To adapt or not in two-way interference channels?

Natasha Devroye (joint work with Ph.D. student Zhiyu Cheng)
Department of Electrical and Computer Engineering
University of Illinois at Chicago

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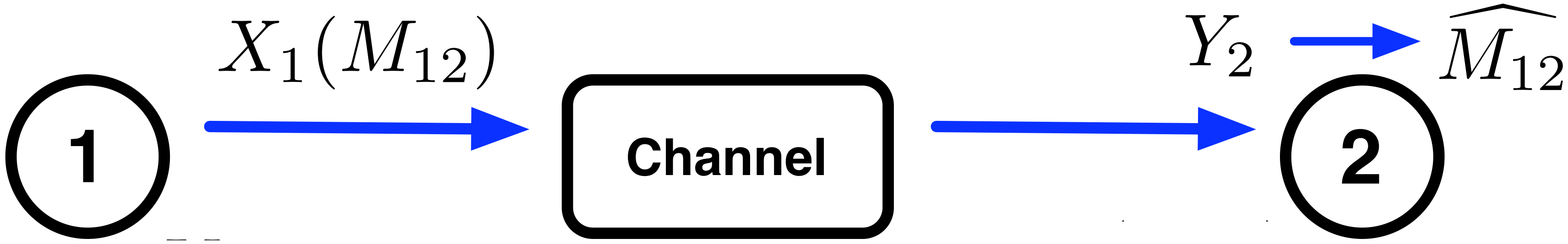
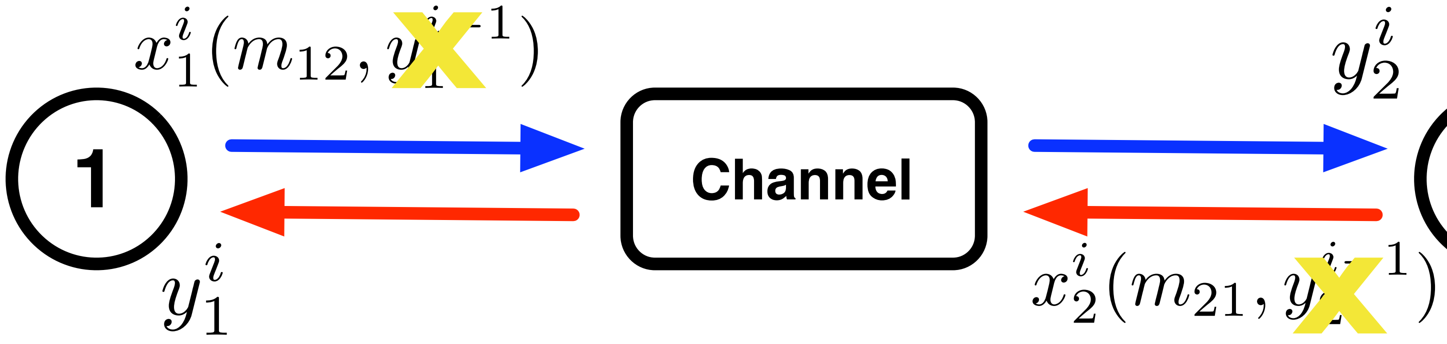
Why is the two-way problem so hard?

Adaptation!

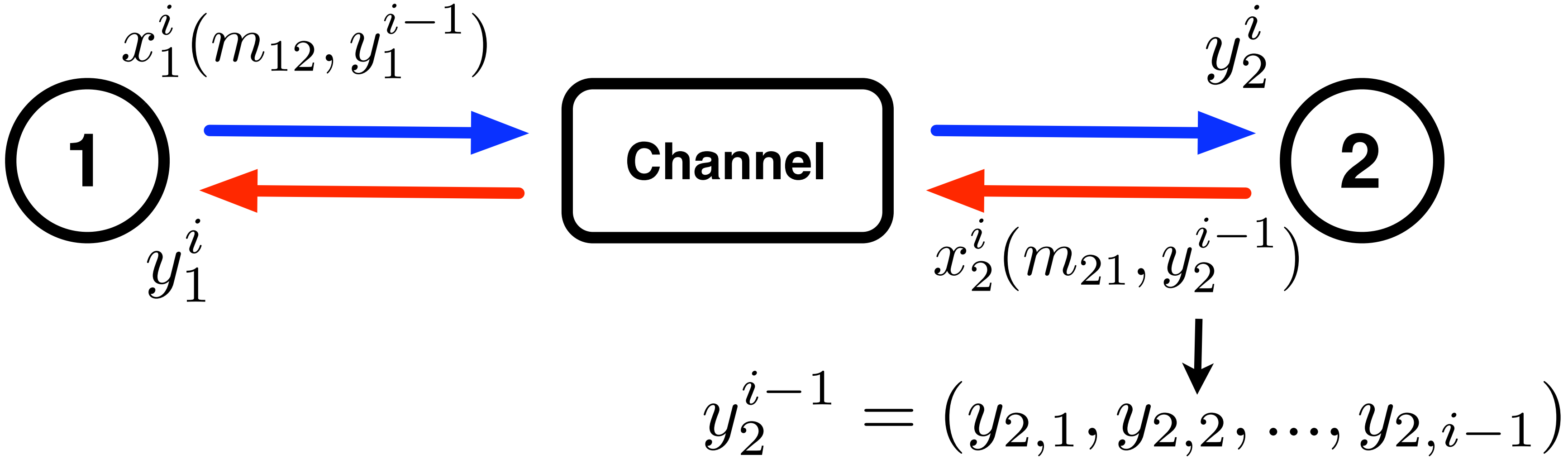
One-way:
no adaptation possible



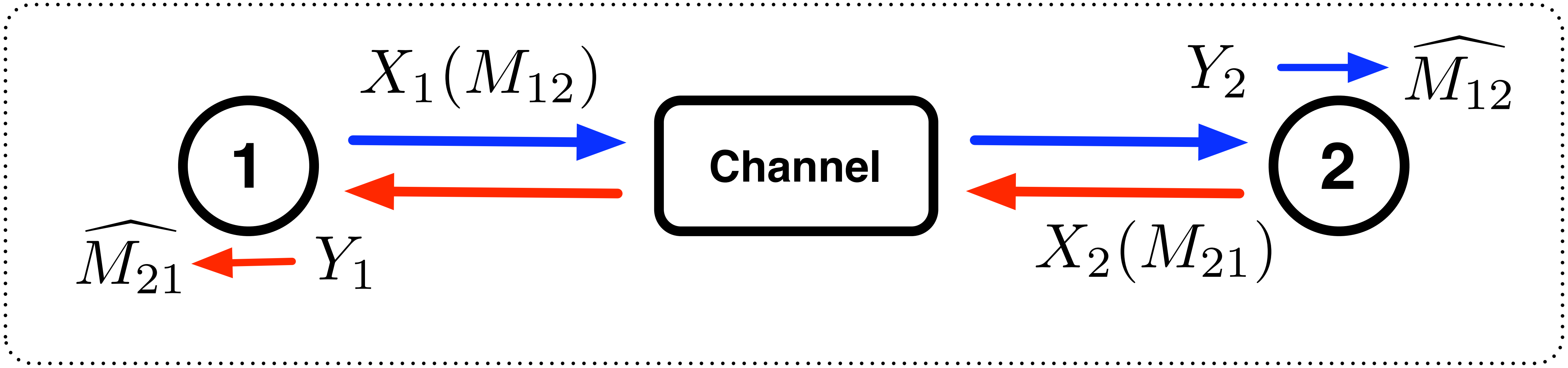
Two-way:
no adaptation = “restricted”



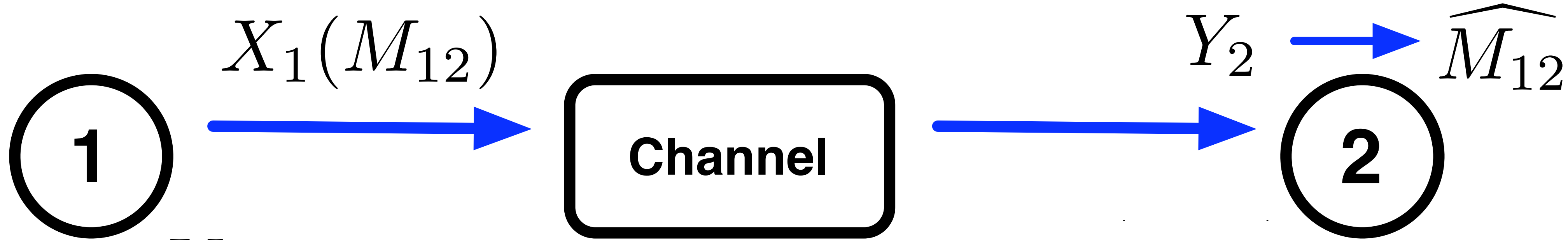
Two-way:
full adaptation



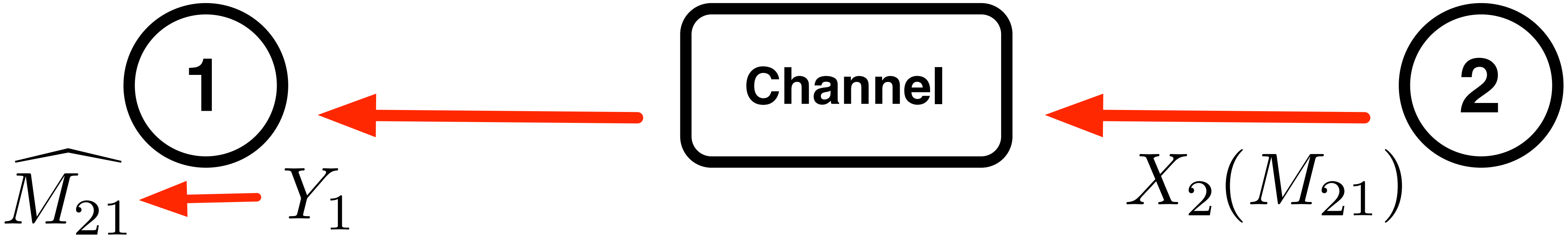
Capacity known for certain examples where



equal to



in parallel with



When is capacity known

Parallel two-way channel

Binary modulo 2 adder channel

Two-way restricted channel

Two-way ``push-to-talk” channel

Two-way Gaussian noise channel

When adaptation is useless (does not increase capacity)!

