Cognitive Radio Communications and Networks

Principles and Practice

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Contents

Chapter 1

Information Theoretical Limits on Cognitive Radio Networks

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1.1 Introduction

In the previous chapter, the benefits, from a theoretical perspective, of cooperative communications were introduced. In this chapter we expand upon cooperative communications by allowing a subset of the nodes to be cognitive radios with their own data to transmit (and not merely relay), thereby forming cognitive radio networks. Cognitive networks, for our purposes, are wireless networks which consist of two types of users:

- **Primary users:** these wireless devices are the primary license-holders of the spectrum band of interest. In general, they have priority access to the spectrum, and and subject to certain Quality of Service (QoS) constraints which must be guaranteed.

- **Secondary users:** these users may access the spectrum which is licensed to the primary users. They are thus secondary users of the wireless spectrum, and are often envisioned to be cognitive radios. For the rest of this chapter, we will assume the secondary users are cognitive radios (and the primary users are not) and will use the terms interchangeably. These cognitive users employ their "cognitive" abilities to communicate while ensuring the communication of the primary users is kept at an acceptable level[1].

[1] *Acceptable* may mean a number of things. Different mathematical models may be useful for different situations.

The study of cognitive networks is relatively new and there are many questions and aspects to be tackled before before cognitive radios can seamlessly and opportunistically employ spectrum licensed to primary user(s). Of both theoretical and practical importance is the question: what are the fundamental limits of communication in a cognitive network? Information theory provides an ideal framework for analyzing this question, as it encompasses a number of tools and metrics suited to such fundamental studies. The limits obtained provide benchmarks for the operation of cognitive networks, where researchers may gauge the efficiency of any practical network as well as draw inspiration as to which direction to pursue in their design.

In this chapter, we outline recent information theoretic advances pertaining to the limits of cognitive networks. We emphasize and explore the impact of *cognition*, defined as extra information (or side information) the cognitive radio nodes have about their wireless environment, on the fundamental limits. We first briefly describe why cognitive networks are of intense contemporary interest before outlining several types of *cognition* which cognitive networks could exploit.

1.1.1 The rise and importance of cognitive networks

Cognitive networks are motivated by the apparent lack of spectrum under the current spectrum management policies. The right to use the wireless spectrum in the United States is controlled by the Federal Communications Commission (FCC) [FCCa]. Most of the frequency bands useful to wireless communication have already been licensed by the FCC [FCC03b]. A few bands have however been designated by the FCC to be unlicensed bands, most notably the Industrial Scientific and Medical bands (ISM bands), over which the immensely popular WiFi devices transmit. These bands are filling up fast, and despite their popularity, the vast majority of the wireless spectrum is in fact licensed. Currently, the primary license holders obtain from the FCC the exclusive right to transmit over their spectral bands. As most of the bands have been licensed out, and the unlicensed bands are also rapidly filling up, it would appear that we are approaching a spectral crisis. This, however, is far from the case. Recent measurements, such as those presented in Chapter 7, [JP08] have shown that for as much as 90% of the time, large portions of the *licensed* bands remain unused. As licensed bands are difficult to reclaim and re-lease, the FCC is considering *dynamic* and *secondary* spectrum licensing [FCCb, FCC03a] as an alternative to reduce the amount of unused spectrum. Bands licensed to primary users could, under certain negotiable conditions, be shared with non-primary users without having the primary licensee release its own license. Whether the primary users would be willing to share their spectrum would depend on a number of factors, including the impact on their own communication.

Cognitive radios, wireless devices with reconfigurable hardware and software (including transmission parameters and protocols) [Mit99], are capable of delivering what these secondary devices would need: the ability to intelligently sense and adapt to their spectral environment. Along with this newfound flexibility comes the challenge of understanding the limits of and designing protocols and transmission schemes to fully exploit these cognitive capabilities. In particular, in order to design practical and efficient protocols, the theoretical limits must be well understood. We next describe different scenarios, assumptions and corresponding types of cognitive behavior, for which information theoretic limits have been considered.

1.1.2 Types of cognitive behavior

Networks which contain cognitive radios should intuitively be able to achieve better performance than networks in which they are absent. We are being intentionally vague at this point as to what *performance* and *fundamental limits* mean as they vary between applications. Different information theoretic metrics will be discussed in the next Section. Cognitive networks should achieve better performance as they are able to (1) exploit their cognitive abilities, i.e. sensing and adapting to their wireless environment, and (2) often (but not necessarily) exploit new policies in secondary spectrum licensing scenarios in which the agile cognitive radios are permitted to share the spectrum with primary users. Naturally, the extent to which the performance of the network can be improved depends on what the cognitive radios know about their spectral environment, and consequently, how they adapt to this. We depart from the assumption that the secondary users are cognitive radios wishing to share the primary users' spectrum. Cognitive behavior, or how the secondary cognitive users employ the primary spectrum, may be grouped into three categories, as also done with slight variations in [DMT06c, DMS$^+$07, DT07, DMT06c, GJMS08], each of which exploits varying degrees of knowledge of the wireless environment at the secondary user(s). Another implicit assumption is that the burden of guaranteeing primary user communication at a pre-determined level is borne by the secondary users. That is, the legacy primary system does not necessarily (but may) adapt to the cognitive users, while the cognitive users definitely adapt to the primary, often legacy, system. The three types of cognitive behavior we consider are:

- **Interference avoiding behavior (spectrum interweave):** the secondary users employ the primary spectrum without interfering with the primary users whatsoever. The primary and secondary signals may be thought of as being orthogonal to each other: they may access the spectrum in a Time-Division-Multiple-Access (TDMA) fashion, in a Frequency-Division-Multiple-Access (FDMA) fashion, or in any fashion that ensures that the primary and secondary signals do not interfere with each other. The cognition required by

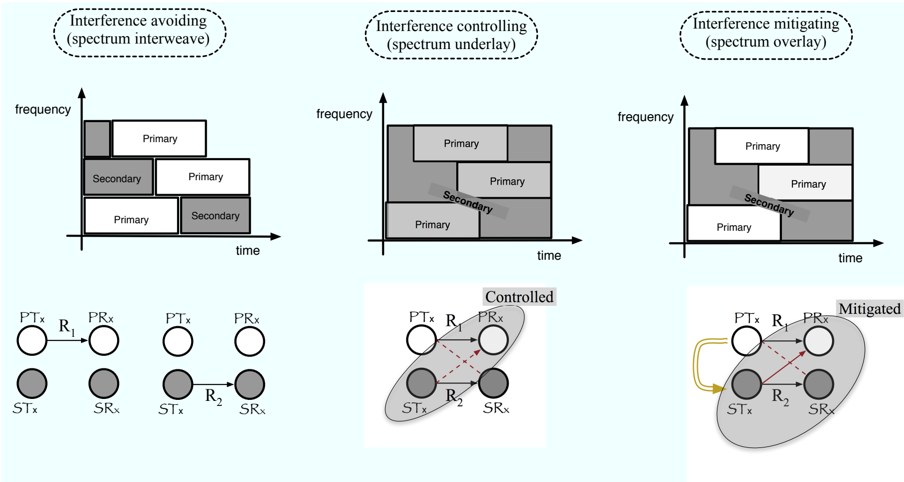

Figure 1.1: Graphic representations of three types of cognitive behavior.

the secondary users to accomplish this is knowledge of the spectral gaps (in for example time, frequency) of the primary system. The secondary users may then fill in these spectral gaps. Notice that this form of behavior is referred to as *spectrum overlay* in Chapter 7; unfortunately information theory uses a different definition of spectrum overlay, as given below.

- **Interference controlling behavior (spectrum underlay):** the secondary users transmit over the same spectrum as the primary users, but do so in a way that the interference seen by the primary users from the cognitive users is controlled to an acceptable level. This acceptable level is captured by primary QoS constraints.[2] This is termed *underlay* as often the cognitive radios transmit in such a fashion that they appear to be noise under the primary signals. The cognition required is knowledge of the "acceptable levels" of interference at primary users in a cognitive user's transmission range as well as knowledge of the effect of the cognitive transmission at the primary receiver. This last assumption boils down, in classical wireless channels, to knowledge of the channel(s) between the cognitive transmitter(s) and the primary receiver(s). In Chapter 7, spectrum underlay is described in a similar manner, with an emphasis on how underlay techniques are achieved: using spread spectrum techniques.

