

The sum-capacity of the symmetric linear deterministic Complete K-user Z-interference channel

Alex Dytso, Natasha Devroye, and Daniela Tuninetti
 Department of Electrical and Computer Engineering
 University of Illinois at Chicago, Chicago IL 60607, USA,
 Email: odytso2, devroye, danielat @ uic.edu

Abstract—We consider a K-user interference channel in which the channel gains from transmitter i to receiver $j > i$ are 0, resulting in a type of Z interference channel which we term the Complete K-user Z Interference Channel which has not been explicitly studied. We first consider the symmetric linear high SNR deterministic approximation of the Gaussian noise channel. For this model, we provide an outer bound which may be achieved, thereby obtaining the sum-capacity. The outer bound is obtained by providing two types of side-information: the first reduces the channel model to one in which each receiver sees a sum of the direct and interference terms, the second is reminiscent of the Etkin-Tse-Wang side-information for the interference channel. It is noted that the sum-capacity is a function of the number of users K and is larger than the “W” curve for the fully connected K-user interference channel, but reduces to it as $K \rightarrow \infty$.

I. INTRODUCTION

The demand for higher data rates appears to be ever increasing. Smart phones and tablets, in conjunction with social network and multimedia applications, have finally given communication engineers a “killer application”. As such, the motivation to determine the ultimate limits of many long standing network models, such as the interference and relay channels, is strong. However, even what appears to be a simple problem at first glance, such as the capacity of the 2-user Interference Channel (IC), has been a formidable challenge for the community for over thirty years.

Recently, in order to make progress in the understanding of networks in general, it has been advocated that approximations of – as opposed to the exact characterizations of – the whole capacity region might be an alternative relevant metric [1]. With this approach, the capacity of the Gaussian 2-user IC was determined to within 1 bit [2]. The ingredients of this new powerful approach are to develop insights from rather simple deterministic models (where the noise is neglected and the interaction of signals is the main concern, such as in [3]) and then translate them to the Gaussian noise network at high SNR (where the impact of the noise is of secondary importance as compared to the interference). This has led to the important and insightful metric – the *generalized degrees of freedom* (gDoF) of a network, i.e., the pre-log of the sum-capacity in the regime where both INR and SNR are large and their ratio is dB is kept constant. In this paper we follow the same approach

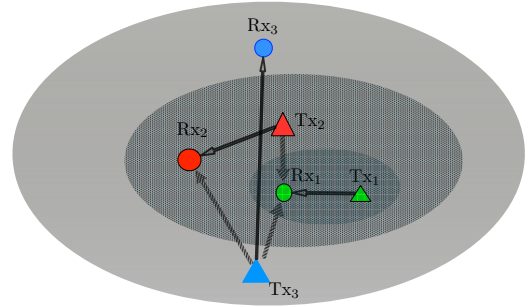


Fig. 1. An example of layered network with 3 users/cells. Transmitter i , denoted as Tx_i , communicates with receiver i , denoted as Rx_i , $i \in [1 : K]$, and is interfered by users/cells with higher index only. Circles represent the coverage area of the corresponding transmitter.

and we find the sum-capacity of a deterministic channel which in turn will hopefully offer insight into the gDoF and capacity for the Gaussian channel.

In this work we deal with the K-user IC. Instead of considering the most general problem, which is characterized by K^2 channel gains/parameters, we focus on what we term the *K-user complete Z-IC* (we drop the K-user from now on for brevity). The complete Z-IC is a channel where the receiver with index $k \in [1 : K]$ only suffers interference from its “downstream” users, that is, transmitter with higher index. Motivation for this model comes from the communication scenario depicted in Fig. 1. Often a transmitter-receiver pair must be introduced inside of the coverage area of an existing system. In Fig. 1 the smallest user/cell 3 falls inside of the coverage area of larger cells 1 and 2, while cell 2 falls inside of the coverage area of cell 1. Because of the relative radii of the cells, receiver 3 experiences interference from all transmitters, receiver 2 only from receiver 1 (because outside the communication range of transmitter 3) and receiver 1 in not interfered at all. This cell arrangement makes sense in pico-cell networks [4].

