Achieving Fairness in Distributed Cognitive Radio Networks Using a Timer Mechanism

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Abstract—We consider the issue of fairness in scheduler design for distributed cognitive radio networks (CRN). We first distinguish between intra- and inter-class fairness as a result of hierarchical resource access right in a secondary spectrum allocation regime. Further, we develop a timer mechanism to achieve round-robin, max-min and proportional fairness for intra-class scheduling in a CRN. The case of inter-class fairness is a addressed by defining a time bias to prioritize the resource access of primary links. Numerical results show that proportional fair scheduling sustains a good level of performance for both intra and inter-class fairness.

Index Terms—Fairness, Scheduling, Spectrum Sharing, Virtual Timer, Cognitive Radio Networks.

I. INTRODUCTION

In the span of a decade, Cognitive Radio (CR) has evolved from a theoretical concept [1] towards a real-life communication platform serving a variety of applications. Standardization efforts, such as by IEEE 802.22 [2] and IEEE SCC41 [3], together with favorable regulatory reforms such as by the Federal Communication Commission (FCC) [4] and similar bodies in other countries, are paving the way for a wide spread deployment of this promising technology. The main advantage of exploiting CRs is to facilitate accessing underutilized licensed spectrum bands without imposing prohibitive interference to the incumbent technologies. As such a CR Network (CRN) should determine existence of spectrum holes, determines its interfering effect on the existing primary users of the band and accordingly adapts its transmission characteristics. The scheduling policy of a CRN is thus affected not only by the RF environment within the CRN but also by the radio scene containing the primary network too.

Information-theoretic capacity limits of a CR channel was first studies by [5]. Further, the capacity of a link under received-power constraint at a third party receiver assuming an AWGN channel, pertinent to primary-secondary spectrum sharing scenarios, was first analyzed in [6]. The extension of this analysis to fading channels was proposed in [7]. Scheduling mechanisms to approach these established CR capacity limits were also studied in the literature. The existence of the so-called interference diversity gain when the received power limit at the primary receiver is on the average power as opposed to the instantaneous received power was determined by [8]. Several other resource allocation strategies for CRNs are proposed including [9], [10].

The issue of fairness in designing schedulers for CRNs has been relatively less explored. Most studies in the literature, including [11] and [12], simply focus on the issue of fairness in resource allocation within the CRN, in a similar manner to legacy networks. So far no quantitative model to gauge the fairness in interaction of primary and secondary networks has been proposed in the literature. The aim of this paper is to establish a framework to quantify intra and inter-class fairness pertinent to cognitive spectrum access regimes. Furthermore, mechanisms to materialize fair resource allocation strategies in distributed CRNs will be developed in this study.

Our focus in this study is on scenarios where the secondary network employs a similar PHY standard to the primary users of the band. Further, to utilize the licensed resources, CRN is synchronized to the primary network in the resource allocation process. The underlying assumptions in this setting can capture for instance scenarios of femto-cell macro-cell coexistence in an UMTS Long Term Evolution (LTE) scenario. The plugand-play nature of femto-cell base-stations makes detailed cell planning, widely employed in previous generations of cellular technologies, a challenging task in LTE context. On the other hand, if femto-cells posses CR capabilities they can detect the transmission of UMTS macro-cell and neighboring femtocells and avoid interfering with them on a secondary spectrum access basis. In this scenario the service provider is able to set specific network control parameters, as will be discussed later in the paper, which determine the level of fairness between the primary (macro-cell) and secondary (femto-cell) users of the band. It is therefore understood that primarysecondary spectrum access scenarios, such as accessing TV white spaces or radar bands, in which the primary networks are not communication systems and can not accommodate primary-secondary interaction control mechanisms are out of the scope of this study.

The rest of this paper is organized as follows. First we introduce our assumptions and notation in Section II. The problem formulation for intra-class fair scheduling policies are introduced in Section III. The case of inter-class cognitive fairness is addressed in Section IV. Extensive numerical analysis of the proposed scheduling schemes are presented in

Section V, before the paper concludes in Section VI.