- **Interference mitigating behavior (spectrum overlay):** the secondary users transmit over the same spectrum as the primary users but in addition

[2] What constitutes an acceptable level will be described later and may vary from system to system.

> to knowledge of the channels between primary and secondary users (nature), the cognitive nodes have additional information about the primary system and its operation. Examples are knowledge of the primary users' codebooks, allowing the secondary users to decode primary users' transmissions, or in certain cases even knowledge of the primary users' message. In Section 1.5 we will discuss why these assumptions may be plausible and realizable in cognitive networks. Spectrum overlay in Chapter 7 refers to the spectrum interweave concept here; there is presently no analogous concept of overlay in dynamic spectrum access systems in agile transmission techniques.

To illustrate the effect of different types of cognition, in this chapter we take as an example that simple channel in which a primary transmitter-receiver pair (white, $\mathcal{PT}_x, \mathcal{PR}_x$) and a cognitive transmitter-receiver pair (grey, $\mathcal{ST}_x, \mathcal{SR}_x$) share the same spectrum, shown in Figs. 1.1 and 1.2. We will derive fundamental limits on the communication possible under each type of cognition. One information theoretic metric that lends itself well to illustrative purposes and is central to many studies is the capacity region of the channel in Fig. 1.2. Under Gaussian noise, we will illustrate different examples of cognitive behavior and in Sections 1.3 - 1.5 we will build up to the right illustration in Fig. 1.2, which corresponds to the rates achieved under different levels of cognition.

The basic and natural conclusion is that, the higher the level of cognition at the cognitive terminals, the higher the achievable rates. However, increased cognition often translates into increased complexity. At what level of cognition future secondary spectrum licensing systems will operate will depend on the available side information and network design constraints. We next outline the chapter in more detail.

1.1.3 Chapter preview

We start the study of information theoretic limits of cognitive networks in Section 1.2, where we define classical information theoretic channels and measures of interest. In Section 1.3 we outline challenges and trends in interference avoiding cognitive behavior. In Section 1.4 we first outline the limits some spectrum underlay techniques in small networks and then pursue throughput scaling laws in large cognitive underlay networks. In Section 1.5 we explore the communication possibilities when the cognitive radios are able to decode some of the primary's messages, allowing for behavior that borders and overlaps with *cooperative* behavior. Of particular interest in cognitive networks is the lack of symmetry in the cooperation. These sections will progressively build the reader up to the achievable rate regions of Fig. 1.2, which illustrates the main point: different levels of cognition result in different fundamental limits.

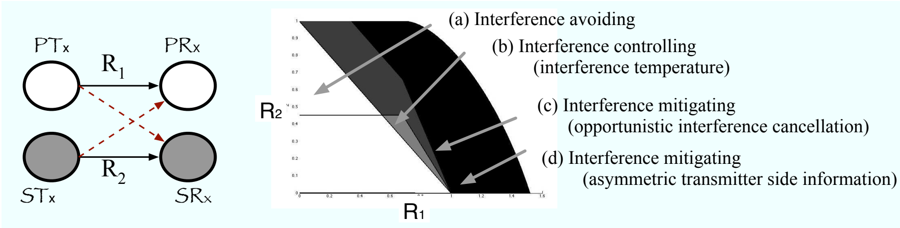

Figure 1.2: The primary users (white) and secondary users (grey) wish to transmit over the same channel. Solid lines denote desired transmission, dotted lines denote interference. The achievable rate regions under four different cognitive assumptions and transmission schemes are shown on the right. (a) - (d) are in order of increasing cognitive abilities.

1.2 Information theoretic basics

One of information theory's main contributions is the characterization of fundamental limits of communication. We first define two of the most common types of communication channels: the discrete memoryless and additive white Gaussian noise channels. We then outline two information theoretic metrics of interest: the capacity (region) and the sum-throughput scaling law, which will be examined for cognitive networks. We then briefly outline classical information theoretic channels which will form a nice basis from which to explore channels with primary and secondary users.

1.2.1 Communications channels

A channel is modeled as a set of conditional probability density functions relating the inputs and outputs of the channel. Communication over this channel takes place through the use of an encoder, which may be viewed as a function which maps a message into an encoded channel *input (sequence)* and a decoder, which may be viewed as a function which takes the channel *output (sequence)* and tries to recover the sent message. Two of the most common information theoretic channels are Discrete Memoryless Channels (DMCs) and Additive White Gaussian Noise (AWGN) channels. While we will outline information theoretic results for both types of channels, the latter lends itself well to examples and illustrations and so will be more heavily focussed on.

Discrete Memoryless Channels

Communication over a discrete memoryless channel takes place in a discrete number of "channel uses", indexed by the natural number $i \in \mathbb{N}$. We illustrate concepts

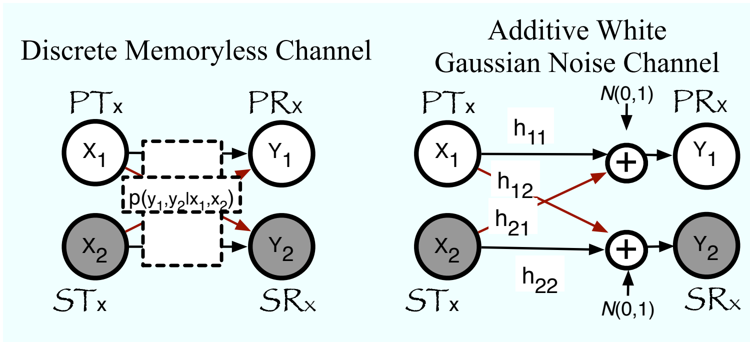

Figure 1.3: The primary users (white, $\mathcal{PT}_x \to \mathcal{PR}_x$) and secondary users (grey, $\mathcal{ST}_x \to \mathcal{SR}_x$) wish to transmit over the same channel. We show a discrete memoryless channel, described by $p(y_1, y_2|x_1, x_2)$ and an additive white Gaussian noise channel with channel coefficients $h_{11}, h_{12}, h_{21}, h_{22}$.

using the simple two transmitter, 2 receiver channel shown in Fig. 1.3. The primary (secondary) transmitter $\mathcal{PT}_x$ ($\mathcal{ST}_x$) wishes to communicate a message to a single primary (secondary) receiver $\mathcal{PR}_x$ ($\mathcal{SR}_x$). The transmitters communicate their messages by transmitting *codewords* which span n channel uses (one input symbol per channel use). The receivers independently decode the received signals, often corrupted by noise according to the statistical channel model, to obtain the desired message. One quantity of fundamental interest in such communication is the maximal *rate*, typically cited in (bits/channel use) at which communication can take place. Most information theoretic results of interest are asymptotic in the number of channel uses, that is, hold in the limit as $n \to \infty$.

A *discrete channel* has finite input alphabets and output alphabets $\mathcal{X}_1, \mathcal{X}_2$ and $\mathcal{Y}_1, \mathcal{Y}_2$ respectively which are related though a collection of conditional probability mass functions $p(y_1, y_2|x_1, x_2)$. This conditional distribution defines the DMC. Transmitter $\mathcal{PT}_x$ ($\mathcal{ST}_x$) wishes to send a message $W_1 \in \{1, 2, \cdots, 2^{nR_1}\}$ ($W_2 \in \{1, 2, \cdots, 2^{nR_2}\}$), where R_1 (R_2) are the transmission rates, encoded as the n-sequence $\mathbf{x}_1^n$ ($\mathbf{x}_2^n$) to its receivers in n channel uses. The received signal which the decoder uses to obtain the transmitted signal is y_1^n (y_2^n). We consistently use the notation x as an instance of the random variable X which takes on value in the alphabet $\mathcal{X}$. Vectors and matrices are denoted using bold fonts, and we omit super and sub-scripts n when it is clear from context.

