A Lattice Compress and Forward Scheme

Natasha Devroye, *Assistant Professor, UIC*
Yiwei Song, *Ph.D. candidate, UIC*

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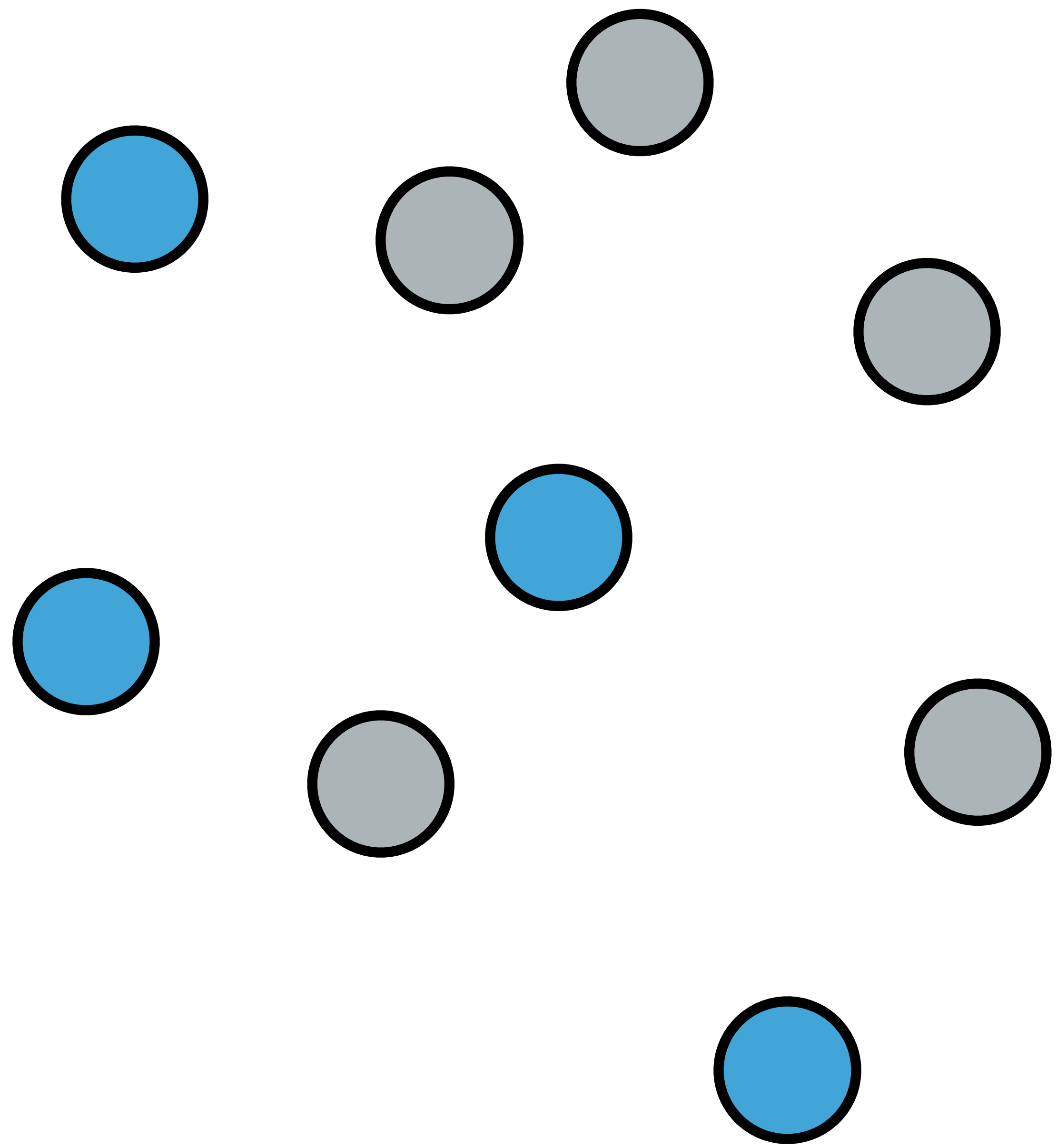
Gaussian networks



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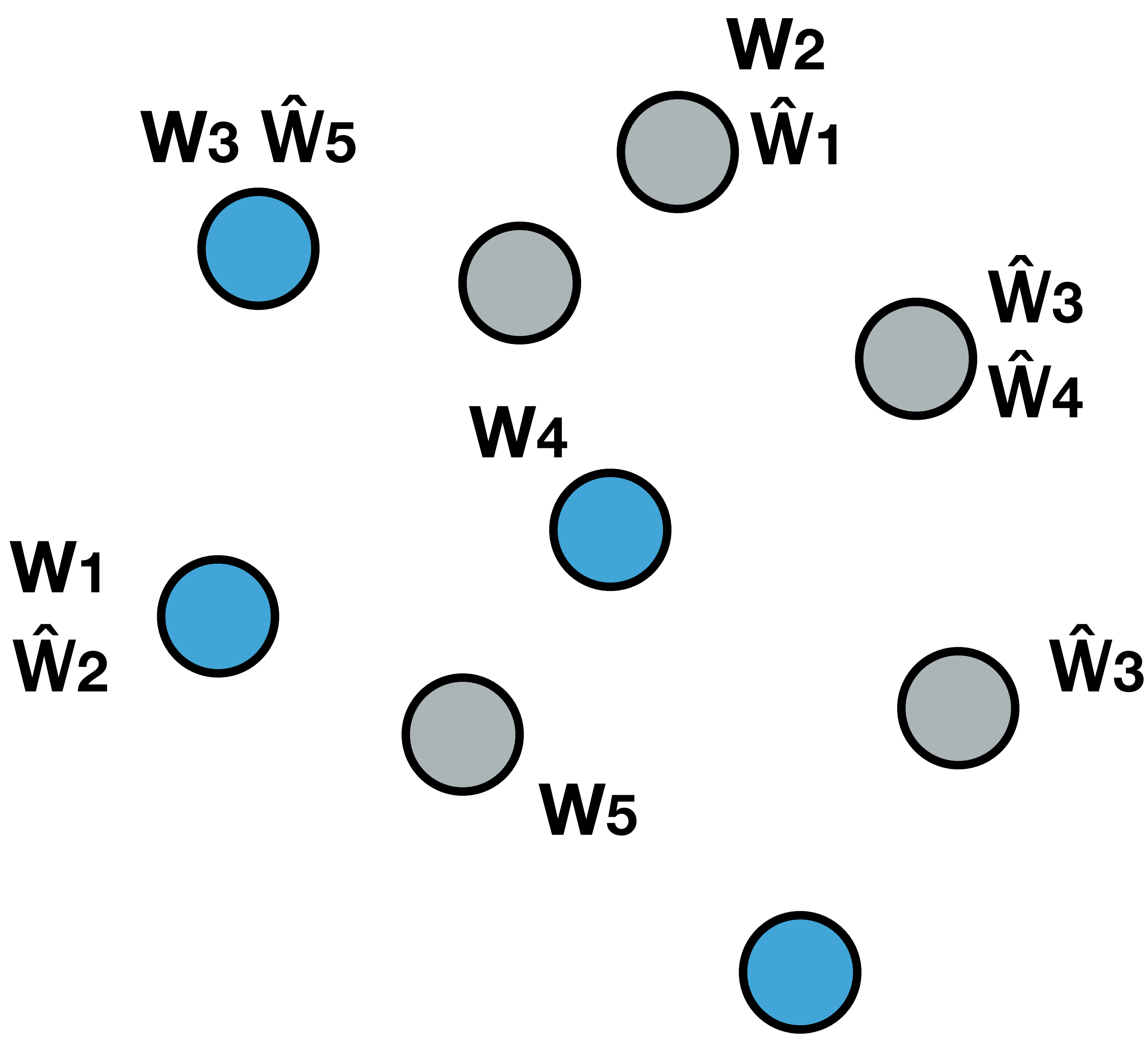
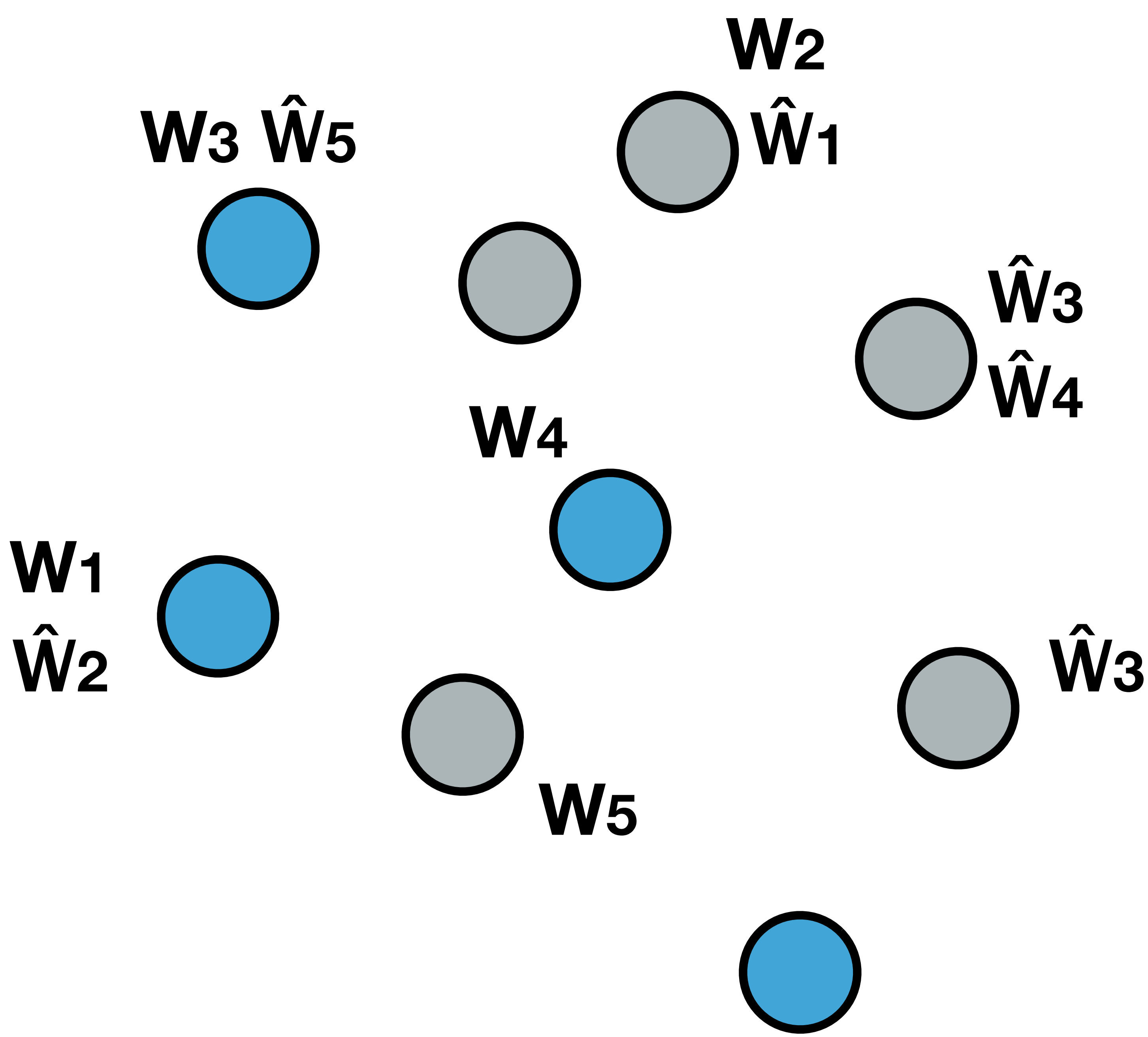


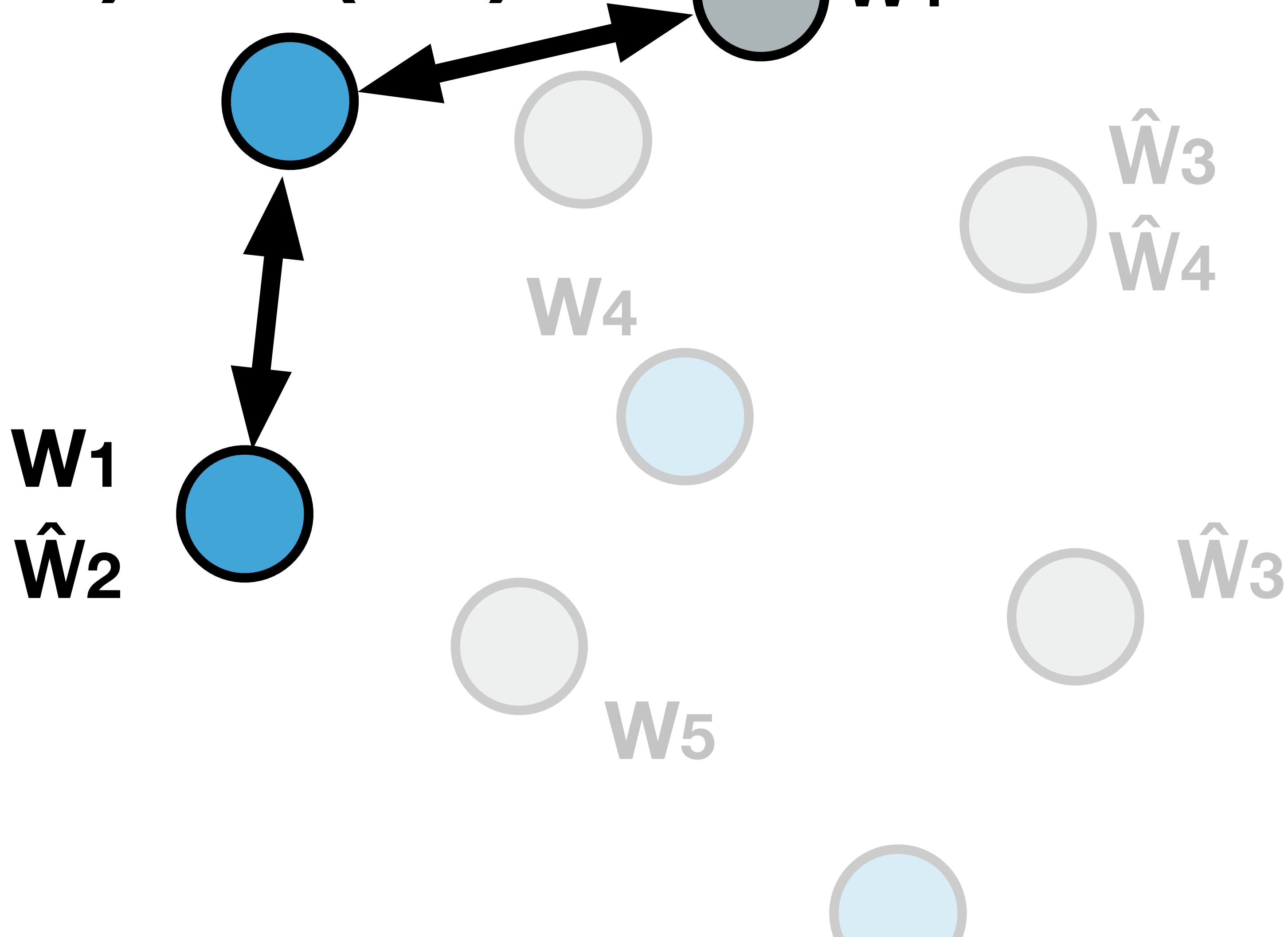
Structured codes for Gaussian networks



Structured codes for Gaussian networks

→ **decode the sum**

$$X_1(W_1) + X_2(W_2)$$



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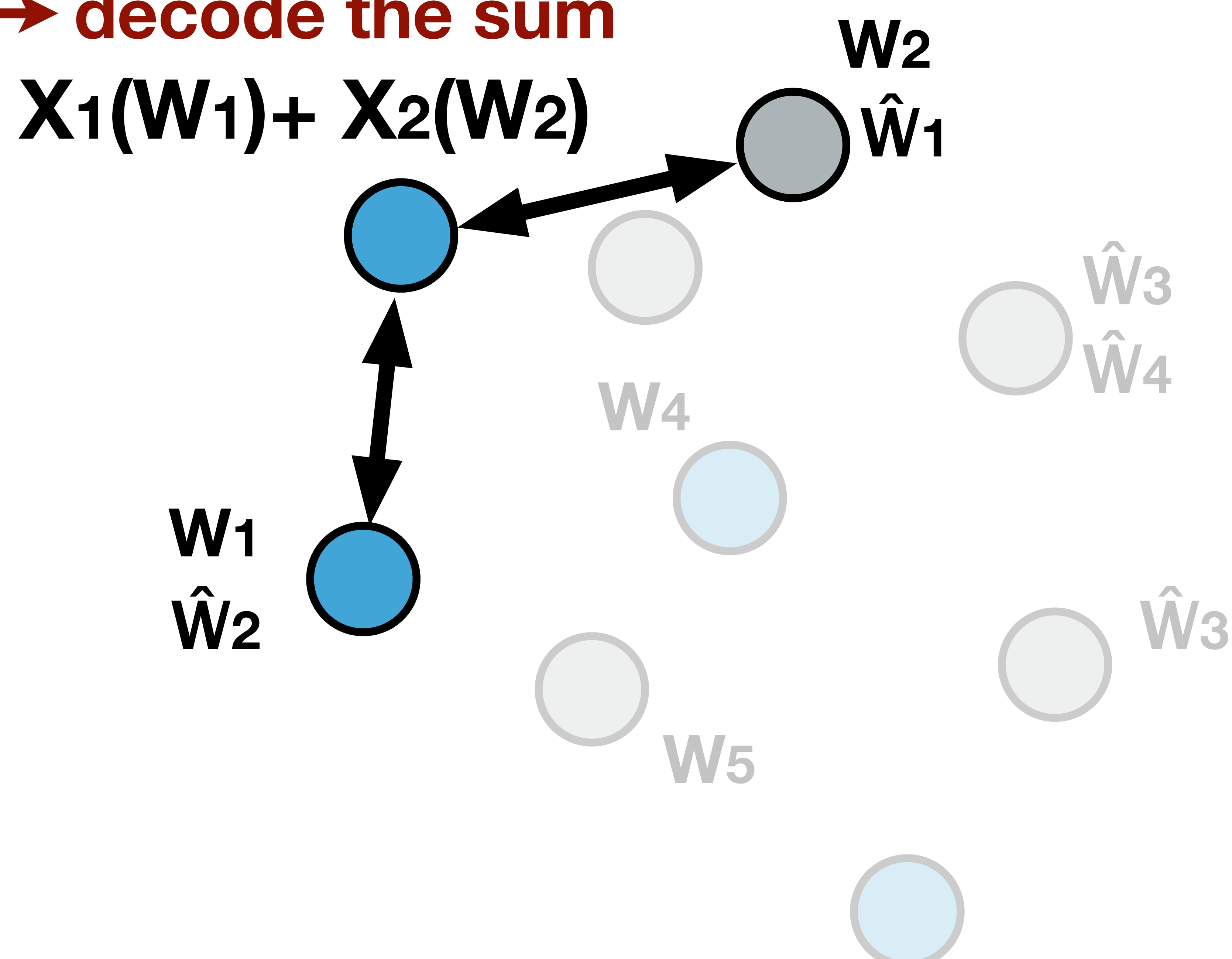

General "decode the sum" → Compute-and Forward

[Nazer, Gastpar, Trans IT, 2011]

Random codes for
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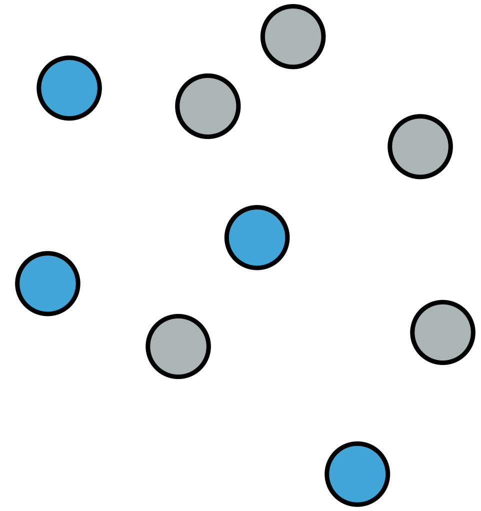

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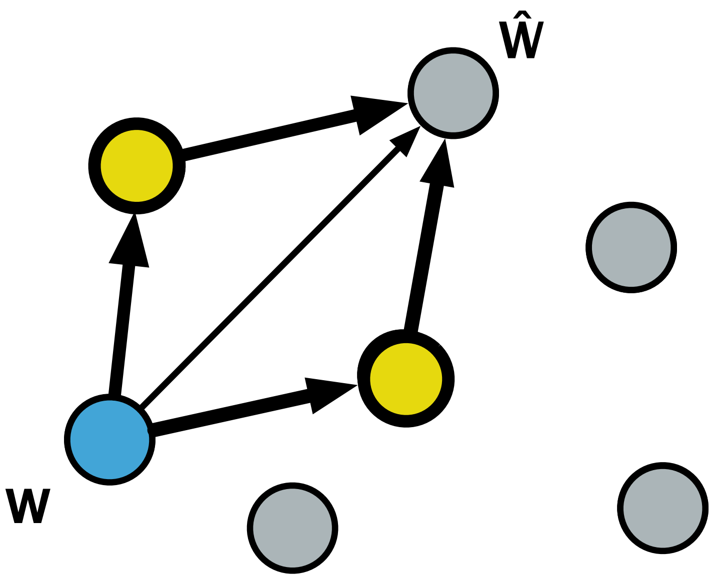