When is capacity unknown

General discrete memoryless channel

Binary multiplier channel

When adaptation is useful?

What about two-way interfering *networks*?

Do there exist / when is

Two-way capacity

$\equiv$

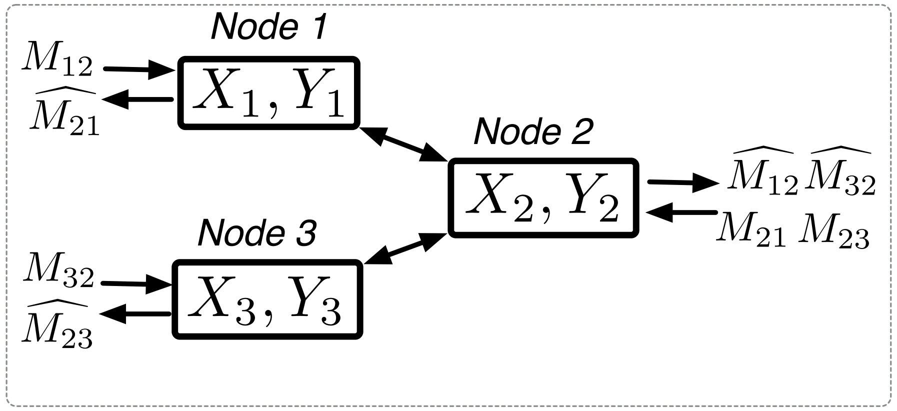
One-way →

One-way ←

(as in two-way binary adder, two-way Gaussian channels)

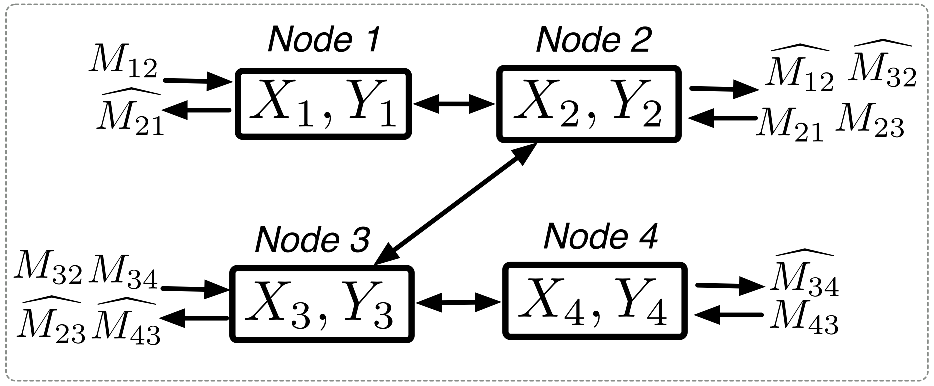
Results: Capacity achieved without adaptation for:

[Cheng, Devroye, Allerton 2011]
[Cheng, Devroye, ISIT 2012]
[Cheng, Devroye, submitted IT Trans., 2012]



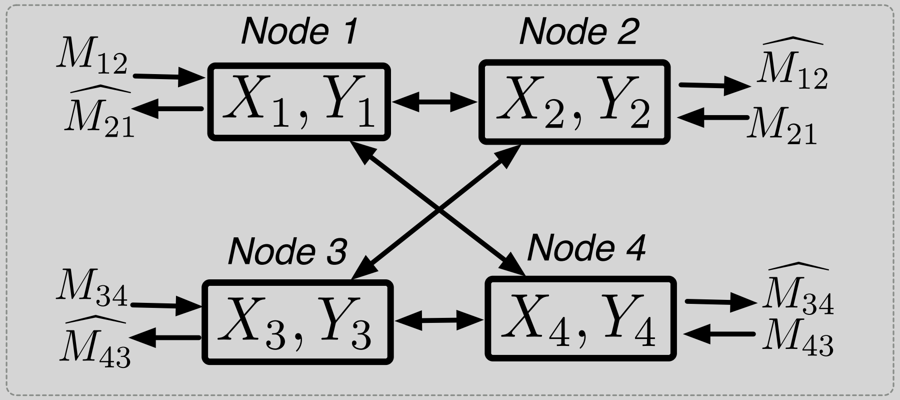
(a) Two-way MAC/BC

- Binary adder channel
- Linear deterministic channel (models Gaussian noise channel at high SNR)



(b) Two-way Z channel

- Binary adder channel
- Linear deterministic channel

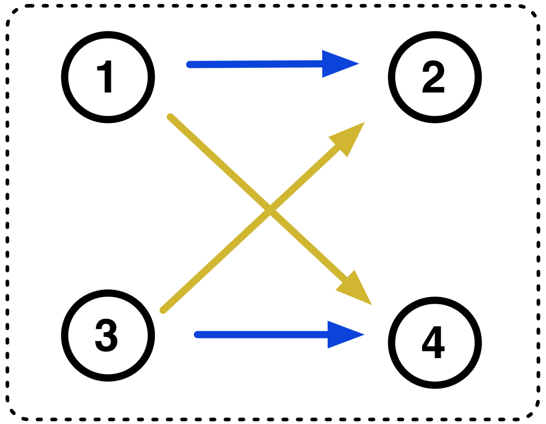


(c) Two-way interference channel

- Binary adder channel (always)
- Linear deterministic channel (only with restricted interaction!)
- Constant gap certain Gaussian channels (with/without restricted interaction!)

Two-way interference channel related work

One-way IC



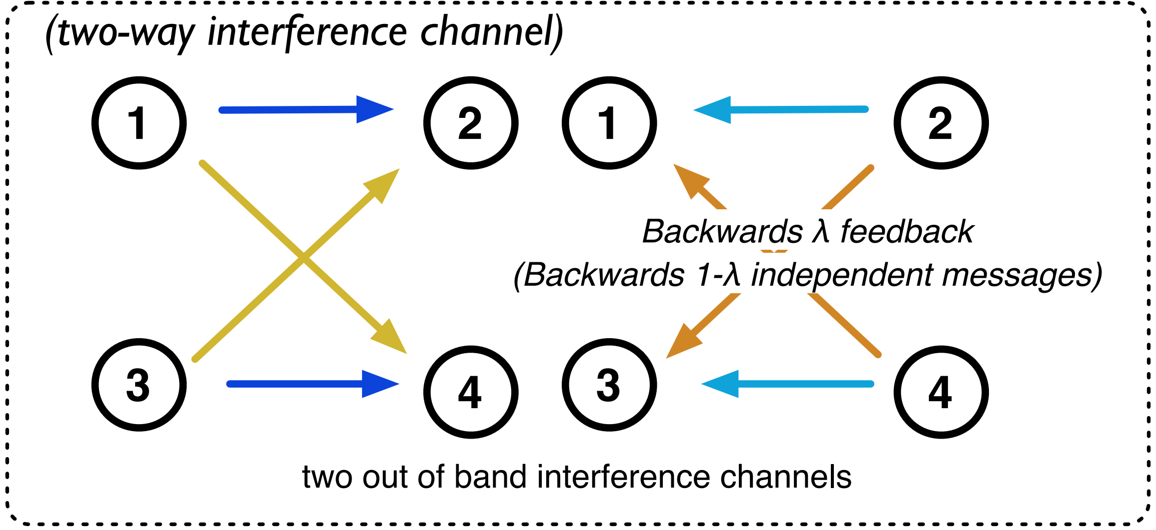
2 messages

[El Gamal, Costa 1982]

[Bresler, Tse 2008]

[Etkin, Tse, Wang 2008]

One-way IC with interfering FB *[Suh, Wang, Tse 2012]*



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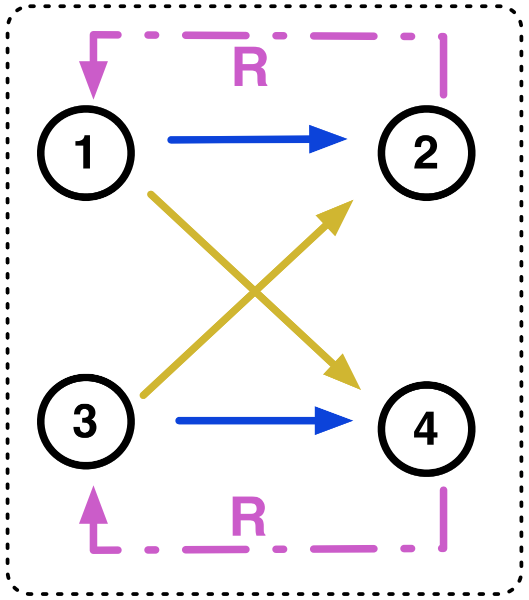
One-way IC with rate-limited FB

[Vahid, Suh, Avestimehr 2012]