II. PRELIMINARIES: PROBLEM ASSUMPTIONS

We assume there exist in total K links in the network; a link refers to a transmitter paired with a receiver. Links belonging to the primary network are denoted by the set U_p and those in the secondary network are denoted by U_s . Depending on the context, some links might have a shared transmitter, e.g., in a Broadcast Channel (BC), or a shared receiver, e.g., in a Multiple Access Channel (MAC). We propose usage of a timer-based mechanism to achieve the resource allocation goals of each link.

Definition 1: a timer-based mechanism in this paper refers to a distributed means of resource allocation whereby the resource admission control is realized by setting specific timer values for each coexisting link. These timers are set in a distributed fashion and depend only on local information available at that node. Upon the expiry of this virtual timer, which may or may not be dependent on the resource-block value, the given link may initiate its resource access procedure.

A. Resource Access

The set of all available resource blocks, shared among the coexisting links, is denoted by $\mathcal{R} = \{r_1, r_2, \cdots, r_N\}$. While the resource blocks may correspond to time slots, frequency bands, codes or any combination of them, we limit the scope of the paper to the case of OFDMA-based systems in which the resource blocks are sub-channels. The resources allocated to link *i* comprise the set \mathcal{R}_i , where $\mathcal{R}_i \subseteq \mathcal{R}$, $\forall i \in \{1, 2, \ldots, K\}$. The resources allocated to all links, except link *i* are denoted by \mathcal{R}_{-i} , where $\mathcal{R}_{-i} \subseteq \mathcal{R}$.

In this paper we only address scenarios where $\mathcal{R}_i \cap \mathcal{R}_{-i} = \emptyset$, i.e., each sub-channel is exclusively allocated to one link, primary or secondary, thus alleviating mutual interference in the network. Further, assume each link has ordered the set of sub-channels based on their channel gains into an ordered set \mathcal{G}_i , the *n*th element of which is defined as $\mathcal{G}_i[n] = g_{i,n}$, where $g_{i,n}$ is the channel gain of sub-channel *n* for link *i*.

B. Resource Allocation Time Frame

We assume the resource allocation is repeated periodically and all the links are synchronized in the resource allocation process. The possibly infinite set of the discrete time indices of primary network resource allocation process are given by,

$$t \in \left(\cdots, t_0, t_0 + \frac{1 \times \tau}{\tau}, t_0 + \frac{2 \times \tau}{\tau}, \cdots\right), \qquad (1)$$

where t_0 is an arbitrary resource access instant. As mentioned before, secondary network synchronizes its resource allocation process with the primary. Designing the synchronization mechanism for our distributed network setting is out of the scope of this paper; existing work in this direction may be found for instance in [13]. We note that the result obtained here may thus be seen as idealized, providing upper bounds on what may be achieved in the absence of synchronization.

C. Utility Function and Related Definitions

Define the utility function of each link as its throughput, given by

$$\mathcal{U}_{i}\left(\boldsymbol{\mathcal{R}}_{i},\boldsymbol{\mathcal{R}}_{-i}\right) = R_{i} = \sum_{n \in \boldsymbol{\mathcal{R}}_{i}} \frac{B}{N} \log\left(1 + \frac{p_{i,n}g_{i,n}}{\sigma_{i}^{2}}\right), \quad (2)$$

where $p_{i,n}$ is the transmission power, $g_{i,n}$ is the channel gain between the transmitter and the receiver in link *i* and subchannel *n* and σ_i^2 is the noise power at the receiver *i*. We assume a block fading channel model whereby the channel gains are assumed fixed in each period, but randomly vary from one period to another. Two related functions of utility, exploited in later sections, are defined below.

