Approximate Sum-Capacity of K-user Cognitive Interference Channels with Cumulative Message Sharing

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***Abstract*—This paper considers the K-user cognitive interference channel with one primary and $K-1$ secondary/cognitive transmitters with a *cumulative message sharing* structure, i.e., cognitive transmitter $i \in [2:K]$ has non-causal knowledge of the messages of users with index less than i. A computable outer bound valid for any memoryless channel is proposed. The sum-rate outer bound is evaluated first for the high-SNR linear deterministic approximation of the Gaussian noise channel. This is shown to be both the sum capacity for the 3-user channel with arbitrary channel gains, and the sum-capacity for the symmetric K-user channel. Interestingly, for the K user channel, cognition at transmitters 2 to $K-1$ is not needed, and knowledge of all messages at the K-th transmitter only is sufficient to achieve the sum-capacity. Next, the sum-capacity of the symmetric Gaussian noise channel is characterized to within a constant additive and multiplicative gap, both of which are functions of K. As opposed to other multiuser interference channel models, a single scheme (in this case based on dirty-paper coding) suffices for both the weak and strong interference regimes. The generalized degrees of freedom (gDoF) are then derived and are shown, unlike interference and broadcast channels, to be a function of K. Interestingly, it is shown that as the number of users grows to infinity the gDoF of the K-user cognitive interference channel with cumulative message sharing tends to the gDoF of a broadcast channel with a K-antenna transmitter and K single-antenna receivers. Finally, numerical evaluations show that the actual gaps between the presented inner and outer bounds are significantly smaller than the analytically derived gaps.**

***Index Terms*—Cognitive interference channel, generalized degrees-of-freedom, sum-capacity, linear deterministic channel, symmetric Gaussian channel, MIMO broadcast channel, multiplicative gap, additive gap.**

I. Introduction

COGNITIVE radio technology has been used to improve spectral management by allowing artificially intelligent secondary users (cognitive radios) to utilize the same frequency band as primary / licensed users. Cognitive radios may search for available unused spectrum (interweave), may operate simultaneously with primary users as long as the interference caused is within an acceptable level (underlay), or may exploit knowledge of the messages of primary users through encoding schemes to cancel interference (overlay) [1].

The cognitive radio channel, first introduced in [2], falls into the overlay category, and consists of two source-destination pairs in which one of the transmitters called the *secondary transmitter* has non-causal knowledge of the message of the other transmitter known as the *primary transmitter*. This non-causal message knowledge idealizes a cognitive radio's ability to overhear other transmissions and exploit them to either cancel these at their own receiver, or aid in their transmission. For the state-of-the-art on the two-user cognitive channel we refer the reader to [3], [4]. In particular, the capacity of the semi-deterministic two-user cognitive channel is known [3]; the capacity of the Gaussian noise channel is known exactly for most channel parameters, and to within one bit otherwise [4].

In this paper we consider the extension of the two-user cognitive interference channel to K users in which there is one primary and $K-1$ secondary, or cognitive, users. We assume a *cumulative* message cognition structure introduced in [5] for the three-user channel, and extended here to K users, whereby user 1 is the primary user, and cognitive user i, $i \in [2:K]$, knows the messages of user 1 through $i-1$.

The cognition model may be motivated as:

1) First and foremost, it is inspired by the concept of overlaying, or layering, cognitive networks. In particular, we consider multiple types of devices sharing the spectrum. The first "layer" consists of the primary users. Each additional cognitive layer transmits simultaneously with the previous layers (overlay) given the lower layers' codebooks. This may enable them to learn the lower layers' messages and use this to aid the lower layers' transmission, or to combat interference at their own receivers. This *non-causal* message knowledge idealizes this ability of higher layers to learn the messages of lower layers before transmission, and may thus be viewed as a channel whose performance upper bounds the performance of a more realistic causal network.

2) This model may alternatively be used to model a coordinated multipoint network where there is a high capacity backhaul (sufficiently high to exchange messages perfectly) between the various transmitters in a network [6]. These transmitters could be used to model distributed base-stations connected through backhaul, and the receivers would model mobile users. The message knowledge would then be shared over the backhaul between the base-stations, and studying this model would reveal what type of message knowledge structure is useful. Many types of message structures are possible; the cumulative message knowledge structure considered here is again inspired by having certain "legacy" networks which do not change, while others are sequentially built on top of these, again forming a layered network.

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3) Finally, non-causal message knowledge may further be justified in a network with re-transmissions. That is, suppose messages are simultaneously transmitted over a network with multiple transmitters and receivers. Under certain channel conditions, it is conceivable that certain receivers are not able to decode a message and demand retransmission, but that other transmitters are able to decode the message(s) of other transmitters. In the re-transmission phase, certain transmitters would then have non-causal message knowledge of other users. This channel model is one particular example of the message knowledge structure in a re-transmission phase [7].

For this model, we are interested in the impact the cumulative message cognition has on the sum-capacity, or network throughput, and how it extends known results for the two-user case [4]. We furthermore seek to determine how K-user cumulative message cognition differs from other K-user channel models such as the K-user interference channel (with no cognition) or the K-antenna broadcast channel (where every user knows all messages).

A. Past Work

The literature on the fundamental performance of multi-user cognitive interference channels is limited, in part due to the fact that the two-user counterpart is not yet fully understood [3], [4]. The only other work on a K-user cognitive interference channel with $K > 3$ is, to the best of our knowledge, that of [8]. In [8] the channel model consists of one primary user and $K - 1$ parallel cognitive users; each cognitive user only knows the primary message in addition to their own message (thus not a cumulative message structure); the cognitive users do not cause interference to one another but only to the primary receiver and are interfered only by the primary transmitter (whereas we consider here a fully connected K-user interference channel model); for this channel model the capacity in the "very strong" interference regime is obtained by using lattice codes [8]. Related as well to K-user cognitive channels is the work in [6] where the Degrees of Freedom (DoF) of a K-user interference channel (K independent messages) in which each transmitter, in addition to its own message, has access to a subset of the other users' messages, is obtained. We will be interested in characterizing the extension of the DoF – the Generalized DoF (gDoF), as well as capacity to within a constant gap – for one particular message knowledge structure.

While not much work on $K > 3$ channels exists, in [5], [9]–[12] different three-user cognitive channels are considered; we note that the models differ from the one considered here either in the number of transmitter/receivers, or in the message sharing/cognition structure in all but [5], [9]. In the more comprehensive [9], several types of 3-user cognitive interference channels are proposed: that with "cumulative message sharing" (CMS) as considered here, that with "primary message sharing" where the message of the single primary user is known at both cognitive transmitters (who do not know each others' messages), and finally "cognitive only message sharing" (CoMS) where there are two primary users who do not know each others' message and a single cognitive user which knows both primary messages. Achievable rate regions are obtained which are evaluated in Gaussian noise. The CoMS mechanism yields almost the same message structure as in the interference channel with a cognitive relay – identical if the relay were to further have a message of its own (see [13], [14] and references therein for the interference channel with a cognitive relay). In [10] the CoMS was first introduced. In [11] the CoMS structure is assumed and the cognitive user is furthermore assumed not to interfere with the primary users; an inner and an outer bound are obtained. In [12] capacity under "strong interference" for the CoMS is obtained. We thus emphasize that the channel considered here is more general than others studied as we consider K users, a fully connected interference channel, and consider the less studied CMS sharing structure.

B. Contributions

The main contributions of this work are:

1) We derive a novel and general outer bound region that reduces to the outer bound of [3] for the two-user case. The bound is valid for any memoryless channel and any number of users. The bound does not contain auxiliary random variables and is therefore computable for many channels of interest, including the Gaussian channel.

2) We determine the sum-capacity the 3-user Linear Deterministic Approximation of the Gaussian noise channel at high-SNR for any channel parameters. This optimal scheme inspires a scheme for the K-user symmetric channel. This latter scheme only requires cognition of all messages at one transmitter; all the others need only knowledge of their own message.

3) We derive the sum-capacity for the symmetric Gaussian noise channel with K users to within a constant additive and multiplicative gap. The additive gap is a function of the number of users and grows as $(K-2)\log_2(K-2)$. The proposed achievable scheme is based on Dirty Paper Coding (DPC) and may be thought of as a Gaussian MIMO-broadcast channel scheme where only one encoding order is possible due to the cumulative message sharing mechanism. As opposed to other multiuser interference channel models, a single scheme suffices for both the weak and strong interference regimes. Moreover, no interference alignment of structured coding seems to be needed. Numerical evaluations show that the actual gap is less than the analytical one; this is so because of necessary crude bounding steps needed to obtain analytically tractable sum-rate expressions. The multiplicative gap is K and is achieved by having all users beamform to the primary user.

4) The normalized gDoF, defined as the pre-log of the sum-capacity as a function of SNR is shown to be a function of K. This is in contrast with other channel models, like the non-cognitive case or the broadcast channel, where the gDoF are the same for any K. Interestingly, it is shown that as the number of users grows to infinity the gDoF of the K-user cognitive interference channel with CMS tends to the gDoF of a broadcast channel with a K-antenna transmitter and K single-antenna receivers.