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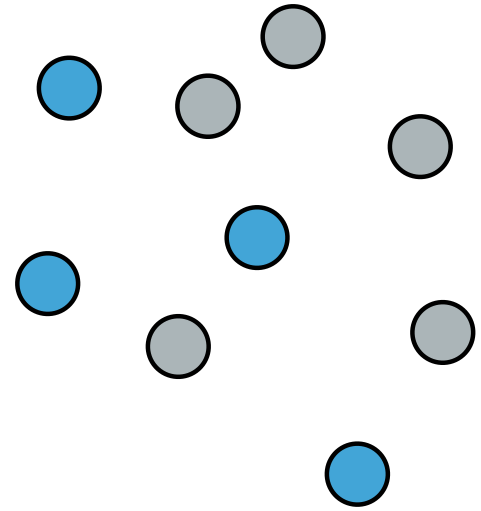

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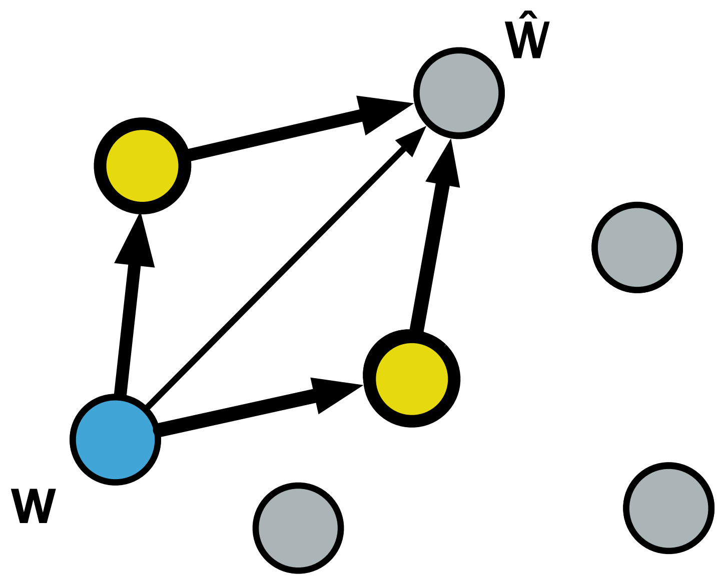

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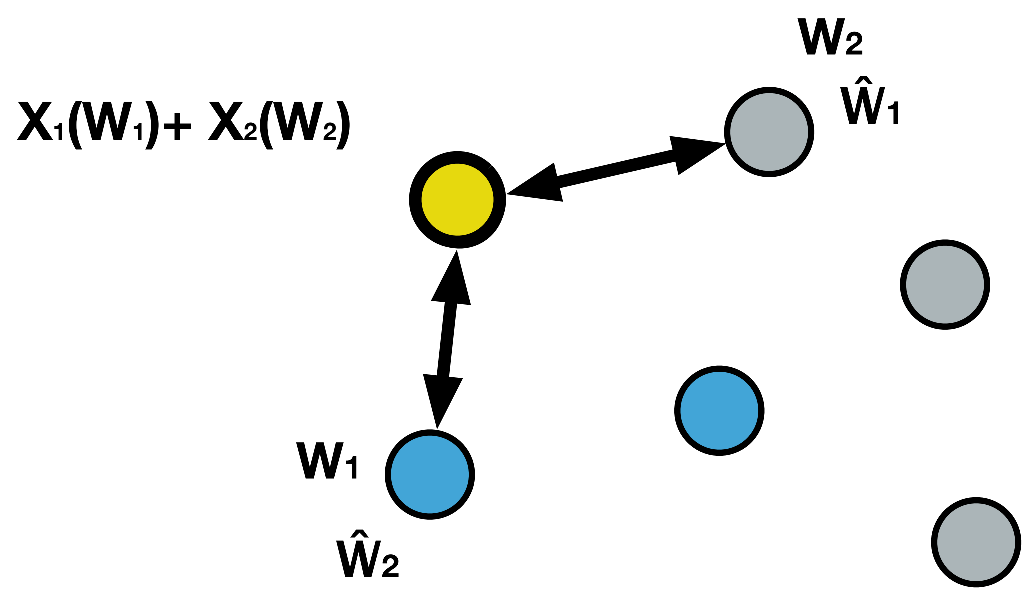

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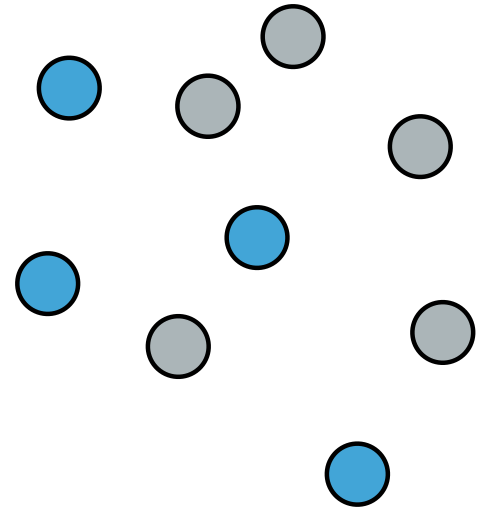

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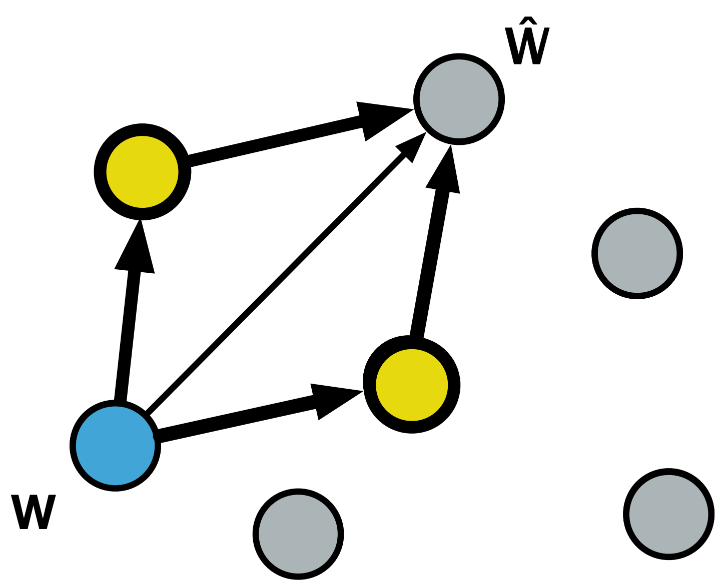

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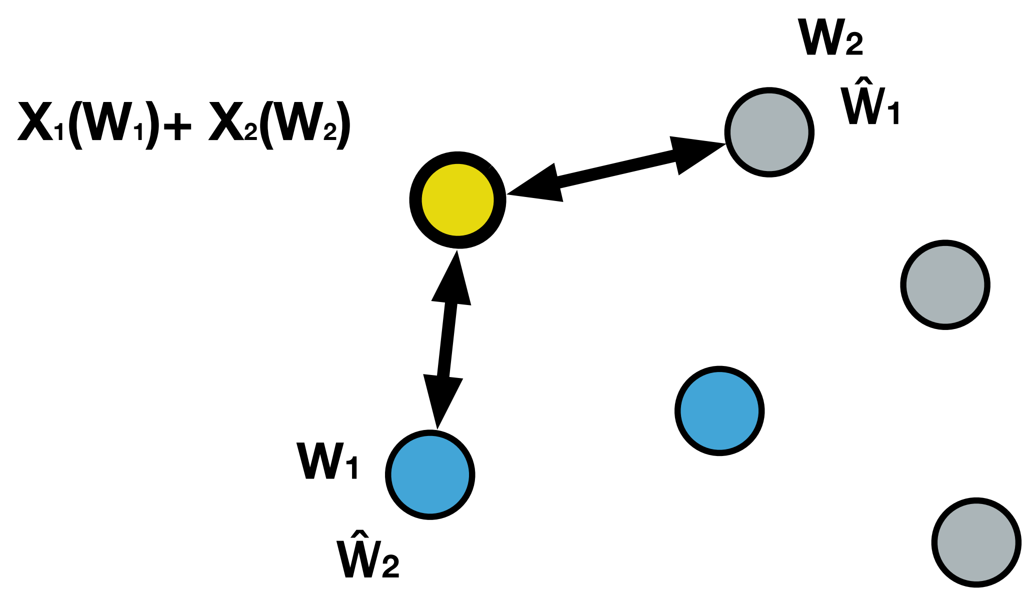

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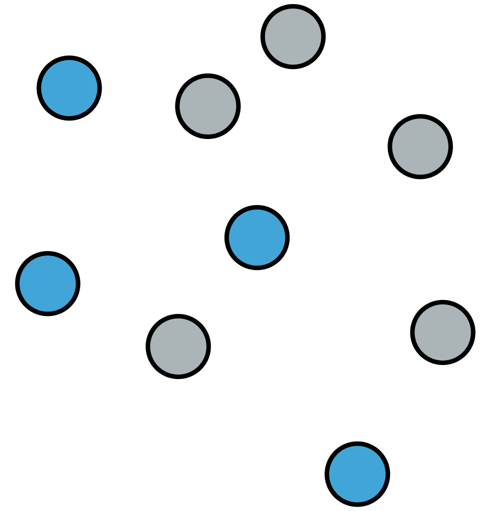

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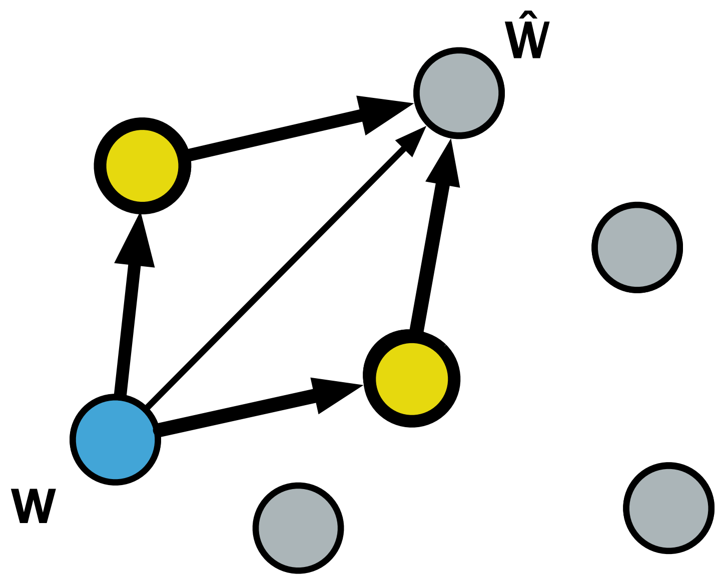

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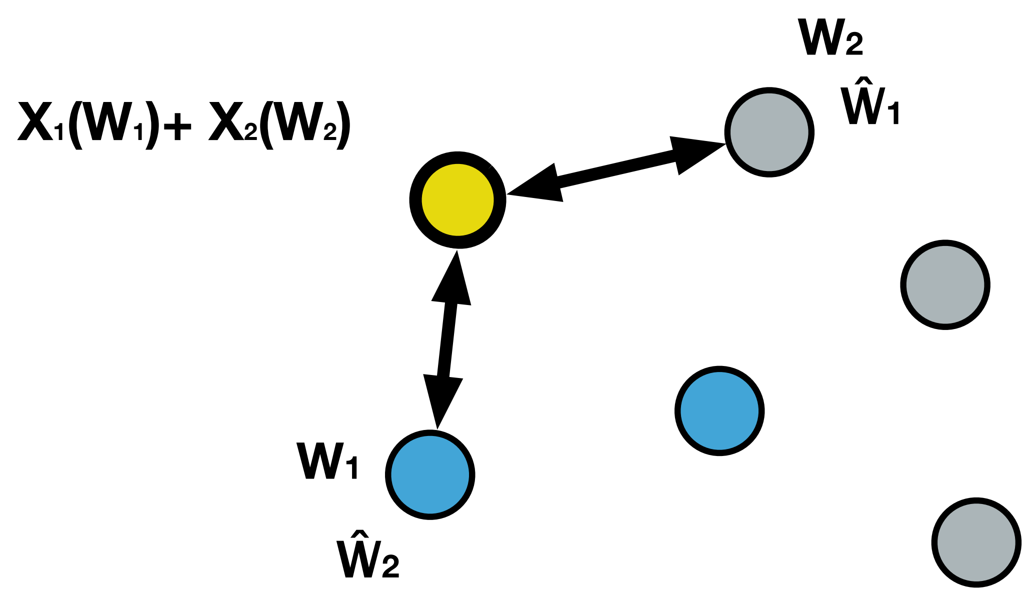

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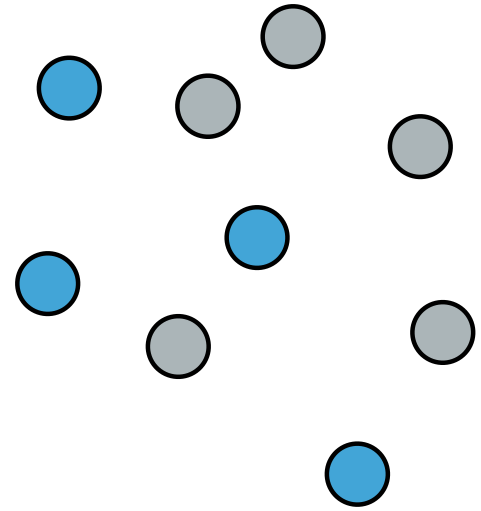

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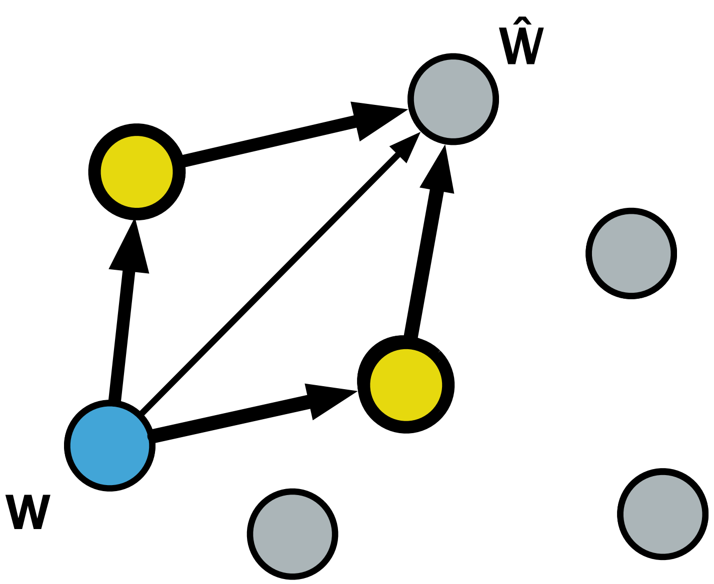

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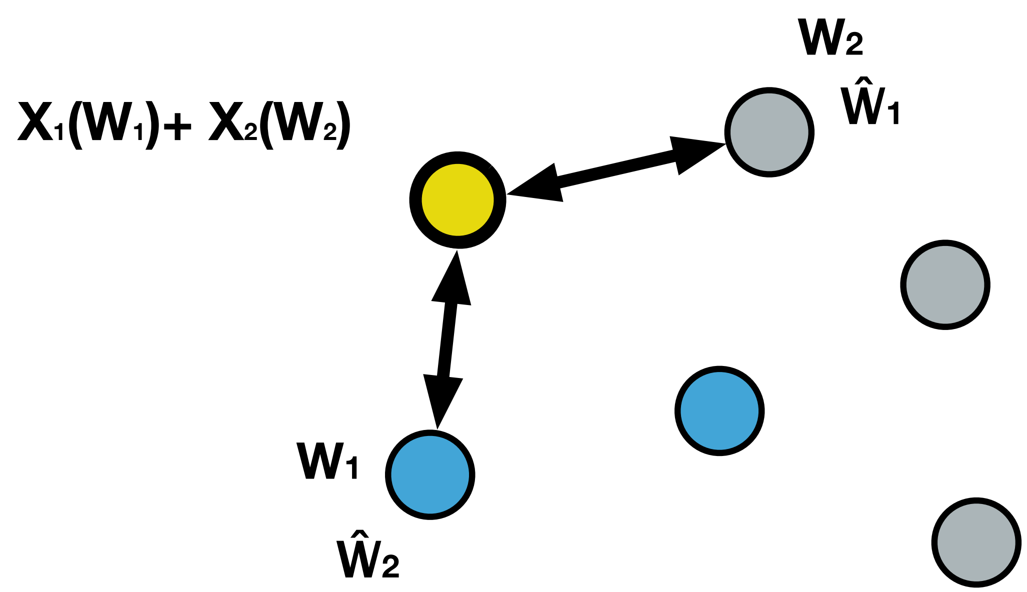

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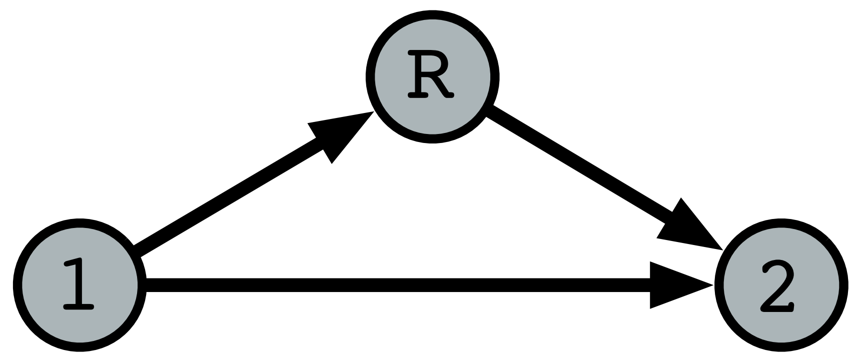
Simplest form of Cooperation *(in Gaussian)*

- AWGN relay channel



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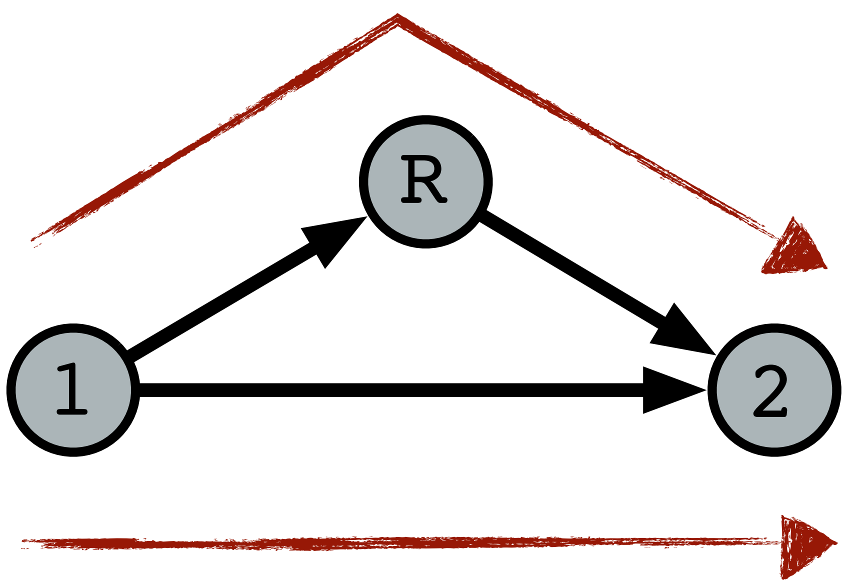

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Various links carry same message!

