Optimization of Two-way Communication with ARQ Feedback

Besma Smida, Natasha Devroye and Tian Li

***Abstract*—In this paper, we study ARQ feedback in the context of two-way wireless communications. In particular, we consider two nodes which wish to exchange data over a frequency division duplex, time-varying wireless additive white Gaussian noise with Rayleigh fading, channel. In two-way scenarios, unlike the more well studied one-way data scenarios, the data and resources allocated to feedback and channel estimation may share the same link, leading to interesting tradeoffs. To analyze these, we present a two-way framework in which 1) training (estimation of channel state), 2) feedback (in the form of ARQ), and 3) data are taken into account, and share the *same* noisy fading channel. We obtain an expression which captures the tradeoffs between allocating resources for these three tasks on the overall throughput achievable in each direction, which we numerically evaluate. In particular, we obtain the optimal resource allocations corresponding to different channel conditions, SNR regimes, and receiver feedback protocols under fast and slow fading conditions.**

***Index Terms*—bi-directional communication, feedback, throughput, time-varying channel**

I. Introduction

Feedback in wireless systems may improve data rates in several ways: it may enable the collaborative encoding or decoding of messages, allow terminals to learn the channel state, or request the re-transmission of a failed reception. In general, feedback has been studied from a one-way perspective, meaning data travels in one direction, and feedback – often assumed to be perfect – in the other. We propose to extend the study of perfect one-way feedback to *noisy* feedback in *two-way communications*. When no data besides feedback is conveyed over the reverse link, it may appear to be "free" as it does not utilize system resources for the forward direction. However, when pairs of nodes wish to exchange messages in a two-way fashion, the forward and reverse links may each be used to transmit data in one direction, or to feedback bits for the user in the other direction. Allocating more resources to training reduces the number of retransmissions needed for successful decoding, and the system performance may be dominated by the channel estimation accuracy. In contrast, by allocating less resources to training, the number of retransmissions may increase and the performance may be dominated by the number of retransmissions. We aim to explore the inter-dependent forward and reverse link rates in a two-way communication with feedback, something about which relatively little is known despite its immediate practical relevance in wireless networks.

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Related work. While most research on wireless systems with feedback has considered one-way data, some interesting exceptions include [1]–[8]. In the most related [1], a two-way beamforming system is considered, where data and CSI flow in both directions between two multi-antenna transceivers. For this system, bounds on the feedback rate for maximizing "net throughput" – throughput minus average feedback cost – are derived under block fading assumptions, and without the possibility of re-transmission. Similarly, the authors of [3], [4] took into account 1) all resources used to obtain CSI at the source and destination, and 2) that channel estimates at the source and destination are mismatched and noisy. They derive the full diversity-multiplexing tradeoff.

The key departure from prior work is that a) our model combines a time-varying channel model and limited feedback in one framework, while previous models have considered only block fading or static channels, and 2) forward and feedback (ARQ) links are modeled as fading wireless channels and all training / feedback resources are accounted for; prior works have not considered re-transmission which is crucial to exploring the tradeoffs particular to the two-way network.

Contributions. Our main contributions are the:
1) The study of feedback in two-way networks which captures time-variation of the channel rather than block fading, accounts for training, feedback, and data bits all of which are over noisy, imperfect channels.
2) Derivation of an achievable throughput and tradeoff between training, data, and ARQ bits in a two-way scenario.

II. System Setup: Framework

We first propose a transmission protocol that captures the tradeoff between feedback and data in two-way scenarios. Then, we outline the time-varying fading channel model before deriving an expression for the throughput which demonstrates many of the two-way tradeoffs involved.

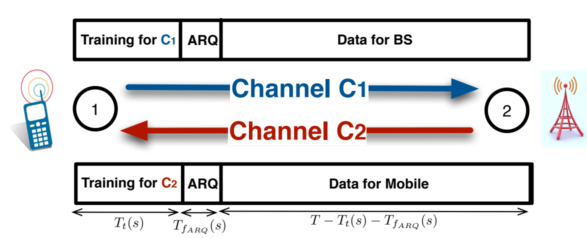

Fig. 1. The two-way packet-structure and inter-dependence of streams. Node 1 is the "mobile", node 2 is the "base-station" (BS).