Additive White Gaussian Noise Channels

The additive white Gaussian noise (AWGN) channel is typically considered the most important continuous alphabet channel [CT91]. For the purpose of this chapter, at each channel use, we assume that outputs at the primary and cognitive receivers, Y_1 and Y_2 respectively, are related to the inputs at the primary and cognitive transmitters X_1 and X_2, respectively, as shown in Fig. 1.3 as

$$Y_1 = X_1 + h_{21}X_2 + N_1, \quad N_1 \sim \mathcal{N}(0,1)$$
$$Y_2 = h_{12}X_1 + X_2 + N_2, \quad N_2 \sim \mathcal{N}(0,1).$$

Here h_{12}, h_{21} are the quasi-static [Mol05] fading coefficients assumed to be known to all transmitters and receivers and we by normalizing, we may assume w.l.o.g. that $h_{11} = h_{22} = 1$. The rate achieved by the primary and cognitive Tx-Rx pairs are R_1, and R_2 respectively, measured in (bits/channel use). In large cognitive networks, we will be assuming that each transmitter-receiver pair in the wireless network sees independent additive white Gaussian noise, and when applicable, independent fading. In networks, we refer to the *sum-rate* or *sum-thorughput* as the sum of all the rates of the different transmitter-receiver pairs in the network that may be simultaneously achieved (we will define what it means for a rate to be achieved shortly). For more extended definitions of standard information theoretic quantities, see [CT91, Yeu02, CK81].

Multiple Input Multiple Output Channels

Wireless channels in which transmitters and receivers employ multiple antennas, or Multiple-Input Multiple-Output (MIMO) channels have been extensively studied in recent years due to their ability to (theoretically) combat fading, increase data rates, or allow one to place beamform signals to desired users or spatial subspaces[Tel99, FG98]. In cognitive radio networks, the use of MIMO adds another dimension to the problem, and could for example, allow MIMO cognitive users to transmit in the null space of the primary channels.

1.2.2 Information theoretic metrics of interest

Given a probabilistic characterization of a channel, the fundamental limits of communication over the channel may be expressed in terms of a number of metrics. In this chapter, we will be considering the following two commonly considered and powerful metrics:

1. **Capacity/capacity regions:** Largest rate / rate tuples at which reliable communication may be ensured.

2. **Sum-throughput scaling:** How the sum-rate of the network scales with the number of nodes n, as $n \to \infty$.

Capacity/capacity regions

The capacity of a point-to-point channel (single transmitter, single receiver) is defined as the supremum over all rates (expressed in bits/channel use) for which reliable communication may take place. Reliable communication is achieved when the probability of decoding error may be made arbitrarily small, and is usually achieved in an asymptotic sense as the number of channel uses tends to infinity. Shannon's pioneering work [Sha48] proved that for a simple discrete memoryless point-to-point channel with inputs x of the input alphabet $\mathcal{X}$ to the outputs y of the output alphabet $\mathcal{Y}$, the capacity C is given by the supremum over all input distributions $p(x)$ of the mutual information

$$I(X;Y) = \sum_{x,y} p(x,y) \log_2 \left(\frac{p(x,y)}{p(x)p(y)} \right).$$

Naturally, C will depend on the conditional distributions $p(y|x)$ which define the DMC. The mutual information $I(X;Y)$ intuitively measures how much information the variables X and Y share, that is how much one can tell you about the other. One of the most challenging aspects in obtaining the capacity of a channel is determining what input distribution $p(x)$ maximizes the mutual information.

In the point-to-point AWGN channel, the output Y is related to the input X according to $Y = hX + N$, where h is a fading coefficient (often modeled as a Gaussian random variable), and N is the noise which is $N \sim \mathcal{N}(0,1)$. Under an average input power constraint $E[|X|^2] \leq P$, it is known that the optimal input distribution is Gaussian as well, allowing one to obtain the well-known capacity

$$C = \frac{1}{2} \log_2 \left(1 + |h|^2 P\right) = \frac{1}{2} \log_2 (1 + \text{SINR}) := \mathcal{C}(\text{SINR})(\text{bits/channel use}).$$

Here SINR is the received signal to interference plus noise ratio, and $\mathcal{C}(x) := \frac{1}{2}\log_2(1+x)$. Gaussian noise channels have the computationally convenient property that the optimal, capacity achieving input distribution $p(x)$ is often Gaussian as well. Thus, in Gaussian noise channels, even when the capacity achieving input distribution of the channel is unknown, achievable rate regions are often computed assuming Gaussian input distributions.

While capacity is central to many information theoretic studies, it is often challenging to determine. Inner bounds, or achievable rates, as well as outer bounds to the capacity may be more readily available. Inner bounds (or inner bound regions when there are multiple simultaneous data streams) lie inside the capacity region and are obtained obtained by suggesting a coding scheme and proving that it achieves asymptotically small error as the block length $n \to \infty$. The suggested scheme may not be the best transmission scheme. The capacity (or capacity region for multiple data streams) is the supremum over all achievable rates (rate regions, respectively). An outer bound to the capacity is a rate above which reliable communication (in the sense of the probability of error $\to 0$ as the blocklength $n \to \infty$) may be shown to be impossible. If one can determine an outer bound on the capacity region and show that a particular scheme achieves all points on the outer bound of a channel, then that encoding scheme is said to be *capacity-achieving.*

Capacity regions have been particularly difficult to obtain for channels in which multiple transmitters and multiple receivers simultaneously wish to communicate. Indeed, *multi-user information theory* or *network information theory* is a challenging field with a plethora of open questions. As an example, one of the central, and simplest of multi-user channels is the information theoretic *interference channel.* This channel, introduced over 30 years ago [Car78, Sat77] consists of two independent transmitters which wish to communicate independent messages to two independent receivers, much like the simple channel depicted in Fig. 1.3. Although the channel capacity region[3] is known in certain cases, the general capacity region, despite promising recent advances [ETW07, Kra04, SKC07, CMGG06], is still an open problem. At the crux of this lies the information theory community's lack of understanding of how to deal with interference and overheard, undesired information.

Sum throughput scaling laws

When we have a network of nodes, the exact capacity region of the network is currently out of reach. The sum-throughput scaling law of a network is a more tractable asymptotic approximation commonly used in describing wireless channels. The study of throughput scaling laws was initiated by the work of Gupta and Kumar [GK00] and has expanded to consider a variety of wireless channel models and communication protocol assumptions [GT02, XK04, XK06a, KV04, AK04, LT05, AJV06, JKV06, XK06b, FDTT07, OLP07, dLT07]. One typically assumes n (transmitter-receiver) pairs of randomly located devices wish to communicate and asks how their sum-rate scales as a function of n. The number of nodes n is allowed to grow to ∞ by either letting the density of nodes stay fixed and the area increase with n (extended network), or by fixing the network area and letting the density increase with n (dense network). Due to node limitations such as power constraints, multiple-hops may be needed for a specific message to reach a distant destination.

As expected, the throughput scaling in ad hoc networks depends greatly on the node distribution and the physical-layer processing capability, more specifically the ability to cooperate among nodes. Some specific examples are:

[3]Capacity regions and achievable rate regions are natural extensions of the notions of capacity and achievable rate to higher dimensions.

- In the interference-limited regime, in which no cooperation is allowed (except simple decode-and-forward), and all nodes treat other signals as interference, the per-node throughput scales at most as $1/\sqrt{n}$ [GK00].
- If the nodes are uniformly distributed, a simple nearest-neighbor forwarding scheme achieves a $1/(n \log(n))$ per-node throughput [GK00].
- When the nodes are distributed according to a Poisson point process, a backbone-based routing scheme achieves the per-node scaling of $1/\sqrt{n}$ [FDTT07], meeting the upper bound.
- When nodes are able to cooperate in a MIMO-like fashion, a novel hierarchical scheme can achieve a *linear* grow in the sum rate, corresponding to a constant per-node throughput [dLT07].

The development of these scaling laws show that the assumptions about the network and the nodes' signal processing capability are crucial to the scaling law. We will later be outlining scaling law results for *cognitive networks* in which primary and secondary devices co-exist. This scenario differs from existing scaling law results in that the network is no longer homogeneous, and the cognitive nodes must transmit in a way that is acceptable to the primary nodes.

1.2.3 Classical channels

We briefly outline four classical multi-user channels, illustrated in Fig. 1.4, which are relevant to the study of cognitive networks.

