this channel model, and the capacity region is that of a modulo 2 adder MAC and a binary BC channel in parallel.

Remark 3: We notice that capacity is achieved by time-sharing amongst the nodes/messages transmitting in the same "direction" (i.e. between nodes 1 and 3, and between messages M_{21} and M_{23}) but not between the two directions themselves.

That is, transmission may take place simultaneously between the two directions, as is the case in the point-to-point modulo 2 adder and Gaussian channel models, where no time-sharing is needed between the two transmission directions $\rightarrow$ and $\leftarrow$.

IV. The Capacity Region of the Two-way Modulo 2 Adder Interference Channel

The two-way modulo 2 adder interference channel is discrete and memoryless and at each channel use is described by

$$\begin{aligned}Y_1 &= X_1 \oplus X_2 \oplus X_4\\ Y_2 &= X_1 \oplus X_2 \oplus X_3\\ Y_3 &= X_2 \oplus X_3 \oplus X_4\\ Y_4 &= X_1 \oplus X_3 \oplus X_4.\end{aligned}$$

If all nodes employ adaptation, then at channel use i

$$\begin{aligned}X_{1,i} &= f_1(M_{12}, Y_1^{i-1})\\ X_{2,i} &= f_2(M_{21}, Y_2^{i-1})\\ X_{3,i} &= f_3(M_{34}, Y_3^{i-1})\\ X_{4,i} &= f_4(M_{43}, Y_4^{i-1}).\end{aligned}$$

The capacity region of this channel is stated as follow:

Theorem 4: The capacity region of the two-way modulo 2 adder interference channel is the set of non-negative rate tules $(R_{12}, R_{21}, R_{34}, R_{43})$ such that

$$R_{12} + R_{34} \leq 1 \quad (3)$$
$$R_{21} + R_{43} \leq 1. \quad (4)$$

Proof: Clearly we may achieve this region using two time-sharing random variables; one between nodes 1 and 3, and a second between nodes 2 and 4. For the converse,

Proof of bound (3)*:*

$$\begin{aligned}
&n(R_{12} + R_{34} - \epsilon)\\
&\overset{(a)}{\leq} I(M_{12}; Y_2^n|M_{21}, M_{43}) + I(M_{34}; Y_4^n, Y_2^n|M_{12}, M_{21}, M_{43})\\
&= I(M_{12}; Y_2^n|M_{21}, M_{43}) + I(M_{34}; Y_2^n|M_{21}, M_{12}, M_{43})\\
&+ I(M_{34}; Y_4^n|M_{21}, M_{12}, M_{43}, Y_2^n)\\
&\overset{(b)}{\leq} I(M_{12}; Y_2^n|M_{21}, M_{43}) + I(M_{34}; Y_2^n|M_{21}, M_{12}, M_{43})\\
&+ H(Y_4^n|M_{21}, M_{12}, M_{43}, Y_2^n)\\
&\overset{(c)}{=} I(M_{12}; Y_2^n|M_{21}, M_{43}) + I(M_{34}; Y_2^n|M_{21}, M_{12}, M_{43})\\
&+ \sum_{i=1}^{n}[H(X_{1,i} \oplus X_{3,i} \oplus X_{4,i}|M_{21}, M_{12}, M_{43}, Y_4^{i-1}, X_4^i, Y_2^n, X_2^n)]\\
&\overset{(d)}{=} I(M_{12}; Y_2^n|M_{21}, M_{43}) + I(M_{34}; Y_2^n|M_{21}, M_{12}, M_{43})\\
&+ \sum_{i=1}^{n}[H(X_{1,i} \oplus X_{3,i}|M_{21}, M_{12}, M_{43}, Y_4^{i-1}, X_4^i, X_1^n \oplus X_3^n, X_2^n)]\\
&= I(M_{12}; Y_2^n|M_{21}, M_{43}) + I(M_{34}; Y_2^n|M_{21}, M_{12}, M_{43})\\
&\overset{(e)}{\leq} \sum_{i-1}^{n}[H(Y_{2,i}) - H(Y_{2,i}|Y_2^{i-1}, M_{12}, M_{21}, M_{43})\\
&+ H(Y_{2,i}|Y_2^{i-1}, M_{12}, M_{21}, M_{43})]\\
&= \sum_{i=1}^{n}[H(Y_{2,i})]\\
&\leq n
\end{aligned}$$

where (a) follows from Fano's inequality and the introduction of a genie Y_2^n in the second mutual information term. We drop a negative entropy term in inequality (b). In (c), we apply the chain rule on the entropy term and we add X_4^i and X_2^n in the conditioning part since $X_4^i = f_4(M_{43}, Y_4^{i-1})$ and $X_2^n = f_2(M_{21}, Y_2^{n-1})$. In (d), we cancel $X_{4,i}$ in the entropy term since we know X_4^i. In addition, $X_1^n \oplus X_3^n$ is decoded from $Y_2^n = X_1^n \oplus X_2^n \oplus X_3^n$ since X_2^n is known. Thus, the last entropy term is zero. We apply the chain rule again in step (e) and drop the conditioning part of the first entropy term and a negative entropy term. The bound (4) follows by symmetry. ■

Remark 5: The capacity region of the two-way modulo 2 adder interference channel with full adaptation is the same as the combination of two one-way modulo 2 adder interference channels. Capacity is achieved using time sharing and nodes need not adapt. Thus, adaptation is again useless in this scenario.