A. Transmission Protocols

We consider Frequency Division Duplex (FDD) two-way systems with limited feedback. Two users exchange packets whose structure is shown in Figure 1. Each packet is divided into three phases: training, feedback (ARQ only), and data transmission. We fix the power allocated to each channel use and study the throughput tradeoffs by varying the duration of each phase. We consider a completely symmetric system[1], i.e. both users have the same transmit SNR and channels will have identical models (though not realizations). We next describe these three phases in more detail. By symmetry, we need only focus on the forward link. Let each packet contain T symbols, or channel uses. The transmission s is divided into:

Training Phase: $T_t(s)$ channel uses are employed by user 1 for transmitting training symbols known to both users, enabling the user 2 to estimate the channel gain. We assume imperfect CSI at the receiver and no CSI at the transmitter.

Feedback Phase: user 2 asks for a re-transmission using an ARQ ($T_{f_{ARQ}}(s)$ symbols). The receiver sends a one-bit ACK or NAK to indicate the success/failure of transmission (which may be repeated or coded over more symbols for extra protection). The transmitter moves on to the next message in the transmission queue if it receives an ACK and re-transmits if it receives a NAK. Receivers may employ the re-transmissions in an ARQ system in different ways, three of which are considered in this work: 1) in the simplest "basic ARQ" used in the ALOHA protocol (ALO), the transmitter sends the same packet and the receiver discards the erroneous packets. 2) In diversity-combining systems, several noisy observations of the same packet are combined [9]–[12]. A popular Hybrid-ARQ (HARQ) scheme called chase combining HARQ (CC-HARQ [12]) performs coherent combining of all retransmissions, thus improving the probability of successful decoding. 3) With incremental redundancy HARQ (IR-HARQ), each retransmission typically uses a different set of coded bits than the previous transmission. Thus, at every retransmission the receiver gains extra information [13]–[15].

Data Transmission Phase: user 1 uses the remaining $T - T_t(s) - T_{f_{ARQ}}(s)$ symbols to transmit data to user 2.

III. Channel Model: Basis Expansion Model

A. Modeling of Time Varying Channel Gain

Accurate radio channel modeling is important when studying tradeoffs in two-way communications with feedback. In particular, different models lead to different evaluations of the minimum channel estimation error which is needed to derive the throughput as a function of the resources dedicated to channel training. We consider a time-selective fading channel model which is able to capture static, quasi-static, block fading and fast fading channels. The input-output relationship for the i-th symbol or channel use of the s-th transmission (or packet), is given by the additive Gaussian noise channel[2]

$$y_s(i) = h_s(i)x_s(i) + w_s(i), i \in [0, T-1] \tag{1}$$

where $y_s(i)$ is the received signal (at user 2), $x_s(i)$ is the transmitted signal (at user 1) of maximal power P, $w_s(i)$ is additive white Gaussian noise (AWGN) with mean zero and variance N_0, and the channel gain $h_s(i)$ is a time varying random variable to be specified. The time-varying channel gain is often approximated as a wide-sense-stationary narrow-band complex Gaussian process, with the so-called Jake's power spectrum [16]. Since the number of channel realizations (one per channel use) to estimate in each packet is greater than the number of training symbols, it is not efficient to estimate the time varying gain directly.

Instead, we consider the Complex Exponential-Basis Expansion Model (CE-BEM) [17], [18] for $h_s(i)$, which allows us to represent the channel variation by: a) small number $(Q+1)$ of random coefficients that remain invariant per packet transmission, but may change with each transmission s; and b) finite Fourier basis that captures the time variation, but is common to all transmissions / packets. That is, we model

$$h_s(i) = \sum_{q=0}^{Q} c_q(s) e^{jw_q i} \tag{2}$$

where $w_q := 2\pi(q - Q/2)/T$, $Q := 2\lceil f_{ND} T \rceil$ and f_{ND} is the Doppler spread normalized by the sampling frequency [3]. We assume that the CE-BEM coefficients $c_q(s)$ are zero-mean, complex Gaussian random variables [19] (Rayleigh fading).