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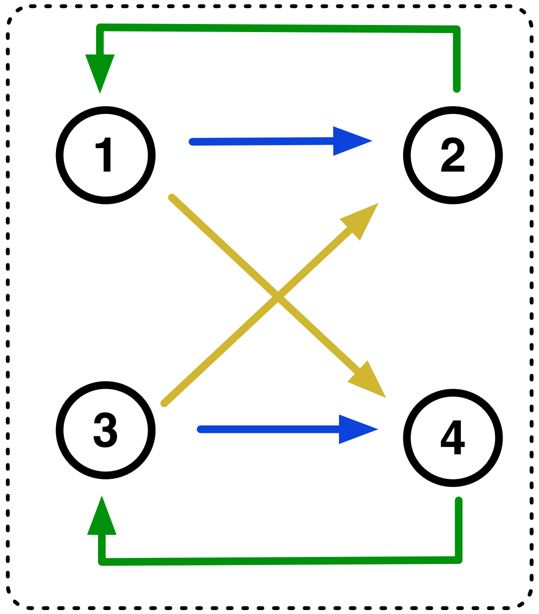
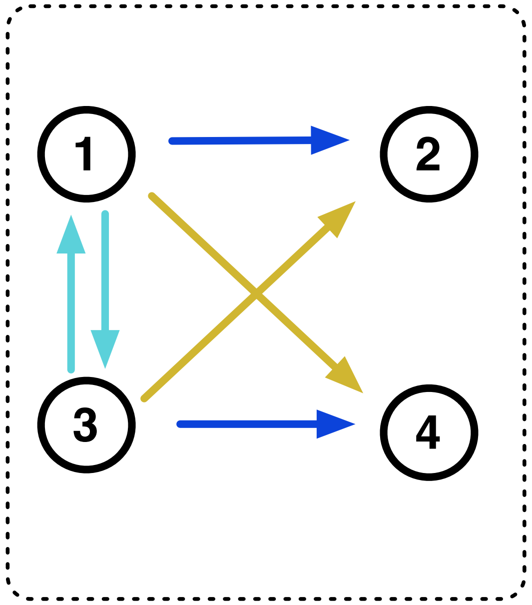
One-way IC with FB



2 messages

[Suh, Tse 2011]

One-way IC with generalized FB



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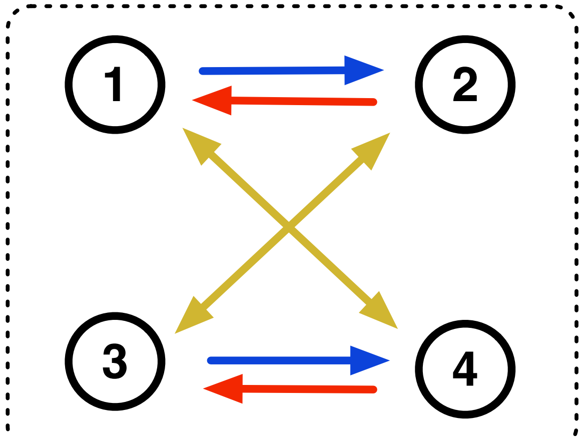
[Yang, Tuninetti 2011]

[Prabhakaran, Viswanath 2011]

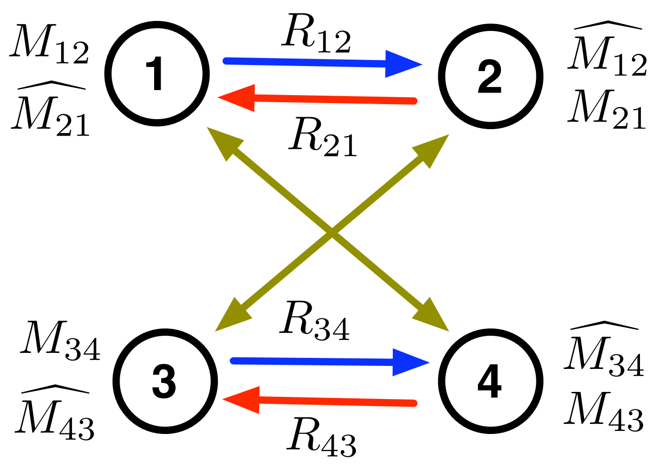
Two-way IC



4 messages *true adaptation*

[Cheng, Devroye sub. Trans IT, 2012. On Arxiv]

Two-way Modulo 2 Adder IC



$$
\begin{aligned}
Y_1 &= X_1 \oplus X_2 \oplus X_4 \\
Y_2 &= X_1 \oplus X_2 \oplus X_3 \\
Y_3 &= X_2 \oplus X_3 \oplus X_4 \\
Y_4 &= X_1 \oplus X_3 \oplus X_4.
\end{aligned}
$$

Theorem 2: The capacity region of the two-way modulo 2 adder interference channel is the set of non-negative rate tules $(R_{12}, R_{21}, R_{34}, R_{43})$ such that

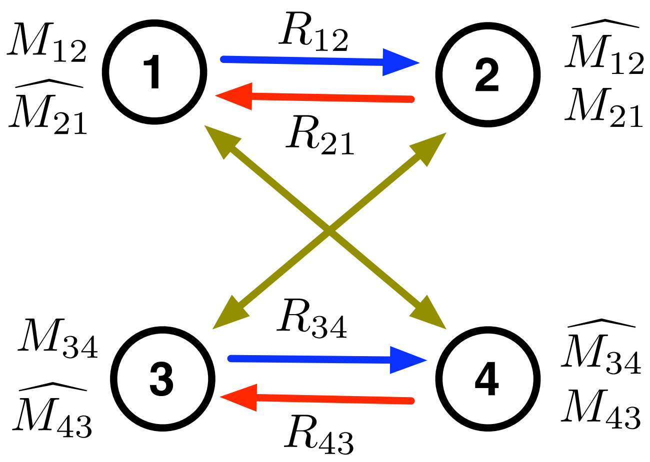
$$\longrightarrow \quad R_{12} + R_{34} \leq 1 \qquad (1)$$

$$\longleftarrow \quad R_{21} + R_{43} \leq 1. \qquad (2)$$

Adaptation useless (can not increase capacity)!

Achievable via time-sharing, cut-set outer bound or directly via alternative proof (see arxiv)

Two-way linear deterministic IC



$$N = \max(n_{11}, n_{22}, n_{33}, n_{44}, n_{12}, n_{21}, n_{32}, n_{23}, n_{14}, n_{41}, n_{34}, n_{43})$$

$$Y_1 = S^{N-n_{11}} X_1 + S^{N-n_{21}} X_2 + S^{N-n_{41}} X_4$$

$$Y_2 = S^{N-n_{12}} X_1 + S^{N-n_{22}} X_2 + S^{N-n_{32}} X_3$$

$$Y_3 = S^{N-n_{23}} X_2 + S^{N-n_{33}} X_3 + S^{N-n_{43}} X_4$$

$$Y_4 = S^{N-n_{14}} X_1 + S^{N-n_{34}} X_3 + S^{N-n_{44}} X_4$$

If partial adaptation:

$$X_{1,i} = f_1(M_{12}), \quad X_{2,i} = f_2(M_{21}, Y_2^{i-1})$$

$$X_{3,i} = f_3(M_{34}), \quad X_{4,i} = f_4(M_{43}, Y_4^{i-1})$$

Then capacity:

$R_{12} \leq n_{12}, \quad R_{34} \leq n_{34},$

$R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+$

$R_{12} + R_{34} \leq \max(n_{34}, n_{14}) + [n_{12} - n_{14}]^+$

$R_{12} + R_{34} \leq \max([n_{12} - n_{14}]^+, n_{32}) + \max([n_{34} - n_{32}]^+, n_{14})$

$2R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{12} - n_{14}]^+ + \max([n_{34} - n_{32}]^+, n_{14})$

$R_{12} + 2R_{34} \leq \max(n_{34}, n_{14}) + [n_{34} - n_{32}]^+ + \max([n_{12} - n_{14}]^+, n_{32})$

(A) IC in $\rightarrow$ direction

$R_{21} \leq n_{21}, \quad R_{43} \leq n_{43}$

$R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{43} - n_{41}]^+$

$R_{21} + R_{43} \leq \max(n_{43}, n_{23}) + [n_{21} - n_{23}]^+$

$R_{21} + R_{43} \leq \max([n_{21} - n_{23}]^+, n_{41}) + \max([n_{43} - n_{41}]^+, n_{23})$

$2R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{21} - n_{23}]^+ + \max([n_{43} - n_{41}]^+, n_{23})$

$R_{21} + 2R_{43} \leq \max(n_{43}, n_{23}) + [n_{43} - n_{41}]^+ + \max([n_{21} - n_{23}]^+, n_{41})$

(B) IC in $\leftarrow$ direction

Partial adaptation key lemma:

$$X_{1,i} = f_1(M_{12}), \;\; X_{2,i} = f_2(M_{21}, Y_2^{i-1})$$
$$X_{3,i} = f_3(M_{34}), \;\; X_{4,i} = f_4(M_{43}, Y_4^{i-1})$$

Partial adaptation conditions

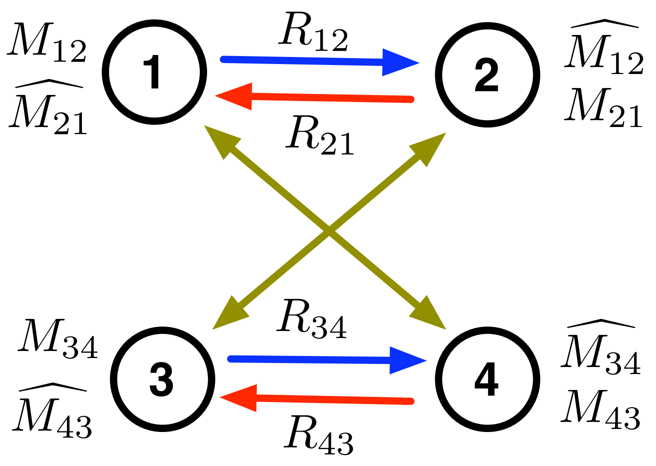
Lemma: Under partial adaptation conditions, for some deterministic functions f_5 and f_6,

$$X_{2,i} = f_5(M_{12}, M_{21}, M_{34}) \perp M_{43}, \;\; \forall i$$
$$X_{4,i} = f_6(M_{43}, M_{34}, M_{12}) \perp M_{21}, \;\; \forall i$$

where $\perp$ denotes independence.

Central to many of the partial adaptation converses!

