Cognitive Multiple Access Networks

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***Abstract*— A cognitive radio can sense the transmission of other users in its environment and possibly extract the corresponding messages. It can use this information to transmit over the same channel while reducing interference from, and to other users. In this paper, we define *inter/intra*-cluster *competitive, cooperative*, and *cognitive* behavior in wireless networks. We define *inter-cluster cognitive behavior* as simultaneous transmissions by two or more clusters in which some clusters know the messages to be transmitted by other clusters, and so can act as relays or use a Gel'fand-Pinsker coding-like technique to mitigate interference. We construct an achievable region for the inter-cluster behavior of two multiple access channels. In the Gaussian case, we compare our achievable region to that of competitive behavior as well as that of cooperative behavior.**

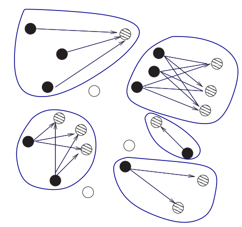

Fig. 1. A wireless network consisting of cognitive and/or non-cognitive devices. Black nodes are senders ($\mathbf{S_i}$), striped nodes are receivers ($\mathbf{R_i}$), and white nodes are inactive. A directed edge is placed between each desired sender-receiver pair at each point/period in time. Here the active links have been decomposed into subsets $(\mathbf{S_i}, \mathbf{R_i})$ of *generalized MIMO channels.*

I. Motivation

Current FCC measurements [9] indicate that about 90% of the time certain portions of licensed spectra remain unused. However, the emergence of cognitive radio technology [7], [13], along with recent FCC announcements on secondary wireless spectrum licensing [8], promise significant improvements to spectral efficiency. To do so, new proposals suggest allowing users to sense voids in the spectrum and, under certain conditions, opportunistically employ them. In current proposals, devices can only transmit or "borrow" spectrum when it is unused [12].

In [4], [5], we proposed a more flexible approach. We assumed that cognitive radios could sense the presence and obtain the messages of other already transmitting users. Then, rather than waiting for a silence in the spectrum, we suggested that these incoming users simultaneously transmit with the already active user(s) and employ Gel'fand-Pinsker coding-like techniques to mitigate the known interference. We demonstrated an achievable region for such a *cognitive radio channel*: a 2 sender, 2 receiver case in which one user knows the other user's message non-causally and simultaneously transmits. We also suggested protocols which allow the second user to causally obtain the first user's message, and compared these to the non-causal (or genie-aided) case.

In this paper, we extend our results and motivate our problem from a more global wireless network perspective. We demonstrate an achievable region for a specific building block of the global picture: the *cognitive radio multiple access channel*.

A. Cognitive Network Decomposition

In this work, we consider an arbitrary wireless network consisting of cognitive and (possibly) non-cognitive radio devices. At each point/period in time, certain devices in sending mode wish to transmit to other devices in receiving mode (a radio device cannot simultaneously send and receive data). At each point/period in time, the wireless network can be represented as a graph by drawing a directed edge between every sender-receiver pair, as in Fig. 1. We then have the following obvious lemma.

Lemma 1: All active links of a cognitive network can be decomposed into sets of *generalized MIMO channels* $(\mathbf{S_i}, \mathbf{R_i})$ where each sender node in $\mathbf{S_i}$ only transmits to a subset of the receiver nodes $\mathbf{R_i}$.

Proof: Any graph can be partitioned into a set of weakly connected components. Each connected component is bi-partite, since each node is either a sender or a receiver but not both. ■

When a wireless network is partitioned in this fashion, we can speak of three types of *intra/inter-cluster behavior.* Within each cluster, or amongst clusters, nodes can compete for resources (*competitive behavior*), can fully cooperate (*cooperative behavior*), or can partially cooperate in what we call *cognitive behavior*. Interference channels are an example of *competitive behavior* between the sending nodes, while MIMO channels and relays demonstrate *cooperative behavior*. In this paper, we consider the less well-studied *cognitive behavior*. *Intra-cluster cognitive behavior* is when certain sending nodes within one cluster obtain the messages of other nodes within that cluster, and use this to mitigate interference. *Inter-cluster cognitive behavior* refers to when some interfering clusters obtain the messages to be transmitted by other cluster(s). The former can then use this knowledge to mitigate or reduce interference from the latter. Note that there is an inherent asymmetry to this problem: one cluster or node knows the messages of another, but not vice-versa.