C. Paper Organization

The paper is organized as follows. Section II describes the channel model. Section III contains the novel, computable outer bound region; first the 3-user case is considered to highlight the key 'side information' idea, which is then extended to any number of users. In Section IV we first derive the sum-capacity of the 3-user Linear Deterministic Approximation of the Gaussian noise channel at high-SNR for any channel parameters and then extend it to the symmetric K-user case; we also compare the sum-capacity of the interference channel with cumulative message sharing mechanisms with other interference channel models. In Section V we derive the sum-capacity of the symmetric Gaussian noise channel to within an additive and multiplicative gap; we use a DPC-based scheme inspired by the MIMO-BC with only one possible encoding order due to the cumulative message sharing mechanism. We further show by numerical optimization of inner and outer bounds that the actual gap is less than the analytical one presented. The gDoF is also derived and shown to be a function of the number of users. As is the case for other interference models, the gDoF and the sum-capacity of the Linear Deterministic Approximation of the Gaussian noise channel at high-SNR coincide. Section VI concludes the paper.

II. Channel Model

The general memoryless K-user cognitive interference channel with cumulative message sharing (K-CIFC-CMS) consists of K source-destination pairs sharing the same physical channel, where some transmitters have non-causal knowledge of the messages of other transmitters. Here transmitter 1 is referred to as the primary user and is assumed to have no cognitive abilities. Transmitter i, $i \in [2:K]$, is non-causally cognizant of the messages of the users with index smaller than i. More formally, the K-CIFC-CMS channel consists of

- Channel inputs $X_i \in \mathcal{X}_i,\ i \in [1:K]$,
- Channel outputs $Y_i \in \mathcal{Y}_i,\ i \in [1:K]$,
- A memoryless channel with joint transition probability $P(Y_1, \ldots, Y_K | X_1, \ldots, X_K)$,
- Messages W_i known to users $1, 2, \ldots, i$, $i \in [1:K]$.

A code with non-negative rate vector $(R_1, \ldots, R_K)$ and blocklength N is defined by

- Messages $W_i,\ i \in [1:K]$, uniformly distributed over $[1:2^{NR_i}]$ and independent of everything else,
- Encoding functions $f_i^{(N)} : [1:2^{NR_1}] \times \ldots \times [1:2^{NR_i}] \to \mathcal{X}_i^N$ such that $X_i^N := f_i^{(N)}(W_1, \ldots, W_i)$, $i \in [1:K]$,
- Decoding functions $g_i^{(N)} : \mathcal{Y}_i^N \to [1:2^{NR_i}]$ such that $\widehat{W}_i = g_i^{(N)}(Y_i^N)$, $i \in [1:K]$,
- Probability of error $P_e^{(N)} := \max_{i \in [1:K]} \mathbb{P}[\widehat{W}_i \neq W_i]$.

The capacity of the K-CIFC-CMS channel consists of all non-negative rate tuples $(R_1, \ldots, R_K)$ for which there exist a sequence of codes indexed by the block length N such that $P_e^{(N)} \to 0$ as $N \to \infty$. Since the decoders cannot cooperate and the channel is used without feedback, the capacity may be shown to depend only on the marginal noise distributions rather than the joint noise distribution by an argument similar to that used for the broadcast channel (BC) [15].

In this work we focus on two channel models.

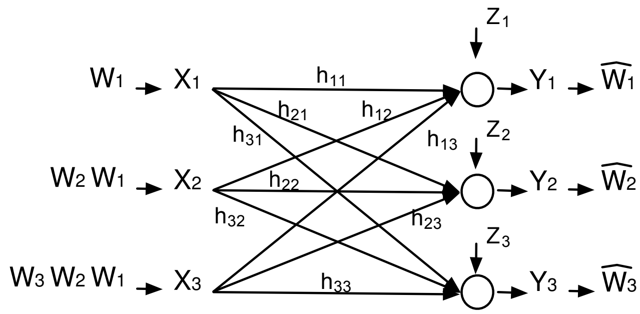

Fig. 1. The Gaussian 3-CIFC-CMS.

A. The Gaussian Noise Channel

The single-antenna complex-valued K-CIFC-CMS with Additive White Gaussian Noise (AWGN), shown in Fig. 1 for $K = 3$, has input-output relationship

$$Y_\ell = \sum_{i \in [1:K]} h_{\ell i} X_i + Z_\ell,\ \ell \in [1:K], \tag{1a}$$

where, without loss of generality, the inputs are subject to the power constraint

$$\mathbb{E}[|X_i|^2] \le 1,\ i \in [1:K], \tag{1b}$$

and the noises are marginally proper-complex Gaussian random variables with parameters

$$Z_\ell \sim \mathcal{N}(0,1),\ \ell \in [1:K]. \tag{1c}$$

The channel gains h_{ij}, $(i,j) \in [1:K]^2$, are constant and therefore known to all terminals. Without loss of generality we may assume the direct links h_{ii}, $i \in [1:K]$ to be real-valued and non-negative since the receiver i can always compensate for the phase of one channel gain.

The Generalized Degrees-of-Freedom (gDoF) of the *symmetric* Gaussian channel is a performance metric that characterizes the high-SNR behavior of the sum-capacity and is defined as follows. Let SNR be a non-negative number and parameterize

$$|h_{ii}|^2 := \mathsf{SNR},\ i \in [1:K], \tag{2a}$$
$$|h_{\ell i}|^2 := \mathsf{SNR}^\alpha,\ (\ell, i) \in [1:K]^2, \ell \neq i, \tag{2b}$$

for some non-negative α. The gDoF is

$$d(\alpha) := \lim_{\mathsf{SNR} \to +\infty} \frac{C_\Sigma}{\log(1+\mathsf{SNR})}, \tag{3}$$

where $C_\Sigma := \max\{R_1 + \ldots + R_K\}$ and where the maximization is over all achievable rates. The sum-capacity is said to be known to within a constant gap of b bits if one can show rates $R_\Sigma^{(\mathrm{in})}$ and $R_\Sigma^{(\mathrm{out})}$ such that

$$R_\Sigma^{(\mathrm{in})} \le C_\Sigma \le R_\Sigma^{(\mathrm{out})} \le R_\Sigma^{(\mathrm{in})} + \mathsf{b} \log(2). \tag{4}$$

The gDoF and constant gap characterization of the symmetric sum capacity imply that

$$C_\Sigma = d(\alpha) \log(1+\mathsf{SNR}) + o(1),$$

where $o(1)$ indicates a quantity that is finite at all SNR.

B. Linear Deterministic Approximation of the Gaussian Noise Channel at High SNR

The Linear Deterministic approximation of the Gaussian Noise Channel at high SNR (LDC) was first introduced in [16] to allow focussing on the effect of signal interactions between users rather than on the effect of additive noise. The proposed framework has been powerful in revealing key issues for the problem of communicating over interfering networks. The insights gained from the LDC have often translated into Gaussian capacity results to within a constant gap for any finite SNR [4], [17], [18]. In light of these success stories we also start our investigation from the LDC. The LDC has input-output relationship

$$Y_\ell = \sum_{i\in[1:K]} \mathbf{S}^{m-n_{\ell i}} X_i,\ \ell \in [1:K], \tag{5}$$

where $m := \max\{n_{ij}\}$, $\mathbf{S}$ is the binary shift matrix of dimension m, all inputs and outputs are binary column vectors of dimension m, the summation is bit-wise over of the binary field, and the channel gains $n_{\ell i}$ for $(\ell, i) \in [1 : K]^2$, are positive integers. In a *symmetric* LDC all direct links have the same strength $n_{ii} = n_{\rm d} \geq 0, i \in [1 : K]$, and all the interfering links have the same strength $n_{\ell i} = n_{\rm i} = \alpha\ n_{\rm d} \geq 0, (\ell, i) \in [1 : K]^2,\ \ell \neq i$. Note that the subscript i (roman font) of $n_{\rm i}$ stands for "interference" and is not an index; as such it should not be confused with index i (italic font).

The channel in (5) can be thought of as the high SNR approximation of the channel in (1) with their parameters related as $n_{ij} = \lfloor \log(1 + |h_{ij}|^2)\rfloor,\ (i, j) \in [1 : K]^2$.

III. Outer Bound

In this section we derive an outer-bound region for the general memoryless K-CIFC-CMS. We start with the case of $K = 3$ users to highlight the main proof techniques and ease the reader into the extension to any number of users.

Theorem 1. *The capacity region of the general memoryless 3-CIFC-CMS is contained in the region defined by*

$$R_1 \leq I(Y_1; X_1, X_2, X_3), \tag{6a}$$
$$R_2 \leq I(Y_2; X_2, X_3|X_1), \tag{6b}$$
$$R_3 \leq I(Y_3; X_3|X_1, X_2), \tag{6c}$$
$$R_2 + R_3 \leq I(Y_2; X_2, X_3|X_1) + I(Y_3; X_3|X_1, X_2, Y_2), \tag{6d}$$
$$R_1 + R_2 + R_3 \leq I(Y_1; X_1, X_2, X_3) + I(Y_2; X_2, X_3|X_1, Y_1) + I(Y_3; X_3|X_1, Y_1, X_2, Y_2), \tag{6e}$$

for some input distribution P_{X_1,X_2,X_3}. The joint conditional distribution $P_{Y_1,Y_2,Y_3|X_1,X_2,X_3}$ can be chosen so as to tighten the different bounds as long as the conditional marginal distributions $P_{Y_i|X_1,X_2,X_3}$, $i \in [1 : 3]$, are preserved.