Past Work. Recently, a large body of work has focused on the K-user IC with $K > 2$ [5]–[12]. In [5] it was shown that the gDoF of the symmetric K-user IC (a fully connected IC where all the direct links have strength SNR and all interfering links have strength INR) is the same as for the 2-user case and is

given by the so-called W-curve of [2]. In [6] it was shown that the gDoF of the cyclic symmetric Z-channel (a very sparse IC where all the direct links have strength SNR and each receiver experiences interference from one transmitter only with strength INR; the channel is completely symmetric with respect to the users) is given by the W-curve of [2] and does not depend on K . The model of [6] was analyzed with output feedback from a receiver to its intended transmitter in [8] and it was shown that the gDoF is non-increasing in K and goes from the V-curve of [13] for $K = 2$ to the W-curve of [2] for $K \rightarrow \infty$. The sum-capacity of the generalization of [6] in a deterministic 3-user setting was studied in [12] where the sum-capacity was determined for certain ranges of parameters. The K -user cascade Z-channel (where the circular symmetry of the cyclic symmetric Z-channel is broken by removing one interfering link) was studied in [11], where sum-capacity is obtained for most channel parameters; constant gaps are shown otherwise. In [9] a general capacity outer bound was derived, which was shown to be tight for the sum-capacity of certain complete Gaussian Z-ICs.

Contributions. The main contribution of this paper is to determine the sum-capacity for the linear high SNR deterministic approximation of the complete K -user Z-IC (the linear deterministic channel or LDC), which is shown to be a function of K . We provide outer bound for this channel model which may be extended to general K -user interference channels. For the symmetric scenario we show that the sum-capacity is non-increasing in K and goes from the V-curve of [13] for $K = 2$ to the W-curve of [2] for $K \rightarrow \infty$. We note that our channel is not cyclic and hence the results of [6] do not immediately apply.

Paper Organization. The remainder of the paper is organized as follows. In Section II we formally introduce our channel model. In Section III we derive a new sum-rate outer bound and show schemes that achieve this outer bound for the LDC symmetric K -user Complete Z-IC. In Section IV we make some remarks. In Section V we conclude our results and point out future directions.

II. CHANNEL MODEL

The Linear Deterministic approximation of the Gaussian noise Channel (LDC) at high SNR was first introduced in [14] and allows one to focus on signal interactions rather than on the additive noise. This framework has been very powerful in revealing the behavior of interference networks, and the insights gained for the LDC have often been translated into capacity results to within a constant gap for any finite SNR [2], [13], [15]. In light of these success stories we also start our investigation from the LDC.

The LDC Complete Z-IC is shown in Fig. 2. The channel is characterized by K transmitter-receiver pairs. Transmitter i has input X_i and intends to communicate message W_i , uniformly distributed over $[1 : 2^{NR_i}]$ and independent of everything else, to receiver i with output Y_i , $i \in [1 : K]$. In the LDC channel model, the input-output relationship is given by, for $u \in [1 :$

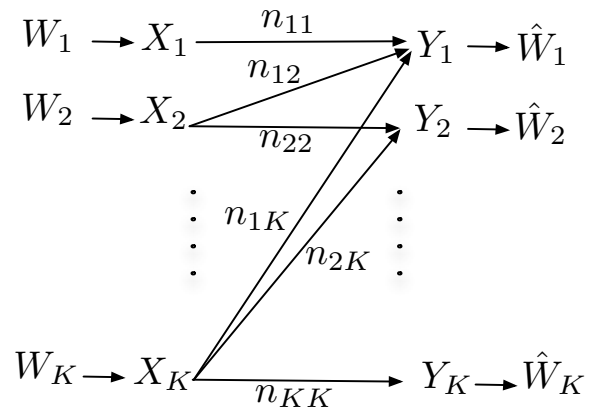


Fig. 2. Deterministic Complete Z-Interference Channel with K users.

