The Capacity Region of the Gaussian Cognitive Radio Channels at High SNR

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***Abstract*— Deterministic channel models have recently proven to be powerful tools for obtaining capacity bounds for Gaussian multiuser networks that are tight in the high SNR regime. In this work we apply this technique to the Gaussian cognitive radio channel: a 2×2 interference channel in which one transmitter has non-causal knowledge of the message of the other, whose capacity region in general is still unknown. We approximate the Gaussian cognitive radio channel at high SNR by an underlying linear deterministic channel model for which we determine the exact capacity region, or generalized degrees of freedom region. Explicit achievable schemes and outer bounds are illustrated and future directions are discussed.**

I. Introduction

In recent years a number of communication paradigms which aim at exploiting the agile and flexible capabilities of cognitive radio technology have emerged [1]. One of the more sophisticated, in terms of assumptions as well as transmission capabilities, cognitive communication models is the *cognitive radio channel* [2]. This channel[1] consists of a primary transmitter-receiver (Tx-Rx) pair and a secondary or cognitive Tx-Rx pair whose transmissions interfere with each other, similar to the classical interference channel. The cognitive Tx's "cognitive" capabilities are modeled by the idealized assumption that it has full a-priori knowledge of the primary message.

The capacity region of the cognitive radio channel, both for discrete memoryless as well as Gaussian noise channel remains unknown in general. Tools such as rate-splitting, binning, cooperation and superposition coding have been used in deriving achievable rate regions. Capacity is known in the weak interference [3], [4] and the strong interference [5] regimes[2]. Unfortunately, there are no general capacity results and the most comprehensive achievable rate regions as well as outer bounds in general involve the union over a number of auxiliary random variables. In the achievable rate regions, this reflects a lack of understanding of which transmission schemes are capable of approaching capacity. Similarly, it is not clear what genie-aided receiver side information will yield the tightest outer bound. In this work we seek intuition on the capacity achieving schemes and outer bounds by focussing on a deterministic cognitive radio channel model.

Deterministic channel models have recently been introduced to obtain capacity bounds for Gaussian multiuser networks that are tight in the high SNR regime, that is to say, the difference between inner and outer bounds is upper bounded by a finite (small) constant at all SNR's. The main intuition is to neglect the additive noise and concentrate on the interaction among intended and interfering signals in a deterministic setting. The capacity of deterministic high-SNR approximation of Gaussian channels can be determined exactly[3]. Once the deterministic approximation's channel capacity is obtained, inner and outer bounds on the capacity region of the original Gaussian channel model follow relatively easily. The approach has allowed, for example, to solve within a constant gap the capacity regions of channels that have been long standing open problems, such as Gaussian interference channels [7], [8], [**?**] and Gaussian relay channels [9].

While deterministic models accurately model Gaussian noise channels at high SNR, they may also be of interest in and of their own right, and may in some cases lead to capacity results which are unknown for the more general channel. Prime examples of this are the capacity regions of the deterministic broadcast channel [10], [11] and a class of interference channels [12]. In such channels, one can exploit the determinism in the channels to simplify achievable rate region expressions as well as correctly select auxiliary random variables to provide tight outer bounds.

[1]This channel has also been termed the interference channel with unidirectional cooperation or the interference channel with degraded message sets.

[2]For a comprehensive survey see [1], [6]

[3]This capacity region may also be considered to be a generalized degrees of freedom region.

Motivated by the successes of deterministic channels, we consider the deterministic cognitive radio channel. After briefly describing our channel model in Section II, in Section III we consider the high SNR deterministic approximation of the Gaussian cognitive radio channel, where we explicitly construct capacity achieving transmission schemes. We conclude and address future directions in Section IV.

II. Channel Models

A. Gaussian cognitive interference channel

We consider the cognitive radio channel model of [2], [6], a memoryless and time invariant channel with finite input alphabets $\mathcal{X}_1, \mathcal{X}_2$ and finite input alphabets $\mathcal{Y}_1, \mathcal{Y}_2$ and transition probability distribution $p(y_1 \ y_2|x_1 \ x_2)$. Encoder $i \in \{1,2\}$ has message W_i, independent uniformly distributed over $\{1, \ldots, 2^{NR_i}\}$, which it wishes to transmit to decoder i over N channel uses at rate R_i bits per channel use. Cognition is modeled by the fact that message W_2 is known at encoder 1. Standard definitions of achievable rate regions and outer bounds apply, see for example Section 3 of [6] whose notation corresponds exactly. The Gaussian cognitive radio channel is defined analogously and is shown in Figure 1 and described by the input-output relations

$$\begin{aligned} Y_1 &= h_{31}X_1 + h_{41}X_2 + Z_1, \quad Z_1 \sim \mathcal{N}(0,1) \text{ iid} \\ Y_2 &= h_{32}X_1 + h_{42}X_2 + Z_2, \quad Z_2 \sim \mathcal{N}(0,1) \text{ iid.} \end{aligned}$$

