Information theoretic limits of two-way (relay) networks

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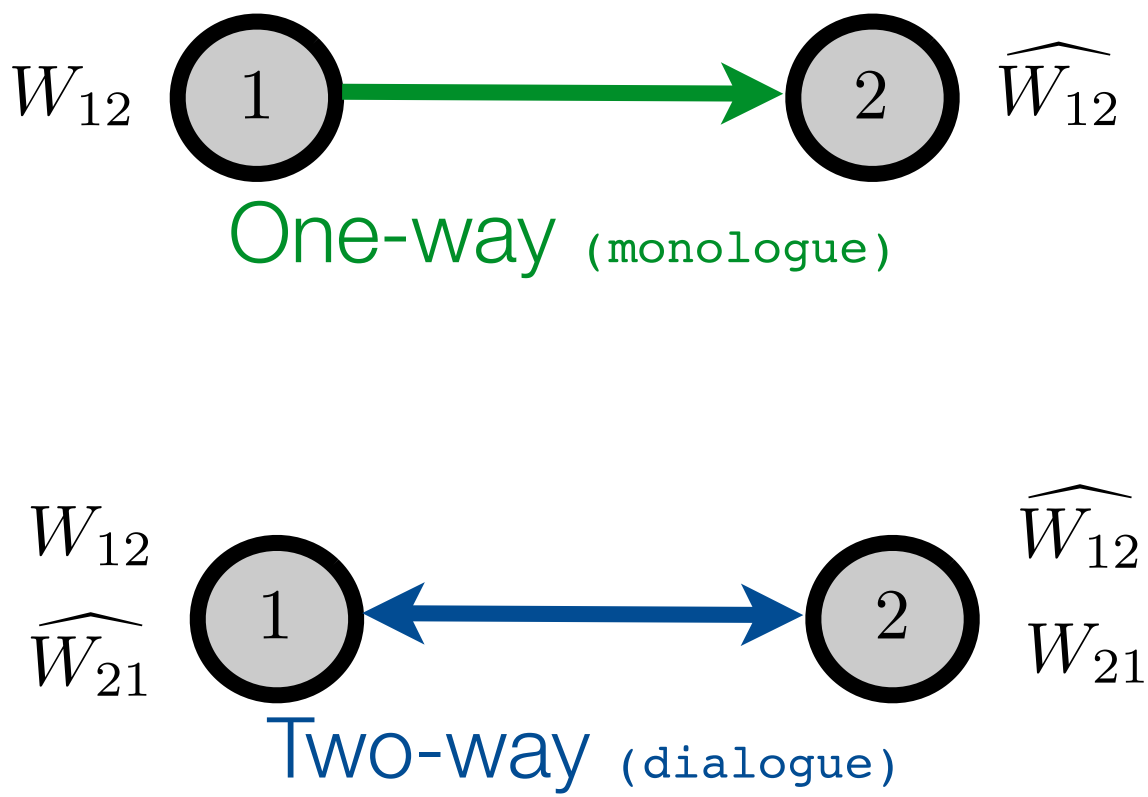
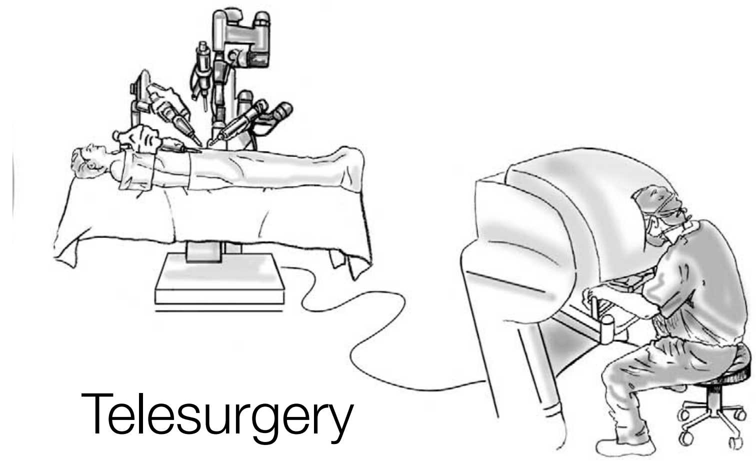
One-way (monologue) vs. Two-way (dialogue)

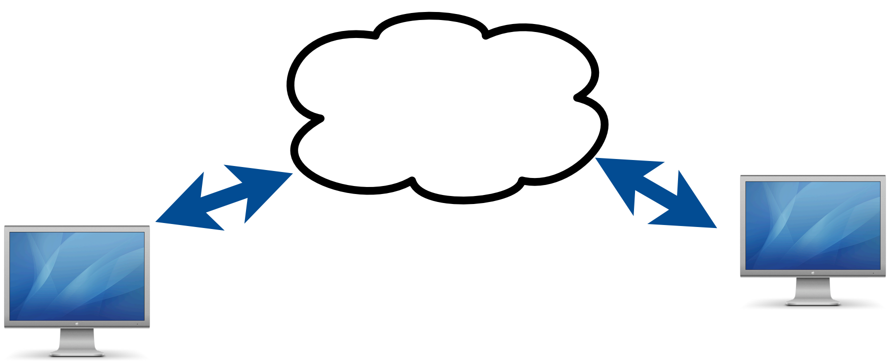


Two-way communication applications - wired

Video conferencing

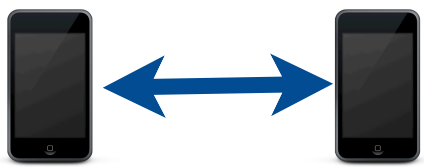


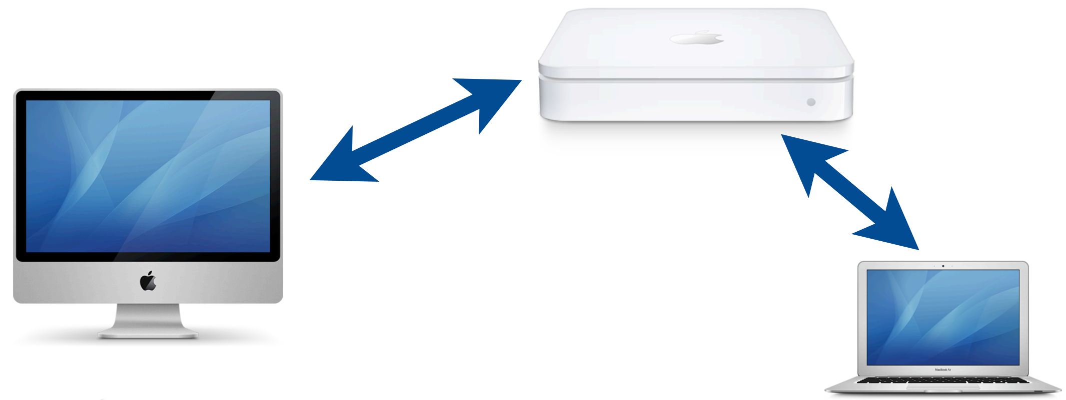
Telesurgery

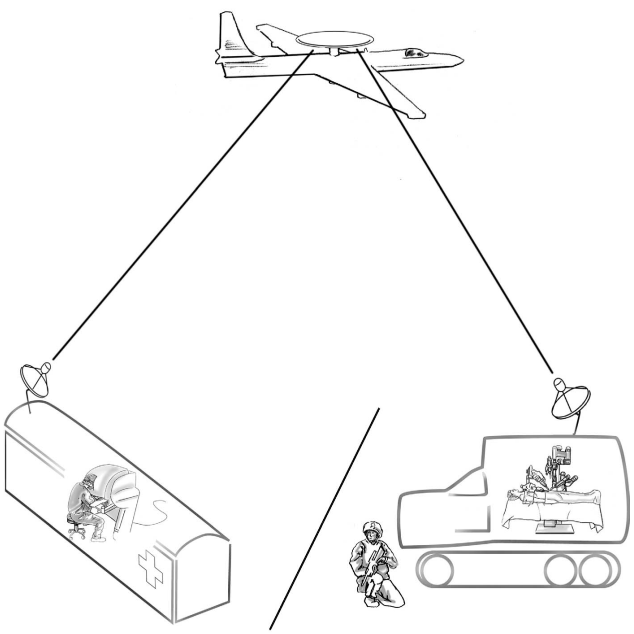


Data synchronization

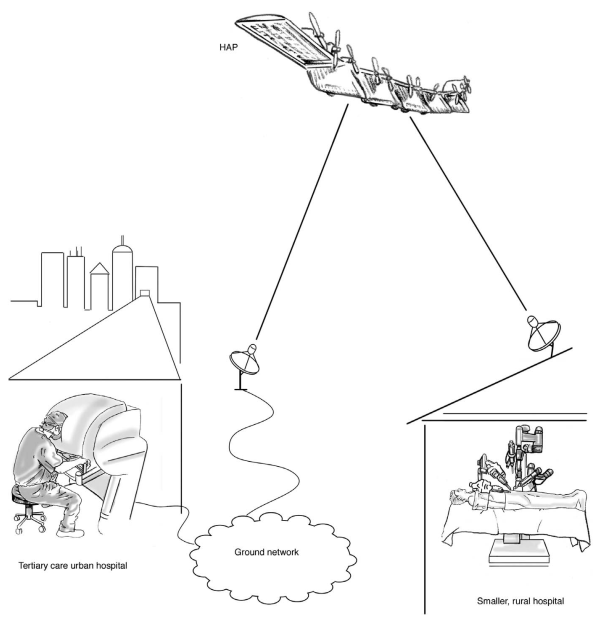
Two-way communication applications - wireless







Battlefield telesurgery



Rural telesurgery

Video conferencing

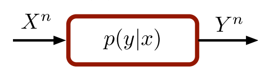
1 Dialogue **≠ or =** 2 Monologues ?

It depends....

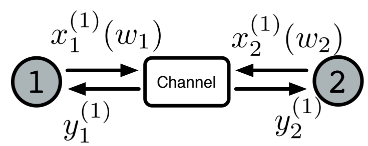
we will use information theory to find out.

Outline

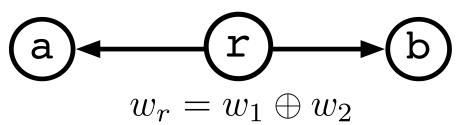
$$C = \max_{p(x)} I(X;Y)$$



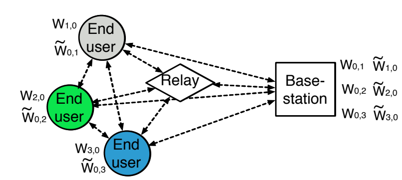
- **Information theory - *what, why, when***



- Two-way channel - *channel coding*



- Two-way relay channels
  - *single flow - canonical example of wireless network coding*
  - *multiple flows with a base-station - pairwise wireless network coding*



Information theory’s claims to fame

Source coding

- Source = random variable
- Ultimate **data compression** limit is the source’s entropy H

zip
mp3
jpg

Channel coding

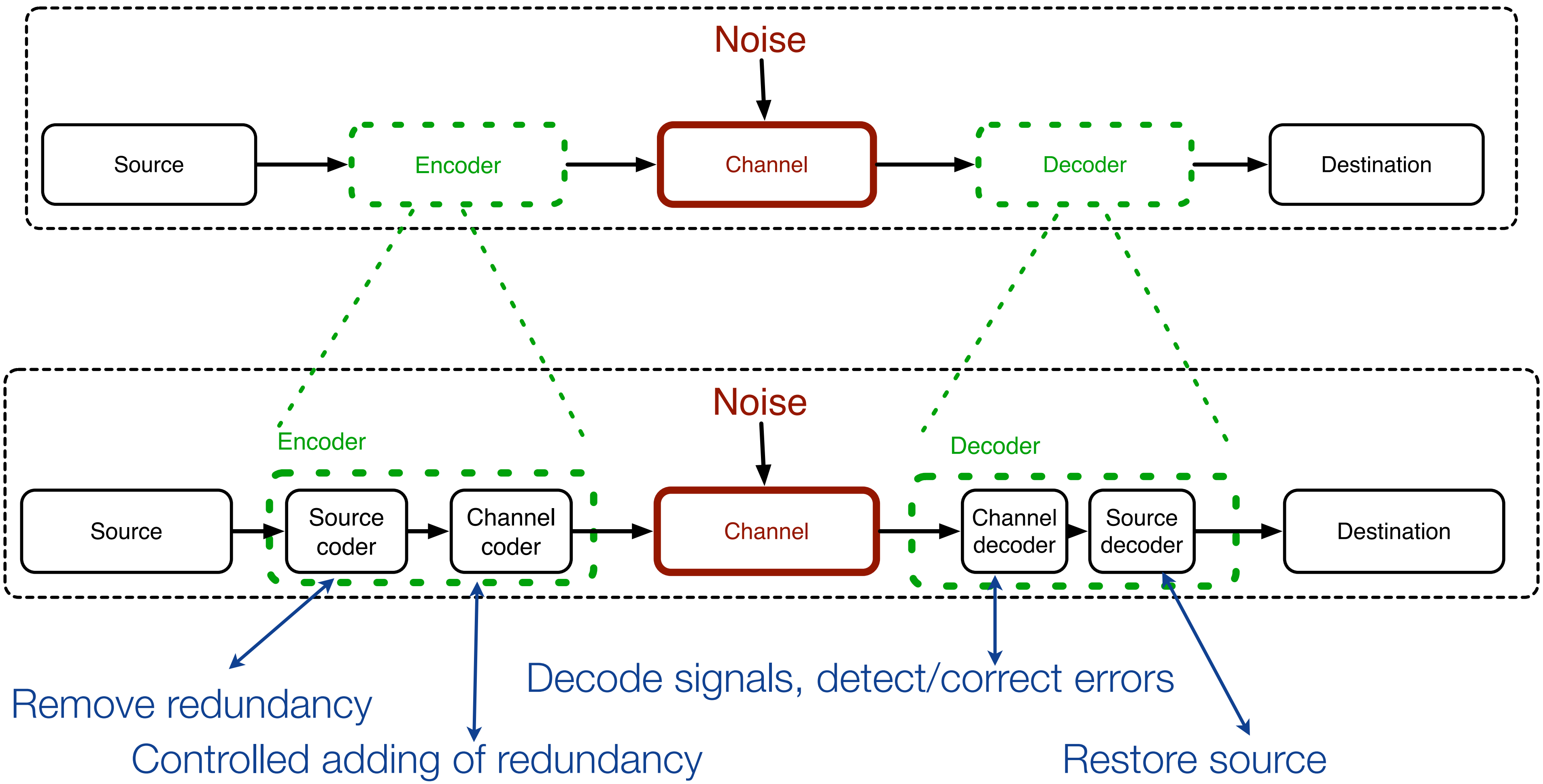
Turbo-codes

- Channel = conditional distributions
- Ultimate **transmission rate** is the channel capacity C

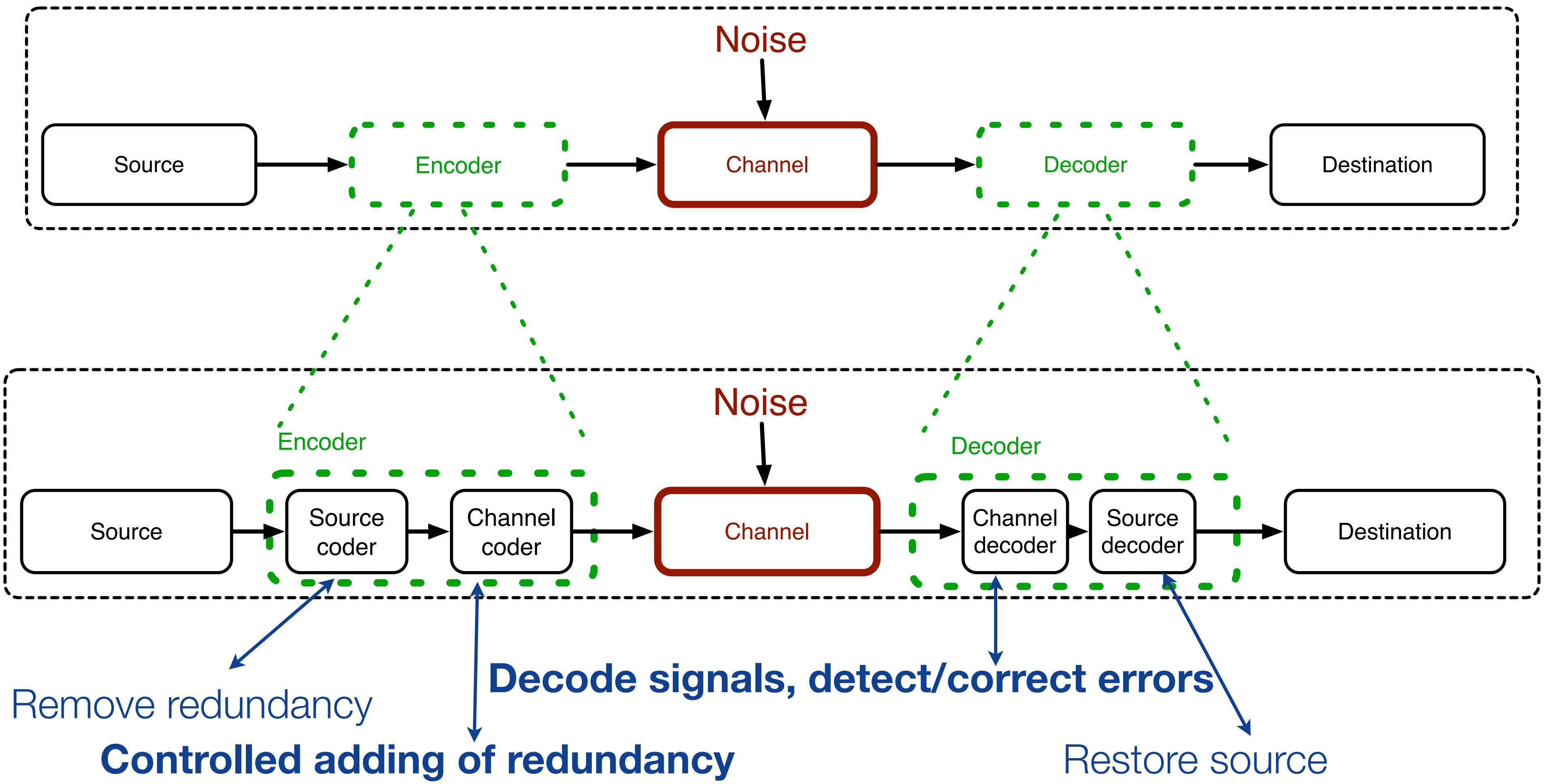
FADING CHANNEL

Reliable communication possible ↔ $H<C$

Source vs. channel coding

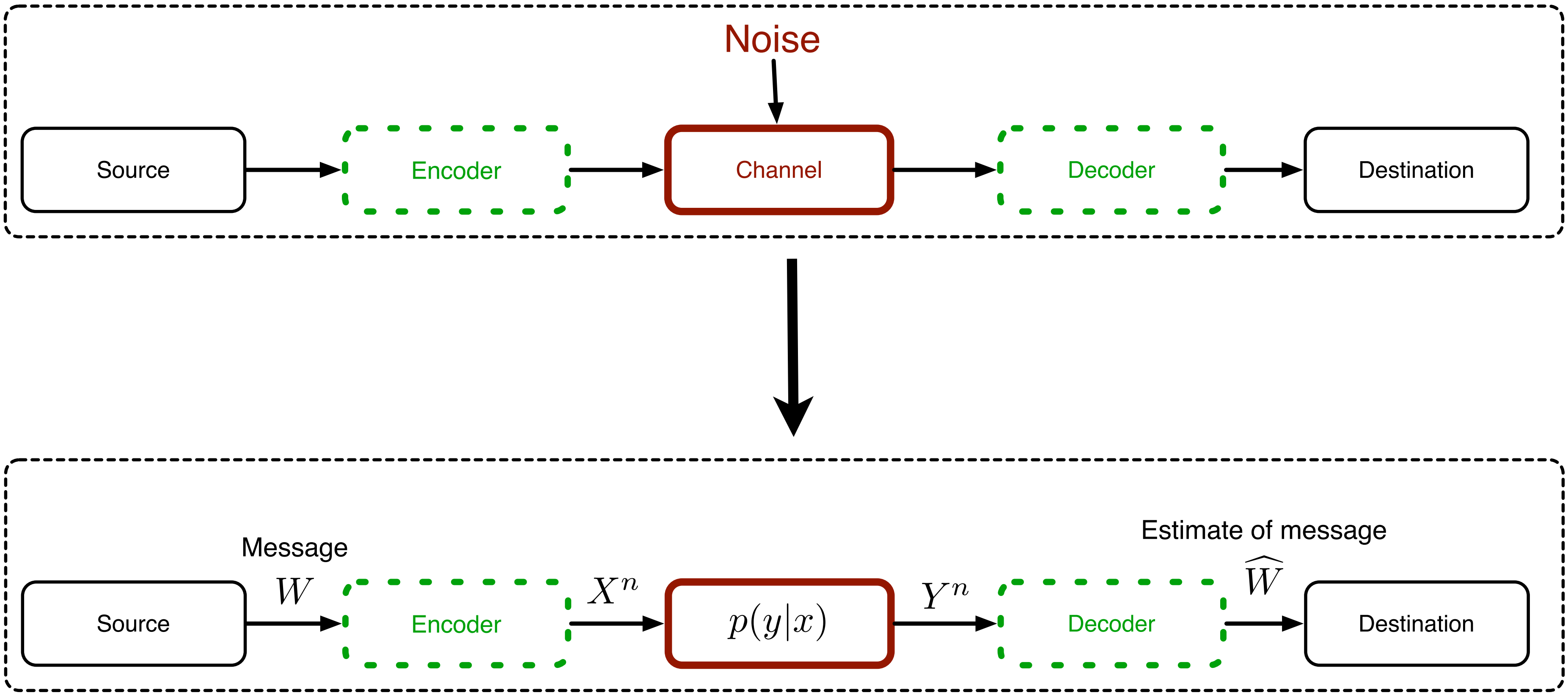


Source vs. channel coding



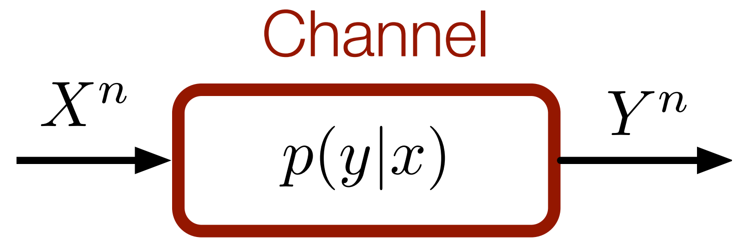
Limits, not constructions

Communication system model



What is the capacity of this channel?

Channel capacity



- Information channel capacity:

$$C = \max_{p(x)} I(X;Y)$$

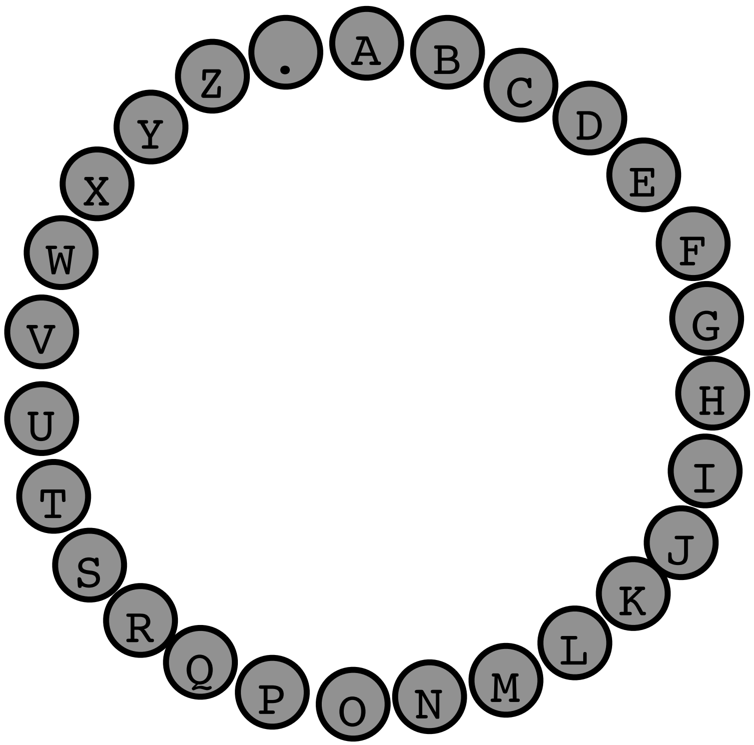
- Operational channel capacity:

```
Highest rate (bits/channel use) that can
communicate at reliably
```

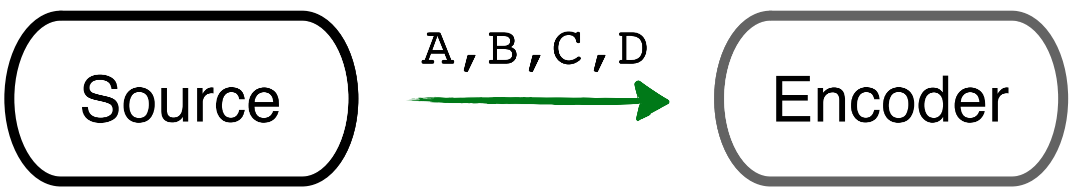
- Channel coding theorem says: information capacity = operational capacity

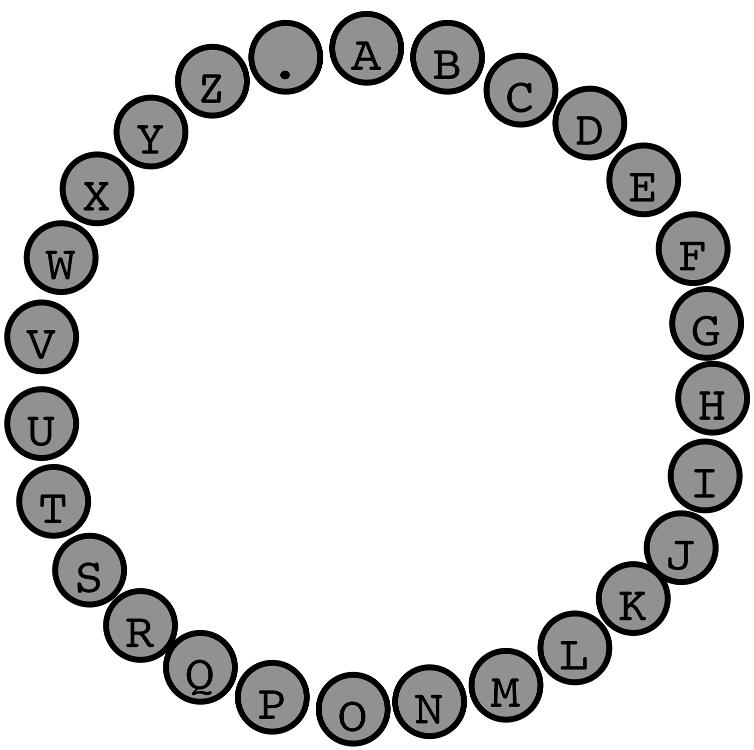
Channel capacity: a cute example

Source



Channel capacity: a cute example

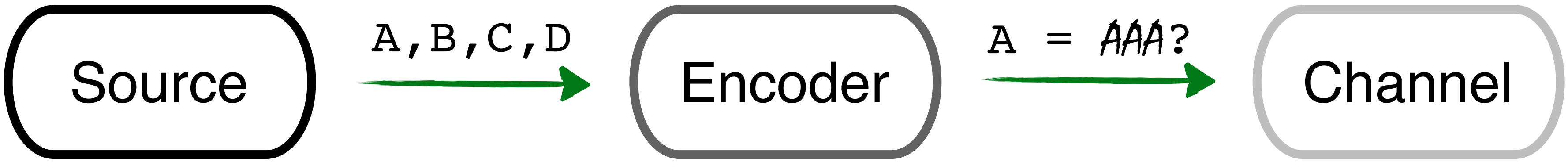




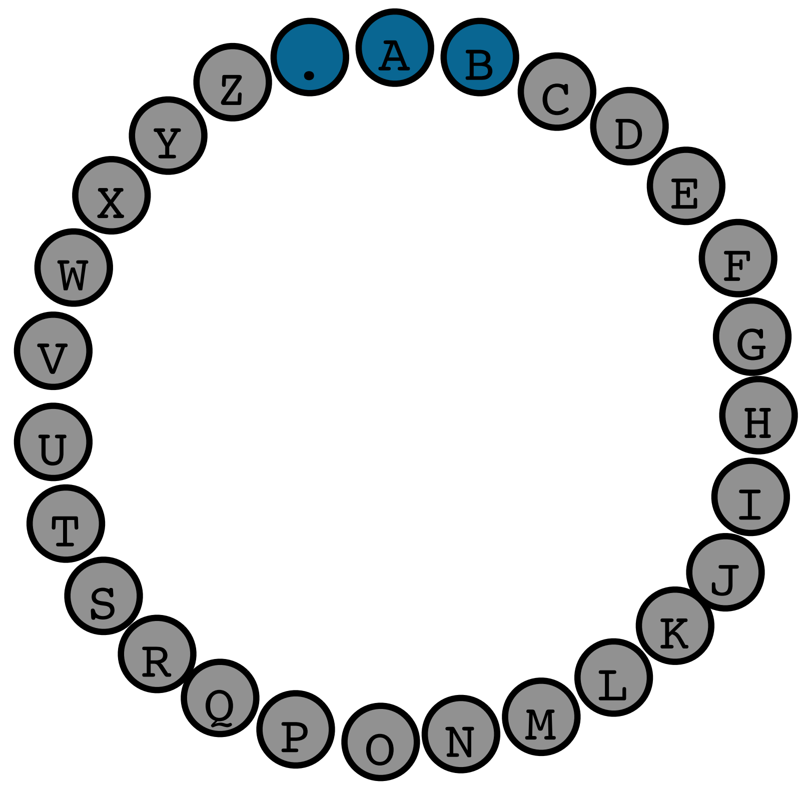
A = A?