Two-way linear deterministic IC



$$N = \max(n_{11}, n_{22}, n_{33}, n_{44}, n_{12}, n_{21}, n_{32}, n_{23}, n_{14}, n_{41}, n_{34}, n_{43})$$

$$Y_1 = S^{N-n_{11}} X_1 + S^{N-n_{21}} X_2 + S^{N-n_{41}} X_4$$

$$Y_2 = S^{N-n_{12}} X_1 + S^{N-n_{22}} X_2 + S^{N-n_{32}} X_3$$

$$Y_3 = S^{N-n_{23}} X_2 + S^{N-n_{33}} X_3 + S^{N-n_{43}} X_4$$

$$Y_4 = S^{N-n_{14}} X_1 + S^{N-n_{34}} X_3 + S^{N-n_{44}} X_4$$

If FULL adaptation:

$$X_{1,i} = f_1(M_{12}, Y_1^{i-1}) \quad X_{2,i} = f_1(M_{21}, Y_2^{i-1})$$

$$X_{3,i} = f_1(M_{34}, Y_3^{i-1}) \quad X_{4,i} = f_1(M_{43}, Y_4^{i-1})$$

Then we still have the outer bounds:

~~$R_{12} \leq n_{12}, \quad R_{34} \leq n_{34},$~~ $\boxed{R_{12} \leq \max(n_{12}, n_{43}), R_{34} \leq \max(n_{34}, n_{21})}$

$R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+$

$R_{12} + R_{34} \leq \max(n_{34}, n_{14}) + [n_{12} - n_{14}]^+$

~~$R_{12} + R_{34} \leq \max([n_{12} - n_{14}]^+, n_{32}) + \max([n_{34} - n_{32}]^+, n_{14})$~~

~~$2R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{12} - n_{14}]^+ + \max([n_{34} - n_{32}]^+, n_{14})$~~

~~$R_{12} + 2R_{34} \leq \max(n_{34}, n_{14}) + [n_{34} - n_{32}]^+ + \max([n_{12} - n_{14}]^+, n_{32})$~~

~~(A) IC in → direction~~

~~$R_{21} \leq n_{21}, \quad R_{43} \leq n_{43}$~~ $\boxed{R_{21} \leq \max(n_{21}, n_{34}), R_{43} \leq \max(n_{43}, n_{12})}$

$R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{43} - n_{41}]^+$

$R_{21} + R_{43} \leq \max(n_{43}, n_{23}) + [n_{21} - n_{23}]^+$

~~$R_{21} + R_{43} \leq \max([n_{21} - n_{23}]^+, n_{41}) + \max([n_{43} - n_{41}]^+, n_{23})$~~

~~$2R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{21} - n_{23}]^+ + \max([n_{43} - n_{41}]^+, n_{23})$~~

~~$R_{21} + 2R_{43} \leq \max(n_{43}, n_{23}) + [n_{43} - n_{41}]^+ + \max([n_{21} - n_{23}]^+, n_{41})$~~

~~(B) IC in ← direction~~

One open problem (least ambitious):

Existing outer bounds under FULL adaptation:

~~$R_{12} \leq n_{12}, \quad R_{34} \leq n_{34},$~~ $\boxed{R_{12} \leq \max(n_{12}, n_{43}), R_{34} \leq \max(n_{34}, n_{21})}$

$R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+$

$R_{12} + R_{34} \leq \max(n_{34}, n_{14}) + [n_{12} - n_{14}]^+$

~~$R_{12} + R_{34} \leq \max([n_{12} - n_{14}]^+, n_{32}) + \max([n_{34} - n_{32}]^+, n_{14})$~~

~~$2R_{12} + R_{34} \leq \max(n_{12}, n_{32}) + [n_{12} - n_{14}]^+ + \max([n_{34} - n_{32}]^+, n_{14})$~~

~~$R_{12} + 2R_{34} \leq \max(n_{34}, n_{14}) + [n_{34} - n_{32}]^+ + \max([n_{12} - n_{14}]^+, n_{32})$~~

~~(A) IC in → direction~~

~~$R_{21} \leq n_{21}, \quad R_{43} \leq n_{43}$~~ $\boxed{R_{21} \leq \max(n_{21}, n_{34}), R_{43} \leq \max(n_{43}, n_{12})}$

$R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{43} - n_{41}]^+$

$R_{21} + R_{43} \leq \max(n_{43}, n_{23}) + [n_{21} - n_{23}]^+$

~~$R_{21} + R_{43} \leq \max([n_{21} - n_{23}]^+, n_{41}) + \max([n_{43} - n_{41}]^+, n_{23})$~~

~~$2R_{21} + R_{43} \leq \max(n_{21}, n_{41}) + [n_{21} - n_{23}]^+ + \max([n_{43} - n_{41}]^+, n_{23})$~~

~~$R_{21} + 2R_{43} \leq \max(n_{43}, n_{23}) + [n_{43} - n_{41}]^+ + \max([n_{21} - n_{23}]^+, n_{41})$~~

~~(B) IC in ← direction~~

What is the general capacity region of linear deterministic two-way IC?

Key technical issues:

Fully adaptive version of ETW-type outer bounds not clear (tried!)

What type of adaptive scheme achieves capacity?

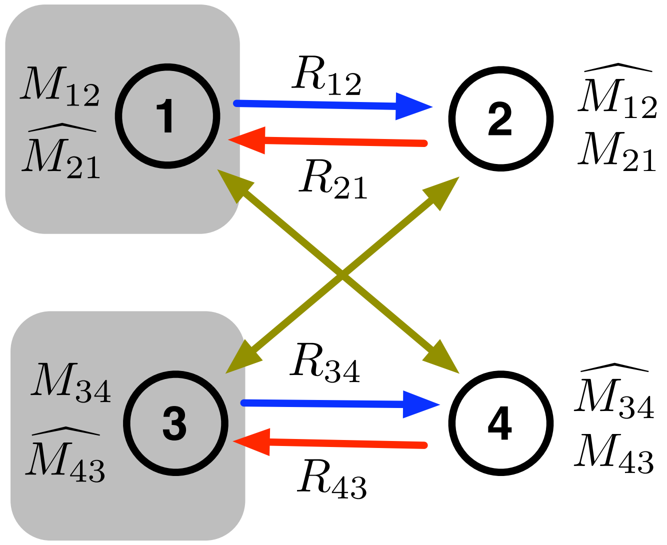
So many channel gains in asymmetric case: how to make sense?

Full adaptation example converse:

$$
\begin{aligned}
& n(R_{12} + R_{34} - \epsilon) \\
& \overset{(a)}{\leq} I(M_{12}; Y_2^n | M_{21}, M_{43}) + I(M_{34}; Y_4^n, Y_2^n | M_{12}, M_{21}, M_{43}) \\
& \leq I(M_{12}; Y_2^n | M_{21}, M_{43}) + I(M_{34}; Y_2^n | M_{21}, M_{12}, M_{43}) + H(Y_4^n | M_{21}, M_{12}, M_{43}, Y_2^n) \\
& \overset{(b)}{=} I(M_{12}; Y_2^n | M_{21}, M_{43}) + I(M_{34}; Y_2^n | M_{21}, M_{12}, M_{43}) \\
& + \sum_{i=1}^{n} [H(S^{N-n_{34}} X_{3,i} | M_{21}, M_{12}, M_{43}, Y_4^{i-1}, X_4^i, Y_2^n, X_2^n, X_1^i)] \\
& \leq \sum_{i-1}^{n} [H(Y_{2,i} | Y_2^{i-1}, M_{21}, X_2^i) - H(Y_{2,i} | Y_2^{i-1}, M_{12}, M_{21}, M_{43}) + H(Y_{2,i} | Y_2^{i-1}, M_{12}, M_{21}, M_{43}) \\
& + H(S^{N-n_{34}} X_{3,i} | M_{21}, M_{12}, M_{43}, Y_4^{i-1}, X_4^i, S^{N-n_{12}} X_1^n + S^{N-n_{22}} X_2^n + S^{N-n_{32}} X_3^n, X_2^n, X_1^i)] \\
& \leq \sum_{i=1}^{n} [H(S^{N-n_{12}} X_{1,i} + S^{N-n_{32}} X_{3,i}) + H(S^{N-n_{34}} X_{3,i} | S^{N-n_{32}} X_{3,i})] \\
& \leq n(\max(n_{12}, n_{32}) + [n_{34} - n_{32}]^+)
\end{aligned}
$$

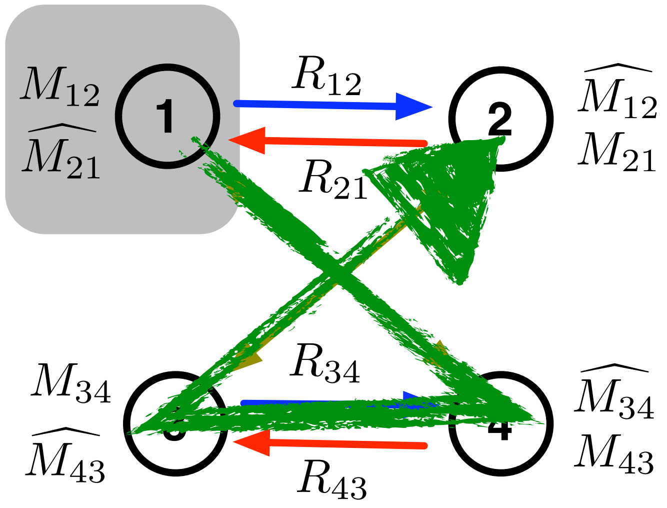
Given M_{12}, X_2^n, X_4^i, we can construct X_1^i.