$K]$

$$Y_u = \sum_{i \in [u:K]} \mathbf{S}^{m-n_{ui}} X_i, \quad m := \max\{n_{ij}\} \quad (1)$$

where \mathbf{S} is the binary shift matrix of dimension m , all inputs and outputs are binary column vectors of dimension m , and the summation is bit-wise over the binary field. We will be interested in a symmetric model in which $n_{ii} = n_S$, $n_{ij} = 0$ for $j < i$, and $n_{ij} = \alpha n_{ii}$ for $j > i$ and $\alpha \geq 0$, $r_i = R_i/n_S$, and $d_K(\alpha) = \frac{1}{K} \sum_{i=1}^K r_i$. Define $[x]^+ := \max\{0, x\}$.

Transmitter i takes message W_i and encodes it into a codeword X_i^N which is then sent over the channel in N channel uses. Receiver i uses the decoding function $\theta_i^{(N)}$ to map its channel output Y_i^N into an index $\widehat{W}_i \in [1 : 2^{NR_i}]$. The probability of error is defined as

$$P_e^{(N)} := \max_{i \in [1:K]} \mathbb{P}[\theta_i^{(N)}(Y_i^N) \neq W_i]. \quad (2)$$

The rate-tuple (R_1, R_2, \dots, R_K) is achievable if one may demonstrate a sequence of encoding and decoding functions such that $P_e^{(N)} \rightarrow 0$ as the blocklength $N \rightarrow \infty$. The capacity region is the closure of the set of all achievable rate tuples.

III. MAIN RESULTS

In this section we present and derive our main results. We first present an outer bound on the sum-rate of K -user LDC Complete Z-IC. We then achieve this sum-rate for general K .

A. Sum-rate upper bound

We now present an outer bound for the LDC channel which is based on two key ideas:

- providing additional genie message information to reduce the Complete Z-IC into a Cascade-IC as in [11]. This ensures that at each receiver i , instead of seeing a sum of its own signal and $K - i$ interference terms, it sees a sum of two terms: its desired signal and one interference term. This alleviates the problem of evaluating the entropy of vectors conditioned on a sum of interference terms.

- providing additional side-information of the form seen in Etkin-Tse-Wang [2] which allows one to reduce the outer bound by canceling a portion of one's own information.

We note that while we present the outer bound for the LDC channel model, this may be extended to the general K -user interference channel and amounts to reducing it to a Cascade Z-IC. For each Cascade Z-IC contained in the original K -user channel one may obtain a new outer bound in this fashion, as long as certain invertibility constraints of the type first seen in [3] are met, i.e. in the following we need $T_i^N = f(X_i^N, L_i^N)$ to be invertible given X_i^N , where we select and provide certain receivers with the side-information L_i^N .

Theorem 1. *The sum-rate is upper-bounded by*

$$\sum_{i=1}^K r_i \leq \max\{1, \alpha\} + (K-2) \max\{1-\alpha, \alpha\} + [1-\alpha]^+. \quad (3)$$

Proof: Let $T_k^N := S^{m-n_{k,k}} X_k^N + S^{m-n_{k,k+1}} X_{k+1}^N$, i.e. T_k^N is the sum of the desired signal (at receiver k) and the interference created by transmitter $k+1$ (modulo k). Furthermore, let $S_k^N := S^{m-n_{k-1,k}} X_k^N$, i.e. this is the interference transmitter k generates at receiver $k-1$. Note that $T_k^N = S^{m-n_{k,k}} X_k^N + S_{k+1}^N$. Then,