Generalized MIMO channels reduce to well-studied channels in certain cases. When a cluster consists of a single sender, it becomes a broadcast channel. When a cluster consists of a single receiver, it becomes a multiple access channel (MAC). In [4], we studied the *inter-cluster cognitive behavior* of two $1 \to 1$ clusters, also known as a 2×2 interference channel [1]. In this work, we extend our methods and consider the *inter-cluster* behavior of two clusters which are both MAC channels. Specifically, an achievable region for two MAC channel clusters that simultaneously transmit and interfere is computed in the case that one MAC cluster knows the messages to be sent by the other MAC cluster. Future work [6] considers the *inter-cluster cognitive behavior* of both MAC clusters. These small pieces will provide building blocks from which an overall picture of cognitive networks will emerge.

B. Paper outline

The paper is structured as follows: Section II defines the *genie-aided cognitive radio multiple access channel* as two MAC channel clusters in which one cluster is non-causally given the other cluster's message. This serves as an outer bound to the causal cognitive case, for which protocols can be devised as done in [4]. Section II also states the main result: achievability of a certain rate region in Theorem 3 and Lemma 2. Our methods borrow ideas from Gel'fand and Pinsker [10], Costa's dirty-paper coding [2], the interference channel [1], the Gaussian MIMO broadcast channel [14], and an achievable region of the interference channel [11]. Here, we give a brief sketch of the proofs to be found in [6]. The significance of our result is shown in Section III, where numerical methods are used to compute an achievable region in the additive white Gaussian noise case. The achievable region described here is compared to the *competitive* (lower bound) and *cooperative* (upper bound, $(p+q) \times 2$ Gaussian MIMO broadcast channel [14]) cases. In Section IV, we summarize the main contributions of this paper: providing a global view of cognitive radio behavior in wireless networks and identifying and studying one of the building blocks: the *cognitive radio multiple access channel*.

II. Genie-aided Cognitive Radio Multiple Access Channel Definition

We define a (p,q) *genie-aided cognitive radio multiple access channel* MAC_G, as in Fig. 2, to be two MAC channels, $\mathbf{S}_1 := (\mathcal{S}_{11}, \mathcal{S}_{12}, \ldots, \mathcal{S}_{1p}) \to \mathcal{R}_1$ and $\mathbf{S}_2 := (\mathcal{S}_{21}, \mathcal{S}_{22}, \ldots, \mathcal{S}_{2q}) \to \mathcal{R}_2$ in which the senders in $\mathbf{S}_2$ are given, in a non-causal manner (i.e., by a genie), a function $g(\mathbf{x_1})$ of the encoded messages $\mathbf{x_1} := (x_{11}^n, x_{12}^n, \ldots, x_{1p}^n)$ which the senders $\mathbf{S}_1$ will transmit. Let X_{1i}, $i = 1, 2, \ldots, p$ and X_{2j}, $j = 1, 2, \ldots, q$, be the random variable inputs to the channel, and let Y_1 and Y_2 be the random variable outputs of the channel. The conditional probabilities of the discrete memoryless MAC_G are fully described by $P(y_1|\mathbf{x_1}, \mathbf{x_2})$ and $P(y_2|\mathbf{x_1}, \mathbf{x_2})$. Under suitable conditions on $g(\mathbf{x_1})$, $\mathbf{S_2}$

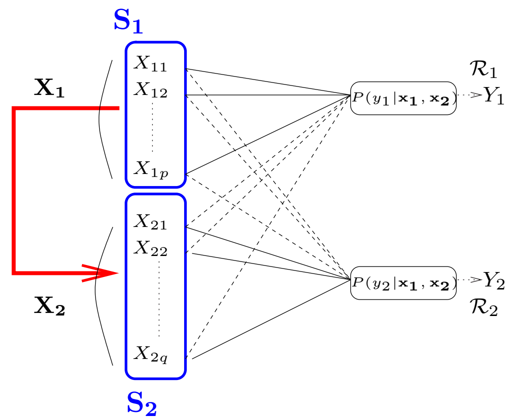

Fig. 2. The genie-aided inter-cluster cognitive radio MAC channel with inputs $\mathbf{X_1} := (X_{11}, \ldots, X_{1p})$, $\mathbf{X_2} := (X_{21}, X_{22}, \ldots, X_{2q})$, and outputs Y_1 and Y_2.

equivalently knows $\mathbf{x_1}$ and could potentially improve its overall transmission rates by Gel'fand-Pinsker coding against $\mathbf{x_1}$ [4]. In the following, an achievable rate region for such a *cognitive radio multiple access channel* is constructed. In a more realistic scenario, other users' messages must be causally obtained. Protocols which exploit the geometric gain (assuming the second transmitter is relatively close to the first transmitter) can be devised in ways similar to [4]. However, in this paper the ideal assumption will be made in order to explore the limits of such *cognitive radio channels*.