Relay channels

In a relay channel [CG79], the communication between a source (grey) and a destination (black) may be aided through the use of a relay node (white), which has no information of its own to convey. Despite its simplicity, the capacity of this channel remains unknown in general. A comprehensive survey of relay channels may be found in [KGG05], while interesting approximation results for relay networks are presented in [ADT]. The notion of having one node relay information for another is a fundamental one, and techniques used in relays channels (for example, Block Markov coding) may be used in cognitive channels. In addition, cognitive nodes may act as relays for other cognitive or even primary nodes.

Multiple-access channels

In the classical multiple-access channel (MAC) two independent nodes wish to communicate to a single common receiver. The capacity of this channel, and techniques employed such as super-position coding and successive decoding are well

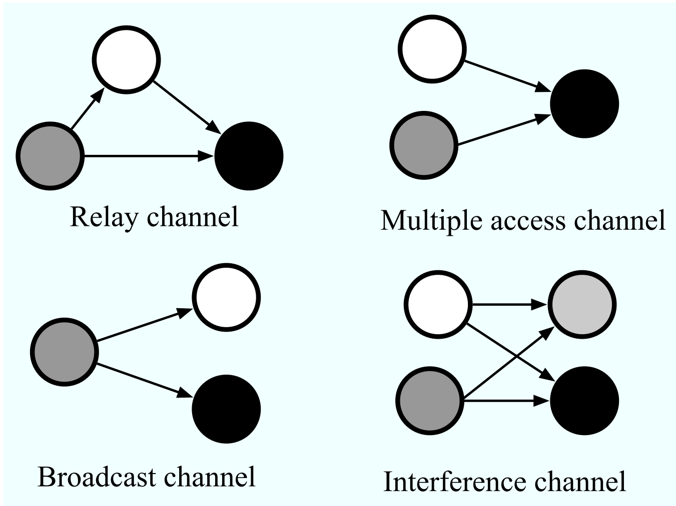

Figure 1.4: Four classical information theoretic multi-user channels. Contemporary cognitive channels encompass and may make use of clever techniques used in these well studied channels.

understood [CT91, Yeu02, CK81] and form the building block for achievable rate regions of other closely related channels including the interference channel. Interesting aspects and properties of the capacity region of MAC may be found in [CGS80, RU96, Car82, Wil82].

Broadcast channels

In the classical broadcast channel a single transmitter wishes to communicate independent messages[4] to two independent receivers. Its capacity region is in general unknown. For the DMC, some of the best achievable rate regions and outer bounds may be found in [Mar79, Cov98]. The capacity region of the Gaussian MIMO broadcast channel has recently been unveiled [WSS04, MC06], where it was shown that encoders using Gaussian codebooks with successive dirty paper coding [Cos83, CS03] are capacity achieving.

Interference channels

In the classical interference channel [Sat77, Car78] two independent transmitters wish to transmit two independent messages to two independent receivers. The capacity region of this channel is in general unknown, though there are capacity

[4]In general the transmitter may wish to transmit a common message destined to both receivers as well as independent ones to each of the receivers.

results for a few special cases [Sat81, CG87]. The best inner bounds for discrete memoryless interference channels may be found in [HK81, CMGG08] and tight outer bounds are obtained in [NG07]. Some of the main techniques used in interference channels include rate-splitting, superposition coding and binning, all of which are useful for cognitive channels. In Gaussian noise, recent advances are bringing us close to the capacity [Sas04, ETW07, Kra04, SKC07]. The degrees of freedom in MIMO interference channels may be found in [JF06, ETW07, PBT08].

Cognitive channels

We now turn to a novel class of channels: *cognitive channels*. In these channels a subset of the nodes may be cognitive radios which wish to access the spectrum licensed out to primary user(s). A simple example is shown in Fig. 1.3. As stated before, cognitive channels seek to exploit the cognition enabled by the cognitive radios; i.e exploit the fact that cognitive radios are able to sense their spectral environment and adapt to it. Various types of cognition, or side information may be available to the cognitive radios. Their behavior will vary depending on this side information. We now use some of the tools, metric and channels outlined in this section to explore three types of cognitive behavior: interference avoiding, interference controlling and interference mitigating cognitive behavior.

1.3 Interference avoiding behavior: spectrum interweave

Secondary spectrum licensing and cognitive radio was arguably conceived with the goal and intent of implementing interference-avoiding behavior [J.M00, Hay05]. Indeed, in this intuitive approach to secondary spectrum licensing cognitive radios sense the spatial, temporal, or spectral voids and adjust their transmission to fill in the sensed *white spaces*. Cognition in this setting corresponds to the ability to accurately detect the presence of other wireless devices; the cognitive side-information is knowledge of the spatial, temporal and spectral gaps a particular cognitive Tx-Rx pair would experience. The cognitive radios would adjust their transmission to fill in the spectral (or spatial/temporal) void, as illustrated in Fig. 1.1, with the potential to drastically increase the spectral efficiency of wireless systems.

The cognition required for this type of behavior is knowledge of the spectral gaps. In a realistic system the secondary transmitter would spend some of its time sensing the the channel to determine the presence of the primary user. As an illustrative example and idealization, we assume that knowledge of the spectral gaps is perfect: when primary communication is present the cognitive devices are able to precisely determine it, instantaneously. While such assumptions may be valid for the purpose of theoretical study, and provide outer bounds on what can be realistically achieved, practical methods for detecting primary signals have also been of great recent interest. A theoretical framework for determining the limits of communication as a function of the sensed cognitive transmitter and receiver gaps is formulated in [JS07, SJJ06]. Studies on how detection errors may affect the cognitive and primary systems are found in [SBNS07, TS08, Tka07]. Because current secondary spectrum licensing proposals demand detection guarantees of primary users at levels at extremely low levels in harsh fading environments, a number of authors have suggested improving detection capabilities through allowing multiple cognitive radios to collaboratively detect the primary transmissions [MSB06, GL05, dSCK07, GS05].

Under our idealized assumptions, the rates R_1 of the primary Tx-Rx pair and R_2 of the cognitive Tx-Rx pair achieved through ideal white-space filling are shown as the inner white triangle of Fig. 1.2. When a single user transmits the entire time in an interference-free environment, the axes intersection points are attained. The convex hull of these two interference-free points may be achieved by time-sharing (TDMA fashion). Where on this line a system operates depends on how often the primary user occupies the specific band. If the primary and secondary power constraints are P_1 and P_2 respectively, then the white-space filling rate region may be described as:

$$\text{White-space filling region (a)} \tag{1.1}$$
$$= \{(R_1, R_2) | 0 \le R_1 \le \eta \mathcal{C}(P_1), 0 \le R_2 \le (1-\eta)\mathcal{C}(P_2), 0 \le \eta \le 1\}. \tag{1.2}$$

Interference avoidance through MIMO

In addition to detecting the spectral white-spaces, interference at the primary user may be avoided or controlled if the cognitive user is equipped with multiple antennas, and is able to place its transmit signal in the null space of the primary users receive channel. In this scenario, the exact channel between the secondary transmit antennas and the primary receive antennas must be known. In [ZL], the authors study the fundamental tradeoff a cognitive transmitter faces between maximizing its own transmit throughput and minimizing the amount of interference it produces at each primary receiver. They address this from an information-theoretic perspective by characterizing the secondary user's channel capacity under both its own transmit-power constraint as well as a set of interference-power constraints each imposed at one of the primary receivers. In particular, this paper exploits multi-antennas at the secondary transmitter to effectively balance between spatial multiplexing for the secondary transmission and interference avoidance at the primary receivers. In [ZXL07] a sum-rate maximization problem for single-input multiple output multiple access channels (SIMO-MAC) under interference constraints for the primary users as well as a peak transmission power constraint for each secondary user is considered. The authors wish to maximize the rate of the secondary users subject to interference constraints on the primary users as well as peak power constraints for the secondary users. Other works which similarly exploit multiple antennas to avoid or control the interference seen by primary users may be found in [ILH07, YV08, ZLXP, HHJN03]. The scenarios considered in these papers can be considered an *interference-avoiding* scheme if the tolerable interference at the primary receivers is set to zero, other-wise it falls under the *interference-controlled* paradigm we look at in the next section.