Definition 2: m-slot expected utility is defined as

$$\mathbb{E}_{t=m} \left\{ \mathcal{U}_{i} \left(\mathcal{R}_{i}, \mathcal{R}_{-i} \right) \right\} = \begin{cases} \frac{1}{m} \sum_{t=t_{0}-m}^{t_{0}-1} \mathcal{U}_{i} \left(\mathcal{R}_{i} \left(t \right), \mathcal{R}_{-i} \left(t \right) \right), & \text{If } m \neq 0, \\ \mathcal{U}_{i} \left(\mathcal{R}_{i} \left(t_{0} \right), \mathcal{R}_{-i} \left(0 \right) \right), & \text{If } m = 0, \end{cases}$$
(3)

for any $m \in \mathbb{N}^+$ and where t_0 is the arbitrary starting point of averaging window.

Definition 3: Maximal utility of link i is defined as

$$\widehat{\mathcal{U}}_{i} = \sum_{n \in \widehat{\mathcal{R}}_{i}} \frac{B}{N} \log \left(1 + \frac{p_{i,n}g_{i,n}}{\sigma_{i}^{2}} \right), \qquad (4)$$

where $\widehat{\mathcal{R}}_i \subseteq \mathcal{R}$ denotes the (hypothetic) optimum set of resource blocks for link *i* assuming no other links were competing to access those resource blocks.

III. INTRA-CLASS FAIR STRATEGIES

A. Round-Robin Fairness

A Round-Robin Fair (RRF) policy allocates a random but equal-size set of resource blocks to each link, i.e.,

$$\mathbb{E}_{t=m}\left\{\left|\mathcal{R}_{1}\left(t\right)\right|\right\} = \mathbb{E}_{t=m}\left\{\left|\mathcal{R}_{2}\left(t\right)\right|\right\} = \dots = \mathbb{E}_{t=m}\left\{\left|\mathcal{R}_{K}\left(t\right)\right|\right\},\tag{5}$$

where $|\mathcal{X}|$ denotes the cardinality of the set \mathcal{X} .

In practice a straight forward solution to address constraint (5) is used, whereby at each resource allocation period all the available resource blocks are allocated to one randomly selected link, resulting in a linearly increasing length of the averaging window with respect to the number of coexisting links, i.e., m = K in (5), where K is the total number of links.

A more complex RRF policy can be developed if at each resource allocation period several links are scheduled such that on a shorter averaging window the RRF policy is achieved. An interesting extension of this multiple link scheduling approach is when the averaging window attains its smallest possible value, i.e., $m \rightarrow 1$ in (5). This extreme averaging choice results in an *instantaneous* RRF resource allocation strategy and can be implemented by allocating

$$|\mathcal{R}_{inst,i}| = \left\lfloor \frac{N}{\mathcal{K}} \right\rfloor, \text{ if } \mathcal{K} \le \min(K, N),$$
 (6)

resource blocks to a given link *i*, where $i \in \{1, 2, ..., K\}$, $\mathcal{R}_{inst,i}$ is the set of resources allocated to this link using instantaneous RRF policy and \mathcal{K} is the number of scheduled links. Clearly, $\mathcal{K} \leq K$ and $\frac{N}{\mathcal{K}} \geq 1$ for the instantaneous RRF policy to be feasible.

Timer Mechanism. In order to accomplish the RRF resource allocation in a distributed manner, a virtual timer $t_{i,n} = C_{RRF}^{i,n}$ is selected by link *i* for accessing sub-channel *n*, where $C_{RRF}^{i,n}$ is a random variable from a uniform distribution over the interval $[0, c_{max}]$. The value of c_{max} is chosen such that the longest timer value is short enough compared with the rate of change in sub-channel gains.

Depending on the desired period of RRF operation, i.e., m in (5), which consequently determines the value of \mathcal{K} in (6), different number of links will be able to access the shared band in each resource allocation period. After expiry of its timer, each link will check the shared band to determine how many links have already been scheduled. This measurement may be based on detecting the unique ID of each link, broadcasted in their occupied sub-channels. The new link will only access idle sub-channels if the number of already scheduled links is at most $N - \lfloor \frac{N}{\mathcal{K}} \rfloor$. Each scheduled link will not initiate its timer again for a period of $\mathcal{K} \times \tau$ (sec).

