On The Capacity of the Symmetric Interference Channel with a Cognitive Relay at High SNR

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***Abstract*—The capacity of the Interference Channel with Cognitive Relay, a channel model which generalizes the broadcast, interference and cognitive interference channels, is still an open question. To make progress towards understanding this complex channel, we first consider the binary linear deterministic model that approximate the Gaussian channel at high SNR. We consider symmetric channel gains and show achievability of a tightened version of a previously known outer bound for almost all channel parameters. Of particular interest in this channel model is how the cognitive relay may be used to simultaneously relay as well as cancel/neutralize interference at two receivers. The achievability schemes used to prove capacity use combinations of three main strategies at the cognitive relay that we term bit cancellation, bit sharing, and bit (self)cleaning. We highlight the capacity achieving schemes in the different regimes, pointing out some of the interesting new behaviors seen at the cognitive relay.**

I. Introduction

The Interference Channel with a Cognitive Relay (IFC-CR) consists of a two-user interference channel, which models the communication between two interfering source-destination pairs, in which communication is aided through the presence of a relay. This relay is *cognitive* in the sense that it non-causally, or before transmission commences, knows the messages of both sources. The Cognitive Relay (CR) may thus be exploited to relay the messages to the sources, or may use its cognitive message knowledge to simultaneously cancel/neutralize the interference at one, or both destinations. The main technical insight from the study of such a channel is how this non-causal knowledge of the sources' messages at the CR may be best exploited.

The IFC-CR generalizes numerous well studied networks. In particular, the IFC-CR reduces to: (a) the interference channel [1] by not using the CR; (b) the broadcast channel [2] by not using the two sources; (c) the cognitive interference channel [3] by not using one the two sources.

The IFC-CR model is of theoretical interest because of its generality and as a further step towards an understanding of the capacity region for general multi-terminal network problems. It is also practically motivated in the context of femto-cell networks. In particular, the interference channel models a legacy or existing cellular system with two base-stations wishing to communicate to two terminal nodes. One might be interested in understanding how the addition of a third base-station (e.g. a femto-cell) may aid in the communication by, for example, mitigating interference, introducing user cooperation, etc. In this new system, the assumption that the new base-station has full, a priori knowledge of both the legacy base stations' messages provides an outer bound on what may be achieved in practice with partial and causal message knowledge.

Past work. Limited work exists on the IFC-CR, whose capacity in general remains unknown. Here we focus on the past work for the case where the relay has *non-causal message knowledge* and is *in-band*, that is, the CR shares the same channel as the two source-destination pairs. We note however that significant work exists in this channel model with causal cognition at the relay (see for example [4]–[6] and references therein) which we shall not review here for sake of space.

The IFC-CR was first considered in [7], where an achievable rate region was proposed. This rate region was improved upon in Gaussian noise in [8], and again for a general discrete memoryless channel in [9]. In terms of outer bounds, [8] first proposed a sum-rate outer bound for the Gaussian noise channel. The first outer bound for a general memoryless IFC-CR was derived in [10], which was tightened for a class of semi-deterministic channels in the spirit of [11]. The tightened outer bound was shown to be capacity for a class of high-SNR linear deterministic approximation [12] to the Gaussian IFC-CR in the absence of interfering links, and for several other special cases; the insight from the capacity achieving schemes was used to show capacity to within 3 bits/sec/Hz in the Gaussian IFC-CR without interfering links in [13]. Finally, in [14], capacity of the general IFC-CR with very strong interference at one destination was proven.

Contributions. To make progress towards the capacity of this relatively complex channel, here we seek the full capacity region of the linear deterministic high SNR approximation of the Gaussian IFC-CR, which correspond to the generalized degrees of freedom of the Gaussian channel [1]. For sake of space, we focus on the *symmetric scenario* in which the two direct links, the two interfering links, and the two cognitive links are equal (yielding two normalized parameters). One may view the results obtained here as a generalization of the one-dimensional W-curve of [1] for the interference channel, which describe the normalized sum-rate versus the normalized strength of the interfering links, to a second dimension which represents the relative strength of the CR-to-destination links.

