Maximum Likelihood Time Delay Estimation and Cramér-Rao Bounds for Multipath Exploitation

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***Abstract*—In this paper, time delay estimation using the maximum likelihood principle is addressed for the multipath exploitation problem, and the corresponding Cramér-Rao bounds are derived. A single wideband radar, and a target in a known reflecting geometry are assumed. If the multipath is indeed detectable and resolvable, it is shown here that multipath exploitation, firstly, permits estimating the angle of arrival (AoA) of the target with a single sensor, and secondly, improves estimation accuracy of the direct path time delay. Both these are possible because the multipath's time delay is a deterministic function of the time delay of the direct path as well as its AoA, as is demonstrated here. The multipath caused from reflections from surfaces yields virtual radar sensors observing the target from different aspects, thereby allowing AoA estimation.**

I. Introduction

The objective in the multipath exploitation radar is improving the radar system performance by incorporating the additional information, about either targets or their environments, embedded in the multipath returns. The multipath exploitation hypothesis rests on the fact that "multipath exists because of the environment," which in turn requires that multipath returns are distinguishable.

In this paper, a single wideband radar sensor observes a target in *a priori* known reflecting geometry, consisting of a ground plane. Accordingly, the multipath returns are caused by specular reflections of the radar signal from a smooth surface, an assumption seen for example in [1] - [5] and references therein.

The novelty of this approach is that, using a ray tracing analysis [7], the multipath time delay is parameterized as a function of the geometrical direct path time-delay and its AoA; in particular, this approach is applicable even when the direct path is obstructed. Since multipath time delay on its own is not directly useful, by employing this parametrization, the maximum likelihood estimator (MLE) and the Cramér-Rao lower bounds (CRLB) are derived for estimating the direct path time delay as well as its AoA. The CRLBs are derived in the frequency domain, and are shown to be a function of the SNR as well as the operating bandwidth.

The multipath exploitation problem has been studied in the recent past in, *e.g.* [1] - [6], and references therein. In [5], [6], detection using the generalized likelihood ratio test (GLRT) was employed for the multipath exploitation problem, assuming knowledge of the multipath and direct path time delays, obtained from a priori knowledge of the environment where the radar operates. A multipath model and exploitation technique is addressed in [3], which properly detects and utilizes the target ghosts in through-the-wall and urban radar sensing applications. Using the multipath exploitation, authors of [4] also demonstrated that localization can be achieved with a single sensor. Examples of targets in urban canyons and through-the-wall radar were employed to demonstrate non-coherent localization. Target tracking and ground moving target indication (GMTI) applications of multipath exploitation were explored in [2], and [1], respectively.

The paper is organized as follows, in Section II the model is presented, and in Section III the problem is presented formally. The maximum likelihood (ML) technique and the CRLB are presented in Sections VI and V, respectively. Representative simulation results and conclusions are presented in Section VI and VII.

II. Multipath Propagation Model

In this Section we describe the radar-target scenario that involves the multipath propagation. The geometry of the radar-target environment is illustrated in Fig. 1. We formulate the mathematical expression for the propagation model of the radar scene using ray-tracing techniques. The advantage of the ray-tracing approach is that each individual trajectory is explicitly associated with all the mechanisms of wave propagation so that a clear description of all the physical phenomena is available, [7]. A two-ray model is considered at first to remain tractable. The radar is assumed to be located at the center of the polar coordinate system. The transmitted pulse is assumed to be

$$s(t) = \begin{cases} \frac{1}{\sqrt{T_d}} & 0 \le t \le T_d \\ 0 & \text{Otherwise} \end{cases}$$

so that the received signal is given as

$$r(t) = \alpha_1(t)s(t-\tau_1) + \alpha_2(t)s(t-\tau_2) + w(t),$$

where $r(t)$, $s(t)$ and $w(t)$ are the baseband equivalents of the received signal, transmitted signal and noise, respectively. Parameters $\alpha_1(t)$ and $\alpha_2(t)$, which are complex and deterministic, are the strengths of the direct and reflected multipath returns, of time delays τ_1 and τ_2, respectively.