B. Max-Min Fairness

A Max-Min Fair (MMF) policy is the result of a resource allocation such that $U_i(\mathcal{R}_i, \mathcal{R}_{-i}), \forall i \in \{1, 2, ..., K\}$, can not be increased without decreasing the utility of at least another link, say *j*, who has an equal or lower utility value, i.e., $U_j(\mathcal{R}_j, \mathcal{R}_{-j}) \leq U_i(\mathcal{R}_i, \mathcal{R}_{-i})$ [14].

Assuming a fixed transmission power, for mathematical tractability, the MMF policy can be implemented using

Maximize
$$\min_{i} \left(\sum_{n \in \mathcal{R}_{i}} \frac{B}{N} \log \left(1 + \frac{Pg_{i,n}}{\sigma_{i}^{2}} \right) \right),$$
 (7)

subject to

$$\sum_{n \in \mathcal{R}_i} P \le P_{max,i}, \quad \forall \ i \in \{1, 2, \dots, K\}.$$
(8)

Timer Mechanism. We note that MMF policy penalizes links with higher utility in favor of those with lower utility. Therefore, the virtual timer values should be selected such that lower utility will have priority in accessing resource blocks. To this end, we can use the maximal utility defined in (4).

Let us define $\underline{i} = \underset{i}{\operatorname{argmin}} \widehat{U}_i$. It is obvious that link \underline{i} which has the lowest \widehat{U}_i in the absence of coexisting links, will also have the lowest utility in presence of coexisting links, since with a high probability there might exist at least one link with

a better channel condition in at least one of the $\left|\widehat{\mathcal{R}}_{\underline{i}}\right|$ resource blocks of link \underline{i} , denying link \underline{i} of accessing that resource block.

Therefore, a reasonable virtual timer value to achieve MMF policy is given by

$$t_i = \mathcal{C}_{MMF} \times \widehat{U}_i, \quad \forall i \in \{1, 2, \dots, K\},$$
(9)

where C_{MMF} is a constant which keeps the value of t_i short enough. To access the resources, all links start their virtual timer, given by (9) and upon expiry of their timer, they try to access $|\widehat{\mathcal{R}}_i|$ sub-channels out of the *N* available bands. If any of the candidate sub-channels are already occupied, link *i* will try to access one of the following lower-order sub-channels from the set of channels, \mathcal{G}_i . This process continues to either access enough sub-channels by each link or if there are no more idle bands available.

C. Proportional Fairness

The last fairness paradigm studied in this paper is a Proportional-Fair (PF) policy [15] which is defined as the result of a choice of resources $(\mathcal{R}_i, \mathcal{R}_{-i}), \forall i \in \{1, 2, ..., K\}$, such that for any other resource allocation $(\mathcal{R}_i^*, \mathcal{R}_{-i}^*)$ we have

$$\sum_{i=1}^{K} \frac{\mathcal{U}_{i}\left(\boldsymbol{\mathcal{R}}_{i}^{*}, \boldsymbol{\mathcal{R}}_{-i}^{*}\right) - \mathcal{U}_{i}\left(\boldsymbol{\mathcal{R}}_{i}, \boldsymbol{\mathcal{R}}_{-i}\right)}{\mathcal{U}_{i}\left(\boldsymbol{\mathcal{R}}_{i}, \boldsymbol{\mathcal{R}}_{-i}\right)} \leq 0.$$
(10)

From (10), we see that this fairness policy results in an equilibrium where the sum of the proportional changes in the utility of all network links would be equal to or smaller than the current allocation.

One can reach this equilibrium through the use of a normalized expected utility perspective [16]. The expected utility, for a given, fixed m is defined in (3). Then, define the normalized expected utility of link i as

$$\bar{\mathcal{U}}_{i}\left(\boldsymbol{\mathcal{R}}_{i},\boldsymbol{\mathcal{R}}_{-i}\right) = \frac{\widehat{\mathcal{U}}_{i}\left(\widehat{\boldsymbol{\mathcal{R}}}_{i},\widehat{\boldsymbol{\mathcal{R}}}_{-i}\right)}{\mathbb{E}_{t=m}\left\{\mathcal{U}_{i}\left(\boldsymbol{\mathcal{R}}_{i},\boldsymbol{\mathcal{R}}_{-i}\right)\right\}}.$$
(11)

where the maximal utility and m-slot expected utility for link i are defined by (4) and (3), respectively.