The channel inputs are subject to the usual power constraints $E[|X_i|^2] \leq P_i, \ i = 1,2$ respectively.

B. Deterministic channel for the high SNR approximation of the Gaussian cognitive interference channel

The deterministic approximation for the linear Gaussian interference network was introduced in [13] and [9]. The key idea is considering a transmitted signal X as a sequence of bits $\boldsymbol{x}$ at different power levels, which relate to the SNR according to the relationship $X = 2^{\frac{1}{2}\log SNR} \sum_{i=0}^{\infty} \boldsymbol{x}_i 2^{-i}$. If we assume that the noise Z has unitary peak power, the received signal over an additive noise link is $Y = 2^{\frac{1}{2}\log SNR} \sum_{i=0}^{\infty} \boldsymbol{x}_i 2^{-i} + \sum_{i=1}^{\infty} \boldsymbol{z}_i 2^{-i}$, where $\boldsymbol{z}$ is again the power expansion of the noise realization. The high SNR approximation ignores the carry-over of the bitwise addition, thus obtaining $Y \approx 2^n \sum_{i=1}^{n} \boldsymbol{x}_i 2^{-i} + \sum_{i=1}^{\infty} (\boldsymbol{x}_{i+n} + \boldsymbol{z}_i) 2^{-i}$. With this approximation is clear that the receiver will be able to correctly decode the first n bits sent by the transmitter, where $n = \frac{1}{2}\log(SNR)$.

Extending this notion to multiple inputs and outputs, we define

$$n_{ji} = \frac{1}{2}\log\left(|h_{ji}|^2 P_i\right), \quad m = \max_{i,j} n_{ij},$$

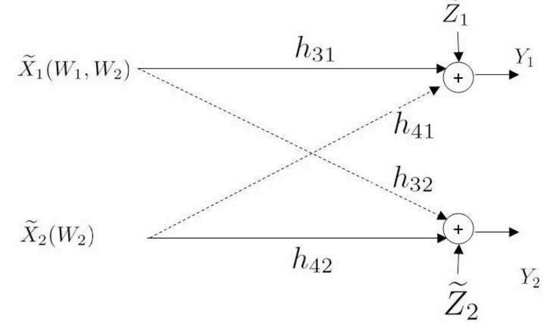

Fig. 1. Gaussian cognitive interference channel.

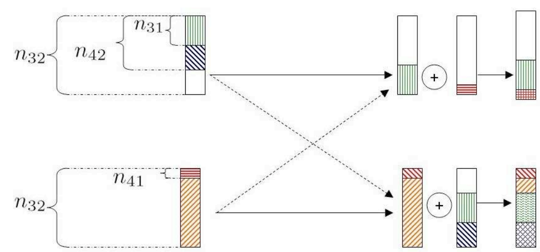

Fig. 2. Block representation of the high SNR approximation of Gaussian cognitive interference channel with $m = n_{32} > n_{42} > n_{31} > n_{41}$

and the $m \times m$ binary shift matrices $\boldsymbol{S}$ with components $\boldsymbol{S}_{ij} = \delta_{i-1,j}$ for $(i,j) \in \{1,...,m\} \times \{1,...,m\}$. The high SNR deterministic approximation to the Gaussian cognitive radio channel is then given by

$$\boldsymbol{y}_i = \mathbf{S}^{m-n_{i1}} \boldsymbol{x}_1 + \mathbf{S}^{m-n_{i2}} \boldsymbol{x}_2, \ i = 1,2. \tag{1}$$

where $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are binary vectors of length m. All operations are defined on GF(2).

We will use block representations as in Figure 2, which allow for an intuitive grasp of the effect of the shift matrices $\boldsymbol{S}^{m-n_{ij}}$. In particular, (a) the zero padding in the transmitted signal, represented as a white block, is shifted below the level of the noise over every link; (b) blocks of bits shift downward according to the SNR on that link, and (c) the interference is represented as a binary sum between the two shifted signal vectors that align at the noise lovel.