Proof: The proof is found in Appendix A. ■

Remarks

1) The region in Th. 1 reduces to the outer bound in [3, Th. 6] by setting $X_3 = Y_3 = \emptyset$.

2) The outer bound region in (6) does not contain auxiliary random variables. Moreover, every mutual information term contains all the inputs. These two facts imply that the outer bound region in Th. 1 can be easily evaluated for many channels of interest. For example, for the Gaussian noise channel in Section II-A, the "Gaussian maximizes entropy" principle suffices to show that jointly Gaussian inputs exhaust the outer bound.

3) The sum-capacity bound in (6e) is obtained by giving S_i as side information to receiver i, $i \in [1 : K]$, where $S_i = [S_{i-1}, W_{i-1}, Y_{i-1}^N]$ starting with $S_1 = \emptyset$. With this "nested" side information, the mutual information terms can be expressed in terms of entropies which may be recombined in ways that can be easily single-letterized. This form of the side information allows us to extend the result from the 3-user case to any number of users.

4) The mutual information terms in (6e) have the form $I(Y_i; X_i, \ldots, X_K|X_1, Y_1, \ldots, X_{i-1}, Y_{i-1})$, $1 \leq i \leq K$, which can be given the following interpretation.
Since message W_i is available at transmitters i through K, inputs $(X_i, \ldots, X_K)$ are "informative" for receiver i, while inputs $(X_1, \ldots, X_{i-1})$ are independent of W_i; receiver i decodes from Y_i the information carried in $(X_i, \ldots, X_K)$ that could not be recovered by users with lesser index as represented by $(X_1, Y_1, \ldots, X_{i-1}, Y_{i-1})$.

Th. 1 can be extended to any K as follows.

Theorem 2. *The capacity region of the general memoryless K-CIFC-CMS is contained in the region defined by*

$$R_i \leq I(Y_i; X_i, \ldots, X_K|X_1, \ldots, X_{i-1}), \tag{7a}$$
$$\sum_{j=i}^{K} R_j \leq \sum_{j=i}^{K} I(Y_j; X_j, \ldots, X_K|X_1, \ldots, X_{j-1}, Y_i, \ldots, Y_{j-1}), \tag{7b}$$
$$i \in [1 : K],$$

for some input distribution $P_{X_1,\ldots,X_K}$. Moreover, each rate bound in (7b) *may be tightened with respect to the channel conditional distribution as long as the channel conditional marginal distributions are preserved.*

Proof: The proof is found in Appendix B. ■

In the following section we shall derive achievable schemes matching the sum-capacity outer bound in Th. 2 for the LDC in (5) and schemes that achieve the sum-capacity outer bound to within a constant bounded gap regardless of the channel parameters for the Gaussian channel in (1).

IV. Sum-Capacity for the Linear Deterministic K-CIFC-CMS

In Sections IV-A and IV-B we determine the sum-capacity of the LDC with $K = 3$ users and any value of the channel gains. In Sections IV-D and IV-E we derive the sum-capacity for any K but for symmetric channel gains only. The main results of this section are

Theorem 3. *The sum capacity bound in* (6e) *is achievable for the LDC 3-CIFC-CMS with generic channel gains.*

Theorem 4. *The sum capacity bound with $i = 1$ in* (7b) *is achievable to the LDC K-CIFC-CMS with symmetric channel gains. The capacity achieving scheme only requires cognition of all messages at one single transmitter.*

The rest of the section is devoted to their proofs.

A. Sum-capacity Outer Bound for the 3-user Case and Generic Channel Gains

The sum capacity outer bound in Th. 1 specialized to a deterministic 3-CIFC-CMS (i.e., $H(Y_i|X_1, X_2, X_3) = 0, i \in [1:3]$) gives the following sum capacity outer bound

$$R_\Sigma := R_1 + R_2 + R_3$$
$$\leq \max\Big\{H(Y_1) + H(Y_2|X_1, Y_1) + H(Y_3|X_1, Y_1, X_2, Y_2)\Big\},$$

where the maximization is over all possible joint distributions P_{X_1,X_2,X_3}. For the LDC in (5) with $K = 3$ we obtain

$$R_1 + R_2 + R_3 \leq \max\{n_{11}, n_{12}, n_{13}\} \quad \text{(8a)}$$
$$+ f(n_{22}, n_{23}|n_{12}, n_{13}) \quad \text{(8b)}$$
$$+ [n_{33} - \max\{n_{13}, n_{23}\}]^+, \quad \text{(8c)}$$

where $f(c, d|a, b)$ in (8b) follows from [19, eq.(5)] and is defined as $f(c, d|a, b) := \max\{c + b, a + d\} - \max\{a, b\}$ if $c-d \neq a-b$ and $\max\{a, b, c, d\}-\max\{a, b\}$ if $c-d = a-b$.

The bound in (8) follows by maximizing each mutual information term individually as the second equation at the top of the next page, where $[x]^+ := \max\{0, x\}$. Notice that i.i.d. Bernoulli(1/2) input bits simultaneously maximize each entropy terms.

B. Achievability of the Sum Capacity Outer Bound for the 3-user Case and Generic Channel Gains

In the following, depending on whether $[n_{33} - \max\{n_{13}, n_{23}\}]^+$ in (8c) is zero or positive, different interference scenarios are identified and transmission schemes that are capable of achieving the sum-capacity upper bound in (8) are proposed. In particular:

Case 1: If the signal sent by the most cognitive transmitter is received the weakest at the intended destination, that is, if

$$n_{33} \leq \max\{n_{13}, n_{23}\}, \quad (9)$$

the sum-capacity in (8) becomes

$$R_1 + R_2 + R_3 \leq \max\{n_{11}, n_{12}, n_{13}\} + f(n_{22}, n_{23}|n_{12}, n_{13}).$$

The condition in (9) corresponds to $H(Y_3|X_1, Y_1, X_2, Y_2) = 0$, i.e., conditioned on (X_1, X_2) the signal received at the most cognitive receiver is a degraded version of the signal received at the other two receivers. Recall that user 3 may send information to all receivers as it knows all messages. The condition in (9) implies that the signal X_3 may convey more information to receivers 1 and 2 than it can to the receiver 3. In this case, one might thus suspect that $R_3 = 0$ is optimal and that the best use of the cognitive capabilities of user 3 is to broadcast to the receivers. We will next show that this is indeed the case.

We set $R_3 = 0$ and we therefore convert the LCD 3-CIFC-CMS into a deterministic 2-CIFC-CMS where user 1 is the primary user (with input X_1 and output Y_1) and the cognitive user has vector input $[X_2, X_3]$ and output Y_2. The capacity of a general deterministic 2-user cognitive interference channel is [3, Th. 12]

$$R_1 \leq H(Y_1), \ R_2 \leq H(Y_2|X_1),$$
$$R_1 + R_2 \leq H(Y_1) + H(Y_2|X_1, Y_1),$$

for some input distribution $P_{X_1,[X_2,X_3]}$. Hence the sum-capacity is

$$R_1 + R_2 = \max_{P_{X_1,[X_2,X_3]}} \Big\{H(Y_1) + H(Y_2|X_1, Y_1)\Big\}$$
$$= \max\{n_{11}, n_{12}, n_{13}\} + f(n_{22}, n_{23}|n_{12}, n_{13}),$$

which proves our claim.

Case 2: In the regime not covered by the condition in (9), that is, for

$$n_{33} > \max\{n_{13}, n_{23}\}, \quad (10)$$

the sum-capacity in (8) becomes

$$R_1 + R_2 + R_3 \leq \max\{n_{11}, n_{12}, n_{13}\} + f(n_{22}, n_{23}|n_{12}, n_{13})+$$
$$n_{33} - \max\{n_{13}, n_{23}\}.$$

In this case, the condition in (10) suggests that the intended signal at receiver 3 is sufficiently strong to be able to support a non-zero rate. The form of the sum-capacity also suggests that a plausible strategy is to use the optimal strategy for Case 1 and "sneak in" extra bits for user 3 in such a way that they do not appear at the other receivers. We next show that this is optimal.

We split the signal of transmitter 3 in two parts

$$X_3 := X_{3a} + X_{3b},$$

where X_{3a} is intended to mimic the scheme for Case 1 (i.e., as if user 2 had input $[X_2, X_{3a}]$) and X_{3b} carries the information to Y_3, possibly "pre-coded" against the interference of (X_1, X_2, X_{3a}), and such that X_{3b} is not received at receivers 1 and 2. We define

$$X_{3b} := \mathbf{S}^{\max\{n_{13},n_{23}\}} V_3,$$

for some vector V_3 defined in the following. Note that the shift caused by $\boldsymbol{S}^{\max\{n_{13},n_{23}\}}$ is such that V_3 is not received at Y_1 and at Y_2. We note that V_3 is "private information" for receiver 3 that is dirty paper coded against the interference caused by $[X_1, X_2, X_{3a}]$ at receiver 3; with this receiver 3 is virtually interference-free. We then implement the optimal strategy for Case 1 with $[X_1, X_2, X_{3a}]$ and with the remaining bits in X_{3b} we transmit to receiver 3 thereby achieving the sum-capacity in (8).

C. Example of Sum-capacity Optimal Schemes for the 3-user Case and Symmetric Channel Gains

We now present several concrete examples of the achievability scheme presented in Section IV-B.