$$\begin{aligned} N \sum_{k=1}^K R_k &= \sum_{k=1}^K H(W_k) = \sum_{k=1}^K H(W_k | W_{k+2}, \dots, W_K) \\ &\leq \sum_{k=1}^K I(W_k; Y_k^N | W_{k+2}, \dots, W_K) + N\epsilon_N \\ &\leq \sum_{k=1}^K I(X_k^N; Y_k^N | X_{k+2}^N, \dots, X_K^N) + N\epsilon_N \\ &= I(X_1^N; T_1^N) + \sum_{k=2}^{K-1} I(X_k^N; T_k^N) + I(X_K^N; T_K^N) + N\epsilon_N \\ &\leq I(X_1^N; T_1^N) + \sum_{k=2}^{K-1} I(X_k^N; T_k^N, S_k^N) + I(X_K^N; T_K^N) + N\epsilon_N \\ &= H(T_1^N) - H(T_1^N | X_1^N) + \\ &+ H(S_2^N) + H(T_2^N | S_2^N) - H(T_2^N | X_2^N) \\ &+ \dots \\ &+ H(S_{K-1}^N) + H(T_{K-1}^N | S_{K-1}^N) - H(T_{K-1}^N | X_{K-1}^N) \\ &+ H(T_K^N) - H(T_K^N | X_K^N) + N\epsilon_N \\ &\stackrel{(a)}{=} H(T_1^N) + \sum_{k=2}^{K-1} H(T_k^N | S_k^N) \\ &+ H(T_K^N) - H(T_{K-1}^N | X_{K-1}^N) + N\epsilon_N \end{aligned}$$

where we note that for (a) $H(S_i^N | T_i^N, X_i^N) = 0$ by definition of S_i^N , $H(S_{i+1}^N) = H(T_i^N | X_i^N)$, which leads to much cancellation. For the LDC model, note that $H(T_1^N) \leq Nn_S \max\{1, \alpha\}$, $H(T_k^N | S_k^N) \leq Nn_S \max\{1-\alpha, \alpha\}$ (the utility of the Etkin-Tse-Wang type side-information). To evaluate $H(T_K^N) - H(T_{K-1}^N | X_{K-1}^N)$ notice that the last user

does not suffer any interference in our channel model and hence $T_K^N = S^{m-n_{K,K}} X_K^N$, and that given $X_{K-1}^N, T_{K-1}^N = S^{m-n_{K-1,K}} X_K^N$. Then, with some abuse of notation (shift matrices apply to the vector X_k^N at each channel use), we split X_K into a length $[n_S - \alpha n_S]^+$ portion X_a , and a length αn_S portion X_b , and write

$$X_K^N = [X_a^N, X_b^N], \quad S^{m-\alpha n_S} X_K^N = X_b^N,$$

we see that

$$\begin{aligned} H(T_K^N) - H(T_{K-1}^N | X_{K-1}^N) &= H(S^{m-n_S} X_K^N) - H(S^{m-\alpha n_S} X_K^N) \\ &= H([X_a^N, X_b^N] | X_b^N) \leq H(X_a^N) \leq Nn_S [1-\alpha]^+. \end{aligned}$$

Normalizing by $N \cdot n_S$ completes the proof of (3). \blacksquare

Next, by using Theorem 1 and the fact that the capacity of a K -user IC is contained into the capacity of any 2-user IC obtained by silencing all but 2 users in the original channel, the sum-rate outer bound in (3) reduces to:

For $\alpha \neq 1$

$$\begin{aligned} d_K(\alpha) &\leq \min \left\{ d_2(\alpha), \frac{1}{K} (\max\{1, \alpha\} + [1-\alpha]^+ \right. \\ &\quad \left. + (K-2) \max\{1-\alpha, \alpha\}) \right\} \\ &= \begin{cases} 1 - \alpha + \alpha/K & \alpha \in [0, 1/2) \\ \alpha + \frac{2-3\alpha}{K} & \alpha \in [1/2, 2/3) \\ 1 - \alpha/2 & \alpha \in [2/3, 1) \\ \alpha/2 & \alpha \in (1, 2) \\ 1 & \alpha \in [2, \infty) \end{cases} \quad (4) \end{aligned}$$

where we also added the sum-rate upper bound for 2-user Z-IC

$$r_i + r_j \leq \min\{2, \max\{2-\alpha, \alpha\}\} = 2d_2(\alpha),$$

for $i \neq j \in [1 : K]$. This corresponds to an interference network where all but users i and j have been silenced, i.e., $d_2(\alpha)$ coincides with the W-curve of [2] for $\alpha > 2/3$. By summing all such bounds on pairs of rates we obtain $d_K(\alpha) \leq d_2(\alpha)$. For $K=2$, the bound in (4) reduces to the V-curve of [13] for $\alpha \leq 2$ and it is equal to the interference free capacity for $\alpha > 2$ (the so-called very strong interference regime).