A = AAA?

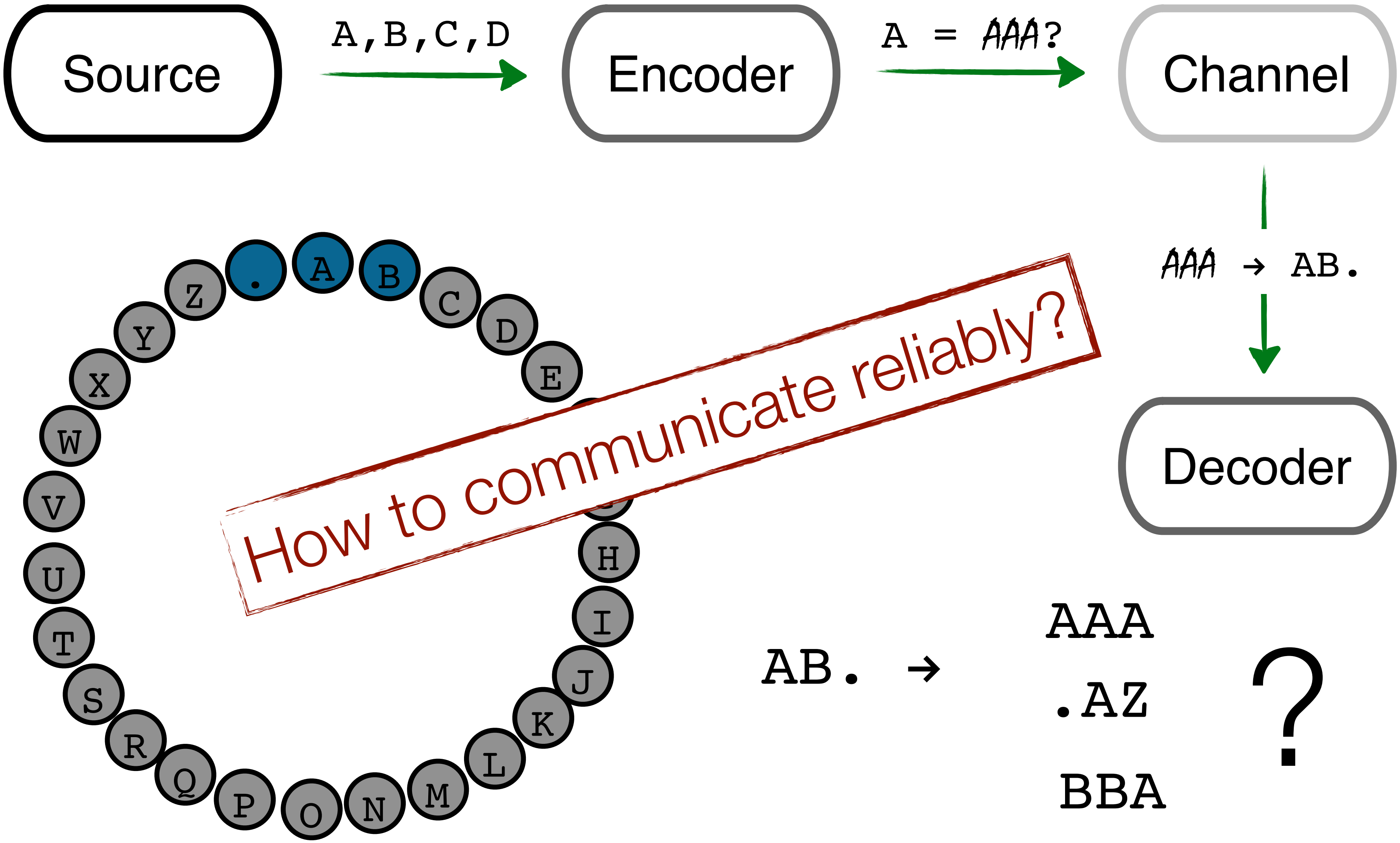
Channel capacity: a cute example



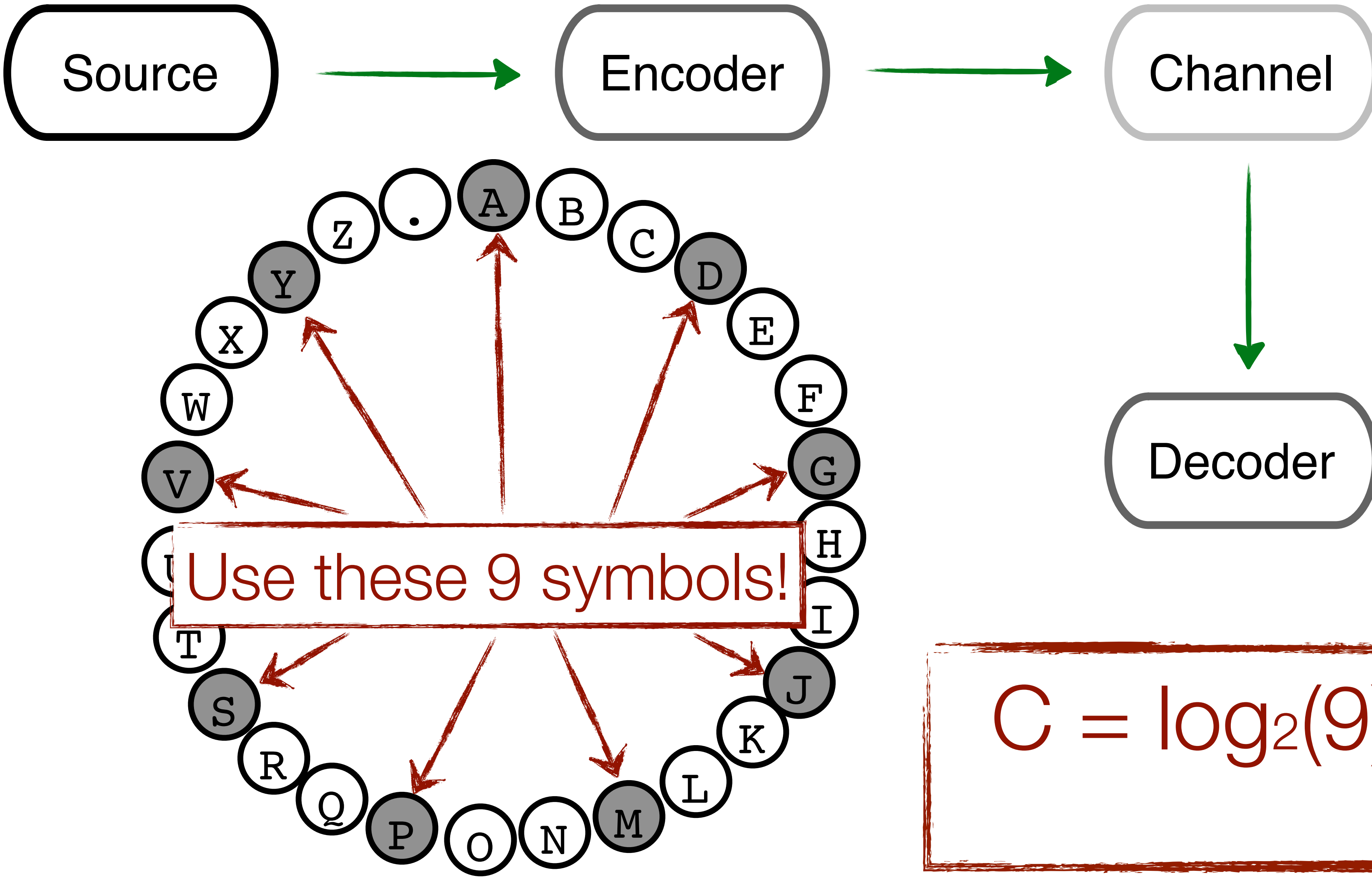
AAA → AB.



Channel capacity: a cute example



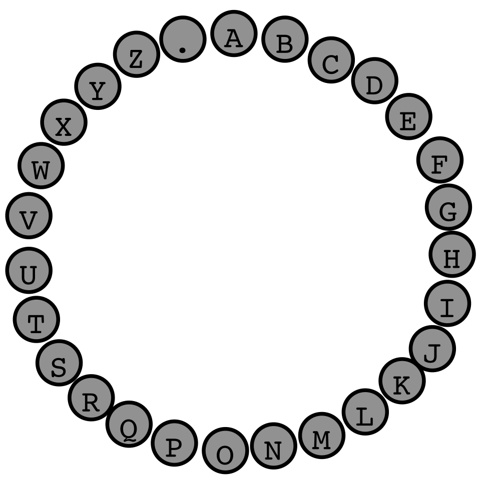
Channel capacity: a cute example



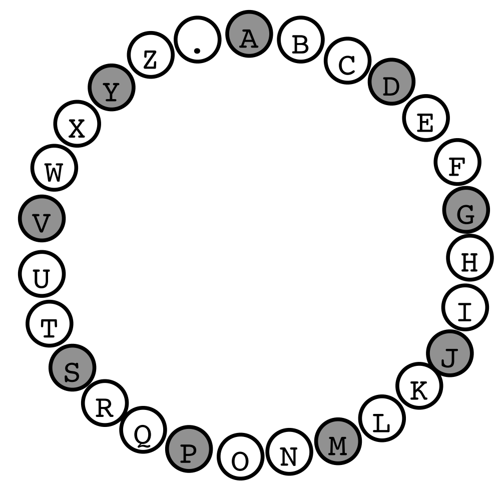
$$C = \log_2(9)$$

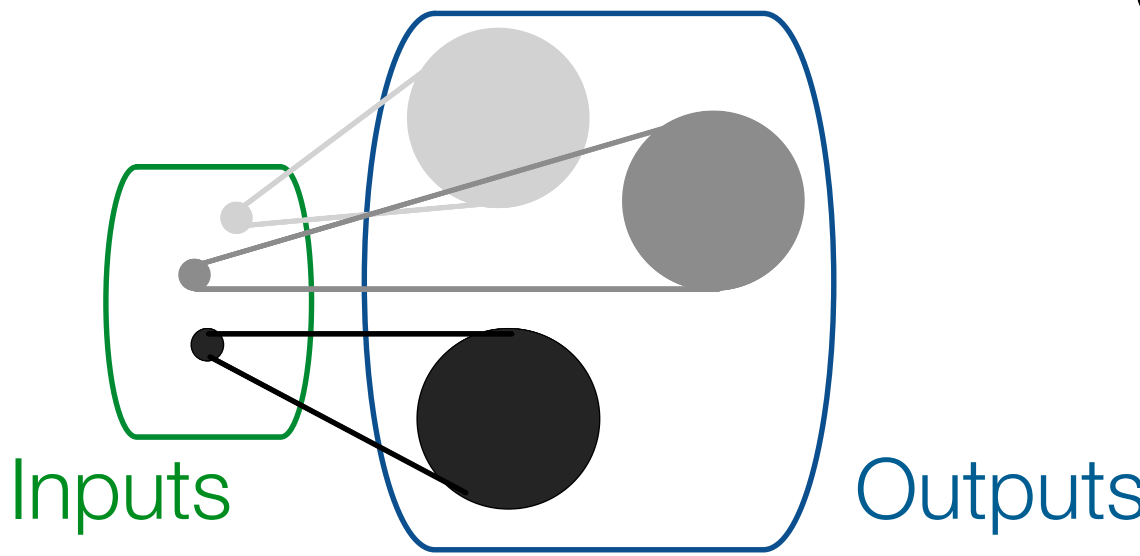
Capacity in general

- Main idea was to reduce the rate (from a 27-letter input per channel use to a 9-letter input per channel use) so as to produce



Non-overlapping outputs!

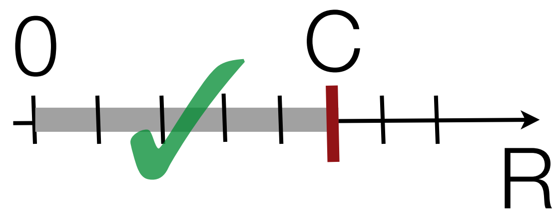




Mathematical description of capacity

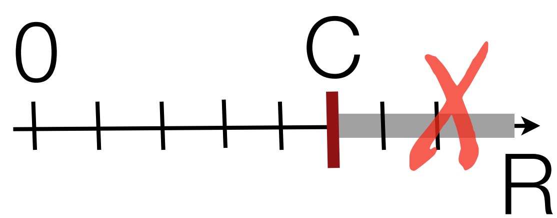
- Can achieve reliable communication for all transmission rates R:

$R < C$



- BUT, probability of decoding error always bounded away from zero if

$R > C$



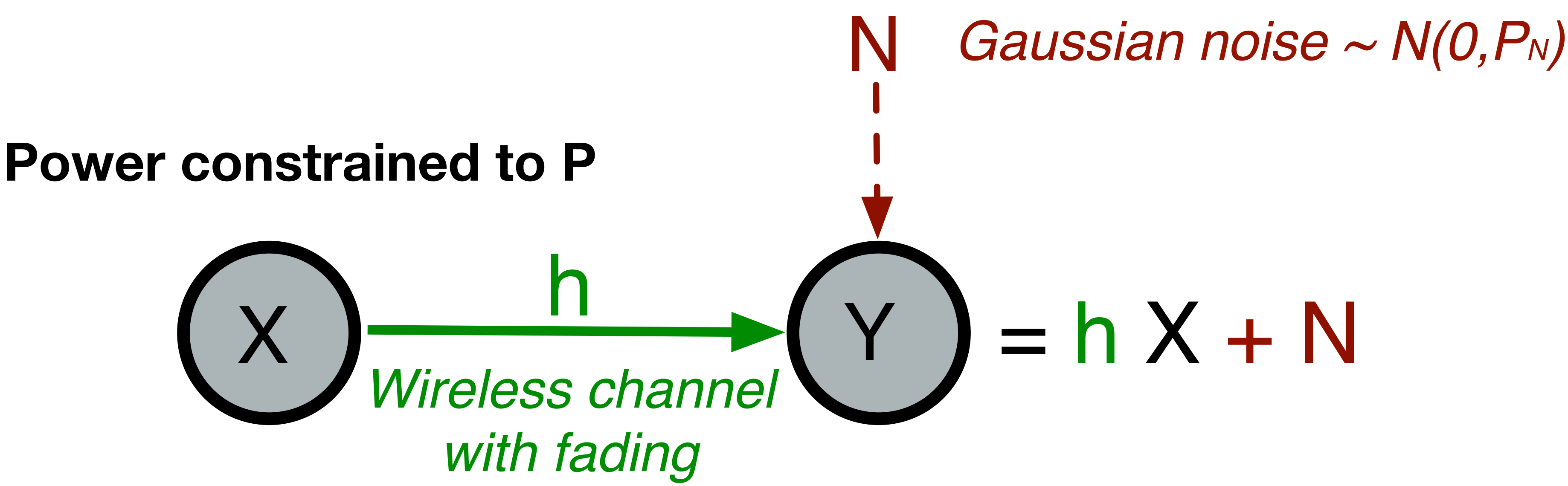
Continuous alphabet channel capacity

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

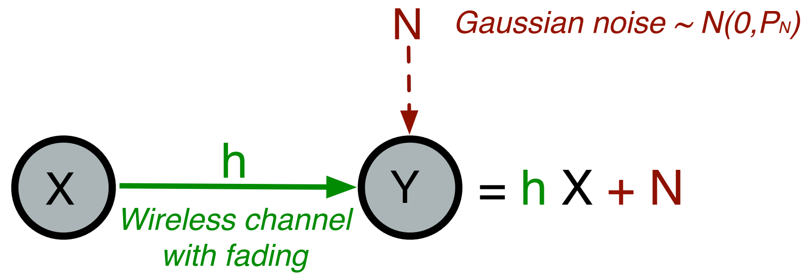
"mutual information" between X and Y

What if
X and Y are not bits, but real numbers?

AWGN channel capacity



AWGN channel capacity

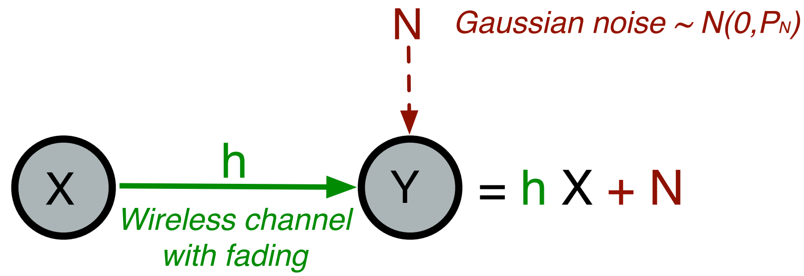


$$
\begin{aligned}
C &= \max_{p(x):E[XX^T]\le P} I(X;Y) \\
&= \max_{p(x):E[XX^T]\le P} h(X) - h(X|Y) \\
&= \max_{p(x):E[XX^T]\le P} h(Y) - h(Y|X) \\
&= \max_{p(x):E[XX^T]\le P} h(hX+N) - h(hX+N|X) \\
&= \max_{p(x):E[XX^T]\le P} h(hX+N) - h(N) \\
&= \frac{1}{2}\log(2\pi e(|h|^2 P + P_N)) - \frac{1}{2}\log(2\pi e P_N) \\
&= \frac{1}{2}\log\left(\frac{|h|^2 P + P_N}{P_N}\right)
\end{aligned}
$$

signal power at Rx

noise power at Rx

AWGN channel capacity



$$C = \frac{1}{2} \log \left(\frac{|h|^2 P + P_N}{P_N} \right)$$
$$= \frac{1}{2} \log (1 + SNR) \;\; \text{(bits/channel use)}$$

What about bits/second and bandwidth of the channel?

$$C = W \log_2 \left(1 + \frac{P}{W N_0} \right) \;\; \text{(bits/second)}$$

[Bandwidth W, h=1, spectral density $N_0/2$]

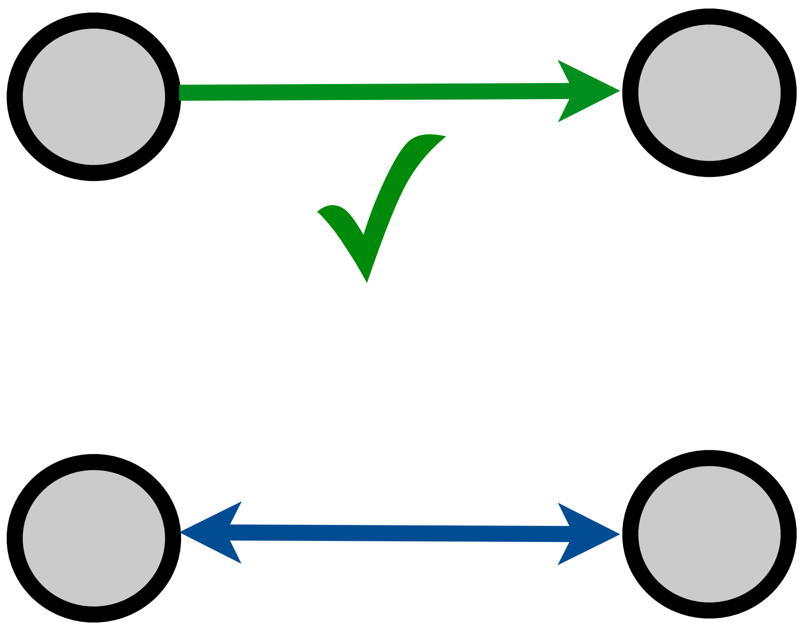
Use?

- Benchmark for performance of practical systems

- Guideline in designing systems - what’s worth shooting for?

- Theoretical insights can lead to practical insights

So now what?

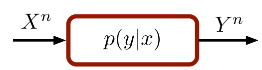


Unsolved

Fundamental

Outline

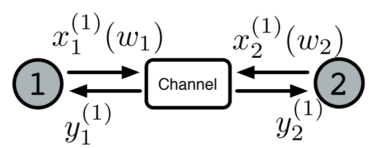
$$C = \max_{p(x)} I(X;Y)$$



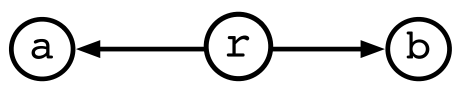
- Information theory - *what, why, when*

  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel

- **Two-way channel - *channel coding***



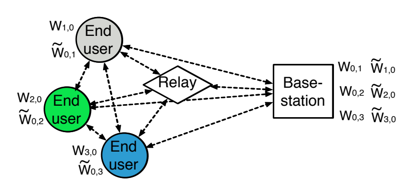
- Two-way relay channels



$w_r = w_1 \oplus w_2$

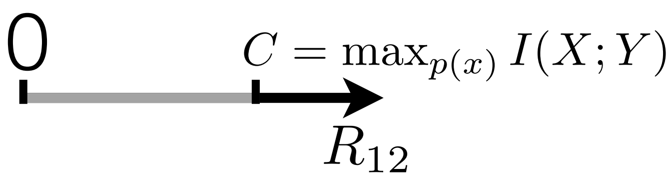
  - *single flow - canonical example of wireless network coding*

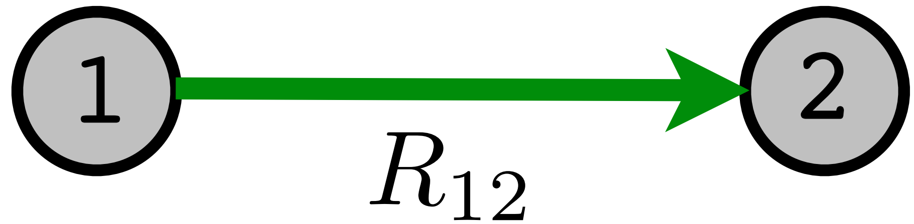
  - *multiple flows with a base-station - pairwise wireless network coding*



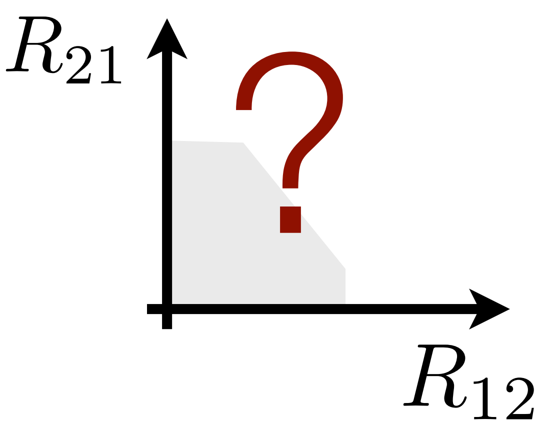
Two-way channel capacity region

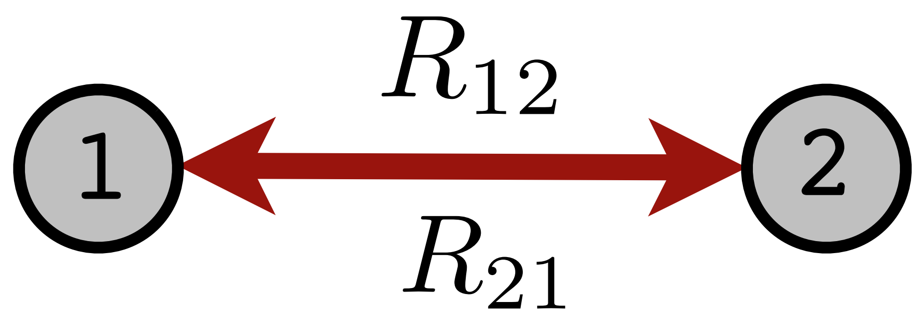
One-way ***Capacity***



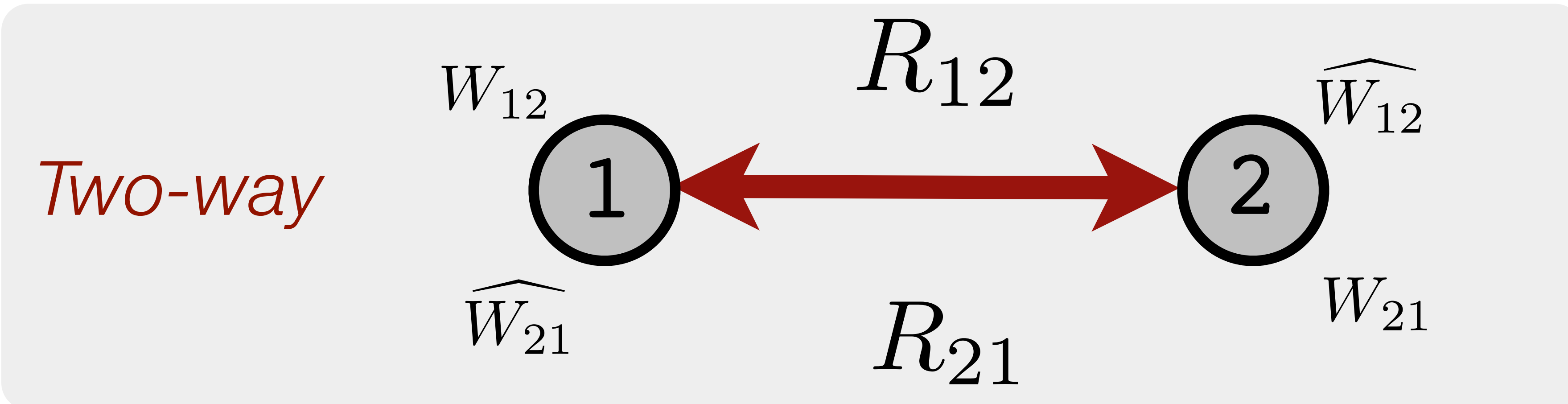


Two-way ***Capacity Region***

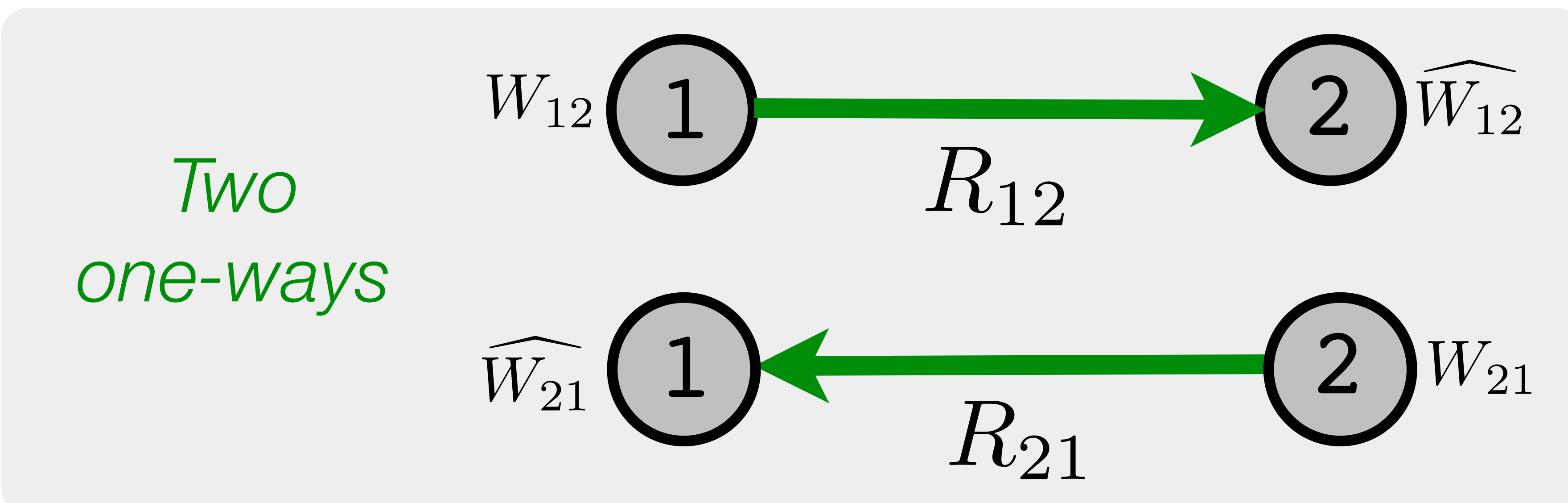




When is

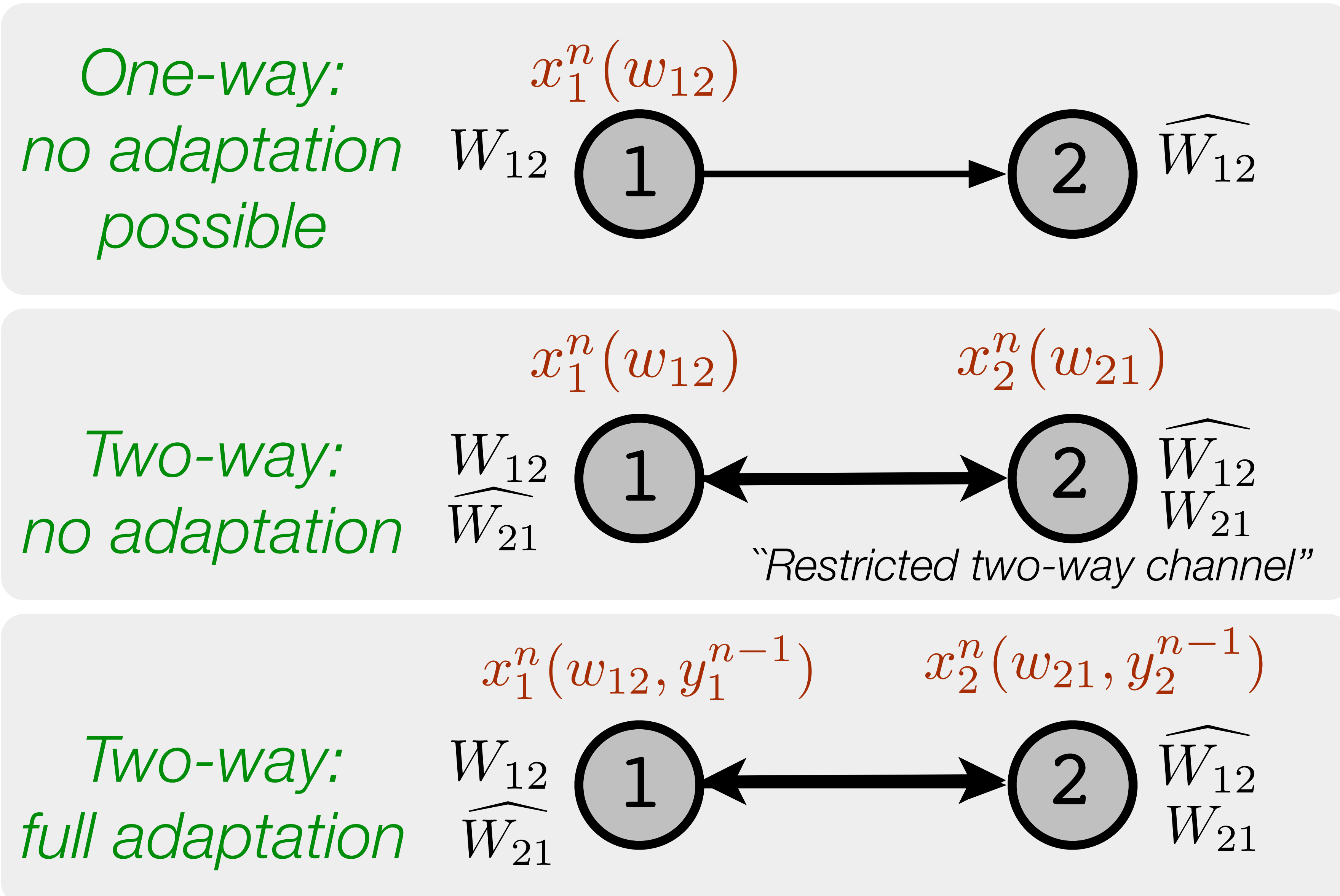


equal to



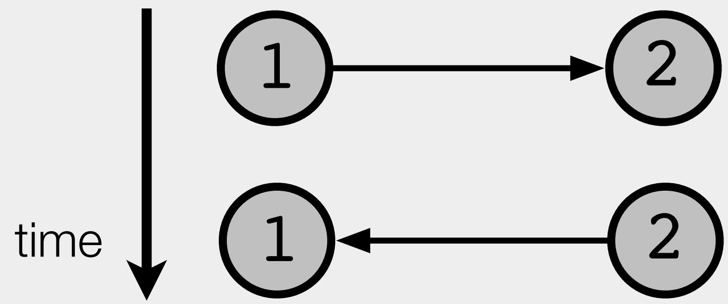
?