We consider first the symmetric scenario with $n_\mathrm{d} > 0, \ n_\mathrm{i} = n_\mathrm{d}\ \alpha, \ \alpha \geq 0$. Define the normalized sum-capacity as

$$d_\Sigma(\alpha; 3) := \frac{\max\{R_1 + R_2 + R_3\}}{n_\mathrm{d}}.$$

Note that when $n_\mathrm{d} = 0$ the channel reduces to a broadcast channel from transmitter $[X_2, X_3]$ to receivers Y_1 and Y_2

$$H(Y_1) = H(\mathbf{S}^{m-n_{11}}X_1 + \mathbf{S}^{m-n_{12}}X_2 + \mathbf{S}^{m-n_{13}}X_3) \leq \max\{n_{11}, n_{12}, n_{13}\},$$
$$H(Y_2|X_1, Y_1) = H(\mathbf{S}^{m-n_{22}}X_2 + \mathbf{S}^{m-n_{23}}X_3|X_1, \mathbf{S}^{m-n_{12}}X_2 + \mathbf{S}^{m-n_{13}}X_3)$$
$$\leq H(\mathbf{S}^{m-n_{22}}X_2 + \mathbf{S}^{m-n_{23}}X_3|\mathbf{S}^{m-n_{12}}X_2 + \mathbf{S}^{m-n_{13}}X_3) f(n_{22}, n_{23}|n_{12}, n_{13}),$$
$$H(Y_3|X_1, Y_1, X_2, Y_2) = H(\mathbf{S}^{m-n_{33}}X_3|X_1, X_2, \mathbf{S}^{m-n_{13}}X_3, \mathbf{S}^{m-n_{23}}X_3)$$
$$\leq H(\mathbf{S}^{m-n_{33}}X_3|\mathbf{S}^{m-\max\{n_{13},n_{23}\}}X_3)[n_{33} - \max\{n_{13}, n_{23}\}]^+$$

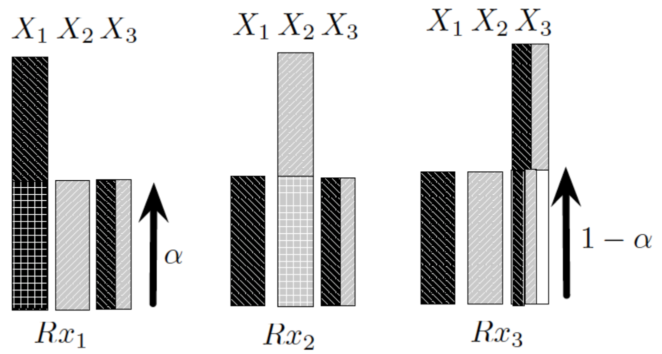

Fig. 2. LDC 3-CIFC-CMS in weak interference with $\alpha = 1/2$. The achievable rates are $R_1/n_d = R_2/n_d = 1, R_3/n_d = 1 - \alpha$ thereby achieving the sum-capacity upper bound in (8) under the condition in (10). Dark black bits are intended to Rx_1, gray bits are intended to Rx_2, and white bits are intended to Rx_3.

(receiver 3 cannot be reached by its transmitter and hence $R_3 = 0$ is optimal; similarly the primary user cannot reach its intended destination and cannot deliver any information to the other destinations, hence $X_1 = 0$ is optimal); the capacity region of a deterministic broadcast channel is known [20] and for the symmetric LDC with $n_\mathrm{d} = 0$ it reduces to $R_1 + R_2 = 2n_\mathrm{i}$.

When $n_\mathrm{d} > 0$ the sum-capacity can be expressed as

$$d_\Sigma(\alpha; 3) = \max\{1, \alpha\} + \frac{f(n_\mathrm{d}, n_\mathrm{d}\ \alpha; n_\mathrm{d}\ \alpha, n_\mathrm{d}\ \alpha)}{n_\mathrm{d}} + [1-\alpha]^+$$
$$= \begin{cases} 3\max\{1, \alpha\} - \alpha & \text{for } \alpha \neq 1, \\ 1 & \text{for } \alpha = 1. \end{cases}$$

Fig. 2 shows an example of the achievable strategy for *weak interference* defined as $\alpha < 1$ (corresponding to Case 2 in Section IV-A). The case $\alpha = 1$ corresponds to a channel where all received signals are statistically equivalent and therefore its capacity region is that of a 3-user Multiple Access Channel. The *strong interference* regime defined as $\alpha > 1$ (corresponding to Case 1 in Section IV-A) is not explicitly considered as the achievable strategy is the same as for the weak interference regime except for the fact that the most cognitive user sends at zero rate, as its bits would create interference at the non-intended receivers. Notice the important role of cognition in Fig. 2: the third transmitter (cognitive of all 3 messages) sends a linear combination of the messages of users 1 and 2 in such a way that the effect of the aggregate interference is neutralized at all receivers, i.e., leaving the receivers 1 and 2 interference-free. The third transmitters also sends some "private" information bits in such a way that these bits do not appear at the other receivers. It is important also to observe that user 2, who is cognizant of the message of user 1, does not use this message knowledge. In other words, user 2 need not be cognizant in order to achieve the sum-capacity in the symmetric case.

D. Sum Capacity Outer Bound for the K-user Case and Symmetric Channel Gains

For the K-user symmetric LDC the sum-capacity is upper bounded by

$$d_\Sigma(\alpha; K) \leq \begin{cases} K\max\{1, \alpha\} - \alpha & \text{for } \alpha \neq 1, \\ 1 & \text{for } \alpha = 1. \end{cases} \quad (11)$$

The proof that the sum-capacity upper bound in Th. 2 evaluates to the expression in (11) is provided next. For the K-user symmetric LDC with $m = n_\mathrm{d}\max\{1, \alpha\}$ the sum-capacity is upper bounded by

$$\sum_{k=1}^{K} R_k \leq \sum_{k=1}^{K} H\Big(Y_k|X_1, \ldots, X_{k-1}, Y_1, \ldots, Y_{k-1}\Big)$$
$$= \sum_{k=1}^{K-1} H\Big(\mathbf{S}^{m-n_\mathrm{d}}X_k + \mathbf{S}^{m-n_\mathrm{i}}(\sum_{i=k+1}^{K} X_i)$$
$$|X_1, \ldots, X_{k-1}, \mathbf{S}^{m-n_\mathrm{i}}(\sum_{i=k}^{K} X_i)\Big)$$
$$+ H\Big(\mathbf{S}^{m-n_\mathrm{d}}X_K|X_1, \ldots, X_{K-1}, \mathbf{S}^{m-n_\mathrm{i}}X_K\Big)$$
$$\leq \sum_{k=1}^{K-1} H\Big((\mathbf{S}^{m-n_\mathrm{d}} + \mathbf{S}^{m-n_\mathrm{i}})X_k\Big) + H\Big(\mathbf{S}^{m-n_\mathrm{d}}X_K|\mathbf{S}^{m-n_\mathrm{i}}X_K\Big)$$
$$\leq (K-1)\max\{n_\mathrm{d}, n_\mathrm{i}\} + [n_\mathrm{d} - n_\mathrm{i}]^+$$
$$\leq n_\mathrm{d}\Big(K\max\{1, \alpha\} - \alpha\Big).$$

The discontinuity at $\alpha = 1$ in (11) is because when $n_\mathrm{d} = n_\mathrm{i}$ all received signal are equivalent, i.e., $Y_1 = \ldots = Y_K = \sum_{i=1}^{K} X_i$, and the channel reduces to a K-user MAC with sum-capacity $\max H(Y_1) = n_\mathrm{d}$.

E. Achievability of the Sum Capacity Outerbound for the K-user Case and Symmetric Channel Gains

The schemes which were shown to be optimal for the LCD 3-CIFC-CMS in Section IV-C may be extended to any arbitrary number of users. Let U_j, $j \in [1 : K]$, be the signal intended for receiver j, that is, U_j is only a function of message W_j, and composed of i.i.d. Bernoulli(1/2) bits. Let the transmit signals be

$$X_j = U_j,\ j \in [1:K-1],$$
$$X_K = \begin{bmatrix} I_{n_\mathrm{i}} & 0_{n_\mathrm{i}\times[n_\mathrm{d}-n_\mathrm{i}]^+} \\ 0_{[n_\mathrm{d}-n_\mathrm{i}]^+\times n_\mathrm{i}} & 0_{[n_\mathrm{d}-n_\mathrm{i}]^+\times[n_\mathrm{d}-n_\mathrm{i}]^+} \end{bmatrix} \left(\sum_{j=1}^{K-1} U_j\right) + \begin{bmatrix} 0_{n_\mathrm{c}\times n_\mathrm{i}} & 0_{n_\mathrm{i}\times[n_\mathrm{d}-n_\mathrm{i}]^+} \\ 0_{[n_\mathrm{d}-n_\mathrm{i}]^+\times n_\mathrm{i}} & I_{[n_\mathrm{d}-n_\mathrm{i}]^+} \end{bmatrix} U_K,$$

so that

$$\sum_{j=1}^{K} X_j = \begin{bmatrix} 0_{n_\mathrm{i}\times n_\mathrm{i}} & 0_{n_\mathrm{i}\times[n_\mathrm{d}-n_\mathrm{i}]^+} \\ 0_{[n_\mathrm{d}-n_\mathrm{i}]^+\times n_\mathrm{i}} & I_{[n_\mathrm{d}-n_\mathrm{i}]^+} \end{bmatrix} \left(\sum_{j=1}^{K} U_j\right),$$

where $0_{n\times m}$ indicates the all zero matrix of dimension $n\times m$ and I_n the identity matrix of dimension n. With these choices, the signal at receiver ℓ, $\ell\in[1:K]$, is

$$Y_\ell = (\mathbf{S}^{m-n_\mathrm{d}} + \mathbf{S}^{m-n_\mathrm{i}})X_\ell + \mathbf{S}^{m-n_\mathrm{i}}\left(\sum_{j=1}^{K} X_j\right) = (\mathbf{S}^{m-n_\mathrm{d}} + \mathbf{S}^{m-n_\mathrm{i}})X_\ell, \quad m=\max\{n_\mathrm{d}, n_\mathrm{i}\}.$$