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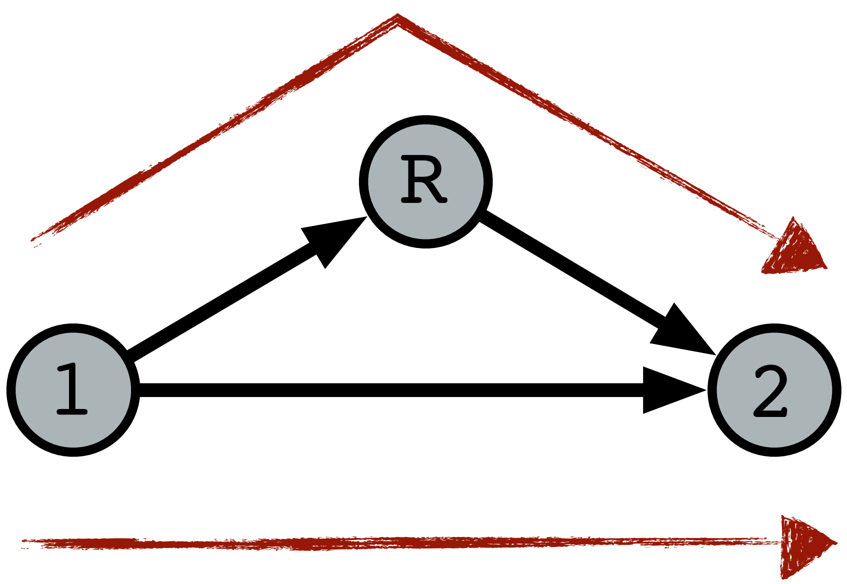

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- Compress and Forward (CF)
- Amplify and Forward (AF)
- Quantize and Forward

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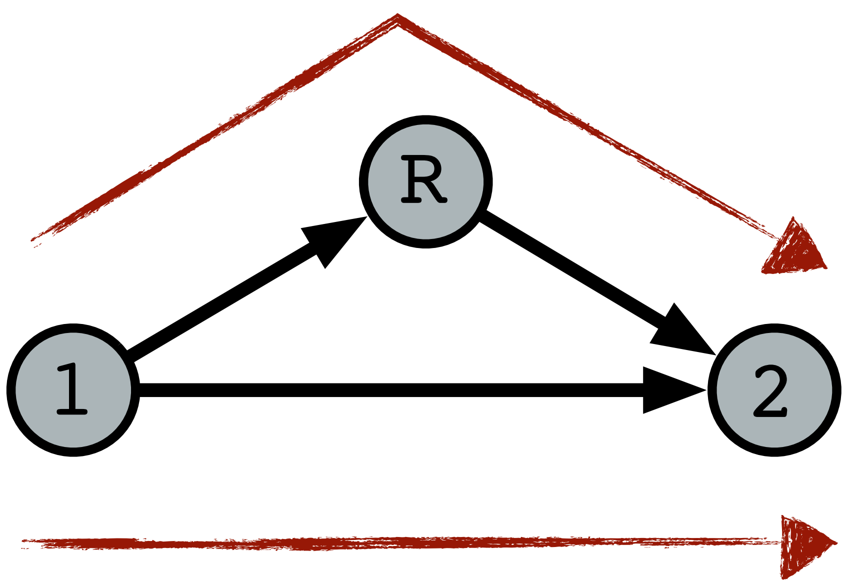

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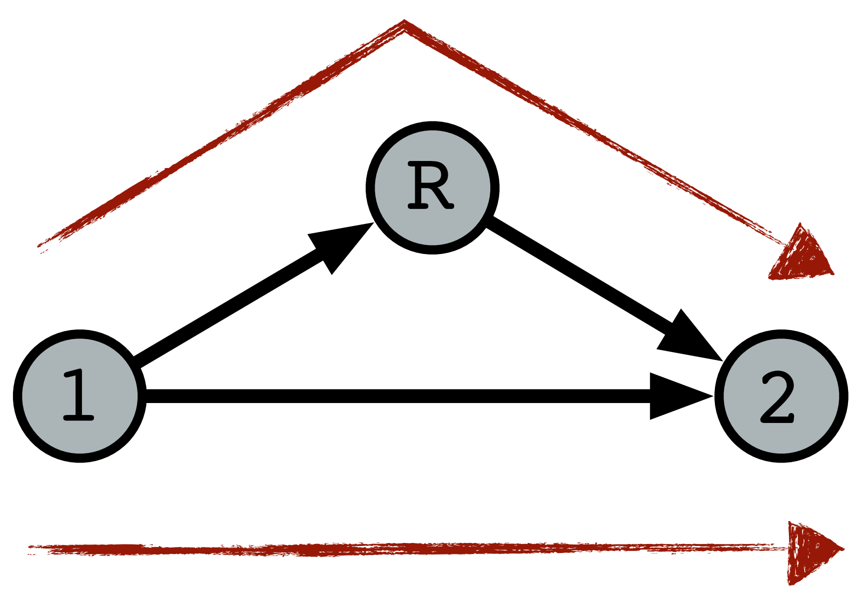

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Lattice versions?

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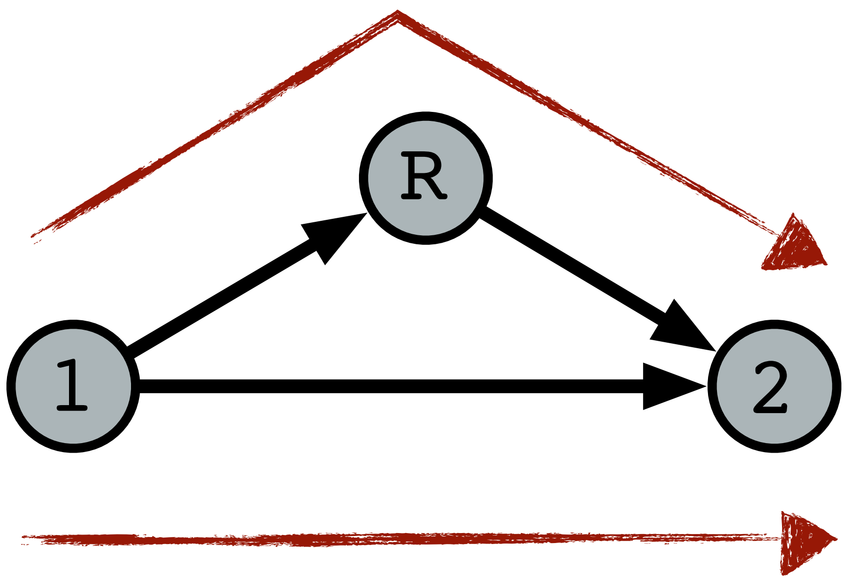
Central contribution

- AWGN relay channel



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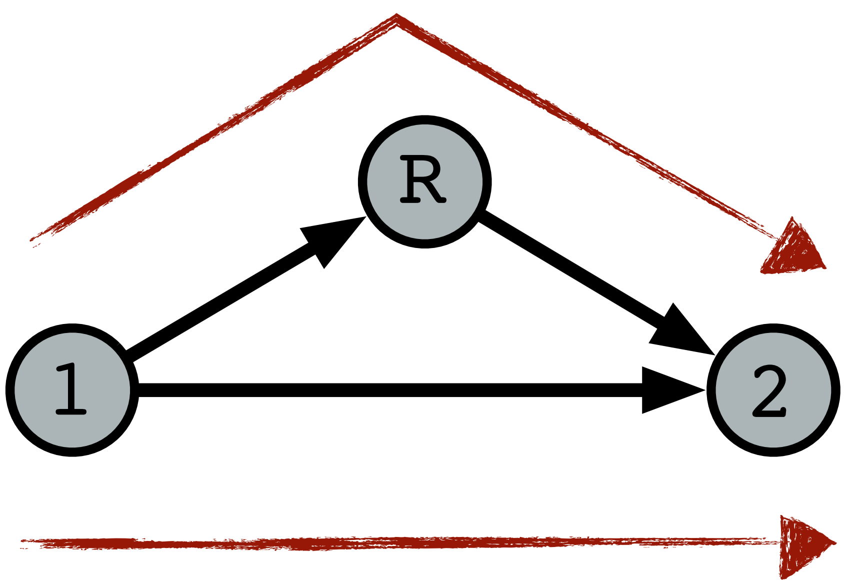

- Decode and Forward (DF) → lattice codes achieve **DF** rate of random Gaussian codes

[Song, Devroye ``List decoding for nested lattices and applications to relay channels" Allerton 2010]

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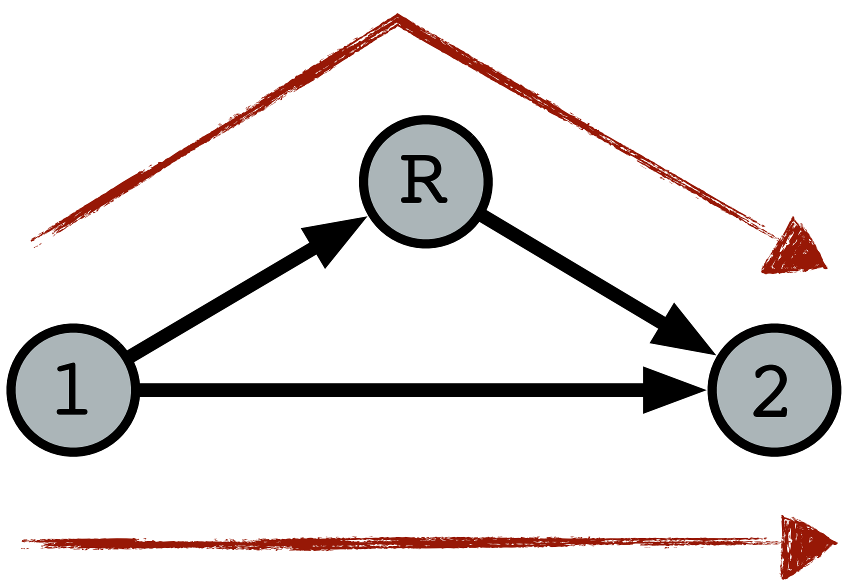

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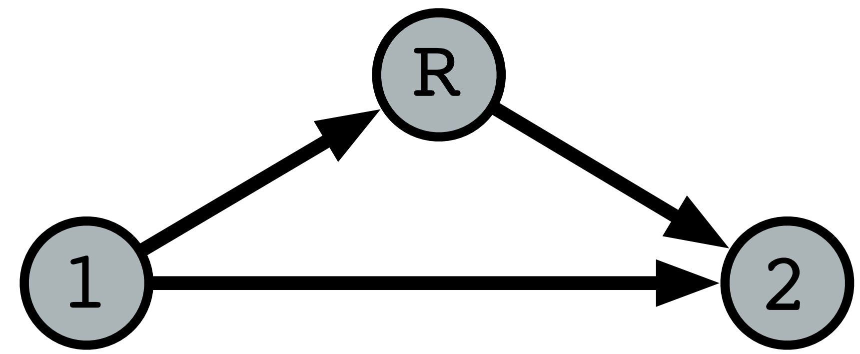
- **This paper** → lattice codes achieve **CF** rate of random Gaussian codes

General relay network theorems

- AWGN relay channel DF and CF schemes first considered in *[Cover, El Gamal, 1979]*



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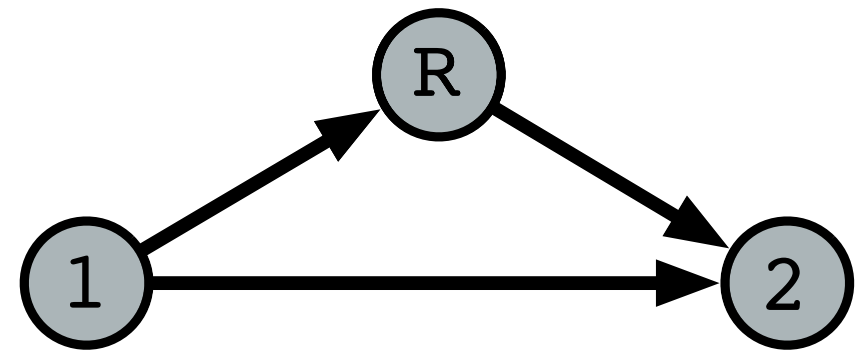

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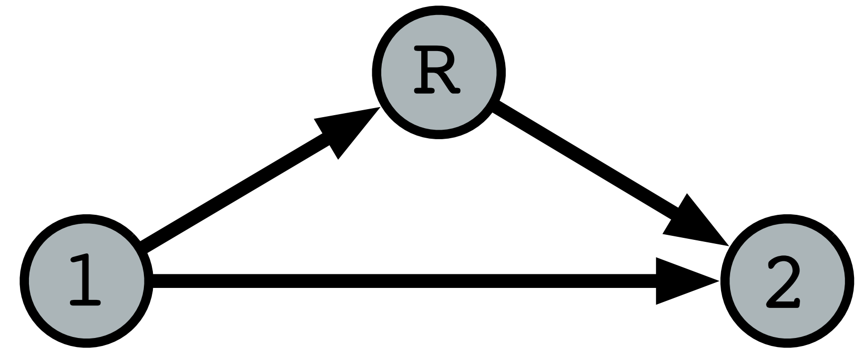

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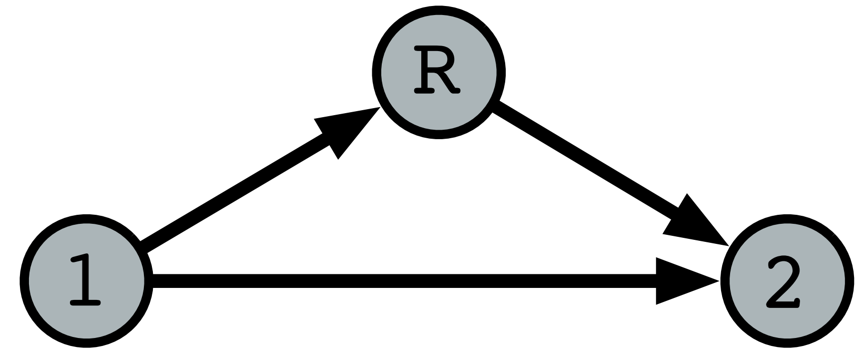

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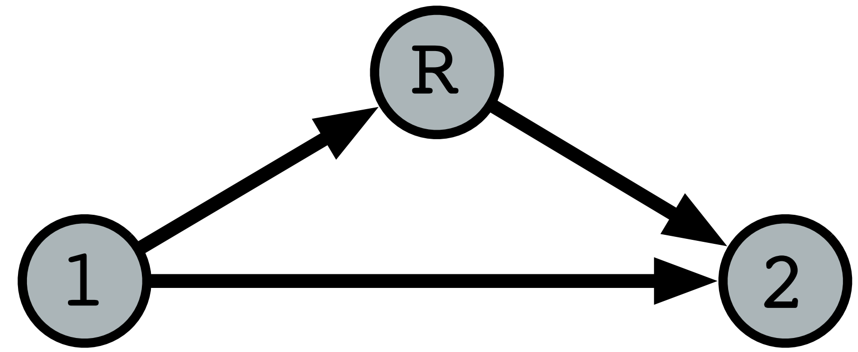

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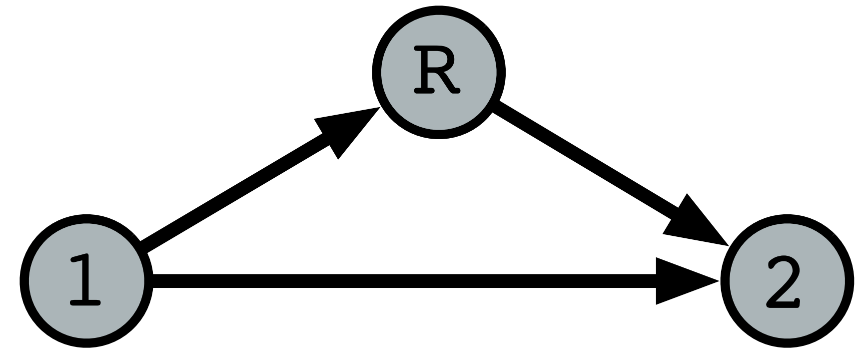

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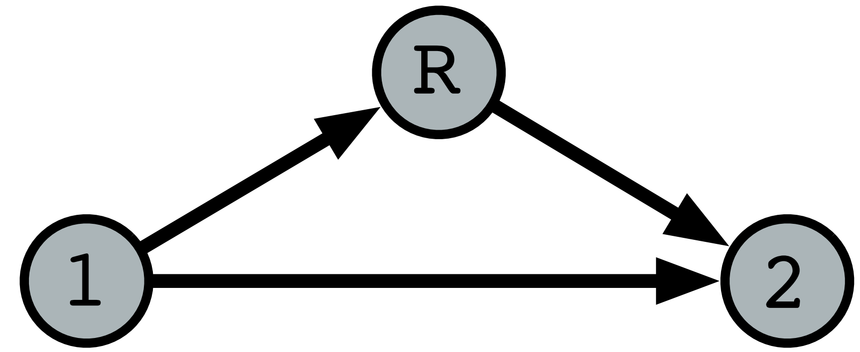

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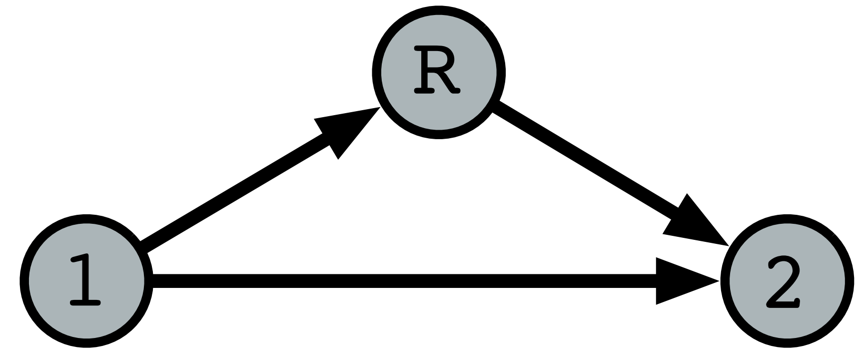

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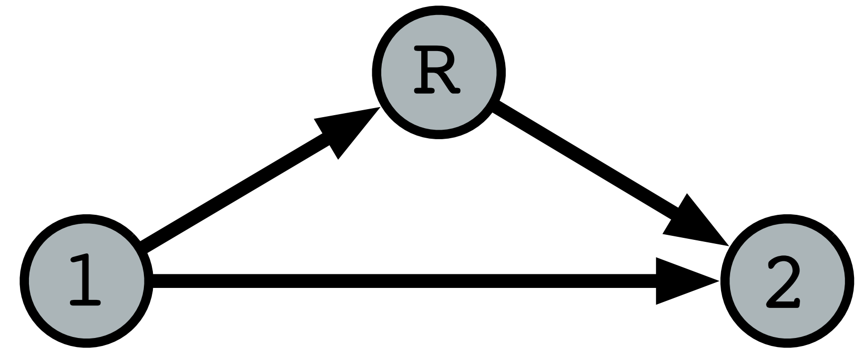

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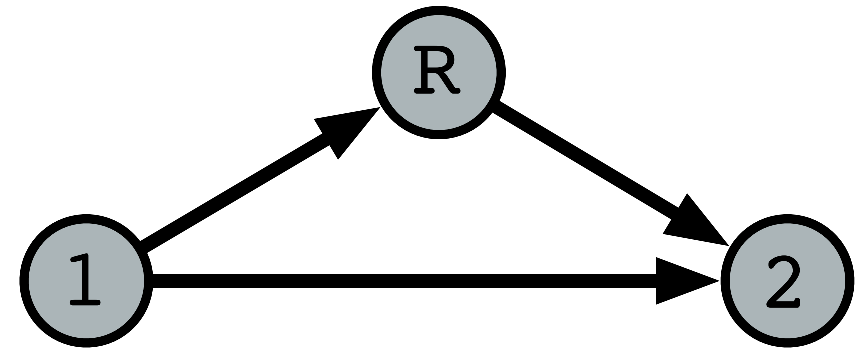

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    - Lattice DF single relay channel *[Song, Devroye, Allerton 2010]*

Outline - a lattice CF scheme

- Lattice notation

- Compress and forward review

- Lattice ($X+Z_1$, $X+Z_2$) Wyner-Ziv coding scheme

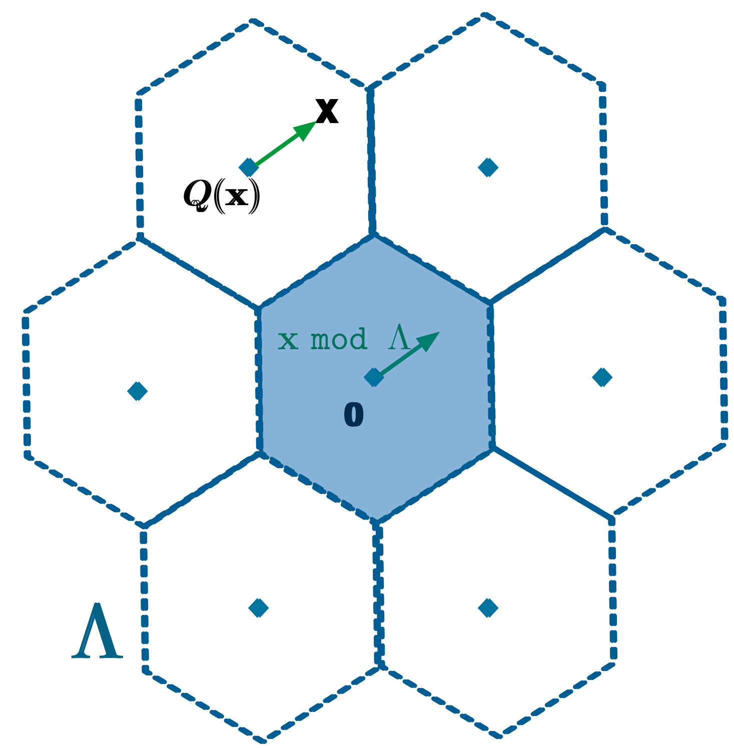
- Lattices achieve CF rate for AWGN relay