Models for two-way adaptation



Duplex

Two-way:
half duplex



Two-way:
full duplex

(All on same frequency band)

When is capacity known

- Parallel two-way channel
- Mod-2 adder
- Two-way restricted channel
- Two-way “push-to-talk” channel
- Two-way Gaussian noise channel (full & half duplex, restricted & unrestricted)

When is capacity unknown

- **General** unrestricted discrete memoryless channels
- Binary multiplier channel (BMC)

General results

Inner bound

$$R_1 \leq I(X_1; Y_2 | X_2)$$
$$R_2 \leq I(X_2; Y_1 | X_1)$$

where X_1 and X_2 follow the joint distribution $p(x_1, x_2) = p(x_1)p(x_2)$.

Not in general equal!

Outer bound

$$R_1 \leq I(X_1; Y_2 | X_2)$$
$$R_2 \leq I(X_2; Y_1 | X_1)$$

where the joint distribution of random variables X_1 and X_2 is $p(x_1, x_2)$.

[Shannon '61]

Capacity: two parallel channels

[Shannon '61]

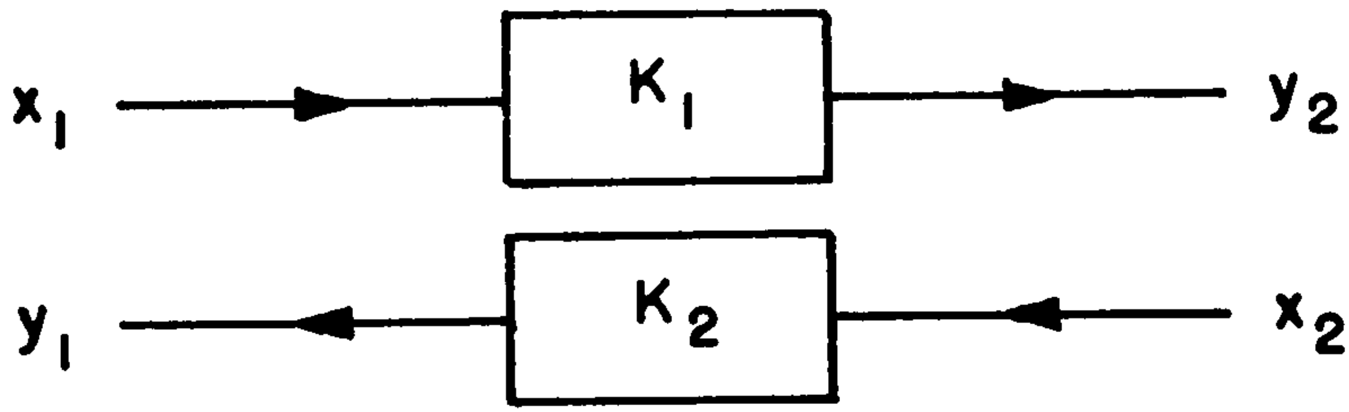
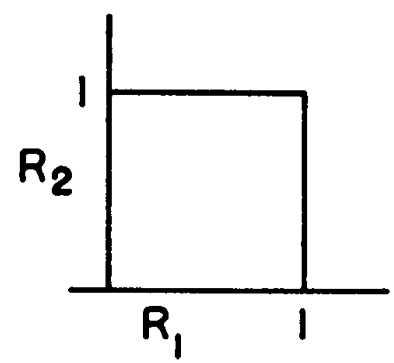


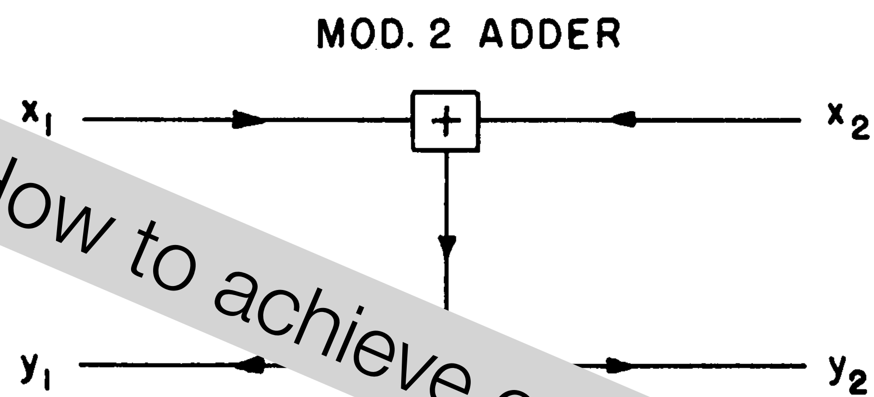
FIGURE 2

$$R_{12} \leq C_{K_1}$$
$$R_{21} \leq C_{K_2}$$



Capacity: binary mod-2 adder channel

[Shannon '61]



FIGUR...

How to achieve capacity region?

$y_1 = y_2 = x_1 + x_2$ (mod ...

$Y_1=Y_2$	X_2 = 0	X_2 = 1
X_1 = 0	0	1
X_1 = 1	1	0

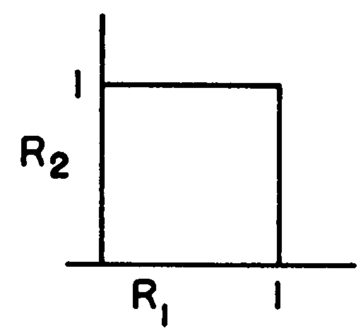


FIGURE 3

Achieving mod 2 adder channel capacity

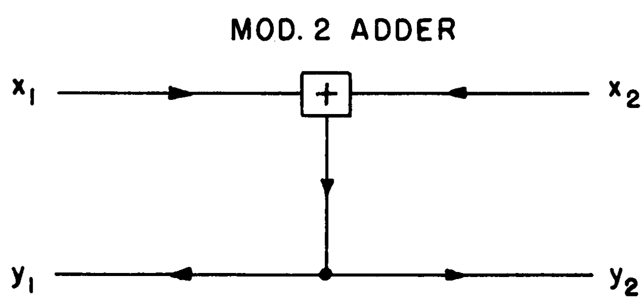


FIGURE 4

$$y_1 = y_2 = x_1 + x_2 \pmod 2.$$

Receiver 1:

$$\begin{aligned}\hat{x}_2 &= y_1 \oplus x_1 \\ &= x_1 \oplus x_2 \oplus x_1 \\ &= x_2\end{aligned}$$

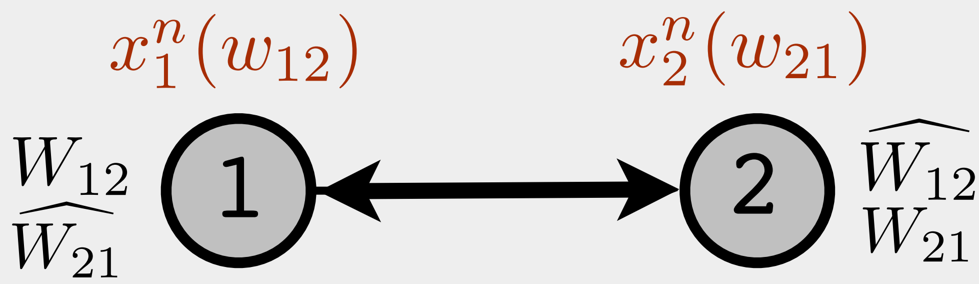
$Y_1 = Y_2$	X_2 = 0	X_2 = 1
X_1 = 0	0	1
X_1 = 1	1	0

Receiver 2:

$$\begin{aligned}\hat{x}_1 &= y_2 \oplus x_2 \\ &= x_1 \oplus x_2 \oplus x_2 \\ &= x_1\end{aligned}$$

EXPLOIT TWO-WAY!

Capacity: restricted channel



Capacity region:

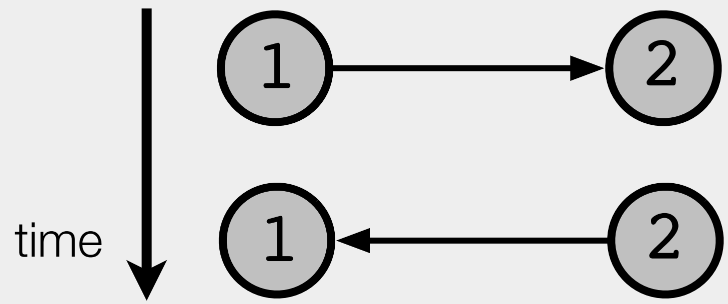
$$R_1 \leq I(X_1; Y_2 | X_2)$$
$$R_2 \leq I(X_2; Y_1 | X_1)$$

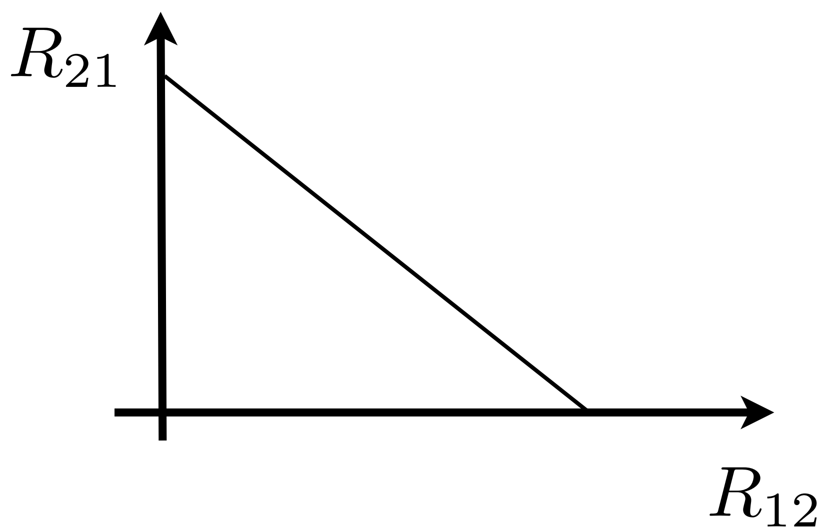
where X_1 and X_2 follow the joint distribution $p(x_1, x_2) = p(x_1)p(x_2)$.

[Shannon '61]

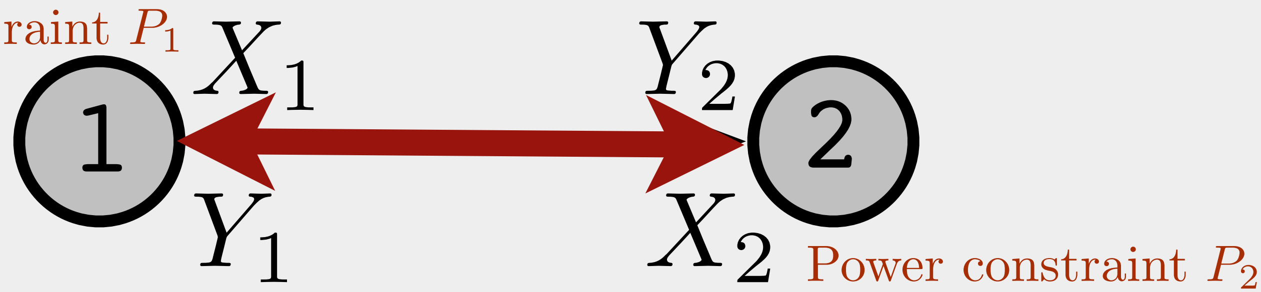
Capacity: "push-to-talk" channel

Two-way:
half duplex





Capacity: Gaussian noise channel



$$Y_1 = aX_1 + bX_2 + N_1 \sim \mathcal{N}(0, \sigma_1^2)$$

$$Y_2 = cX_1 + dX_2 + N_2 \sim \mathcal{N}(0, \sigma_2^2)$$

Capacity region:

$$R_1 \leq (1/2)\log(1 + c^2 P_1/\sigma_2^2)$$

$$R_2 \leq (1/2)\log(1 + b^2 P_2/\sigma_1^2)$$

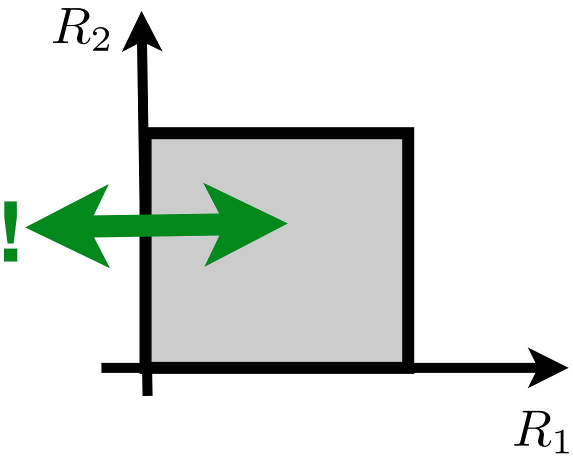
No dependence on ``a" or ``d"

[Han '84]

$$Y_1 = aX_1 + bX_2 + N_1$$
$$Y_2 = cX_1 + dX_2 + N_2$$

Capacity: Gaussian noise channel

$$R_1 \leq (1/2)\log(1 + c^2 P_1/\sigma_2^2)$$
$$R_2 \leq (1/2)\log(1 + b^2 P_2/\sigma_1^2)$$



- **TWO PARALLEL CHANNELS!!**
- Achieved by Gaussian inputs
- ``Feedback" does not help here

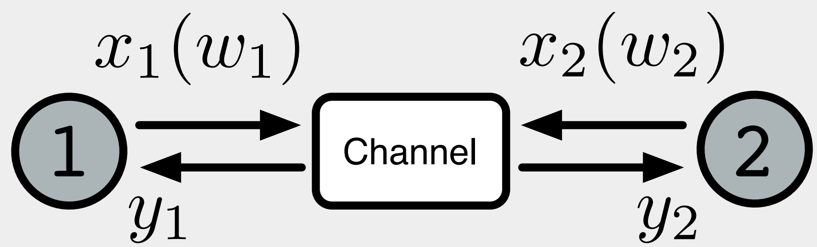
Why so hard?

Adaptation

Adaptive codewords

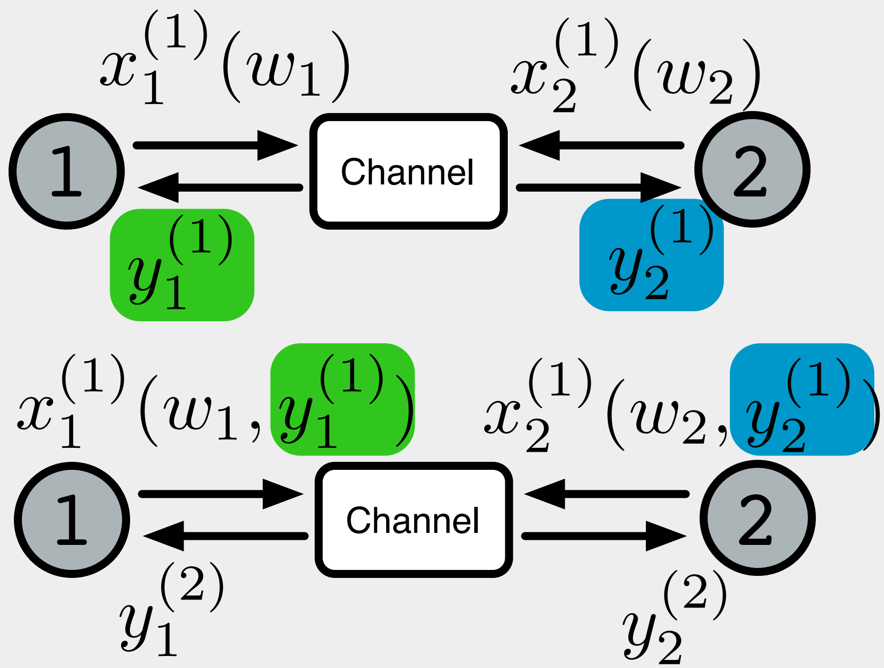
Single-letter

$C = f(X_1, X_2)$



Two-letter

$C = f(X_1^{(1)}, X_1^{(2)}, X_2^{(1)}, X_2^{(2)})$

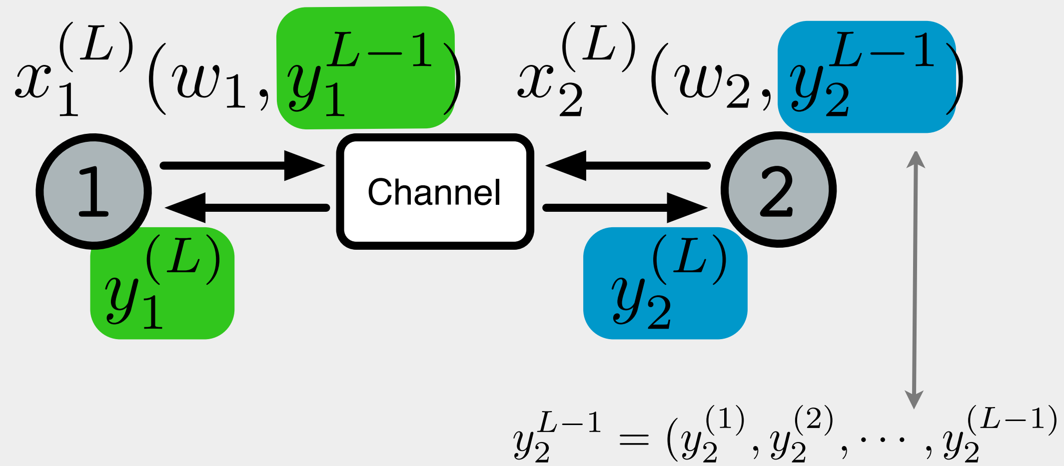


G. Kramer, "Capacity results for the discrete memoryless network," *IEEE Trans. Inf. Theory*, vol. 49, no. 1, pp. 4–21, Jan. 2003.

S. M. S. Tatikonda, "The capacity of channels with feedback," *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 323–349, Jan. 2009.

Adaptive codewords

L-letter



The space over which we can code (x's) is enormous!

G. Kramer, "Capacity results for the discrete memoryless network," *IEEE Trans. Inf. Theory*, vol. 49, no. 1, pp. 4–21, Jan. 2003.

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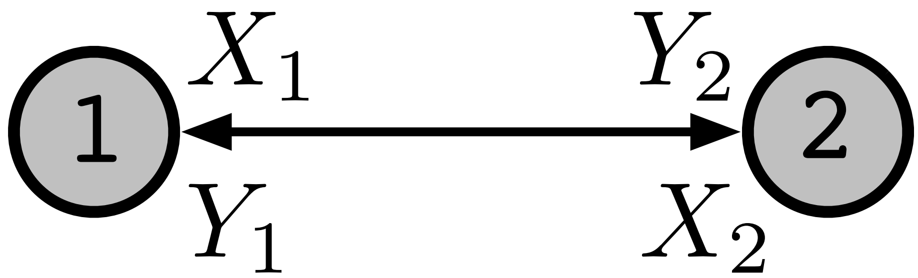
Non-adaptive codewords:

$X_1 \in \{0,1\}$

$Y_1 = X_1 X_2$



$Y_2 = Y_1 \mod 2$

$X_2 \in \{0,1,2\}$

Code of user 1

$u_{11} = 0$ 0 0 0

$u_{11} = 1$ 1 1 1

Code of user 2

$u_{21} = 0$ 1 0 2

$u_{21} = 1$ 0 2 0

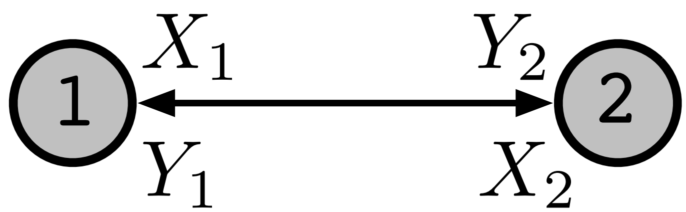
L=N=3 channel uses

G. Kramer, "Directed information for channels with feedback," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 1998.

Adaptive codewords:

$X_1 \in \{0, 1\}$

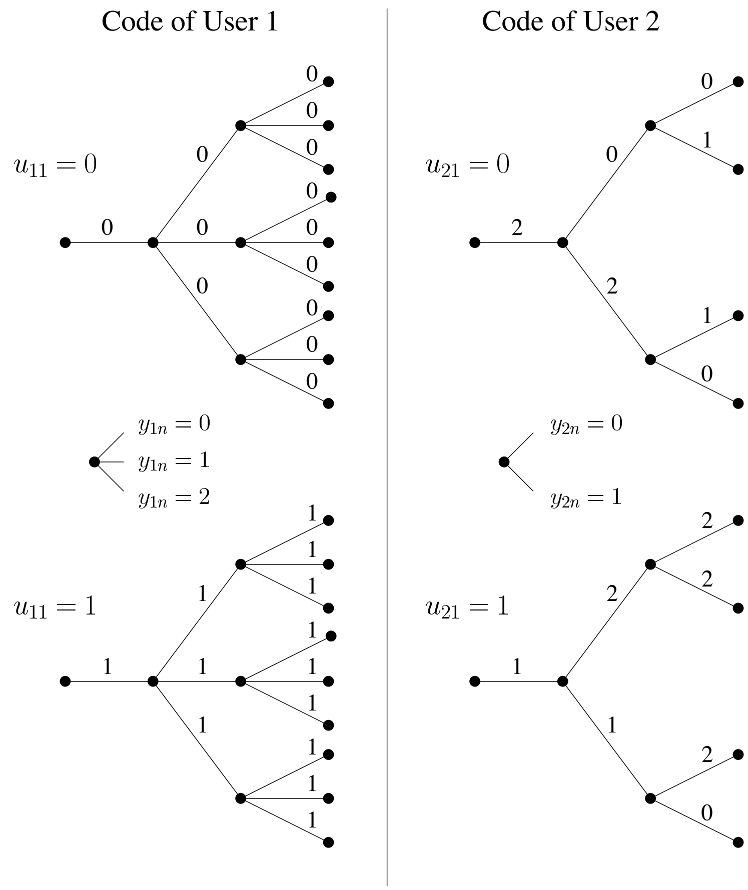
$Y_1 = X_1 X_2$



$Y_2 = Y_1 \mod 2$

$X_2 \in \{0, 1, 2\}$

Adaptive codewords $\mathbf{A}_1^3$



Adaptive codewords $\mathbf{A}_2^3$

L=N=3 channel uses

G. Kramer, "Directed information for channels with feedback," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 1998.

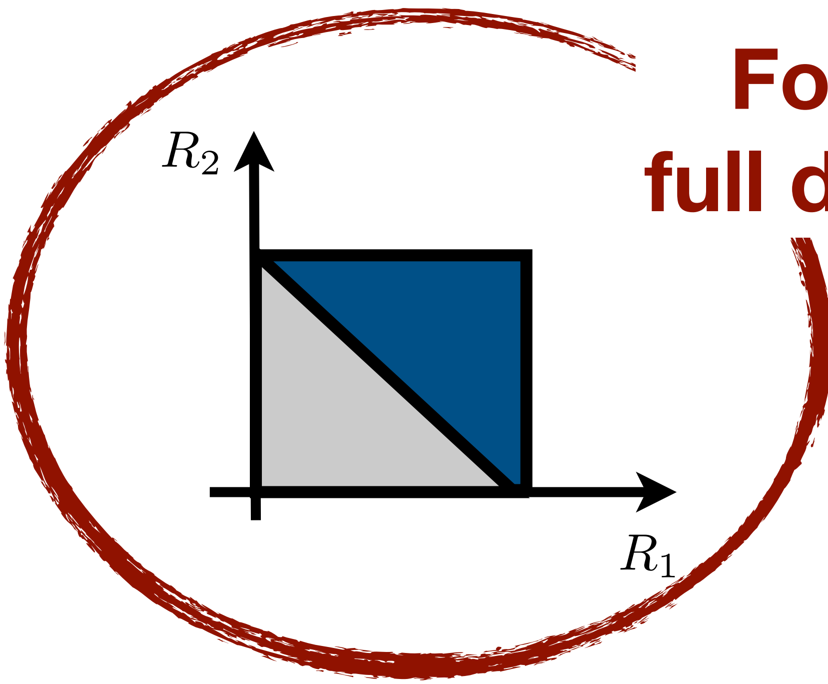
Can we take this adaptation into

CAUSAL adaptation - complex and generally deemed unsatisfactory

Take away points - AWGN two-way channel

- If have half-duplex constraint and memoryless channels, time-share

- If have full-duplex - obtain two parallel clean channels



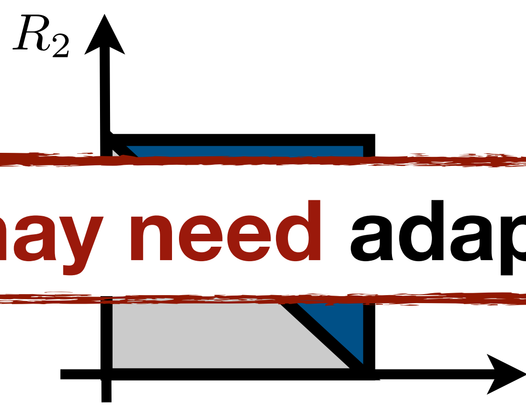
For applications - full duplex gains a lot!