III. The capacity region of the high SNR approximation to the Gaussian cognitive radio channel

Using the high SNR approximation to the Gaussian linear interference channel in (1), we are able to derive the following capacity result:

Theorem 3.1: The capacity region of the channel in (1) is given by the set of rates (R_1, R_2) satisfying

$$0 \le R_1 \le n_{31} \quad (2)$$
$$0 \le R_2 \le \max\{n_{41}, n_{42}\} \quad (3)$$
$$0 \le R_1 + R_2 \le \max\{n_{41}, n_{42}\} + [n_{31} - n_{41}]^+ \quad (4)$$

Proof:

Outer bound: By standard arguments we have:

$$\begin{aligned} NR_1 &\le I(W_1; Y_1^N) + N\epsilon_N \\ &\le^{(a)} I(W_1; Y_1^N | W_2) + N\epsilon_N \\ &\le H(Y_1^N | W_2, X_2^N(W_2)) \\ &\quad - H(Y_1^N | W_1, W_2, X_1^N(W_1, W_2), X_2^N(W_2)) + N\epsilon_N \\ &\le^{(b)} H(Y_1^N | X_2^N(W_2)) + N\epsilon_N \\ &\le \sum_{t=1}^{N} H(Y_{1,t} | X_{2,t}) + N\epsilon_N \\ &\le N(H(\mathbf{S}^{m-n_{31}} X_1) + \epsilon_N) \le N(n_{31} + \epsilon_N) \end{aligned}$$

where (a) follows from the independence of W_1 and W_2 and (b) from the deterministic nature of the channel. Similarly,

$$\begin{aligned} NR_2 &\le I(W_1; Y_2^N) + N\epsilon'_N \\ &= H(Y_2^N) - H(Y_2^N | W_1) + N\epsilon'_N \\ &\le H(Y_2^N) - H(Y_2^N | W_1, X_1^N, X_2^N) + N\epsilon'_N \\ &\le \sum_{t=1}^{N} H(Y_{2,t}) + N\epsilon'_N \\ &\le N(H(\mathbf{S}^{m-n_{41}} X_1 + \mathbf{S}^{m-n_{42}} X_2) + \epsilon_N) \\ &\le N(\max\{n_{41}, n_{42}\} + \epsilon_N). \end{aligned}$$

For the sum rate,

$$\begin{aligned} N(R_1 + R_2) &\le I(W_1; Y_1^N) + I(W_2; Y_2^N) + N\epsilon''_N \\ &\le I(W_1; Y_1^N | W_2) + I(W_2; Y_2^N) + N\epsilon''_N \\ &\le I(W_1; Y_1^N, V_1^N | W_2)|_{V_1 = \mathbf{S}^{m-n_{41}} X_1} \\ &\quad + I(W_2; Y_2^N) + N\epsilon''_N \\ &=^{(c)} H(Y_2^N) + H(Y_1^N | W_2, X_2^N(W_2), V_1^N) + N\epsilon''_N \\ &\le H(Y_2^N) + H(Y_1^N | X_2^N(W_2), V_1^N) + N\epsilon''_N \\ &\le \sum_{t=1}^{N} H(Y_{2,t}) + H(Y_{1,t} | X_{2,t}, V_{1,t}) + N\epsilon''_N \\ &\le^{(d)} N(\max\{n_{41}, n_{42}\} + [n_{31} - n_{41}]^+ + \epsilon''_N) \end{aligned}$$

where (c) follows from the deterministic nature of the channel, and (d) since $H(Y_{1,t} | X_{2,t}, V_{1,t}) \le H(\mathbf{S}^{m-n_{31}} X_{1,t} | \mathbf{S}^{m-n_{41}} X_{1,t}) \le [n_{31} - n_{41}]^+$.