Since the matrix $\mathbf{S}^{m-n_\mathrm{d}} + \mathbf{S}^{m-n_\mathrm{i}}$ is full rank for $n_\mathrm{d}\neq n_\mathrm{i}$, receiver ℓ, $\ell\in[1:K]$, decodes U_ℓ from $(\mathbf{S}^{m-n_\mathrm{d}} + \mathbf{S}^{m-n_\mathrm{i}})^{-1}Y_\ell = X_\ell$. Hence receiver ℓ, $\ell\in[1:K-1]$, decodes $m=\max\{n_\mathrm{d}, n_\mathrm{i}\}$ bits since $X_\ell = U_\ell$, while receiver K decodes the lower $[n_\mathrm{d}-n_\mathrm{i}]^+$ bits of U_K from X_K.

Interestingly, receivers from 1 to $K-1$ are interference free, while receiver K decodes n_i bits of the "interference function" $\sum_{j=1}^{K-1} U_j$. Notice that cognition is only needed at one transmitter in all interference regimes. This implies that this sum-capacity result holds for all cognitive channels where user i is cognizant of any subset (including the empty set) of the messages of users with index less than i. We suspect that the fact that only the last user need cognition of all the other messages is a consequence of: 1) the extreme symmetry in the channel model (which is needed for analytical tractability), which naturally aligns the interfering signals at all users. Thus, if the most cognitive user cancels interference at one receiver, it essentially cancels it at all receivers by symmetry. 2) the LDA channel model in which "coherent" gains often seen in Gaussian channels, when two users have the same message may beamform that message to a particular receiver at higher rates, is not possible. That is, the modulo 2 addition at a bit-wise level prohibits such coherent gains and as such it may not be useful to share the messages with other transmitters since the last fully cognitive user is already eliminating interference and additional gains are not possible. We note that these are heuristic rather than rigorous statements, and we do not expect this to hold for Gaussian channels where coherent gains are possible.

F. Comparison between Different Channel Models

We compare the symmetric sum-capacity of channels with different levels of cognition. Our base line for comparison is the K-user interference channel without any cognition, whose sum-capacity is [21]

$$d_\Sigma^{(\mathrm{IFC})}(\alpha;K) = \frac{K}{2} d_\Sigma^{(\mathrm{IFC})}(\alpha;2) \tag{12}$$

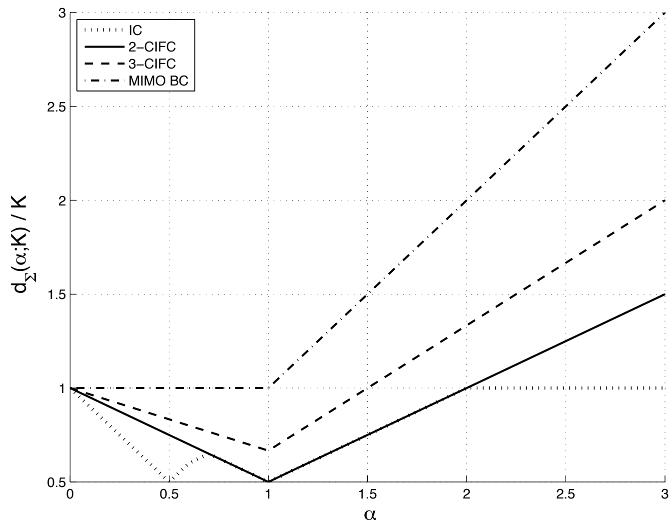

Fig. 3. $d_\Sigma(\alpha;K)/K$ for different channel models. The discontinuity at $\alpha=1$ is not shown where the value is $\frac{1}{K}$.

and where $d_\Sigma^{(\mathrm{IFC})}(\alpha;2)$ is the so-called W-curve of [17] except for a discontinuity at $\alpha=1$ where $d_\Sigma^{(\mathrm{IFC})}(\alpha;K)=1$ for all K [21]. Note that, except at $\alpha=1$, the normalized sum-capacity $\frac{1}{K}d_\Sigma^{(\mathrm{IFC})}(\alpha;K)$ does not depend on K.

At the other extreme of message cognition, consider the case where all users are cognitive of all messages. In this case the channel is equivalent to a MIMO-BC with K transmit antennas and K single-antenna receivers. The system may zero-force the interference to obtain

$$d_\Sigma^{(\mathrm{BC})}(\alpha;K) = K\max\{1,\alpha\}, \tag{13}$$

except for a discontinuity at $\alpha=1$ where $d_\Sigma^{(\mathrm{BC})}(\alpha;K)=1$, since in this case all the receivers are statistically equivalent and time-sharing is optimal. When $\alpha\neq1$, the normalized sum-capacity $\frac{1}{K}d_\Sigma^{(\mathrm{BC})}(\alpha;K)$ does not depend on K.

The sum-capacity of the symmetric LDC K-CIFC-CMS is given by (11), which is a function of K even after normalization by K, i.e.,

$$\frac{1}{K}d_\Sigma^{(\mathrm{CIFC-CMS})}(\alpha;K) = \max\{1,\alpha\} - \frac{\alpha}{K}. \tag{14}$$

This has the interesting interpretation that CMS looses α/K with respect to $d_\Sigma^{(\mathrm{BC})}(\alpha;K)/K$. In other words, as the number of cognitive users increases the CMS sum-capacity approaches the sum-capacity of a fully coordinated broadcast channel, which is intuitive.

Fig. 3 shows the sum-capacity normalized by the number of users for different channel models; we do not show the discontinuity at $\alpha=1$. We note the increase in performance in all interference regimes when compared to that of the 2-user CIFC-CMS and the K-user interference channel, but a loss with respect to the K-user broadcast channel (BC) with K transmit antennas and K single antenna receivers.

V. Sum-Capacity for the Gaussian K-CIFC-CMS to within a Constant Gap

In this section we derive the sum-capacity for the symmetric Gaussian channel with an arbitrary number of users to within a constant gap. For notational convenience we denote the direct link gains as $|h_{\rm d}|$, which can be taken to be real-valued and non-negative without loss of generality, and the interference link gains as $h_{\rm i}$, so that the channel in (1) can be rewritten as

$$Y_\ell = \Big(|h_{\rm d}| - h_{\rm i}\Big)X_\ell + h_{\rm i}\Big(\sum_{j=1}^{K} X_j\Big) + Z_\ell,\ \ell \in [1:K].$$

The main results of this section are

Theorem 5. *The generalized Degrees-of-Freedom of the symmetric K-user Gaussian noise channel are*

$$d(\alpha) = K\max\{1,\alpha\} - \alpha,$$

with a discontinuity at $\alpha = 1$ in the special case where all channel gains are the same (in modulo and phase), in which case $d(1) = 1$.

Theorem 6. *The sum-capacity bound in* (7b) *for $i = 1$ is achievable for the symmetric Gaussian K-CIFC-CMS to within 6 bits per channel use for $K = 3$ and to within $(K-2)\log_2(K-2) + 3.88$ bits per channel use for $K \geq 4$.*

Theorem 7. *The sum-capacity bound in* (7b) *for $i = 1$ is achievable to within a factor K by beamforming to the primary user.*

A. Sum-capacity Upper Bound for the K-user Case and Symmetric Channel Gains

For the K-user symmetric Gaussian channel with $|h_{\rm d}| \neq h_{\rm i}$ the bound in (7b) for $i = 1$ can be further bounded as follows. We note that we may tighten the bound by choosing the "worst noise covariance matrix", but for simplicity, here we use independent noises.

$$\begin{aligned}
\sum_{k=1}^{K} R_k &\leq \sum_{u=1}^{K} I\Big(X_u, \cdots, X_K; Y_u \Big| X_1, Y_1, \cdots, X_{u-1}, Y_{u-1}\Big)\\
&= I\Big(X_1, \cdots, X_K; |h_{\rm d}|X_1 + h_{\rm i}\sum_{i=2}^{K} X_i + Z_1\Big)\\
&+ \sum_{u=2}^{K-1} I\Big(X_u, \cdots, X_K; |h_{\rm d}|X_u + h_{\rm i}\sum_{i=u+1}^{K} X_i + Z_u\\
&\qquad \Big| X_\ell,\ h_{\rm i}\sum_{i=u}^{K} X_i + Z_\ell,\ \ell \in [1:u-1]\Big)\\
&+ I\Big(X_K; |h_{\rm d}|X_K + Z_K \Big| X_\ell,\ h_{\rm i}X_K + Z_\ell,\ \ell \in [1:K-1]\Big)\\
&\leq h\Big(|h_{\rm d}|X_1 + h_{\rm i}\sum_{i=2}^{K} X_i + Z_1\Big) - h(Z_1)\\
&+ \sum_{u=2}^{K-1} h\Big([|h_{\rm d}| - h_{\rm i}]X_u + Z_u - Z_{u-1}\Big) - h(Z_u)\\
&+ h\Big(|h_{\rm d}|X_K + Z_K \Big| h_{\rm i}X_K + \frac{1}{K-1}\sum_{\ell=1}^{K-1} Z_\ell\Big) - h(Z_K).
\end{aligned}$$

