

Adaptive waveform scheduling in radar: an information theoretic approach

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ABSTRACT

In this paper, the problem of adaptively selecting radar waveforms from a pre-defined library of waveforms is addressed from an information theoretic perspective. Typically, radars transmit specific waveforms periodically, to obtain for example, the range and Doppler of a target. Although modern radars are capable of transmitting different waveforms during each consecutive period of transmission, it is hitherto unclear as to how these waveforms must be scheduled to best understand the dynamic radar scene. In this paper, a new information theoretic metric – *directed information* – is employed for waveform scheduling, and is shown to incorporate the past radar returns to effectively schedule waveforms. We formulate this waveform scheduling problem in a Gaussian framework, derive the corresponding maximization problem, and illustrate several special cases.

Keywords: Waveform diversity, Cognitive radar, Waveform scheduling, Directed information, Mutual information

1. INTRODUCTION

Active sensing systems such as radar transmit one, or a burst of waveforms to illuminate a scene. While it is envisioned that waveform diversity would aid in better understanding the radar scene,¹ no general consensus yet exists on how to “best” schedule various waveforms. Radar scenes are dynamic, and the radar returns not only contain information about the target, but also contain information about the scene itself, and interactions of the target with the scene. In this paper, waveform scheduling, as is possible using cognitive radar, is addressed. The objective is to adaptively transmit waveforms having incorporated the scene from the prior radar echoes; utilizing *feedback*, and hence closing the loop in cognitive radar sensing. To do so, an information theoretic metric is proposed.

It is argued here that the directed information (DI)^{2–4} between the target impulse response and the radar returns should be maximized when scheduling over multiple epochs. This allows us to naturally incorporate feedback, and in the presence of it, may be a more appropriate measure of the radar-target channel than the more common mutual information (MI) metric. Typical radar scenes comprise clutter, and it is readily acknowledged that EM target-clutter interactions exist.^{5–7} When such interactions are ignored and are not used in feedback, then maximizing the directed information is equivalent to maximizing the mutual information and is demonstrated here.

In the past literature, Bell’s seminal work formalized the relation between information theory and radar waveform design.⁸ Mutual information was employed in designing waveforms for one measurement epoch. Waveform design in the presence of signal dependent interference for one measurement epoch was also treated using MI in.⁹ Previous approaches to waveform scheduling have been proposed using MI, see for example.¹⁰ Other metrics for waveform scheduling in target tracking can be seen in,¹¹ and references therein. Maximizing signal to noise ratio as a metric for waveform design was analytically treated in,^{8,9} and references therein. However, as stated by Bell who in turn summarizes Woodward: “there is no straightforward mathematical framework which implies that maximizing signal to noise ratio ensures maximal information gain”. It is emphasized that we are interested in selecting waveforms from a pre-determined library of waveforms to maximize information gain, and not to design waveforms for this purpose. Nevertheless, our formulation allows for waveform design, when the maximization is performed on the waveforms themselves rather than over the waveform choice.

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2. MODEL

A single complex target is assumed consisting of many point scatterers whose spatial extent spans multiple range cells. Further for simplicity, we assume that we are given a waveform library consisting of two waveforms, $\{s_1(t), s_2(t)\}$, where t indexes continuous time. If waveform $s_i(t)$ is transmitted at the first scheduling instant, the radar return in baseband is

$$y_1^{(i)}(t) = \alpha_1(t) * s_i(t) + \beta_1(t) * s_i(t) + v_1(t), \quad (1)$$

for $t \in [\tau_{\min}, \tau_{\max}]$, are the set of time delays (or equivalently range) under consideration, $*$ denotes the convolution operator and the noise $v_1(t)$ is a zero mean complex stationary Gaussian random processes independent of $\alpha_1(t)$ and $\beta_1(t)$.

The impulse responses, $\alpha_1(t)$ and $\beta_1(t)$ are complex, finite duration and finite energy Gaussian random processes, modelling the reflectivities of the target, and the clutter plus the target interactions with the clutter (or its environment), respectively. Hence $\alpha_1(t)$ and $\beta_1(t)$ may in general be correlated. Multipath scenarios are one example where multiple radar returns manifest due to interactions of the target with the clutter and are therefore generally correlated with the target response. For ease of analysis, we will assume that the processes, $\alpha_1(t), \beta_1(t)$ are locally covariance stationary within their respective temporal supports. A simple example of non-overlapping target and “clutter plus target” responses in the range domain is shown in Fig.1 for a single measurement epoch.

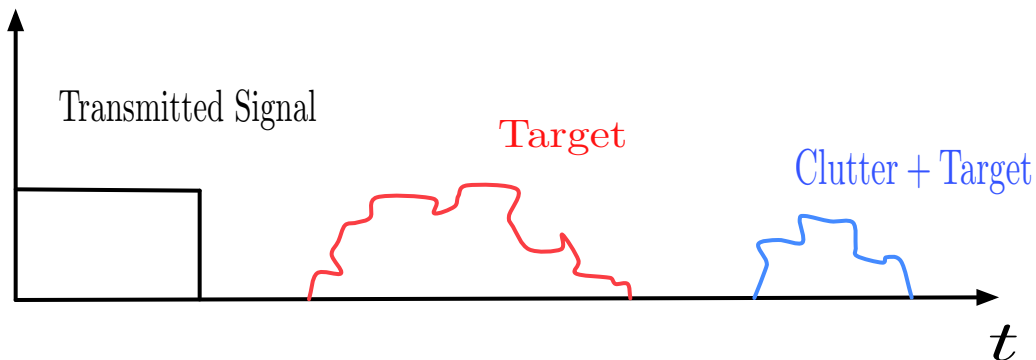


Figure 1: Transmitted and received returns from the target, and clutter and target interactions