Lattice notation

- $\Lambda = \{\lambda = G\,\mathbf{i} : \mathbf{i} \in \mathbb{Z}^n\}$, G the generator matrix

- *lattice quantizer* of Λ:

$$Q(\mathbf{X}) = \arg\min_{\lambda \in \Lambda} ||\mathbf{X} - \lambda||$$



- $\mathbf{x} \mod \Lambda := \mathbf{x} - Q(\mathbf{x})$

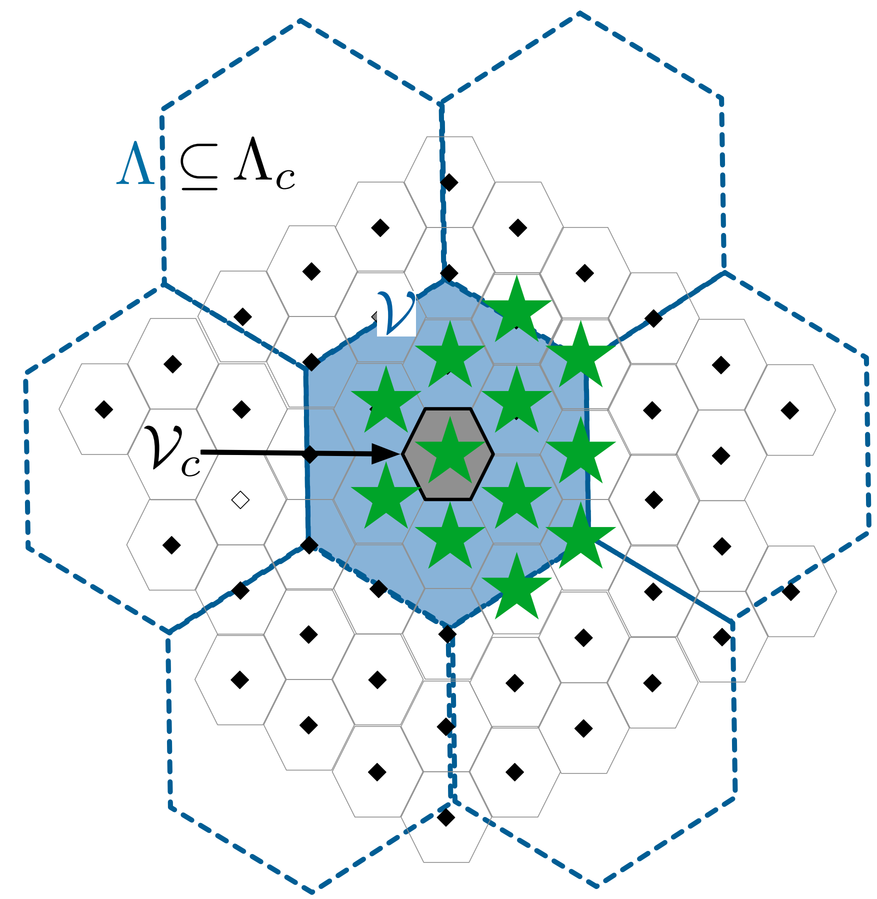
- *fundamental region* $\mathcal{V} := \{\mathbf{x} : Q(\mathbf{x}) = \mathbf{0}\}$ of volume V

- *second moment per dimension of a uniform distribution over* $\mathcal{V}$:

$$\sigma^2(\Lambda) := \frac{1}{V} \cdot \frac{1}{n} \int_{\mathcal{V}} ||\mathbf{x}||^2 d\mathbf{x}$$

Nested lattice codes

- Nested lattice pair : $\Lambda \subseteq \Lambda_c$ (Λ is Rogers-good and Poltyrev-good, Λ_c is Poltyrev-good)



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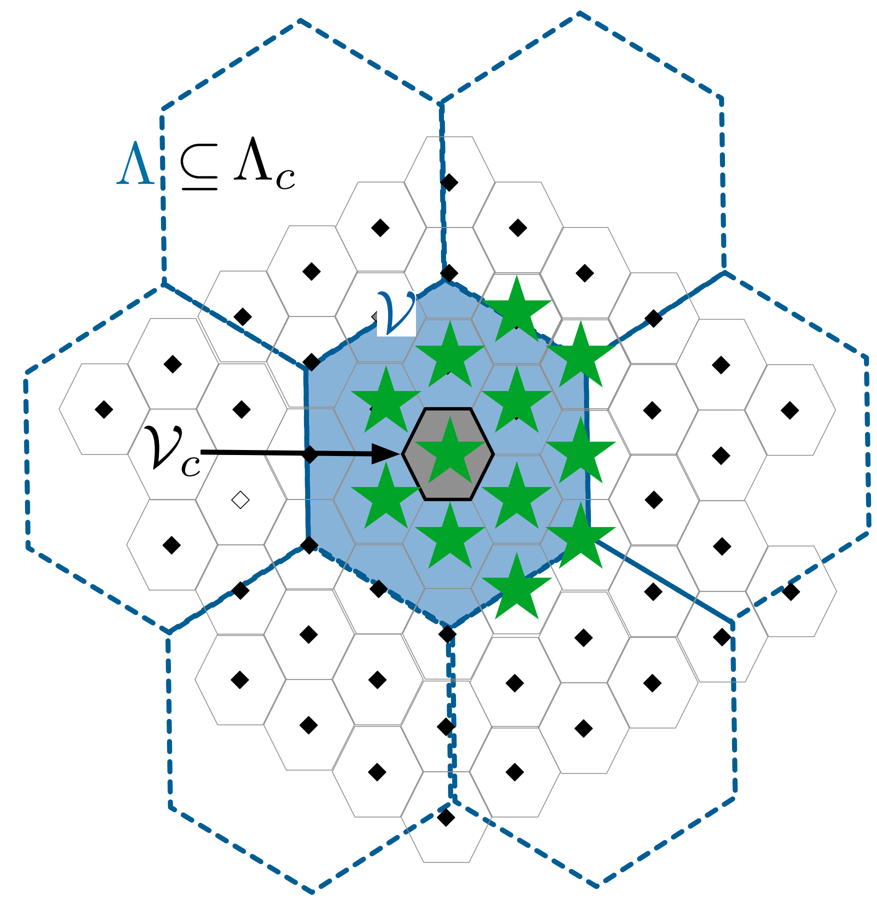

- The (dithered) code book ★ $\mathcal{C} = \{\Lambda_c \cap \mathcal{V}(\Lambda)\}$ ★ is used to achieve the capacity of AWGN channel and used to achieve the quadratic Gaussian R(D) in *[Erez, Litsyn, Zamir, Trans. IT, 2005] [Erez+Zamir, Trans. IT, 2004]*

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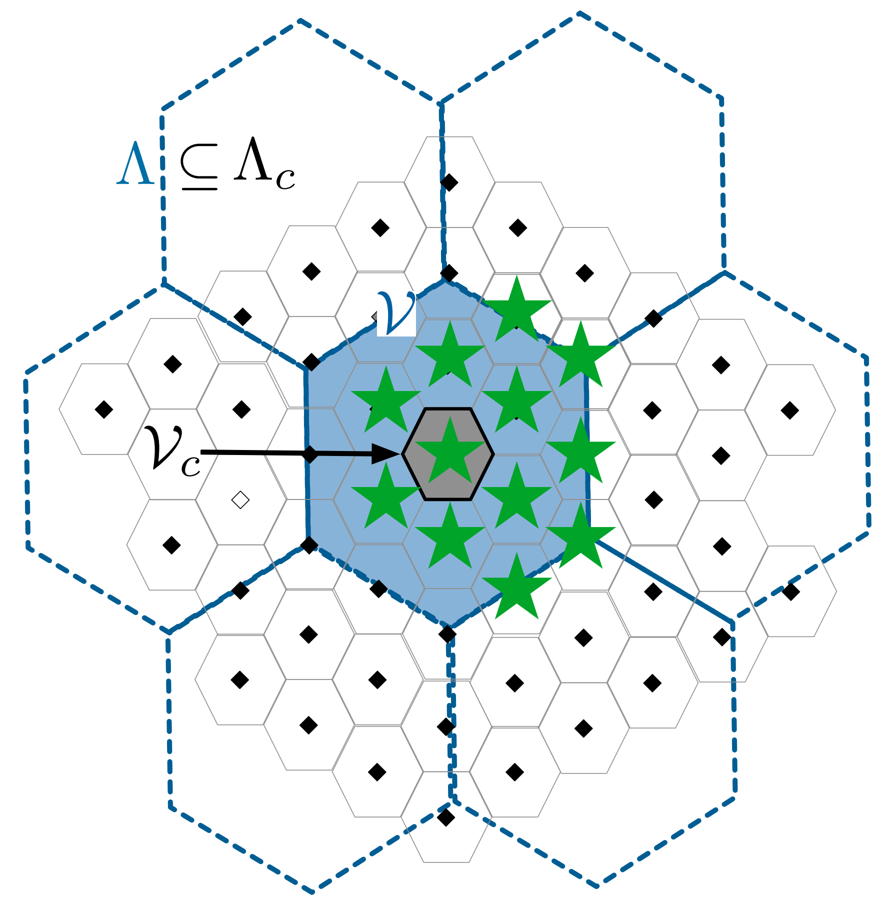

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- Coding rate: $R = \frac{1}{n}\log|\mathcal{C}| = \frac{1}{n}\log\frac{V(\Lambda)}{V(\Lambda_c)}$ arbitrary *(# of ★)*

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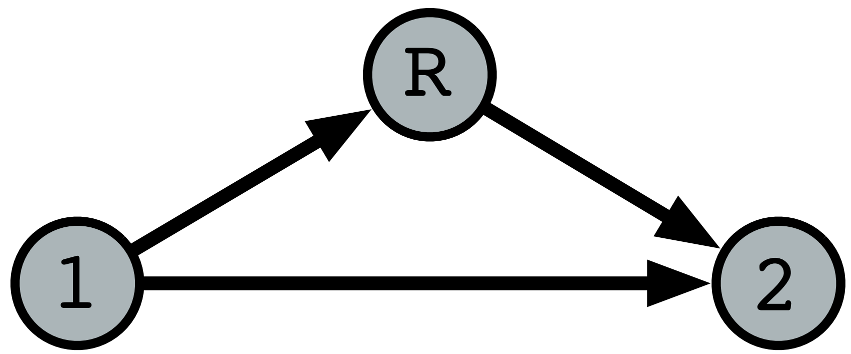

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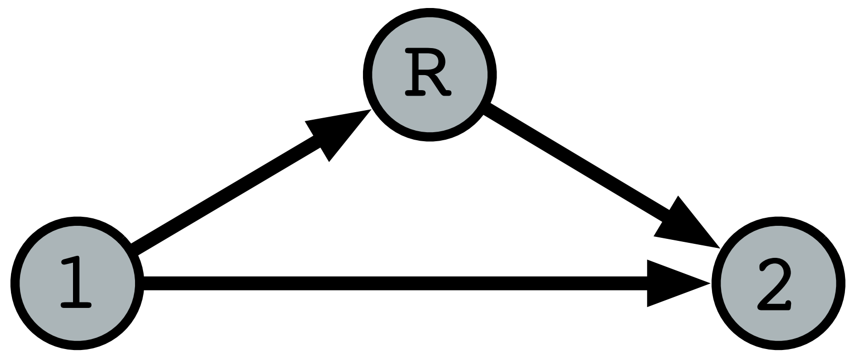

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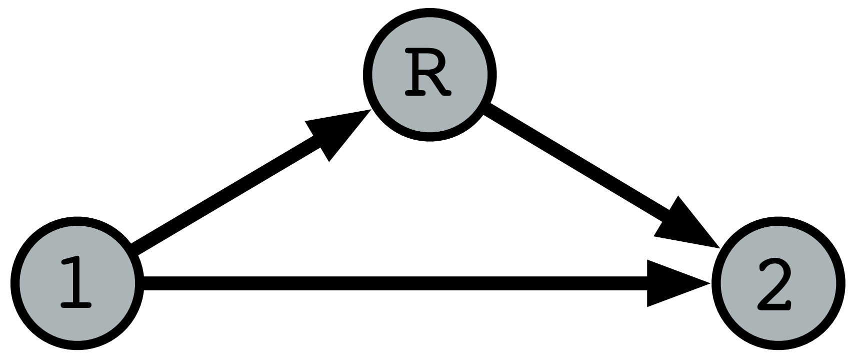

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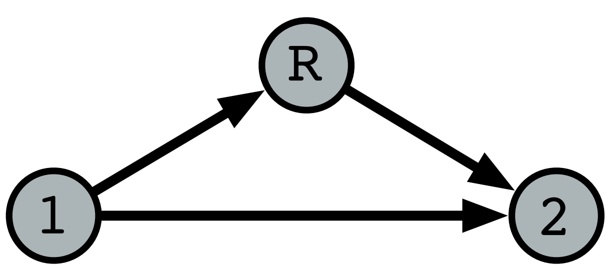
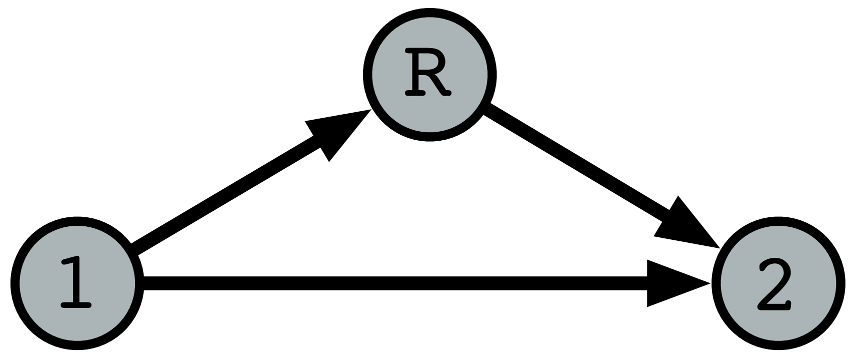


direct link side-information

Compress and forward (CF)



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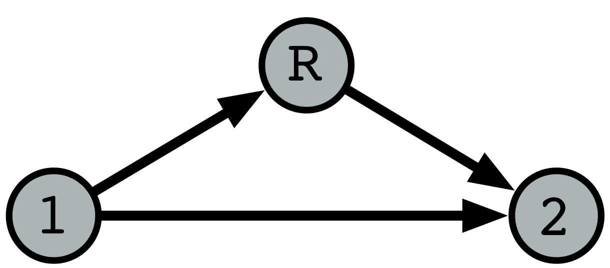
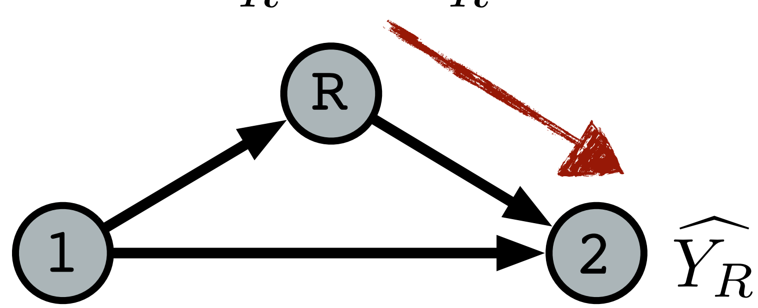
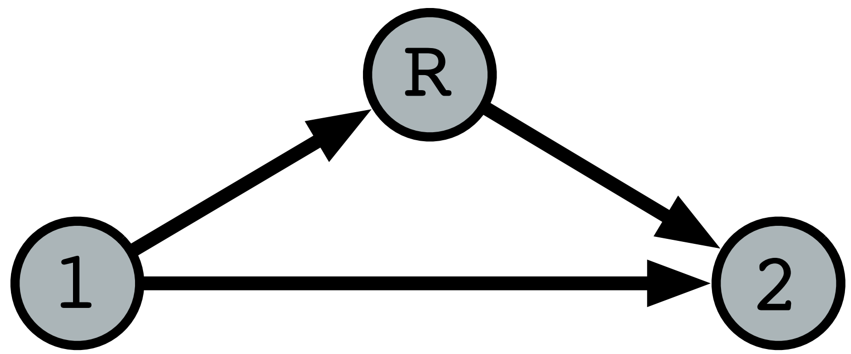


direct link side-information

$\widehat{Y_R} \rightarrow X_R$

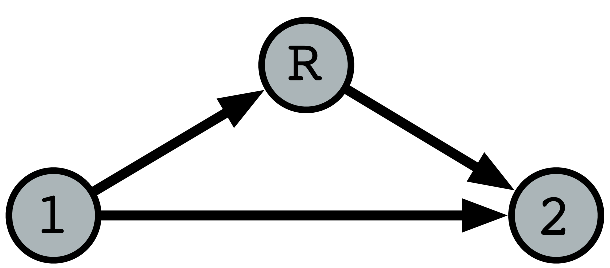


Compress and forward (CF)



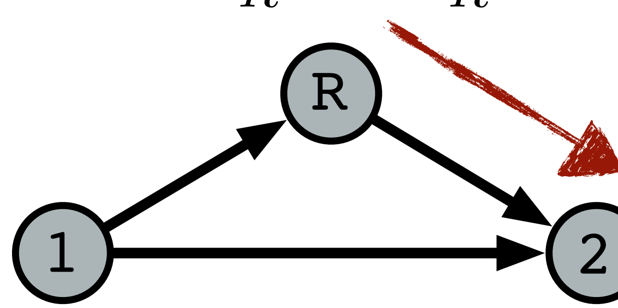
- DF limited by need to decode at relay $R < I(X_1; Y_R | X_R)$
- CF is NOT limited in this fashion

$Y_R \rightarrow \widehat{Y_R}$ Wyner Ziv

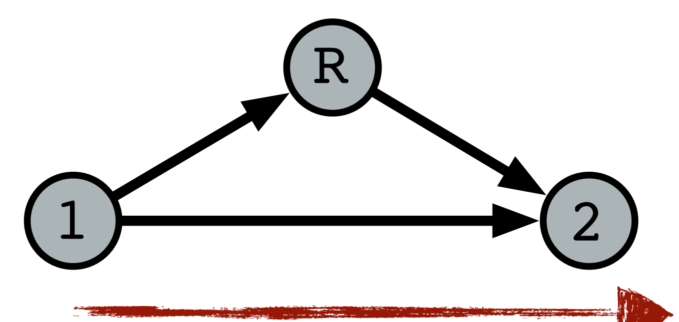


direct link side-information

$\widehat{Y_R} \rightarrow X_R$



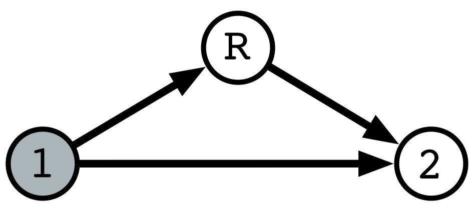
$\widehat{Y_R}$



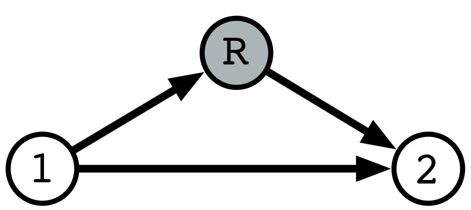
combine with direct link

Mimic all steps with lattice codes

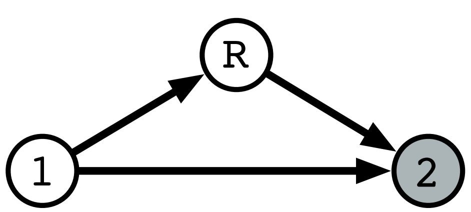
Encoding



Compress + forward



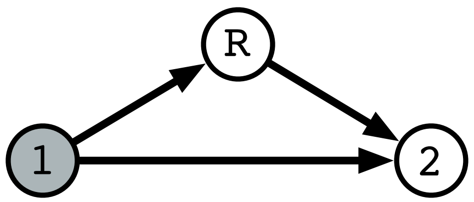
Decoding

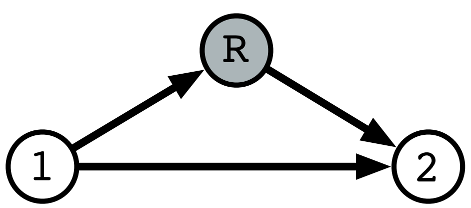


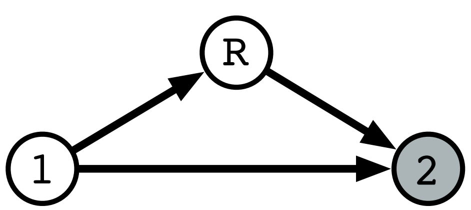
	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$			
Compress + forward				
Decoding				

Mimic all steps with lattice codes

	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$			
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$			
Decoding				