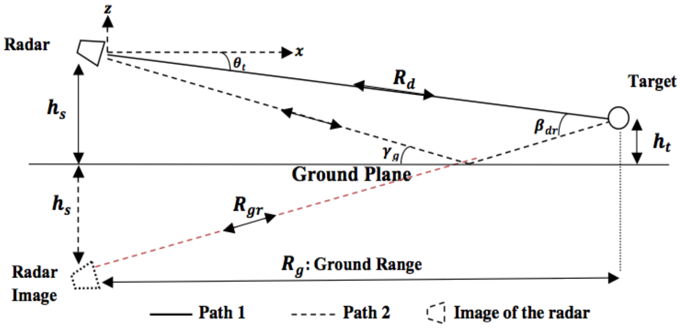

Fig. 1. Geometry of the problem: Radar-Target over a Ground Plane

III. Problem Formulation

In this section, we assume $w(t)$ is a stationary zero-mean complex white Gaussian random process with power spectral density σ^2. Since the pulse duration, T_d, is considered small compared to the coherence time of the radar-target channel, $\alpha_1(t)$ and $\alpha_2(t)$ are approximated with unknown complex deterministic parameters α_1 and α_2 respectively. Then, $r(t)$ can be written as, [5],

$$r(t) = \alpha_1 s(t-\tau_1) + \alpha_2 s(t-\tau_2) + w(t).$$

In terms of geometric parameters, time delays τ_1 and τ_2 are obtained as

$$\tau_1 = \frac{2R_d}{c} \quad \text{and} \quad \tau_2 = \frac{2R_{gr}}{c},$$

where R_d and R_{gr} are the ranges of target with respect to the radar and radar image respectively. Furthermore τ_2 can be written as a function of τ_1 and θ_t with *a priori* knowledge of h_s, which is the height of the radar above the planar reflecting surface,

$$\tau_2 = g(\tau_1, \theta_t) = \sqrt{(\tau_1 \cos\theta_t)^2 + (4h_s/c + \tau_1 \sin\theta_t)^2}.$$

Thus, for the estimation problem, the received signal can be written as

$$\begin{aligned} r(t) &= \alpha_1 s(t-\tau_1) + \alpha_2 s(t - g(\tau_1, \theta_t)) + w(t) \\ &= s_1(t,\ \mathbf{\Theta}) + s_2(t,\ \mathbf{\Theta}) + w(t), \end{aligned}$$

where $t \in [0, T_o]$ is the observation interval and $\mathbf{\Theta} := [\tau_1,\ \theta_t,\ \alpha_1,\ \alpha_2]^T$ is the vector of parameters to be estimated.

The novelty of this approach is that we estimate two geometrical parameters, $[\tau_1, \theta_t]^T$ with a single sensor by exploiting the multipath and *a priori* knowledge of the reflecting environment. In other words, h_s is assumed to be known.

IV. Maximum Likelihood Estimation

The MLE formulation adopted here is similar to the one taken in [8], but unlike our approach, the authors of [8] estimate the multipath time delay for multipath mitigation in global positioning systems (GPS). The log-likelihood function that needs to be maximized with respect to $\mathbf{\Theta}$, and is readily shown to be

$$\ln \Lambda[r(t), \mathbf{\Theta}] \propto -\frac{1}{\sigma^2}\int_0^{T_o} |r(t) - s_1(t,\mathbf{\Theta}) - s_2(t,\mathbf{\Theta})|^2\, dt \tag{1}$$

In general, for an efficient unbiased estimator we must have [9], [10],

$$\frac{\partial \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \Theta_i} = [\hat{\Theta}_i(r(t)) - \Theta_i] J_{ii}(\Theta_i), \tag{2}$$

where J_{ij} is the (i,j)th element of the Fisher information matrix (FIM) $\mathbf{J}$ as in (4), $\hat{\Theta}_i$ is the ith element of the estimator vector $\hat{\mathbf{\Theta}}$ which is a function of the received data, whereas Θ_i is the i^{th} element of unknown parameter vector $\mathbf{\Theta}$. In this particular problem the equality (2) does not hold for time-delay τ_1 and angle of arrival θ_t estimation but only for α_1 and α_2, [9], [10]. Nevertheless, the maximum likelihood estimation is considered here due to its asymptotically efficient properties.