Remark on partial adaptation:



$$X_{1,i} = f_1(M_{12}), \quad X_{2,i} = f_2(M_{21}, Y_2^{i-1})$$
$$X_{3,i} = f_3(M_{34}), \quad X_{4,i} = f_4(M_{43}, Y_4^{i-1})$$

Blocks routing at node 1,3

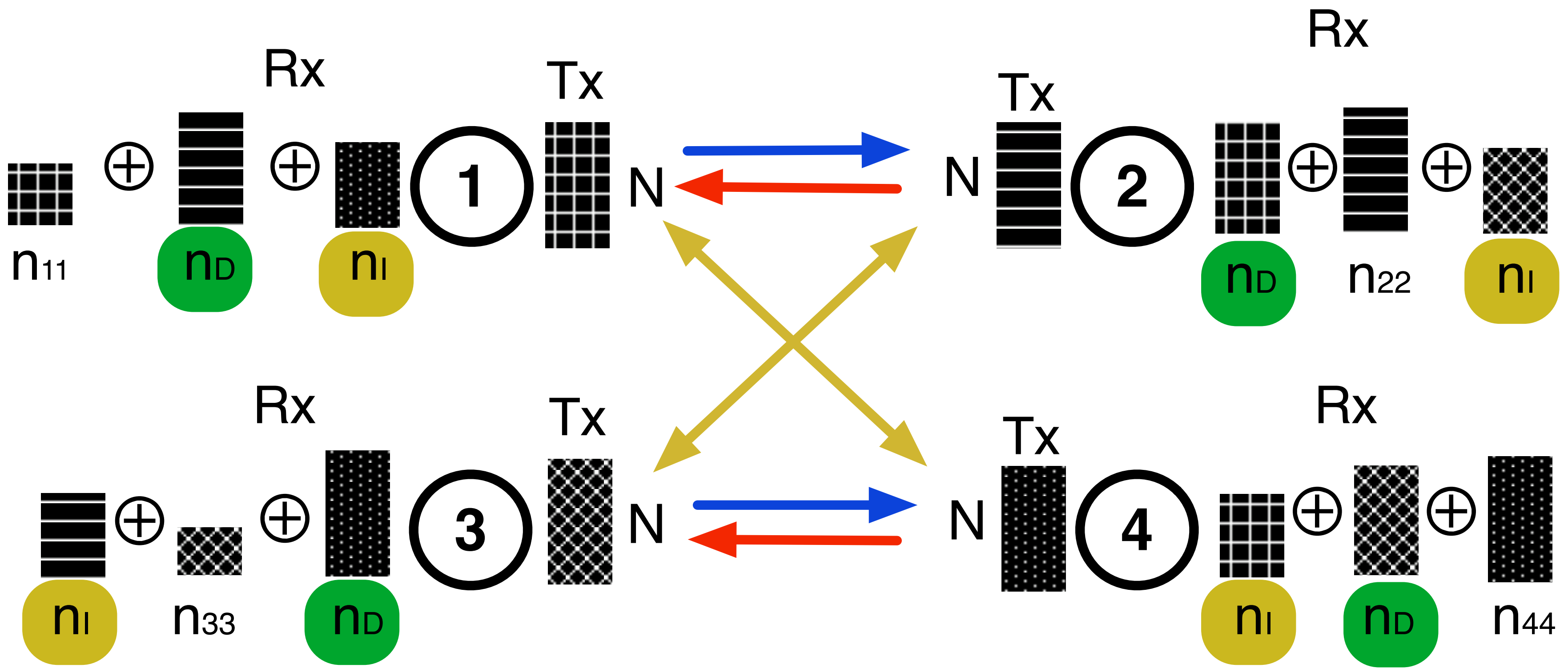


Adaptation useful in general!

Routing via interference!

However, it is sometimes useless!

Symmetric two-way linear deterministic IC

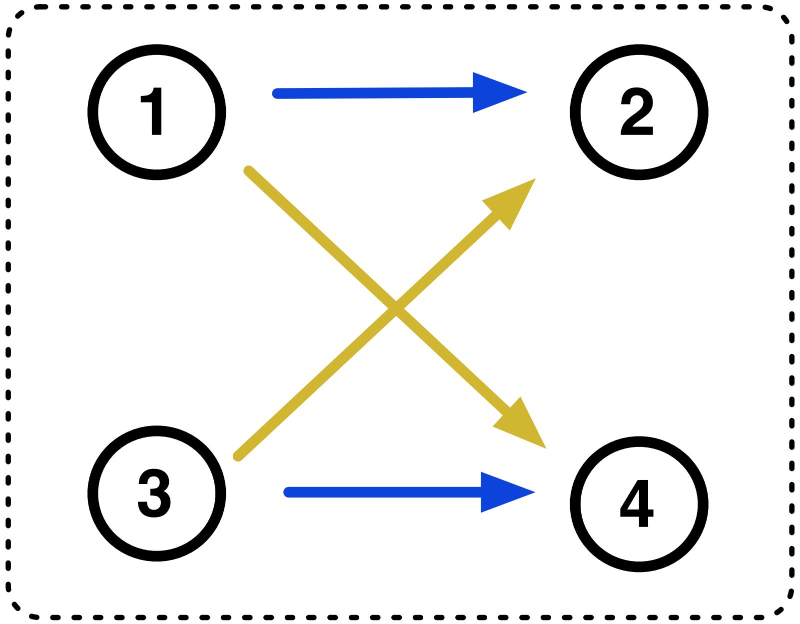


Notice symmetry!

Define: $C_{sym}(\alpha) := \frac{R_{12}+R_{34}}{2} = \frac{R_{21}+R_{43}}{2}, \ \alpha := \frac{n_I}{n_D}$

Let's compare C_{sym} for 4 models:

One-way IC

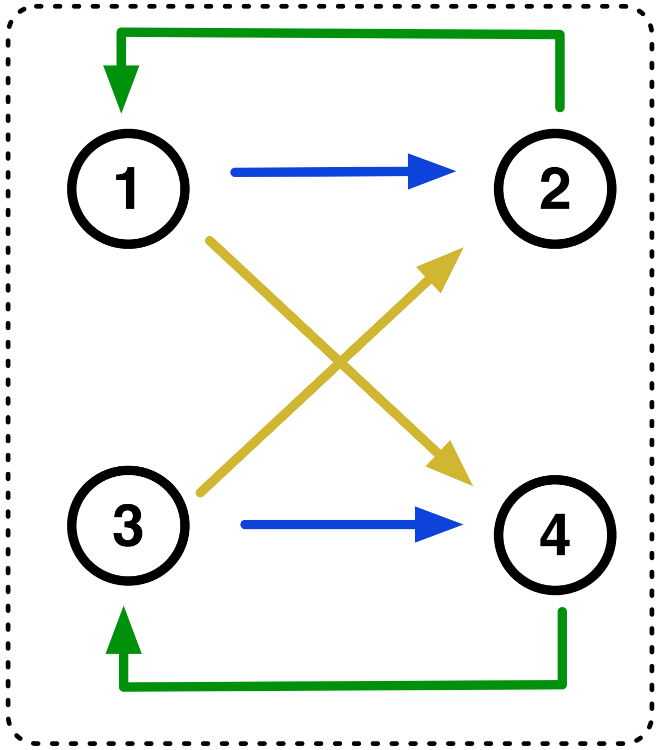


[Bresler, Tse 2008]
[Etkin, Tse, Wang 2008]

2 messages

One-way IC with FB

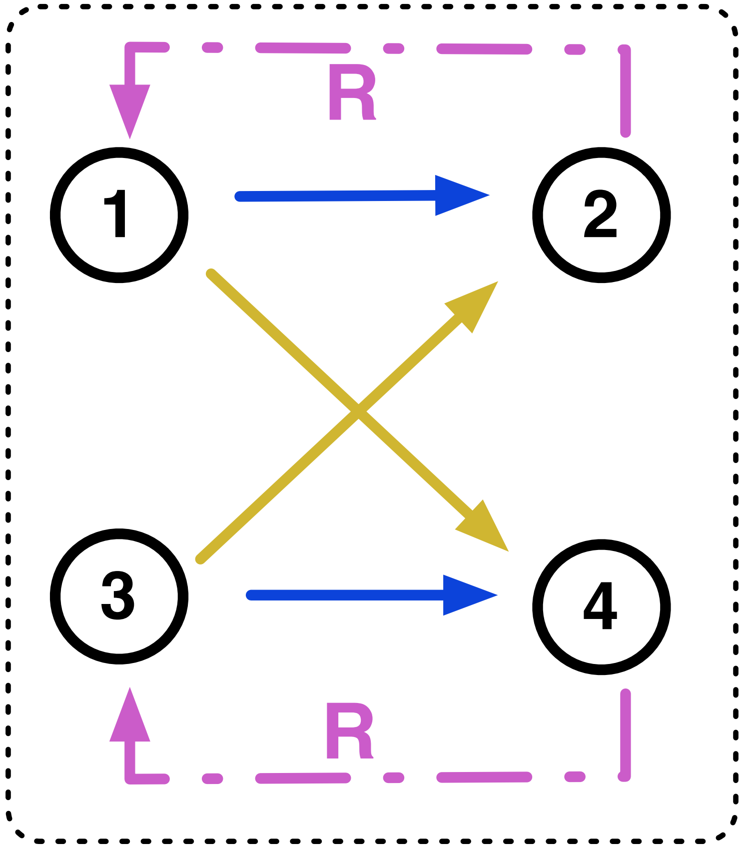
[Suh, Tse 2011]



2 messages

One-way IC with rate-limited FB

[Vahid, Suh,
Avestimehr 2012]