For $\alpha = 1$ (singularity)

$$d_K(\alpha) \leq \frac{1}{K}. \quad (5)$$

This corresponds to the special case when all signals are statistically equivalent. Hence, all the messages can be decoded by each receiver. Thus the sum capacity is the same as capacity of the K -user multiple-access channel.

B. Achievability

In this section we show how one can achieve the sum-rate in (4). For each regime we will develop a coding scheme which we illustrate graphically. In all coding scheme plots, we show the different linear combinations of the shifted transmit signals

X_1, X_2, \dots, X_K received at the K receivers. Each user uses a different hatching for its bits. If the same hatch is used in different places in a block it is meant to represent repetition of that block of bits. White blocks denote empty blocks of bits, sometimes accompanied by the letter ‘R’. We note that the height of the block received on the direct link is always 1 and those of the “interfering” blocks is always α in the symmetric scenario under consideration. At user i , the length of the hatched block from transmitter i that may be extracted from the combination yields the (normalized) achievable rate $r_i \ i \in [1 : K]$.

Very Weak Interference Regime: $0 \leq \alpha \leq \frac{1}{2}$. In this regime the optimal strategy is to treat interference as noise. Fig. 3(a) shows the achievable strategy for K users. As seen from Fig. 3(a), $r_K = 1$ and $r_i = 1 - \alpha$ for $i \in [1 : K - 1]$ are achievable. Thus the sum rate is $d_K(\alpha) = \frac{1+(K-1)(1-\alpha)}{K} = 1 - \alpha + \alpha/K$. Note that in this regime the normalized sum-capacity (with respect to the number of users) is decreasing with K . For $K \rightarrow \infty$ it becomes the “ $1 - \alpha$ ”-part of the W-curve [2]. Note also the $d_K(\alpha)$ does not represent the symmetric rate achievable by all users as user K (the one who does not suffer from interference) may achieve a larger rate than the other users.

Moderately Weak-I Interference Regime: $\frac{1}{2} < \alpha \leq \frac{2}{3}$. In this regime treating interference as noise is no longer optimal and inspired by the 2-user case we use the Han-Kobayashi rate splitting technique [16]. The sum-capacity achieving strategy is shown in Fig. 3(b). As it can be seen, $r_K = 2 - 2\alpha$ and $r_i = \alpha$ for $i \in [1 : K - 1]$ is achievable. Thus $d_K(\alpha) = \frac{2-2\alpha+(K-1)\alpha}{K} = \alpha + \frac{2-3\alpha}{K}$. Note that in this regime too the normalized sum-capacity (with respect to the number of users) is decreasing with K . For $K \rightarrow \infty$ it becomes the “ $1 - \alpha$ ”-part of the W-curve [2].

Moderately Weak-II Interference Regime: $\frac{2}{3} < \alpha < 1$. We note that for $\alpha > 2/3$ the sum-capacity does not depend on K except at the singularity point $\alpha = 1$ and is the same as for the case $K = 2$. In this regime in order to achieve capacity we may use repetitions. The achievable strategy in this regime is show in Fig. 3(c). As it can be seen, every user achieves $r_i = 1 - \alpha + 2\alpha - \frac{3}{2}\alpha - 1 = 1 - \frac{\alpha}{2}$. Thus $d_K(\alpha) = 1 - \frac{\alpha}{2}$. However, one may worry that for, for example, $\alpha \geq \frac{4}{5}$ the repeating bits of interfering signals overlap. We illustrate how to deal with this we consider the example $\alpha = \frac{6}{7} > \frac{4}{5}$, shown of Fig. 4. By flipping around the repeated bits, one may still achieve $1 - \frac{\alpha}{2}$ at each user. Consider for example the decoding at receiver $k - 1$ (at desires “b” bits): b_1 is cleanly decoded, as is a_1 because of the flipped bits. This a_1 is used to decode b_2 from $b_2 + a_1$, and b_1 is used to decode a_2 from $b_1 + a_2$. Then a_2 is used to decode b_3 , and b_4 is clean. Thus, Rx $k - 1$ is able to decode 4 bits out of 7 as needed.