1.4 Interference controlled behavior: spectrum underlay

When the interference caused by the secondary users on the primary users is permitted when below a certain level, or while guaranteeing a certain level of quality of service, the more flexible *interference controlled behavior* emerges. We look at spectrum underlay techniques and an example of the resulting achievable rate region in small networks, as well as the resulting throughput scaling laws in two different types of large networks. We note that this type of interference controlled behavior covers a large spectrum of cognitive behavior and we highlight only three examples, while referring to only a small subset of all the possible references. In Fig. 1.1 spectrum underlay is graphically depicted as having primary users slightly grey, as opposed to the interference-free white color illustrated in interference avoiding.

1.4.1 Underlay in small networks: achievable rates

Interference temperature

Rather than detecting white spaces, in spectrum underlay, a cognitive radio simultaneously transmits with the primary user(s) while using its cognitive abilities to control the amount of harm it inflicts upon them. While the definition of harm may be formulated mathematically in a number of ways, one common definition involves the notion of *interference temperature*, a term first introduced by the FCC [Kol06] to denote the average level of interference power seen at a primary receiver. In secondary spectrum licensing scenarios, the primary receiver's interference temperature should be kept at a level that will satisfy the primary user's desired quality of service. That is, primary transmission schemes may be designed to withstand a certain level of interference, which cognitive radios or secondary nodes may exploit for their own transmission. Provided the cognitive user knows (1) the maximal interference temperature for the surrounding primary receivers, (2) the current interference temperature level, and (3) how its own transmit power will translate to received power at the primary receiver, then the cognitive radio may adjust its own transmission power so as to satisfy any interference temperature constraint the primary user(s) may have. The work [Gas07, GS06, WPW07, XMH+07] all consider the capacity of cognitive systems under various receive-power (or interference-temperature-like) constraints.

As an illustrative example, we consider a very simple interference-temperature based cognitive transmission scheme. Assume in the channel model of Fig. 1.3 that each receiver treats the other user's signal as noise, a lower bound to what may be achieved using more sophisticated decoders [Ver03]. The rate region obtained is shown as the light grey region (b) of Fig. 1.2. This region is obtained as follows: we assume the primary transmitter communicates using a Gaussian codebook of constant average power P_1. We assume the secondary transmitter allows its power to lie in the range $[0, P_2]$ for P_2 some maximal average power constraint. The rate region obtained may be expressed as:

$$\begin{aligned} &\text{Simultaneous-transmission rate region (b)} \\ &= \left\{ (R_1, R_2) | 0 \leq R_1 \leq \mathcal{C}\left(\frac{P_1}{h_{21}^2 P_2^* + 1}\right), \right. \\ &\qquad \left. 0 \leq R_2 \leq \mathcal{C}\left(\frac{P_2^*}{h_{12}^2 P_1 + 1}\right),\ 0 \leq P_2^* \leq P_2 \right\}. \end{aligned} \tag{1.3}$$

The actual value of P_2^* chosen by the cognitive radio depends on the interference-temperature, or received power constraints at the primary receiver.

1.4.2 Underlay in large networks: scaling laws

Information theoretic limits of interference controlled behavior has also been investigated for large networks, i.e. networks whose number of nodes $n \to \infty$. We illustrate two types of networks: single hop networks and multi-hop networks. In the former, secondary nodes transmit subject to outage-probability-like constraints on the primary network. In the latter, the multi-hop secondary network is permitted to operate as long as the scaling law of the primary network is kept the same as in the absence of the cognitive network.

Single-hop cognitive networks

The planar network model considered in [VT08] is depicted in Figure 1.5, where multiple primary and secondary users co-exist in a network of radius R (the number of nodes grows to ∞ as $R \to \infty$). Around each receiver, either primary or cognitive, we assume a protected circle of radius $\epsilon_c > 0$, in which no interfering transmitter may operate. Other than the receiver protected regions, the primary transmitter and receiver locations are arbitrary, subject to a minimum distance R_0 between any two primary transmitters. This scenario corresponds to a broadcast network, such as the TV or the cellular networks, in which the primary transmitters are base-stations. The cognitive transmitters, on the other hand, are uniformly and randomly distributed with constant density λ. We assume that each cognitive receiver is within a $D_{\max}$ distance from its transmitter, as shown in the cognitive user model of Fig. 1.5 and transmit with constant power P. We assume that the channel gains are path-loss dependent only (no fading or shadowing) and that each user treats unwanted signals from all other users as noise.

The quality of service guarantee of the primary users is of the form
Pr[primary user's rate $\leq C_0$] $\leq \beta$. That is, the secondary users must transmit so as to guarantee that the probability that the primary users' rates fall below C_0 is less than a desired amount β. This may be done by appropriate selection of the network parameters $P, \epsilon_c, R_0, \lambda$ as done in [VDT08].

The questions answered in [VT08] and [VDT08] that relate to this single-hop cognitive network setting may be summarized as:

- **What is the scaling law of the secondary network?** By showing that the average interference to the cognitive users remains bounded due to the finite transmission ranges D_{max} of the cognitive users and R_0 of the primary users, one can show that the lower and upper bounds to each user's average transmission rate are constant and thus the *average* network throughput grows linearly with the number of users [VT08] .

- **How should the network parameters be chosen to guarantee** Pr[**primary user's rate** $\leq C_0$] $\leq \beta$**?** This interesting question is addressed in [VDT08, HS05], and is omitted here for brevity.

- **May the cognitive nodes scale their power depending on the distance from the primary network?** Yes, under the assumption that there is a single primary network at the center of the cognitive network, the further away the cognitive users are from the center, the larger their transmit power may be (or alternatively the larger the distance D_{max} may get. Specifically, suppose that a cognitive user at distance r transmits with power $P = P_c r^{\gamma}$, where P_c is a constant. Then, provided that $0 \leq \gamma < \alpha - 2$, the total interference from the cognitive users to the primary user is still bounded, making the power scaling an attractive option for the cognitive users. With the power-scaling, the maximum distance $D_{\max}$ between a cognitive Tx and Rx can now grow with the network size as $D_{\max} \leq K_d r^{\gamma/\alpha} < K_d r^{1-2/\alpha}$. where r again is the distance from the cognitive transmitter to the primary transmitter and K_d is a constant. Thus depending on the path loss α, the cognitive Tx-Rx distance can grow with an exponent of up to $1 - 2/\alpha$. For a large α, this growth is almost at the same rate as the network.

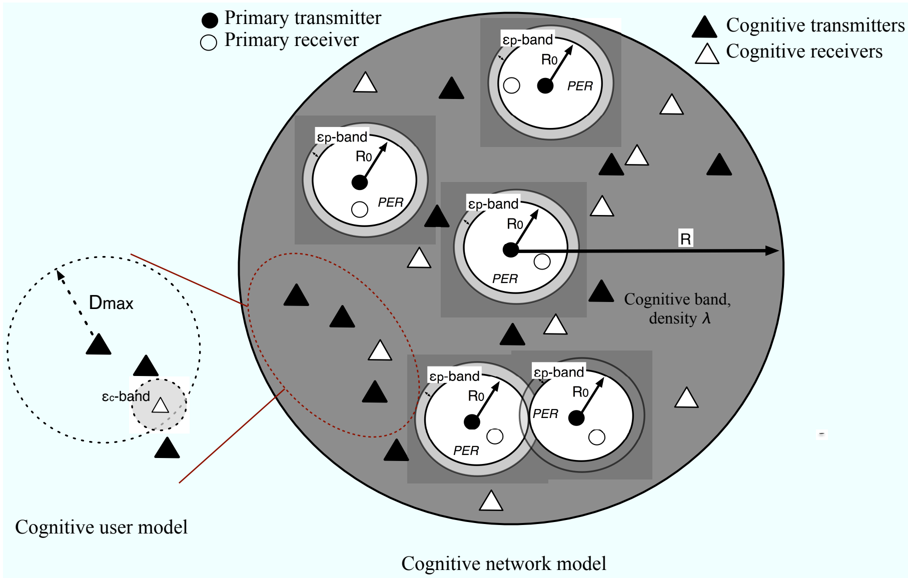

Figure 1.5: Network model: A cognitive network consists of multiple primary and cognitive users. The primary users locations are arbitrary with a minimum distance R_0 between any two primary transmitters. The cognitive transmitters are distributed randomly and uniformly with density λ. Cognitive user model: Each cognitive transmitter wishes to transmit to a single cognitive receiver which lies within a distance $\leq D_{max}$ away. Each cognitive receiver has a protected circle of radius $\epsilon_c > 0$, in which no interfering transmitter may operate.