B. Channel estimation

The $T_t(s)$ training symbols are used by user 2 to estimate $c_q(s)$ for $q = 0, 1, \cdots Q$ which in turn yield estimates for $h_s(i)$ for all i in the data transmission phase. In the sequel, we evaluate the minimum channel estimation error of the channel vector $\mathbf{h}_s := [h_s(0), \ldots, h_s(T)]^T$, as a function of $T_t(s)$, which is needed to subsequently derive the throughput. The vector counterpart of (2) is

$$\mathbf{h}_s = \mathbf{T}\mathbf{c}_s^{BEM}, \tag{3}$$

where $\mathbf{c}_s^{BEM} := [c_0(s), c_1(s), \ldots, c_Q(s)]^T$ is the channel BEM coefficient vector, and the matrix

$$\mathbf{T} := \begin{bmatrix} 1 & \ldots & 1 \\ e^{jw_0} & \ldots & e^{jw_Q(T-1)} \\ \vdots & & \vdots \\ e^{jw_0(T-1)} & \ldots & e^{jw_Q(T-1)} \end{bmatrix}.$$

Let $\tilde{x} = x - \hat{x}$ denote the mismatch between x and its estimate $\hat{x}$. Based on Equation (3), we derive the minimum mean squared estimation error of the channel vector $\mathbf{h}_s$ as

$$\sigma^2_{\tilde{\mathbf{h}}_s} = \text{tr}\left(\mathbf{T}\sigma^2_{\tilde{\mathbf{c}}_\mathbf{s}^{BEM}}\mathbf{T}^T\right),$$

where $\sigma^2_{\tilde{\mathbf{c}}^{BEM}}$ is minimum mean squared estimation error of the channel coefficient vector $\mathbf{c_s}^{BEM}$.

Now let us derive $\sigma^2_{\tilde{\mathbf{c}}^{BEM}}$ using the training input-output relationship. The i-th received pilot symbol of the s-th transmission can be written as

$$y_s(t_i) = \sum_{q=0}^{Q} c_q(s) e^{jw_q t_i} p_s(i) + w_s(t_i), \tag{4}$$

[1]Due to the inherent complexity of the problem, we address the symmetric case first and leave the asymmetric case for future work.

[2]For sake of simplicity we assume a frequency flat fading channel, the generalization to frequency selective fading channel is straight-forward.

[3]Typical values for practical systems range from $f_{ND} = 10^{-3}$(very slow fading) to $f_{ND} = 10^{-1}$ (very fast fading).

where $p_s(i)$ is pilot symbol send at time slot t_i. Given the Gaussian channel model, the linear MMSE (LMMSE) estimator of CE-BEM coefficients attains the minimum square error [20] given by

$$\left[\sigma^2_{\tilde{\mathbf{c}}_s^{BEM}}\right]_q := \left[\left(R^{-1}_{\mathbf{c}_s^{BEM}} + \frac{1}{\sigma_n^2}\boldsymbol{\Phi}_s^H\boldsymbol{\Phi}_s\right)^{-1}\right]_{qq}, \qquad (5)$$

where $[\mathbf{x}]_q$ denotes the q-th element of the vector x, $[\mathbf{X}]_{kn}$ denotes the $[k,n]$ element of the matrix $\mathbf{X}$, the $T_t(s) \times (Q+1)$ matrix $\boldsymbol{\Phi}_s := [\mathbf{D}(w_0)\mathbf{p}_s, \ldots, \mathbf{D}(w_Q)\mathbf{p}_s]$, $\mathbf{p}_s := [p_s(1), \ldots, p_s(T_t(s))]^T$ is the pilot vector, $\mathbf{D}(w_q) := \text{diag}[e^{jw_q(t_1)}, \ldots, e^{jw_q(t_{T_t(s)})}]$, and

$$R_{\mathbf{c}_s^{BEM}} = (\mathbf{T}^T\mathbf{T})^{-1}\mathbf{T}^T E[\mathbf{h}_s\mathbf{h}_s^H]\mathbf{T}(\mathbf{T}^T\mathbf{T})^{-1}.$$