A. Terminology and definitions

An $(n, \mathbf{K_1}, \mathbf{K_2}, \lambda)$ *code* for the *genie-aided cognitive radio multiple access channel*, where $\mathbf{K_1} := (K_{11}, K_{12}, \ldots, K_{1p})$ and $\mathbf{K_2} := (K_{21}, K_{22}, \ldots, K_{2q})$, consists of $K_{1i} \geq 1$ codewords $x_{1i}^n \in \mathcal{X}_{1i}^n$ for sender $\mathcal{S}_{1i}$, $i = 1, 2, \ldots, p$, and $(K_{11} \times K_{12} \times \cdots \times K_{1p}) \times K_{2j}$ codewords $x_{2j}^n \in \mathcal{X}_{2j}^n$ for sender $\mathcal{S}_{2j}$, $j = 1, 2, \ldots, q$. Together, these form the *codebook*, revealed to all senders and receivers, which has the property that the maximum (over $\mathcal{R}_1$ and $\mathcal{R}_2$) of the average error probabilities under some decoding scheme is less than λ. A rate tuple $(\mathbf{R_1}, \mathbf{R_2})$, where $\mathbf{R_1} := (R_{11}, \ldots, R_{1p})$ and $\mathbf{R_2} := (R_{21}, \ldots, R_{2q})$ is said to be *achievable* for the MAC_G if there exists a sequence of $(n, \mathbf{K_1^n}, \mathbf{K_2^n}, \lambda_n)$ codes with $K_{1i}^n = 2^{nR_{1i}}$ and $K_{2j}^n = 2^{nR_{2j}}$ such that $\lambda_n \to 0$ as $n \to \infty$. An *achievable region* is the closure of a subset of achievable rate pairs, and the capacity region is defined analogously.

The interference channel capacity, in the most general case, is still an open problem. This is also the case for the genie-aided cognitive radio multiple access channel. In [11], an achievable region for the interference channel is found by first considering a modified problem and then establishing a correspondence between the achievable rates of the modified and the original channel models. A similar modification is made in the next subsection.

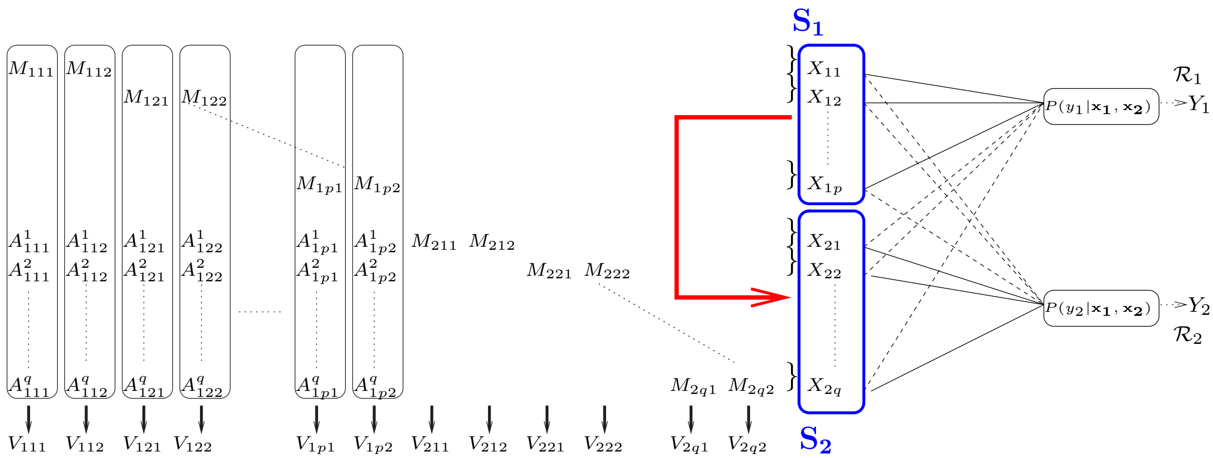

Fig. 3. The modified cognitive radio multiple access channel with auxiliary random variables $\mathbf{M_1}$ and $\mathbf{M_2}$, inputs $\mathbf{X_1}$ and $\mathbf{X_2}$, and outputs Y_1 and Y_2. The auxiliary random variable A^j_{1ik} associated with $\mathcal{S}_{2j}$, aids in the transmission of M_{1ik}. The vectors V_{1ik} and V_{2jk} denote the effective random variables encoding the transmission of the private and public messages. Solid lines indicate desired paths, dashed lines indicate interference.