Multi-hop cognitive networks

We now consider a cognitive network consisting of multiple primary and multiple cognitive users, where there is no restriction on the maximum cognitive Tx-Rx distance. We assume Tx-Rx pairs are selected randomly, as in a classical [GK00] stand-alone ad hoc network. Both types of users are ad hoc, randomly distributed according to Poisson point processes with different densities. Here the quality of service guarantee to the primary users states that the scaling law of the primary ad hoc network does not diminish in the presence of the secondary network.

In [JDV+08] it is shown that provided that the cognitive node density is higher than the primary node density, using multi-hop routing, **both** types of users, primary and cognitive, can achieve a throughput scaling as if the other type of users were not present. Specifically, the throughput of the m primary users scales as $\sqrt{m/\log m}$, and that of the n cognitive users as $\sqrt{n/\log n}$.

What is of particular interest in this result is that to achieve these throughput scalings, the primary network need not change anything in its protocols; it is *oblivious* to the secondary network's presence. The cognitive users, on the other hand, rely on their higher density and a clever routing technique (in the form of *preservation regions* [JDV+08]) to avoid interfering with the primary users.

1.5 Interference mitigating behavior: spectrum overlay

Thus far, the side-information available to the cognitive radios has been (a) knowledge of the primary spectral gaps and (b) knowledge of the primary interference constraints and secondary to primary channel gains. In this section we increase the level of cognition even further. In interference-mitigating cognitive behavior, the cognitive user transmits over the same spectrum as the primary user, but makes use of this additional cognition to mitigate (1) interference it causes to the primary receiver and (2) interference the cognitive receiver experiences from the primary transmitter.

In order to mitigate interference, the cognitive nodes must have the primary system's *codebooks*. This will allow the cognitive transmitter and/or receiver to opportunistically decode the primary users' messages, which in turn may lead to gains for both the primary and secondary users, as we will see. We consider two types of interference-mitigating behavior in this section:

1. **Opportunistic interference cancellation:** The cognitive nodes have the codebooks of the primary users. The cognitive receivers opportunistically decode the primary users' messages which they pull off of their received signal, increasing the secondary channel's transmission rates.

2. **Asymmetrically cooperating cognitive radio channels:** The cognitive nodes have the codebooks of the primary users, and the cognitive transmitter(s) has knowledge of the primary user's message. The cognitive transmitter may use this message knowledge to carefully mitigate interference at the cognitive receiver as well as cooperate with the primary in boosting its signal at its receiver.

Opportunistic interference cancellation

We assume the cognitive link has the same knowledge as in the interference-temperature case (b) and has some additional information about the primary link's communication: the primary user's *codebook*. Primary codebook knowledge translates to being able to decode primary transmissions; we suggest a scheme which exploits this extra knowledge next.

In *opportunistic interference cancellation*, as first outlined in [PYN$^+$07] the cognitive receiver opportunistically decodes the primary user's message, which it then subtracts off its received signal. This intuitively cleans up the channel for the cognitive pair's own transmission. The primary user is assumed to be oblivious to the cognitive user's operation, and so continues transmitting at power P_1 and rate R_1. When the rate of the primary user is low enough relative to the primary signal power at the cognitive receiver (or $R_1 \leq \mathcal{C}\left(h_{12}^2 P_1\right)$) to be decoded by $\mathcal{SR}_x$, the channel ($\mathcal{PT}_x, \mathcal{ST}_x \rightarrow \mathcal{SR}_x$) will form an information theoretic multiple-access channel, whose capacity region is well known [CT91]. In this case, the cognitive receiver will first decode the primary's message, subtract it off its received signal, and proceed to decode its own. When the cognitive radio cannot decode the primary's message, the latter is treated as noise.

The region (c) of Fig. 1.2 illustrates the gains opportunistic decoding may provide over the former two strategies. It is becoming apparent that higher rates are achievable when there is a higher level of cognition in the network which is properly exploited. What type of cognition is valid to assume will naturally depend on the system/application.

1.5.1 Asymmetrically cooperating cognitive radio channels

We increase the cognition even further and assume the cognitive node(s) has the primary codebooks as well as the message to be transmitted by the primary sender(s). For simplicity of presentation we consider again the two transmitter, two receiver channel shown in Figs. 1.2 and 1.7. This additional message knowledge allows for a form of *asymmetric cooperation* between the primary and cognitive transmitters. This asymmetric form of transmitter cooperation, first introduced in [DMT05b, DMT06a], can be motivated in a cognitive setting in a number of ways.

- Depending on the device capabilities, as well as the geometry and channel gains between the various nodes, certain cognitive nodes may be able to hear and/or obtain the messages to be transmitted by other nodes. For example, if $\mathcal{ST}_x$ is geographically close to $\mathcal{PT}_x$ (relative to $\mathcal{PR}_x$), then the wireless channel ($\mathcal{PT}_x \rightarrow \mathcal{ST}_x$) could be of much higher capacity than the channel ($\mathcal{PT}_x \rightarrow \mathcal{PR}_x$). Thus, in a fraction of the transmission time, $\mathcal{ST}_x$ could listen to, and obtain the message transmitted by $\mathcal{PT}_x$. These messages would need to be obtained in real time, and could exploit the geometric gains between cooperating transmitters relative to receivers in, for example, a 2 phase protocol [DMT06a].

- In an Automatic Repeat reQuest (ARQ) system, a cognitive transmitter, under suitable channel conditions (if it has a better channel to the primary transmitting node than the primary receiver), could decode the primary user's transmitted message during an initial transmission attempt. In the event that the primary receiver was not able to correctly decode the message, and it must be re-transmitted, the cognitive user would already have the to-be-transmitted message, or asymmetric side information, at no extra cost (in terms of overhead in obtaining the message).

- The authors in [WVA07] consider a network of wireless sensors in which a sensor $\mathcal{S}_2$ has a better sensing capability than another sensor $\mathcal{S}_1$ and thus is able to sense two events, while $\mathcal{S}_1$ is only able to sense one. Thus, when they wish to transmit, they must do so under an asymmetric side-information assumption: sensor $\mathcal{S}_2$ has two messages, and the other has just one.

The main question that information theory helps in answering is: how can the cognitive system best exploit this extra level of cognition, i.e. knowledge of the primary user's message?

Background: exploiting transmitter side information

A key idea behind achieving high data rates in an environment where two senders share a common channel is interference cancelation or mitigation. The capacity of a discrete memoryless channel $p(y|x,s)$ when side-information s (which may be thought of as interference) is known non-causally at the transmitter, but not the receiver was first considered by Gel'fand and Pinsker [GP80]. They showed that the capacity of this discrete memoryless channel is given by,

$$C = \max_{p(u|s)p(x|u,s)} I(U;Y) - I(U;S), \tag{1.4}$$

for an auxiliary random variable U distributed jointly with X and S.

The result of Gel'fand and Pinsker was later generalized by Costa [Cos83] to real alphabets in his well-known paper entitled "Writing on Dirty Paper". There, he showed that in a Gaussian noise channel with noise N of power Q, input X power constraint $E[|X|^2] \leq P$, and additive interference S of arbitrary power known non-causally to the *transmitter* but not the receiver,

$$Y = X + S + N, \quad E[|X|]^2 \leq P, \quad N \sim \mathcal{N}(0,Q)$$

the capacity is that of an interference-free channel, or

$$\begin{aligned} C &= \max_{p(u|s)p(x|u,s)} I(U;Y) - I(U;S) && (1.5)\\ &= \frac{1}{2}\log_2\left(1 + \frac{P}{Q}\right). && (1.6)\end{aligned}$$

This remarkable and surprising result has found its application in numerous domains including data storage [KT74, HG83], watermarking/steganography [SM01], and most recently, *dirty-paper coding* has been shown to be the capacity achieving technique in Gaussian MIMO broadcast channels [WSS04, CS03]. We now apply dirty-paper coding techniques to the Gaussian cognitive channel.

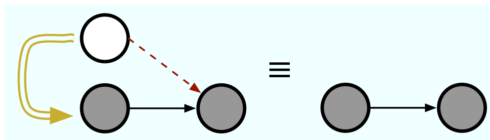

Figure 1.6: A channel with non-causal knowledge of the interference has, in Gaussian noise, capacity equivalent to an interference-free channel.