Paper Organization. After defining the channel model in Section II and stating a tightened version of the outer

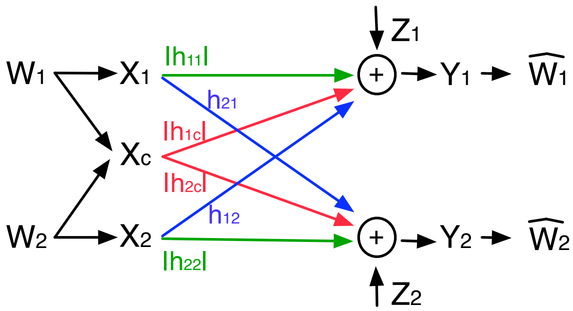


Fig. 1. The symmetric high SNR approximation of the Gaussian IFC-CR.

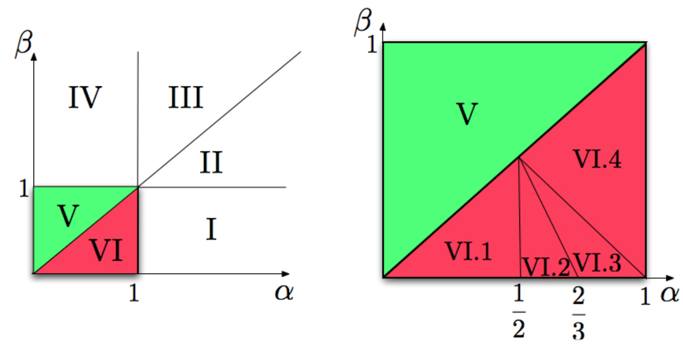


Fig. 2. Regimes of operation for the linear deterministic IFC-CR, where $\alpha := n_I/n_S$ and $\beta := n_C/n_S$.

bound in [10] we outline three key strategies employed by the CR, namely, bit sharing, bit cancellation, and bit (self) cleaning, to achieve capacity for almost all channel parameters. Several interesting new achievability strategies emerge whose Gaussian counterparts would be of interest to understand in the longer term. Section IV concludes the papaer.

II. Channel Model and Known Outer Bound

We first introduce the linear deterministic IFC-CR channel model, which captures the behavior of the Gaussian IFC-CR at high SNR, before outlining a tightened version of the outer bound in [10]. This outer bound will naturally lead to dividing the channel parameters into several regimes where the outer bound simplifies.

Channel Model. The high SNR approximation of the Gaussian IFC-CR is the deterministic channel

$$Y_u = \mathbf{S}^{m-n_{uu}} X_u \oplus \mathbf{S}^{m-n_{uc}} X_c \oplus \mathbf{S}^{m-n_{u\overline{u}}} X_{\overline{u}}, \;, \qquad (1)$$

for $(u, \overline{u}) \in \{1,2\} \times \{1,2\}$, $\overline{u} \neq u$, where $\mathbf{S}$ is the binary shift matrix of dimension $m := \max\{n_{11}, n_{12}, n_{1c}, n_{21}, n_{22}, n_{2c}\}$, where all inputs and outputs are binary column vectors of dimension m, and where the symbol $\oplus$ denotes the component-wise modulo-2 addition of the binary vectors [12]. Source 1 (resp. 2) encodes message W_1 of rate R_1 (resp. W_2 of rate R_2) into its channel input X_1 (resp. X_2); the messages are assumed independent. Message W_1 (resp. W_2) is destined for destination 1 (resp. 2) that must decode it from its channel output Y_1 (resp. Y_2). The two sources are aided by a CR that encodes both messages into the channel input X_c. We adopt standard definitions for codebooks, probability of error, and achievable rate pair (R_1, R_2) [15]. The capacity is the set of all achievable rates, which we seek to determine.