V. The Capacity Region of the Two-way Modulo 2 Adder Z Channel

The two-way modulo 2 adder Z channel is discrete and memoryless and at each channel use is described by

$$\begin{aligned}Y_1 &= X_1 \oplus X_2\\ Y_2 &= X_1 \oplus X_2 \oplus X_3\\ Y_3 &= X_2 \oplus X_3 \oplus X_4\\ Y_4 &= X_3 \oplus X_4.\end{aligned}$$

If all nodes employ adaptation, then at channel use i

$$\begin{aligned}X_{1,i} &= f_1(M_{12}, Y_1^{i-1})\\ X_{2,i} &= f_2(M_{21}, M_{23}, Y_2^{i-1})\\ X_{3,i} &= f_3(M_{32}, M_{34}, Y_3^{i-1})\\ X_{4,i} &= f_4(M_{43}, Y_4^{i-1}).\end{aligned}$$

The capacity region of this channel is stated as follow:

Theorem 6: The capacity region of the two-way modulo 2 adder Z channel is the set of non-negative rate tuples $(R_{12}, R_{21}, R_{23}, R_{32}, R_{34}, R_{43})$ such that

$$R_{12} + R_{32} + R_{34} \leq 1 \quad (5)$$
$$R_{21} + R_{23} + R_{43} \leq 1 \quad (6)$$

Proof: Time-sharing may again be used to achieve this region. For the converse,

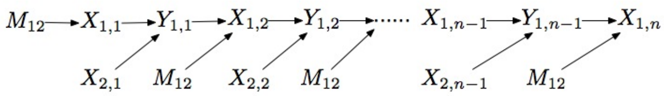

Fig. 4. The Markov chain used in the outer bound proof of Theorem 6.

Proof of bound (5)*:*

$$
\begin{aligned}
& n(R_{12}+R_{32}+R_{34}-\epsilon) \\
& \leq I(M_{12};Y_2^n|M_{21},M_{23},M_{43}) \\
& + I(M_{32},M_{34};Y_4^n,Y_2^n|M_{43},M_{12},M_{21},M_{23}) \\
& \overset{(a)}{\leq} I(M_{12};Y_2^n|M_{21},M_{23},M_{43}) + I(M_{32},M_{34};Y_2^n|M_{43},M_{12}, \\
& \quad M_{21},M_{23}) + I(M_{32},M_{34};Y_4^n|M_{43},M_{12},M_{21},M_{23},Y_2^n) \\
& \overset{(b)}{\leq} H(Y_2^n|M_{21},M_{23},M_{43}) - H(Y_2^n|M_{12},M_{21},M_{23},M_{43}) \\
& + H(Y_2^n|M_{12},M_{21},M_{23},M_{43}) \\
& + H(Y_4^n|M_{43},M_{12},M_{21},M_{23},Y_2^n) \\
& \overset{(c)}{\leq} \sum_{i=1}^{n}[H(Y_{2,i}) + H(Y_{4,i}|M_{12},M_{21},M_{23},M_{43},Y_4^{i-1},Y_2^n)] \\
& \overset{(d)}{=} \sum_{i=1}^{n}[H(Y_{2,i}) + H(X_{3,i}\oplus X_{4,i}|M_{12},M_{21},M_{23},M_{43},Y_4^{i-1}, \\
& \qquad X_4^i, X_3^{i-1}, Y_2^n, X_2^n)] \\
& \overset{(e)}{=} \sum_{i=1}^{n}[H(Y_{2,i}) + H(X_{3,i}|M_{12},M_{21},M_{23},M_{43},Y_4^{i-1}, \\
& \qquad X_4^i, X_3^{i-1}, X_1^n\oplus X_2^n\oplus X_3^n, X_2^n, X_1^n)] \\
& = \sum_{i=1}^{n}[H(Y_{2,i})] \\
& \leq n
\end{aligned}
$$

where (a) follows from the chain rule. We drop two negative entropy terms in inequality (b) and notice that the second and the third entropy terms cancel each other. In (c), we apply the chain rule first, then we drop the conditioning part of the first entropy term. In (d), we first add X_4^i in the conditioning part of the second entropy term since $X_4^i = f_4(M_{43}, Y_4^{i-1})$. Note X_3^{i-1} is decoded from $Y_4^{i-1} = X_3^{i-1} \oplus X_4^{i-1}$ since X_4^{i-1} is known. Adding X_2^n follows from the fact $X_2^n = f_2(M_{21}, M_{23}, Y_2^{n-1})$. In (e), we cancel $X_{4,i}$ in the second entropy term since we know X_4^i. In addition, given M_{12} and X_2^n, we may construct X_1^n as illustrated in Fig. 4. Now, we may obtain X_3^n from $Y_2^n = X_1^n \oplus X_2^n \oplus X_3^n$, so that the second entropy term in zero. Bound (6) follows by symmetry. ■

Remark 7: We again notice that since time-sharing achieves the above region, adaptation does not help. We again see that the messages in the $\rightarrow$ and the $\leftarrow$ directions may be simultaneously communicated, but that the messages within one direction must be time-shared.

VI. Conclusion

We obtained the capacity regions of the two-way modulo 2 adder MAC/BC channel, the two-way modulo 2 adder interference channel and the two-way modulo 2 adder Z channel, all with adaptation. We showed that adaptation does *not* affect the capacity regions in these channels and that the capacity region of the two-way channel is that of two parallel one-way channels in the $\rightarrow$ and $\leftarrow$ directions. This relied heavily on being able to construct inputs and outputs at various nodes, which is a direct result of the deterministic, invertible (can undo the modulo 2 sum if know one of the components), and highly symmetric nature of the channels considered. We suspect that when one of these components is missing, adaptation *will* be of use. To answer this, a number of questions regarding whether the adaptation helps in *two-way networks* are the subject of ongoing work, including: 1) multi-user two-way modulo 2 adder channels with noise; 2) the general deterministic two-way channels; and 3) Gaussian channels.

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