The k,n-th element of the matrix $R_{\mathbf{h}_s} = E[\mathbf{h}_s\mathbf{h}_s^H]$ is $[R_{\mathbf{h}_s}]_{kn} = J_0(2\pi f_{ND}(k-n))$, for J_0 the Bessel function of the first kind. For static, or quasi-static channels we assume the CE-BEM coefficients remain invariant over m transmissions, and use the training symbols transmitted over m packets to estimate the channel. Consequently,

$$\mathbf{y} = \boldsymbol{\Phi}\mathbf{c}_s^{BEM} + \mathbf{w}, \qquad (6)$$

where $\mathbf{y} := [\mathbf{y}_1^T, \ldots, \mathbf{y}_m^T]^T$, $\mathbf{w} := [\mathbf{w}_1^T, \ldots, \mathbf{w}_m^T]^T$,

$$\begin{aligned}\boldsymbol{\Phi} \;:=\; & [\mathbf{D}_1(w_0)\mathbf{p}_1 \ldots \mathbf{D}_1(w_Q)\mathbf{p}_1 \ldots\ldots\ldots \\ & \mathbf{D}_m(w_0)\mathbf{p}_m \ldots \mathbf{D}_m(w_Q)\mathbf{p}_m],\end{aligned}$$

and $\mathbf{D}_i(w_q) := \text{diag}[e^{jw_q((t_1+(i-1)T)}, \ldots, e^{jw_q((t_{T_t(i)}+(i-1)T)}]$. We then rely on the Wiener solution of (6) to derive the LMMSE estimator of $\mathbf{h}_s$ and the minimum square error (similar to the previous methodology).

IV. Throughput Analysis

In this section, we focus on our main goal: the derivation of a general throughput expression for this two-way system. Our analysis is valid under the following idealized assumptions: identical packet sizes, capacity achieving IR/CC codes, negligible signaling overhead and fixed rate target R in bits/Hz/s. Due to symmetry, the analysis of the forward and reverse link are identical, and we focus on the forward link.

A. Notations and Preliminaries

To obtain the throughput we first need an expression for average rate per transmission. This depends on the the probability of outage, or that the target rate we are transmitting at, R, is above what the channel may support, modeled as its mutual information. We define the event $A_m := \{I_m > R\}$, where I_m is the mutual information (MI) after the m-th transmission/packet. Under Gaussian inputs and noise, the MIs are functions of the Signal to Interference plus Noise Ratio (SINR) β_m as

$$\begin{aligned}\text{ALO:} \quad & I_m = \alpha(m)\log(1+\beta_m), \\ \text{CC-HARQ:} \quad & I_m = \alpha(m)\log(1+\textstyle\sum_{s=1}^m \beta_s), \\ \text{IR-HARQ:} \quad & I_m = \textstyle\sum_{s=1}^m \alpha(s)\log(1+\beta_s),\end{aligned} \qquad (7)$$

where

$$\beta_s = \frac{|\widehat{\mathbf{h}}_s|^2 P}{N_o + |\widetilde{\mathbf{h}}_s|^2 P}, \text{ and } \alpha(s) = \frac{T - T_t(s) - T_{f_{ARQ}}(s)}{T}. \qquad (8)$$

The mismatch $\widetilde{\mathbf{h}}_s = \mathbf{h}_s - \widehat{\mathbf{h}}_s$ between the true channel $\mathbf{h}_s$ and its estimate $\widehat{\mathbf{h}}_s$, is unknown at the destination, and hence considered unresolvable. Note that the distribution of $\widehat{\mathbf{h}}_s$ and $\widetilde{\mathbf{h}}_s$ vary with the resources allocated to training $T_t(s)$. ALO takes into account only the most recently received signal burst. In CC-HARQ, the SNR after combining m received bursts equals $\sum_{s=1}^m \beta_{k,s}$. In IR-HARQ, the mutual information is the sum of all previous received mutual information (MI) terms corresponding to a given message [21], [22].