In this work we only consider the *symmetric* version of the channel in (1), that is, a channel whose gains satisfy

$$n_{11} = n_{22} := n_S > 0 \qquad (2a)$$
$$n_{12} = n_{21} := \alpha\, n_S, \;\; \alpha \geq 0 \qquad (2b)$$
$$n_{1c} = n_{2c} := \beta\, n_S, \;\; \beta \geq 0 \qquad (2c)$$

We also parameterize the rates as

$$R_u := r_u\, n_S, \;\; r_u \geq 0, \;\; u \in \{1,2\}. \qquad (3)$$

Fig. 1 shows the symmetric high SNR approximation of the Gaussian IFC-CR considered in this work.

Outer Bound. With the parameterization in (2) and (3) the outer bound in [10, Th.III.3] becomes

$$r_1 \leq \max\{1, \beta\}, \qquad (4a)$$
$$r_2 \leq \max\{1, \beta\}, \qquad (4b)$$
$$r_1 + r_2 \leq [1 - \max\{\alpha, \beta\}]^+ + \beta + \max\{1, \alpha\} \qquad (4c)$$
$$r_1 + r_2 \leq \max\{1, \beta\} \;\; \text{apply for } \alpha = 1 \text{ only} \qquad (4d)$$
$$r_1 + r_2 \leq 2\max\{1-\alpha, \alpha, \beta\} + \mathsf{MLP} \qquad (4e)$$
$$2r_1 + r_2 \leq \max\{1, \beta, \alpha\} + \max\{1-\alpha, \alpha, \beta\} + \max\{1-\alpha, \beta\} + \mathsf{MLP} \qquad (4f)$$
$$r_1 + 2r_2 \leq \max\{1, \beta, \alpha\} + \max\{1-\alpha, \alpha, \beta\} + \max\{1-\alpha, \beta\} + \mathsf{MLP} \qquad (4g)$$

with $\mathsf{MLP} := 2\min\{\alpha, \beta\}$.

Remark on Tightening the Outer Bound of [10]. We note that the bounds in (4f) and (4g) are tighter than the ones reported in [10, eq.(5f)-(5g)] where terms of the type $H(Y_1|V_{21}, X_2)$ were trivially upper bounded by $H(Y_1|X_2) \leq n_S \max\{1-\alpha, \alpha, \beta\}$ (please refer to [10] for the definition of V_{21} and for the derivation of the rate bounds in (4)). A more careful bounding yields $H(Y_1|V_{21}, X_2) \leq n_S \max\{1-\alpha, \beta\}$ as in (4f) and (4g).

Parameter Regimes Definition. The outer bound in (4) naturally leads to the division of the channel parameters (α, β) in (2) into six parameter regimes based on the different max and min operations in (4), as shown in Fig. 2. In [10], the outer bound in (4) was shown to be achievable in two cases: a) when $\alpha = 0$, and b) when $1 > \beta > \alpha$ (Regime V in Fig. 2). In this paper we show capacity for all the remaining regimes except for $\alpha \in [1/2, 1]$ and $\beta \in [0, \alpha]$ (Regimes VI.2, VI.3 and VI.4 in Fig. 2) and speculate on what is missing.

For the figures, the signals in blue-ish tones represent bits from Tx1 to Rx1 (interference at Rx2), and red-ish tones represents bits from Tx2 to Rx2 (interference at Rx1). When two different colors are represented side-by-side in the same bit vector it means that these bits are superposed, i.e., their modulo-2 sum is transmitted.

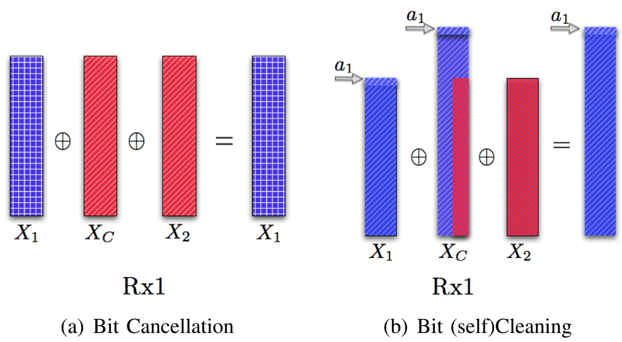


Fig. 3. Basic transmission techniques at the cognitive relay.