TABLE I

POSSIBLE EXPRESSIONS FOR THE OUTER BOUND OF THEOREM 3.1

$n_{41} \ge n_{31}$	$n_{41} \ge n_{42}$	$\begin{cases} R_1 \le n_{31} \\ R_1 + R_2 \le n_{41} \end{cases}$
strong interf.	$n_{41} < n_{42}$	$\begin{cases} R_1 \le n_{31} \\ R_1 + R_2 \le n_{42} \end{cases}$
$n_{41} < n_{31}$	$n_{41} \ge n_{42}$	$\begin{cases} R_2 \le n_{41} \\ R_1 + R_2 \le n_{31} \end{cases}$
weak interf.	$n_{41} < n_{42}$	$\begin{cases} R_1 \le n_{31} \\ R_2 \le n_{41} \\ R_1 + R_2 \le n_{31} + n_{42} - n_{41} \end{cases}$

Achievability:

The outer bound rate region is a pentagon if $n_{31} \le n_{41}$ or $n_{31} > n_{41} \ge n_{42}$, and is a rhomboid otherwise. Depending on the relative value of n_{41} and n_{42}, and of n_{31} and n_{41}, the outer bound can have one of the four expressions listed in Table I. We analytically demonstrate the achievability of the first ($n_{41} \ge n_{31}$, $n_{41} \ge n_{42}$) and last ($n_{41} < n_{31}$, $n_{41} < n_{42}$) cases, and illustrate the middle two cases in Figures 4 and 5. The achievability of corner points involving one of the rates being zero is a trivial flow argument. We only show the achievability of the corner points in which both the rates are non-zero since time sharing between corner points will give any rate in the convex hull.

Case $n_{41} \ge n_{31}$ (strong interference) and $n_{41} \ge n_{42}$: In this case the achievable region is a rhomboid so we only have to show the achievability of the point $(R_1, R_2) = (n_{31}, n_{41} - n_{31})$. A block representation of this scenario will help understanding the transmission scheme scheme that achieve this point. The least significant bits of the transmitted signal will by erased from the every link's shift matrix. Referring to Figure 3 it is clear that if the transmitter 2 is silent transmitter 1 can send n_{31} bits to decoder 2 and $n_{41} - n_{31}$ bits to decoder 2. Let $\boldsymbol{b}_1^{n_{31}}$ be a vector containing n_{31} bits to be decoded by decoder 1 and $\boldsymbol{b}_2^{n_{31}-n_{41}}$ be a vector containing $n_{41} - n_{31}$ bits to be decoded by decoder 2. Then if $\boldsymbol{x}_1 = [\boldsymbol{b}_1^{n_{31}} \ \boldsymbol{b}_2^{n_{41}-n_{31}} \ \mathbf{0}^{m-n_{41}}]^T$, $\boldsymbol{x}_2 = \mathbf{0}_m$ the channel output will be

$$\begin{aligned} y_1 &= S^{m-n_{31}} \boldsymbol{x}_1 = [\mathbf{0}^{m-n_{31}} \ \boldsymbol{b}_1^{n_{31}}]^T \\ y_2 &= S^{m-n_{41}} \boldsymbol{x}_1 = [\mathbf{0}^{m-n_{41}} \ \boldsymbol{b}_1^{n_{31}} \ \boldsymbol{b}_2^{n_{41}-n_{31}}]^T \end{aligned}$$

so decoder 1 can correctly decode n_{31} bits and decoder 2 can decode $n_{41} - n_{31}$ bits.

Case $n_{31} > n_{41}$ (weak interference) and $n_{42} > n_{41}$: In this case we prove the achievability of the two points:

$$\begin{aligned} A &= (R_1, R_2) = (n_{31}, n_{42} - n_{41}) \\ B &= (R_1, R_2) = (n_{31} - n_{41}, n_{42}) \end{aligned}$$

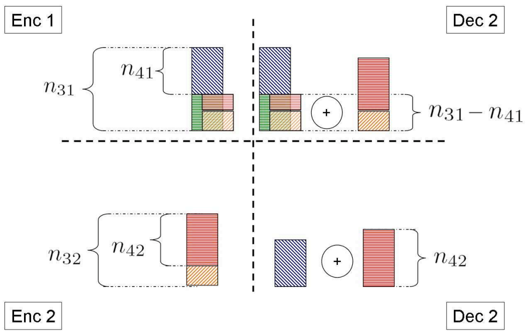

Fig. 3. Block representation of the high SNR approximation of Gaussian cognitive interference channel with $n_{41} \geq n_{31}$ and $n_{41} \geq n_{42}$

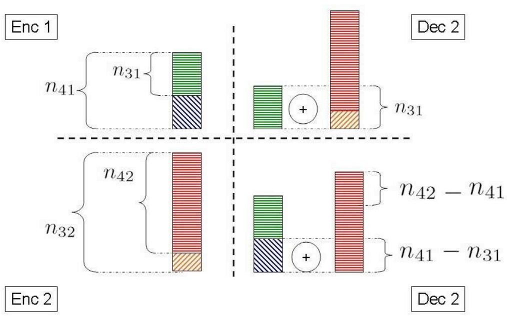

Fig. 4. Block representation of the high SNR approximation of Gaussian cognitive interference channel with $n_{42} > n_{41} \geq n_{31}$