We state the probability $p(m)$ that the random sequence $I_1, I_2, \ldots, I_m$ of mutual information at the user decoder did not cross the level R at the m-th step, $p(m) = \text{P}_r\{\overline{A_1}, \overline{A_2}, \ldots, \overline{A_m}\}$. We also define the probability $q(m)$ that the the random sequence $I_1, I_2, \ldots, I_m$ of mutual information at the user decoder crosses the level R at the m-th step (and not before), $q(m) = \text{P}_r\{\overline{A_1}, \ldots, \overline{A_{m-1}}, A_m\}$. Note that, for $p(m) = 1 - \sum_{l=1}^m q(l)$, that

$$\begin{aligned} q(m) &= \text{P}_r\{\overline{A_1}, \ldots, \overline{A_{m-1}}, A_m\} \\ &= \text{P}_r\{\overline{A_1}, \overline{A_2}, \ldots, \overline{A_{m-1}}\} - \text{P}_r\{\overline{A_1}, \overline{A_2}, \ldots, \overline{A_m}\} \\ &= p(m-1) - p(m). \end{aligned}$$

We also take into account the number of additional slots due to redundant retransmissions (from feedback errors) of a packet. Error is incorporated as follows: an unreliable feedback message (not decoded) is always assumed to be a NAK, i.e. a NAK is never incorrectly received as an ACK. On the other hand, an ACK incorrectly received as a NAK will cause a redundant retransmission of a correctly received packet. By setting the feedback rate to 1 bit, we define the probability of error of feedback as $p_f(m) = \frac{1}{2}\text{Pr}\{T_{f_{ARQ}}(s)\log_2(1+\beta_m) < 1\}$. Note that to increase the feedback reliability, the feedback ACK/NAK data may be coded over a larger block length (increase $T_{f_{ARQ}}(s)$).

B. Derivation of outage probabilities $p(m)$ and $p_f(m)$

For a normalized Rayleigh fading channel ($|\mathbf{h}_s|^2 = 1$), the random variables $|\widetilde{\mathbf{h}}_s|^2$ and $|\widehat{\mathbf{h}}_s|^2$ have an exponential distribution with variance $\sigma^2_{\widetilde{\mathbf{h}}_s}$ and $\sigma^2_{\widetilde{\mathbf{h}}_s} + 1$, respectively. We now derive $p(m)$ for each protocol ($p(0) = 1$) as

$$\begin{aligned}\text{ALO:} \quad & p(m) = \textstyle\prod_{s=1}^m \Pr\left(\alpha(s)\log(1+\beta_s) < R\right), \\ \text{CC-HARQ:} \quad & p(m) = \Pr\left(\alpha(m)\log(1+\textstyle\sum_{s=1}^m \beta_s) < R\right), \\ \text{IR-HARQ:} \quad & p(m) = \Pr\left(\textstyle\sum_{s=1}^m \alpha(s)\log(1+\beta_s) < R\right).\end{aligned}$$

<u>ALO:</u> The $p(m)$ is derived as follows:

$$\begin{aligned} p(m) &= \prod_{s=1}^m \Pr\left(\alpha(s)\log(1+\beta_s) < R\right), \\ &= \prod_{s=1}^m \Pr\left(\beta_s < \gamma(s)\right), \\ &= \prod_{s=1}^m \Pr\left(|\widehat{h}_s|^2 < \frac{N_0\gamma(s)}{P} + \gamma(s)|\widetilde{h}_s|^2\right), \end{aligned}$$

$$= \prod_{s=1}^{m} \left(1 - \frac{1+\sigma^2_{\tilde{\mathbf{h}}_s}}{1+\sigma^2_{\tilde{\mathbf{h}}_s} + \gamma(s)\sigma^2_{\tilde{\mathbf{h}}_s}} \exp\left(-\frac{N_0\gamma(s)}{P + P\sigma^2_{\tilde{\mathbf{h}}_s}} \right) \right),$$

where $\gamma(s) = \exp\left(\frac{R}{\alpha(s)}\right) - 1$.

CC-HARQ: The $p(m)$ is derived as follows:

$$\begin{aligned} p(m) &= \Pr\left(\alpha(m)\log(1+\sum_{s=1}^{m}\beta_s) < R \right) \\ &= \Pr\left(\sum_{s=1}^{m}\beta_s < \gamma(m) \right) \\ &= \int_0^{\gamma(m)} f_1(x) * \cdots * f_m(x)dx \end{aligned}$$

where

$$f_s(x) = \left(\frac{\sigma^2_{\tilde{\mathbf{h}}_s}(1+\sigma^2_{\tilde{\mathbf{h}}_s})}{(1+\sigma^2_{\tilde{\mathbf{h}}_s} + x\sigma^2_{\tilde{\mathbf{h}}_s})^2} + \frac{N_0}{P(1+\sigma^2_{\tilde{\mathbf{h}}_s} + x\sigma^2_{\tilde{\mathbf{h}}_s})} \right) \times \exp\left(-\frac{N_0 x}{P(1+\sigma^2_{\tilde{\mathbf{h}}_s})} \right), \quad (9)$$

and $*$ denotes convolution.