III. Capacity for the linear deterministic IFC-CR

To show capacity we will use achievability schemes where the general behavior of the CR may be grouped into three different coding strategies, which we refer to as: Bit Cancellation, Bit Sharing and Bit (self) Cleaning, as described next.

a) Bit Cancellation: The CR may be used to cancel/neutralize unwanted interference, as illustrated in Fig. 3(a). For illustrative purposes we only show the received signal at receiver Rx1. The CR carries the interference message such that it overlaps exactly with the interference message. Due to the component-wise modulo 2 additions of the receiver, the interference is completely removed.

b) Bit Sharing: When the relay link is stronger than both the interference and direct links, i.e., $\beta > \max\{\alpha, 1\}$, the CR may transmit bits above the interference and direct links, which are thus received cleanly at both receivers (due to symmetry). The CR can share these bits among the users.

c) Bit (self) Cleaning: An interesting new achievability strategy, which we term Bit Cleaning, is shown by example in Fig.3(b) for Rx1. Here, the CR transmits a linear combination of the two signals such that interference (in red in this case, at Rx1) is canceled. At first glance, the desired information (in blue) appears to be interfering with itself. A closer look shows that X_1 and the desired portion of X_c are strictly shifted versions of each other. This means that if at least one bit, shown as a_1 on Fig. 3(b), is cleanly received from the CR, this bit may be used to sequentially clean and decode the remaining bits. In this case, the reason both Tx1 and the CR transmit the same messages (one might imagine having Tx1 send nothing in this example) is because of what happens at Rx2; we must really consider both received signals and this is an illustrative example only. Bit (self) cleaning often arises when the CR wishes to cancel interference at one receiver, which ends up interfering with itself at the other. In this case, this self-interference must be "cleaned".

We now show achievability of the outer bound in (4) using a combination of the bit cancellation, bit sharing and bit (self) cleaning strategies. Due to space constraints, we group some of the regimes which have similar achievability strategies, mentioning how these examples may be tailored to the related regimes. We also highlight some of the most conceptually interesting strategies, where we see behavior not encountered in other channels and which highlights the dual role of the CR: *to relay for and cancel interference at two receivers simultaneously.* In the remaining Figs. 4 and 5 for clarity we drop the $\oplus$ symbol between the three received bit-vectors at each receiver, which is understood as in Fig. 3. In Figs. 4 and 5, blocks of bits denoted by A_i are desired at Rx1, blocks of bits B_j are desired at Rx2.

Capacity for $\alpha = 1$. The capacity for $\alpha = 1$ is $r_1 + r_2 \leq \max\{1, \beta\}$ (i.e., bound (4d) only) which is trivially achieved by time division between the cases where one source is silent and the CR fully helps the other source; in this case the two channel outputs are statistically equivalent and the IFC-CR is effectively a compound Multiple Access Channel (MAC).

Outer Bound for $\alpha \neq 1$. We divide the parameters into two cases. When (4f), (4g) and (4e) are redundant, that is for $\max\{\alpha, \beta\} \geq 1$ (all but Regimes V and VI in Fig. 2), the region in (4) simplifies to

$$r_1 \leq \max\{1, \beta\}, \tag{5a}$$
$$r_2 \leq \max\{1, \beta\}, \tag{5b}$$
$$r_1 + r_2 \leq \beta + \max\{1, \alpha\}. \tag{5c}$$

and otherwise, that is for $\max\{\alpha, \beta\} < 1$, in which case only Regime VI must be considered since capacity for Regime V was proved in [10]. The outer bound region in (4) for Regime VI, that is for $0 < \beta < \alpha < 1$, simplifies to