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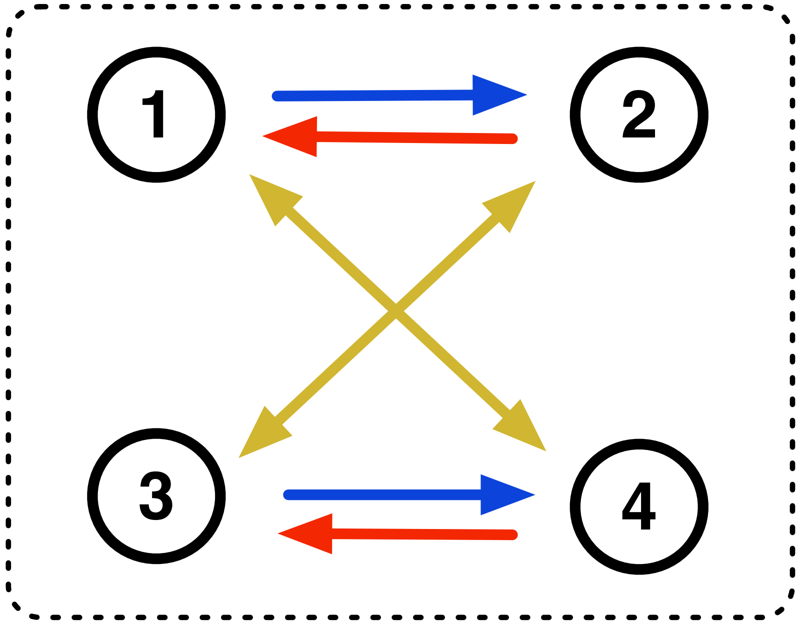
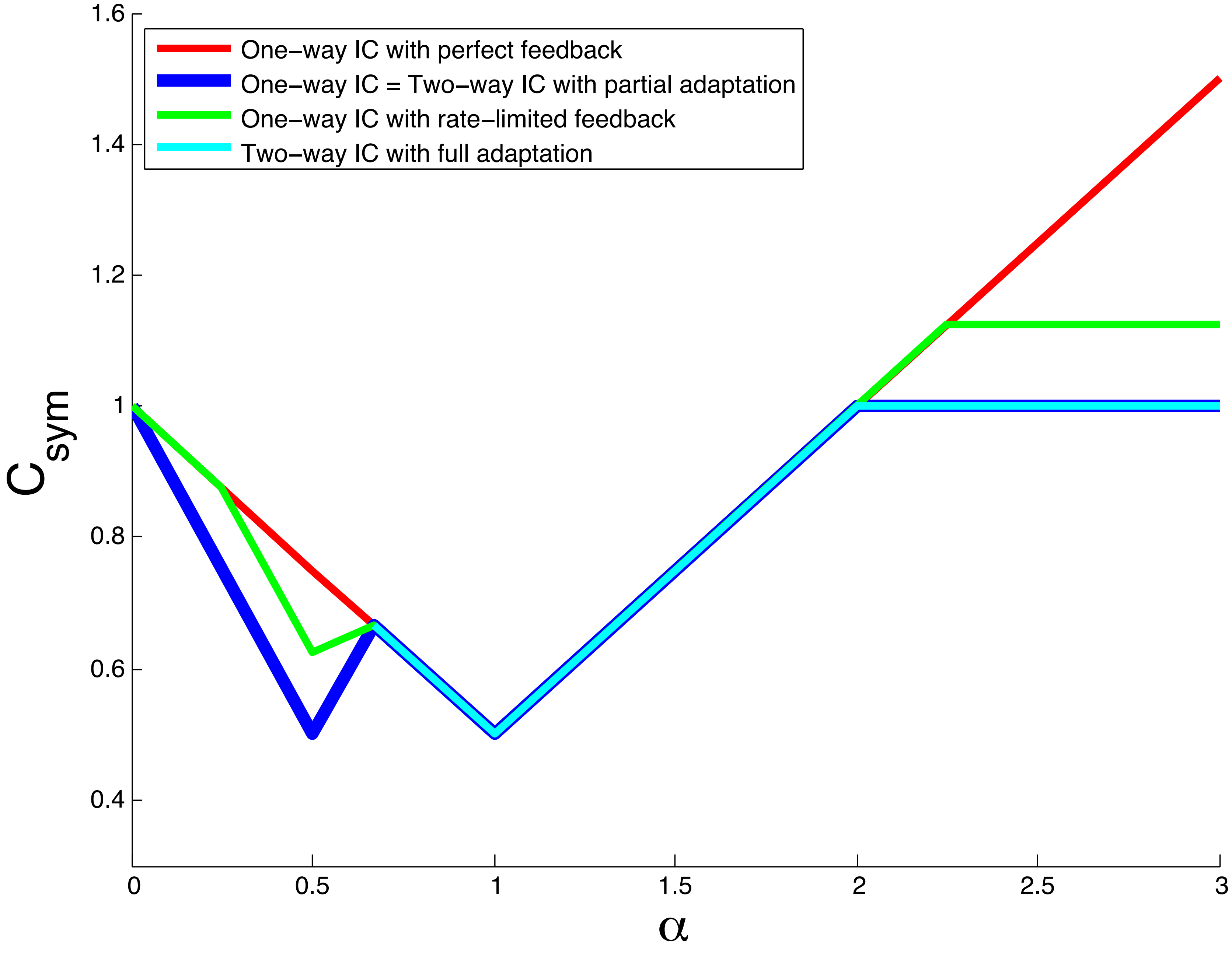
Two-way IC

[Cheng, Devroye 2012]



4 messages

Symmetric sum-rate capacity comparison:



So far all

Two-way capacity

≡

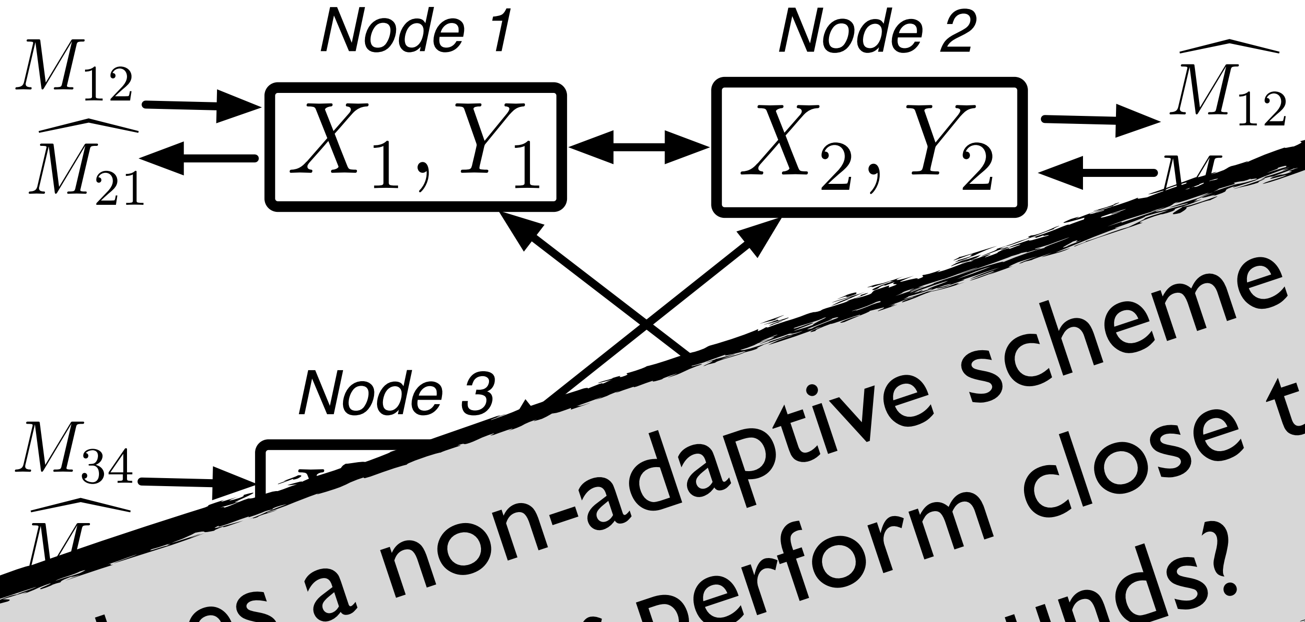
One-way →

One-way ←

for *deterministic* models

What about noisy channels?

Gaussian two-way IC



Question: does a non-adaptive scheme (Han +Kobayashi) ever perform close to the adaptive outer bounds?

$$+ g_{41}X_4 + Z_1$$

$$+ g_{22}X_2 + g_{32}X_3 + Z_2$$

$$= g_{23}X_2 + g_{33}X_3 + g_{43}X_4 + Z_3$$

$$Y_4 = g_{14}X_1 + g_{34}X_3 + g_{44}X_4 + Z_4,$$

where g_{jk} are the complex channel gains, $E[|X_j|^2] \leq 1$, i.i.d. $Z_j \sim \mathcal{CN}(0,1)$ and $\texttt{SNR}_{jk} = |g_{jk}|^2, \texttt{INR}_{jk} = |g_{jk}|^2$

Theorem: Outer bound: full adaptation. For the Gaussian two-way symmetric IC under full adaptation, any achievable symmetric rate $R_{sym} = \frac{R_{12}+R_{34}}{2} = \frac{R_{21}+R_{43}}{2}$, achievable by each user, satisfies,

$$R_{sym} \leq \frac{1}{2}\log\left(1 + \mathtt{SNR} + \mathtt{INR} + 2\sqrt{\mathtt{SNR} \times \mathtt{INR}}\right) + \frac{1}{2}\log\left(1 + \frac{\mathtt{SNR}}{1 + \mathtt{INR}}\right) \quad (1)$$

same as perfect output feedback sum-rate, though arrived at differently!