Bounds on the capacity of cognitive radio channels

Although in practice the primary message must be obtained causally, as a first step, numerous works have idealized the concept of message knowledge: whenever the cognitive node $\mathcal{ST}_x$ is able to hear and decode the message of the primary node $\mathcal{PT}_x$, it is assumed to have full *a-priori* knowledge.[5] The one way double arrow in Fig. 1.7 indicates that $\mathcal{ST}_x$ knows $\mathcal{PT}_x$'s message but not vice versa. This is the simplest form of asymmetric non-causal cooperation at the transmitters. The term *cognitive* is used to emphasize the need for $\mathcal{ST}_x$ to be a device capable of obtaining the message of the first user and altering its transmission strategy accordingly.

This asymmetric transmitter cooperation present in the *cognitive* channel, has elements in common with the *competitive* channel and the *cooperative* channels of Fig. 1.7, which may be explained as follows:

1. **Competitive behavior/channel:** The two transmitters transmit independent messages. There is no cooperation in sending the messages, and thus the two users *compete* for the channel. This is the same channel as the 2 sender, 2 receiver interference channel [Car78]. The largest to-date known general region for the interference channel is that described in [HK81] which has been stated more compactly in [CMGG08]. Many of the results on the cognitive channel, which contains an interference channel if the non-causal side information is ignored, use a similar rate-splitting approach to derive large rate regions [DMT06a, MYK07, JX07].

2. **Cognitive behavior/channel:** Asymmetric cooperation is possible between the transmitters. This asymmetric cooperation is a result of $\mathcal{ST}_x$ knowing

[5]This assumption is often called the *genie assumption*, as these messages could have been given to the appropriate transmitters by a genie.

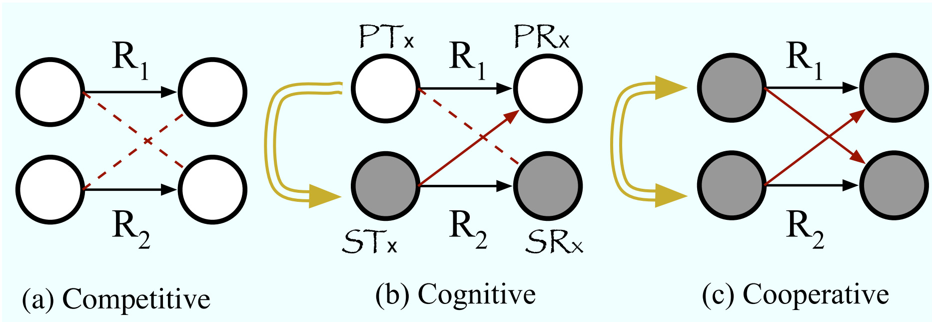

Figure 1.7: Three types of behavior depending on the amount and type of side-information at the secondary transmitter. (a) Competitive: the secondary terminals have no additional side information. (b) Cognitive: the secondary transmitter has knowledge of the primary user's message and codebook. (c) Cooperative: both transmitters know each others' messages. The double line denotes non-causal message knowledge.

$\mathcal{PT}_x$'s message, but not vice-versa. We will discuss this actively researched channel in this Section.

3. **Cooperative behavior/channel:** The two transmitters know each others' messages (two way double arrows) and can thus fully and symmetrically cooperate in their transmission. The channel pictured in Fig.1.7 (c) may be thought of as a two antenna sender, two single antenna receivers broadcast channel, where, in Gaussian MIMO channels, *dirty-paper coding* was recently shown to be capacity achieving [WSS04, CS03].

Cognitive behavior may be modeled as an interference channel with asymmetric, non-causal transmitter cooperation. This channel was first introduced and studied in [DMT05b, DMT06b][6]. Since then, a flurry of results, including capacity results in specific scenarios, of this channel have been obtained. When the interference to the primary user is weak ($h_{21} < 1$), rate region (d) has been shown to be the capacity region in Gaussian noise [JV06] and in related discrete memoryless channels [WVA07]. In channels where interference at both receivers is strong both receivers may decode and cancel out the interference, or where the cognitive decoder wishes to decode both messages, capacity is also known [MYK07, JXG07, LSBP+07]. However, the most general capacity region remains an open question for both the Gaussian noise as well as discrete memoryless channel cases.

[6]It was first called the *cognitive radio channel*, and is also known as the *interference channel with degraded message sets.*

When using an encoding strategy that properly exploits this asymmetric message knowledge at the transmitters, the region (d) of Fig. 1.2 is achievable in AWGN, and in the weak interference regime ($h_{21} < 1$ in AWGN) corresponds to the capacity region of this channel [WVA06, JV06]. The encoding strategy used assumes both transmitters use random Gaussian codebooks. The primary transmitter continues to transmit its message of average power P_1. The secondary transmitter, splits its transmit power P_2 into two portions, $P_2 = \psi s P_2 + (1-\psi)P_2$ for $0 \leq \psi \leq 1$. Part of its power, ψP_2, is spent in a *selfless* manner: on relaying the message of $\mathcal{PT}_x$ to $\mathcal{PR}_x$. The remainder of its power, $(1-\psi)P_2$ is spent in a *selfish* manner on transmitting its own message using the interference-mitigating technique of *dirty-paper coding*. This strategy may be thought of as selfish, as power spent on dirty-paper coding may harm the primary receiver (and is indeed treated as noise at $\mathcal{PR}_x$.) The rate region (d) may be expressed as [Dev07, JV06]:

$$\begin{aligned}&\text{Asymmetric cooperation rate region (d)}\\ &= \left\{ (R_1, R_2) | 0 \leq R_1 \leq \mathcal{C}\left(\frac{(\sqrt{P_1} + h_{12}\sqrt{\psi P_2})^2}{h_{12}^2 (1-\psi)P_2 + 1} \right), \right. \\ &\qquad 0 \leq R_2 \leq \mathcal{C}\left((1-\psi)P_2\right),\ 0 \leq \psi \leq 1 \}\end{aligned} \tag{1.7}$$

By varying ψ, we can smoothly interpolate between strictly selfless behavior to strictly selfish behavior. Of particular interest from a secondary spectrum licensing perspective is the fact that the primary user's rate R_1 may be strictly increased with respect to all other three cases (i.e. the x-intercept is now to the right of all other three cases) That is, by having the secondary user possibly relay the primary's message in a selfless manner, the system essentially becomes a 2×1 multiple-input-single-output (MISO) system which sees all the associated capacity gains over non-cooperating transmitters or antennas. This increased primary could serve as a motivation for having the primary share its codebook and message with the secondary user.

While Fig. 1.2 shows the impact of increasing cognition (or side information at the cognitive nodes) on the achievable rate regions corresponding to protocols which make use of this side information, Fig. 1.8 shows the impact of transmitter cooperation. In that figure, the region achieved through asymmetric transmitter cooperation (*cognitive behavior*) is compared to the (1) Gaussian MIMO broadcast channel region (in which the two transmitters may cooperate, *cooperative behavior*), (2) the achievable rate region for the interference channel region obtained in [HK81] (the largest known to date for the Gaussian noise case, *competitive behavior*)[7], and (3) the time-sharing region where the two transmitters take turns using

[7] The achievable rate region of [HK81] used in these figures (as the "interference channel" achievable region) assumes the same Gaussian input distribution as in [DMT06a] and is omitted for brevity.

the channel (*interference-avoiding behavior*). The capacity region for the Gaussian MIMO broadcast channel with two single antenna receivers and one transmitter with two antennas subject to per antenna power constraints of P_1 and P_2 respectively, is given by Eqn. (1.8), which may be obtained from the general formulation in [WSS04, CS03].

MIMO BC region = Convex hull of

$$\left\{\begin{array}{c} (R_1,R_2): \\ \begin{array}{c} R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \end{array} \bigcup \begin{array}{c} R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger+Q_1}{Q_1}\right) = R_1(\pi_{21}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^\dagger+Q_2}{H_2(B_1)H_2^\dagger+Q_2}\right) = R_2(\pi_{21}) \end{array} \\ B_1,B_2 \succeq 0,\ \ B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22}\end{bmatrix},\ B_2 = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22}\end{bmatrix},\ \ B_1+B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2\end{bmatrix} \end{array}\right\} \tag{1.8}$$

.