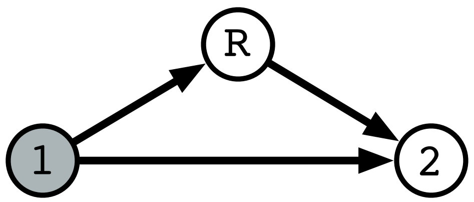


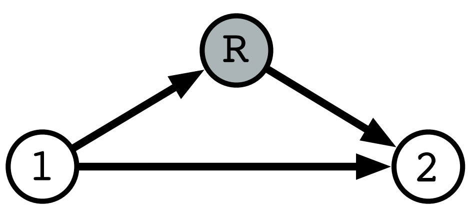


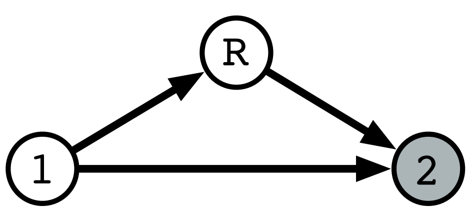


Mimic all steps with lattice codes

	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$			
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$			
Decoding				

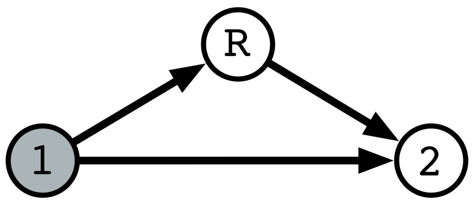




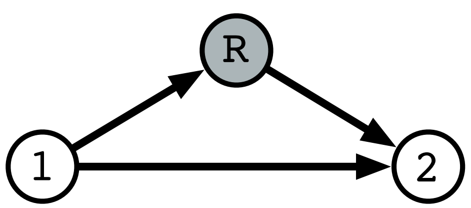


Mimic all steps with lattice codes

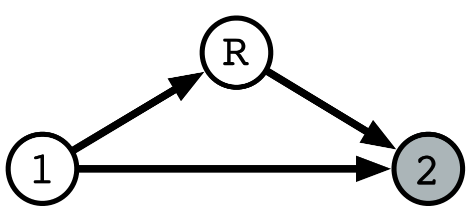
Encoding



Compress + forward



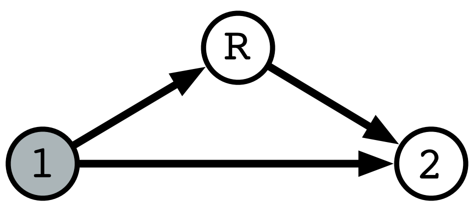
Decoding



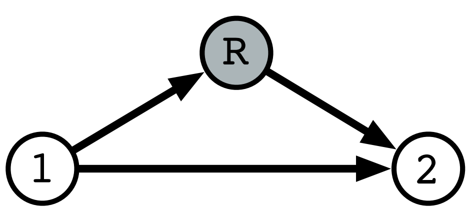
	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$			
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$			
Decoding	$y_{2,1} - x_{R,1}$			

Mimic all steps with lattice codes

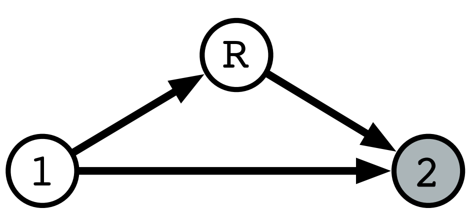
Encoding



Compress + forward

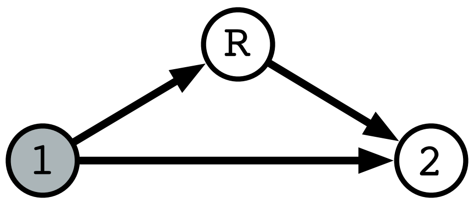


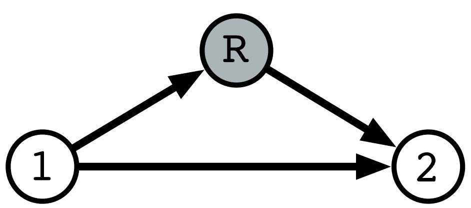
Decoding

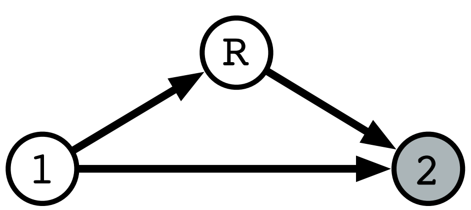


	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$		
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$			
Decoding	$y_{2,1} - x_{R,1}$			

Mimic all steps with lattice codes







	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$		
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$		
Decoding	$y_{2,1} - x_{R,1}$			

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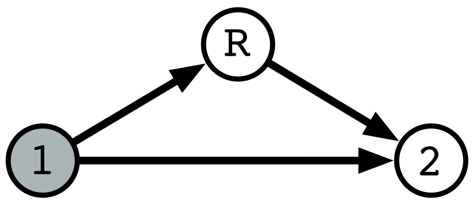
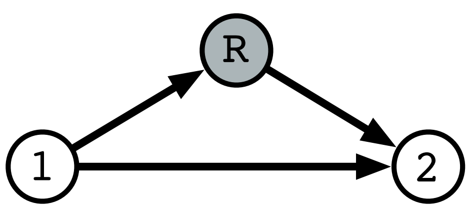
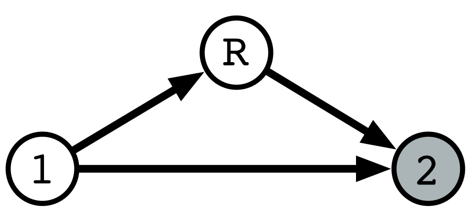
Mimic all steps with lattice codes

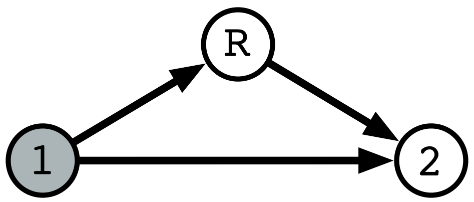
	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$		
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$		
Decoding	$y_{2,1} - x_{R,1}$			

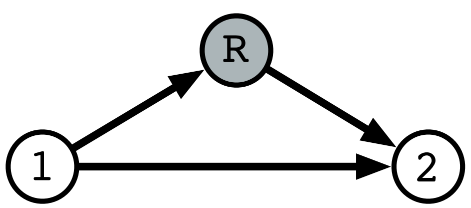


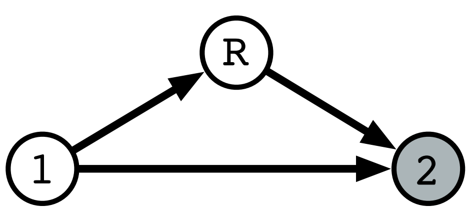




Mimic all steps with lattice codes



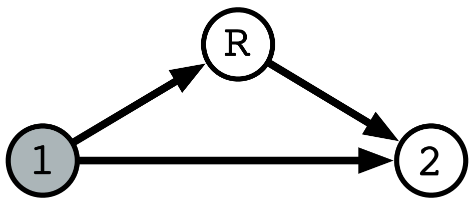




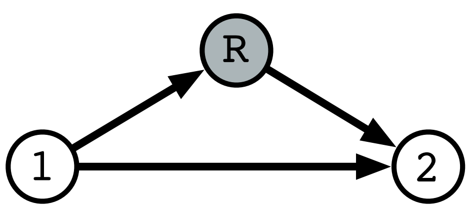
	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$		
Compress + forward	$y_{R,1} \to \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \to \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$		
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$		

Mimic all steps with lattice codes

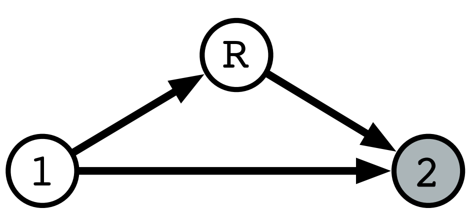
Encoding



Compress + forward



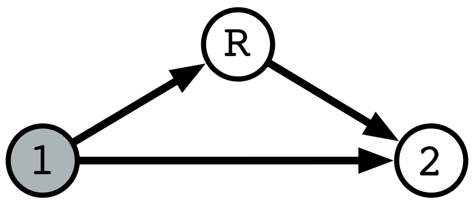
Decoding

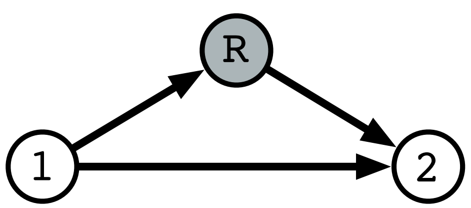


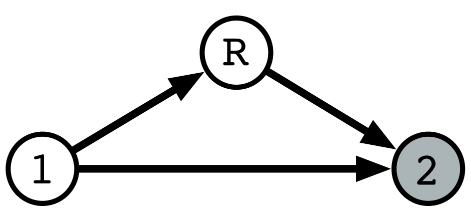
	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$		
Compress + forward	$y_{R,1} \to \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \to \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$		
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$		

Combine

Mimic all steps with lattice codes







	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$		
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$		
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$		

Combine

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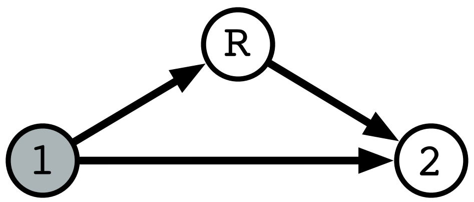
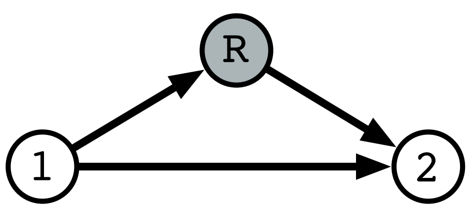
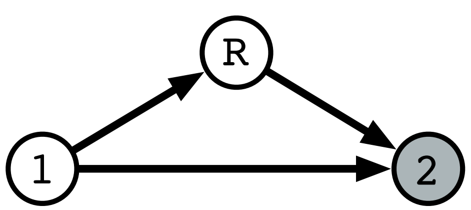
Mimic all steps with lattice codes



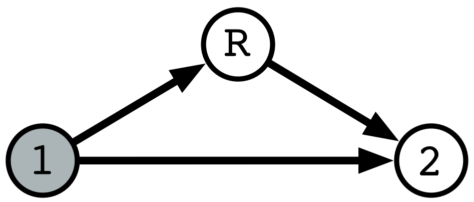
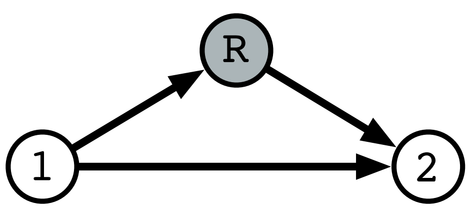
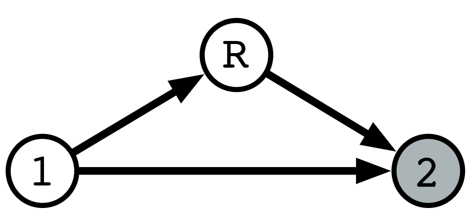




	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$		
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$		

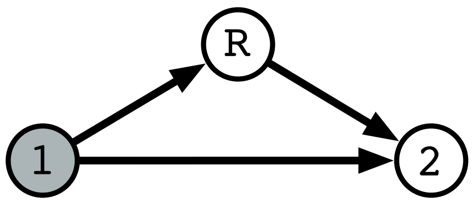
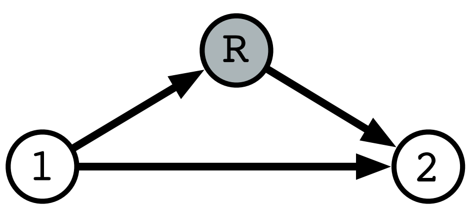
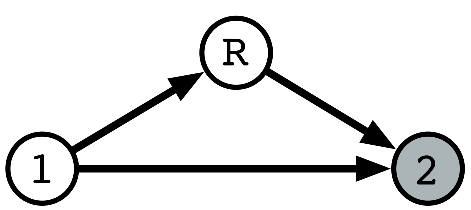
Combine

Mimic all steps with lattice codes

	Block 1	Block 2	Block 3	Block 4
Encoding 	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	
Compress + forward 	$y_{R,1} \to \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \to \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \to \widehat{y_{R,3}}(i_3)$	
Decoding 	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$		

Combine

Mimic all steps with lattice codes

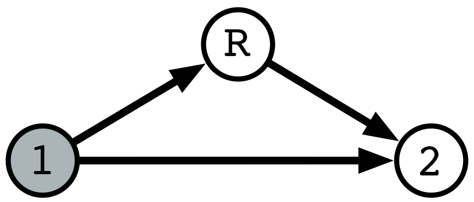
	Block 1	Block 2	Block 3	Block 4
Encoding 	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	
Compress + forward 	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \rightarrow \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	
Decoding 	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$		

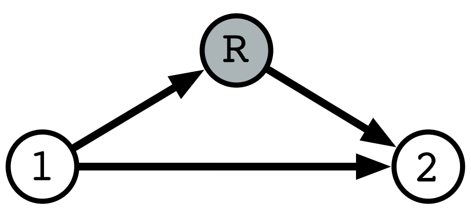
Combine

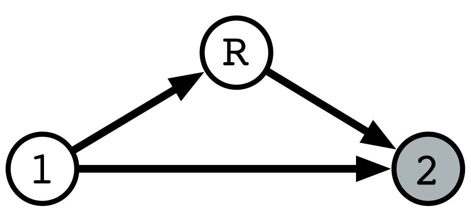
Mimic all steps with lattice codes

	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \rightarrow \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$	$\widehat{y_{R,2}}$	

Combine







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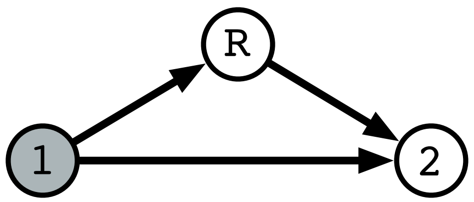
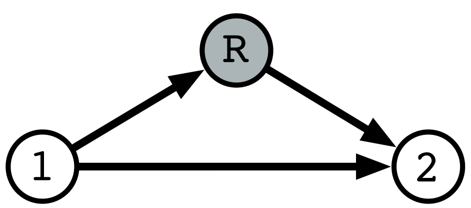
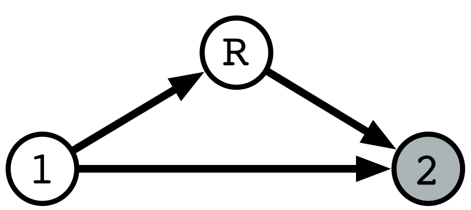
College of Engineering

Mimic all steps with lattice codes

	Block 1	Block 2	Block 3	Block 4
Encoding 	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	
Compress + forward 	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \rightarrow \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	
Decoding 	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$	$\widehat{y_{R,2}}$	

Combine

Combine

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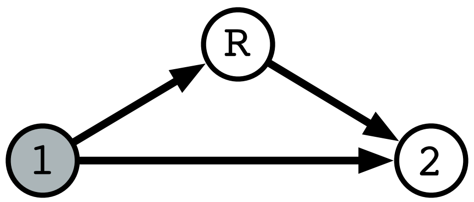
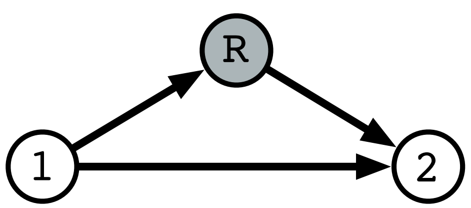
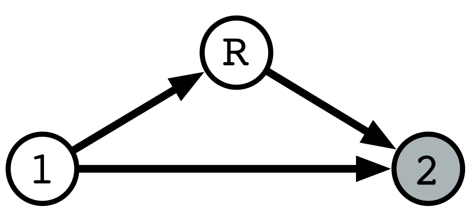
Mimic all steps with lattice codes

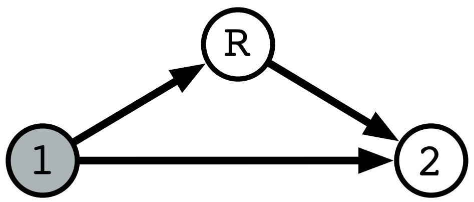
	Block 1	Block 2	Block 3	Block 4
Encoding 	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	
Compress + forward 	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \rightarrow \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	
Decoding 	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$	$\widehat{y_{R,2}}$ $y_{2,3} - x_{R,3}$	

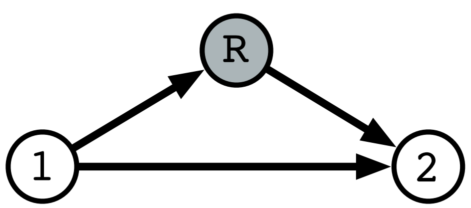
Combine

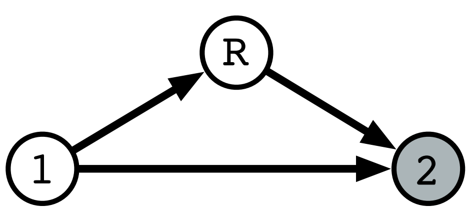
Combine

UIC Department of Electrical and Computer Engineering
UNIVERSITY OF ILLINOIS AT CHICAGO
COLLEGE OF ENGINEERING

Mimic all steps with lattice codes





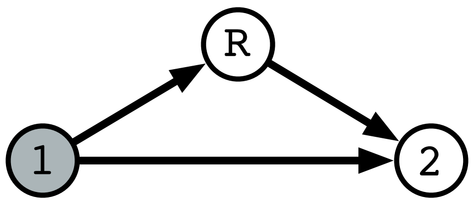


	Block 1	*Block 2*	*Block 3*	*Block 4*
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	$x_{1,4}(1)$
Compress + forward	$y_{R,1} \to \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \to \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \to \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$	$\widehat{y_{R,2}}$ $y_{2,3} - x_{R,3}$	

Combine Combine

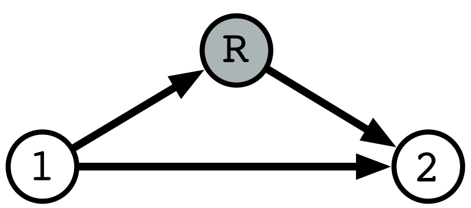
Mimic all steps with lattice codes

Encoding

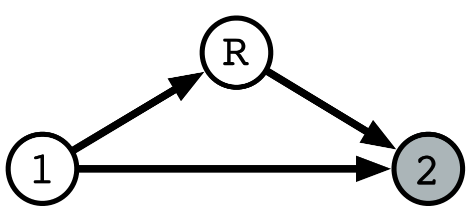


	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	$x_{1,4}(1)$
Compress + forward	$y_{R,1} \to \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \to \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \to \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	$x_{R,4}(i_3)$
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$	$\widehat{y_{R,2}}$ $y_{2,3} - x_{R,3}$	

Compress + forward



Decoding



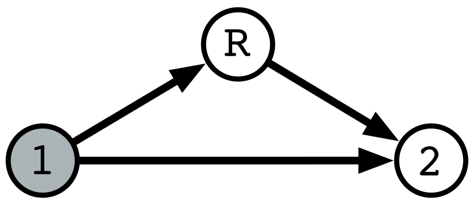
Combine

Combine

Mimic all steps with lattice codes

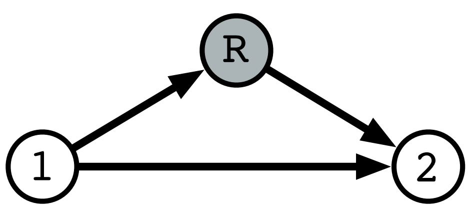
	Block 1	Block 2	Block 3	Block 4

Encoding



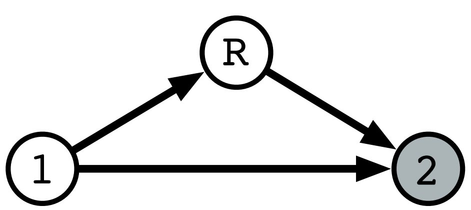
Block 1	Block 2	Block 3	Block 4
$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	$x_{1,4}(1)$

Compress + forward



Block 1	Block 2	Block 3	Block 4
$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$	$y_{R,3} \rightarrow \widehat{y_{R,3}}(i_3)$	
$x_{R,1}(1)$	$x_{R,2}(i_1)$	$x_{R,3}(i_2)$	$x_{R,4}(i_3)$

Decoding

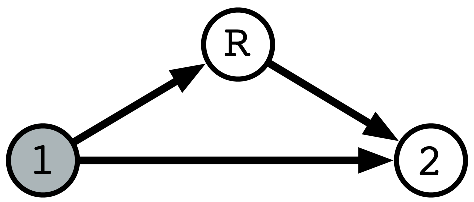


Block 1	Block 2	Block 3	Block 4
	$\widehat{y_{R,1}}$	$\widehat{y_{R,2}}$	$\widehat{y_{R,3}}$
$y_{2,1} - x_{R,1}$	$y_{2,2} - x_{R,2}$	$y_{2,3} - x_{R,3}$	