B. The Modified Genie-aided Cognitive Multiple Access Channel MAC^m_G

We define the modified genie-aided cognitive radio multiple access channel MAC^m_G as in Fig. 3, where $\mathbf{X_1}, \mathbf{X_2}, Y_1$ and Y_2 are defined as in the non-modified MAC_G case. The conditional probabilities of the discrete memoryless MAC^m_G are the same as those of the discrete memoryless MAC_G.

The channel MAC^m_G introduces many new auxiliary random variables, whose purposes can be made intuitively clear by relating them to auxiliary random variables in previously studied channels. The M_{1ik}, M_{2jk} variables $(i = 1, 2, \dots, p,\ j = 1, 2, \dots q,\ k = 1, 2)$ divide the information to be sent into private and public parts, as done in the construction of [11]. The M_{1i1} and M_{2j2} represent the *private* information to be sent from $\mathcal{S}_{1i} \to \mathcal{R}_1$ and $\mathcal{S}_{2j} \to \mathcal{R}_2$ respectively. The variables M_{1i2} and M_{2j1} represent the *public* information to be sent from $\mathcal{S}_{1i} \to (\mathcal{R}_1, \mathcal{R}_2)$ and $\mathcal{S}_{2j} \to (\mathcal{R}_1, \mathcal{R}_2)$ respectively.

The random variables A^j_{1ik} are *auxiliary*, or *aiding* random variables found at sender $\mathcal{S}_{2j}$ that aid in the transmission of the message M_{1ik} of user $\mathcal{S}_{1i}$. The vector $V_{1ik} = (M_{1ik}, A^1_{1ik}, A^2_{1ik}, \dots, A^q_{1ik})$ consists of all variables effectively encoding the transmission of $\mathcal{S}_{1i}$'s private message $(k = 1)$, or public message $(k = 2)$ for transmission to $\mathcal{R}_1$ and $\mathcal{R}_2$. Similarly, V_{2jk} is defined to be the vector consisting of all random variables effectively encoding $\mathcal{S}_{2j}$'s messages. Since $\mathbf{S_1}$ has no knowledge of $\mathbf{S_2}$'s messages (asymmetry), we see that $V_{2jk} = M_{2jk}$. We also define $\mathbf{A^j} := (A^j_{111}, A^j_{112}, \dots, A^j_{1p1}, A^j_{1p2})$, the vector of aiding random variables at sender $\mathcal{S}_{2j}$. Also, we let $\mathbf{A_1}$ be the vector of all A_{1ik}, $i = 1, 2, \dots, p$, $k = 1, 2$.

The V_{2jk} (or equivalently M_{2jk}) also have a second purpose: they act as the auxiliary random variable introduced in coding for channels with side information known to the transmitter, [3], [10]. The 'side information' in our case will be the messages $\mathbf{V_1} := (V_{111}, V_{112}, \dots, V_{1p1}, V_{1p2})$ that are used

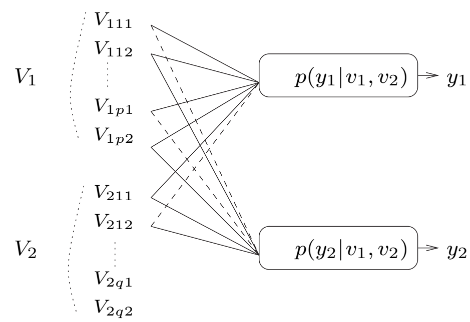

Fig. 4. The equivalent channel with random variables V_{1ik} and V_{2jk}. These are two overlapping, and interfering MAC channels. The solid lines are desireable connections while the dashed lines are interference.