As a comparison point, we recall the celebrated Cramér-Rao inequality

$$Var\left(\hat{\Theta}_{ij}(r(t)) - \Theta_{ij}\right) \geq J^{ij} \tag{3}$$

where J^{ij} is defined as the (i,j)-th element of the square matrix $\mathbf{J}^{-1}$ which is the inverse of the FIM $\mathbf{J}$. Elements of $\mathbf{J}$ are defined as, [9],

$$\mathbf{J} = -E\left[\frac{\partial^2 \ln \mathbf{\Lambda}[r(t), \mathbf{\Theta}]}{\partial \mathbf{\Theta} \partial \mathbf{\Theta}^T}\right] \tag{4}$$

where $E[\cdot]$ denotes the statistical expectation operator.

A. *MLE of Amplitudes*

From equation (1) the MLE score for α_1 and α_2 are obtained as

$$\frac{\partial \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \alpha_1} = \frac{1}{\sigma^2}\left[R_{rs}(\tau_1) - \alpha_1^* - \alpha_2^* \Phi(\tau_2, \tau_1)\right], \tag{5}$$

and similarly

$$\frac{\partial \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \alpha_2} = \frac{1}{\sigma^2}\left[R_{rs}(\tau_2) - \alpha_2^* - \alpha_1^* \Phi(\tau_2, \tau_1)\right], \tag{6}$$

where

$$R_{rs}(\tau) = \int_0^{T_o} r(t) s(t-\tau) dt, \tag{7}$$

$$\Phi(\tau_2, \tau_1) = \int_0^{T_o} s(t-\tau_1) s(t-\tau_2) dt. \tag{8}$$

Thus, the ML estimates for α_1 and α_2 are obtained as, [8],

$$\hat{\alpha}_1 = \frac{R_{rs}(\tau_1) - \Phi(\tau_2, \tau_1) R_{rs}(\tau_2)}{1 - \Phi(\tau_2, \tau_1)^2}, \tag{9}$$

$$\hat{\alpha}_2 = \frac{R_{rs}(\tau_2) - \Phi(\tau_2, \tau_1) R_{rs}(\tau_1)}{1 - \Phi(\tau_2, \tau_1)^2}. \tag{10}$$

Here $\hat{\alpha}_1$ and $\hat{\alpha}_2$ are unbiased efficient estimators that satisfy the equality (2).

B. Estimation of Time Delay τ_1 and Angle of Arrival θ_t

In this section we derive the MLE equations for τ_1 and θ_t. In this case the estimation problem is not linear anymore as in the amplitude estimation. Although there is no efficient unbiased estimator for τ_1 and θ_t, MLE can be implemented numerically where it is asymptotically unbiased and efficient.

The ML score for τ_1 is found as

$$\frac{\partial \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \tau_1} = \frac{2}{\sigma^2} \Re \Bigg\{ \int_0^{T_o} \{r(t) - [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]\} \times \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]^*}{\partial \tau_1} dt \Bigg\}. \tag{11}$$

In a similar manner, the ML score for θ_t is found as

$$\frac{\partial \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \theta_t} = \frac{2}{\sigma^2} \Re \Bigg\{ \int_0^{T_o} \{r(t) - [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]\} \times \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]^*}{\partial \theta_t} dt \Bigg\}. \tag{12}$$

One can obtain the necessary condition for $\hat{\tau}_{1_{ml}}$ and $\hat{\theta}_{t_{ml}}$ respectively by making the right hand side of the equations (11) and (12) equal to zero.

In order to concentrate the likelihood function (1) on τ_1 and θ_t we insert $\hat{\alpha}_1$ and $\hat{\alpha}_2$, which are given in (9) and (10), into the likelihood function and maximize the resulting likelihood function with respect to τ_1 and θ_t as

$$\begin{aligned} &\max_{\tau_1, \theta_t} \Big(\ln \Lambda[r(t), \mathbf{\Theta}] \Big) \\ &= \max_{\tau_1, \theta_t} \left[\frac{-1}{\sigma^2} \int_0^{T_o} \left| r(t) - \sum_{i=1}^{2} \hat{\alpha}_i s(t - \tau_i) \right|^2 dt \right], \end{aligned} \tag{13}$$

where $\tau_2 = g(\tau_1, \theta_t)$. Closed form expressions for (11)-(13) are intractable and the MLE must be evaluated numerically.