If waveform $s_j(t)$ is transmitted in the second epoch, then the radar return can be written similar to (1) as,

$$y_2^{(j)}(t) = \alpha_2(t) * s_j(t) + \beta_2(t) * s_j(t) + v_2(t) \quad (2)$$

for $t \in [T + \tau_{\min}, T + \tau_{\max}]$, where T is the period between the two scheduling epochs, (typically in the order of μs). Here $\alpha_2(t)$ and $\beta_2(t)$ are complex finite energy Gaussian random processes similar to $\alpha_1(t)$ and $\beta_1(t)$ but defined in the second time scheduling instant. The noise in (2) is $v_2(t)$, again assumed to be Gaussian, and it may be correlated with $v_1(t)$. Statistical assumptions on $\alpha_2(t)$ and $\beta_2(t)$ are again possibly correlated Gaussian random processes with assumptions similar to those imposed on $\alpha_1(t)$ and $\beta_1(t)$, and the four impulse responses may in general be correlated. We do distinguish the impulses responses at the first and second scheduling epochs as we allow for moving targets, therefore giving rise to a different radar cross section (RCS) fading process of the target and its interaction with the clutter. As we analyze only two scheduling instants, the Doppler cannot be estimated satisfactorily. Nevertheless, the phase progression arising from the Doppler can be easily absorbed into say $\alpha_m(t), m = 1, 2$. For ease of exposition, we assume that the same set of time delays $[\tau_{\min}, \tau_{\max}]$, are valid for each scheduling instance. In practice they may be different and this may be accounted for in our model. The discrete model is derived next.

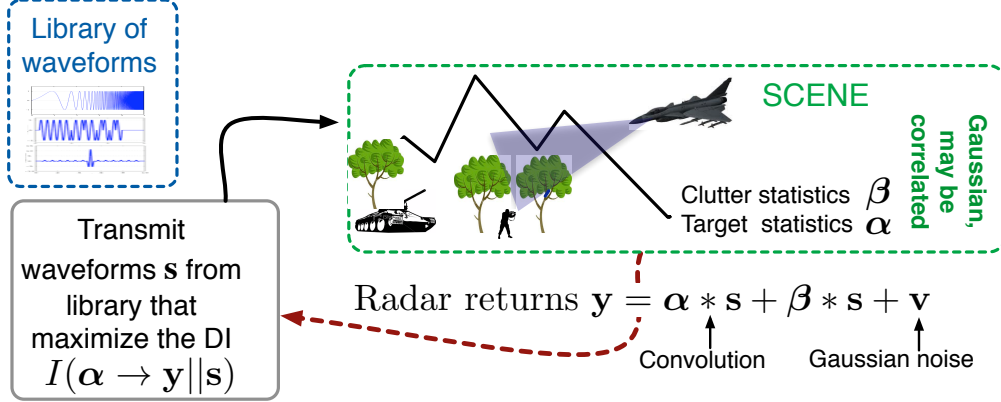


Figure 2: Closed loop radar waveform scheduling to maximize the directed information (DI).

2.1 Discrete model

By sampling the continuous-time model we obtain a discrete model. Without loss of generality, assume that N denotes the data length of both the returns, and there are K discrete scatterers representing the random processes $\alpha_m(t)$ and $\beta_m(t)$, $m = 1, 2$ in each scheduling instant*, then we can write both (1), (2) in an equivalent discrete form as

$$\begin{aligned} \mathbf{y}_1^{(i)} &= \bar{\mathbf{S}}_i [\boldsymbol{\alpha}_1^H, \boldsymbol{\beta}_1^H]^H + \mathbf{v}_1, \quad i = 1, \text{ or } 2 \\ \mathbf{y}_2^{(j)} &= \bar{\mathbf{S}}_j [\boldsymbol{\alpha}_2^H, \boldsymbol{\beta}_2^H]^H + \mathbf{v}_2, \quad j = 1, \text{ or } 2 \end{aligned} \quad (3)$$

where, $\boldsymbol{\alpha}_m = [\alpha_{m1}, \alpha_{m2}, \dots, \alpha_{mK}]^H \in \mathbb{C}^{K \times 1}$ and $\boldsymbol{\beta}_m = [\beta_{m1}, \beta_{m2}, \dots, \beta_{mK}]^H \in \mathbb{C}^{K \times 1}$, $m = 1, 2$ now represent the reflectivities of the scatterers, and the reflectivities of the clutter and target interactions, respectively. We note that the discrete model allows us to consider range cells which are target and target+clutter only. In other words, range cells which consist of noise only contributions are ignored. The matrices, \mathbf{S}_i and \mathbf{S}_j are defined as,

$$\bar{\mathbf{S}}_i := \text{Diag}\{\mathbf{S}_i, \mathbf{S}_i\}, \quad \bar{\mathbf{S}}_j := \text{Diag}\{\mathbf{S}_j, \mathbf{S}_j\},$$

where, $\text{Diag}\{\cdot, \cdot\}$ converts the matrix arguments into a block diagonal matrix. The matrices, \mathbf{S}_i and \mathbf{S}_j consists of the waveform samples, $s_i(\cdot)$ and $s_j(\cdot)$, respectively. Convolution matrices are special cases of \mathbf{S}_i and \mathbf{S}_j , but in general their structure depends on the sparsity of both $\boldsymbol{\alpha}_m$ and $\boldsymbol{\beta}_m$.

The question we seek to answer is: do we transmit waveform $s_1(t)$ or $s_2(t)$ in the first scheduling instant, and likewise transmit $s_1(t)$ or $s_2(t)$ in the second scheduling instant? To answer this, we must state what we seek to maximize when selecting these waveforms over the two slots.

3. WAVEFORM SCHEDULING VIA DIRECTED INFORMATION

Directed information (DI) was derived to analyze the performance of communication systems with feedback.^{2,3} Our goal is to schedule waveforms so as to maximize the amount of information gained over the two scheduling instances. The DI captures the information causally obtained at the receiver about the scene (here captured by $\boldsymbol{\alpha}$) and is the natural choice for maximizing the information gain over multiple time epochs while incorporating the past radar returns. Mutual information on the other hand does not preserve the causality of the information flow.³ However, as we will see, in some relevant scenarios the DI is equal to the (two epoch) MI. See also¹² for other relevant scenarios where DI coincides with the MI.