to send information from $\mathbf{S_1}$ to $\mathcal{R}_1$ or $(\mathcal{R}_1, \mathcal{R}_2)$ as appropriate. These $\mathbf{V_1}$ and $\mathbf{V_2}$ random variables serve as fictitious inputs to an equivalent channel shown in Fig. 4. The $\mathbf{V_1}$ variables do not use any Gel'fand-Pinsker coding techniques, whereas the variables $\mathbf{V_2}$ do. Such channels, whose simplest models are with input X, side information S and output Y have capacity

$$C = \max_{p(u,x|s)} I(U;Y) - I(U;S),$$

where U is an auxiliary random variable that serves as a fictitious input to the channel. There is a penalty in using this approach which will be reflected by a reduction in achievable rates (compared to the fictitious DMC from $U \to Y$) for the links which use the non-causal information. The reduction in the rates is the cost of limiting the fictitious input U to those that are jointly typical to the non-causal side information, or equivalently, $I(U;S)$. In our case, each V_{2jk} variable, which uses the non-causal knowledge of $\mathbf{V_1}$ variables, will suffer a reduction in rate of $I(V_{2jk}; \mathbf{V_1})$.

C. Terminology and definitions

Let $\mathbf{K_1} := (K_{111}, K_{112}, \dots, K_{1p1}, K_{1p2})$ and $\mathbf{K_2} := (K_{211}, K_{212}, \dots, K_{2q1}, K_{2q2})$, $K_{1ik}, K_{2jk} \geq 1$, and define an

$(n, \mathbf{K_1}, \mathbf{K_2}, \lambda)$ code for MAC_G^m as a set of $K_{1i1} \times K_{1i2}$ codewords $x_{1i}^n \in \mathcal{X}_{1i}^n$ for $\mathcal{S}_{1i}$ for $i = 1, 2, \ldots, p$, and $(K_{111} \times K_{112} \times \cdots \times K_{1p1} \times K_{1p2}) \times K_{2j1} \times K_{2j2}$ codewords $x_{2j} \in \mathcal{X}_{2j}^n$ for $\mathcal{S}_{2j}$ for $j = 1, 2, \ldots, q$ such that the average probability of decoding error is less than λ. We say the rate $(\mathbf{R_1^m}, \mathbf{R_2^m})$, where $\mathbf{R_1^m} := (R_{111}, R_{112}, \cdots R_{1p1}, R_{1p2})$ and $\mathbf{R_2^m} := (R_{211}, R_{212}, \cdots R_{2q1}, R_{2q2})$, is achievable if there exists a sequence of $(n, \mathbf{K_1^n}, \mathbf{K_2^n}, \lambda_n)$ codes with $K_{1ik}^n = 2^{nR_{1ik}}$ and $K_{2jk}^n = 2^{nR_{2jk}}$ such that $\lambda_n \to 0$ as $n \to \infty$. An achievable region of MAC_G^m is the closure of a subset of achievable rates.

Let $W \in \mathcal{W}$ be a time-sharing random variable whose n-sequences $w^n \triangleq (w^{(1)}, w^{(2)}, \ldots, w^{(n)})$ are generated independently of the messages, according to $\prod_{t=1}^n P(w^{(t)})$. The n-sequence w^n is given to all senders and receivers. Let T_G be the set of all subscripts of the first MAC channel, and T_1 and T_2 denote the set of all subscripts of all the "V" random variables that $\mathcal{R}_1$ and $\mathcal{R}_2$ respectively wish to receive, i.e.,

$$T_G := \{111, 112, 121, 122, \ldots, 1p1, 1p2\} \quad (1)$$
$$T_1 := \{111, 112, 121, 122, \ldots, 1p1, 1p2, 211, 221, \ldots, 2q1\} \quad (2)$$
$$T_2 := \{112, 122, \ldots, 1p2, 211, 212, 221, 222, \ldots, 2q1, 2q2\}. \quad (3)$$

The paper's main results are given next.

Lemma 2: Let $(\mathbf{R_1^m}, \mathbf{R_2^m})$ be an achievable rate tuple for MAC_G^m. Then the rate tuple $(\mathbf{R_1}, \mathbf{R_2})$ is achievable for MAC_G, where $R_{1i} = R_{1i1} + R_{1i2}$ and $R_{2j} = R_{2j1} + R_{2j2}$.