Open problem: ***more*** *computable Gaussian full-adaptation bounds?*

$$R_{sym} = \frac{R_{12}+R_{34}}{2} \leq \frac{1}{2}\log\left(1+\mathtt{SNR}+\mathtt{INR}+2\sqrt{\mathtt{SNR}\times\mathtt{INR}}\right)+\frac{1}{2}\log\left(1+\frac{\mathtt{SNR}}{1+\mathtt{INR}}\right)$$

$$\begin{aligned}
&n(R_{12}+R_{34}-\epsilon)\\
&\leq I(M_{12};Y_2^n|M_{21},M_{43},Z_1^n)+I(M_{34};Y_4^n,\boxed{Y_2^n}|M_{12},M_{21},M_{43},Z_1^n)\\
&\leq I(M_{12};Y_2^n|M_{21},M_{43},Z_1^n)+I(M_{34};Y_2^n|M_{21},M_{12},M_{43},Z_1^n)+H(Y_4^n|M_{21},M_{12},M_{43},Y_2^n,Z_1^n)-H(Z_4^n)\\
&\overset{(a)}{=} I(M_{12};Y_2^n|M_{21},M_{43},Z_1^n)+I(M_{34};Y_2^n|M_{21},M_{12},M_{43},Z_1^n)\\
&+\sum_{i=1}^{n}[H(g_{34}X_{3,i}+Z_{4,i}|M_{21},M_{12},M_{43},Y_4^{i-1},X_4^i,\boxed{Y_2^n},X_2^n,Z_1^n,\boxed{X_1^i})]-H(Z_4^n)\\
&\overset{(b)}{\leq}\sum_{i=1}^{n}[H(Y_{2,i}|Y_2^{i-1},M_{21},X_{2,i})-\cancel{H(Y_{2,i}|Y_2^{i-1},M_{12},M_{21},M_{43},Z_1^n)}\\
&\quad+\cancel{H(Y_{2,i}|Y_2^{i-1},M_{12},M_{21},M_{43},Z_1^n)}-H(Z_{2,i})+H(g_{34}X_{3,i}+Z_{4,i}|X_{4,i},\boxed{g_{32}X_{3,i}+Z_{2,i}},X_1^i,X_2^n)-H(Z_{4,i})]\\
&\overset{(c)}{\leq}\sum_{i=1}^{n}H(g_{12}X_{1,i}+g_{32}X_{3,i}+Z_{2,i}|X_{2,i})-H(Z_{2,i})+H(g_{34}X_{3,i}+Z_{4,i}|X_{4,i},g_{32}X_{3,i}+Z_{2,i})-H(Z_{4,i})
\end{aligned}$$

(a): X_1^i is constructed from $(M_{12},X_2^n,X_4^i,Z_1^n)$.

Theorem: Outer bound: partial adaptation. For the Gaussian two-way IC under partial adaptation, in addition to the bounds in full-adaptation Theorem, we may also conclude that any achievable rates $(R_{12}, R_{21}, R_{34}, R_{43})$, and $R_{sym\rightarrow} = \frac{R_{12}+R_{34}}{2}$ and $R_{sym\leftarrow} = \frac{R_{21}+R_{43}}{2}$ must satisfy,

$$R_{12} \leq \log(1+\mathtt{SNR}_{12}) \tag{1}$$

$$R_{21} \leq \log(1+\mathtt{SNR}_{21}) \tag{2}$$

$$R_{34} \leq \log(1+\mathtt{SNR}_{34}) \tag{3}$$

$$R_{43} \leq \log(1+\mathtt{SNR}_{43}) \tag{4}$$

same as Etkin, Tse, Wang

$$R_{sym\rightarrow} \leq \log\left(1+\mathtt{INR}+\mathtt{SNR}-\frac{\mathtt{INR}\times\mathtt{SNR}}{1+\mathtt{INR}}\right) \tag{5}$$

$$R_{sym\leftarrow} \leq \begin{cases} \log\left(1+\mathtt{INR}+\frac{\mathtt{SNR}}{\mathtt{INR}}\right), & \text{if } \mathtt{SNR} \leq \mathtt{INR}^3 \\ \log\left(1+\frac{(\sqrt{\mathtt{SNR}}+\sqrt{\mathtt{INR}})^2}{1+\mathtt{INR}}\right), & \text{if } \mathtt{SNR} > \mathtt{INR}^3 \end{cases} \tag{6}$$

If partial adaptation:

$$X_{1,i} = f_1(M_{12}), \quad X_{2,i} = f_2(M_{21}, Y_2^{i-1})$$

$$X_{3,i} = f_3(M_{34}), \quad X_{4,i} = f_4(M_{43}, Y_4^{i-1})$$

Partial adaptation key lemma:

$$X_{1,i} = f_1(M_{12}), \quad X_{2,i} = f_2(M_{21}, Y_2^{i-1})$$

$$X_{3,i} = f_3(M_{34}), \quad X_{4,i} = f_4(M_{43}, Y_4^{i-1})$$

Partial adaptation conditions

Lemma: partial adaptation Gaussian two-way IC. Under partial adaptation, for some deterministic functions f_5 and f_6,

$$X_{2,i} = f_5(M_{12}, M_{21}, M_{34}, Z_2^{i-1}) \perp M_{43}, \quad \forall i$$

$$X_{4,i} = f_6(M_{43}, M_{34}, M_{12}, Z_4^{i-1}) \perp M_{21}, \quad \forall i$$

where $\perp$ denotes independence.

Central to many of the converses!

$$R_{sym\rightarrow} \leq \log\left(1 + \mathtt{INR} + \mathtt{SNR} - \frac{\mathtt{INR} \times \mathtt{SNR}}{1 + \mathtt{INR}}\right)$$

Proof *(details available in arxiv)*:

$$\begin{aligned}
&n(R_{12} + R_{34} - \epsilon) \leq I(M_{12}; Y_2^n, g_{14}X_1^n + Z_4^n, M_{21}, M_{43}) + I(M_{34}; Y_4^n, g_{32}X_3^n + Z_2^n, M_{21}, M_{43}) \\
&\overset{(a)}{=} H(Y_2^n | g_{14}X_1^n + Z_4^n, M_{43}, M_{21}) + H(g_{14}X_1^n + Z_4^n | M_{43}, M_{21}) - H(Y_2^n, g_{14}X_1^n + Z_4^n | M_{12}, M_{21}, M_{43}) \\
&+ H(Y_4^n | g_{32}X_3^n + Z_2^n, M_{43}, M_{21}) + H(g_{32}X_3^n + Z_2^n | M_{43}, M_{21}) - H(Y_4^n, g_{32}X_3^n + Z_2^n | M_{34}, M_{21}, M_{43}) \\
&\overset{(b)}{=} H(Y_2^n | g_{14}X_1^n + Z_4^n, M_{43}, M_{21}) + \sum_{i=1}^{n} [H(g_{14}X_{1,i} + Z_{4,i} | g_{14}X_1^{i-1} + Z_4^{i-1}, M_{43}, M_{21}, M_{34}) \\
&- H(Y_{2,i}, g_{14}X_{1,i} + Z_{4,i} | Y_2^{i-1}, g_{14}X_1^{i-1} + Z_4^{i-1}, M_{12}, M_{21}, M_{43}, X_2^i, X_1^i)] \\
&+ H(Y_4^n | g_{32}X_3^n + Z_2^n, M_{43}, M_{21}) + \sum_{i=1}^{n} [H(g_{32}X_{3,i} + Z_{2,i} | g_{32}X_3^{i-1} + Z_2^{i-1}, M_{43}, M_{21}, M_{12}) \\
&- H(Y_{4,i}, g_{32}X_{3,i} + Z_{2,i} | Y_4^{i-1}, g_{32}X_3^{i-1} + Z_2^{i-1}, M_{34}, M_{21}, M_{43}, X_4^i, X_3^i)] \\
&\overset{(c)}{=} H(Y_2^n | g_{14}X_1^n + Z_4^n, M_{43}, M_{21}) + \sum_{i=1}^{n} [H(g_{14}X_{1,i} + Z_{4,i} | g_{14}X_1^{i-1} + Z_4^{i-1}, M_{43}, M_{21}, M_{34}, X_3^i, Y_4^{i-1}, X_4^i, \\
&\quad g_{32}X_3^{i-1} + Z_2^{i-1}) - H(g_{32}X_{3,i} + Z_{2,i}, Z_{4,i} | Y_2^{i-1}, g_{14}X_1^{i-1} + Z_4^{i-1}, M_{12}, M_{21}, M_{43}, X_2^i, X_1^i, g_{32}X_3^{i-1} + Z_2^{i-1})] \\
&+ H(Y_4^n | g_{32}X_3^n + Z_2^n, M_{43}, M_{21}) + \sum_{i=1}^{n} [H(g_{32}X_{3,i} + Z_{2,i} | g_{32}X_3^{i-1} + Z_2^{i-1}, M_{43}, M_{21}, M_{12}, X_1^i, Y_2^{i-1}, X_2^i, \\
&\quad g_{14}X_1^{i-1} + Z_4^{i-1}) - H(g_{14}X_{1,i} + Z_{4,i}, Z_{2,i} | Y_4^{i-1}, g_{32}X_3^{i-1} + Z_2^{i-1}, M_{34}, M_{21}, M_{43}, X_4^i, X_3^i, g_{14}X_1^{i-1} + Z_4^{i-1})] \\
&\overset{(d)}{=} \sum_{i=1}^{n} [H(Y_{2,i} | Y_2^{i-1}, g_{14}X_1^n + Z_4^n, M_{43}, M_{21}) - H(Z_{2,i}) + H(Y_{4,i} | Y_4^{i-1}, g_{32}X_3^n + Z_2^n, M_{43}, M_{21}) - H(Z_{4,i})] \\
&\leq \sum_{i=1}^{n} [H(g_{12}X_{1,i} + g_{32}X_{3,i} + Z_{2,i} | g_{14}X_{1,i} + Z_{4,i}, X_{2,i}) - H(Z_{2,i}) \\
&\quad + H(g_{34}X_{3,i} + g_{14}X_{1,i} + Z_{4,i} | g_{32}X_{3,i} + Z_{2,i}, X_{4,i}) - H(Z_{4,i})]
\end{aligned} \tag{1}$$

1) Inspired by the outer bounds for the linear deterministic model.

2) Start with Fano's inequality and provide side information.

3) Use the definition of partial adaptation.