Singularity at: $\alpha = 1$. When $\alpha = 1$ we have a singularity in that the channel is rank deficient and all outputs are statistically equivalent. In this case we achieve $d_K(\alpha) = \frac{1}{K}$ by time-sharing or by the strategy shown in Fig. 3(f).

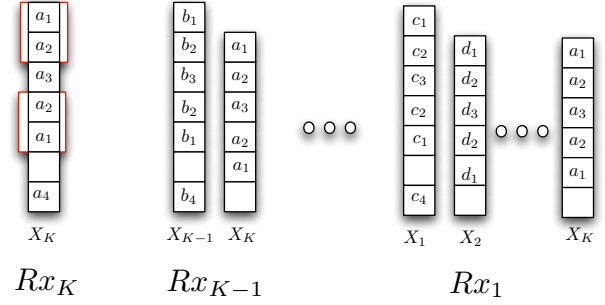


Fig. 4. Strategy for $\alpha = \frac{6}{7}$, overlap strategy. The red blocks show repeated blocks flipped.

Strong Interference Interference Regime: $1 < \alpha \leq 2$. In the strong interference regime as well as in the Moderately Weak-II regime, the achievability schemes require the use of repetition. As it can be seen from Fig. 3(d), every user achieves $r = \frac{\alpha}{2}$. Thus $d_K(\alpha) = \frac{\alpha}{2}$.

Very Strong Interference Interference Regime: $\alpha > 2$. The Very Strong Interference regime $\alpha > 2$ in an interference channel is well understood – one may decode the undesired interference first and cancel it from the received signal to obtain a clean channel – each user achieves one degree of freedom. The strategy is shown in Fig. 3(e)

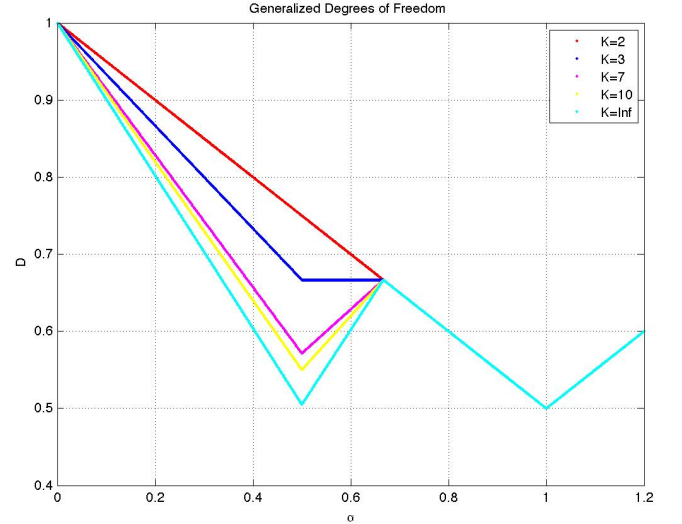


Fig. 5. Sum-capacity of the Complete Z-IC for different values of K .

IV. DISCUSSION

On Complete Z-IC with Output Feedback. It is curious to note that the sum-rate for the complete Z-IC for $\alpha < 2$ is equal to that of for the cyclic symmetric Z-IC with output feedback [8]. Whether there is a fundamental connection or just a coincidence remains to be seen.