V. Cramér Rao Lower Bound

In this section the CRLB for the estimates of τ_1 and θ_t are derived. Here, we assume the perfect knowledge of the noise variance σ^2, α_1 and α_2. In order to assess the CRLB (3) for the estimates, we compute elements of the FIM, $\mathbf{J}$, via (4) and evaluate $\mathbf{J}^{-1}$ numerically.

For τ_1 we differentiate (11) and take the expectation as

$$\begin{aligned} E\left[\frac{\partial^2 \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \tau_1^2}\right] = \\ \frac{2}{\sigma^2} \Re \Bigg\{ E\Bigg[\int_0^{T_o} \{r(t) - [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]\} \\ \times \frac{\partial^2 [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]^*}{\partial \tau_1^2} dt \Bigg] \\ - E\left[\int_0^{T_o} \left| \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]}{\partial \tau_1} \right|^2 dt \right] \Bigg\}. \end{aligned}$$

In the first term one can observe that

$$E\left[r(t) - \{s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})\}\right] = E\left[w(t)\right] = 0.$$

The second term is a non-random term, thus

$$E\left[\frac{\partial^2 \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \tau_1^2}\right] = \frac{-2}{\sigma^2} \int_0^{T_o} \left| \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]}{\partial \tau_1} \right|^2 dt.$$

In a similar manner,

$$E\left[\frac{\partial^2 \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \theta_t^2}\right] = \frac{-2}{\sigma^2} \int_0^{T_o} \left| \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]}{\partial \theta_t} \right|^2 dt.$$

Then J_{11} and J_{22} can be written respectively as

$$\begin{aligned} J_{11} &= -E\left[\frac{\partial^2 \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \tau_1^2}\right] \\ &= \frac{2}{\sigma^2} \int_0^{T_o} \left| \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]}{\partial \tau_1} \right|^2 dt, \end{aligned} \tag{14}$$

and

$$\begin{aligned} J_{22} &= -E\left[\frac{\partial^2 \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \theta_t^2}\right] \\ &= \frac{2}{\sigma^2} \int_0^{T_o} \left| \frac{\partial [s_1(t, \mathbf{\Theta}) + s_2(t, \mathbf{\Theta})]}{\partial \theta_t} \right|^2 dt. \end{aligned} \tag{15}$$

It is also noted that J_{11} in (14) which is exploiting the multipath is always greater than the FIM element J_{11} in [8], which considers the multipath to be independent of the direct path, and shown below.

$$J_{11} = -E\left[\frac{\partial^2 \ln \Lambda[r(t), \mathbf{\Theta}]}{\partial \tau_1^2}\right] = \frac{2}{\sigma^2} \int_0^{T_o} \left| \frac{\partial s_1(t, \mathbf{\Theta})}{\partial \tau_1} \right|^2 dt.$$

This implies that, for this geometry and assumption, multipath exploitation improves the accuracy of τ_1 estimates, at least in the CRLB sense.

Through mathematical manipulations one can write J_{11} and J_{22} in a more explicit form respectively as

$$\begin{aligned} J_{11} = \frac{2}{\sigma^2} \Bigg\{ \left(|\alpha_1|^2 + |\alpha_2|^2 F_1^2\right) \int_{-\infty}^{\infty} (2\pi f)^2 |S(f)|^2 df \\ + 2\Re\left\{ \alpha_1 \alpha_2^* F_1 \int_{-\infty}^{\infty} (2\pi f)^2 e^{-j2\pi f(\tau_1 - \tau_2)} |S(f)|^2 df \right\} \Bigg\} \end{aligned} \tag{16}$$

and

$$J_{22} = \frac{2}{\sigma^2} |\alpha_2|^2 F_2^2 \int_{-\infty}^{\infty} (2\pi f)^2 |S(f)|^2 \, df \tag{17}$$

where

$$\begin{aligned} F_1 &= \frac{\partial \tau_2}{\partial \tau_1} = \frac{\tau_1 + 4 \sin\theta_t h_s / c}{\sqrt{\left(\tau_1 \cos\theta_t\right)^2 + \left(4h_s/c + \tau_1 \sin\theta_t\right)^2}} \\ F_2 &= \frac{\partial \tau_2}{\partial \theta_t} = \frac{4 h_s \tau_1 \cos\theta_t / c}{\sqrt{\left(\tau_1 \cos\theta_t\right)^2 + \left(4h_s/c + \tau_1 \sin\theta_t\right)^2}}. \end{aligned}$$