Combine Combine

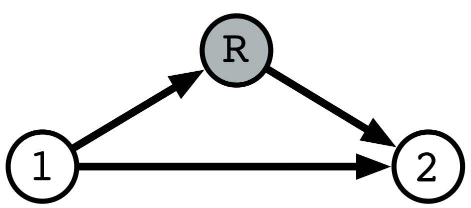
Mimic all steps with lattice codes

Encoding

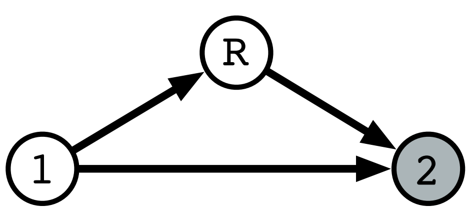


	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	$x_{1,4}(1)$
Compress + forward	$y_{R,1} \rightarrow \widehat{y_{R,1}}(i_1)$ $x_{R,1}(1)$	$y_{R,2} \rightarrow \widehat{y_{R,2}}(i_2)$ $x_{R,2}(i_1)$	$y_{R,3} \rightarrow \widehat{y_{R,3}}(i_3)$ $x_{R,3}(i_2)$	$x_{R,4}(i_3)$
Decoding	$y_{2,1} - x_{R,1}$	$\widehat{y_{R,1}}$ $y_{2,2} - x_{R,2}$	$\widehat{y_{R,2}}$ $y_{2,3} - x_{R,3}$	$\widehat{y_{R,3}}$

Compress + forward



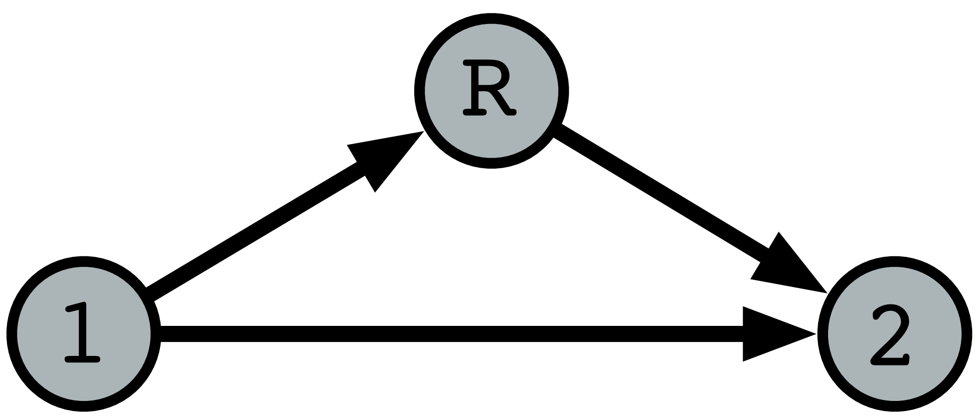
Decoding



Combine Combine Combine

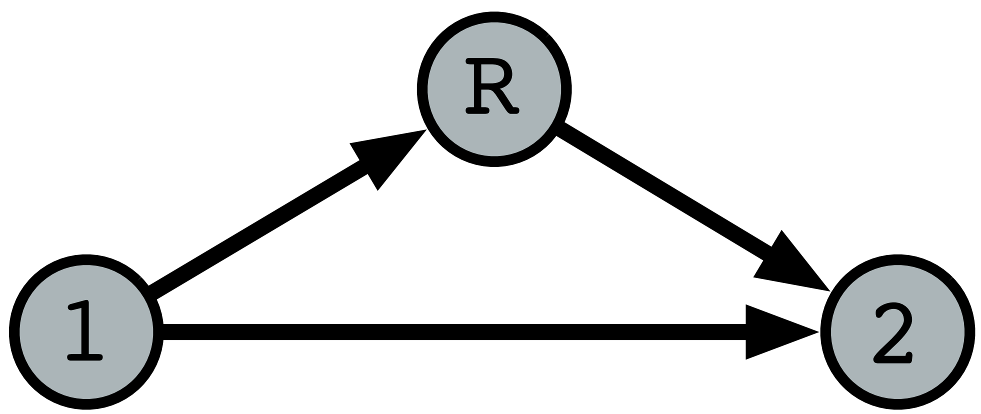
Key issue in CF: how to compress?

$$Y_R = X_1 + Z_R$$



Key issue in CF: how to compress?

$$Y_R = X_1 + Z_R$$



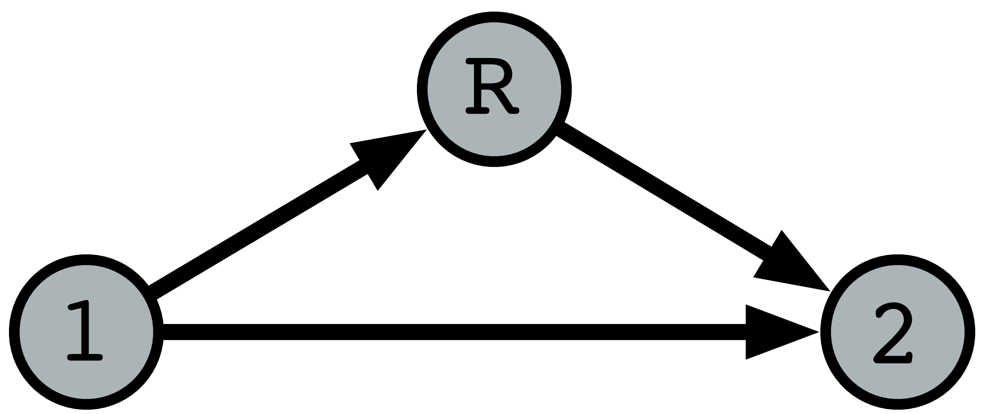
direct link side-information

$$Y_2 = X_1 + Z_2$$

Key issue in CF: how to compress?

$$Y_R = X_1 + Z_R$$

$$Y_R \rightarrow \widehat{Y_R}$$



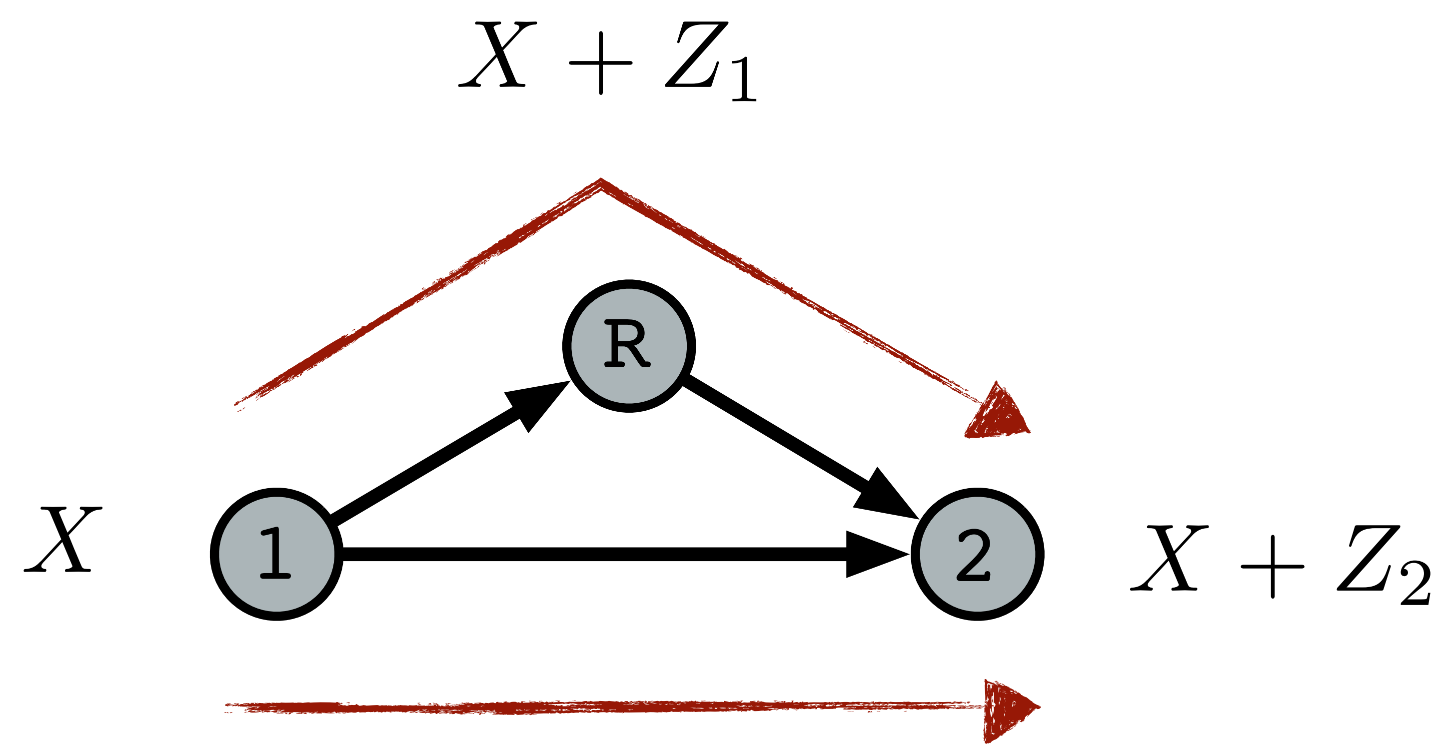
direct link side-information

$$Y_2 = X_1 + Z_2$$

Outline - a lattice CF scheme

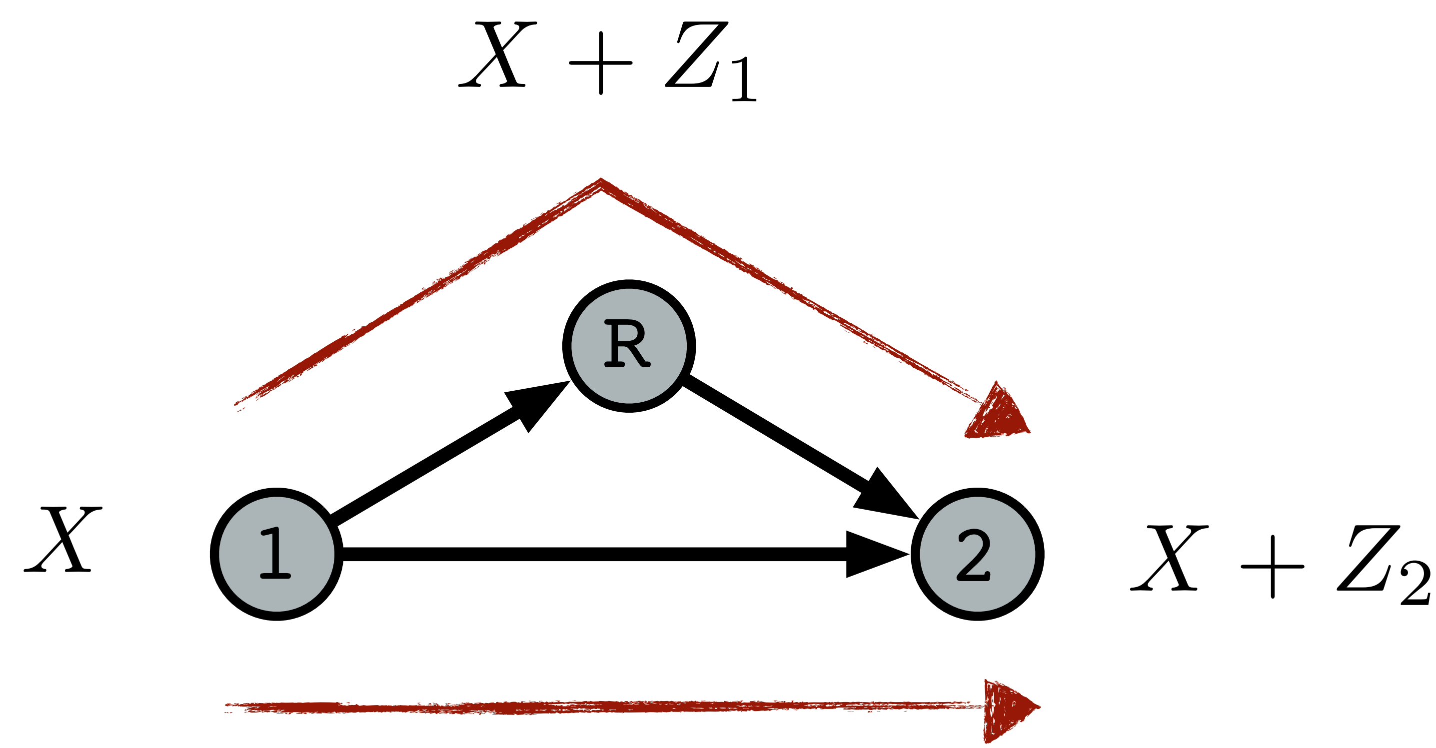
- Lattice notation
- Compress and forward review
- Lattice $(X+Z_1, X+Z_2)$ Wyner-Ziv coding scheme
- Lattices achieve CF rate for AWGN relay

The $(X+Z_1, X+Z_2)$ Wyner-Ziv problem



- Gaussian Wyner-Ziv $(X + Z_1, X + Z_2)$

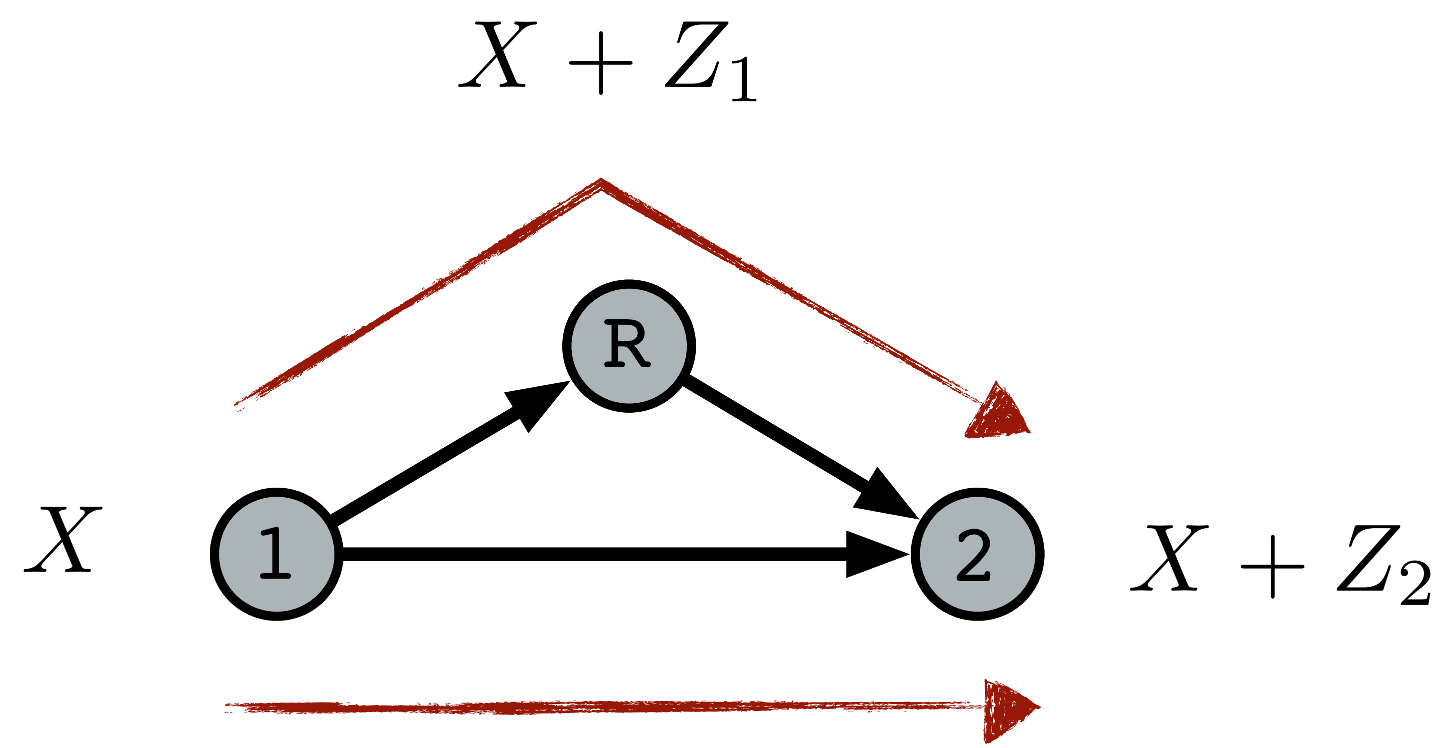
The $(X+Z_1, X+Z_2)$ Wyner-Ziv problem



- Gaussian Wyner-Ziv $(X + Z_1, X + Z_2)$

- in *[Zamir,Shamai,Erez, 2002]* demonstrated a lattice scheme for Gaussian $(X + Z, X)$ Wyner-Ziv which is fully general

The (X+Z1, X+Z2) Wyner-Ziv problem



- Gaussian Wyner-Ziv $(X + Z_1, X + Z_2)$

- in *[Zamir,Shamai,Erez, 2002]* demonstrated a lattice scheme for Gaussian $(X + Z, X)$ Wyner-Ziv which is fully general

- demonstrate $(X + Z_1, X + Z_2)$ for completeness

$$\alpha_1 = \sqrt{1 - \frac{D}{N_1 + \frac{PN_2}{P+N_2}}}, \quad \alpha_2 = \frac{P}{P+N_2}.$$

The $(X+Z_1, X+Z_2)$ Wyner-Ziv problem

Theorem. The following rate-distortion function for the lossy compression of the source $X + Z_1$ subject to the reconstruction side-information $X + Z_2$ and squared error distortion metric may be achieved using lattice codes:

$$\begin{aligned} R(D) &= \frac{1}{2}\log\left(\frac{\sigma^2_{X+Z_1|X+Z_2}}{D}\right), && 0 \le D \le \sigma^2_{X+Z_1|X+Z_2} \\ &= \frac{1}{2}\log\left(\frac{N_1 + \frac{PN_2}{P+N_2}}{D}\right), && 0 \le D \le N_1 + \frac{PN_2}{P+N_2}, \end{aligned}$$

and 0 otherwise.

$$\alpha_1 = \sqrt{1 - \frac{D}{N_1 + \frac{PN_2}{P+N_2}}}, \quad \alpha_2 = \frac{P}{P+N_2}.$$

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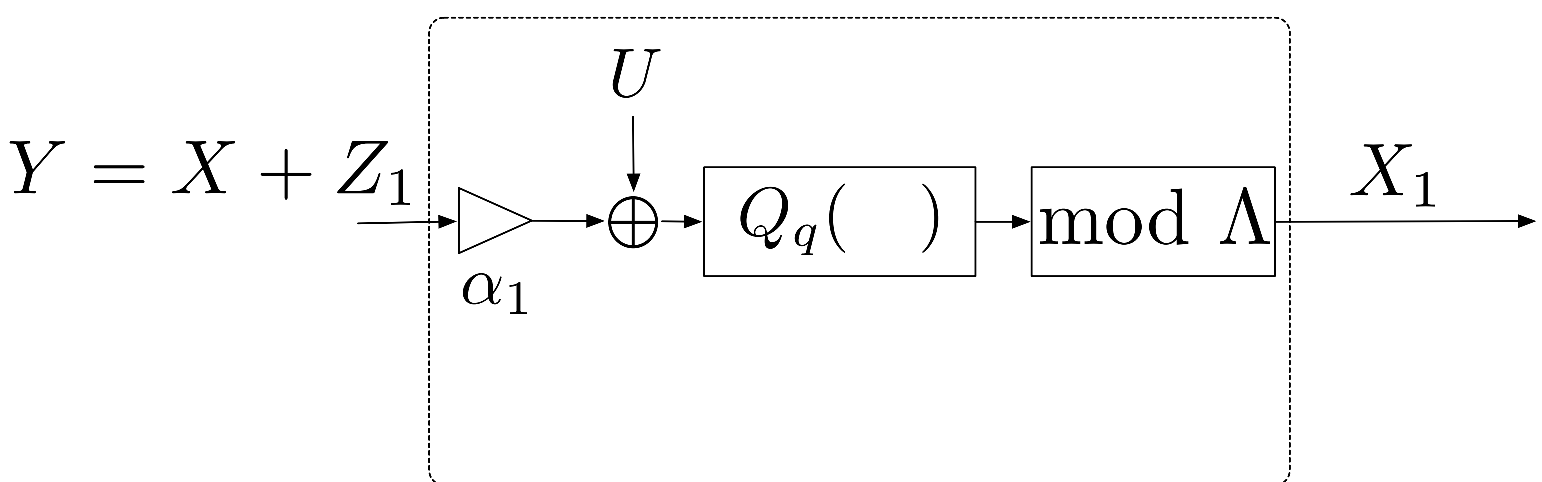