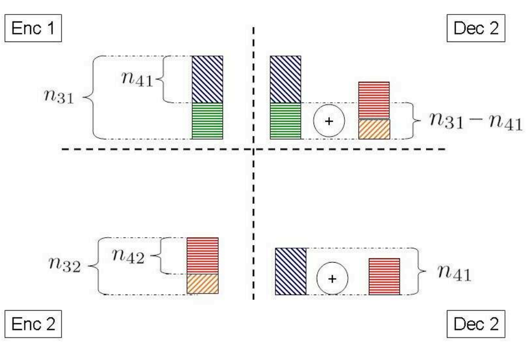

Fig. 5. Block representation of the high SNR approximation of Gaussian cognitive interference channel with $n_{31} > n_{41} \geq n_{42}$

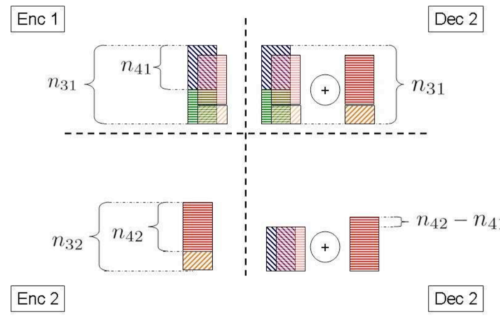

Fig. 6. Block representation of the high SNR approximation of Gaussian cognitive interference channel with $n_{31} > n_{41}$ and $n_{42} > n_{41}$

Consider Figure 6 to see how we can use the condition $n_{42} > n_{41}$ to send additional $n_{42} - n_{41}$ from transmitter 2 to receiver 2 over the interference created by encoder one at receiver 2 for point A. In the vector representation we have

$$
\begin{aligned}
\boldsymbol{u}_1 &= [\boldsymbol{b}_1^{n_{31}} \ \mathbf{0}^{m-n_{31}}]^T \\
\boldsymbol{u}_2 &= [\boldsymbol{b}_2^{n_{42}-n_{41}} \ \mathbf{0}^{m-n_{42}+n_{41}}]^T \\
\boldsymbol{x}_1 &= \boldsymbol{u}_1 + \boldsymbol{S}^{-(n_{32}-n_{31})}\boldsymbol{u}_2 = [\underline{\boldsymbol{b}_1}^{n_{31}} \ \mathbf{0}^{m-n_{31}}]^T \\
\boldsymbol{x}_2 &= \boldsymbol{u}_2.
\end{aligned}
$$

Notice that $\underline{\boldsymbol{b}_1}^{n_{31}}$ is the pre-coded version of $\boldsymbol{b}_1^{n_{31}}$ against the interference generated by encoder 2. The channel outputs are then

$$
\begin{aligned}
y_1 &= \boldsymbol{S}^{m-n_{31}}\boldsymbol{x}_1 + \boldsymbol{S}^{m-n_{32}}\boldsymbol{x}_2 \\
&= [\mathbf{0}^{m-n_{31}}\boldsymbol{b}_1^{n_{31}}]^T \\
y_2 &= \boldsymbol{S}^{m-n_{41}}\boldsymbol{x}_1 + \boldsymbol{S}^{m-n_{42}}\boldsymbol{x}_2 \\
&= [\mathbf{0}^{m-n_{41}} \ \underline{\boldsymbol{b}_1}^{n_{41}}]^T + [\mathbf{0}^{m-n_{42}} \ \boldsymbol{b}_2^{n_{42}-n_{41}}\mathbf{0}^{n_{41}}] \\
&= [\mathbf{0}^{m-n_{42}} \ \boldsymbol{b}_2^{n_{42}-n_{41}} \ \underline{\boldsymbol{b}_1}^{n_{41}}],
\end{aligned}
$$

which shows that A is achievable.