For now, we will discuss only the relevant properties of DI as it pertains to our radar problem. In particular, we wish to select the waveforms \mathbf{s}_i and \mathbf{s}_j at times 1,2 respectively that will maximize the causally conditioned

*Both N and K may assume different values in the two scheduling instants, but for notational simplicity here we assume they are constant over time.

directed information *between the target responses α and the received signal \mathbf{y}* , $DI(i, j)$, defined for $i, j \in \{1, 2\}$ as

$$\begin{aligned} DI(i, j) &:= I(\alpha \rightarrow \mathbf{y} | \mathbf{s}_i, \mathbf{s}_j) \\ &= I(\alpha_1; \mathbf{y}_1^{(i)} | \mathbf{s}_i) + I(\alpha; \mathbf{y}_2^{(j)} | \mathbf{s}_i, \mathbf{s}_j, \mathbf{y}_1^{(i)}), \end{aligned} \quad (4)$$

where, $\alpha = [\alpha_1^H, \alpha_2^H]^H$, $\mathbf{y} = [\mathbf{y}_1^{(i)H}, \mathbf{y}_2^{(j)H}]^H$, $\mathbf{s}_m \in \mathfrak{R}^{(N+1-K) \times 1}$, $m = 1, 2$ represents the vectors comprising the waveform's samples, and $I(X; Y | Z)$ is the mutual information between (X, Y) conditioned on Z defined in the usual manner, see for example.¹³ The mutual information, in contrast, taken over the two time steps is given by

$$MI(i, j) = I(\alpha; \mathbf{y} | \mathbf{s}_i, \mathbf{s}_j) \quad (5)$$

$$\begin{aligned} &= I(\alpha_1, \alpha_2; \mathbf{y}_1^{(i)} | \mathbf{s}_i, \mathbf{s}_j) + I(\alpha; \mathbf{y}_2^{(j)} | \mathbf{s}_i, \mathbf{s}_j, \mathbf{y}_1^{(i)}), \\ &= DI(i, j) + I(\alpha_2; \mathbf{y}_1^{(i)} | \mathbf{s}_i, \mathbf{s}_j, \alpha_1). \end{aligned} \quad (6)$$

Like the MI, DI is non-negative, but unlike MI, DI is not symmetric in its arguments (α and \mathbf{y} in the above). From the above it is clear that $DI(i, j) \leq MI(i, j)$, and that the mutual information is equal to the DI plus the term $I(\alpha_2; \mathbf{y}_1^{(i)} | \mathbf{s}_i, \mathbf{s}_j, \alpha_1)$. This term may heuristically be interpreted as being “non-causal” in the sense that it contains the mutual information between the current received signal (\mathbf{y}_1) and the target response in the *next* time slot (α_2).

The waveform selection or scheduling criteria is then:

$$(\mathbf{s}_i^*, \mathbf{s}_j^*) = \arg \max_{i, j} DI(i, j) \quad (7)$$

After the first scheduling instant, one has some information about α_1 and access to the returns $\mathbf{y}_1^{(i)}$. This is used to select a waveform in the next time slot which will best illuminate α_2 in order to maximize the net information transfer from the target to the radar over the two time-steps. However, for some cases, maximizing the DI is equivalent to maximizing the MI. From (5) and (6), $MI(i, j) \neq DI(i, j)$ when $I(\alpha_2; \mathbf{y}_1^{(i)} | \mathbf{s}_i, \mathbf{s}_j, \alpha_1) \neq 0$ or when

$$H(\alpha_2 | \alpha_1, \mathbf{y}_1^{(i)}, \mathbf{s}_i, \mathbf{s}_j) \neq H(\alpha_2 | \alpha_1, \mathbf{s}_i, \mathbf{s}_j) \quad (8)$$

This holds when knowing $\mathbf{y}_1^{(i)}$ (in addition to α_1) provides partial additional information about α_2 . Let us denote $\text{Cov}\{\mathbf{x}, \mathbf{y}\}$ as the covariance (matrix) between, \mathbf{x} and \mathbf{y} . Consider the term, $H(\alpha_2 | \alpha_1, \mathbf{y}_1^{(i)}, \mathbf{s}_i, \mathbf{s}_j)$. The following can be shown,

$$\begin{aligned} H(\alpha_2 | \alpha_1, \mathbf{y}_1^{(i)}, \mathbf{s}_i, \mathbf{s}_j) &= \ln \det\{\text{Cov}\{\alpha_2, \alpha_2\} - \mathbf{A}\mathbf{B}\mathbf{A}^H\} + \mathcal{K} \\ \mathbf{A} &= [(\text{Cov}\{\alpha_1, \alpha_2\}^H + \text{Cov}\{\alpha_2, \beta_1\})\mathbf{S}_i^H, \text{Cov}\{\alpha_1, \alpha_2\}^H] \\ \mathbf{B} &= \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{12}^H & \mathbf{B}_{22} \end{bmatrix} \end{aligned} \quad (9)$$

where, \mathcal{K} is some arbitrary constant related to the dimensions of the covariance matrix, and the elements of \mathbf{B} are given by,

$$\begin{aligned} \mathbf{B}_{11} &= \left[\begin{array}{c} \mathbf{S}_i(\text{Cov}\{\alpha_1, \alpha_1\} + \text{Cov}\{\beta_1, \beta_1\} + \text{Cov}\{\alpha_1, \beta_1\} + \text{Cov}\{\alpha_1, \beta_1\}^H)\mathbf{S}_i^H + \text{Cov}\{\mathbf{v}_1, \mathbf{v}_1\} \\ - \mathbf{S}_i(\text{Cov}\{\alpha_1, \alpha_1\} + \text{Cov}\{\alpha_1, \beta_1\})\text{Cov}^{-1}\{\alpha_1, \alpha_1\}(\text{Cov}\{\alpha_1, \alpha_1\} + \text{Cov}\{\alpha_1, \beta_1\}^H)\mathbf{S}_i^H \end{array} \right]^{-1} \\ \mathbf{B}_{22} &= \text{Cov}^{-1}\{\alpha_1, \alpha_1\} + \\ &\quad \text{Cov}^{-1}\{\alpha_1, \alpha_1\}(\text{Cov}\{\alpha_1, \alpha_1\} + \text{Cov}\{\alpha_1, \beta_1\})\mathbf{S}_1^H\mathbf{B}_{11}\mathbf{S}_1(\text{Cov}\{\alpha_1, \alpha_1\} + \text{Cov}\{\alpha_1, \beta_1\}^H)\text{Cov}^{-1}\{\alpha_1, \alpha_1\} \\ \mathbf{B}_{12} &= -\mathbf{B}_{11}\mathbf{S}_1(\text{Cov}\{\alpha_1, \alpha_1\} + \text{Cov}\{\alpha_1, \beta_1\})\text{Cov}^{-1}\{\alpha_1, \alpha_1\} \end{aligned} \quad (10)$$