IR-HARQ: The $p(m)$ is derived as follows:

$$\begin{aligned} p(m) &= \Pr\left(\sum_{s=1}^{m}\alpha(s)\log(1+\beta_s) < R \right) \\ &= \int_0^{R} g_1(x) * \cdots * g_m(x)dx \end{aligned}$$

where

$$g_s(x) = \frac{\exp(x/\alpha(s))}{\alpha(s)} f_s(\exp(x/\alpha(s)) - 1).$$

Feedback: The $p_f(m)$ is derived as follows:

$$\begin{aligned} p_f(m) &= \frac{1}{2}\Pr\left(T_{f_{ARQ}}(s)\log_2(1+\beta_s) < 1 \right), \\ &= \frac{1}{2}\Pr\left(\beta_s < \gamma_f(s) \right), \\ &= \frac{1}{2}\left(1 - \frac{1+\sigma^2_{\tilde{\mathbf{h}}_s}}{1+\sigma^2_{\tilde{\mathbf{h}}_s} + \gamma_f(s)\sigma^2_{\tilde{\mathbf{h}}_s}} \exp\left(-\frac{N_0\gamma_f(s)}{P + P\sigma^2_{\tilde{\mathbf{h}}_s}} \right) \right), \end{aligned}$$

where $\gamma_f(s) = 2^{\frac{1}{T_{f_{ARQ}}(s)}} - 1$.

C. Throughput Expressions

Finally, we define the *throughput* as

$$\nu = \frac{\mathcal{R}}{\mathcal{T}}, \quad (10)$$

where, assuming maximum M transmissions[4], the *expected rate* $\mathcal{R}$ in bits/Hz/s is

$$\mathcal{R} = R(1 - p(M)),$$

and the *expected number of transmission per packet* $\mathcal{T}$ is

$$\mathcal{T} = \sum_{m=0}^{M-1} p(m) + \sum_{m=1}^{M-1} q(m) \left[\sum_{l=m}^{M-1} \prod_{n=m}^{l} p_f(n) \right]. \quad (11)$$

[4]We stop transmitting the same packet after M attempts.

Note that if we assume perfect feedback the expected number of transmission per packet becomes $\mathcal{T} = \sum_{m=0}^{M-1} p(m)$. Our definition of throughput assumes all nodes always have packets to send [22], [23]; extensions to bursty traffic remains for future work.

Remark 1: We wish to note the importance of a *two-way* throughput formulation in understanding the value of feedback in practical systems. To see this, consider a point-to-point communication system in which we account for the training symbols used in estimation, and wish to determine the optimal number of pilot symbols. When no re-transmission are assumed, prior work has shown that one would select the number of pilot symbols to be as small as possible [19], [24]–[26], i.e., (L + 1)(Q + 1). This small number can be justified thus: the resources allocated to training directly reduce the amount of data in the forward channel, reducing the rates in the pre-log factor outside the log(1 + SNR), (SNR is Signal to Noise Ratio), of capacity expressions for Gaussian channels. However, SNR-gain due to training improves rates inside the log. With our proposed two-way throughput formulation, the SNR-gain due to training not only improves rates inside the log but also reduces the number of re-transmissions, which leads to more realistic tradeoffs.

V. Performance comparison and tradeoffs

In this section, we numerically evaluate the throughput obtained in the previous section for the case of two nodes which wish to exchange data over a frequency division duplex, time-varying Rayleigh fading channel. We assume that the nodes stop transmitting the same packet after $M = 3$ attempts. We also assume that the pilot symbol are inserted equi-spaced [19]. We first compare the throughput of the three feedback protocols ALO, CC-HARQ, and IR-HARQ for different SNR regimes and channel conditions (quasi-static $f_{ND} = 10^{-4}$ and very fast fading $f_{ND} = 0.01$). We set the noise to $N_0 = 1$, $T_{f_{ARQ}}(s) = 1$, the target rate $R = 2$ bits/Hz/s and vary the transmit power $P = 1(0\text{dB}), 2(3\text{dB}), 3(4.8\text{dB}), 5(7\text{dB})$.