Take away points - Discrete memoryless two-way channel

- If have half-duplex constraint (`push-to-talk”), time-share
- If have parallel two-way channels, mod-2 adder
- If have restricted channel

$$R_1 \leq I(X_1; Y_2 | X_2)$$
$$R_2 \leq I(X_2; Y_1 | X_1)$$

where X_1 and X_2 follow the joint distribution $p(x_1, x_2) = p(x_1)p(x_2)$.

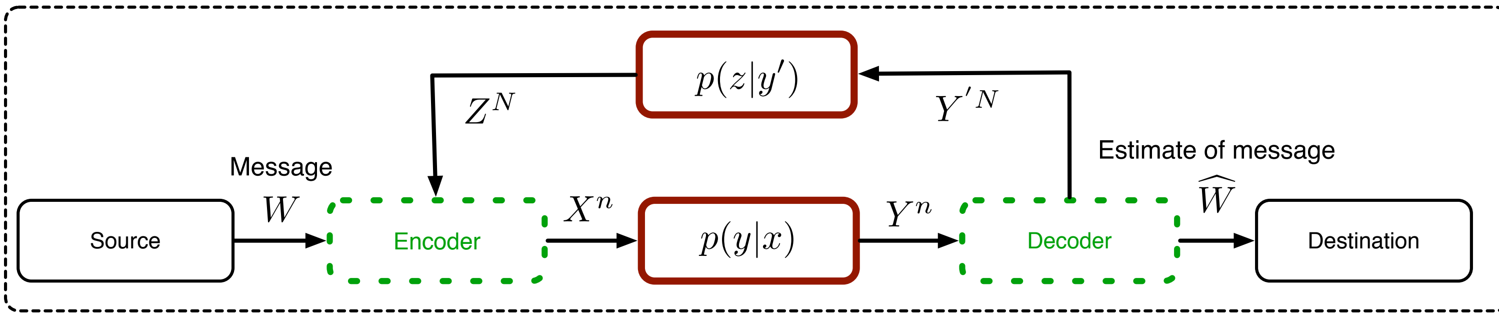


In general may need adaptive codewords

In general OPEN PROBLEM

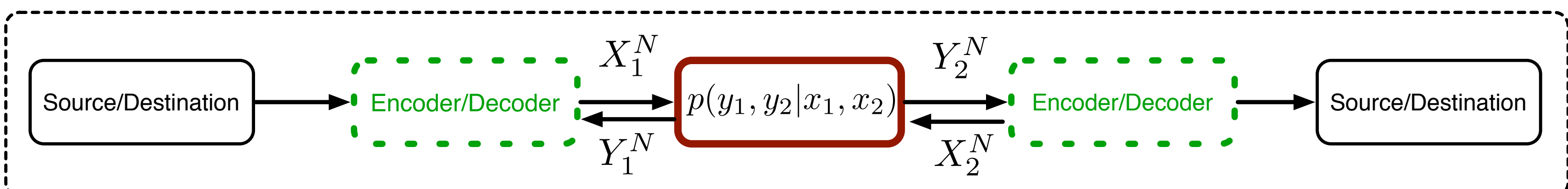
Relationship to feedback channels

- Feedback channel



Information is still one-way!!

- Two-way channel

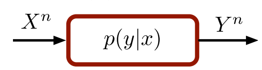


Set rate in ← direction to 0

Properly choose $p(y_1, y_2|x_1, x_2)$

Outline

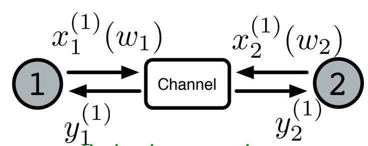
$$C = \max_{p(x)} I(X;Y)$$



- Information theory - *what, why, when*

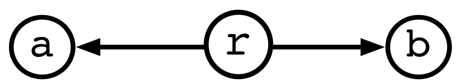
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel

- Two-way channel - *channel coding*



  Adaptive codewords, capacity in Gaussian noise = two parallel channels

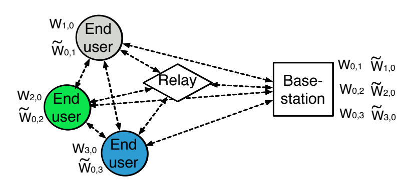
- **Two-way relay channels**



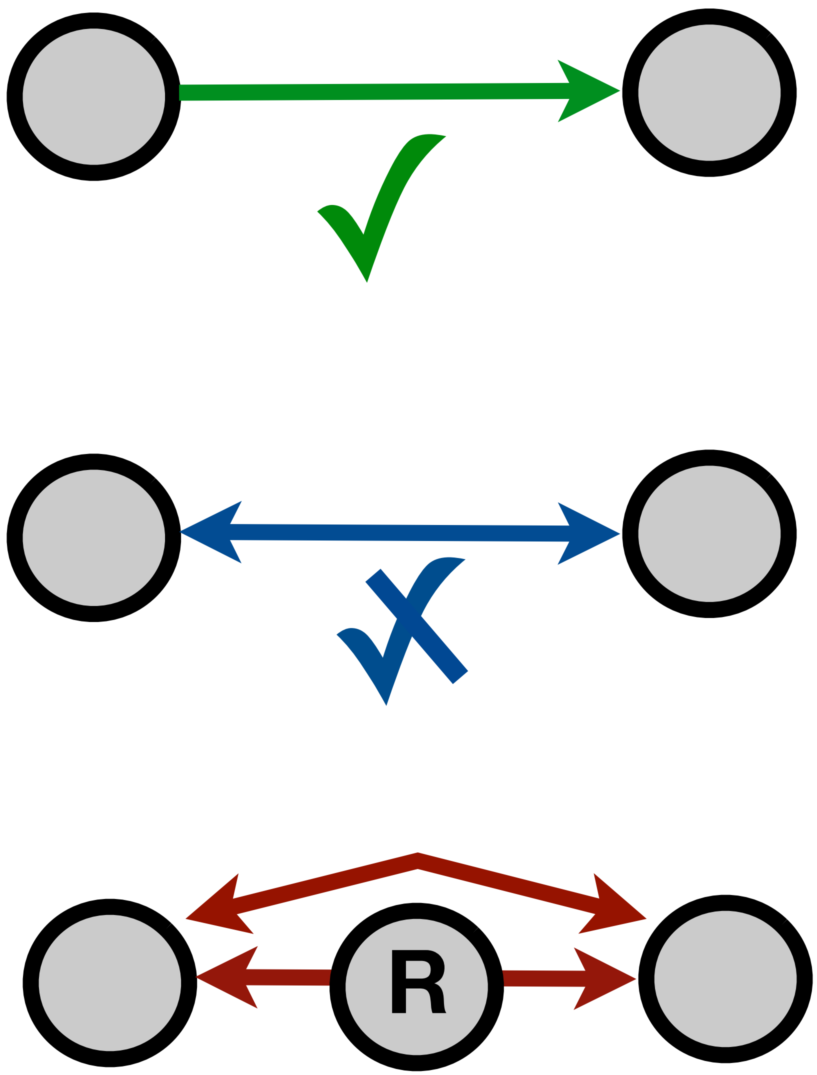
$$w_r = w_1 \oplus w_2$$

  - ***single flow*** *- canonical example of wireless network coding*

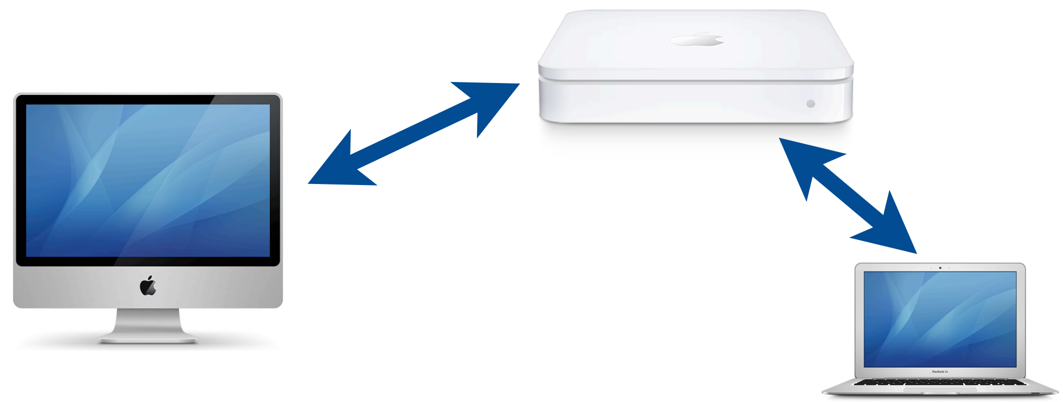
  - *multiple flows with a base-station - pairwise wireless network coding*

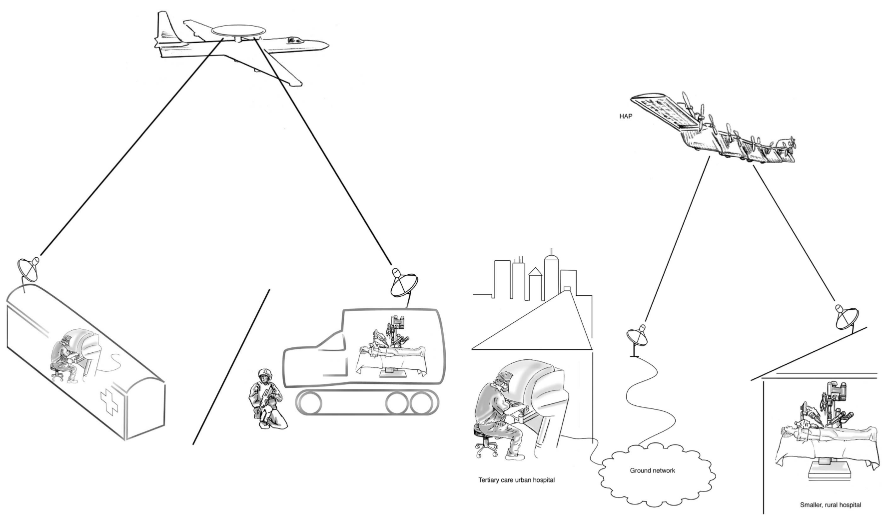


So now what?

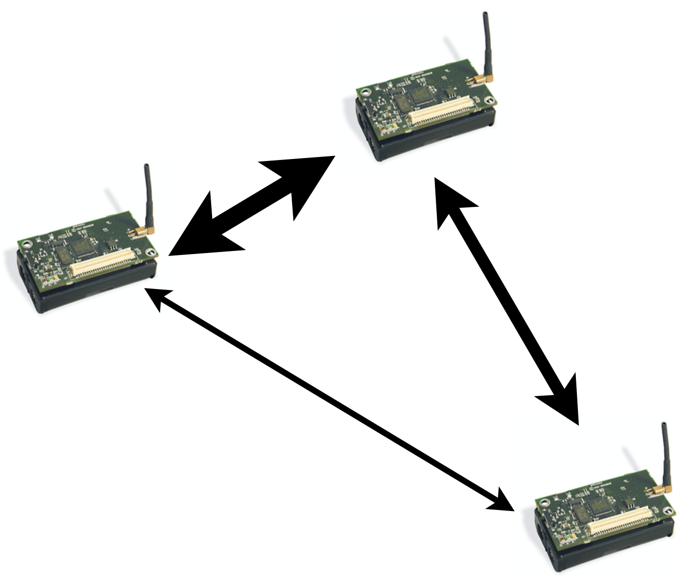


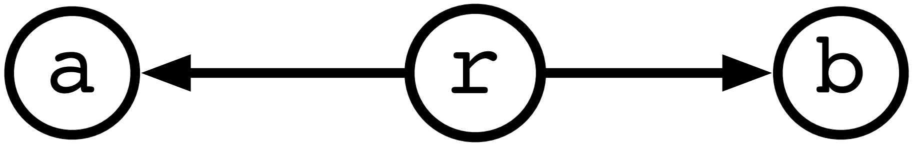
Motivation





Telesurgery





(and extensions)

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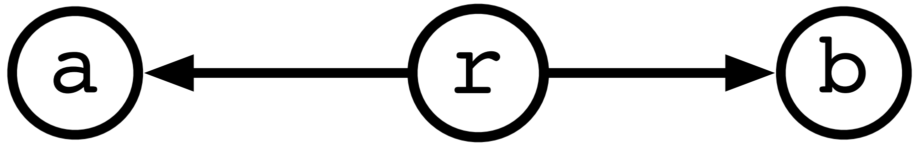

Channel model

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Half duplex

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Full duplex

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S.-Y. C. W. Nam and Y. Lee, “Capacity of the gaussian two-way relay channel to within 1/2 bit,” 2009. [Online]. Available: http://arxiv.org/abs/0902.2438

S. J

prot

Direct link between terminal nodes

L. Ong, S. Johnson, and C. Kellett, “An Optimal Coding Strategy for the Binary Multi-Way Relay Channel,” http://arxiv4.library.cornell.edu/abs/1004.2299.

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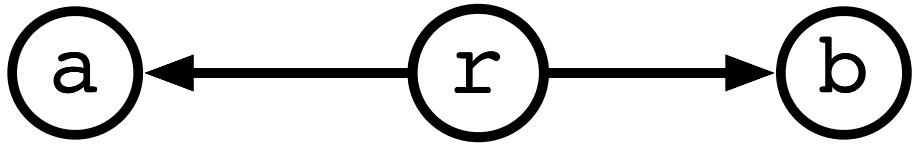

Relaying type

Decode and forward

Compress and forward

Amplify and forward

Lattice codes

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M. P. Wilson, K. Naray... ysical layer coding and network coding for bi-direction... //arxiv.org/abs/0805.0012

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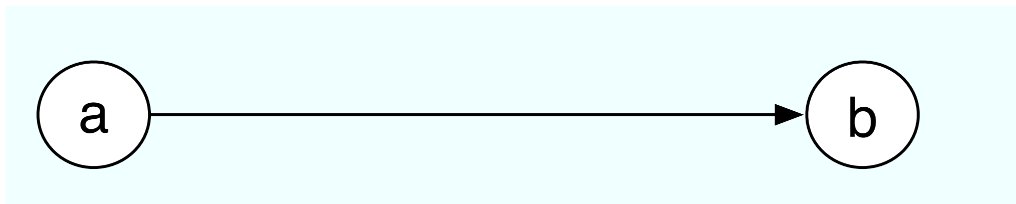
S. Kim, N. Devroye, and V. Tarokh, "Bi-directional half-duplex protocols with multiple relays," 2008. [Online]. Available: http://arxiv.org/abs/0810.1268

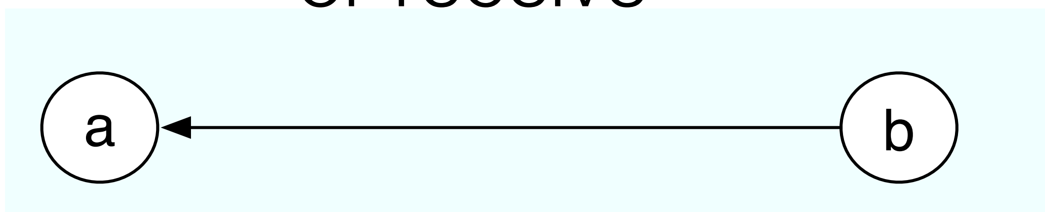
S. Kim, N. Devroye, and V. Tarokh, "A class of bi-directional multi-relay protocols," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, June 2009, pp. 349–353.

Two-way relay channel: half-duplex

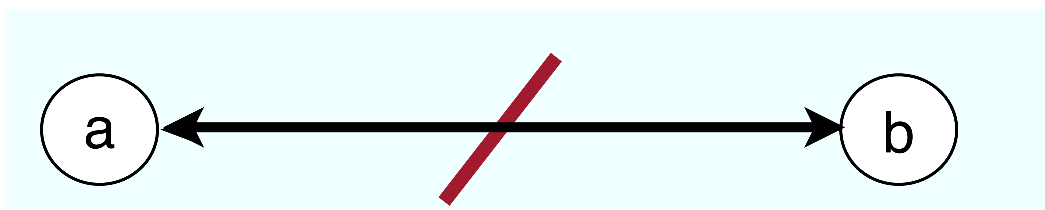
Nodes can either transmit



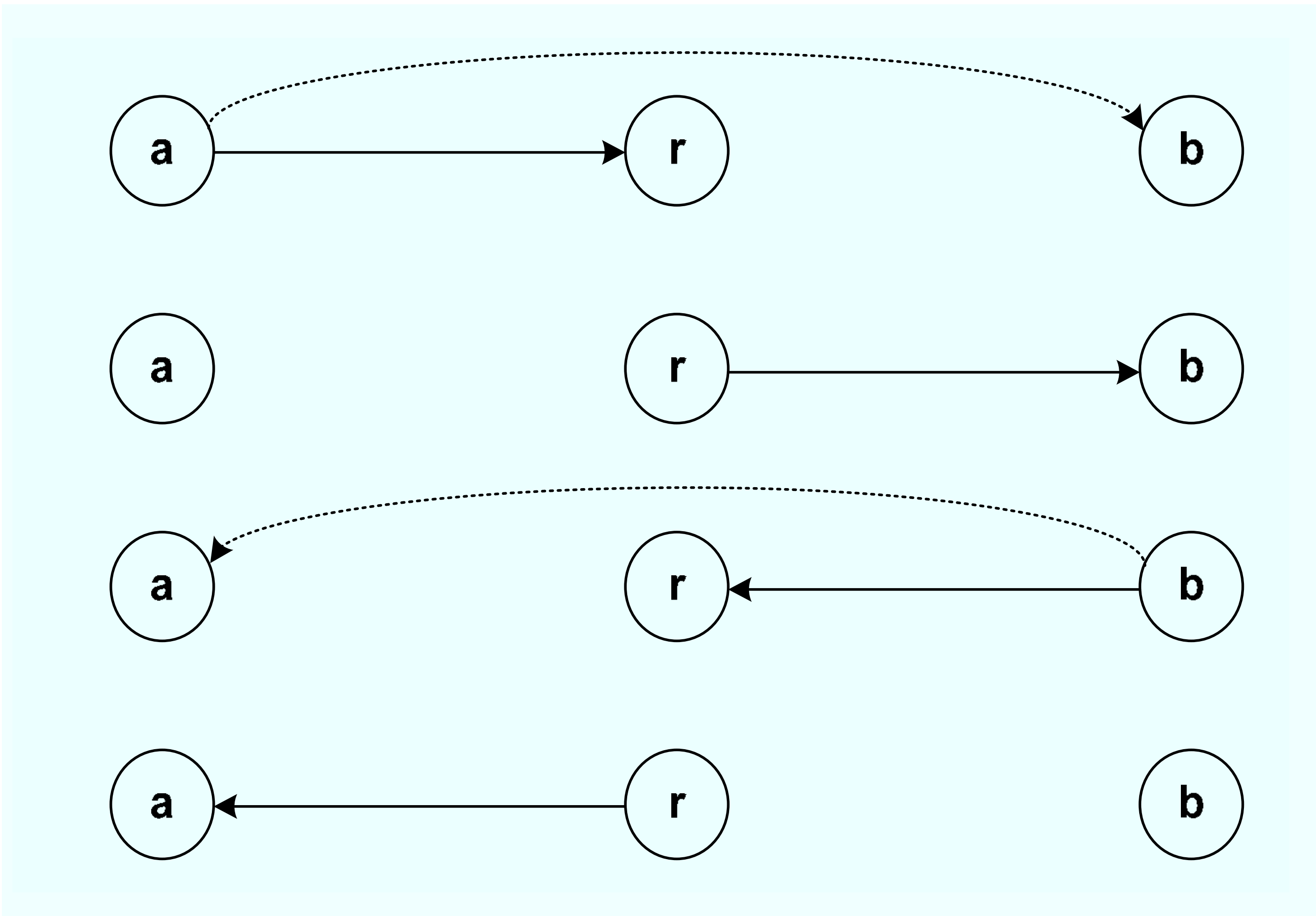
or receive



but not both.

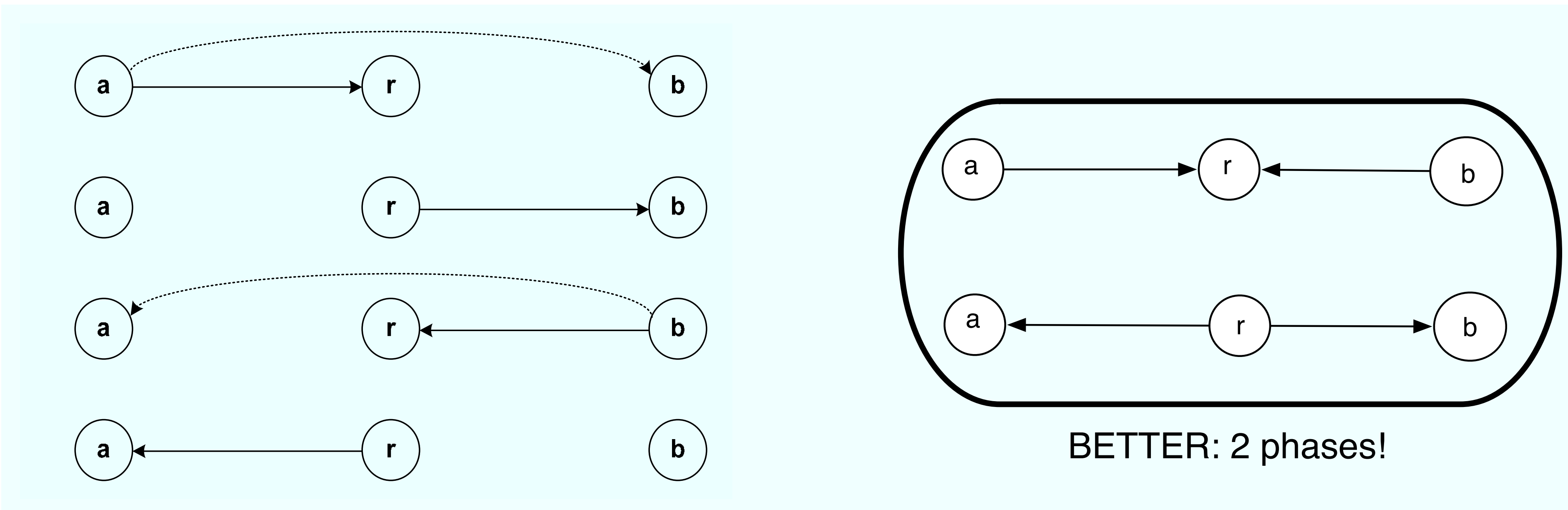


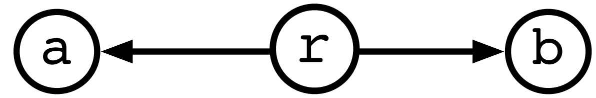
Temporal "phases": who transmits when



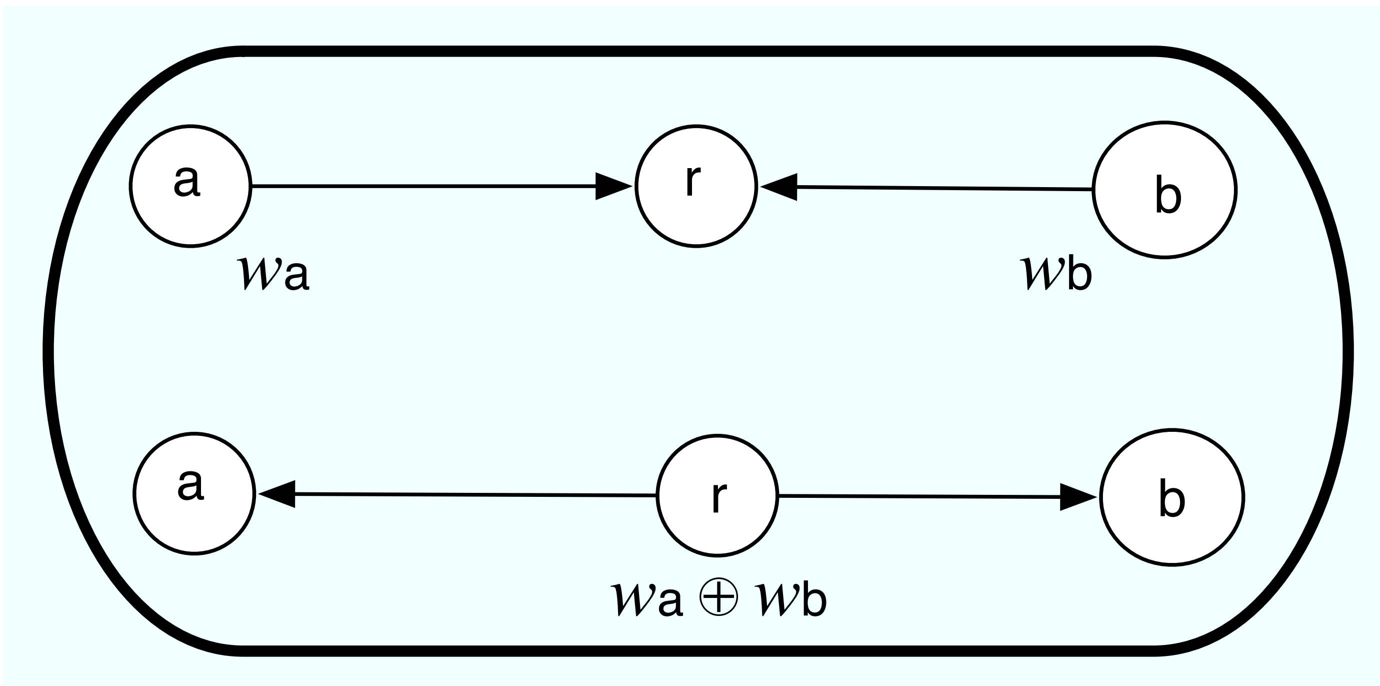
Are 4 phases needed? **NO!**

Better protocol



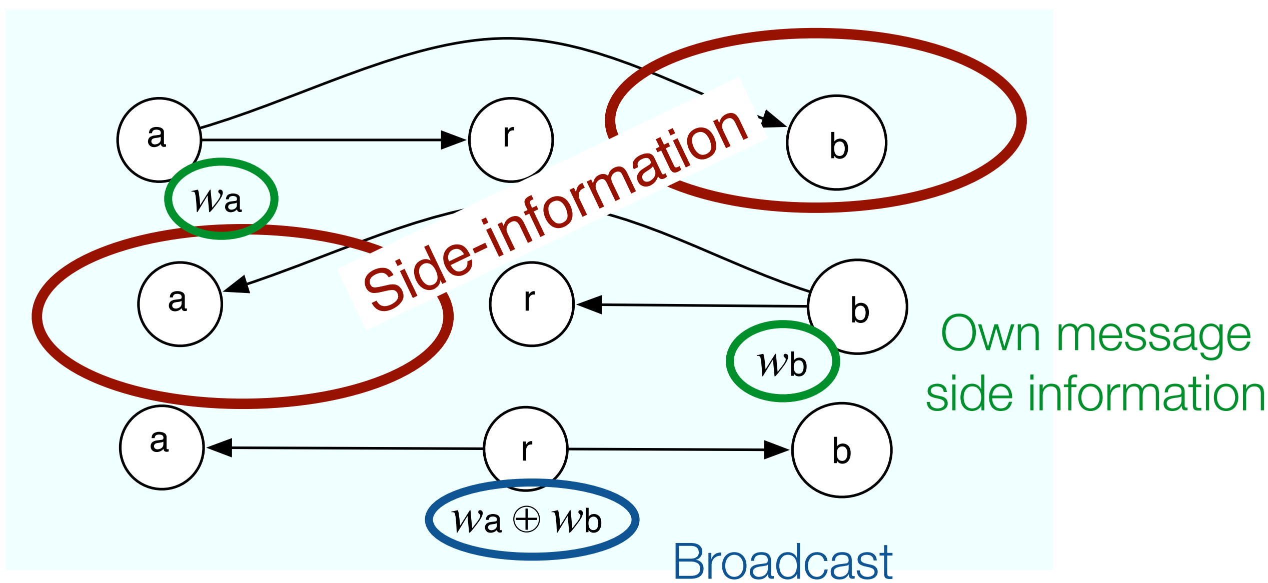


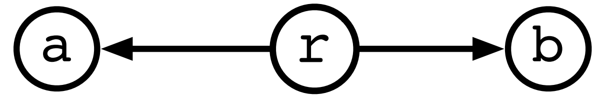
Message-level network coding



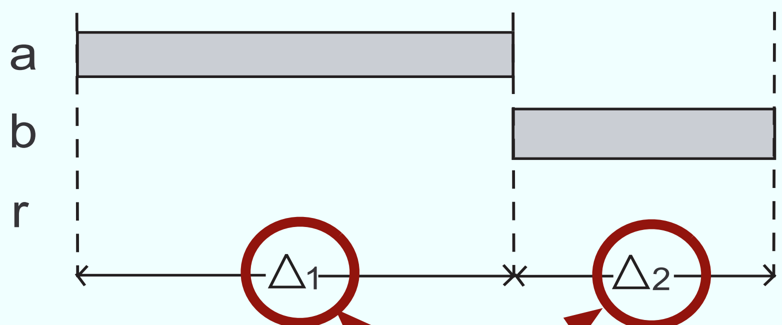
Key exploits

- ``own message side information” at nodes used to cancel out own message
- ``overheard side information” available to nodes when not transmitting
- broadcast nature of wireless channels: relay broadcasts one thing, both nodes hear it.