Using (10) in (9), and after some simplifications it may then be shown that (8) holds when

$$\text{Cov}\{\alpha_1, \beta_1\} \neq \mathbf{0} \text{ and } \text{Cov}\{\alpha_2, \beta_1\} \neq \mathbf{0}, \quad (11)$$

or when the target responses and clutter responses are correlated over both the slots.

4. SPECIAL CASES

In this section we will consider the clutter responses to be independent of the target responses. In the discrete case, it is shown that DI maximization is related to minimizing the Bayesian mean squared error. In the analog model, DI maximization is interpreted in the spectral domain.

4.1 Independent target and clutter responses: discrete

We now consider the special case of when the target is statistically independent of clutter and its interactions with the clutter are not considered, i.e. when $\text{Cov}\{\alpha_1, \beta_1\} = \text{Cov}\{\alpha_2, \beta_1\} = \mathbf{0}$ and hence $MI(i, j) = DI(i, j)$. For brevity, we can now absorb the β_m 's into the noise as they are uncorrelated with the α_m 's. Then, the waveform scheduling criteria becomes,

$$(\mathbf{s}_i^*, \mathbf{s}_j^*) = \arg \max_{i,j} DI(i, j) \quad (12)$$

$$= \arg \min_{i,j} \ln \det\{\text{BMSE}(\alpha|\mathbf{y})\} \quad (13)$$

$$= \arg \min_{i,j} \ln \det\{(\mathbf{C}_\alpha^{-1} + \mathbf{H}^{(ij)H} \mathbf{C}_\mathbf{v}^{-1} \mathbf{H}^{(ij)})^{-1}\}$$

where, BMSE is the minimum Bayesian mean square error,¹⁴ and

$$\mathbf{H}^{(ij)} := \begin{bmatrix} \mathbf{S}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_j \end{bmatrix}$$

$$\mathbf{C}_\alpha := \text{Cov}\{\alpha, \alpha\}, \quad \mathbf{C}_\mathbf{v} = \text{Cov}\{\mathbf{v}, \mathbf{v}\}, \quad \mathbf{v} := [\mathbf{v}_1^H, \mathbf{v}_2^H]^H.$$

We may alternatively view (analog) waveform scheduling (or design) in the spectral domain, which is derived next, while still enforcing independence of target and clutter.

4.2 Independent target and clutter responses: analog

Assume that the radar operates with a bandwidth denoted by W . Let us divide the bandwidth into P consecutive bands each of infinitesimal width denoted by δf . Denote the center frequency of the p -th band as $f_p, p = 1, \dots, P$. Then, consider the following transformation on (1) and (2),

$$y_1^{(i)}(t) = \sum_{p=1}^P y_{1p}^{(i)}(t), \quad y_{1p}^{(i)}(t) := \alpha_{1p}(t) * s_{ip}(t) + v_{1p}(t)$$

$$y_2^{(j)}(t) = \sum_{p=1}^P y_{2p}^{(j)}(t), \quad y_{2p}^{(j)}(t) := \alpha_{2p}(t) * s_{jp}(t) + v_{2p}(t)$$

where $v_{mp}(t)$ and $\alpha_{mp}(t)$ ($m = 1, 2$) have spectral content in the p -th band only and zero elsewhere. Using identical notation, $s_{ip}(t)$ and $s_{jp}(t)$ are constrained to be in the p -th band and have spectral content defined by $S_{ip}(f) = S_i(f_p)r_p(f)$, and $S_{jp}(f) = S_j(f_p)r_p(f)$, respectively. Here, $S_m(f)$ is the Fourier transform of $s_m(t)$, the indicator function is denoted as $\mathbb{1}[\cdot]$, and $r_p(f) := \mathbb{1}[f_p - \delta f/2 \leq f \leq f_p + \delta f/2]$. Let us define the power spectral density (PSD) of $v_{mp}(t)$, $m = 1, 2$ as $V_p^m(f) = V_m(f_p)r_p(f)$ and the energy spectral variance (ESV) of $\alpha_{mp}(t)$ as $\Gamma_p^m(f) = \Gamma_m(f_p)r_p(f)$, where, $V_m(f)$, and $\Gamma_m(f)$ denote the PSD and ESV of $v_m(t)$ and $\alpha_m(t)$, respectively. Similarly, we can define the cross PSD and cross ESV to be $V_{12}^p(f) = V_{12}(f_p)r_p(f)$ and $\Gamma_{12}^p(f) = \Gamma_{12}(f_p)r_p(f)$ of the noise and target impulse responses at the two scheduling instants, respectively, where $V_{12}(f)$ and $\Gamma_{12}(f)$ are the cross PSD and cross ESV of the original random processes. If we denote the DI in the p -th band as $DI_p(i, j)$, then we can now readily show that,

$$DI_p(i, j) = I(\alpha_{1p}(t), \alpha_{2p}(t); y_{1p}^{(i)}(t), y_{2p}^{(j)}(t) | s_{ip}(t), s_{jp}(t)) = \tilde{T} \delta f \ln \left[1 + \frac{\chi_1(f_p, i, j)}{\chi_2(f_p) \tilde{T}^2} \right], \quad (14)$$