Proof: Analogous to Corollary (2.1) of [11]. ■

Theorem 3: Let $Z \triangleq (Y_1, Y_2, \mathbf{X_1}, \mathbf{X_2}, \mathbf{V_1}, \mathbf{V_2}, W)$, as shown in Fig. 3. Let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form

$$P(w) \times \left[\prod_{i=1}^{p} P(m_{1i1}|w) P(m_{1i2}|w) P(x_{1i}|m_{1i1}, m_{1i2}, w)\right]$$
$$\times \left[\prod_{i=1}^{p} P(a_{1i1}^1, a_{1i1}^2, \ldots a_{1i1}^q | m_{1i1}, w) P(a_{1i2}^1, a_{1i2}^2, \ldots, a_{1i2}^q | m_{1i2}, w)\right]$$
$$\times \left[\prod_{j=1}^{q} P(m_{2j1}|\mathbf{v_1}, w) P(m_{2j2}|\mathbf{v_1}, w)\right]$$
$$\times \left[\prod_{j=1}^{q} P(x_{2j}|m_{2j1}, m_{2j2}, \mathbf{a^j}, w)\right] P(y_1|\mathbf{x_1}, \mathbf{x_2}) P(y_2|\mathbf{x_1}, \mathbf{x_2}), \quad (4)$$

where $P(y_1|\mathbf{x_1}, \mathbf{x_2})$ and $P(y_2|\mathbf{x_1}, \mathbf{x_2})$ are fixed by the channel. For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all tuples $\mathbf{R_1} := (R_{111}, R_{112}, R_{121}, R_{122}, \ldots, R_{1p1}, R_{1p2})$, $\mathbf{R_2} := (R_{211}, R_{212}, R_{221}, R_{222}, \ldots, R_{2q1}, R_{2q2})$ of non-negative real numbers such that there exist non-negative reals $\mathbf{L_1} := (L_{111}, L_{112}, L_{121}, L_{122}, \ldots, L_{1p1}, L_{1p2})$ and $\mathbf{L_2} := (L_{211}, L_{212}, L_{221}, L_{222}, \ldots, L_{2q1}, L_{2q2})$ satisfying:

$$\bigcap_{T \subset T_G} \left(\sum_{t \in T} R_t\right) \leq I(g(\mathbf{X_1}); \mathbf{M}_T | \mathbf{M}_{\overline{T}}) \quad (5)$$
$$R_{1ik} = L_{1ik} \quad (6)$$
$$R_{2jk} \leq L_{2jk} - I(V_{2jk}; \mathbf{V_1}) \quad (7)$$
$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1}\right) \leq I(Y_1, \mathbf{V}_{\overline{T}}; \mathbf{V}_T | W) \quad (8)$$

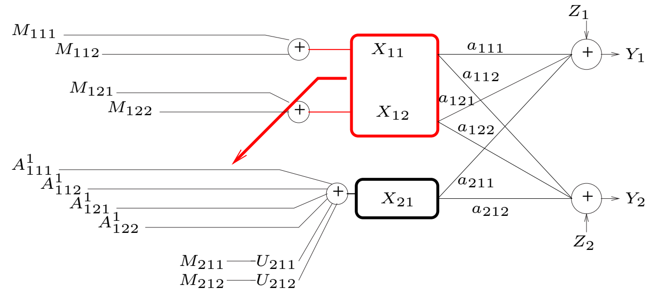

Fig. 6. The modified Gaussian genie-aided cognitive radio multiple access channel for $p = 2$, $q = 1$, with inputs X_{11}, X_{12}, X_{21}, auxiliary random variables $M_{111}, M_{112}, M_{121}, M_{122}, M_{211}, M_{212}, U_{211}$ and U_{212}, outputs Y_1 and Y_2, additive Gaussian noise Z_1 and Z_2 and interference coefficients.

$$\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2}\right) \leq I(Y_2, \mathbf{V}_{\overline{T}}; \mathbf{V}_T | W), \quad (9)$$

for $i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, q$ and $k = 1, 2$. The genie presents the second MAC with some function $g(\mathbf{X_1})$ of the encoded messages of the first MAC channel. $\overline{T}$ denotes the complement of the subset T with respect to T_1 in (8), with respect to T_2 in (9), and $\mathbf{V}_T$ denotes the vector of V_i such that $i \in T$. Let S be the closure of $\cup_{Z \in \mathcal{P}} S(Z)$. Then any element $(\mathbf{R_1}, \mathbf{R_2})$ in S, is achievable for MAC_G^m. □