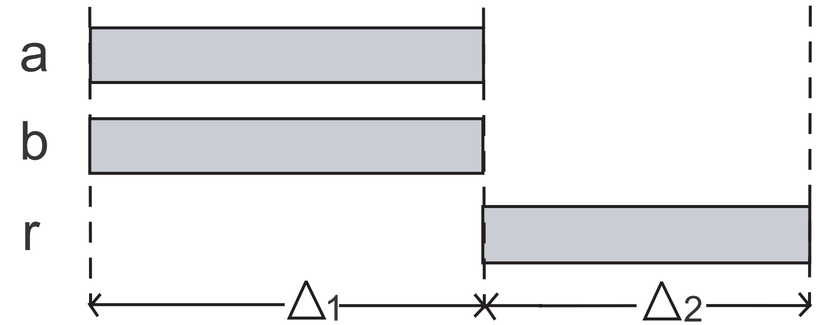


Four possible protocols

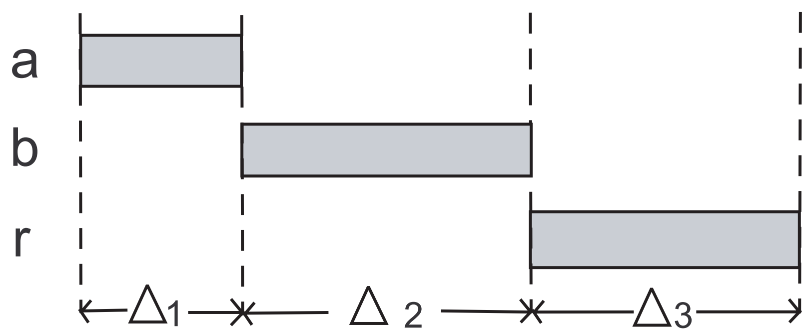


(i) DT protocol

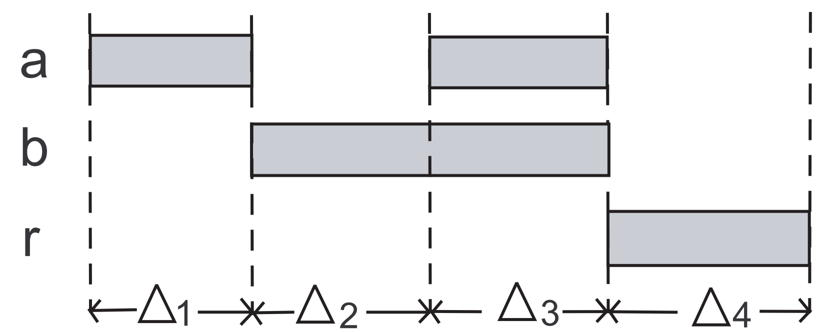
Time of phase 1 and 2, respectively



(ii) MABC protocol

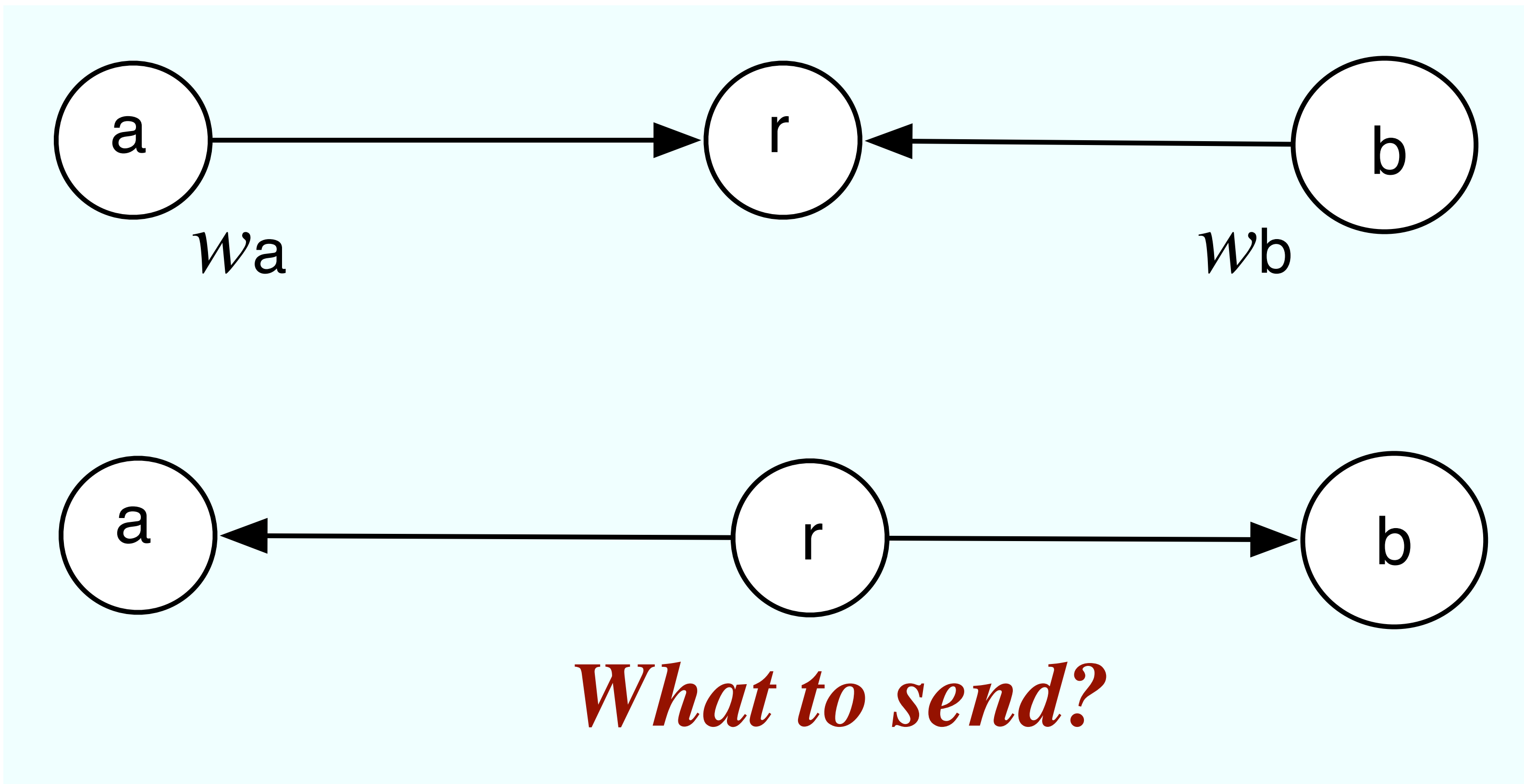


(iii) TDBC protocol



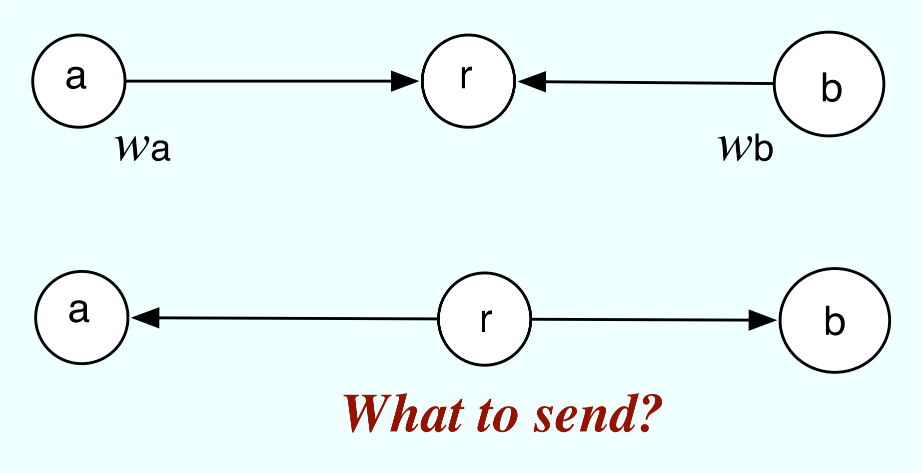
(iv) HBC protocol

Relaying schemes



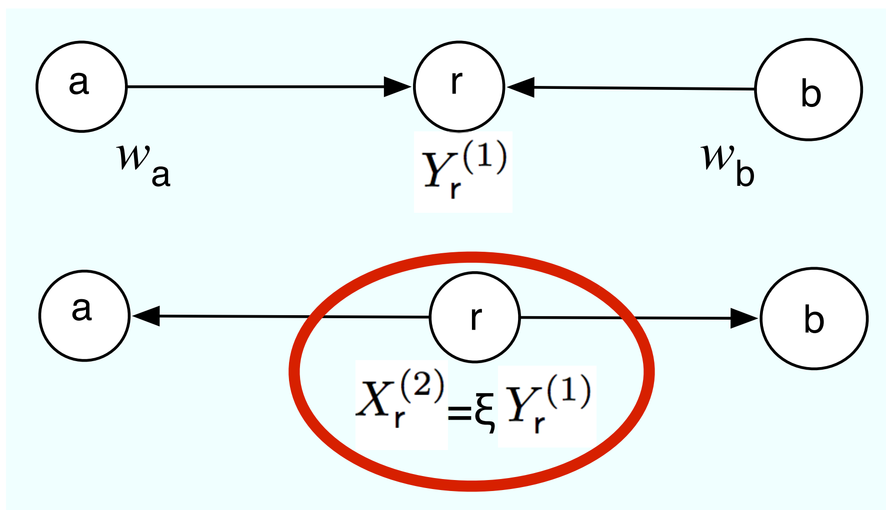
Relaying schemes

- *Amplify and Forward* (AF)
- *Decode and Forward* (DF)
- *Compress and Forward* (CF)
- *Mixed Forward*



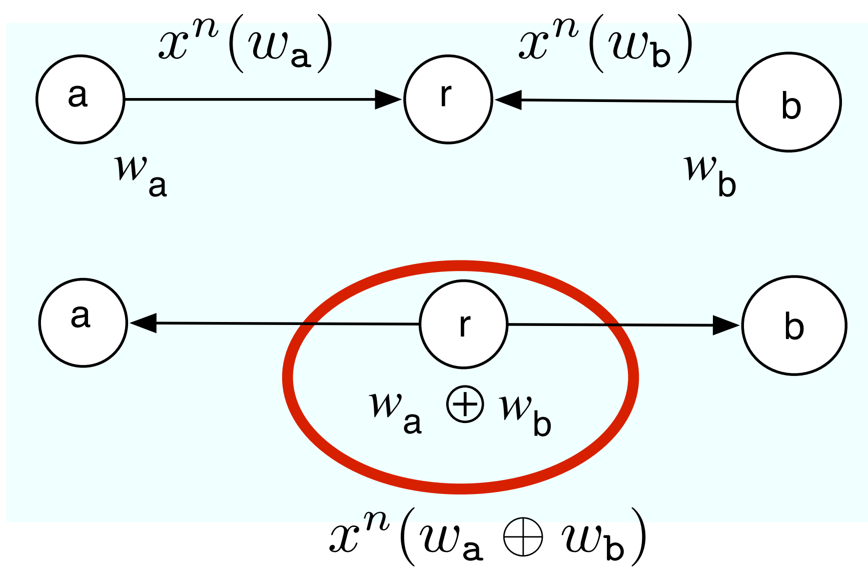
Amplify and forward (AF)

- The relay sends a scaled version of the signal it receives.
- Very little computation is needed.



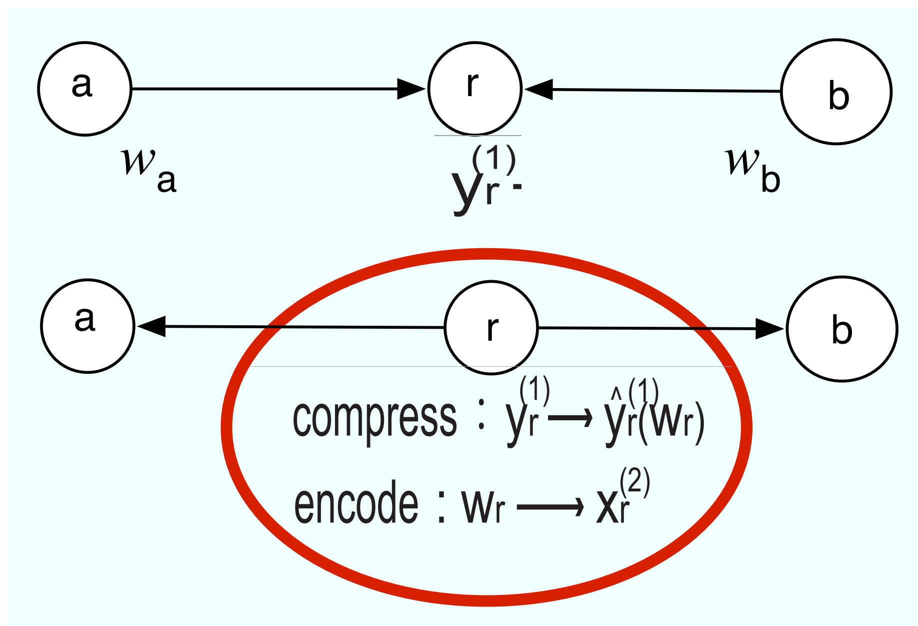
Decode and forward (DF)

- The relay decodes both w_{a} and w_{b}.
- Much computation, and transmitter codebooks are needed at the relay.



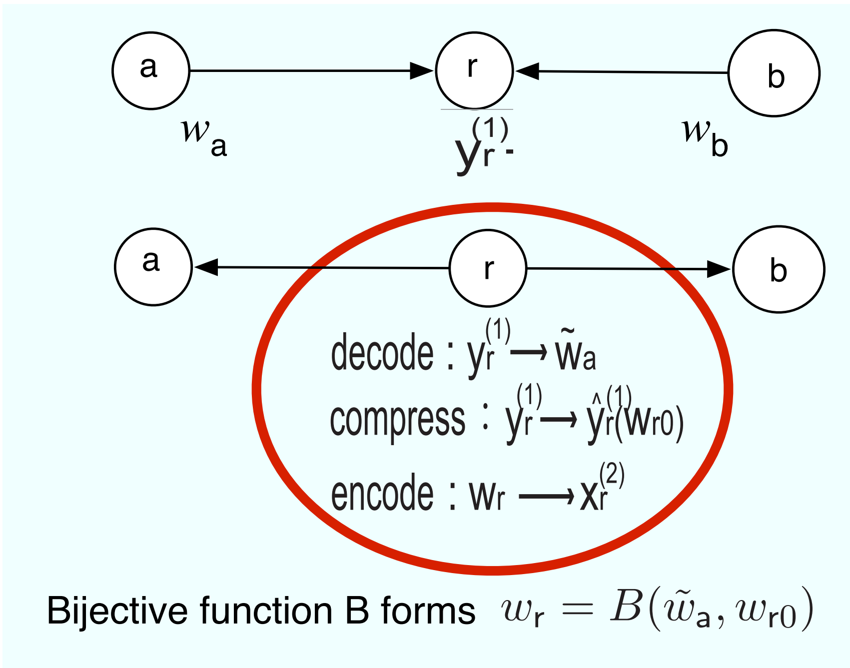
Compress and forward (CF)

- The relay compresses/quantizes the received signal.
- Less computation than DF and transmitter codebooks are not needed at the relay.



Mixed Forward (MF)

- The relay decodes w_{a} and compresses w_{b}, combines them into a new message w_r according to a bijective function, which it encodes and transmits.



Comparison of protocols

Protocol	Side information	Phase	Interference
MABC	not present	2	present
TDBC	present	3	not present
HBC	present	4	present
Relaying	**Complexity**	**Noise**	**Relay needs**
AF	very low	carried	nothing
DF	high	eliminated	full codebooks
CF	low	distortion	$p(y_{\mathsf{r}})$
Mixed	moderate	partially carried	a codebook, $p(y_{\mathsf{r}})$

Achievable rate regions: one example

- **Theorem 1**: The capacity region of the half-duplex bi-directional relay channel with the MABC protocol is the union of

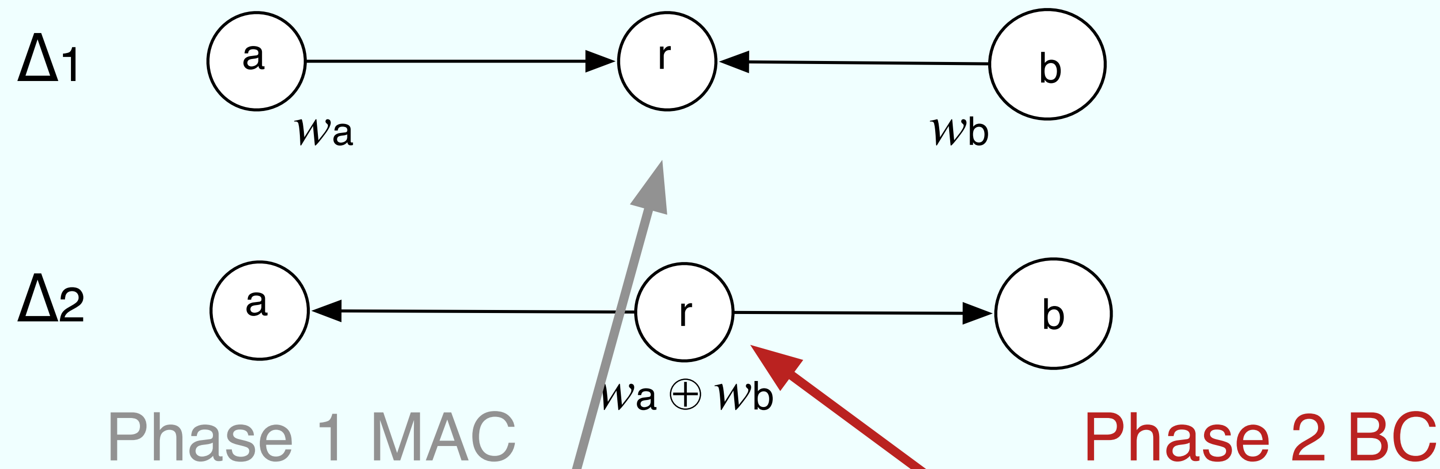
$$R_{\mathsf{a}} < \min\left\{\Delta_1 I(X_{\mathsf{a}}^{(1)}; Y_{\mathsf{r}}^{(1)}|X_{\mathsf{b}}^{(1)}, Q), \Delta_2 I(X_{\mathsf{r}}^{(2)}; Y_{\mathsf{b}}^{(2)}|Q)\right\}$$

$$R_{\mathsf{b}} < \min\left\{\Delta_1 I(X_{\mathsf{b}}^{(1)}; Y_{\mathsf{r}}^{(1)}|X_{\mathsf{a}}^{(1)}, Q), \Delta_2 I(X_{\mathsf{r}}^{(2)}; Y_{\mathsf{a}}^{(2)}|Q)\right\}$$

$$R_{\mathsf{a}} + R_{\mathsf{b}} < \Delta_1 I(X_{\mathsf{a}}^{(1)}, X_{\mathsf{b}}^{(1)}; Y_{\mathsf{r}}^{(1)}|Q)$$

over all joint distributions $p(q)p^{(1)}(x_{\mathsf{a}}|q)p^{(1)}(x_{\mathsf{b}}|q)p^{(2)}(x_{\mathsf{r}}|q)$ with $|\mathcal{Q}| \leq 5$.

Achievable rate regions: an example



$$R_{\mathsf{a}} < \min\left\{\Delta_1 I(X_{\mathsf{a}}^{(1)}; Y_{\mathsf{r}}^{(1)} | X_{\mathsf{b}}^{(1)}, Q), \Delta_2 I(X_{\mathsf{r}}^{(2)}; Y_{\mathsf{b}}^{(2)} | Q)\right\}$$

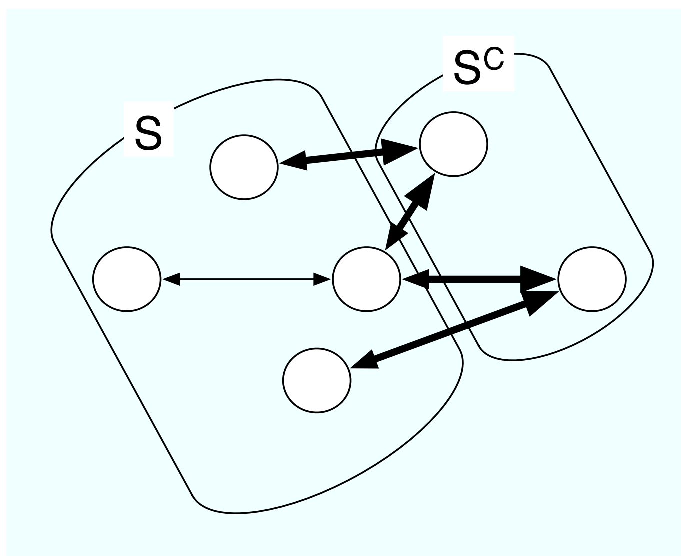
$$R_{\mathsf{b}} < \min\left\{\Delta_1 I(X_{\mathsf{b}}^{(1)}; Y_{\mathsf{r}}^{(1)} | X_{\mathsf{a}}^{(1)}, Q), \Delta_2 I(X_{\mathsf{r}}^{(2)}; Y_{\mathsf{a}}^{(2)} | Q)\right\}$$

$$R_{\mathsf{a}} + R_{\mathsf{b}} < \Delta_1 I(X_{\mathsf{a}}^{(1)}, X_{\mathsf{b}}^{(1)}; Y_{\mathsf{r}}^{(1)} | Q)$$

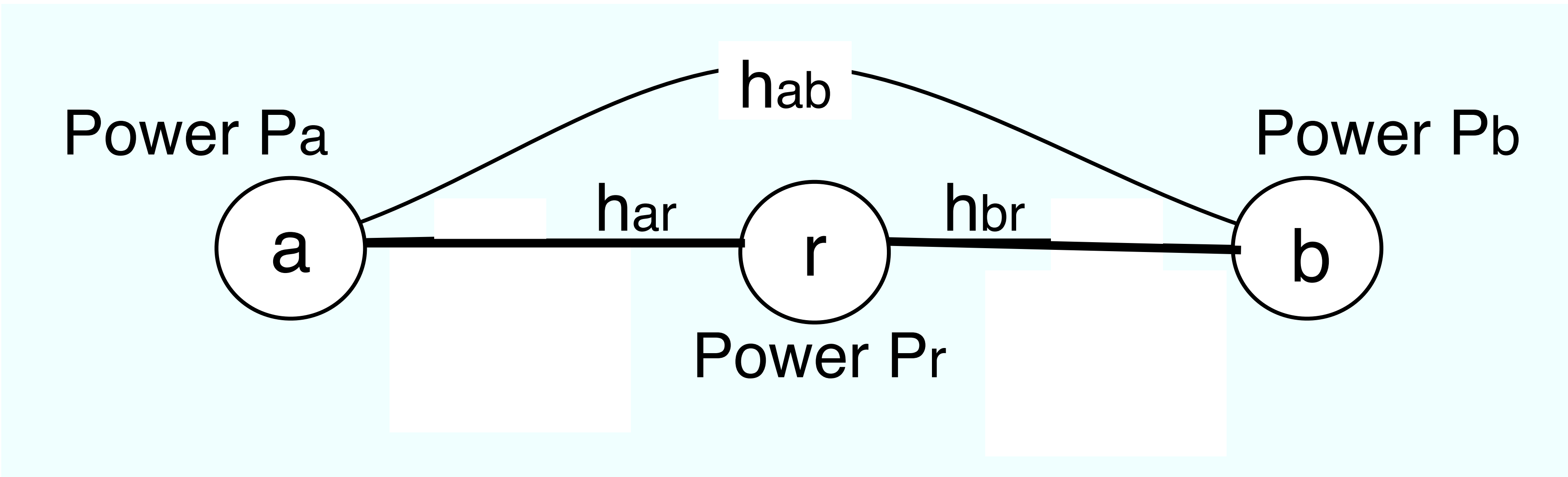
Outer bounds: cut-set bound

If the rates $\{R^{(ij)}\}$ are achievable with a protocol P and $R_\Sigma(S \to S^c)$ denotes the total rate of independent information sent from set S to set S^c then for all sets S:

$$R_\Sigma(S \to S^c) \leq \sum_i \Delta_i I(X^{(i)}_{(S)}; Y^{(i)}_{(S_c)} | X^{(i)}_{(S^c)}, Q).$$



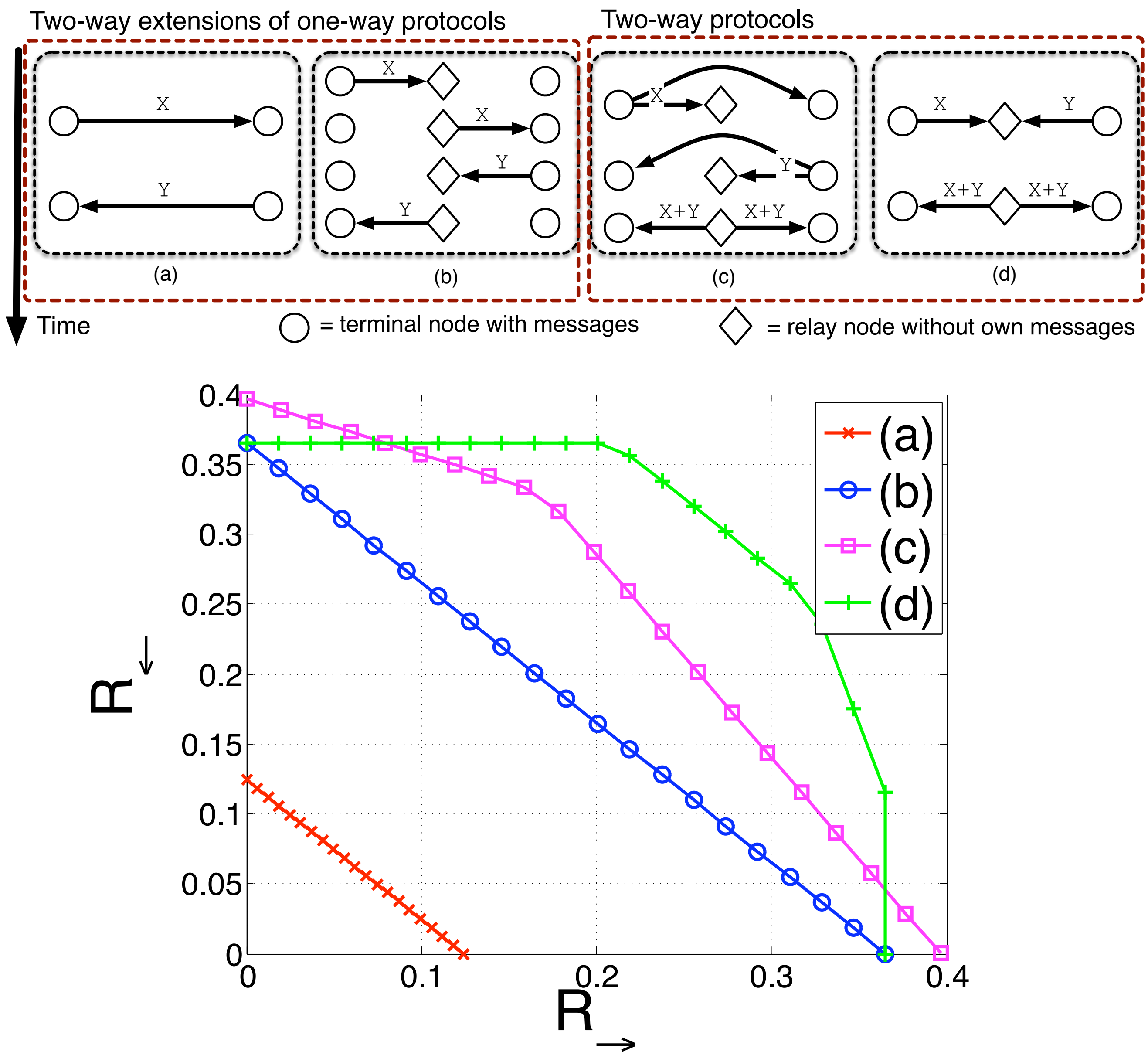
Simulations for the Gaussian noise channel



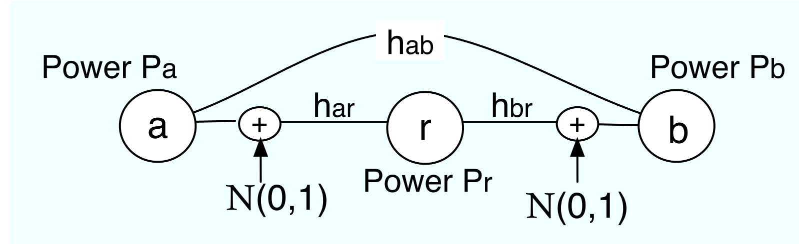
$$Y_r = h_{ar}X_a + h_{br}X_b + N_r, \quad N_r \sim \mathcal{N}(0,1)$$
$$Y_a = h_{ba}X_b + h_{ra}X_r + N_a, \quad N_a \sim \mathcal{N}(0,1)$$
$$Y_b = h_{ab}X_a + h_{rb}X_r + N_b, \quad N_b \sim \mathcal{N}(0,1)$$