where

$$\begin{aligned}\chi_1(f_p, i, j) &= |S_i(f_p)|^2 |S_j(f_p)|^2 \Gamma_1(f_p) \Gamma_2(f_p) + \tilde{T} |S_i(f_p)|^2 \Gamma_1(f_p) V_{22}(f_p) + \tilde{T} |S_j(f_p)|^2 \Gamma_2(f_p) V_{11}(f_p) \\ &\quad - 2\tilde{T} \text{Re}\{S_i(f_p) S_j^*(f_p) \Gamma_{12}(f_p) V_{12}(f_p)\} - |S_i(f_p)|^2 |S_j(f_p)|^2 |\Gamma_{12}(f_p)|^2 \\ \chi_2(f_p) &= V_{11}(f_p) V_{22}(f_p) - |V_{12}(f_p)|^2\end{aligned}$$

where \tilde{T} is the total time duration of $y_1^{(i)}(t)$ and $y_2^{(j)}(t)$. Considering any two non-overlapping bands, and due to independence, the total MI is the sum of their respective MIs. Hence in the limiting case we have,

$$DI(i, j) = I(\alpha_1(t), \alpha_2(t); y_1^{(i)}(t), y_2^{(j)}(t) | s_i(t), s_j(t)) \quad (15)$$

$$= \sum_p \lim_{\delta f \rightarrow 0} DI_p(i, j) \quad (16)$$

$$= \sum_p \lim_{\delta f \rightarrow 0} I(\alpha_{1p}(t), \alpha_{2p}(t); y_{1p}^{(i)}(t), y_{2p}^{(j)}(t))$$

$$= \tilde{T} \int_{\mathcal{W}} \ln \left[1 + \frac{\chi_1(f, i, j)}{\chi_2(f) \tilde{T}^2} \right] df \quad (17)$$

The waveform scheduling criteria now becomes,

$$\arg \max_{i, j} DI(i, j) = \arg \max_{i, j} \int_{\mathcal{W}} \ln \left[1 + \frac{\chi_1(f, i, j)}{\chi_2(f) \tilde{T}^2} \right] df$$

The results of⁸ and⁹ may be obtained as special cases of the framework presented here, when the waveform design is considered rather than waveform scheduling. In particular, for one scheduling epoch only, the first term in the DI is the one-step MI, optimized in⁸ and⁹ where we note that (1) and (2) allow for signal-clutter interactions. It is noted that in waveform design the optimization is over the waveforms rather than over the waveform index i, j .

4.3 An example when waveform diversity is useless.

We consider a special case where transmitting diverse waveforms on the scheduling epochs is unnecessary. Assume that $V_1(f) = V_2(f) = \sigma^2, f \in \mathcal{W}$ and $V_{12}(f) = 0$ (white, Gaussian noise, independent and identical over the two slots). In the same spirit assume $\Gamma_1(f) = \Gamma_2(f) = \sigma_a^2, f \in \mathcal{W}$ and $\Gamma_{12}(f) = 0$. These assumptions imply that the ESV's are flat in the bandwidth, and the cross ESV is zero. Now substituting these assumptions in (15), and using (7), we have

$$\begin{aligned}\arg \max_{i, j} DI(i, j) &= \arg \max_{i, j} \int_{\mathcal{W}} \ln \left[\left(1 + \frac{|S_i(f)|^2 \sigma_a^2}{\tilde{T} \sigma^2} \right) \left(1 + \frac{|S_j(f)|^2 \sigma_a^2}{\tilde{T} \sigma^2} \right) \right] df \\ &= \arg \max_i \int_{\mathcal{W}} \ln \left(1 + \frac{|S_i(f)|^2 \sigma_a^2}{\tilde{T} \sigma^2} \right) df.\end{aligned} \quad (18)$$

From (18), we see that the maximization over the two epochs decouples to a single maximization for one epoch. In other words, pick one waveform which maximizes (18) and schedule it for both transmission epochs. In practice, radar scenes with the aforementioned assumptions are more an exception than the rule.

5. SIMULATIONS

We now illustrate how the spectral domain maximization of the DI i.e, (15) is carried out through several examples. For ease of exposition, the bandwidth is normalized by the sampling frequency and denoted as \mathcal{W} and assumes the normalized value of 0.6 (± 0.3), unless noted otherwise. The normalized frequency axis is between $(-0.5, 0.5]$. To be fair in comparisons, it is emphasized that the waveforms employed in this section have identical

energy and identical bandwidth. Furthermore, we wish to stress here that the waveforms as chosen by the DI in the scheduling instants depend on interactions of the spectral content of the waveforms with the auto and cross target ESV's along with the auto and cross noise PSDs. Their interactions are dictated by (14) in the individual frequency sub-band and (15) over the entire radar bandwidth, respectively.

In the first example, we consider the waveform library comprised of two waveforms, whose Fourier transforms are denoted as $S_i(f)$, and magnitude squared responses are given by

$$\begin{aligned} |S_1(f)|^2 &= |3 - 2.5\text{sech}(f)| \\ |S_2(f)|^2 &= \exp(-10f^2)(8 - 6 \cos(2\pi \times 2f - 2\pi/7)) \end{aligned}$$

The noise auto PSDs at the two scheduling instants, and noise cross PSD are denoted as $V_1(f) = 1$, $V_2(f) = 1$, $V_{12}(f) = 0$, $f \in \mathcal{W}$, respectively. The target ESV at the first scheduling instant is defined as, $\Gamma_1(f)$, and given by

$$\Gamma_1(f) = \begin{cases} -\frac{f}{0.3} + 1 & \text{if } 0 < f \leq 0.3 \\ \frac{f}{0.3} + 1 & \text{if } -0.3 \leq f < 0 \\ 0 & \text{Otherwise} \end{cases}$$

In the same spirit, the target ESV at the second scheduling instant is given by

$$\Gamma_2(f) = \begin{cases} -\frac{f}{0.2} + 1 & \text{if } 0 < f \leq 0.3 \\ \frac{f}{0.2} + 1 & \text{if } -0.3 \leq f < 0 \\ 0 & \text{Otherwise} \end{cases}$$

The target cross ESV denoted as $\Gamma_{12}(f)$ assumes values such that

$$|\Gamma_{12}(f)|^2 = 0.1|\Gamma_1(f)| \times |\Gamma_2(f)|$$

Since there are two waveforms in the library, and we consider scheduling in two instants, the waveform permutations are limited. Let us denote, permutations-1,2,3,4 as waveform choices (s_1, s_1) , (s_2, s_2) , (s_1, s_2) , and (s_2, s_1) , in the two scheduling instants respectively. It is then seen from Fig.3(d) that the choice (s_2, s_1) is selected for transmission. This is surprising given that the DI costs as computed from (14) in each frequency bin and as shown in Fig.3(c) are more or less similar for all the permutations. However, after close inspection of Fig. 3(c) we can see that the DI costs for the chosen permutation is flatter across the radar bandwidth, than the other three permutations. For this simulation scenario, the DI is sensitive to small changes in the costs, and prefers waveform diversity rather than selecting identical waveforms for transmission at the two scheduling instants. It is also surprising to note that the DI cost for the permutation (s_1, s_2) is the least among the other choices.