Proof: The full proof will be given in [6]. The main intuition is as follows: the equations in (5) ensure that when the second MAC channel is presented with $g(\mathbf{x_1})$, the auxiliary variables M_{1ik} can be recovered. Eqs. (8) and (9) correspond to the equations for two overlapping MAC channels seen between the effective random variables $\mathbf{V}_{T_1} \to \mathcal{R}_1$, and $\mathbf{V}_{T_2} \to \mathcal{R}_2$. Eqs. (6) and (7) are necessary for the Gel'fand-Pinsker coding scheme to work. ■

This theorem is of interest because the coding scheme covers in a sense, two limiting possibilities of how $\mathbf{S_2}$ could employ its knowledge of $\mathbf{S_1}$'s message: in one case it could completely aid $\mathbf{S_1}$, which is obtained by selecting $P(x_{2j}|m_{2j1}, m_{2j2}, \mathbf{a^j}, w) = P(x_{2j}|\mathbf{a^j}, w)$, and in the other case it could dirty-paper code against the known interference by selecting $P(x_{2j}|m_{2j1}, m_{2j2}, \mathbf{a^j}, w) = P(x_{2j}|m_{2j1}, m_{2j2}, w) := P(x_{2j}|v_{2j1}, v_{2j2}, w)$, where v_{2j1} and v_{2j2} serve as the fictitious auxiliary inputs in the dirty paper coding argument.

III. The Gaussian Cognitive Multiple Access Channel

Consider the $(2, 1)$ genie-aided cognitive radio multiple access channel, depicted in Fig. 6, with independent additive noise $Z_1 \sim \mathcal{N}(0, Q_1)$, $Z_2 \sim \mathcal{N}(0, Q_2)$ and $g(X_{11}, X_{12}) = X_{11} + X_{12}$. In order to determine an achievable region for the modified Gaussian genie-aided cognitive radio multiple access channel, specific forms of the random variables described in Theorem 1 are assumed. For the purpose of deriving an achievable region, we let W, the time-sharing random variable, be constant.

Consider the case where, for certain $\alpha, \beta \in \mathbb{R}$, $\mu, \nu \in [0, 1]$ and $\lambda, \overline{\lambda}, \gamma, \overline{\gamma}, \eta, \overline{\eta} \in [0, 1]$, with $\lambda + \overline{\lambda} = 1$, $\gamma + \overline{\gamma} = 1$, $\eta + \overline{\eta} =$

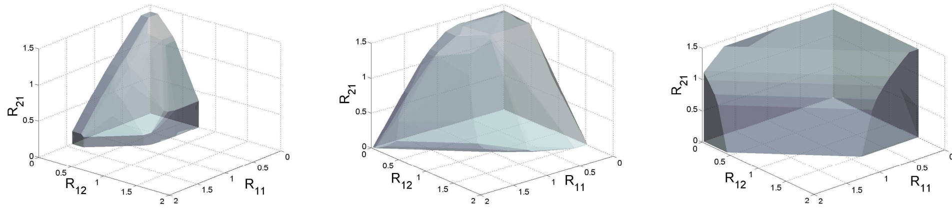

Fig. 5. Left: the Gaussian competitive MAC channel achievable region, obtained setting $\alpha = \beta = 0$ and $P_{31} = P_{32} = 0$. Middle: the Gaussian cognitive MAC channel achievable region of Theorem 3 and Lemma 2. Right: the Gaussian cognitive MAC channel achievable region outer bound obtained by considering the 3×2 MIMO broadcast channel and bounds on R_{11}, R_{12} and R_{21}. In all figures, the parameters used are $a_{111} = a_{212} = 1$, $a_{112} = a_{121} = a_{122} = a_{211} = 0.55$, $Q_1 = Q_2 = 1$, $P_{11} = P_{12} = P_{21} = 6$. The respective volumes of the regions are 0.6536, 1.5064 and 2.9127 $(bits/sample)^3$.