The off-diagonal elements of the FIM are

$$
\begin{aligned}
J_{12} &= -E\left[\frac{\partial^2 \ln \Lambda[r(t), \boldsymbol{\Theta}]}{\partial\tau_1\partial\theta_t}\right] \\
&= -\frac{2}{\sigma^2}\Re\Bigg\{\int_0^{T_o} E\Bigg[-\frac{\partial[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]}{\partial\theta_t}\times \\
&\qquad\qquad \frac{\partial[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]^*}{\partial\tau_1}\Bigg] \\
&\qquad + E\Bigg[\{r(t)-[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]\}\times \\
&\qquad\qquad \frac{\partial^2[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]^*}{\partial\theta_t\partial\tau_1}\Bigg]dt\Bigg\}.
\end{aligned}
\tag{18}
$$

Since

$$
E\left[\{r(t)-[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]\}\frac{\partial^2[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]^*}{\partial\theta_t\partial\tau_1}\right] = 0,
$$

$$
J_{12} = \frac{2}{\sigma^2}\Re\Bigg\{\int_0^{T_o}\frac{\partial[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]}{\partial\theta_t} \times \frac{\partial[s_1(t,\boldsymbol{\Theta})+s_2(t,\boldsymbol{\Theta})]^*}{\partial\tau_1}dt\Bigg\}.
\tag{19}
$$

More explicitly, J_{12} is found as

$$
\begin{aligned}
J_{12} = \frac{2}{\sigma^2}\Re\Bigg\{&\alpha_2\alpha_1^*F_2\int_{-\infty}^{\infty}(2\pi f)^2 e^{-j2\pi f(\tau_2-\tau_1)}|S(f)|^2 df \\
&+ |\alpha_2|^2 F_1F_2\int_{-\infty}^{\infty}(2\pi f)^2|S(f)|^2 df\Bigg\}.
\end{aligned}
\tag{20}
$$

Since the FIM $\mathbf{J}$ is Hermitian symmetric: $J_{21} = J_{12}$.Thus, this completes the elements of FIM of $\boldsymbol{\Theta} := [\tau_1, \theta_t]^T$.

VI. Simulations

In this section, we provide the simulations results for CRLB for τ_1 and θ_t. The actual parameters are assumed to be $\tau_1 = 2R_d/c$ and $\tau_2 = 2R_{gr}$ where $R_d = 26.92\ m$ and $R_{gr} = 194.74\ m$, $\theta_t = -0.2630\ rad$ and $\alpha_1 = \alpha_2 = 1$. The radar is located at $h_s = 100\ m$ above the ground. From our convention negative θ_t implies that the target is below the radar. These values were chosen such that the multipath is resolvable with the direct path. Using (1) the following proves useful in simulating the CRLBS,

$$
S(f) \propto \text{sinc}(fT_d),\ \text{sinc}(x) := \sin(\pi x)/\pi x
$$

Our convention is to let the bandwidth refer to $1/T_d$ instead of the classical $2/T_d$. In all the simulations thrice the Nyquist rate was used in simulating the rectangular radar pulses.

In Fig. 2 the CRLB on τ_1 is shown when multipath is exploited as well as when it is not, for varying radar bandwidths. In other words, we compare the $CRLB(\tau_1)$ derived here and denoted as $CRLB(\tau_1)-exploited$ to the one derived in [8] but treating τ_2 independent of τ_1 and denoted as $CRLB(\tau_1)-independent$. It is readily seen that through multipath exploitation the CRLB performs much better. For this simulation we choose the noise variance $\sigma^2 = 0.01$ which is 20 dB on both the direct and multipath returns. The bandwidths are chosen starting from 1 MHz to 1000 MHz in multiplicative increments of 10 MHz.