Unlike the previous simulation where the DI picks diverse waveforms, in this simulation the DI picks identical waveforms at both the scheduling instants, although it yet is to be decided which one is picked. For this simulation, the magnitude squared response of the waveforms are varied, and are given by

$$\begin{aligned} |S_1(f)|^2 &= |3 - 2.5\text{sech}(100f)| \\ |S_2(f)|^2 &= \exp(-10f^2)(8 - 6 \cos(2\pi \times 10f - 2\pi/7)) \end{aligned}$$

All other simulation parameters and identical to the previous example. The results are shown in Fig.4, and are self explanatory. In particular, we see from Fig.4(d) that (s_2, s_2) is selected for transmission. This is not surprising given the DI costs as in Fig.4(d), where we can see that permutation-2 has the highest cost at and near the vicinity of DC. These costs therefore bias the DI to pick a traditional non-diverse scheme of waveform transmission. Nonetheless, DI picks the second waveform at both scheduling instants rather than the first. This maybe attributed to the the deep null the first waveform has at DC, as shown in Fig.4(a).

In this example, the ESV's unlike the previous simulation were made approximately flat over the radar bandwidth, and the the second waveform's spectral content to be slowly varying. These changes make the DI pick identical waveforms at the scheduling instants, but opposite of what was chosen in the previous example.

In this last simulation example, all parameters are identical to the previous cases, except the waveforms and the auto ESVs which are varied and are given by,

$$|S_1(f)|^2 = |3 - 2.5\text{sech}(f)|$$

$$|S_2(f)|^2 = \exp(-30f^2)(8 - 6 \cos(2\pi \times f - 2\pi/7))$$

$$\Gamma_1(f) = \begin{cases} -\frac{f}{3} + 1 & \text{if } 0 < f \leq 0.3 \\ \frac{f}{3} + 1 & \text{if } -0.3 \leq f < 0 \\ 0 & \text{Otherwise} \end{cases}$$

$$\Gamma_2(f) = \begin{cases} -\frac{f}{1.5} + 1 & \text{if } 0 < f \leq 0.3 \\ \frac{f}{1.5} + 1 & \text{if } -0.3 \leq f < 0 \\ 0 & \text{Otherwise} \end{cases}$$

We can see from Fig.5(a) that the first waveform has a nearly flat spectral response, in contrast to the second waveform, over the radar bandwidth. Given the target auto and target cross ESV which are again more or less flat over the bandwidth, and as seen in Fig.5(b), it is no surprise that the DI prefers transmitting the first waveform over both the scheduling instants as readily seen from Fig.5(c). The following example demonstrates the fact that DI for waveform scheduling weighs in the target auto ESVs, the target cross ESV, as well as the individual waveform's spectral shape, albeit in a complex manner as demonstrated from (14)-(15).

6. CONCLUSIONS

A cognitive radar framework was proposed to adaptively schedule waveforms by extracting information from the past radar returns. The Gaussian model assumed was general encompassing clutter and interference which are correlated with the target. Maximizing the directed information, which incorporates feedback, was proposed for the first time. The optimization problem was considered in several special cases. For simplicity, the analysis assumed a waveform library comprising two distinct waveforms and two scheduling instants. Nevertheless, the conclusions and analysis apply to a larger waveform library and multiple epochs. Simulations in the spectral domain were performed to depict scenarios where waveform adaptation is useful versus those in which it is useless.