1, and additional *independent* auxiliary random variables U_{211} and U_{212} as in Fig. 6, the following hold:

$$
\begin{aligned}
&M_{111} \sim \mathcal{N}(0, \lambda P_{11}), \qquad M_{112} \sim \mathcal{N}(0, \overline{\lambda} P_{11}) \\
&X_{11} = M_{111} + M_{112} \\
&M_{121} \sim \mathcal{N}(0, \gamma P_{12}), \qquad M_{122} \sim \mathcal{N}(0, \overline{\gamma} P_{12}) \\
&X_{12} = M_{121} + M_{122} \\
&P_{31} = \mu P_{21},\ P_{32} = \nu(P_{21} - P_{31}),\ P_{33} = P_{21} - P_{31} - P_{32} \\
&A^1_{111} = \sqrt{(\theta P_{31})/(\lambda P_{11})} M_{111}, \quad A^1_{112} = \sqrt{(\overline{\theta} P_{31})/(\overline{\lambda} P_{11})} M_{121} \\
&A^1_{121} = \sqrt{(\psi P_{32})/(\gamma P_{12})} M_{121}, \quad A^1_{122} = \sqrt{(\overline{\psi} P_{32})/(\overline{\gamma} P_{12})} M_{121} \\
&U_{211} \sim \mathcal{N}(0, \eta P_{33}), \qquad U_{212} \sim \mathcal{N}(0, \overline{\eta} P_{33}) \\
&M_{211} = U_{211} + \alpha(X_{11} + X_{12} + A^1_{111} + A^1_{112} + A^1_{121} + A^1_{122}) \\
&M_{212} = U_{212} + \beta(X_{11} + X_{12} + A^1_{111} + A^1_{112} + A^1_{121} + A^1_{122}) \\
&X_{21} = A^1_{111} + A^1_{112} + A^1_{121} + A^1_{122} + U_{211} + U_{212}
\end{aligned}
$$

Bounds on the rates $R_{111}, R_{112}, R_{121}, R_{122}, R_{211}$ and R_{212} can be calculated as functions of the free parameters $\alpha, \beta, \lambda, \gamma, \eta, \mu, \nu$, the channel coefficients, the noise parameters Q_1 and Q_2, and the power constraints P_{11}, P_{12} and P_{21}.

The achievable region thus obtained by Theorem 3 and Lemma 2 for the Gaussian genie-aided cognitive radio channel is plotted in Fig. 5 (middle). As expected, because of the extra information at the encoder and the partial use of a Gel'fand-Pinsker coding technique, $\mathcal{S}_{21}$ can simultaneously transmit with $\mathcal{S}_{11}$ and $\mathcal{S}_{12}$ at much larger rates than when no collaboration is used.

A. The Competitive and Cooperative Cases

When $\mathbf{S_2}$ does not know or employ $\mathbf{S_1}$'s message, the two MAC clusters behave in a *competitive* manner. We set $\alpha = \beta = 0$ (no Gel'fand-Pinsker coding), and obtain the achievable region for the competitive case shown in Fig. 5 (left). The cooperative case is obtained by considering the 3×2 Gaussian MIMO broadcast channel, whose capacity was recently computed in [14]. This region provides a 2-D region for the broadcast rates R_1 and R_2. We equate $R_2 = R_{21}$, and split write $R_1 = R_{11} + R_{12}$ The 3×2 MIMO broadcast channel provides a loose bound since all users are permitted to cooperate. We tighten the outer bound by noticing that because $\mathbf{S_1}$ cannot aid $\mathbf{S_2}$, the rate R_{21} is bounded by the no-interference case, or $R_{21} \leq 1/2 \log\left(1 + a_{212}^2 P_{21}/Q_2\right)$. Similarly, since S_{12} cannot aid S_{11}, even if $R_{12} = 0$, we see that $R_{11} \leq 1/2 \log\left(1 + \frac{(a_{111}\sqrt{P_{11}} + a_{211}\sqrt{P_{21}})^2}{Q_1}\right)$ and analogously, $R_{12} \leq 1/2 \log\left(1 + \frac{(a_{121}\sqrt{P_{12}} + a_{211}\sqrt{P_{21}})^2}{Q_1}\right)$. We also restrict the diagonal elements of the covariance matrix constraint used to evaluate the 3×2 MIMO broadcast capacity to be P_{11}, P_{12} and P_{21} respectively. The MIMO 3×2 broadcast channel intersected with the bounds on R_{11}, R_{12} and R_{21} is plotted in Fig. 5 (right), and provides an outer bound on the *cognitive behavior*.

IV. Conclusion

We have defined *inter/intra-cluster cognitive behavior*, and have derived an achievable region for the *cognitive radio multiple access channel*. In the Gaussian case, this region was compared to the achievable regions under competitive as well as cooperative behavior. These results provide a foundation for theoretical studies of the fundamental, information theoretic limits of cognitive radio channels.

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