Fig. 2(a) shows how one may optimize the bits allocated to training during the first transmission as a function of SNR for a quasi-static channel. We show the performance of the quasi-static channel after the first transmission only because the resources allocated for training are minimal after that. Notice that, interestingly, the optimum resource allocations vary with channel conditions and SNR regimes. In the low SNR regime, adding extra bits to the training only slightly reduces the channel estimation error while reducing the data transmission phase, which results in a loss in performance. But in the high SNR regime, the extra training bits considerably improve the channel estimate and are worth the loss in data.

Fig. 2(b) shows the throughput versus the allocated training bits over the three (possible) retransmissions for a fast fading channel. Note that we optimize the throughput over the total allocated resources for three retransmissions. The simulations show that the performance of IR-HARQ outperforms CC-HARQ, which in turn outperforms ALO, as expected. In addition, allocating much resources to training does not appear to be helpful, which may be explained by the fact that under fast fading, it is challenging to learn the channel.

	S/IR	S/CC	S/ALO	F/IR	F/CC	F/ALO
$T_t(1)$	6	12	8	3	3	3
$T_t(2)$	2	0	0	3	3	3
$T_t(3)$	0	0	0	3	3	3

TABLE I
OPTIMUM ALLOCATION OF TRAINING BITS.

Next, we examined the optimum allocation of training phases for each transmission in Table I. The "S" and "F" indicate slow or fast fading. For the quasi-static channel, allocating more training symbols for channel estimation to the first transmission, and less afterwards is intuitive. On the other hand, for fast fading channels the optimum training bits are more homogeneous over the three transmissions.

We have also analyzed the expression of the throughput for some specific channels[5], by examining the derivative of equation (11). For a static channel, it may be shown that the optimum $T_t(M)$ is a decreasing function of the sum of all bits dedicated to training $\sum_{i=1}^{M-1} T_t(i)$ and is not a function of the $T_t(i)$ individually. In the case of very fast fading channel, it may be shown that the optimum $T_t(M)$ is independent of the number of bits dedicated to training in previous transmissions. In addition, if the feedback is very reliable (i.e. $p_f(m) \approx 0$), under very fast fading, the optimum allocation of training $T_t(i)$ at each transmission i minimizes the outage probability $p(i)$, and may be optimized in a greedy fashion, i.e. for each time slot individually.

Finally, we have also examined the optimum allocation of $T_{f_{ARQ}}(s)$ feedback bits. Due to lack of space we omit the plot but for the fast fading channel, $P = 5$, and IR-HARQ the optimum number of $T_{f_{ARQ}}(s)$ is 2 bits (rather than 1), which renders the feedback more reliable, and hence improves performance.

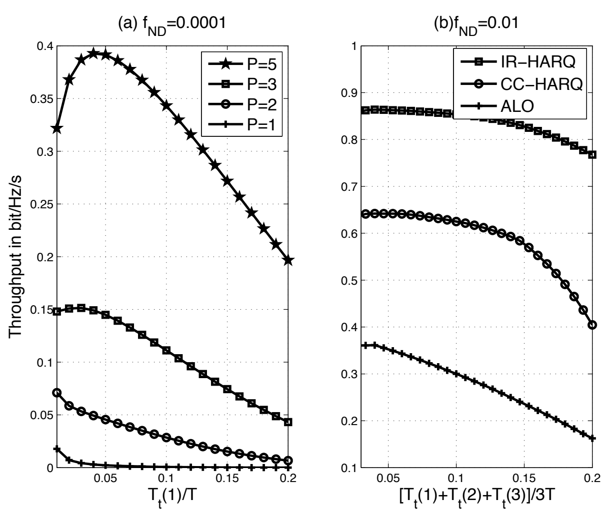

Fig. 2. Throughput vs training phase lengths.

VI. CONCLUSION

A framework for the study of two-way communications with ARQ in fading channels was developed, and throughput expressions which characterize the tradeoff between resources allocated to training (learning channel gains), feedback (ARQ) and data transmission were derived and numerically evaluated.

[5]The complete analysis will be addressed in future work

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