Timer Mechanism. The PF scheduling policy prioritizes links with higher value of normalized expected utility as given by (11). Hence, the corresponding virtual timer value for link *i* is derived as,

$$\mathcal{L}_{i} = \frac{\mathcal{C}_{PF}}{\bar{\mathcal{U}}_{i}\left(\mathcal{R}_{i}, \mathcal{R}_{-i}\right)}, \quad \forall \ i \in \{1, 2, \dots, K\}, \qquad (12)$$

where C_{PF} is an arbitrary constant to scale the value of timers as appropriate. Each link will check the availability of subchannels after expiry of its timer and will follow a selfish resource access procedure if idle sub-channels are available. Finally, at the end of resource allocation period *m*, the expected utility in (3) is updated by [16],

$$\mathbb{E}_{t=(m+1)} \left\{ \mathcal{U}_i\left(\mathcal{R}_i, \mathcal{R}_{-i}\right) \right\} = \left(1 - \frac{1}{m \times \tau}\right) \mathbb{E}_{t=m} \left\{ \mathcal{U}_i\left(\mathcal{R}_i, \mathcal{R}_{-i}\right) \right\} + \frac{1}{m \times \tau} \mathcal{U}_i\left(\mathcal{R}_i, \mathcal{R}_{-i}\right).$$
(13)

IV. PROBLEM FORMULATION: INTER-CLASS FAIR STRATEGIES

Resource allocation in a hierarchically-shared resource domain, as manifested by secondary spectrum access in cognitive communication, brings about a new dimension of fairness, i.e., the interaction of primary and secondary networks. In a synchronized spectrum sharing setting, such as coexistence of femto-cell and macro-cell LTE base-stations, the primary network can set up a time advance in resource access process to prioritize the primary links. The inter-class fairness is influenced by extent to which the primary network increase/decreases its priority in accessing the channel, with respect to the secondary network. To this end we can define the normalized *time bias* towards primary, $\Upsilon_p \in [0, \tau_{p,cnt}]$, as shown in Fig. 1, where $\tau_{p,cnt}$ and $\tau_{s,cnt}$ denote the contention time for primary and secondary links, respectively. During the contention period, each network follows specific resource allocation guidelines to sustain certain intra-class fairness goals. After determining the resource allcation in the contention period, allocated links will proceed to exchange traffic during access period, denoted by $\tau_{p,accs}$ and $\tau_{s,accs}$.

To capture the fairness-efficiency trade-off in inter-class domain, we define the inter-class fairness index by

$$\mathcal{I}_{inter-class}\left(\Upsilon_{p}\right) = \frac{\sum\limits_{i \in \mathcal{U}_{s}} \mathcal{U}_{i}\left(\mathcal{R}_{i}, \mathcal{R}_{-i}\right)}{\sum\limits_{i' \in \mathcal{U}_{p}} \mathcal{U}_{i'}\left(\mathcal{R}_{i'}, \mathcal{R}_{-i'}\right)}.$$
 (14)

Note that the utility value of primary and secondary links will depend on the efficient or fair resource allocation strategy that is used *intra-class*.

We can also use a time-average of inter-class fairness index defined as

$$\bar{\mathcal{I}}_{inter-class}\left(\Upsilon_{p}\right) = \mathbb{E}_{t=m}\left\{\mathcal{I}_{inter-class}\left(\Upsilon_{p}\right)\right\}.$$
 (15)

Therefore, the tweaking of two parameters will affect the interclass fairness as defined in (15): these are the bias towards primary users, Υ_p and the window of averaging, *m*. As will be shown by numerical results, the aforementioned parameters will only demonstrate a rough level of control over interclass fairness, due to the decoupling of intra and inter-class scheduling policies in an interweave cognitive regime.