4) For evaluation, algebra with correlation coefficients.

$$R_{sym\leftarrow} \leq \begin{cases} \log\left(1+\texttt{INR}+\frac{\texttt{SNR}}{\texttt{INR}}\right), & \text{if } \texttt{SNR} \leq \texttt{INR}^3 \\ \log\left(1+\frac{(\sqrt{\texttt{SNR}}+\sqrt{\texttt{INR}})^2}{1+\texttt{INR}}\right), & \text{if } \texttt{SNR} > \texttt{INR}^3 \end{cases}$$

$$\begin{aligned}
&n(R_{21}+R_{43}-\epsilon) \leq I(M_{21};Y_1^n, g_{23}X_2^n+Z_3^n, M_{12}, M_{34}) + I(M_{43};Y_3^n, g_{41}X_4^n+Z_1^n, M_{12}, M_{34}) \\
&= H(Y_1^n|g_{23}X_2^n+Z_3^n, M_{34}, M_{12}) + H(g_{23}X_2^n+Z_3^n|M_{34}, M_{12}) - H(Y_1^n, g_{23}X_2^n+Z_3^n|M_{21}, M_{12}, M_{34}) \\
&+ H(Y_3^n|g_{41}X_4^n+Z_1^n, M_{34}, M_{12}) + H(g_{41}X_4^n+Z_1^n|M_{34}, M_{12}) - H(Y_3^n, g_{41}X_4^n+Z_1^n|M_{43}, M_{12}, M_{34}) \\
&\overset{(a)}{=} H(Y_1^n|g_{23}X_2^n+Z_3^n, M_{34}, M_{12}) + \sum_{i=1}^{n}[H(g_{23}X_{2,i}+Z_{3,i}|g_{23}X_2^{i-1}+Z_3^{i-1}, M_{34}, M_{12}, M_{43}) \\
&- H(Y_{1,i}, g_{23}X_{2,i}+Z_{3,i}|Y_1^{i-1}, g_{23}X_2^{i-1}+Z_3^{i-1}, M_{21}, M_{12}, M_{34}, X_1^i, Z_2^{i-1}, X_2^i)] \\
&+ H(Y_3^n|g_{41}X_4^n+Z_1^n, M_{34}, M_{12}) + \sum_{i=1}^{n}[H(g_{41}X_{4,i}+Z_{1,i}|g_{41}X_4^{i-1}+Z_1^{i-1}, M_{34}, M_{12}, M_{21}) \\
&- H(Y_{3,i}, g_{41}X_{4,i}+Z_{1,i}|Y_3^{i-1}, g_{41}X_4^{i-1}+Z_1^{i-1}, M_{43}, M_{12}, M_{34}, X_3^i, Z_4^{i-1}, X_4^i)] \\
&= H(Y_1^n|g_{23}X_2^n+Z_3^n, M_{34}, M_{12}) + \sum_{i=1}^{n}[H(g_{23}X_{2,i}+Z_{3,i}|g_{23}X_2^{i-1}+Z_3^{i-1}, M_{34}, M_{12}, M_{43}, Z_4^{i-1}, g_{41}X_4^{i-1}+Z_1^{i-} \\
&\quad X_3^i, X_4^i, Y_3^{i-1}) - H(g_{41}X_{4,i}+Z_{1,i}, Z_{3,i}|Y_1^{i-1}, g_{23}X_2^{i-1}+Z_3^{i-1}, M_{21}, M_{12}, M_{34}, Z_2^{i-1}, X_1^i, X_2^i, g_{41}X_4^{i-1}+Z_1^{i-1})] \\
&+ H(Y_3^n|g_{41}X_4^n+Z_1^n, M_{34}, M_{12}) + \sum_{i=1}^{n}[H(g_{41}X_{4,i}+Z_{1,i}|g_{41}X_4^{i-1}+Z_1^{i-1}, M_{34}, M_{12}, M_{21}, Z_2^{i-1}, g_{23}X_2^{i-1}+Z_3^{i-} \\
&\quad X_1^i, X_2^i, Y_1^{i-1}) - H(g_{23}X_{2,i}+Z_{3,i}, Z_{1,i}|Y_3^{i-1}, g_{41}X_4^{i-1}+Z_1^{i-1}, M_{43}, M_{12}, M_{34}, Z_4^{i-1}, X_3^i, X_4^i, g_{23}X_2^{i-1}+Z_3^{i-1})] \\
&= \sum_{i=1}^{n}[H(Y_{1,i}|Y_1^{i-1}, g_{23}X_2^n+Z_3^n, M_{34}, M_{12}) - H(Z_{1,i}) + H(Y_{3,i}|Y_3^{i-1}, g_{41}X_4^n+Z_1^n, M_{34}, M_{12}) - H(Z_{3,i})] \\
&\leq \sum_{i=1}^{n}[H(g_{21}X_{2,i}+g_{41}X_{4,i}+Z_{1,i}|g_{23}X_{2,i}+Z_{3,i}, X_{1,i}) - H(Z_{1,i}) \\
&\quad + H(g_{43}X_{4,i}+g_{23}X_{2,i}+Z_{3,i}|g_{41}X_{4,i}+Z_{1,i}, X_{3,i}) - H(Z_{3,i})] \qquad (1)
\end{aligned}$$

Proof *(details available in arxiv)*:

Differences compared to the previous outer bound:

1) Use the lemma.

2) Evaluation: The correlation coefficient between X_2 and X_4 is not 0.

$$|\lambda_{24}| = \min(1, \frac{\sqrt{\texttt{SNR}\times\texttt{INR}}}{\texttt{INR}^2})$$

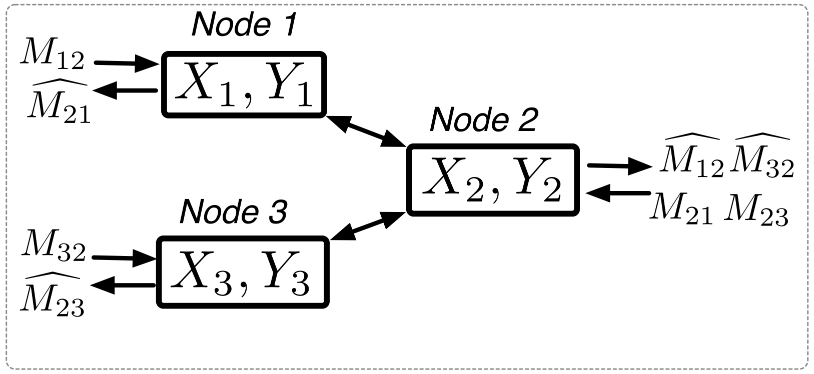
Symmetric H+K scheme of Etkin, Tse, Wang achieves the following gaps:

Two-way Interference	**Constant Gaps per user per direction, in bits (to outer bound)**				
Very Strong					0 (partial)
Strong					1 (full)
Weak	$\mathtt{INR} < 1$				1 (full)
	$\mathtt{INR} \geq 1$	HK1 is active			1.5 (full)
		HK2 is active	$\rightarrow$ direction		1 (partial)
			$\leftarrow$ direction	$\mathtt{SNR} \leq \mathtt{INR}^3$	1 (partial)
				$\mathtt{SNR} > \mathtt{INR}^3$	2 (partial)

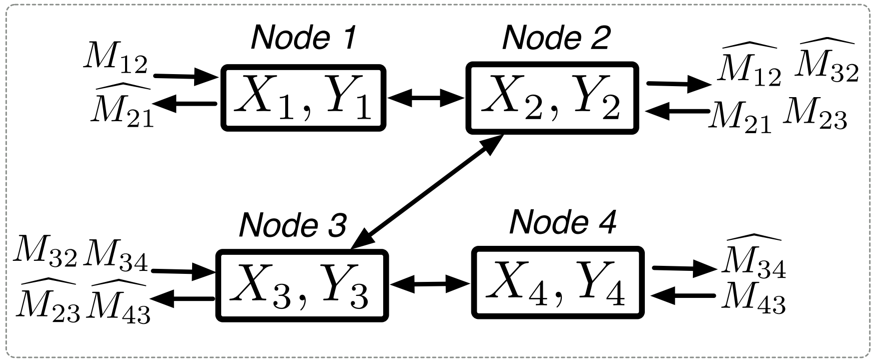
TABLE I

CONSTANT GAPS BETWEEN NON-ADAPTIVE SYMMETRIC HAN AND KOBAYASHI SCHEMES IN EACH DIRECTION AND PARTIALLY OR FULLY ADAPTIVE OUTER BOUNDS FOR THE TWO-WAY GAUSSIAN IC.

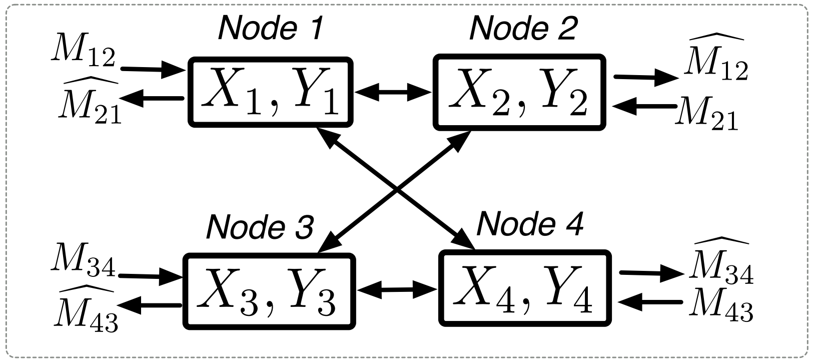
Conclusion



(a) Two-way MAC/BC



(b) Two-way Z channel



(c) Two-way interference channel

Adaptation appears to be useless when:

(a) Can cancel “self-interference”

(b) No coherent gains to be had

(c) No “routing” (interference is good!) possible

In general, when in adaptation useless in two-way networks?

devroye@uic.edu