Finally, by the "Gaussian maximizes entropy" principle, we obtain

$$\sum_{k=1}^{K} R_k \leq \log\left(1 + \Big(|h_{\rm d}| + (K-1)|h_{\rm i}|\Big)^2\right) \quad (15a)$$

$$+ (K-2)\log(2) + (K-2)\log\left(1 + \frac{\big||h_{\rm d}| - h_{\rm i}\big|^2}{2}\right) \quad (15b)$$

$$+ \log\left(1 + \frac{|h_{\rm d}|^2}{1 + (K-1)|h_{\rm i}|^2}\right). \quad (15c)$$

For $h_{\rm i} = |h_{\rm d}|$ all received signals are statistically equivalent, therefore the K-CIFC-CMS is equivalent to a K-user Multiple Access Channel, whose sum-capacity is

$$\begin{aligned}
\sum_{k=1}^{K} R_k &\leq I(X_1, \ldots, X_K; |h_{\rm d}|\sum_{i=1}^{K} X_i + Z_1)\\
&\leq \log(1 + K^2|h_{\rm d}|^2).
\end{aligned}$$

In the limit for high SNR and with the channel parameterization as in (2), the above outer bound can be further bounded

$$\begin{aligned}
\sum_{k=1}^{K} R_k &\leq \log(K^2) + (K-1)\log(2)\\
&+ (K-1)\log\Big(1 + \max\{|h_{\rm d}|^2, |h_{\rm i}|^2\}\Big)\\
&+ \log\left(1 + \frac{|h_{\rm d}|^2}{1 + (K-1)|h_{\rm i}|^2}\right),
\end{aligned}$$

to obtain the following gDoF outer bound

$$d(\alpha) \leq (K-1)\max\{1,\alpha\} + [1-\alpha]^+ = K\max\{1,\alpha\} - \alpha.$$

This gDoF remains valid for $\alpha = 1$ as long as $h_{\rm i} = |h_{\rm d}|\exp(j\theta)$ for $\exp(j\theta) \neq 1$; when $\exp(j\theta) = 1$ the K-user MAC sum-capacity gives $d(\alpha = 1) = 1$. This proves the converse part of Th. 5.

B. Achievable Rate Region for K-CIFC CMS

We now present a scheme which will be used in Section V-C to show that the symmetric outer bound derived in Section V-A is achievable to within a constant gap.

Inspired by the capacity achieving strategy for the Gaussian MIMO-BC, we introduce a scheme that uses Dirty Paper Coding (DPC) with encoding order $1 \to 2 \to 3 \to \cdots K$. We denote by $\mathbf{\Sigma}_\ell$ the covariance matrix corresponding to the message intended for decoder ℓ, $\ell \in [1:K]$, as transmitted across the K antennas/transmitters. The overall input covariance matrix is

$$\mathrm{Cov}[X_1, \ldots, X_K] = \sum_{\ell=1}^{K} \mathbf{\Sigma}_\ell : \left[\sum_{\ell=1}^{K} \mathbf{\Sigma}_\ell\right]_{k,k} \leq 1,\ k \in [1:K], \quad (16a)$$

where the constraints on the diagonal elements correspond to the input power constraints. Moreover, since message ℓ can only be broadcasted by transmitters with index larger than ℓ, we further impose

$$\left[\mathbf{\Sigma}_\ell\right]_{k,k} = 0 \text{ for all } 1 \leq k < \ell \leq K. \quad (16b)$$

The achievable rate region is then the set of non-negative rates $(R_1, \ldots, R_K)$ that satisfy

$$R_\ell \le \log\left(1 + \frac{\mathbf{h}_\ell^\dagger \mathbf{\Sigma}_\ell \mathbf{h}_\ell}{\mathbf{h}_\ell^\dagger \left(\sum_{k=\ell+1}^{K} \mathbf{\Sigma}_k\right) \mathbf{h}_\ell}\right), \quad (17)$$
$$\mathbf{h}_\ell^\dagger := [h_{\ell,1} h_{\ell,2} \ldots h_{\ell,K}],\ \ell \in [1:K],$$

for all possible $\text{Cov}[X_1, \ldots, X_K]$ complying with (16), with the convention that $\sum_{k=K+1}^{K} \mathbf{\Sigma}_k = 0$.

In particular we consider the transmit signals

$$X_1 = \alpha_1 U_1,$$
$$X_j = \gamma_j U_j + \beta_j U_j^{(\text{ZF})} + \alpha_j U_1,\ j \in [2:K-1],$$
$$X_K = \gamma_K U_K - \beta_K \sum_{j=2}^{K-1} U_j^{(\text{ZF})} + \alpha_K U_1,$$

where $U_\ell, U_\ell^{(\text{ZF})}$ are i.i.d. $\mathcal{N}(0,1), \ell \in [1:K]$, and the coefficients $\{\alpha_1, \alpha_j, \beta_j, \gamma_j\}_{j\in[2:K]}$ are such that

$$|\alpha_1|^2 \le 1,$$
$$|\gamma_j|^2 + |\beta_j|^2 + |\alpha_j|^2 \le 1,\ j \in [2:K-1],$$
$$|\gamma_K|^2 + |\beta_K|^2(K-2) + |\alpha_K|^2 \le 1,$$

in order to satisfy the power constraints. Notice the negative sign for β_K, which we shall use to implement zero-forcing of the aggregate interference $\sum_{j=2}^{K-1} U_j^{(\text{ZF})}$. Moreover, all transmitters cooperate in beam forming U_1 to receiver 1. These two facts can be easily seen by observing that for $\beta_1 = \ldots = \beta_K := \beta$

$$\sum_{\ell=1}^{K} X_\ell \Big|_{\beta_1=\ldots=\beta_K} = \sum_{\ell=1}^{K} \gamma_\ell U_\ell, \quad \gamma_1 := \sum_{\ell=1}^{K} \alpha_\ell.$$

With these choices the message covariance matrices are

$$\mathbf{\Sigma}_1 = \mathbf{a}\mathbf{a}^\dagger,\ \mathbf{a} := [\alpha_1, \ldots, \alpha_K]^T,$$
$$\mathbf{\Sigma}_j = |\gamma_j|^2\ \mathbf{e}_j \mathbf{e}_j^\dagger + |\beta|^2\ (\mathbf{e}_j - \mathbf{e}_K)(\mathbf{e}_j - \mathbf{e}_K)^\dagger,\ j \in [2:K],$$

where $\mathbf{e}_j$ indicates a length-K vector of all zeros except for a one in position j, $j \in [1:K]$, $\dagger$ indicates the Hermitian transpose, and where $\beta = \beta_1 = \ldots = \beta_K$.

We next express the channel vectors $\mathbf{h}_\ell$ for the symmetric Gaussian channel as

$$\mathbf{h}_\ell = (|h_\text{d}| - h_\text{i})\ \mathbf{e}_\ell + h_\text{i}\ \left(\sum_{k=1}^{K} \mathbf{e}_k\right),\ \ell \in [1:K].$$

By noticing that $\mathbf{h}_\ell \mathbf{e}_j^\dagger = \delta[\ell - j](|h_\text{d}| - h_\text{i}) + h_\text{i},\ \ell \in [1:K]$, where $\delta[k]$ is the Kronecker's delta function, the following rates are achievable

$$R_1 = \log\left(1 + \frac{\left||h_\text{d}| + |h_\text{i}| \sum_{j=2}^{K} \alpha_j\right|^2}{1 + |h_\text{i}|^2 \sum_{k=2}^{K} |\gamma_k|^2}\right), \quad (18a)$$

$$R_j = \log\left(1 + \frac{\left||h_\text{d}| - h_\text{i}\right|^2 |\beta|^2 + |h_\text{d}|^2\ |\gamma_j|^2}{1 + |h_\text{i}|^2 \sum_{k=j+1}^{K} |\gamma_k|^2}\right), j \in [2:K-1], \quad (18b)$$

$$R_K = \log\left(1 + |h_\text{d}|^2\ |\gamma_K|^2\right), \quad (18c)$$

where we chose $\alpha_1 = \exp(j\angle h_\text{i})$, notice the phase of α_1, which coherently combines all signals carrying U_1 at receiver 1.