$$r_1 \leq 1, \tag{6a}$$
$$r_2 \leq 1, \tag{6b}$$
$$r_1 + r_2 \leq 2 - \alpha + \beta, \tag{6c}$$
$$r_1 + r_2 \leq 2\max\{1 - \alpha, \alpha\} + \mathsf{MLP} \tag{6d}$$
$$2r_1 + r_2 \leq 1 + \max\{1 - \alpha, \alpha\} + \max\{1 - \alpha, \beta\} + \mathsf{MLP} \tag{6e}$$
$$r_1 + 2r_2 \leq 1 + \max\{1 - \alpha, \alpha\} + \max\{1 - \alpha, \beta\} + \mathsf{MLP} \tag{6f}$$

with $\mathsf{MLP} := 2\beta$.

Capacity for Regimes I and II: $\alpha > \max\{1, \beta\}$. We start with Regime I and II, whose achievability schemes are relatively straightforward. We only show the optimal strategy for Regime I ($\beta \leq 1$), and describe how this may be extended to Regime II ($\beta \leq \alpha$). Since the relative amount of cognition represented by β is fairly small in Regime I ($\beta \leq 1$), the channel in this regime resembles an IFC. As such we further divide Regime I into two sub-cases, in a manner similar to the sub-divisions seen in the W-curve in [1]. In particular, there is a conceptual boundary between the Strong (i.e., capacity is a pentagon) and the Very Strong (i.e., capacity is a square) interference regimes at $\alpha = 2$ in the IFC; in our case a similar split occurs at $\alpha + \beta = 2$ (where (5a)+(5b) = (5c) for $\beta < 1$). For $\alpha + \beta \geq 2,\ \beta < 1$ capacity may be achieved by $X_c = 0$ or alternatively by the bit cancellation scheme in Fig. 4(a).

An almost identical strategy to that in Fig. 4(a), with some bit repetition in X_1 and X_2 as for the IFC [16], is capacity achieving in Regime I when $\alpha < 2$ and in Regime II (where more interference can be cancelled since β is larger in Regime II than in Regime I).

Capacity for Regimes III and IV: $\beta > \max\{1, \alpha\}$. We now look at Regimes III and IV, focussing on the more involved Regime III from which we can easily derive achievability for Regime IV. To show capacity we only show achievability of the corner point $r_1 = \beta,\ r_2 = \alpha$ of the region in (5), which is sufficient by symmetry of the channel. Fig. 4(b) shows the strategy that achieves capacity at this corner point, which is one of the most involved cases where all three general strategies being used at the same time. In particular, in Fig. 4(b) at Rx1: the portion of the cognitive message from $\beta - \alpha$ to 0 is easily decoded (bit-sharing); the portion from $\beta - \alpha + 1$ to $\beta - \alpha$ uses interference/bit cancellation by the CR; this last decoded portion is then used to clean the self-interference in the bottom portion (from β to $\beta - 1$) of the received signal, while the remaining portion is able to be decoded thanks to interference cancelation by the CR. Thus, Rx1 receives the full $r_1 = \beta$. At Rx2, interference cancelation and bit (self) cleaning is used to obtain the full $r_2 = \alpha$ bits. This scheme may be extended to Regime IV which uses only the top and bottom portions of Regime III's strategies – using only bit canceling and bit sharing and no bit (self) cleaning.

Capacity for Regime V: $0 < \alpha < \beta < 1$. We pay special attention to Regime V because the strategy that achieves capacity is slightly counter-intuitive. The strategy we propose here is similar to that shown in [10]. The capacity region in (4) has only one corner point given by $r_1 = r_2 = 1$. The capacity achieving scheme is shown in Fig. 4(c). One of the most interesting aspects of this strategy is that the top portion of cognitive message is left empty, which may seem counter-intuitive as the CR, with knowledge of all messages, should be able to use all its bits without harm. However, including bits here would not improve rates as the direct link is already able to convey these bits directly, and the CR is only really needed to simultaneously cancel the interference at both receivers. The receivers then use bit (self) cleaning to recover the bits.