ACKNOWLEDGMENTS

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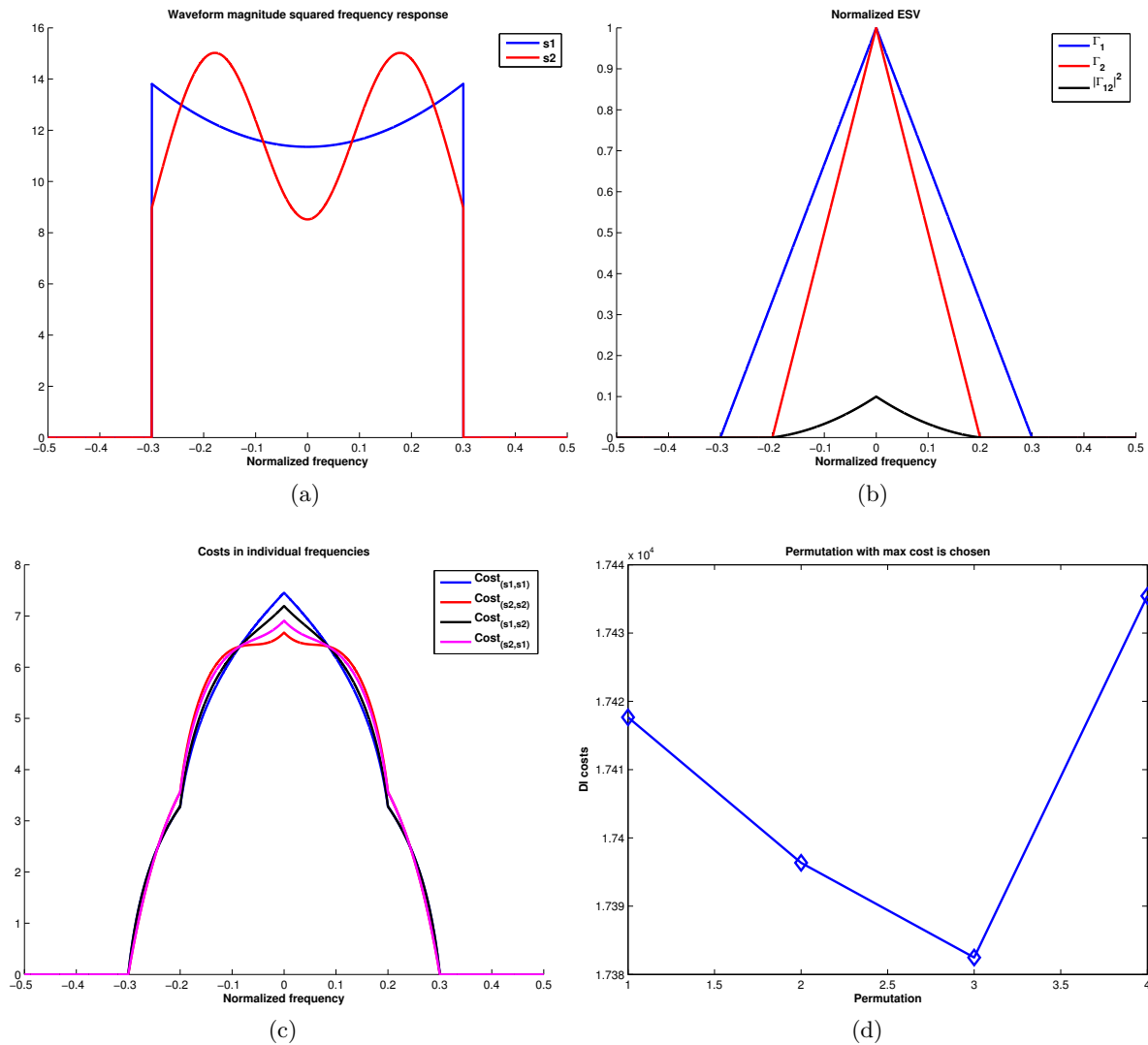
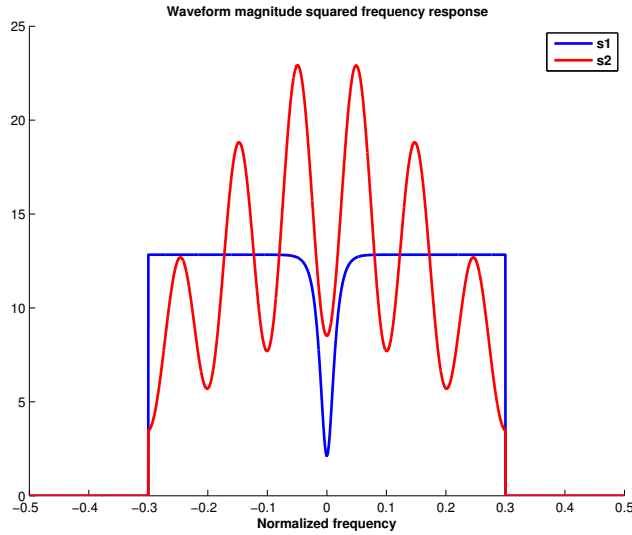
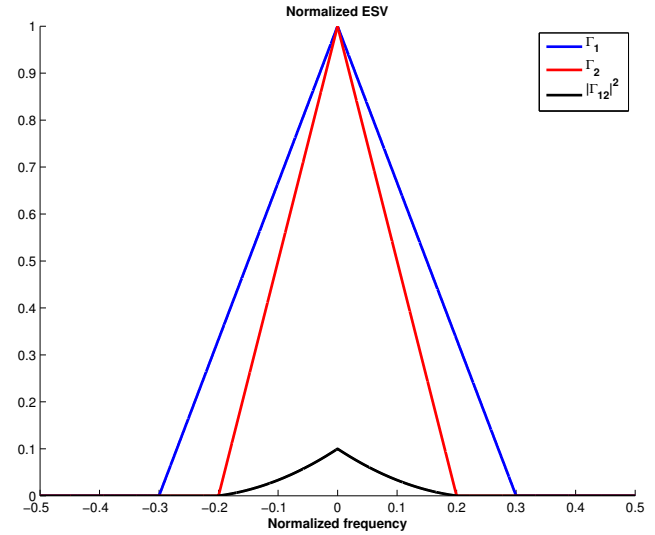


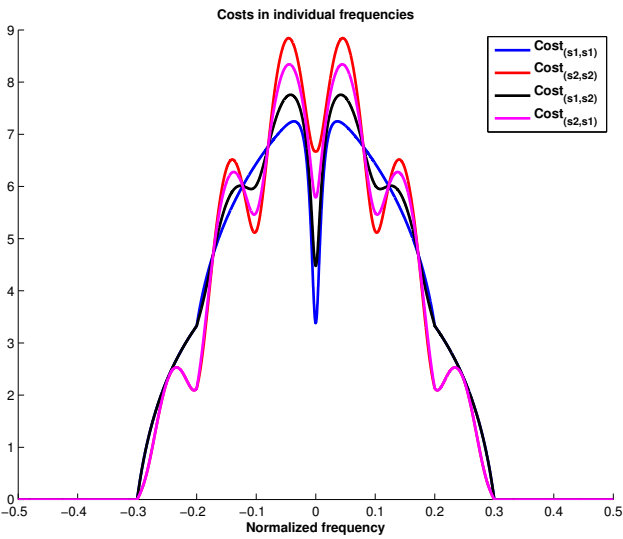
Figure 3: (a) $|S_i(f)|^2, i = 1, 2$, (b) Auto and cross target ESVs, (c) DI costs in each frequency bin, (d) DI costs v.s waveform choice permutations