C. Additive Constant Gap Results for the Symmetric Gaussian Channel

We now choose the parameters in (18) so as to match the upper bound in (15). Due to the presence of the term $(K-1)|h_\text{i}|^2$ in the denominator of the equivalent SNR for receiver K, one might be tempted to suggest that the bound in (15c) would mean that the most cognitive user should treat all the other signals as noise. However we recall that user K is the most cognitive user and can therefore "pre-code" the whole interference seen at its receiver using DPC; by doing so, receiver K would not have anything to treat as noise besides the Gaussian noise itself. We therefore interpret the term $\frac{1}{1+(K-1)|h_\text{i}|^2} \le 1$ as the fraction of power transmitter K dedicates to the transmission of its own signal. This amounts to setting

$$|\gamma_K|^2 = \frac{1}{1 + (K-1)|h_\text{i}|^2}$$

in (18c). This choice guarantees that the achievable rate for user K exactly matches the term in (15c) in the outer bound. Next we would like to match the outer bound term in (15b) to the achievable rates in (18b) by setting

$$\gamma_j = 0,\ j \in [2, K-1], \quad \frac{1}{2} = \frac{|\beta|^2}{1 + |h_\text{i}|^2 |\gamma_K|^2}.$$

However, from the power constraint for user K, we must satisfy

$$|\beta|^2 \le \frac{1 - |\gamma_K|^2}{K-2},$$

which imposes the following condition

$$\frac{K-4}{K-2} + \left(|h_\text{i}|^2 + \frac{2}{K-2}\right) |\gamma_K|^2 \le 0.$$

The above condition cannot be satisfied for $K \ge 4$; for $K = 3$ it requires that

$$|\gamma_3|^2 = \frac{1}{1 + 2|h_\text{i}|^2} \le \frac{1}{|h_\text{i}|^2 + 2}$$

which can be satisfied by $|h_\text{i}|^2 \ge 1$. Therefore, in the following we shall assume $|h_\text{i}|^2 \ge 1$ and set $\gamma_j = 0,\ j \in [2, K-1]$ and

$$|\beta|^2 = \begin{cases} \frac{1-|\gamma_K|^2}{K-2} = \\ \frac{1}{K-2}\left(1 - \frac{1}{1+(K-1)|h_\text{i}|^2}\right) & K \ge 4 \\ \frac{1+|h_\text{i}|^2|\gamma_3|^2}{2} = \frac{1+3|h_\text{i}|^2}{2(1+2|h_\text{i}|^2)} & K = 3 \end{cases},$$

which implies

$$|\alpha_K|^2 = \begin{cases} 0 & K \ge 4 \\ 1 - |\beta|^2 - |\gamma_K|^2 = \frac{-1+|h_\text{i}|^2}{2(1+2|h_\text{i}|^2)} & K = 3 \end{cases}.$$

Finally, for $j \in [2:K-1]$

$$|\alpha_j|^2 = 1-|\beta_j|^2 = \begin{cases} \frac{K-3}{K-2} + \frac{1}{K-2}\ \frac{1}{1+(K-1)|h_\mathrm{i}|^2} & K \geq 4 \\ \frac{1+|h_\mathrm{i}|^2}{2(1+2|h_\mathrm{i}|^2)} & K = 3 \end{cases}.$$

The rates then become: for $K \geq 4$

$$R_K = \log\left(1 + \frac{|h_\mathrm{d}|^2}{1+(K-1)|h_\mathrm{i}|^2}\right)$$

$$R_j = \log\left(1 + \big||h_\mathrm{d}| - h_\mathrm{i}\big|^2 \frac{\frac{1}{K-2}\ \frac{(K-1)|h_\mathrm{i}|^2}{1+(K-1)|h_\mathrm{i}|^2}}{1+\frac{|h_\mathrm{i}|^2}{1+(K-1)|h_\mathrm{i}|^2}}\right),$$

$$\geq \log\left(1 + \frac{\big||h_\mathrm{d}| - h_\mathrm{i}\big|^2}{K-2}\frac{K-1}{K+1}\right),\ j \in [2:K-1],$$

since $|h_\mathrm{i}|^2 \geq 1,$ and for $K = 3$

$$R_1 = \log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}|\sqrt{(K-3)(K-2) + \frac{K-2}{1+(K-1)|h_\mathrm{i}|^2}}\right|^2}{1+\frac{|h_\mathrm{i}|^2}{1+(K-1)|h_\mathrm{i}|^2}}\right)$$

$$\geq \log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}|\sqrt{(K-3)(K-2)}\right|^2}{2}\right),$$

since $|h_\mathrm{i}|^2 \geq 1,$

$$R_3 = \log\left(1 + \frac{|h_\mathrm{d}|^2}{1+2|h_\mathrm{i}|^2}\right), R_2 = \log\left(1 + \big||h_\mathrm{d}| - h_\mathrm{i}\big|^2\frac{1}{2}\right)$$

$$R_1 = \log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}|\ \left(\sqrt{\frac{1+|h_\mathrm{i}|^2}{2(1+2|h_\mathrm{i}|^2)}} + \sqrt{\frac{-1+|h_\mathrm{i}|^2}{2(1+2|h_\mathrm{i}|^2)}}\right)\right|^2}{1+\frac{|h_\mathrm{i}|^2}{1+2|h_\mathrm{i}|^2}}\right)$$

$$\geq \log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}|\frac{1}{2}\right|^2}{2}\right),\ \text{since } |h_\mathrm{i}|^2 \geq 1.$$

By taking the difference between the outer bound in (15) and the lower bounds on the derived achievable rates we find that the gap is upper bounded by: for $K \geq 4$

$$\mathsf{GAP} \leq (K-2)\log(2) + (K-2)$$
$$\left(\log\left(1 + \frac{\big||h_\mathrm{d}| - h_\mathrm{i}\big|^2}{2}\right) - \log\left(1 + \frac{\big||h_\mathrm{d}| - h_\mathrm{i}\big|^2}{K-2}\frac{K-1}{K+1}\right)\right)$$
$$+ \log\left(1 + \Big(|h_\mathrm{d}| + (K-1)|h_\mathrm{i}|\Big)^2\right)$$
$$- \log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}|\sqrt{(K-3)(K-2)}\right|^2}{2}\right)$$
$$\leq (K-2)\log(2) + (K-2)\log\left(\frac{(K+1)(K-2)}{2(K-1)}\right)$$
$$+ \log\left(\frac{2(K-3)(K-2)}{(K-1)^2}\right) \leq (K-2)\log(K-2) +$$
$$\log(2\exp(2)),$$

(where we used $K\log_\mathrm{e}(1+1/K) \leq 1$) and for $K \geq 3$

$$\mathsf{GAP} \leq \log(2) + \log\left(1 + \Big(|h_\mathrm{d}| + 2|h_\mathrm{i}|\Big)^2\right) -$$

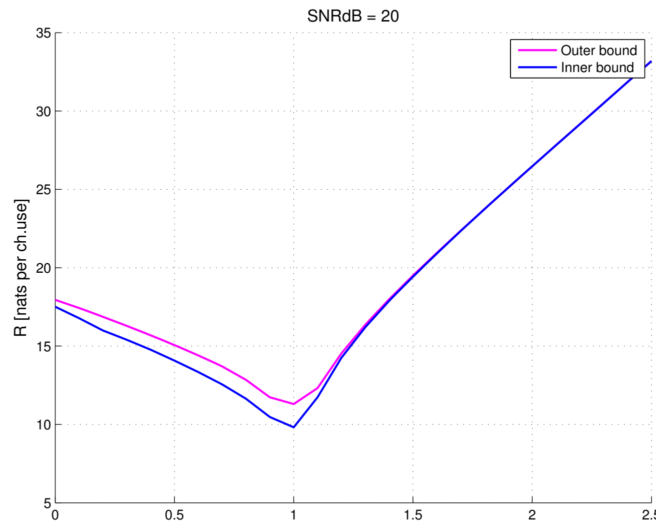

Fig. 4. Comparison of the numerically optimized inner and outer bounds for $K = 3$ users at SNR= 20dB as a function of $\alpha = \frac{\log(|h_\mathrm{d}|)}{\log(|h_\mathrm{i}|)}$; notice a smaller gap than the worst case predicted 6 bits per channel use.

$$\log\left(1 + \frac{\left||h_\mathrm{d}| + |h_\mathrm{i}|\frac{1}{2}\right|^2}{2}\right) \leq 6\log(2).$$

For $|h_\mathrm{i}|^2 < 1$, we set $\beta_j = \alpha_j = 0, \gamma_j = 1$ for $j \in [2:K]$ to obtain

$$\sum_{\ell=1}^{K} R_\ell = \sum_{\ell=1}^{K} \log\left(1 + \frac{|h_\mathrm{d}|^2}{1+(K-\ell)|h_\mathrm{i}|^2}\right).$$

The gap to the outer bound is at most

$$\mathsf{GAP} \leq (K-2)\log(2) + 2\log(K-1) + \sum_{\ell=2}^{K-1}\log\left(\frac{K-\ell}{2}\right),$$

which is smaller than the gap previously obtained for $|h_\mathrm{i}|^2 \geq 1$. This proves Th. 6 and the direct part of Th. 5.

D. Multiplicative Gap Result for the Symmetric Gaussian Channel

In order to provide a complete characterization of the sum-capacity of the symmetric Gaussian channel we next consider approximating the sum-capacity to within a multiplicative gap, which is more relevant at low SNR than an additive gaps. To this end, note that the rate of user j is upper bounded by $C_j := \log(1 + (|h_\mathrm{d}| + (K-j)|h_\mathrm{i}|)^2), j \in [1:K]$ which in turn is upper bounded by $K \times C_1$. Consider an achievability scheme in which all users beamform to user 1: this achieves the sum-rate $R_1 + \cdots R_K = C_1$. This is to within a factor K of the upper bound, proving Th. 7.