Capacity for Regime VI.1 Due to the complexity of the outer bound region in (6), Regime VI is further divided it into four sub-regimes, which also correspond to a generalization of the division of the W-curve in [1] as β is again relatively small in this regime. The boundary between Regimes VI.1 and VI.2 occurs at $2\alpha = 1$, that between Regimes VI.2 and VI.3 at $\beta + 3\alpha = 2$, and that between Regimes VI.3 and VI.4 at $\beta + \alpha = 1$. We note that these boundaries reduce to those of the W-curve in weak interference for $\beta = 0$.

The capacity for Regime VI is so far only known for $\alpha \leq \frac{1}{2}$, that is, for Regime VI.1. Achievability of the corner point $r_1 = 1,\ r_2 = 1 - 2\alpha + 2\beta$ in (6) may be shown with a strategy of bit cancelation and bit (self) cleaning at both receivers, as illustrated in Fig. 5(a).

What could be missing for Regimes VI.2, VI.3 and VI.4? The capacity region for the remaining Regimes VI.2, VI.3 and VI.4 remain unknown. These regimes are related to the most involved area of the W-curve in [1] for the IFC in moderately weak interference (i.e., for $\alpha \in [1/2, 1]$) and as such it is not surprising that these are also the most difficult cases for the IFC-CR. At this point we conjecture that the way [10, Theorem III.2] was bounded is too loose, and that it is in these sub-regimes that it manifests. In particular, the multi-letter portion (MLP) of the outer bounds in [10, Theorem III.2], given by

$$\mathsf{MLP} := 1/N\big(I(V_{12}(X_2^N); V_{1c}(X_c^N))I(V_{21}(X_1^N); V_{2c}(X_c^N))\big),$$

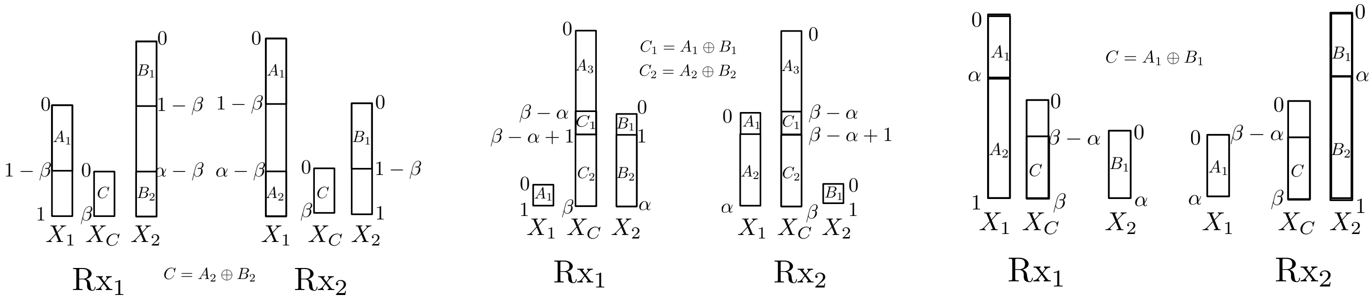
(we defer all definitions and notation to [10]), was bounded in the symmetric case as

$$\begin{aligned}\mathsf{MLP} &\leq \min\{H(V_{12}(X_2)), H(V_{1c}(X_c))\} \\ &+ \min\{H(V_{21}(X_1)), H(V_{2c}(X_c))\} \leq n_S\ 2\min\{\alpha, \beta\}, \quad (7)\end{aligned}$$

noting that the entropy of a discrete random variable is non-negative. We believe that the bound in (7) does not accurately capture the correlation between X_c and (X_1, X_2). Essentially the bound in (7) asserts that X_c can be simultaneously maximally correlated with both X_1 and X_2. However, if X_c is maximally correlated with X_1, i.e., $X_c = X_1$, then it is independent of X_2 (recall that X_1 and X_2 are independent because carry independent messages); in this case the MLP expression would be $\min\{\alpha, \beta\}$ rather than $2\min\{\alpha, \beta\}$. Fig. 5(b) shows a strategy for Regime VI.2 that achieves $r_1 = 1,\ r_2 = \beta$, rather than the corner point $r_1 = 1,\ r_2 = 2\beta$ from the outer bound region in (6).