On the dependence of K . We would like to point out that the reason that the sum-capacity is dependent on K is due to the very anti-symmetric topology of the channel. However, as

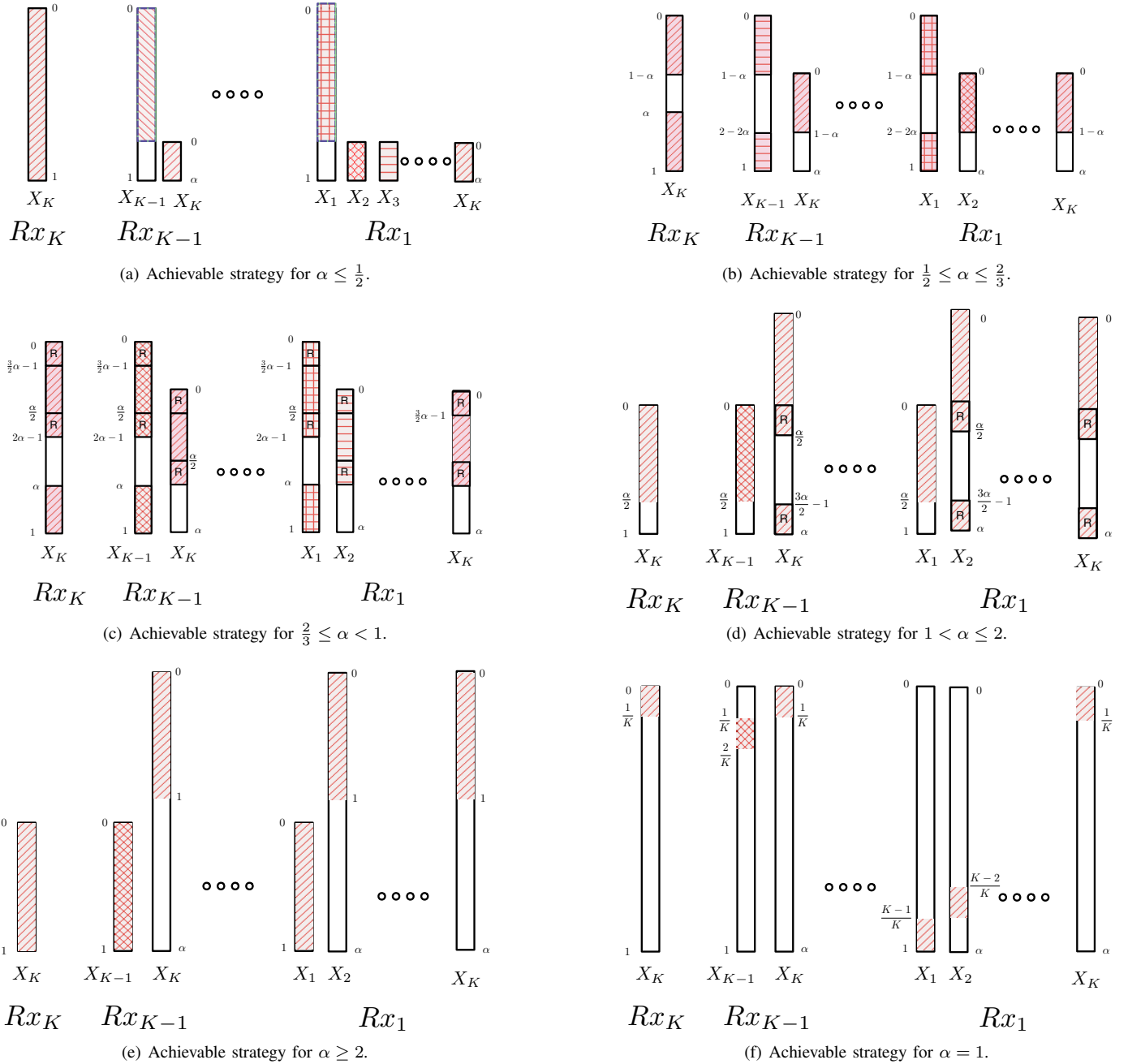


Fig. 3. Achievable strategies for the linear deterministic channel.