E. Numerical Optimization of Inner and Outer Bounds for the Symmetric 3-user Case

Fig. 4 shows the proposed upper and lower bounds for the symmetric channel with $K = 3$ users at SNR= 20dB. In this case the outer and lower bounds where optimized numerically so as to obtain a larger achievable rate and a tighter outer

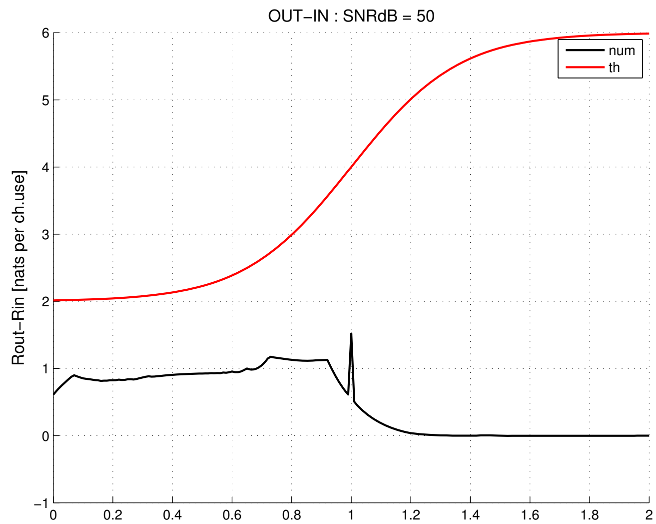

Fig. 5. Analytical and numerical additive gaps for $K = 3$ users at SNR= 50dB.

bound than those used for the analytical evaluation of the gap. We notice that the gap between the bounds is much less than the theoretical gap of 6 bits. In particular, for strong interference the bounds are extremely close to one another, showing again that the analytically provable gap of 6 bits is a worst case scenario, which is the result of crude bounding techniques rather than a poor achievability scheme. Fig. 5 shows the additive gap for $K = 3$ users at SNR= 50dB; notice the gap between the analytical upper and lower bounds (curve labeled 'th') converging to 6 bits for large α while the gap between the numerically optimized upper and lower bounds (curve labeled 'num') going to zero in the same regime; the largest gap is at $\alpha = 1$ where the channel matrix becomes rank deficient; overall the gap is at most around 1 bit, 5 bits smaller than the analytically provable gap.

VI. Conclusion

In this paper we studied the K-user cognitive interference channel with cumulative message sharing. A computable, general outer bound valid for any number of users and any memoryless channel is obtained. For the linear deterministic approximation of the Gaussian channel at high SNR we obtained the sum-capacity for all channel gains in the case of three users, and the symmetric sum-capacity for any K. For the Gaussian channel, we provided a unified achievability scheme which achieves the sum-capacity to within a constant additive and multiplicative gap. In the linear deterministic channel, the sum-capacity was achieved by a scheme which only required cognition at one single user. This begs the question of whether, for the Gaussian channel, one may achieve to within a constant gap of capacity by only having one fully cognitive user; our current achievability scheme does require cognition at intermediate transmitters for dirty paper coding.

Appendix A
Proof of Th. 1

By Fano's inequality $H(W_i|Y_i^N) \leq N\epsilon_N$ with $\epsilon_N \to 0$ as $N \to \infty$ for all $i \in [1:3]$. The bounds in equation (6a) through (6c) are a simple application of the cut-set bound. The bound in (6d) is obtained as follows:

$$\begin{aligned}
&N(R_2+R_3-2\epsilon_N) \overset{(a)}{\leq} I(Y_2^N;W_2)+I(Y_3^N;W_3)\\
&\overset{(b)}{\leq} I(Y_2^N,W_1;W_2)+I(Y_3^N,Y_2^N,W_1,W_2;W_3)\\
&\overset{(c)}{=} I(Y_2^N;W_2|W_1)+I(Y_3^N,Y_2^N;W_3|W_1,W_2)\\
&\overset{(d)}{=} I(Y_2^N;W_2|W_1)+I(Y_2^N;W_3|W_1,W_2)\\
&\quad +I(Y_3^N;W_3|W_1,W_2,Y_2^N)\\
&\overset{(e)}{=} I(Y_2^N;W_2,W_3|W_1)+I(Y_3^N;W_3|W_1,W_2,Y_2^N)\\
&\overset{(f)}{=} I(Y_2^N;W_2,W_3|W_1,X_1^N)\\
&\quad +I(Y_3^N;W_3|W_1,W_2,Y_2^N,X_1^N,X_2^N)\\
&\overset{(g)}{\leq} \sum_{t=1}^{N} H(Y_{2,t}|X_{1,t})-H(Y_{2,t}|X_{1,t},X_{2,t},X_{3,t})\\
&\quad +H(Y_{3,t}|X_{1,t},X_{2,t})-H(Y_{3,t}|X_{1,t},X_{2,t},X_{3,t})\\
&\overset{(h)}{=} \sum_{t=1}^{N} I(Y_{2,t};X_{2,t},X_{3,t}|X_{1,t})+I(Y_{3,t};X_{3,t}|X_{1,t},X_{2,t}),
\end{aligned}$$

where (a) follows from Fano's inequality, (b) the non-negativity of mutual information, (c) from the independence of the messages, (d) and (e) from the chain rule (note the side information allows one to recombine different entropy terms), (f) because the inputs are deterministic functions of the messages, (g) follows since conditioning reduces entropy, and (h) definition of mutual information. Using similar steps (give enough messages to reconstruct the inputs, and give outputs to recombine terms by using the chain rule of mutual information) we obtain the bound in (6e) at the top of the next page.

Appendix B
Proof of Th. 2

By Fano's inequality $H(W_i|Y_i^N) \leq N\epsilon_N$ with $\epsilon_N \to 0$ as $N \to \infty$ for all $i \in [1:K]$. For (7a) we have the equation at the top of the next page.

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$$
\begin{aligned}
&N(R_1+R_2+R_3-3\epsilon_N) \leq I(Y_1^N;W_1)+I(Y_2^N;W_2)+I(Y_3^N;W_3)\\
&\leq I(Y_1^N;W_1)+I(Y_2^N,Y_1^N,W_1;W_2)+I(Y_3^N,Y_1^N,W_1,Y_2^N,W_2;W_3)\\
&\leq I(Y_1^N;W_1,W_2,W_3)+I(Y_2^N;W_2,W_3|Y_1^N,W_1)+I(Y_3^N;W_3|Y_1^N,W_1,Y_2^N,W_2)\\
&\leq \sum_{t=1}^{N} I(Y_{1,t};X_{1,t},X_{2,t},X_{3,t})+I(Y_{2,t};X_{2,t},X_{3,t}|X_{1,t},Y_{1,t})+I(Y_{3,t};X_{3,t}|X_{1,t},X_{2,t},Y_{1,t},Y_{2,t})
\end{aligned}
$$

For (7a) we have

$$
\begin{aligned}
&N(R_i-\epsilon_N) \leq I(Y_i^N;W_i) \leq I(Y_i^N;W_i|W_1,\ldots,W_{i-1}) \leq \sum_{t=1}^{N} h(Y_{i,t}|W_1,\ldots,W_{i-1})-h(Y_{i,t}|W_1,\ldots,W_K,Y_i^{t-1})\\
&\leq \sum_{t=1}^{N} h(Y_{i,t}|X_{1,t},\ldots,X_{i-1,t})-h(Y_{i,t}|X_{1,t},\ldots,X_{K,t}) = \sum_{t=1}^{N} I(Y_{i,t};X_{i,t},\ldots,X_{K,t}|X_{1,t},\ldots,X_{i-1,t}).
\end{aligned}
$$

For (7b) we have

$$
\begin{aligned}
&N\sum_{j=i}^{K}(R_j-\epsilon_N) \leq \sum_{j=i}^{K} I(Y_j^N;W_j) \leq \sum_{j=i}^{K} I(Y_j^N,\ \underbrace{W_1,\ldots,W_{i-1}}_{=\emptyset \text{ for } i=1},\ \underbrace{Y_i^N,W_i,\ldots Y_{j-1}^N,W_{j-1}}_{=\emptyset \text{ for } j=i};W_j)\\
&= \sum_{j=i}^{K} I(Y_i^N,\ldots Y_j^N;W_j|W_1,\ldots,W_{j-1}) = \sum_{j=i}^{K}\sum_{k=i}^{j} I(Y_k^N;W_j|W_1,\ldots,W_{j-1},\ Y_i^N,\ldots,Y_{k-1}^N)\\
&= \sum_{k=i}^{K} I(Y_k^N;W_k,\ldots,W_K|\underbrace{W_1,\ldots,W_{i-1}}_{=\emptyset \text{ for } i=1},\ \underbrace{W_i,Y_i^N,\ldots,W_{k-1},Y_{k-1}^N}_{=\emptyset \text{ for } k=i})\\
&\leq \sum_{k=i}^{K}\sum_{t=1}^{N} I(Y_{k,t};X_{k,t},\ldots,X_{K,t}|X_{1,t},\ldots,X_{k-1,t},Y_{i,t},\ldots,Y_{k-1,t}).
\end{aligned}
$$

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