For point B we have the situation depicted in Figure 7: in this scenario encoder one cannot pre-code against the whole interference because this will create interference at the second decoder. The strategy that achieves the outer bound is then for encoder 2 to transmit at capacity and for encoder 1 to precode for the interference only in the bits that are not received at the second decoder. This translates into:

$$
\begin{aligned}
\boldsymbol{u}_1 &= [\mathbf{0}^{n_{41}} \ \boldsymbol{b}_1^{n_{31}-n_{41}} \ \mathbf{0}^{m-n_{31}}]^T \\
\boldsymbol{u}_2 &= [\boldsymbol{b}_2^{n_{42}} \ \mathbf{0}^{m-n_{42}}]^T \\
\boldsymbol{x}_1 &= \boldsymbol{u}_1 + \boldsymbol{S}^{-(n_{32}-n_{31})}\boldsymbol{u}_2 = [\mathbf{0}^{n_{41}} \ \underline{\boldsymbol{b}_1}^{n_{31}-n_{41}} \ \mathbf{0}^{m-n_{31}}]^T \\
\boldsymbol{x}_2 &= \boldsymbol{u}_2
\end{aligned}
$$

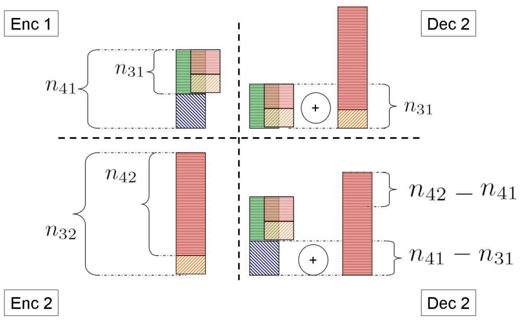

Fig. 7. Block representation of the high SNR approximation of Gaussian cognitive interference channel with $n_{31} > n_{41}$ and $n_{42} > n_{41}$

where again $\underline{\boldsymbol{b}_1}^{n_{31}-n_{41}}$ are the pre-coded bits from transmitter 1. The channel output is then:

$$\begin{aligned} y_1 &= \boldsymbol{S}^{m-n_{31}}\boldsymbol{x}_1 + \boldsymbol{S}^{m-n_{32}}\boldsymbol{x}_2 \\ &= [\mathbf{0}^{m-n_{31}+n_{41}} \ \boldsymbol{b}_1^{n_{31}-n_{41}}]^T \\ y_2 &= \boldsymbol{S}^{m-n_{41}}\boldsymbol{x}_1 + \boldsymbol{S}^{m-n_{42}}\boldsymbol{x}_2 \\ &= [\mathbf{0}^{m}]^T + [\mathbf{0}^{m-n_{42}} \ \boldsymbol{b}_2^{n_{42}}] \\ &= [\mathbf{0}^{m-n_{42}} \ \boldsymbol{b}_2^{n_{42}}] \end{aligned}$$

which shows the achievability of point B. ■

Remark: From the above derivation is clear that the outer bound is valid for a more general class deterministic cognitive channel. In particular, the capacity region for a deterministic cognitive channel for which $Y_1 = f_1(X_1, X_2)$, $Y_2 = f_2(V_1, X_2)$, $V_1 = g(X_1)$ such that $H(Y_2|X_2) = H(V_1)$, for some deterministic functions f_1, f_2 and g, is given by the union over all distributions $p_{X_1X_2}$ of the rate pairs (R_1, R_2) satisfying:

$$\begin{aligned} &R_1 \leq H(Y_1|X_2) \\ &R_2 \leq H(Y_2) \\ &R_1 + R_2 \leq H(Y_2) + H(Y_1|X_2, V_1). \end{aligned}$$

IV. Conclusion

In this work we derived the capacity region of a class of deterministic cognitive channels which approximate Gaussian cognitive channels in the high SNR regime. We first developed a general outer bound and then showed achievable schemes for weak and strong interference separately.

We then showed that our outer bound holds for any deterministic cognitive channel for which $Y_1 = f_1(X_1, X_2)$ and $Y_2 = f_2(g(X_1), X_2)$ such that $H(Y_2|X_2) = H(g(X_1))$. Under certain conditions, our outer bound can be showed to be easily achievable by a superposition scheme requiring the cognitive decoder to decode the primary messages too. We are currently investigating achievable scheme based on Gelfand-Pinsker binning [14, Th.1] to show that our outer bound is indeed the capacity region.

Our outer bound for deterministic cognitive channels can be straightforward generalized to obtain an outer bound for Gaussian cognitive channels. An interesting open question is whether a simple choice of the auxilary random variables in [14, Th.1] is within a finite gap of our outer bound. Preliminary investigation suggests that the answer is in the positive.

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