Here $X \succeq 0$ denotes that the matrix X is positive semi-definite, and we define $H_1 = [1\ h_{21}]$ and $H_2 = [h_{12}\ 1]$. The framework for the above Gaussian MIMO broadcast channel region may also be used to express an achievable rate region for the Gaussian asymmetrically cooperating channel [Dev07]. We notice the similarity with the MIMO broadcast region: the differences lie in the fact that only one dirty-paper coding order is permitted, and the transmit covariance matrix B_2 corresponding to the cognitive user's message, is constrained, reflecting the asymmetry of the cooperation. We note that this region is equivalent to (1.7).

Cognitive region = Convex hull of

$$\left\{\begin{array}{c} (R_1,R_2): \\ R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \\ B_1,B_2 \succeq 0,\ \ B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22}\end{bmatrix},\ B_2 = \begin{bmatrix} 0 & 0 \\ 0 & c_{22}\end{bmatrix},\ \ B_1+B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2\end{bmatrix},\ \ z^2 \leq P_1P_2 \end{array}\right\}$$

This 2×2, non-causal cognitive radio channel has been extended in a number of ways. The effect of generalized feedback has been studied in [YT08] and

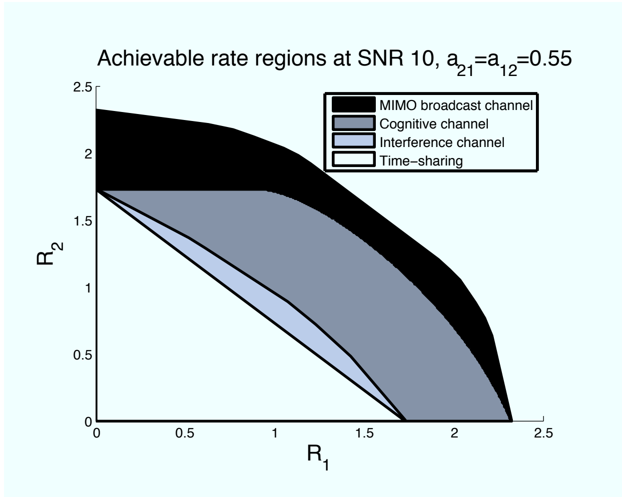

Figure 1.8: Capacity region of the Gaussian 2×1 MIMO two receiver broadcast channel (outer), cognitive channel (middle), achievable region of the interference channel (second smallest) and time-sharing (innermost) region for Gaussian noise powers $N_1 = N_2 = 1$, power constraints $P_1 = P_2 = 10$ at the two transmitters, and channel parameters $h_{12} = 0.55$, $h_{21} = 0.55$.

that of partial message knowledge in [MYK07]. While the above channel assumes non-causal message knowledge, a variety of two-phase half-duplex causal schemes have been presented in [DMT06a, KG07], while a full-duplex rate region was studied in [aXL07]. Many achievable rate regions are derived by having the primary transmitter exploit knowledge of the *exact* interference seen at the receivers (e.g. dirty-paper coding in AWGN channels). The performance of dirty-paper coding when this assumption breaks down has been studied in the context of a compound channel in [MDT06] and in a channel in which the interference is partially known [GS07].

Cognitive channels have also been explored in the context of multiple nodes and/or antennas. Extensions to channels in which both the primary and secondary networks form classical multiple-access channels has been considered in [DMT05a, CYZ$^+$]. Cognitive versions of the X channel [MAMK06] have been considered in [DS07, JS08], while cognitive transmissions using multiple-antennas, without asymmetric transmitter cooperation has been considered in [ZL].

Finally, while we have outlined some results on the exact rate regions for cognitive radio channels, how these rate scale at high signal to noise ratio (SNR $\to \infty$) is also a measure of interest. The multiplexing gain, m, of a cognitive network[8] de-

[8]Multiplexing gain, degrees of freedom and pre-log are all terms which are used interchangeably

fines how the sum-rate of a network, $C_{sum}(\log(\text{SNR}))$, grows as a function of SNR, i.e. $C_{sum}(\text{SNR}) = m\log(\text{SNR}) + o(\log(\text{SNR}))$ as $\text{SNR} \to \infty$. The multiplexing gain is of particular interest in networks in which exact capacity expressions are lacking, and may be thought of intuitively as the number of independent streams of information that may be simultaneously transmitted at high SNR. Great strides have been made in characterizing the degrees of freedom of interference networks [JF06], cognitive and X channels [DS07, JS08], and wireless networks in which cooperation is causally enabled [HMN05, HJ].

1.6 Chapter Summary

In this chapter, we have outlined recent results on the information theoretic limits of cognitive networks. Two main metrics were used: achievable rate / capacity regions for small networks as well as throughput scaling laws, as the number of nodes $n \to \infty$ for large networks. The general conclusion has been that increasing the amount of cognition, or side information available to the cognitive transmitters and/or receivers, increases the amount and quality of the communication. In interference-avoiding behavior, the cognitive nodes transmit in an orthogonal fashion to the primary users, thereby avoiding any mutual interference. Spectral efficiency may however be increased if cognitive nodes transmit over the same spectrum as the primary nodes, as done in the interference controlling and mitigating cognitive behaviors. In the former, cognitive transmitters require knowledge of the impact their transmission will have on the primary system and control their transmissions to stay within the acceptable limits for the primary user. When the secondary users further obtain the codebooks and possibly messages of the primary users, interference-mitigating behavior may be accomplished. The receivers may either opportunistically cancel the primary transmitter's interference or, at the cognitive transmitters, may judiciously select their power levels to either amplify or mitigate the primary user's signal. When building a cognitive network, the issue of obtaining these different types of side information, as well as realistically exploiting it becomes crucial. It will be up to the individual applications to judge whether the promised gains by increasing levels of cognition are worth the effort and cost in obtaining, and properly using it.

1.7 Problems

1. What are three types of cognitive behavior? Can you think of a fourth? How does it relate to the others?

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in the literature.

2. Which types of cognitive behavior does Code-Division-Multiple-Access fit into? Why?

3. Formally define what it means for a:

   (a) rate R to be *achievable* in a discrete memoryless channel

   (b) rate tuple (R_1, R_2) to be *achievable* in a multi-user discrete memoryless channel.

4. State an inner and an outer bound to the capacity region of the relay channel. Under what conditions are the inner or outer bounds tight?

5. State the capacity region of the 2 and 3 user discrete memoryless multiple-access channels.

6. State an inner and an outer bound to the capacity region of the discrete memoryless broadcast channel.

7. State the capacity region of a 2 user Gaussian broadcast channel, where all nodes have single antennas.

8. State the best known inner and outer bounds to the capacity region of the discrete memoryless interference channel.

9. In spectrum interweave cognitive behavior the sensing is idealized: the cognitive transmitters and receivers are able to sense the channel perfectly and instantaneously. Determine the impact on the achievable rate region for spectrum interweave cognitive behavior if perfect sensing of the primary user requires a finite duration of time T and the cognitive transmitter subsequently transmits for a time period T'.

10. What are different types of beamforming? What types are most useful to interference avoidance in cognitive networks with multiple antennas at transmitters and receivers?

11. In spectrum underlay systems in Gaussian noise, the region (b) is achievable. If it is acceptable that the primary user's rate drop to half of its interference-free rate when a cognitive user is present, determine the power P_2* at which the cognitive user may transmit. This P_2* is necessarily a function of the channel gains, the noise, and the primary user power.

12. From the description of opportunistic interference cancellation, mathematically describe the rate region (c) in terms of the channel gains, power constraints, noise power as well as interference temperature constraints of the primary user.

13. Determine two non-identical (and non-inclusive) inner bounds to the capacity region of the discrete memoryless cognitive channel with asymmetric cooperation.

14. What is the best known outer bound to the capacity region of the discrete memoryless cognitive channel with asymmetric cooperation.

15. Using MATLAB, plot the equivalent of Fig. 1.8 for channel gains:

    (a) $h_{12} = 0.2, h_{21} = 0.8$
    (b) $h_{12} = 0.8, h_{12} = 0.2$
    (c) $h_{12} = 1.2, h_{21} = 0.55$
    (d) $h_{21} = 1.2, h_{12} = 0.55$

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