$K \rightarrow \infty$ the sum-capacity reduces to the degrees of freedom of K-User Symmetric Interference Channel in [5]. The reason for this is interference alignment, where all the interference terms seen at a given receiver end up aligning, and may be viewed as interference from a single user. In many cases including the K-User Symmetric Interference Channel it has been shown that this strategy corresponds to using lattice codes for the Gaussian noise case. Fig. 5 shows the normalized sum-capacity $d_K(\alpha)$ for various values of K. It is interesting to note that for $K = 3$ region $\frac{1}{2} \leq \alpha \leq \frac{2}{3}$ is flat.

V. CONCLUSION

We have obtained the sum-capacity for the linear deterministic K -user Complete Z interference channel. Central to proving this result was an outer bound which consisted of reducing the channel to a Cascade Z interference channel and providing Etkin-Tse-Wang-type side-information. We may generalize the bound to provide a new bound for arbitrary K -user interference channels. The normalized sum-capacity is seen to depend on K , and as one might intuitively expect, reduces to the “W” curve as $K \rightarrow \infty$. It was curious to note that the sum-capacity was the same as that of the Cyclic Z -

IC with feedback; whether there is a fundamental connection remains to be seen. Whether the insights obtained here translate to determining the generalized degrees of freedom and a constant-gap-to-capacity result for the Gaussian Complete Z-IC is the subject of ongoing work.

Acknowledgment. The authors would like to thank Dr. Popovski for stimulating discussions. The work of the authors was partially funded by NSF under award 1017436. The contents of this article are solely the responsibility of the authors and do not necessarily represent the official views of the NSF.

REFERENCES

- [1] D. Tse, "ISIT 2009 Plenary: It's Better to Approximate," <http://www.isit2009.info>.
- [2] R. Etkin, D. Tse, and H. Wang, "Gaussian interference channel capacity to within one bit," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.
- [3] A. El Gamal and M. Costa, "The capacity region of a class of deterministic interference channels," *IEEE Trans. Inf. Theory*, vol. 28, no. 2, pp. 343–346, Mar. 1982.
- [4] Wikipedia, "Picocell," <http://en.wikipedia.org/wiki/Picocell>.
- [5] S. A. Jafar and S. Vishwanath, "Generalized degrees of freedom of the symmetric gaussian K user interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 7, pp. 3297 – 3303, 2010.
- [6] L. Zhou and W. Yu, "On the capacity of the k-user cyclic gaussian interference channel," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 1171–1175.
- [7] S. Mohajer, R. Tandon, and H. Poor, "On the feedback capacity of the fully connected k-user interference channel," *Arxiv preprint arXiv:1110.6487*, 2011.
- [8] R. Tandon, S. Mohajer, and H. Poor, "On the symmetric feedback capacity of the k-user cyclic z-interference channel," *Arxiv preprint arXiv:1109.1507*, 2011.
- [9] D. Tuninetti, "K-user interference channels: General outer bound and sum-capacity for certain gaussian channels," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 1166–1170.
- [10] Y. Wu, S. Shamai, and S. Verdú, "Degrees of freedom of the interference channel: A general formula," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 1362–1366.
- [11] Y. Liu and E. Erkip, "On the sum capacity of k-user cascade gaussian z-interference channel," in *Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on*. IEEE, 2011, pp. 1382–1386.
- [12] B. Bandemer, A. El Gamal, and G. Vazquez-Vilar, "On the sum capacity of a class of cyclically symmetric deterministic interference channels," in *Information Theory, 2009. ISIT 2009. IEEE International Symposium on*. IEEE, 2009, pp. 2622–2626.
- [13] C. Suh and D. Tse, "Feedback Capacity of the Gaussian Interference Channel to within 2 Bits," *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2667–2685, May 2011.
- [14] G. Bresler and D. Tse, "The two-user gaussian interference channel: A deterministic view," *European Transactions in Telecommunications*, vol. 19, pp. 333–354, Apr. 2008.
- [15] S. Rini, D. Tuninetti, and N. Devroye, "On the capacity of the gaussian cognitive interference channel: new inner and outer bounds and capacity to within 1 bit," *IEEE Trans. Inf. Theory*, 2012.
- [16] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49–60, Jan. 1981.