(with appropriate half-duplex constraints)

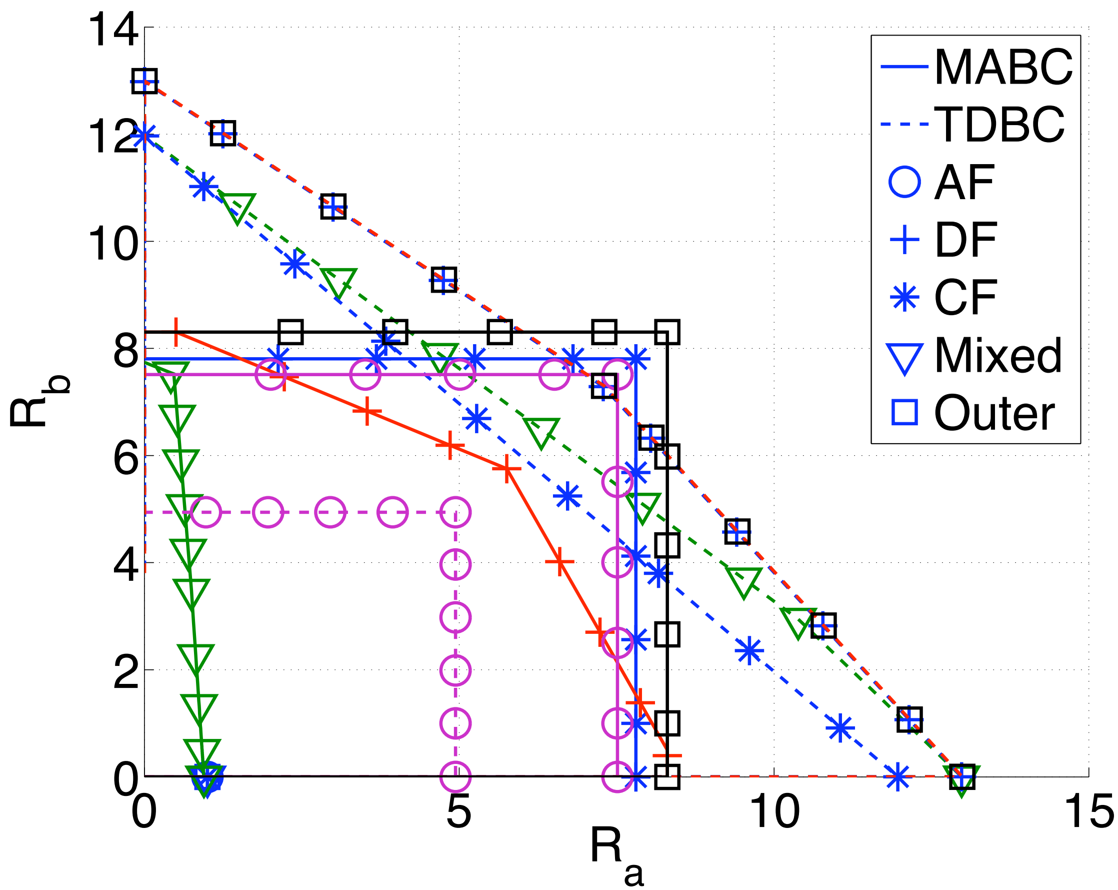
Gaussian simulations



Gaussian simulations



$$h_{\mathsf{ar}} = h_{\mathsf{br}} = 1,\ h_{\mathsf{ab}} = 0.2,\ N = 1, \text{ and } P = 50 \text{ dB}.$$



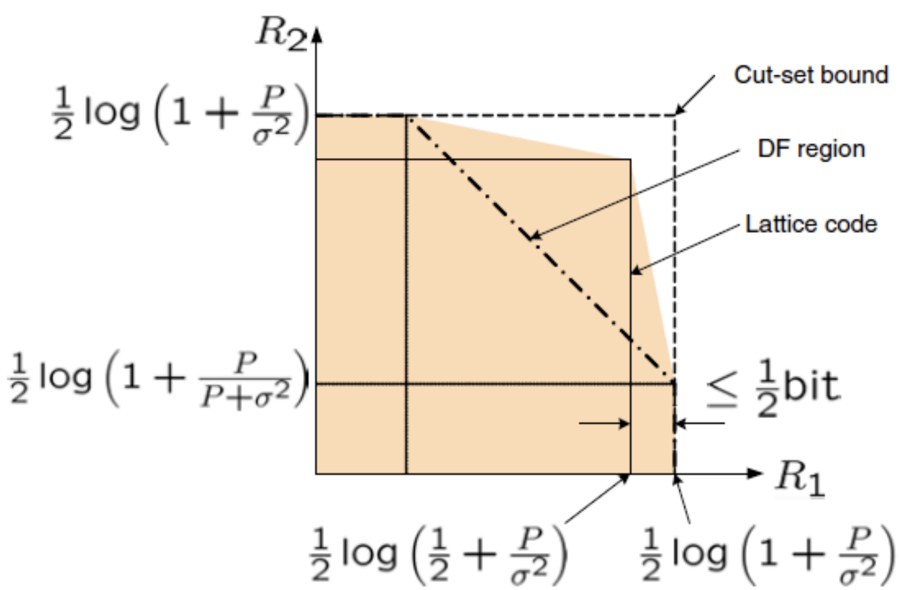
Recent developments: constant gap

- Capacity is known to within a constant # of bits in Gaussian noise *regardless of channel parameters!*

Capacity of the Gaussian two-way Relay channel to within 1/2 bit, W. Nam, S.-Y. Chung and W.H. Lee. Submitted to IEEE Trans. Inf, Theory.

Capacity of the Two Way Relay Channel within a Constant Gap, S. Avestimehr, A. Sezgin, and D. Tse, European Transactions on Telecommunications, to appear.

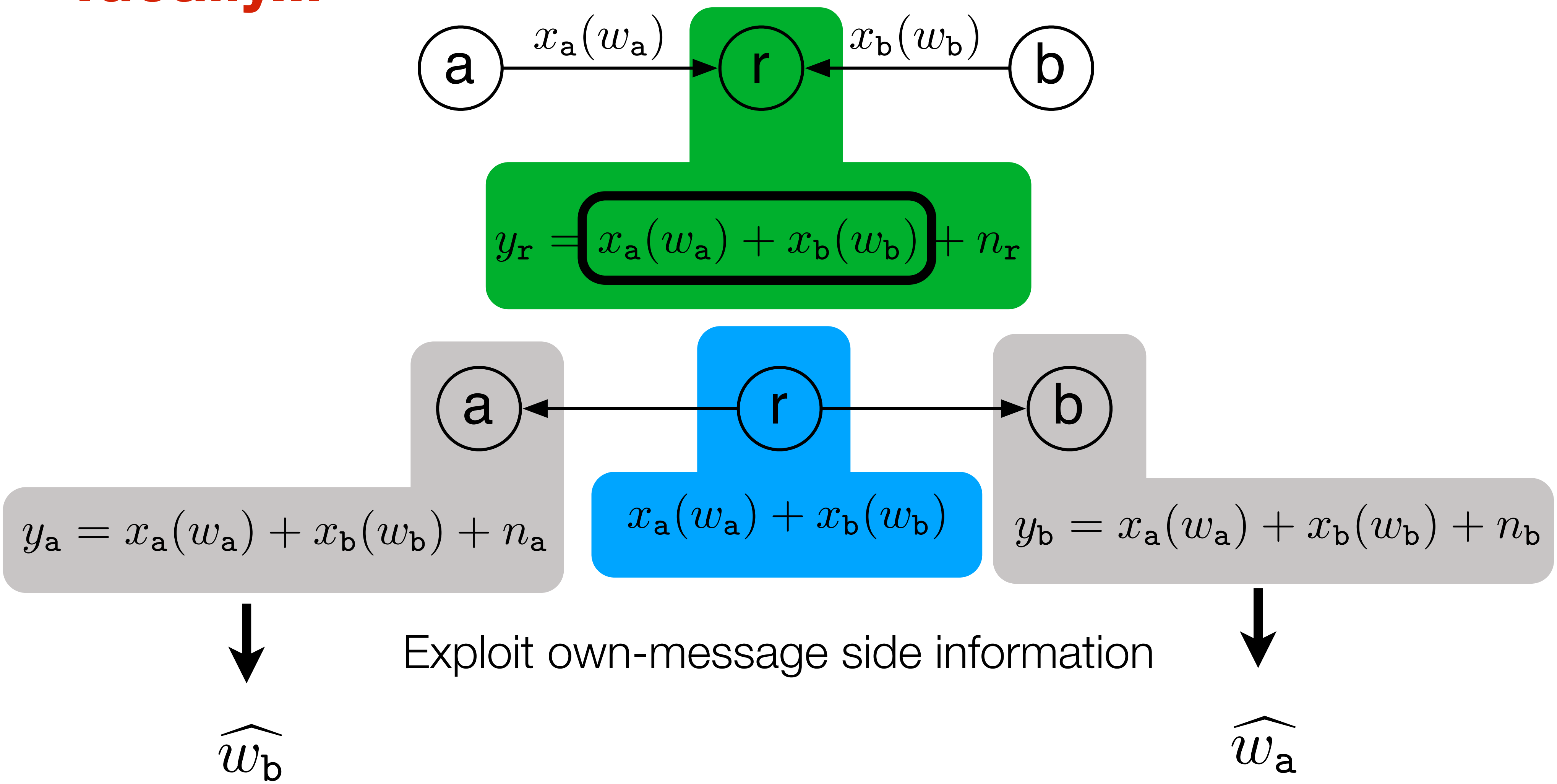
Noisy Network Coding, S.H. Lim, Y.-H. Kim, A. El Gamal and S.-Y. Chung, presented at ISIT 2010, on Arxiv http://arxiv.org/abs/1002.3188



[Capacity of the Gaussian two-way Relay channel to within 1/2 bit](), W. Nam, S.-Y. Chung and W.H. Lee. Submitted to IEEE Trans. Inf, Theory.

Recent developments: usefulness of lattice codes

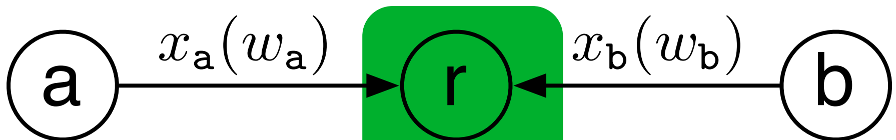
Ideally...



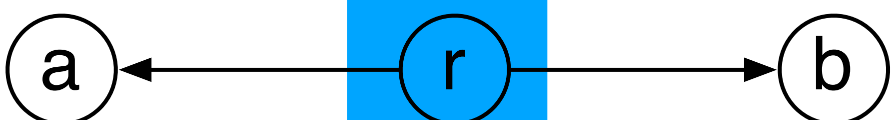
Capacity of the Gaussian two-way Relay channel to within 1/2 bit, W. Nam, S.-Y. Chung and W.H. Lee. Submitted to IEEE Trans. Inf, Theory.

Recent developments: usefulness of lattice codes

Reality....



$$y_{\mathtt{r}} = x_{\mathtt{a}}(w_{\mathtt{a}}) + x_{\mathtt{b}}(w_{\mathtt{b}}) + n_{\mathtt{r}}$$



Random coding

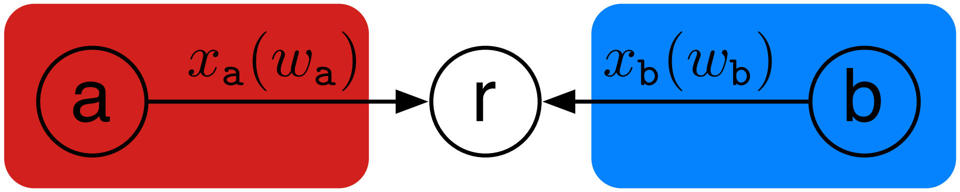
- $x_{\mathtt{a}}(w_{\mathtt{a}}) + x_{\mathtt{b}}(w_{\mathtt{b}})$ **NOT** a codeword
- decode **both** messages
- send $x_{\mathtt{r}}(w_{\mathtt{a}} \oplus w_{\mathtt{b}})$

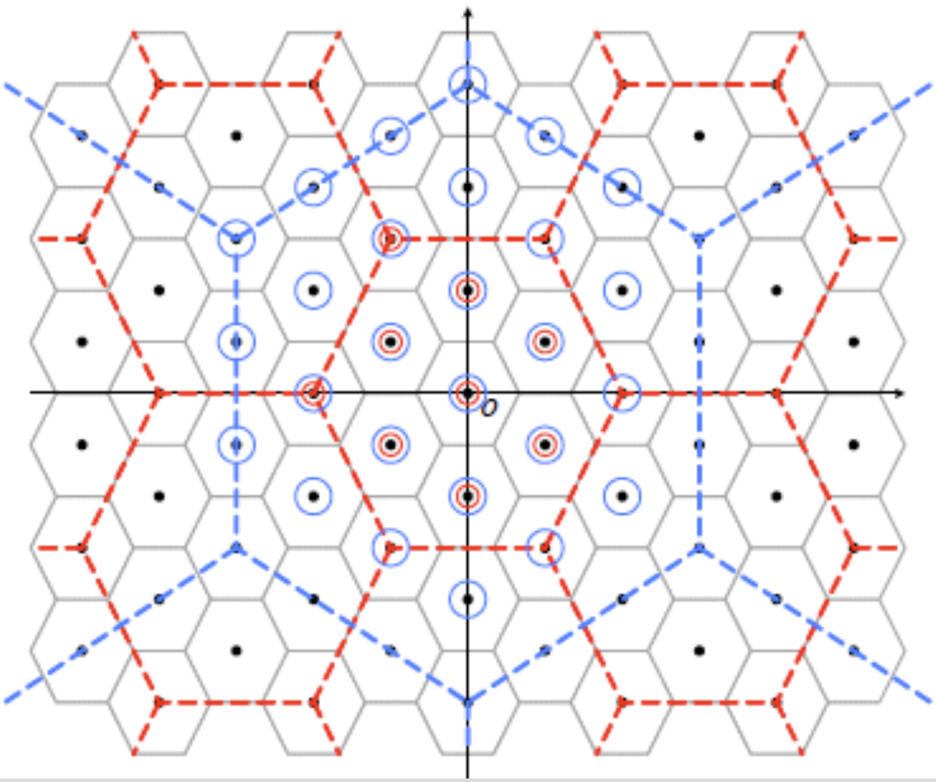
Structured (lattice) coding

- $x_{\mathtt{a}}(w_{\mathtt{a}}) + x_{\mathtt{b}}(w_{\mathtt{b}})$ **IS** a codeword
- no multiple access constraints as decode the sum

Capacity of the Gaussian two-way Relay channel to within 1/2 bit, W. Nam, S.-Y. Chung and W.H. Lee. Submitted to IEEE Trans. Inf, Theory.

Recent developments: nested modulo lattice codes

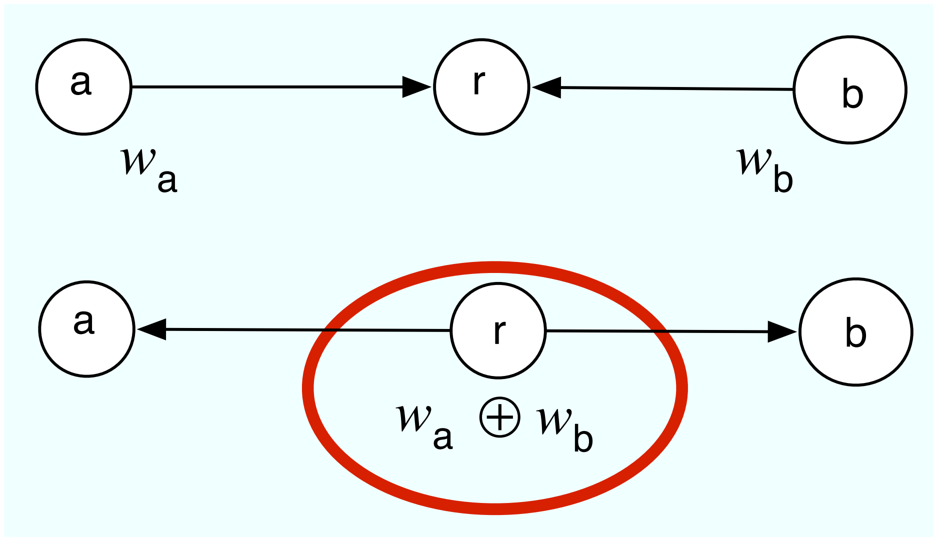




Sum of codewords is a codeword - relay decodes it!!

Relation to network coding?

- bit-level / packet level network coding ➔ **Decode and Forward (DF)**

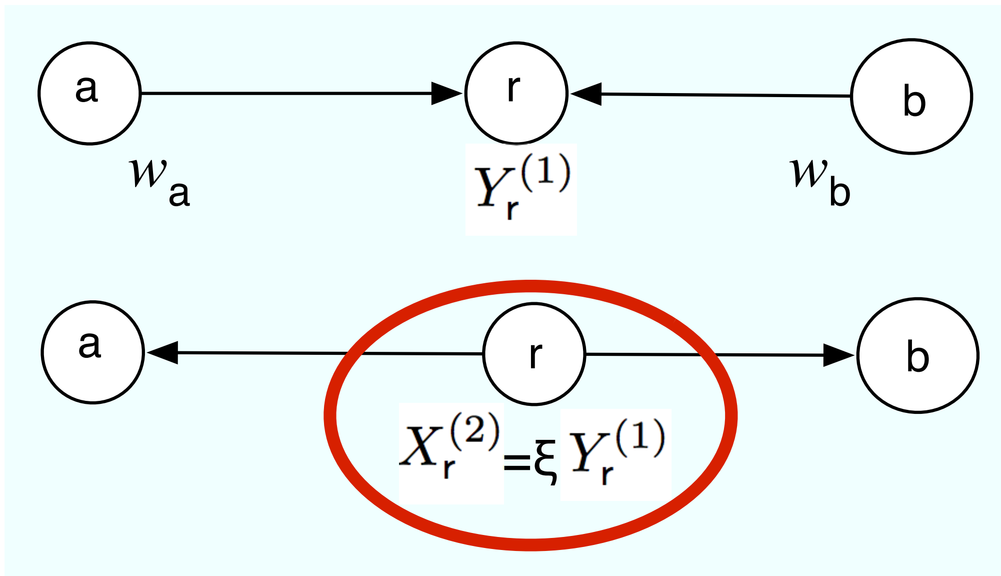


- excellent systems-level demonstration of 2-way relaying gains (all layers, actual testbed)

S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the air: Practical wireless network coding," in *ACM SIGCOMM*, Pisa, Sep. 2006.

Relation to network coding?

- physical / analog network coding ➔ similar to **Amplify and Forward (AF)**

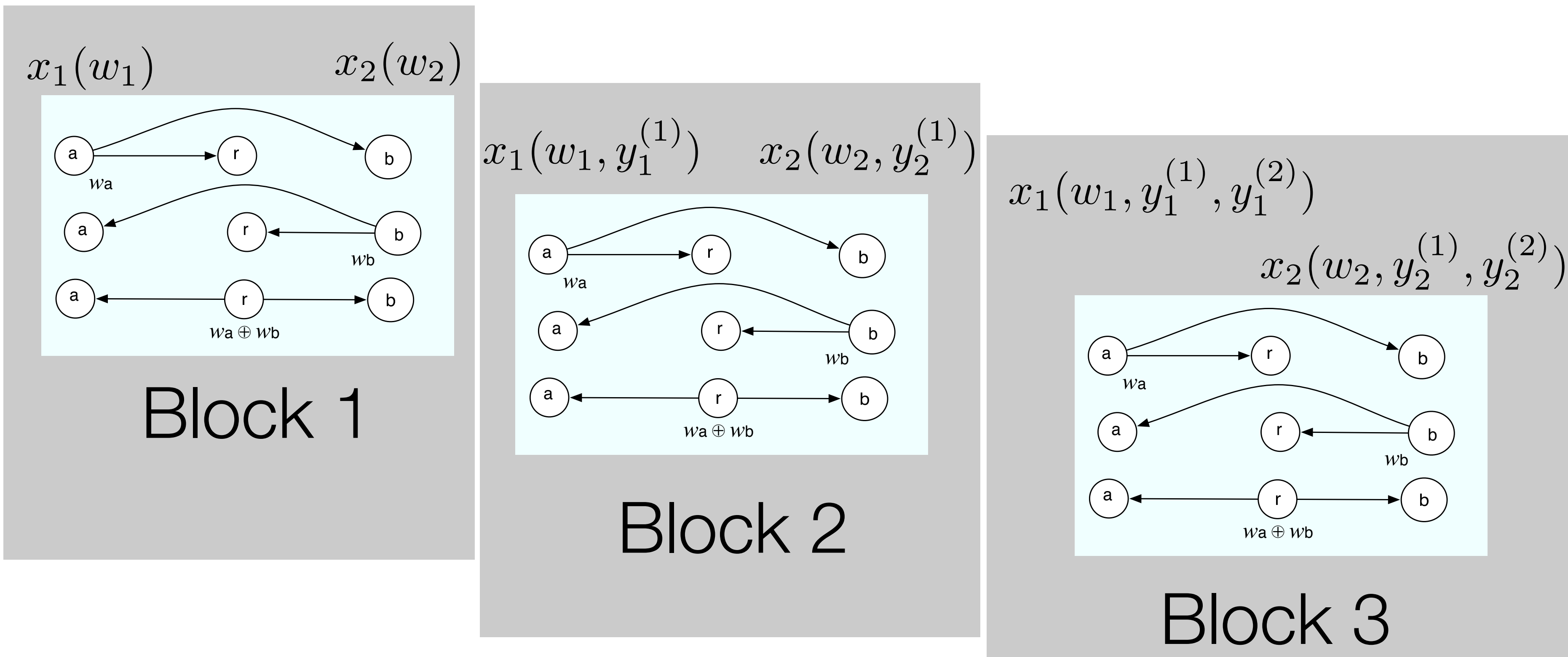


- excellent systems-level demonstration of analog network coding (all layers, actual testbed)

] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *ACM SIGCOMM*, Kyoto, Aug. 2007.

Open questions

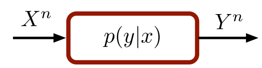
- codeword adaptation?



- **Potential to increase throughput, but will it be used beyond academic demonstrations?**

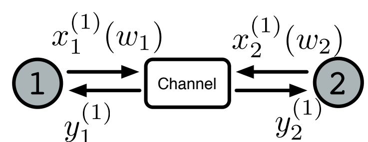
Outline

$$C = \max_{p(x)} I(X;Y)$$



- Information theory - *what, why, when*

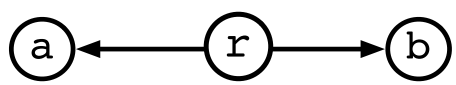
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel



- Two-way channel - *channel coding*

  Adaptive codewords, capacity in Gaussian noise = two parallel channels

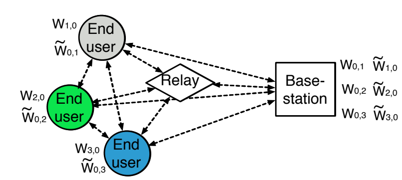
- **Two-way relay channels**



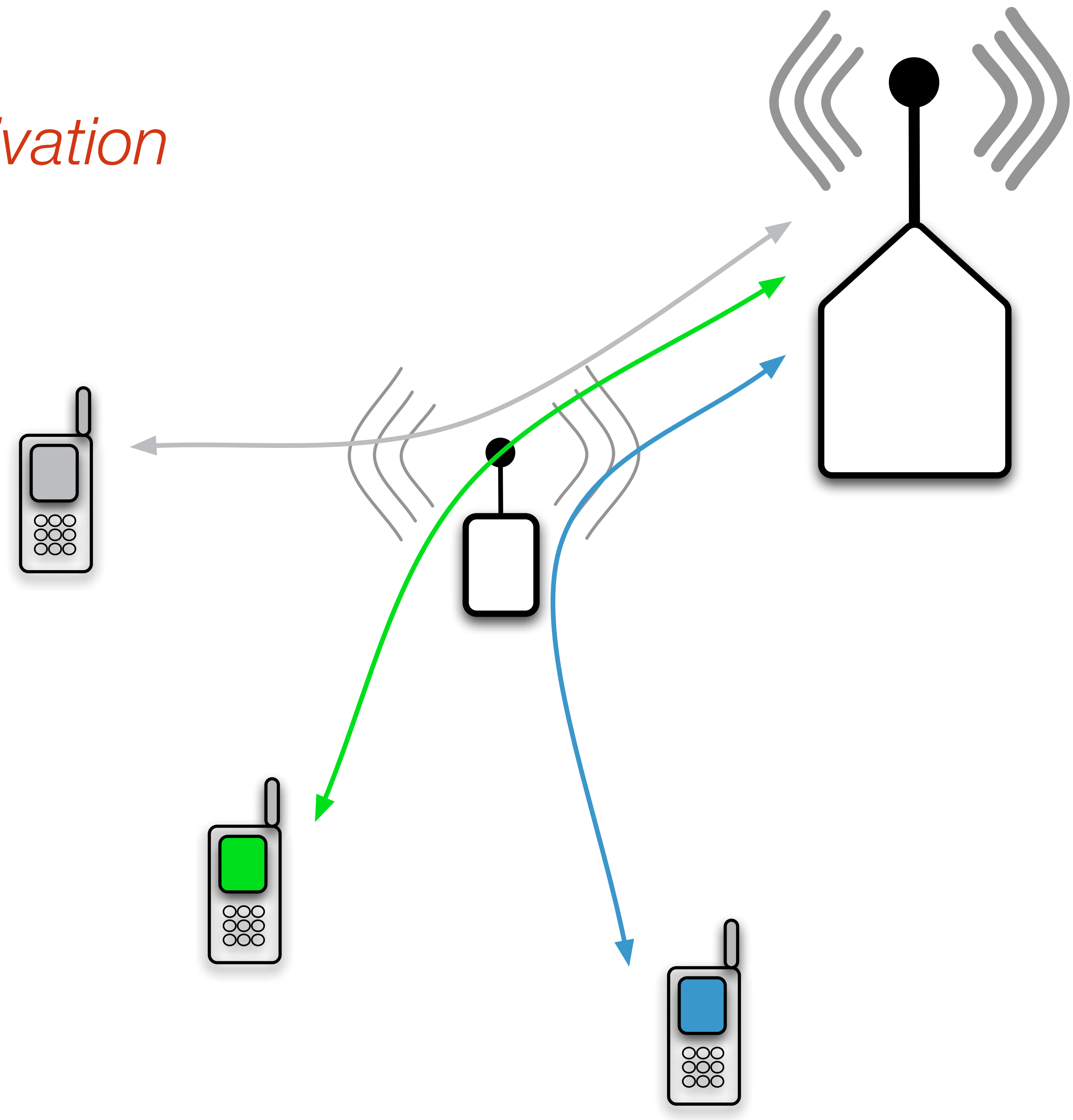
$$w_r = w_1 \oplus w_2$$

  - *single flow - canonical example of wireless network coding*

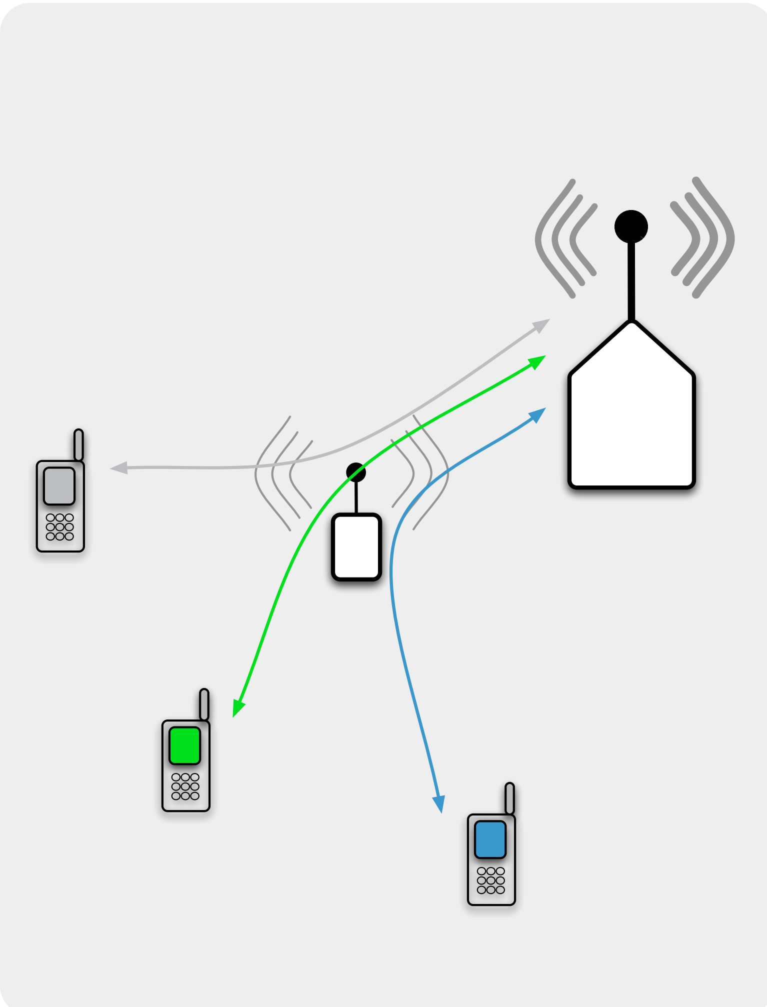
  - ***multiple flows with a base-station*** *- pairwise wireless network coding*

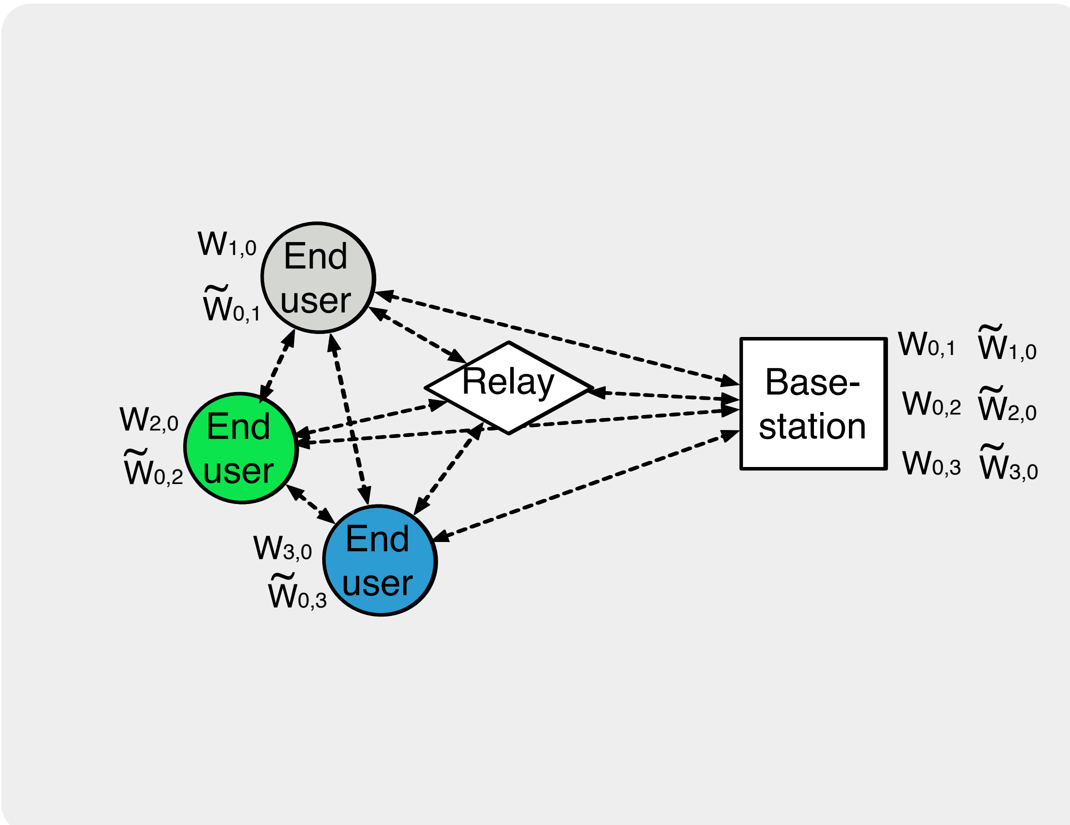


Motivation



Motivation





$W_{1,0}$
$\tilde{W}_{0,1}$
End user
$W_{2,0}$
$\tilde{W}_{0,2}$
End user
$W_{3,0}$
$\tilde{W}_{0,3}$
End user
Relay
Base-station
$W_{0,1}$ $\tilde{W}_{1,0}$
$W_{0,2}$ $\tilde{W}_{2,0}$
$W_{0,3}$ $\tilde{W}_{3,0}$

Multiple terminals: multiple two-way

A. Avestimehr, A. Sezgin, and D. Tse, “Capacity region of the deterministic multi-pair bi-directional relay network,” in *Proc. IEEE Inf. Theory Workshop*, Volos, June 2009.