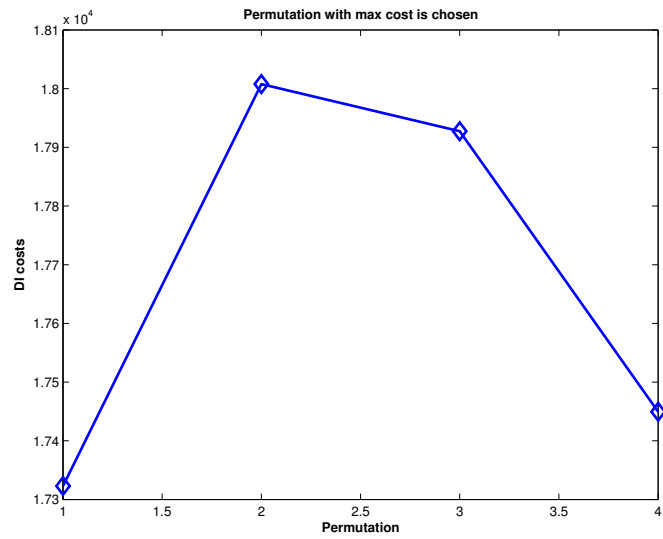
(a)



(b)

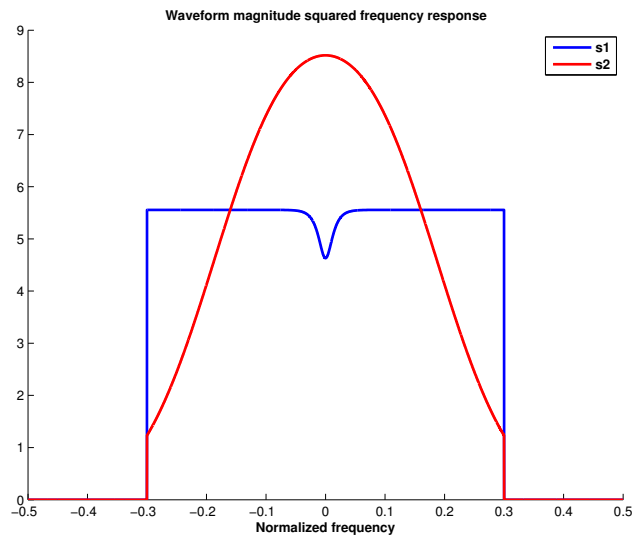


(c)

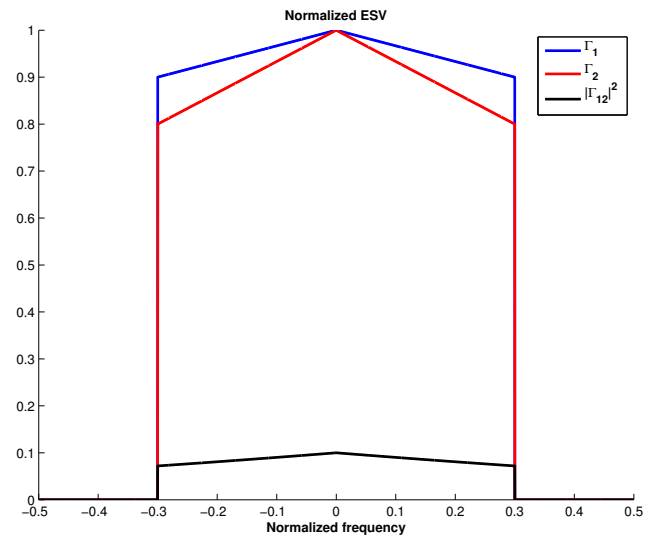


(d)

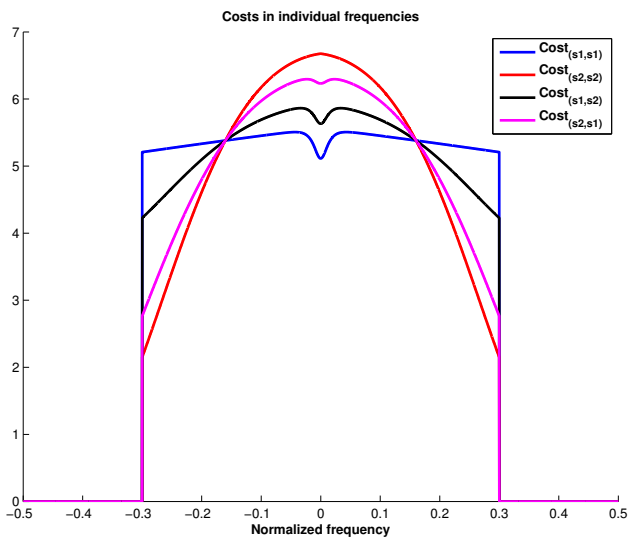
Figure 4: (a) $|S_i(f)|^2, i = 1, 2$, (b) Auto and cross target ESVs, (c) DI costs in each frequency bin, (d) DI costs v.s waveform choice permutations



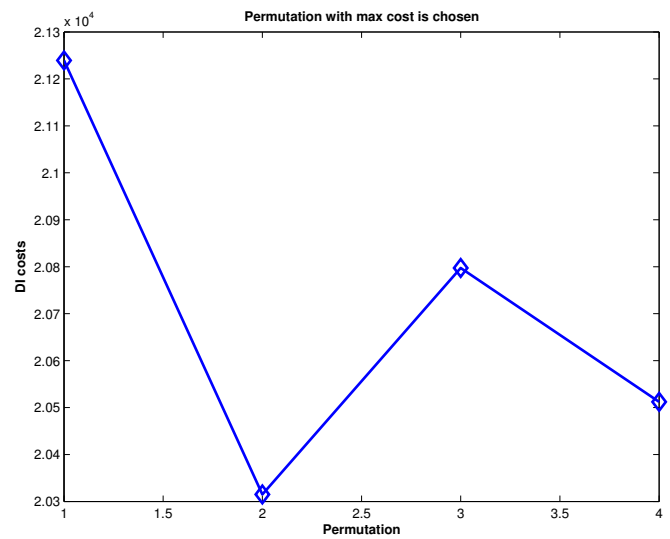
(a)



(b)



(c)



(d)

Figure 5: (a) $|S_i(f)|^2, i = 1, 2$, (b) Auto and cross target ESVs, (c) DI costs in each frequency bin, (d) DI costs v.s waveform choice permutations