V. NUMERICAL RESULTS

The default settings for the numerical results reported in this section are as follows. We consider an isolated square cell of the size $1 \times 1 \text{ Km}^2$, where coexisting links are randomuniformly distributed over the space. There are 25 subchannels shared among coexisting links, where for tractability of results, the link throughputs are normalized to the subchannel bandwidth. The channel pathloss with a patloss exponent equal to 3, 3-dB log-normal shadowing and Rayleigh fading with unit variance are taken into account. Each link has 23 dBm transmit power limit. The results are averaged over 5000 Monte-Carlo realizations of the scenario at hand.

To compare the developed distributed scheduling policies from a fairness perspective, we follow a more comprehensive Cumulative Distribution Function (CDF) view to demonstrate the distribution of utility throughout the network in Fig. 2, when K = 25. We have normalized the utility of links with average network utility and note that a fair distribution of utility in the network is achieved when utility is nearly uniformly distributed throughout the network. As demonstrated by Fig. 2 instantaneous RRF boasts the nearset utility distribution to uniform distribution of utility in the network and thus may be considered to be the fairest practically-feasible resource access regime out of our alternatives. The PF policy also fares well from a fairness perspective, while both MMF and average RRF achieve a similarly low fairness level as seen in Fig. 2.

Furthermore, the effect of the averaging window size, *m*, on the feasible Jain's fairness index of RRF and PF schemes is demonstrated in Fig. 3. For shorter averaging periods, RRF policy approaches instantaneous RRF yielding a higher fairness level whereas longer averaging window values mimics average RRF with a significantly lower fairness index values. On the other hand, the performance of the PF policy is less sensitive, but nevertheless decreasing, with increasing averaging window length, as clear from Fig. 3.

To analyze the inter-class fairness, we limit our attention to scenarios where both primary and secondary networks follow similar *intra-class* scheduling policies. The effect of normalized time-bias towards primary links, i.e., Υ_p , on the average inter-class fairness index defined by (15), is depicted in Fig. 4. In this figure we have added a selfish scheduling policy for comparison purposes, outlined in [17]. Somewhat surprisingly, the selfish intra-class scheduling policy achieves the highest average inter-class fairness with $\overline{\mathcal{I}}_{inter-class} (\Upsilon_p) \leq 1$, with a decreasing utility as $\Upsilon_p \to 1$. The MMF can only sustain a non-zero average inter-class fairness index when $\Upsilon_p \leq 0.25$. Note that the performance of RRF does not depend on the feasible utility of links and thus is independent of Υ_p .

VI. CONCLUSIONS

A large number of studies have covered the resource allocation problem in a cognitive radio networks (CRN) recently. However, the issue of fairness in scheduler design for CRNs is relatively less explored. There is an inherent difference in defining fairness for CRN compared with legacy systems due to secondary spectrum access of CRNs. To this end we can differentiate the intra- and inter-class fairness. Further, achieving specific intra and inter-class fairness policies in a distributed CRN architecture requires novel scheduling mechanisms. In this paper we demonstrated feasibility of exploiting a timer mechanism to achieve round-robin fair (RRF), max-min fair (MMF) and proportional-fair (PF) intra-class scheduling. Also, we proposed a time bias measure to quantify the interaction of primary and secondary networks. Our numerical results shows that instantaneous RRF achieves the highest intra-class fairness while selfish scheduling sustains the highest inter-class fairness level. A reasonable comprise can be achieved by selecting proportional fairness with both a high level of intra- and interclass fairness.

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Fig. 1. The resource allocation period for primary and secondary links.

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Fig. 2. Comparison of fairness capability of various scheduling techniques through distribution of utility in the network.



Fig. 3. The average achievable fairness level, as measured by Jain's fairness index, as a function of time averageing window m.



Fig. 4. The average cognitive fairness index for different scheduling policies. The number of primary and secondary links is assumed similar and is equal to 25.