H. Ghozlan, Y. Mohasseb, H. El Gamal, and G. Kramer, “The MIMO wireless switch: Relaying can increase the multiplexing gain,” 2009. [Online]. Available: http://arxiv.org/abs/0901.2588

A. Sezgin, A. Khajehnejad, A. Avestimehr, and B. Hassibi, “Approximate capacity region of the two-pair bidirectional gaussian relay network,” in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 2018–2022.

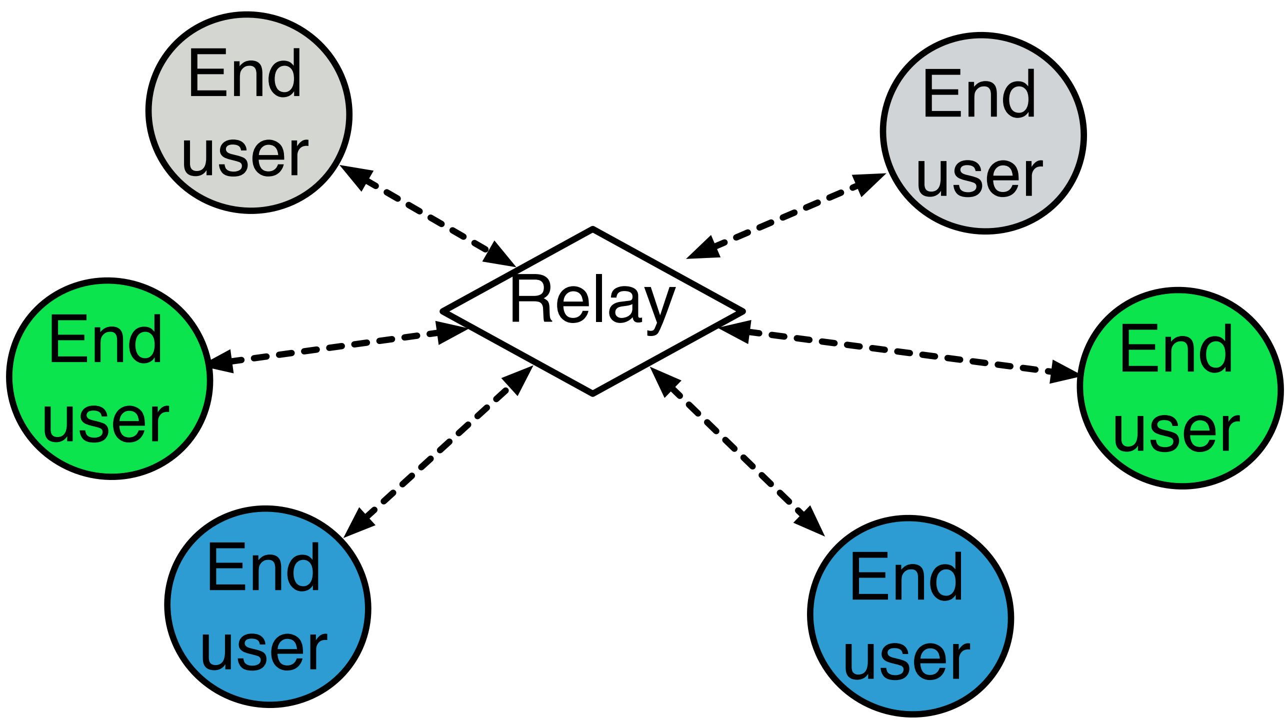
M. Chen and A. Yener, “Power allocation for F/TDMA multiuser two-way relay networks,” *IEEE Trans. Wireless Comm.*, vol. 9, no. 2, pp. 546–551, 2010.

D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, “The multi-way relay channel,” in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 339–343.

D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, “The multi-way relay channel,” http://arxiv.org/abs/1004.2434/.

L. Ong, S. Johnson, and C. Kellett, “An Optimal Coding Strategy for the Binary Multi-Way Relay Channel,” http://arxiv4.library.cornell.edu/abs/1004.2299.

L. Ong, C. Kellett, and S. Johnson, “Capacity Theorems for the AWGN Multi-Way Relay Channel,” http://arxiv4.library.cornell.edu/abs/1004.2300.



Multiple terminals: with base-station

A. Avestimehr, A. Sezgin, and D. Tse, "Capacity region of the deterministic multi-pair bi-directional relay network," in *Proc. IEEE Inf. Theory Workshop*, Volos, June 2009.

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M. Chen and A. Yener, "Interference management for multiuser two-way relaying," in *Proc. Conf. on Inf. Sci. and Sys.*, Princeton, Mar. 2008, pp. 246–251.

A. Sezgin, A. Khajehnejad, A. Avestimehr, and B. Hassibi, "Approximate capacity region of the two-pair bidirectional gaussian relay network," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 2018–2022.

D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 339–343.

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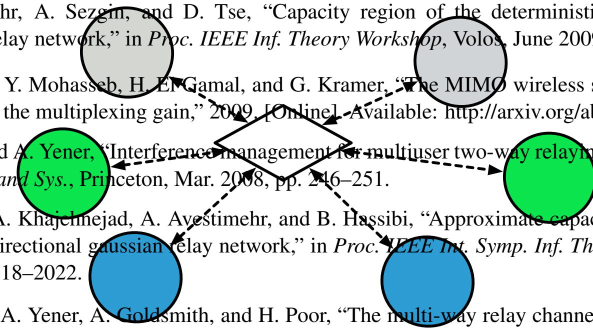
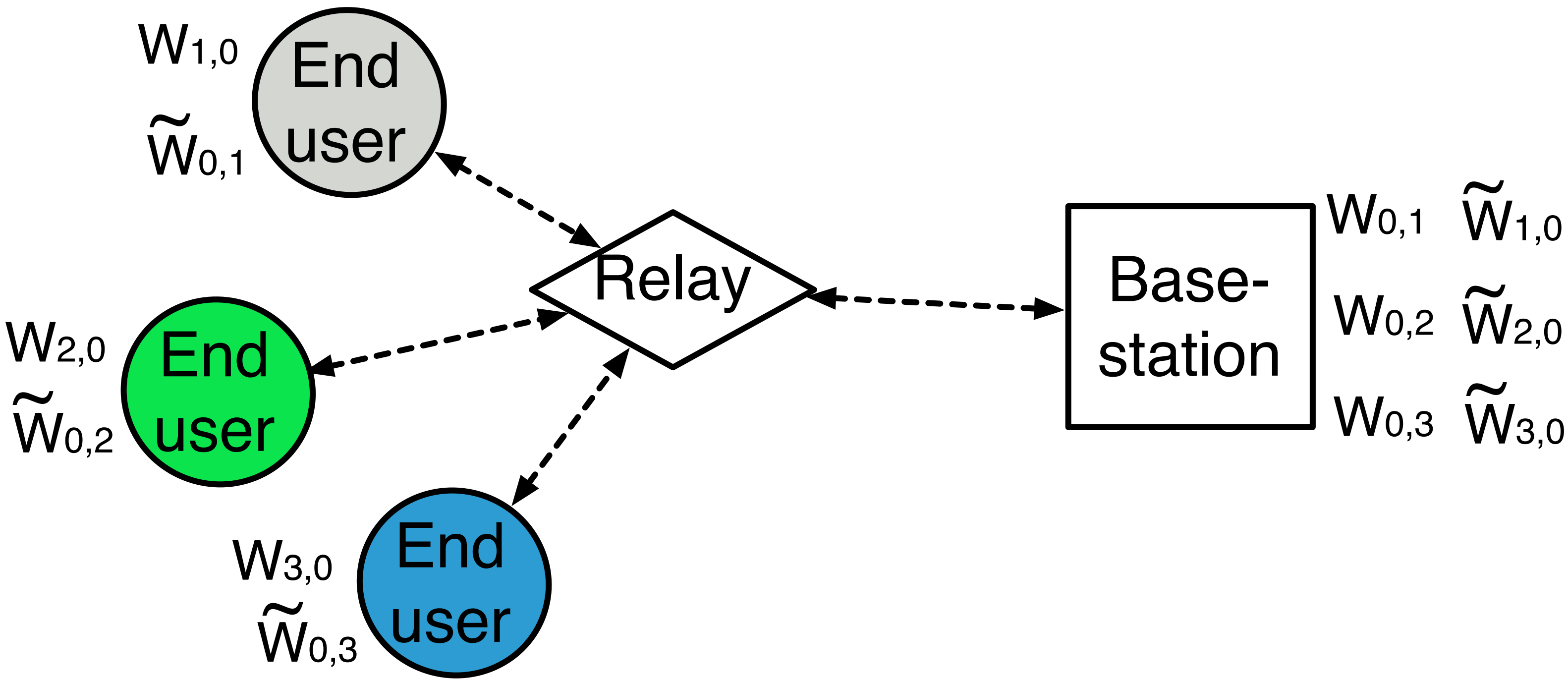
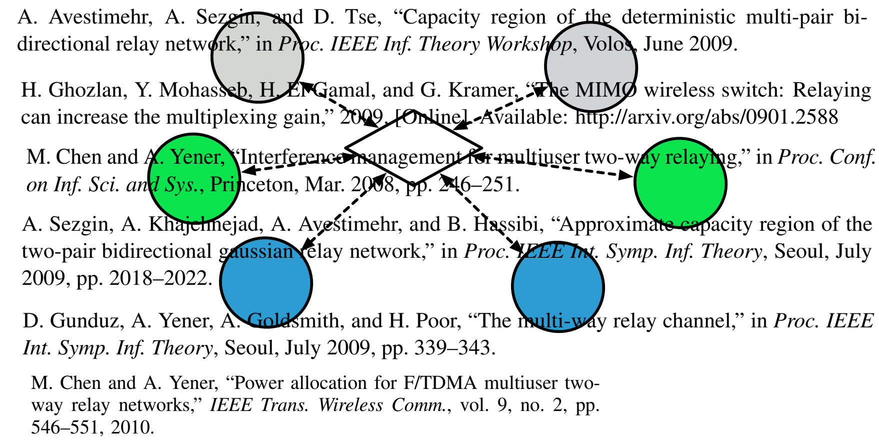




Multiple terminals: with base-station



A. Avestimehr, A. Sezgin, and D. Tse, “Capacity region of the deterministic multi-pair bi-directional relay network,” in *Proc. IEEE Inf. Theory Workshop*, Volos, June 2009.

H. Ghozlan, Y. Mohasseb, H. El Gamal, and G. Kramer, “The MIMO wireless switch: Relaying can increase the multiplexing gain,” 2009. [Online]. Available: http://arxiv.org/abs/0901.2588

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A. Sezgin, A. Khajehnejad, A. Avestimehr, and B. Hassibi, “Approximate capacity region of the two-pair bidirectional gaussian relay network,” in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 2018–2022.

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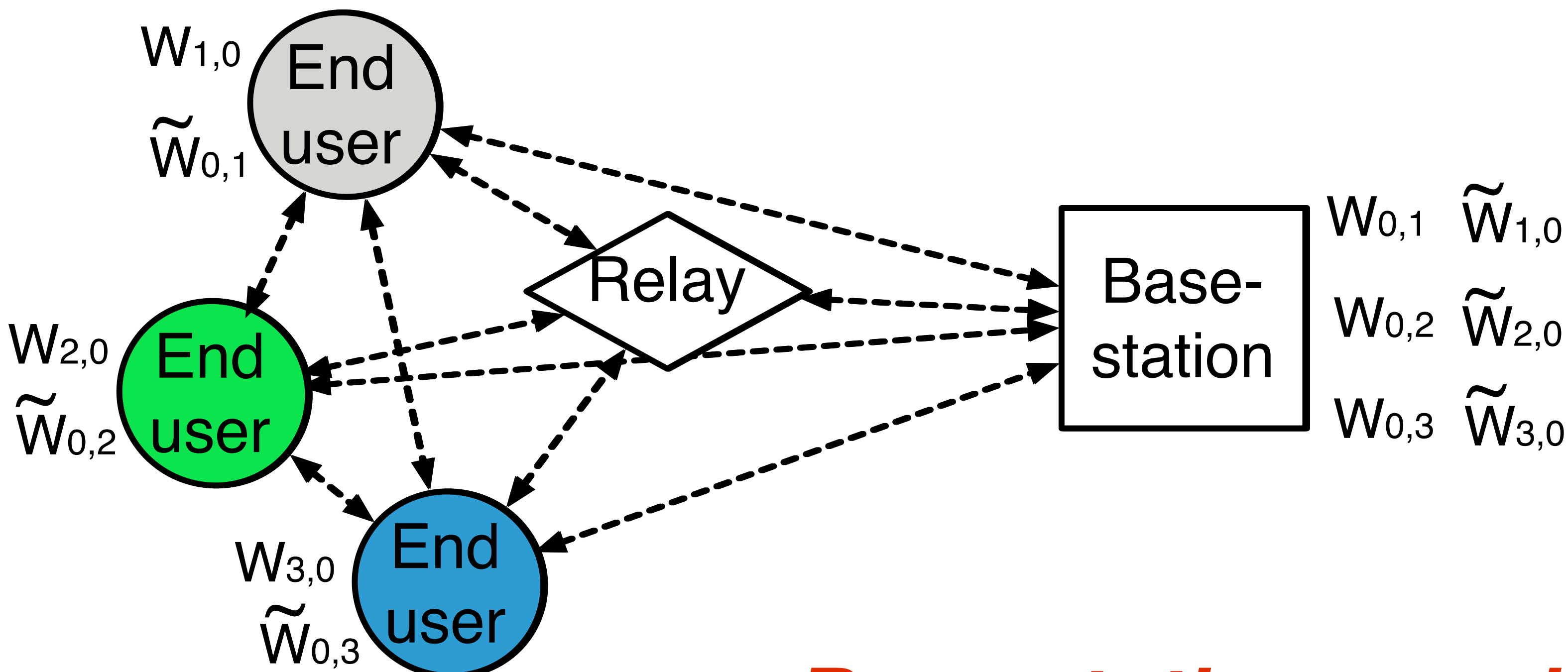
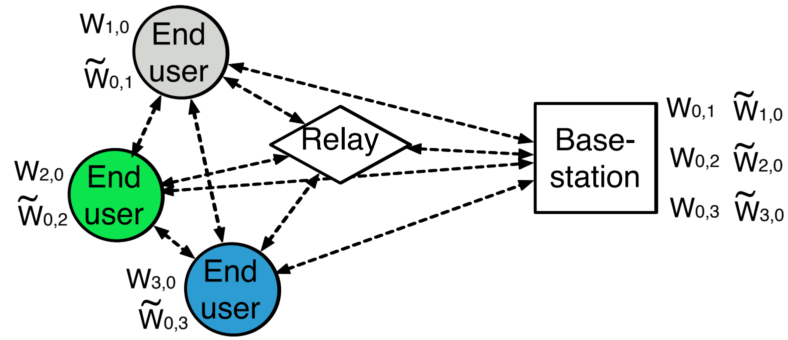
M. Chen and A. Yener, “Power allocation for F/TDMA multiuser two-way relay networks,” *IEEE Trans. Wireless Comm.*, vol. 9, no. 2, pp. 546–551, 2010.



Base-station = node 0

Arbitrary (m) number of end users



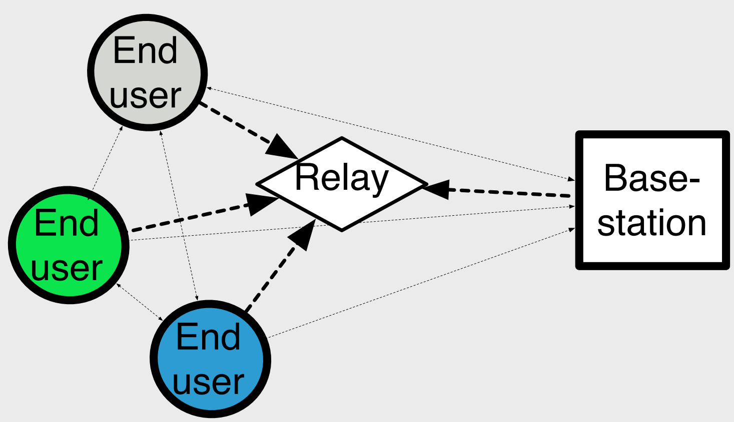
Half-duplex nodes

Decode + forward relay

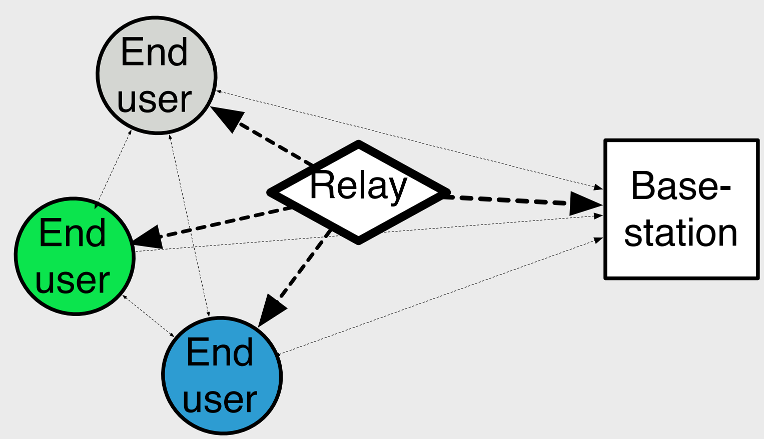
Compress + forward end user cooperation

Per-flow network coding of messages at relay

"Protocols" = time "phases"

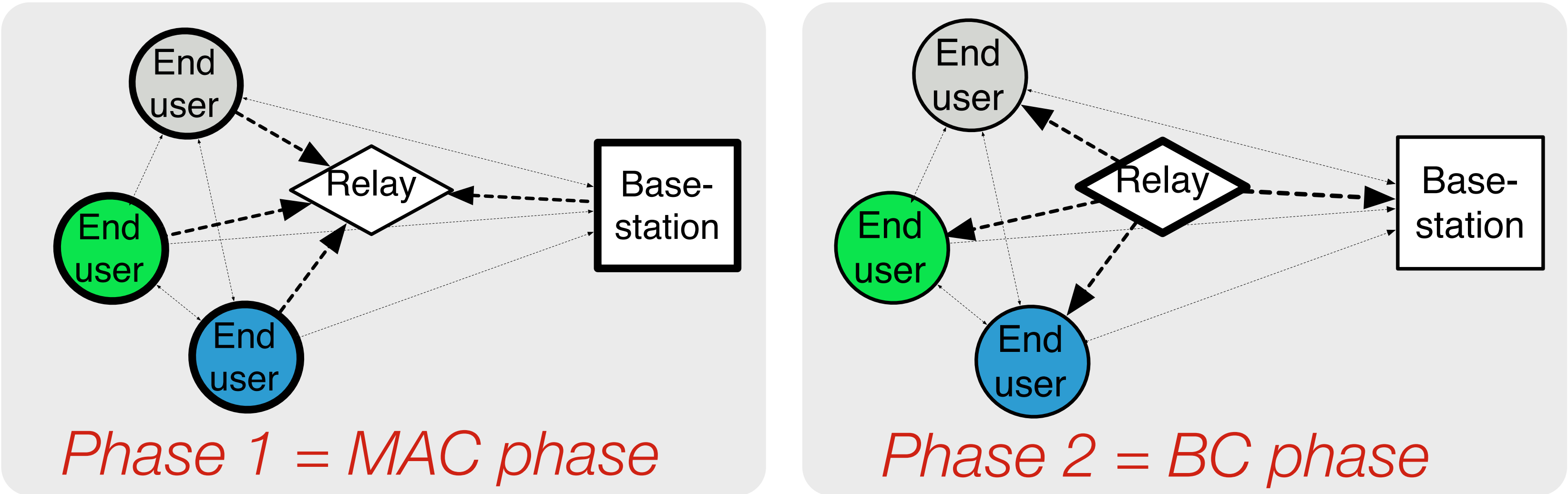


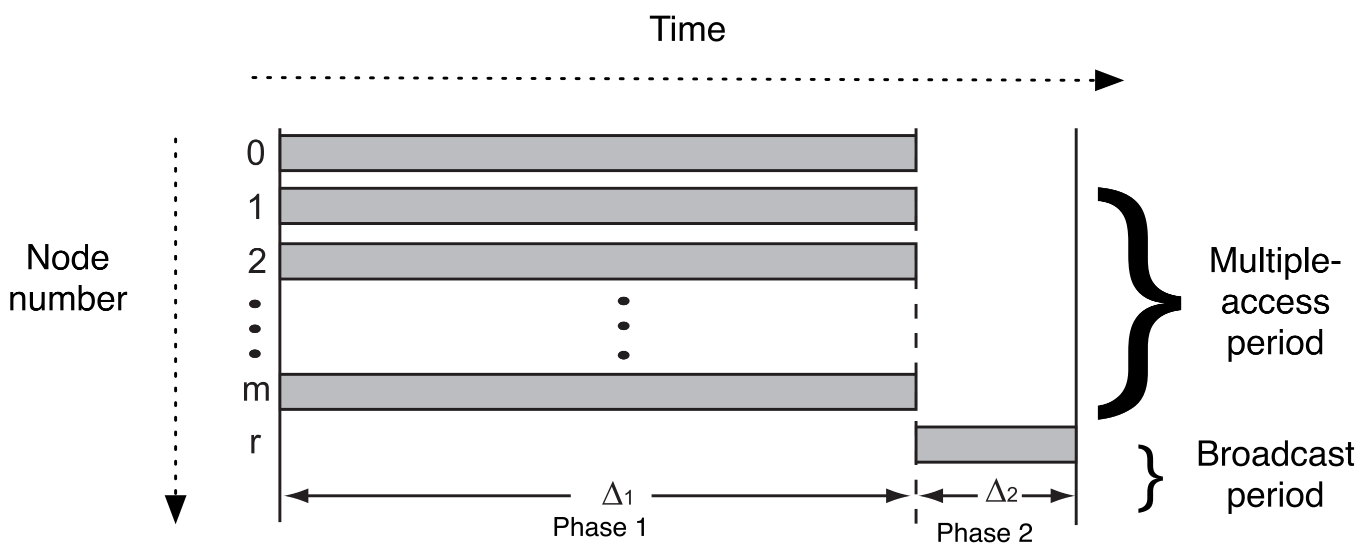
Phase 1 = MAC phase



Phase 2 = BC phase

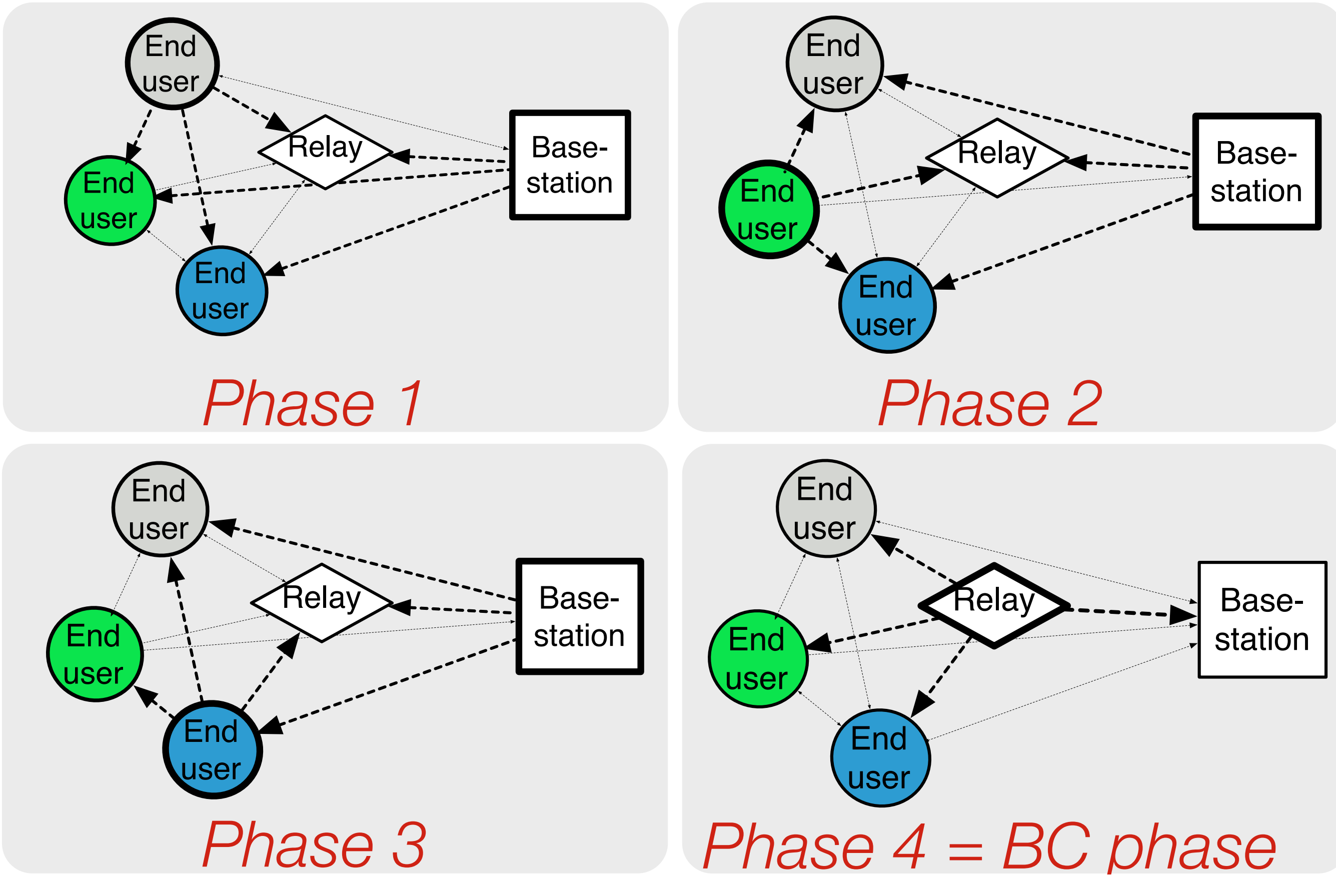
Protocol 1: FMABC (Full MAC then BC)