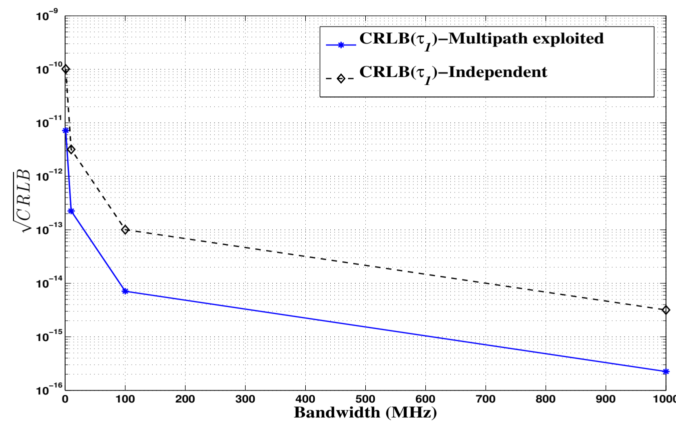

Fig. 2. CLRB comparison

In Fig. 3 the CRLB is shown for varying bandwidths starting from 1 MHz to 1000 MHz in multiplicative increments of 10 MHz. It is readily seen that the CRLB decreases with increasing bandwidths. For this simulation, noise variance, $\sigma^2 = 1$.

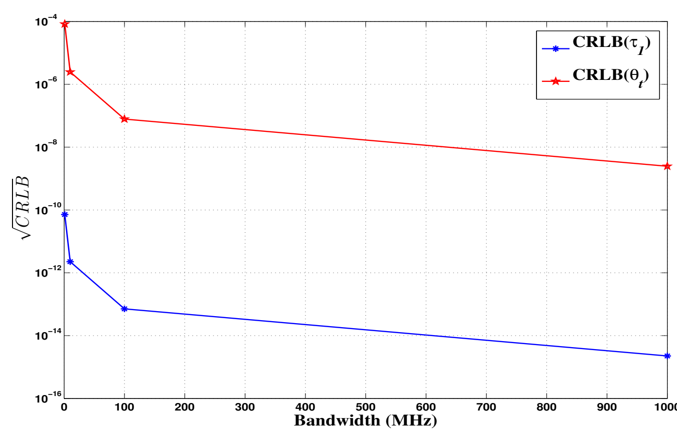

Fig. 3. CRLB vs Bandwidth

In Fig. 4 the CRLB for τ_1 and θ_t are shown for varying noise variance, σ^2. As expected the CRLB increases with increasing σ^2. For this simulation, bandwidth is 10 MHz.

It is well known that the CRLB for time-delay estimation is highly optimistic. Previous studies have shown that the MLE performance for time-delay estimation is much farther

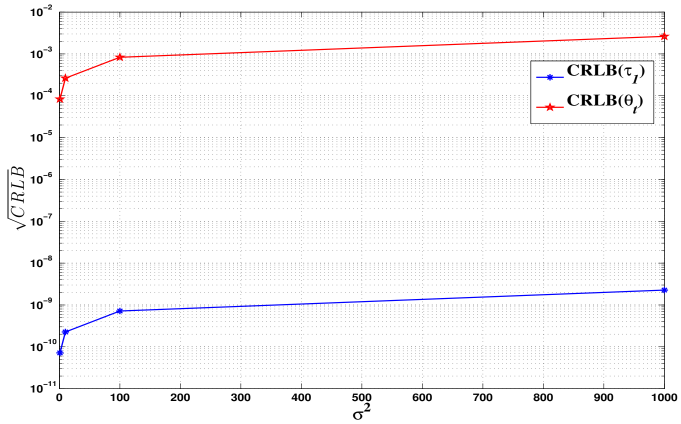

Fig. 4. CLRB vs σ^2

away from the CRLB at low SNRs, see for example [11] and references therein. The MLE converges to the CLRB only at reasonable SNRs. This behavior of the MLE for time-delay estimation has prompted the use of other tighter variance bounds such as the Barankin and Ziv-Zakai bounds which have shown to be much tighter than the CRLB. It remains to be seen however, if the multipath exploited MLE performance is much closer to the multipath exploited CRLB derived here, than their traditional counterpart.

VII. Conclusion

Maximum likelihood and the Cramér-Rao lower bounds were derived for the multipath exploitation problem. A single wideband radar, and a target in a known reflecting geometry were assumed. It was shown here that multipath exploitation offers two advantages, it firstly allows estimation of the AoA, and secondly improves the estimation of the direct path time delay in the CRLB sense, and was shown analytically. The former was possible as multipath gave rise to virtual radar sensors, whereas the latter directly followed from parameterizing the multipath time delay as a function of its direct path.

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