Encoding (function $f(\cdot)$)

$$\alpha_1 = \sqrt{1 - \frac{D}{N_1 + \frac{PN_2}{P+N_2}}}, \quad \alpha_2 = \frac{P}{P+N_2}.$$

The *(X+Z₁, X+Z₂)* Wyner-Ziv problem

Theorem. The following rate-distortion function for the lossy compression of the source $X + Z_1$ subject to the reconstruction side-information $X + Z_2$ and squared error distortion metric may be achieved using lattice codes:

$$\begin{aligned} R(D) &= \frac{1}{2}\log\left(\frac{\sigma^2_{X+Z_1|X+Z_2}}{D}\right), \qquad 0 \leq D \leq \sigma^2_{X+Z_1|X+Z_2} \\ &= \frac{1}{2}\log\left(\frac{N_1 + \frac{PN_2}{P+N_2}}{D}\right), \qquad 0 \leq D \leq N_1 + \frac{PN_2}{P+N_2}, \end{aligned}$$

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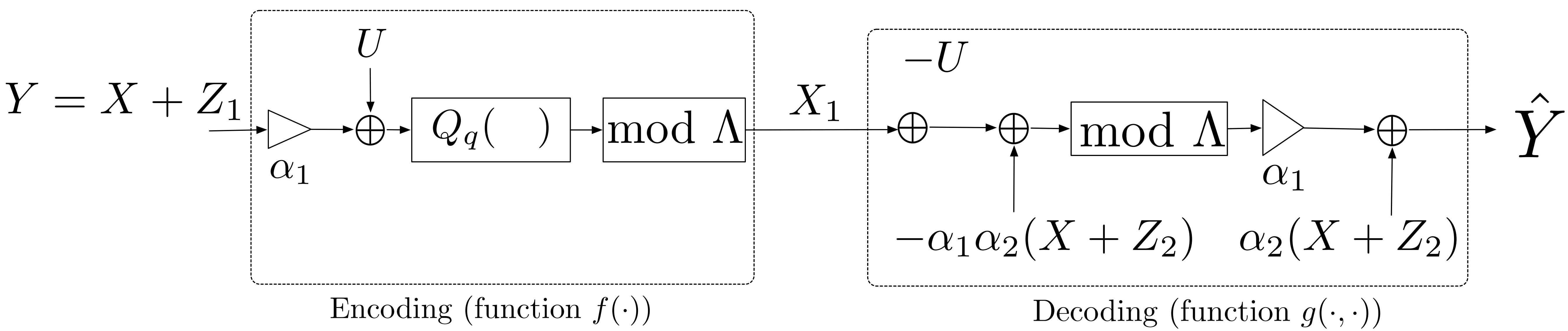


Comparison with *[Zamir, Erez, Shamai, 2002]*

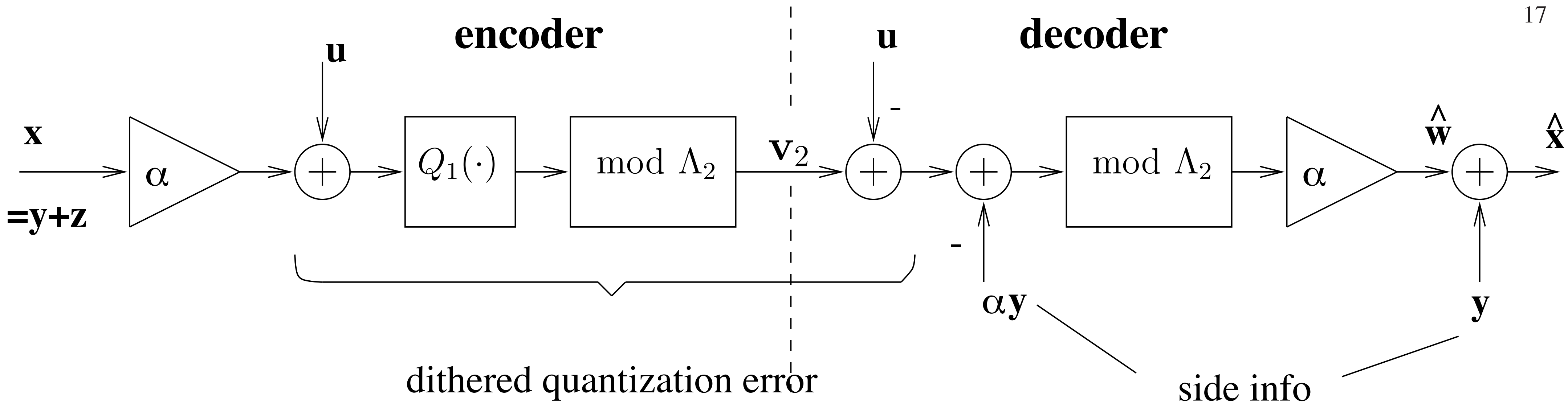


Fig. 5: Wyner-Ziv encoding of a jointly Gaussian source using nested lattice codes. At high resolution $\alpha = 1$.

Comparison with *[Zamir, Erez, Shamai, 2002]*

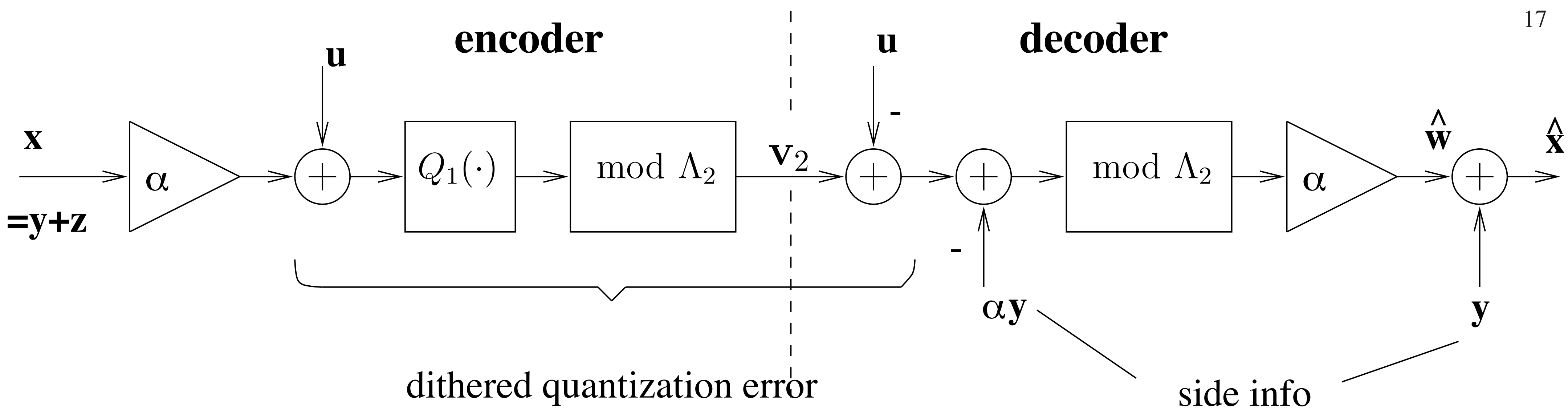
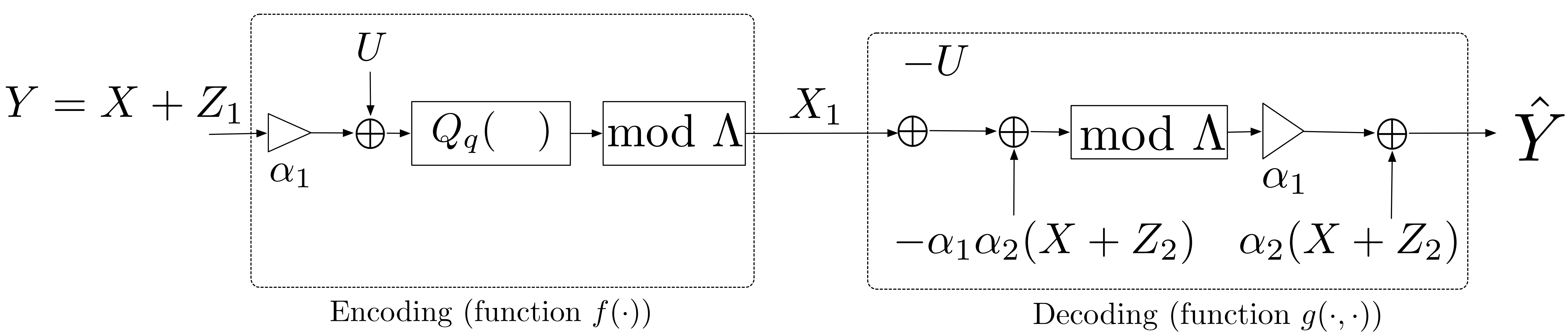
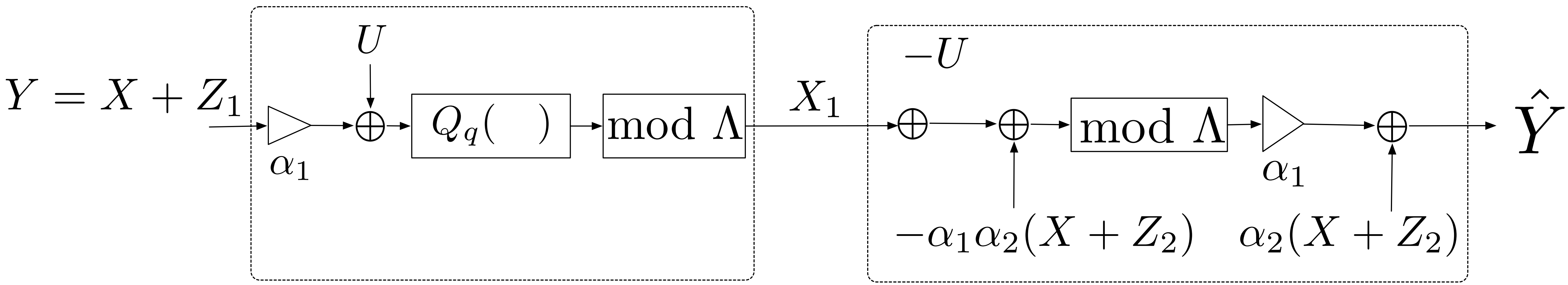


Fig. 5: Wyner-Ziv encoding of a jointly Gaussian source using nested lattice codes. At high resolution $\alpha = 1$.



$$\alpha_1 = \sqrt{1 - \frac{D}{N_1 + \frac{PN_2}{P+N_2}}}, \quad \alpha_2 = \frac{P}{P+N_2}.$$

Comment on α_1, α_2



Encoding (function $f(\cdot)$)

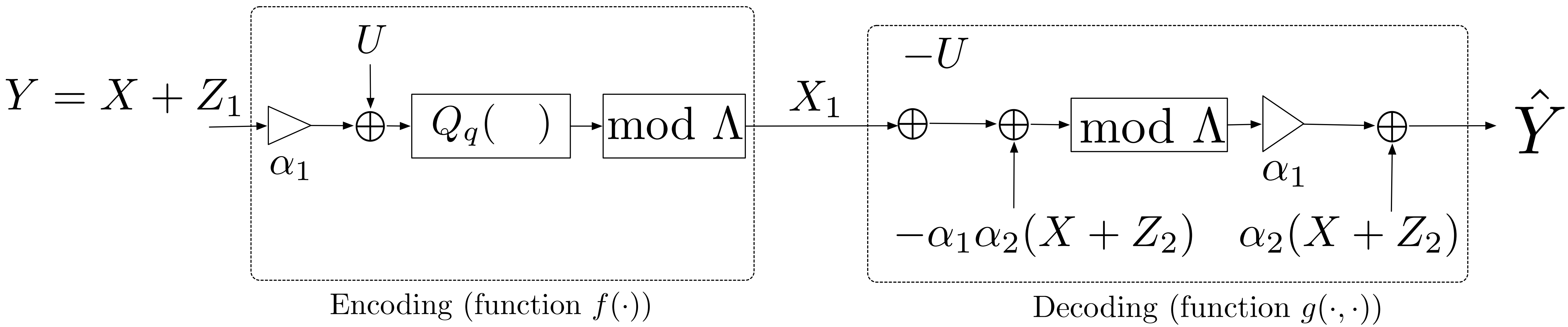
Decoding (function $g(\cdot,\cdot)$)

$$\mathbf{X} + \mathbf{Z_1} = \alpha_2(\mathbf{X} + \mathbf{Z_2}) + (1-\alpha_2)\mathbf{X} + \mathbf{Z_1} - \alpha_2\mathbf{Z_2},$$

$\rightarrow$ choosing $\alpha_2 = \frac{P}{P+N_2}$, then $\mathbf{X} + \mathbf{Z_2} \perp (1-\alpha_2)\mathbf{X} + \mathbf{Z_1} - \alpha_2\mathbf{Z_2}$

$$\alpha_1 = \sqrt{1 - \frac{D}{N_1 + \frac{PN_2}{P+N_2}}}, \quad \alpha_2 = \frac{P}{P+N_2}.$$

Comment on α_1, α_2



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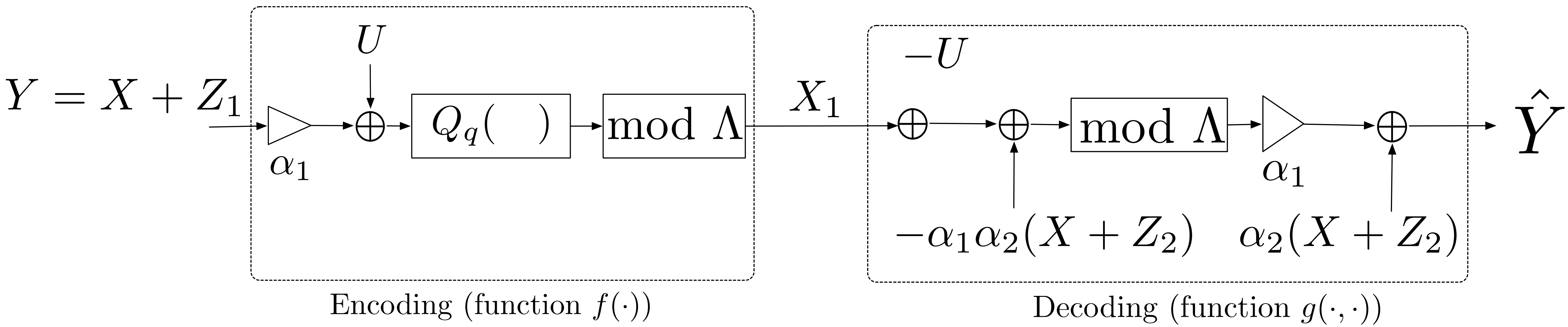
$\rightarrow$ $\alpha_1 \equiv$ source coding MMSE coefficient

$\rightarrow$ $\alpha_2 \equiv$ channel coding MMSE coefficient

$$\alpha_1 = \sqrt{1 - \frac{D}{N_1 + \frac{PN_2}{P+N_2}}}, \quad \alpha_2 = \frac{P}{P+N_2}.$$

Comment on α_1, α_2



$$\mathbf{X} + \mathbf{Z_1} = \alpha_2(\mathbf{X} + \mathbf{Z_2}) + (1 - \alpha_2)\mathbf{X} + \mathbf{Z_1} - \alpha_2\mathbf{Z_2},$$

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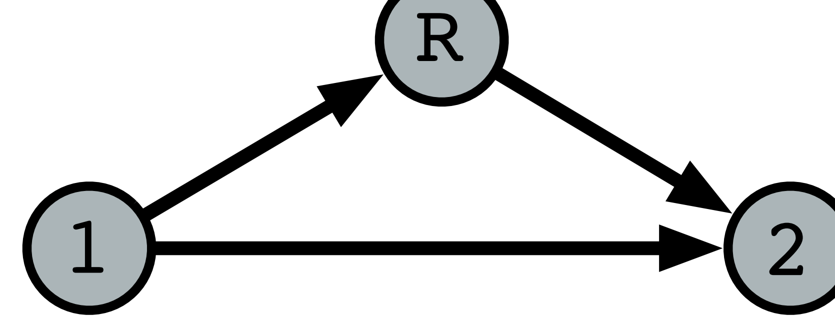
$$\left.\begin{array}{l} \rightarrow \alpha_1 \equiv \text{source coding MMSE coefficient} \\ \rightarrow \alpha_2 \equiv \text{channel coding MMSE coefficient} \end{array}\right\} \quad \textbf{Lose if do not pick these optimally}$$

Outline - a lattice CF scheme

- Lattice notation
- Compress and forward review
- Lattice ($X+Z_1$, $X+Z_2$) Wyner-Ziv coding scheme
- Lattices achieve CF rate for AWGN relay

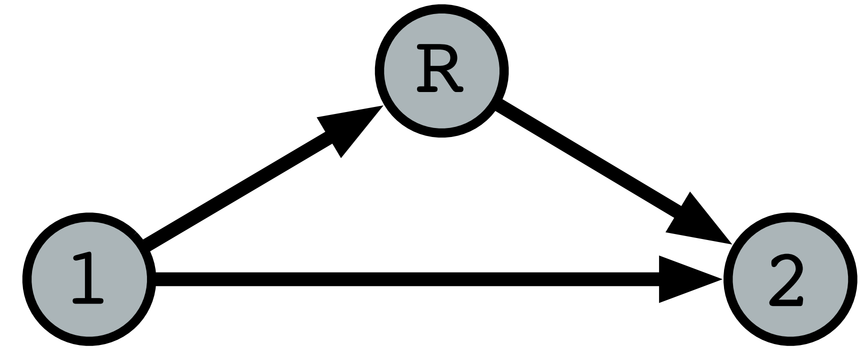
A Lattice CF scheme



Theorem. For the three user Gaussian relay channel described by the input/output equations $Y_R = X_1 + N_R$ at the relay's receiver and $Y_2 = X_1 + X_R + N_2$ at the destination, with corresponding input and noise powers P_1, P_R, N_R, N_2, the following rate may be achieved using lattice codes in a lattice Compress-and-Forward fashion:

$$R < \frac{1}{2}\log\left(1 + \frac{P_1}{N_2} + \frac{P_1 P_R}{P_1 N_R + P_1 N_2 + P_R N_R + N_R N_2}\right).$$

A Lattice CF scheme



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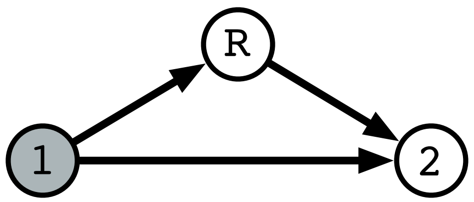
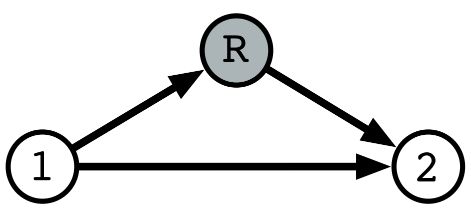
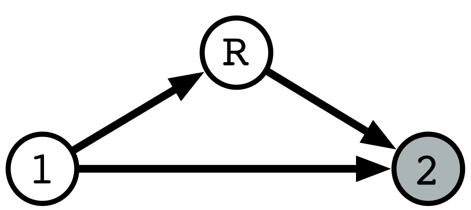
same as that achieved by Gaussian codes in the CF scheme of [Cover, El Gamal, 1979]

Mimic all steps with lattice codes

	Block 1	Block 2	Block 3	Block 4
Encoding	$x_{1,1}(w_1)$	$x_{1,2}(w_2)$	$x_{1,3}(w_3)$	$x_{1,4}(1)$
Compress + forward	$y_{R,1} \to \widehat{y_{R,1}}(i_1)$	$y_{R,2} \to \widehat{y_{R,2}}(i_2)$	$y_{R,3} \to \widehat{y_{R,3}}(i_3)$	
	$x_{R,1}(1)$	$x_{R,2}(i_1)$	$x_{R,3}(i_2)$	$x_{R,4}(i_3)$
Decoding		$\widehat{y_{R,1}}$	$\widehat{y_{R,2}}$	$\widehat{y_{R,3}}$
	$y_{2,1} - x_{R,1}$	$y_{2,2} - x_{R,2}$	$y_{2,3} - x_{R,3}$	