Tightening the bounds in (4e), (4f) and (4g) so as to capture the correlation among transmitted signals, and/or to derive another bound of the form $2R_1 + R_2$ or $R_1 + 2R_2$ (such a bound was needed for the IFC with rate-limited receiver cooperation [17]) is the subject of ongoing investigation.

A possible strategy for Regime VI.2. Of further interest is to note that several of the derived achievable schemes in this section, which may be used to achieve capacity on the boundary between different sub-regimes, use very interesting techniques. As an example, in Fig. 5(c) we show achievability on the boundary of Regime VI.1 and VI.2. In Fig. 5(c) we see that the CR signal, which is received at the lowest level in this regime, sends a combination of four signals rather than two (the maximal of any of the previous known capacity-achieving schemes). In particular, the top A_1 and A_2 portions of the signal at Rx1 are interference free; they must be summed together to recover the B_1 at Rx1 (which is not interfered by the B_2 part of the signal from Tx2 because the CR performed bit cancellation). Then, it uses the B_1 to (self) bit clean and obtain part A_3 of its intended signal. While it is still unclear, it appears that in this more complex region of the parameter space, more sophisticated schemes at the cognitive relay such as this one may be needed. This, as well as deeper connections with network coding, is the subject of ongoing work.

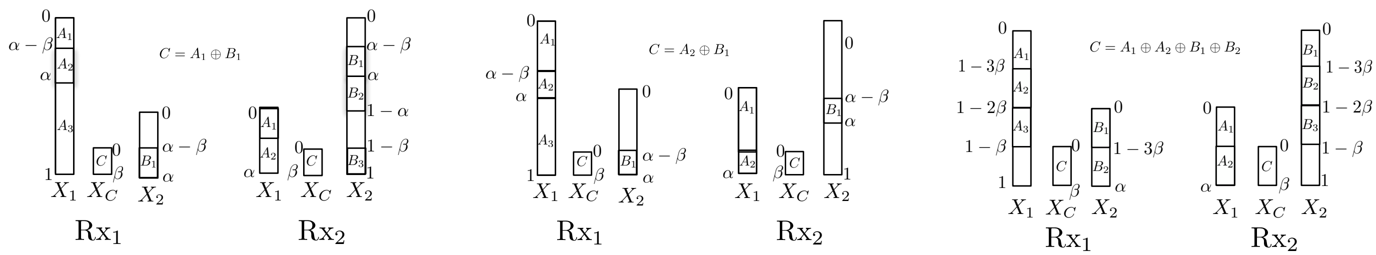


(a) Optimal Strategy for Regime I for $\alpha > 2$.

(b) Optimal Strategy for Regime III.

(c) Optimal Strategy for Regime V.

Fig. 4. Capacity Achieving Strategy for some of the regimes of Fig. 2.



(a) Optimal Strategy for Regime VI.I.

(b) Scheme for Regime VI.2 that achieves $r_1 = 1,\ r_2 = \beta$.

(c) Possible Strategy for Regime VI.2 for $\alpha = 1/2, \beta = 1/4$ that achieves $r_1 = \frac{3}{4},\ r_2 = \frac{3}{4}$.

Fig. 5. Achievability Strategy for Regime VI of Fig. 2.

IV. Conclusion

In this paper we investigated the capacity of the symmetric Gaussian interference channel with a cognitive relay at high SNR by means of the linear deterministic model. We tightened a known outer bound and showed its achievability in almost all parameter regimes by a combination of Bit Cancellation, Bit Sharing and Bit (self) Cleaning at the cognitive relay. The capacity region remains unknown in the case of moderately weak interference and weak cognition. In this case we conjecture that the currently best known outer bound may be loose. Tightening of the outer bound, as well as determining the approximate capacity region of the Gaussian IFC-CR channel at finite SNR is subject of ongoing investigation.

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