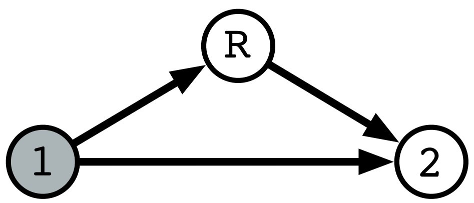


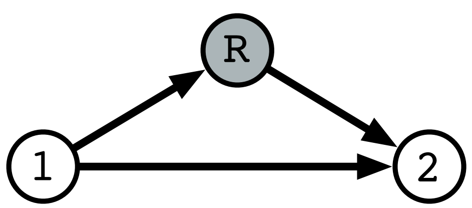


Combine Combine Combine

Encoding

Block b

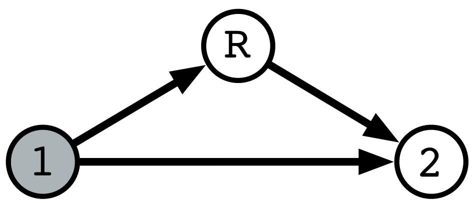


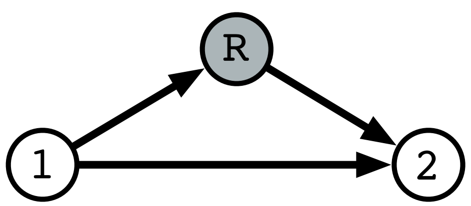


$x_{1,b}(w_b)$

Encoding

Block b



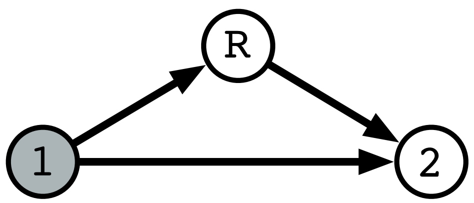


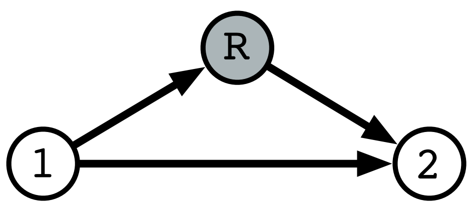
$x_{1,b}(w_b)$

$$\text{rate } R$$
$$w \leftrightarrow t_1,\, \Lambda_1 \subseteq \Lambda_{c1}, \sigma^2(\Lambda_1) = P_1$$

Encoding

Block b





$$x_{1,b}(w_b)$$

$$y_{R,b} \rightarrow \widehat{y_{R,b}(i_b)}$$

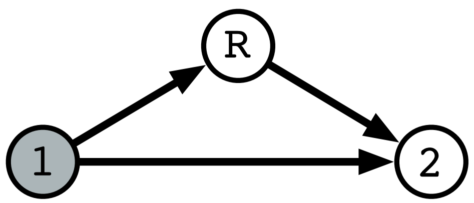
$$\text{send } x_{R,b}(i_{b-1})$$

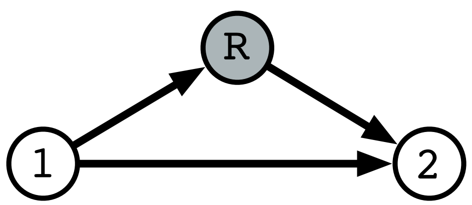
$$\text{rate } R$$

$$w \leftrightarrow t_1,\ \Lambda_1 \subseteq \Lambda_{c1}, \sigma^2(\Lambda_1) = P_1$$

Encoding

Block b





$x_{1,b}(w_b)$
$y_{R,b} \rightarrow \widehat{y_{R,b}(i_b)}$ send $x_{R,b}(i_{b-1})$

rate R

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compression $\Lambda \subseteq \Lambda_q$

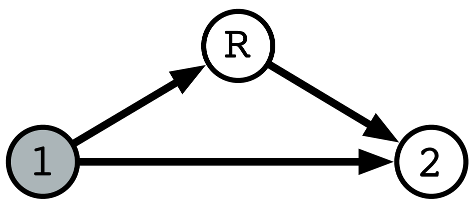
rate $\hat{R}$ $\sigma^2(\Lambda) = N_R + \frac{P_1 N_2}{P_1 + N_2} + D$

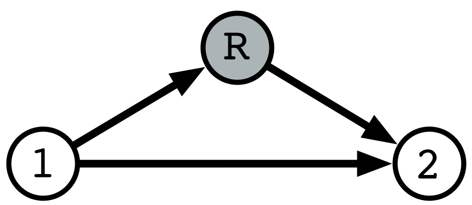
$i \leftrightarrow t_R,\ \Lambda_R \subseteq \Lambda_{cR}$

rate R' $\sigma^2(\Lambda_R) = P_R$

Encoding

Block b





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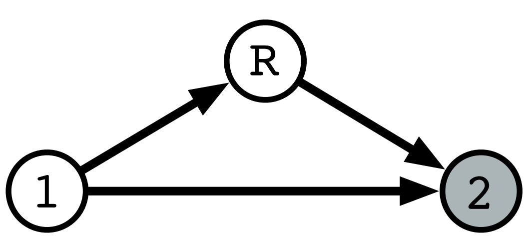
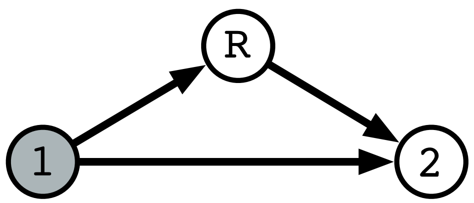
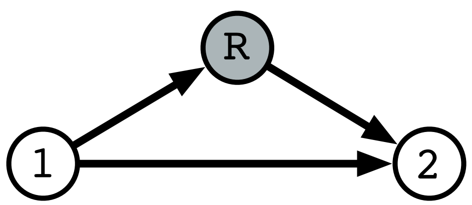
Decoding

Block b-1 *Block b*



Encoding





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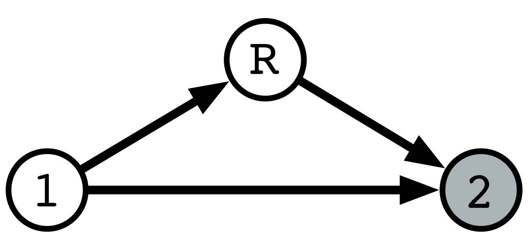
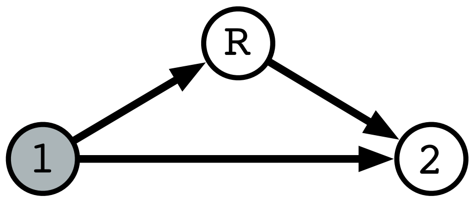
Decoding

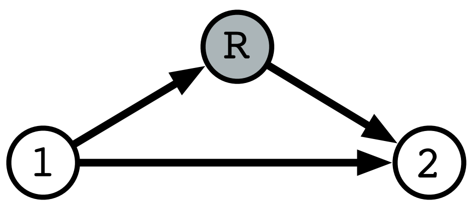


Block b-1	*Block b*
• $Y'_{2,b-1} = X_{1,b-1} + Z_2$	

Encoding

Block b





$x_{1,b}(w_b)$

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send $x_{R,b}(i_{b-1})$

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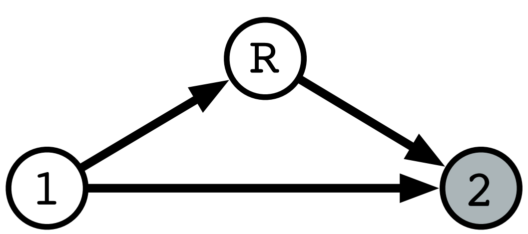
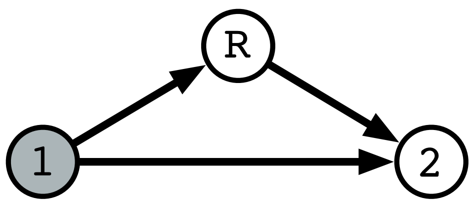
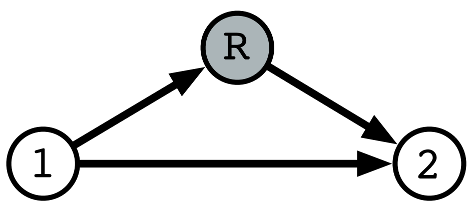
Decoding



Block b-1	*Block b*
• $Y'_{2,b-1} = X_{1,b-1} + Z_2$	• compression index i_{b-1} from $Y_2 = X_1 + X_R + Z_2$

Encoding





Block b

$$x_{1,b}(w_b)$$

$$y_{R,b} \longrightarrow \widehat{y_{R,b}(i_b)}$$

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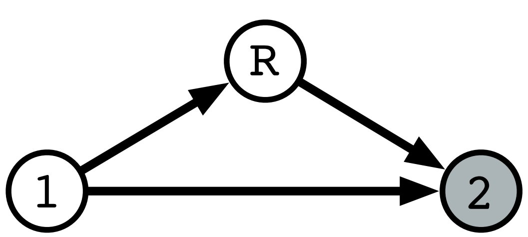
rate R' $\quad \sigma^2(\Lambda_R) = P_R$

Decoding



Block b-1

- $Y'_{2,b-1} = X_{1,b-1} + Z_2$

Block b

- compression index i_{b-1} from $Y_2 = X_1 + X_R + Z_2$
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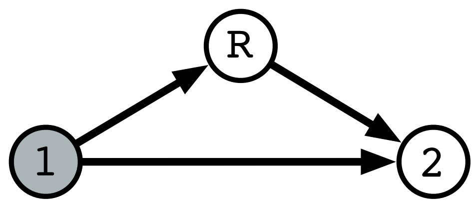
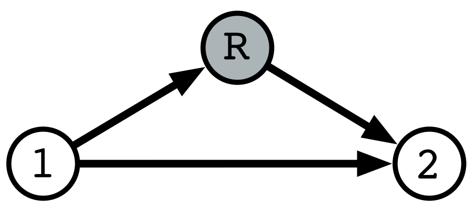
Encoding

Block b





$x_{1,b}(w_b)$

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send $x_{R,b}(i_{b-1})$

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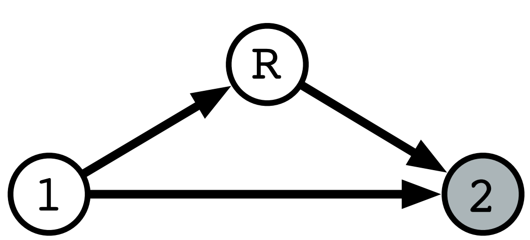
Decoding



Block b-1

- $Y'_{2,b-1} = X_{1,b-1} + Z_2$

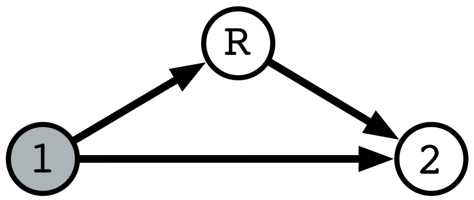
Block b

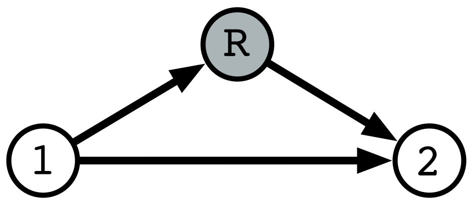
- compression index i_{b-1} from $Y_2 = X_1 + X_R + Z_2$
- $Y'_{2,b} = Y_{2,b} - X_{R,b}(i_{b-1}) = X_{1,b} + Z_2$
- i_{b-1} and $Y'_{2,b-1}$ to reconstruct $\widehat{Y_{R,b-1}}(i_{b-1})$

Wyner-Ziv

UIC Department of Electrical and Computer Engineering
UNIVERSITY OF ILLINOIS AT CHICAGO
COLLEGE OF ENGINEERING

Encoding





Block b

$x_{1,b}(w_b)$
$y_{R,b} \rightarrow \widehat{y_{R,b}(i_b)}$ send $x_{R,b}(i_{b-1})$

rate R

$w \leftrightarrow t_1,\ \Lambda_1 \subseteq \Lambda_{c1}, \sigma^2(\Lambda_1) = P_1$

compression $\Lambda \subseteq \Lambda_q$

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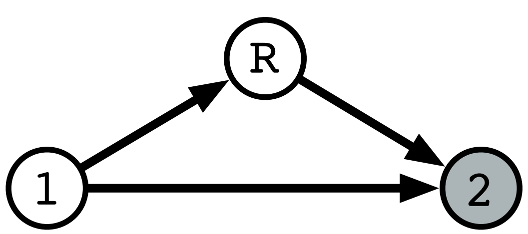
Decoding



Block b-1	*Block b*
• $Y'_{2,b-1} = X_{1,b-1} + Z_2$	• compression index i_{b-1} from $Y_2 = X_1 + X_R + Z_2$ • $Y'_{2,b} = Y_{2,b} - X_{R,b}(i_{b-1}) = X_{1,b} + Z_2$ • i_{b-1} and $Y'_{2,b-1}$ to reconstruct $\widehat{Y_{R,b-1}}(i_{b-1})$ • combine $Y'_{2,b-1}$ and $\widehat{Y_{R,b-1}}(i_{b-1})$ to decode w_{b-1}

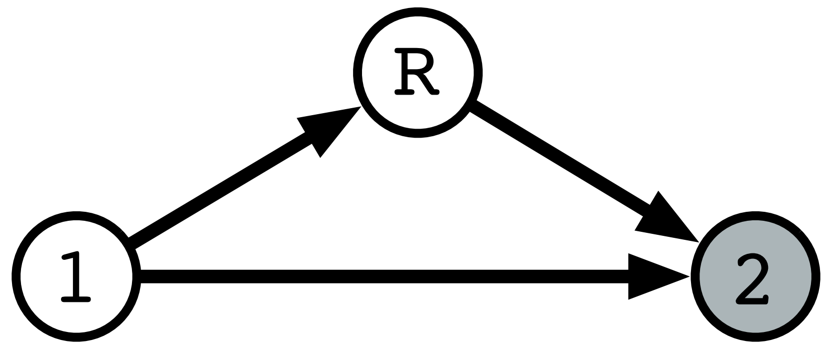
Wyner-Ziv

compression $\Lambda \subseteq \Lambda_q$ rate $\hat{R}$ $\sigma^2(\Lambda) = N_R + \frac{P_1 N_2}{P_1+N_2} + D$

$i \leftrightarrow t_R,\ \Lambda_R \subseteq \Lambda_{cR}$ rate R' $\sigma^2(\Lambda_R) = P_R$

rate R $w \leftrightarrow t_1,\ \Lambda_1 \subseteq \Lambda_{c1}$

$\sigma^2(\Lambda_1) = P_1$



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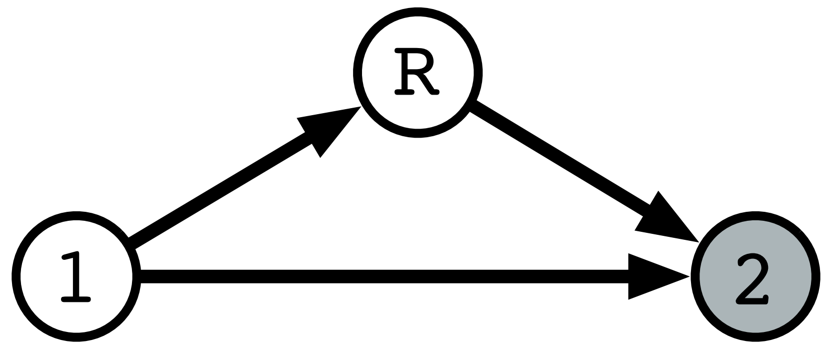

- decodes i from $Y_2 = X_1 + X_R + Z_2$ as long as $R' < \frac{1}{2}\log_2\left(1 + \frac{P_R}{P_1 + N_2}\right)$

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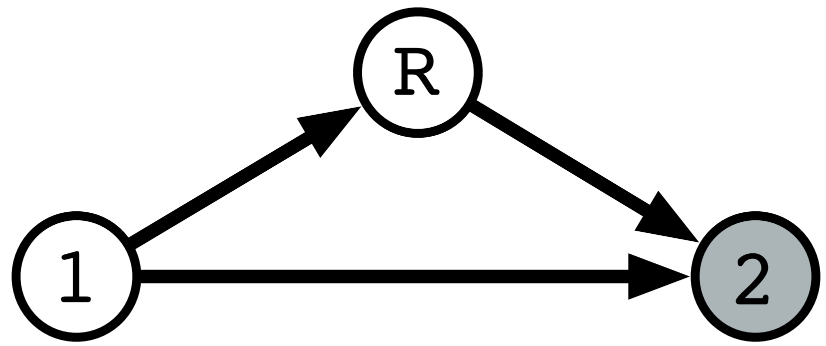

- decodes i from $Y_2 = X_1 + X_R + Z_2$ as long as $R' < \frac{1}{2}\log_2\left(1 + \frac{P_R}{P_1+N_2}\right)$
- source coding < channel coding rate: $\frac{1}{2}\log\left(1 + \frac{N_R + \frac{P_1 N_2}{P_1+N_2}}{D}\right) < \frac{1}{2}\log\left(1 + \frac{P_R}{P_1+N_2}\right)$

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rate R $w \leftrightarrow t_1,\ \Lambda_1 \subseteq \Lambda_{c1}$

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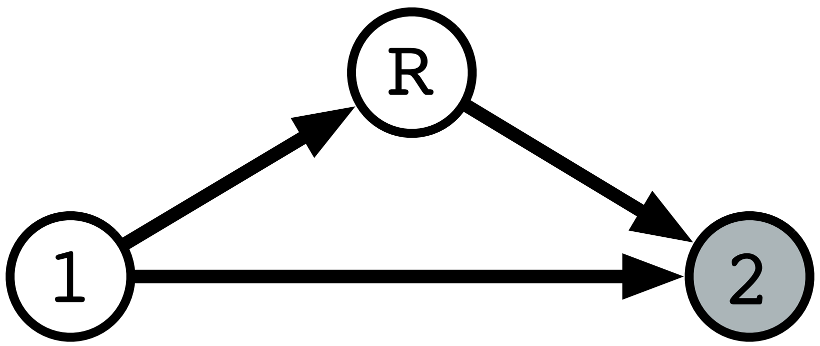

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- use $Y_2' = Y_2 - X_R$ from previous block and X_R from current block to reconstruct $\hat{Y_R}$ as in $(X + Z_R, X + Z_2)$ Wyner-Ziv compression $\Lambda \subseteq \Lambda_q$ rate $\hat{R}$ $\sigma^2(\Lambda) = N_R + \frac{P_1 N_2}{P_1 + N_2} + D$

$i \leftrightarrow t_R,\ \Lambda_R \subseteq \Lambda_{cR}$ rate R' $\sigma^2(\Lambda_R) = P_R$

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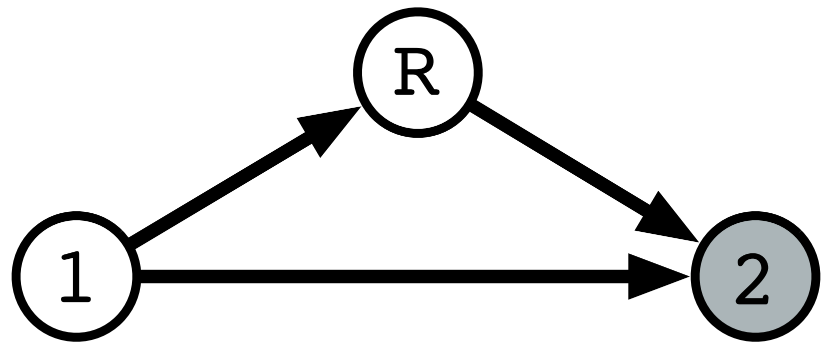

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- use $Y_2' = Y_2 - X_R$ from previous block and X_R from current block to reconstruct $\hat{Y_R}$ as in $(X + Z_R, X + Z_2)$ Wyner-Ziv

- coherently combine Y_2' and $\hat{Y_R}$ to decode w, as long as $R < \frac{1}{2}\log\left(1 + \frac{P_1}{N_2} + \frac{P_1}{N_R + D}\right)$

compression $\Lambda \subseteq \Lambda_q$ rate $\hat{R}$ $\sigma^2(\Lambda) = N_R + \frac{P_1 N_2}{P_1+N_2} + D$

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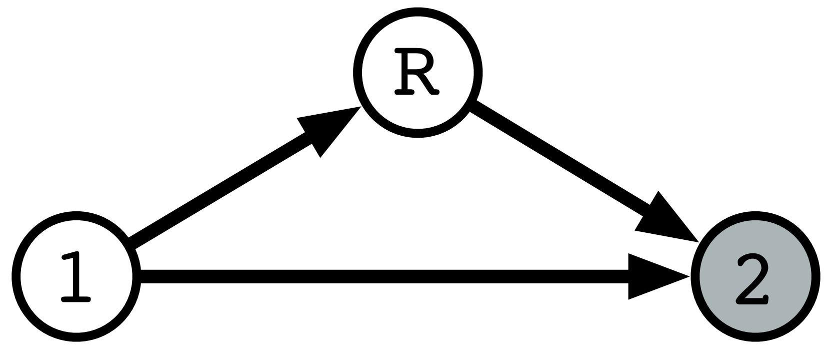

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- note: pick $\alpha_1 = 1$ rather than source coding MMSE to render compression noise independent of all else

Conclusion

- can random codes be replaced by structured codes in Gaussian networks?

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- can random codes be replaced by structured codes in Gaussian networks?
- CF, DF lattices can achieve same rates as random Gaussian codebooks
- can this be combined with linearity of lattices to achieve higher rates in Gaussian networks?
- what is the capacity of relay channels, what are we stuck at?

Questions?

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University of Illinois at Chicago
http://www.ece.uic.edu/Devroye