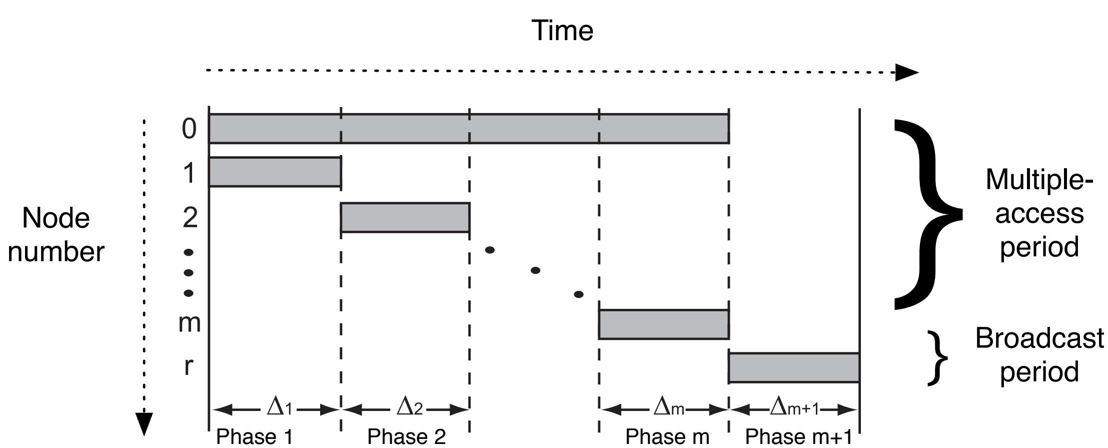




(a) FMABC protocol

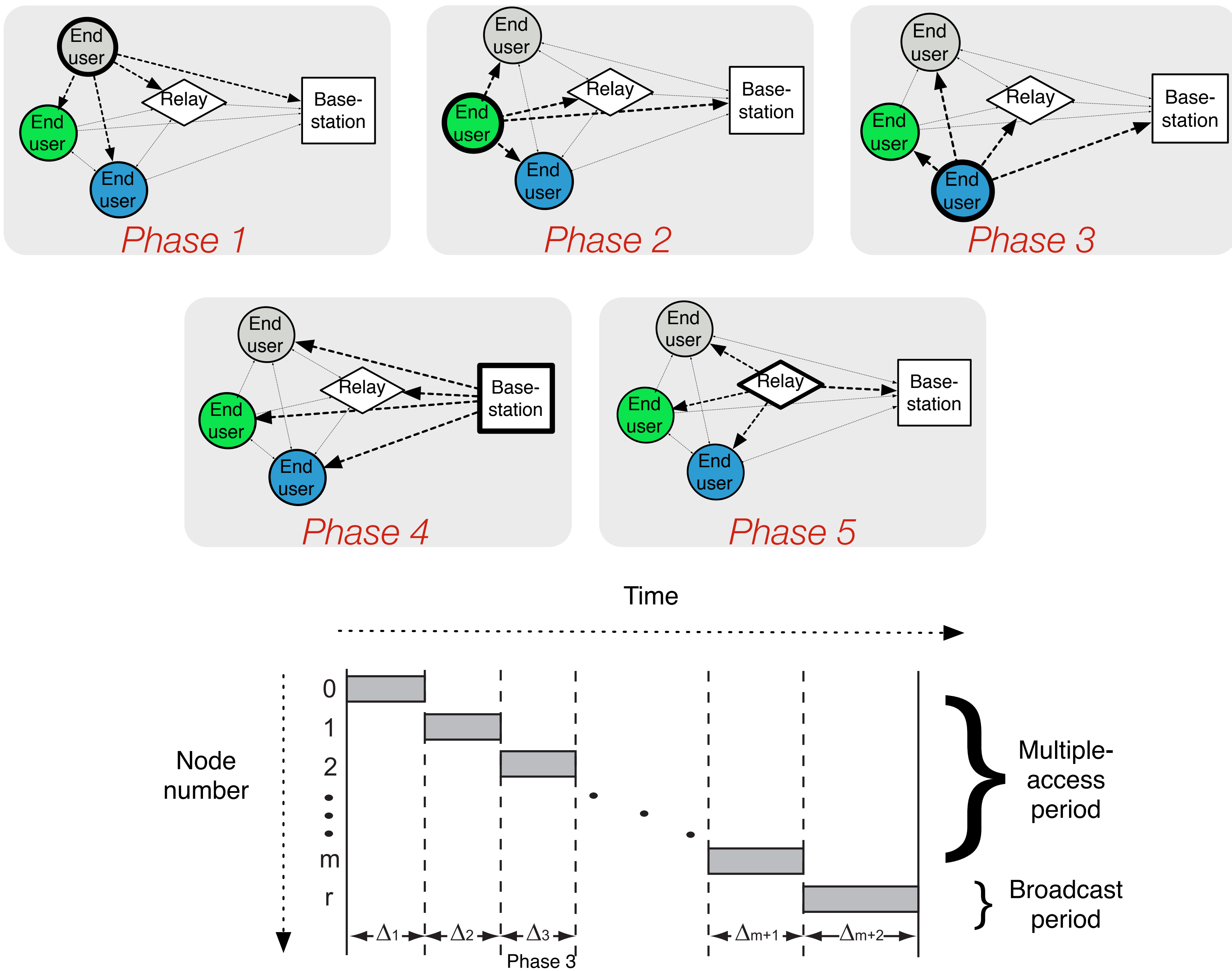
Protocol 2: PMABC (Partial MAC then BC)





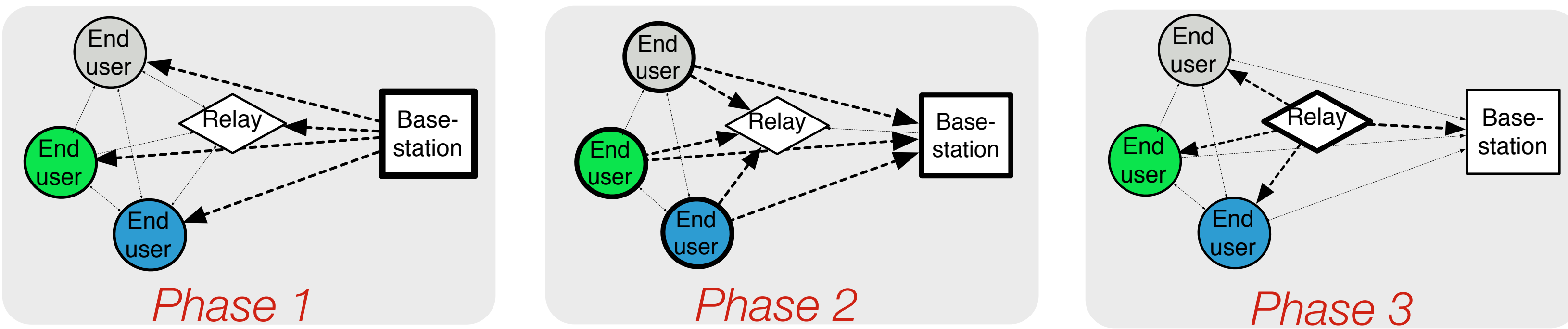
(b) PMABC protocol

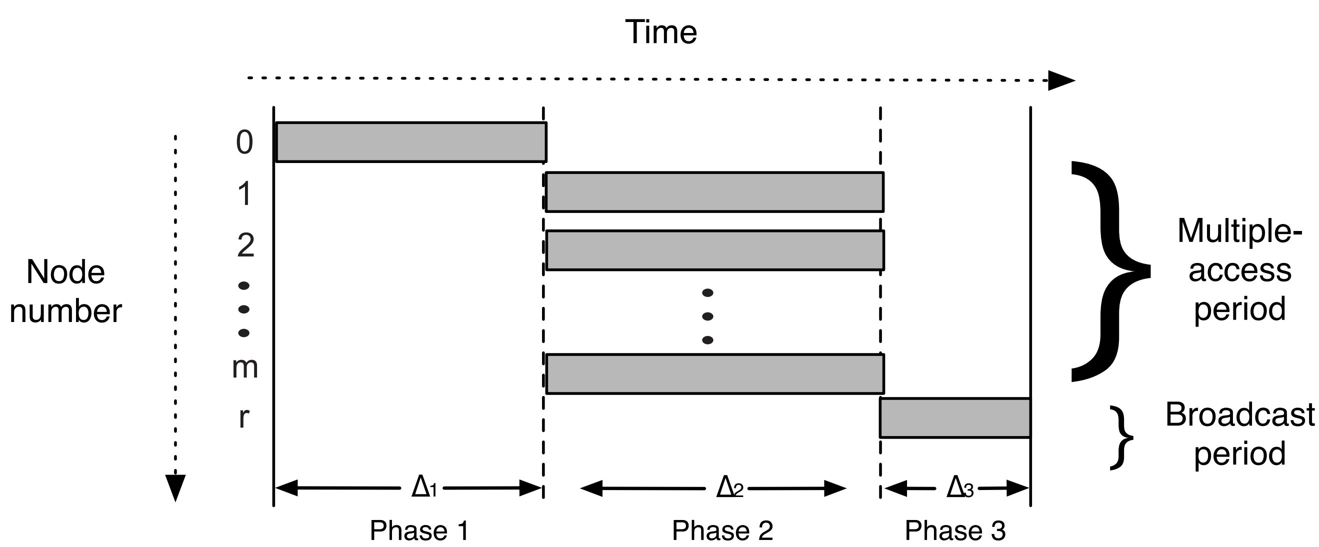
Protocol 3: FTDBC (Full Time Division then BC)



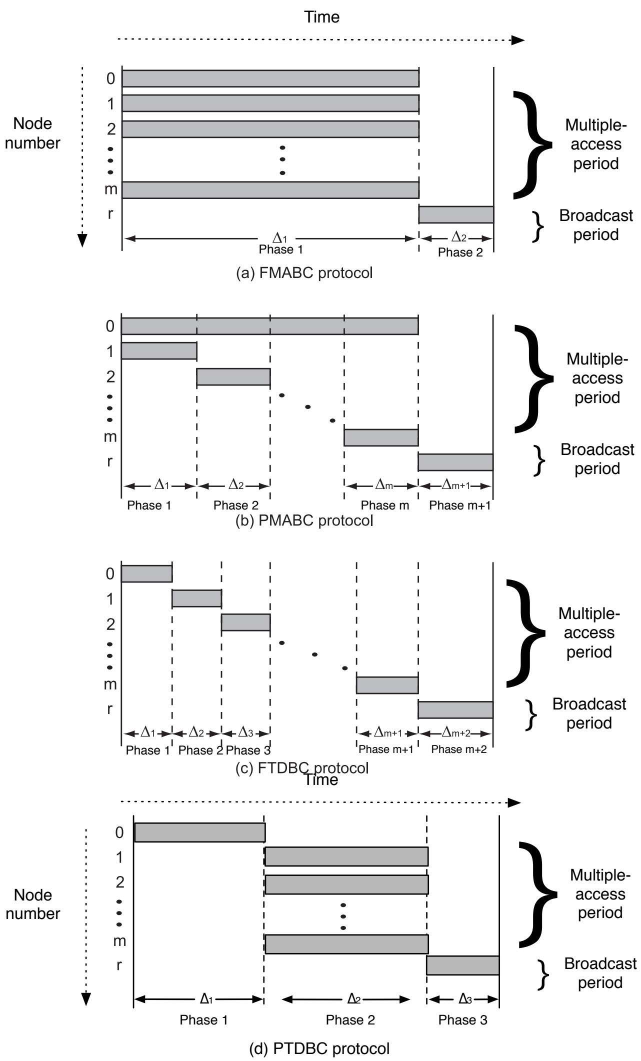
(c) FTDBC protocol

Protocol 4: PTDBC (Partial Time Division then BC)





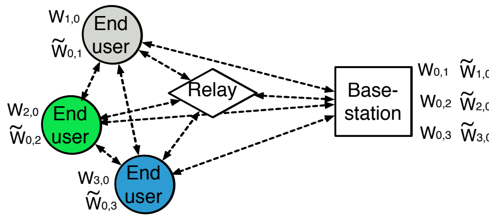
(d) PTDBC protocol

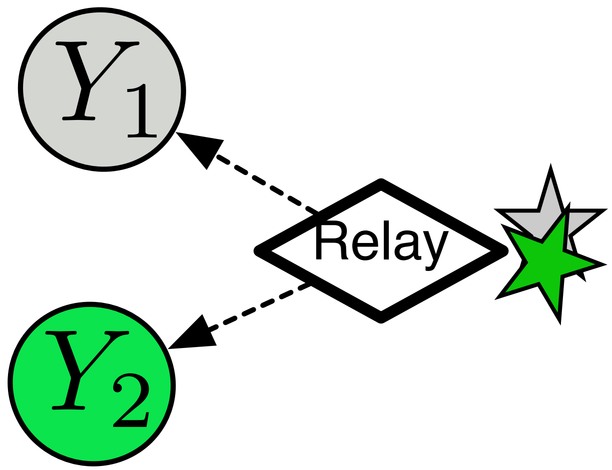


Which protocol is "better"?

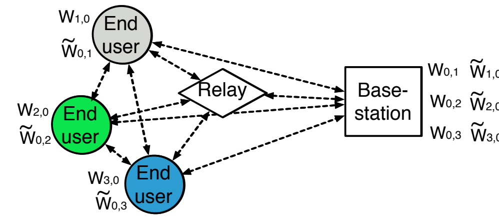
1. Multi-message broadcasting (Marton's region)
2. Per-flow network coding
3. Exploiting side-information (random binning)
4. CF-based terminal node cooperation

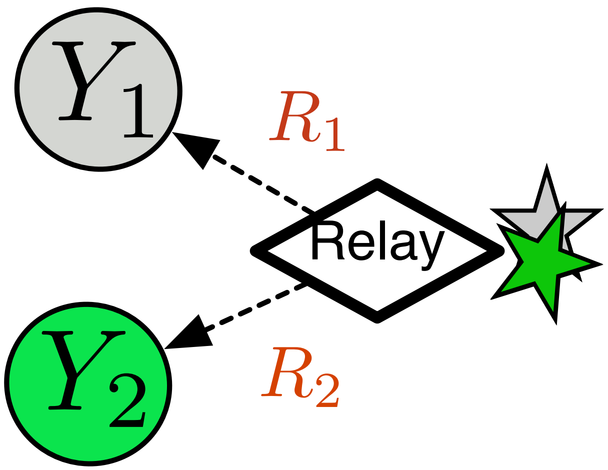
Broadcasting 2 messages





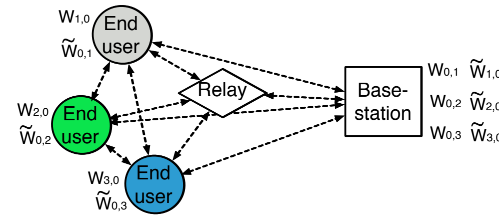
Broadcasting 2 messages

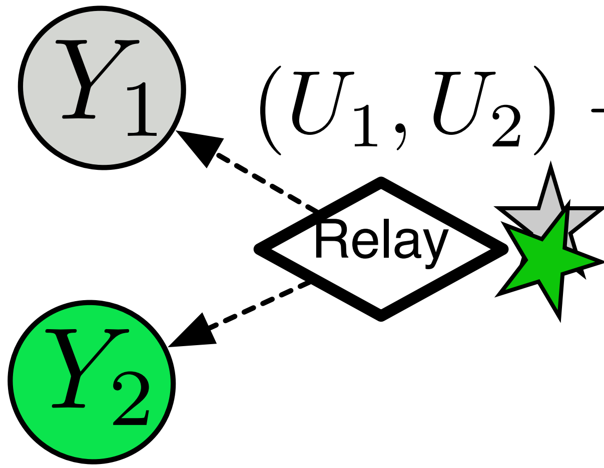




At what rates can we reliably broadcast 2 messages?

Marton's region





$$(U_1, U_2) - X$$

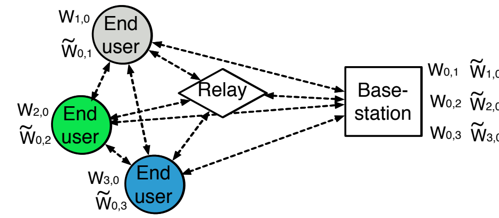
$$R_1 \leq I(U_1; Y_1)$$

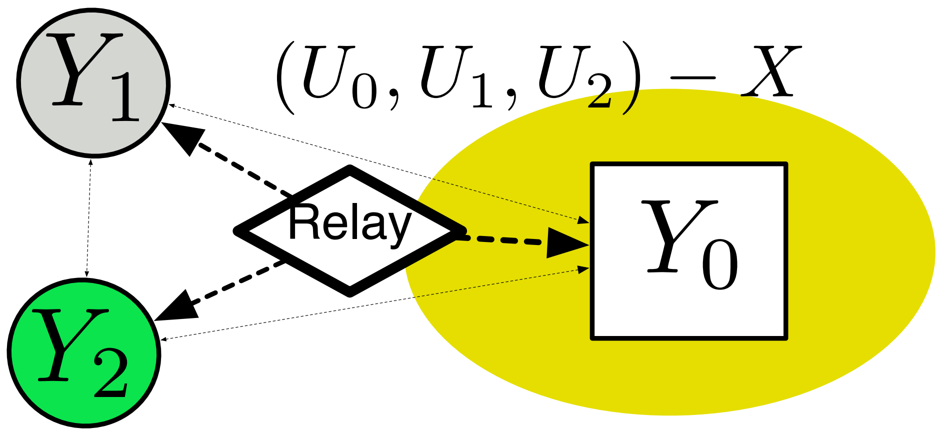
$$R_2 \leq I(U_2; Y_2)$$

$$R_1 + R_2 \leq I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2)$$

over all joint distributions $p(u_1, u_2, x)$

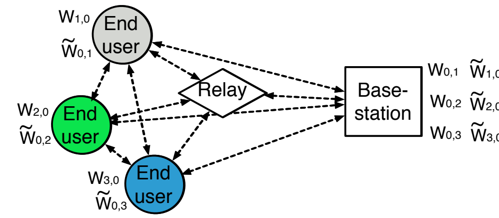
Extended Marton's in our notation





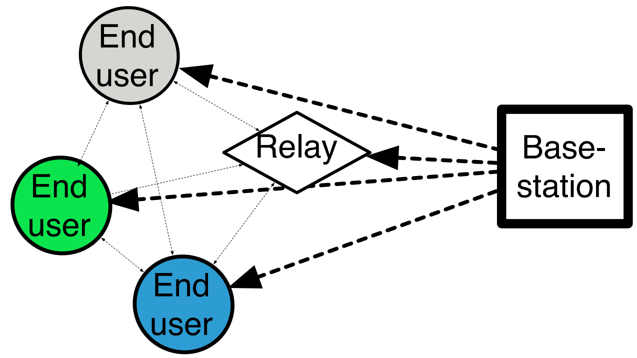
$$
\begin{aligned}
R_{0,1} &\leq I(U_1;Y_1) \\
R_{0,2} &\leq I(U_2;Y_2) \\
R_{1,0} + R_{2,0} &\leq I(U_0;Y_0) \\
R_{1,0} + R_{2,0} + R_{0,1} &\leq I(U_0;Y_0) + I(U_1;Y_1) - I(U_0;U_1) \\
R_{1,0} + R_{2,0} + R_{0,2} &\leq I(U_0;Y_0) + I(U_2;Y_2) - I(U_0;U_2) \\
R_{0,1} + R_{0,2} &\leq I(U_1;Y_1) + I(U_2;Y_2) - I(U_1;U_2) \\
R_{1,0} + R_{2,0} + R_{0,1} + R_{0,2} &\leq I(U_0;Y_0) + I(U_1;Y_1) + I(U_2;Y_2) - I(U_1;U_0) - I(U_2;U_1,U_0)
\end{aligned}
$$

over all joint distributions $p(u_0, u_1, u_2, x)$

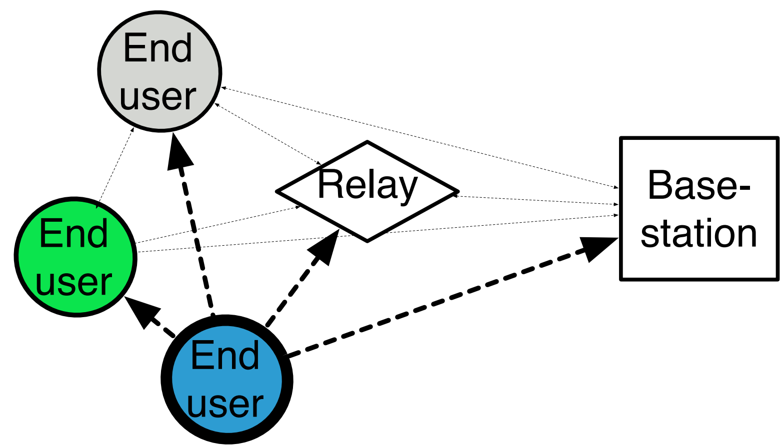


Use extended Marton's region at:

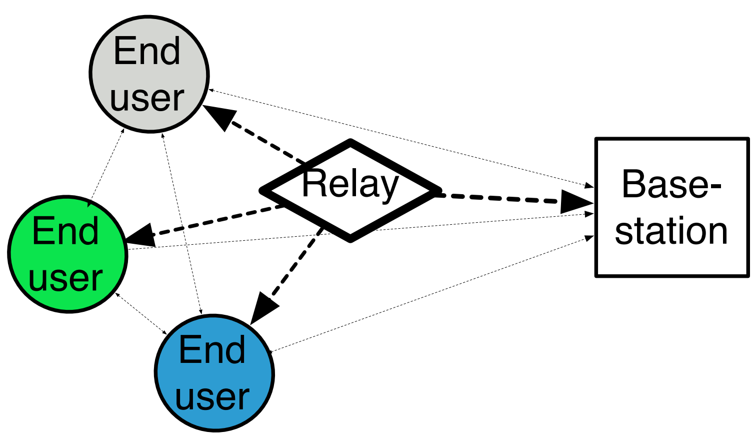
Base station

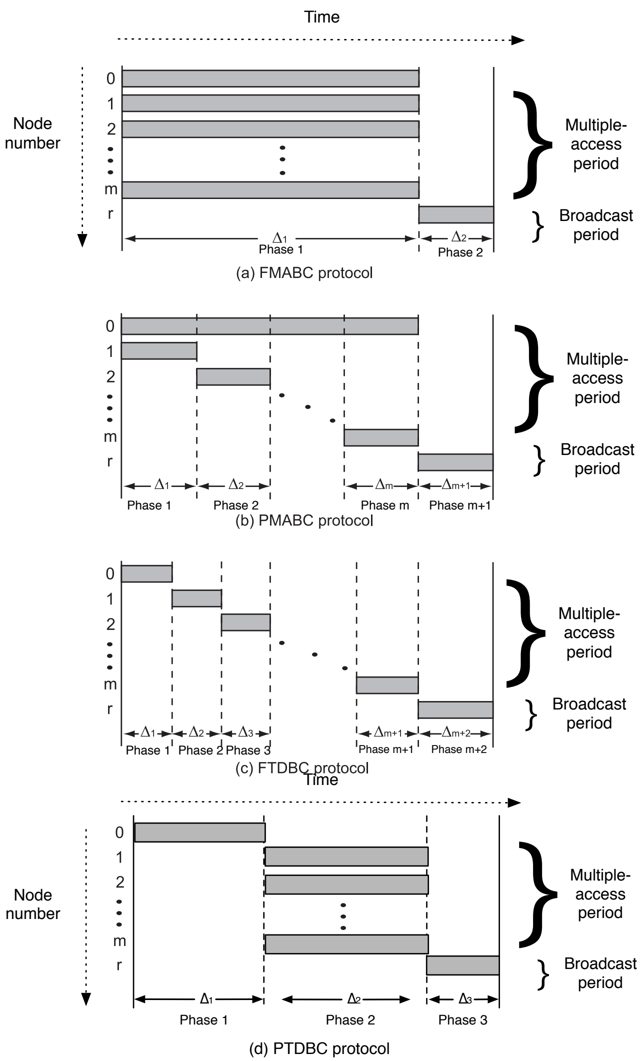


End user



Relay





Which protocol is “better”?

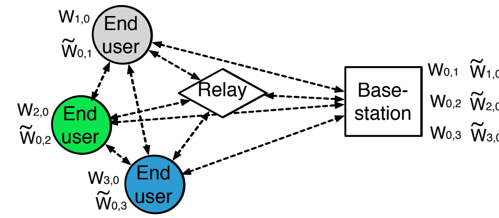
1. Multi-message broadcasting (Marton’s region)

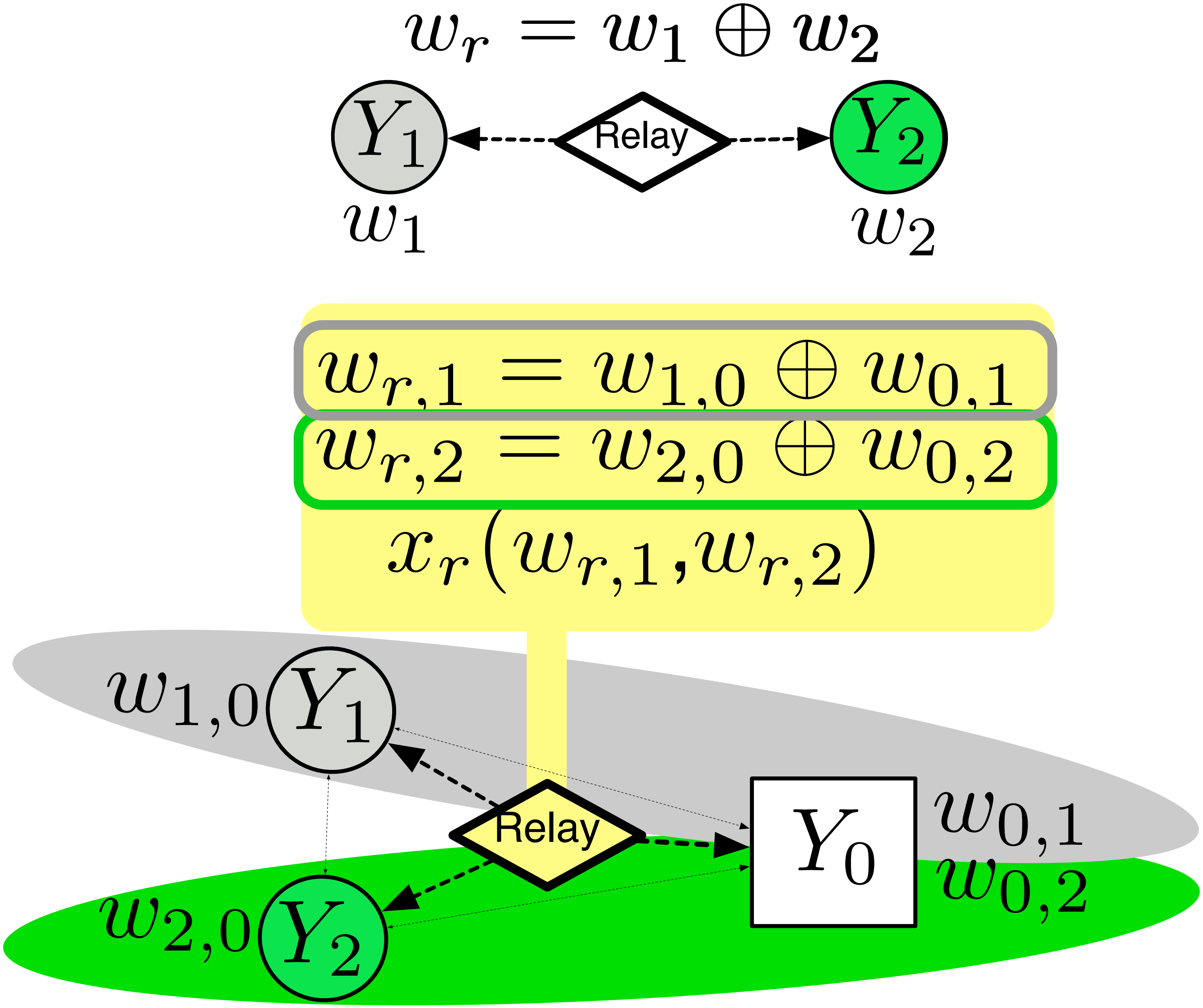
2. Per-flow network coding

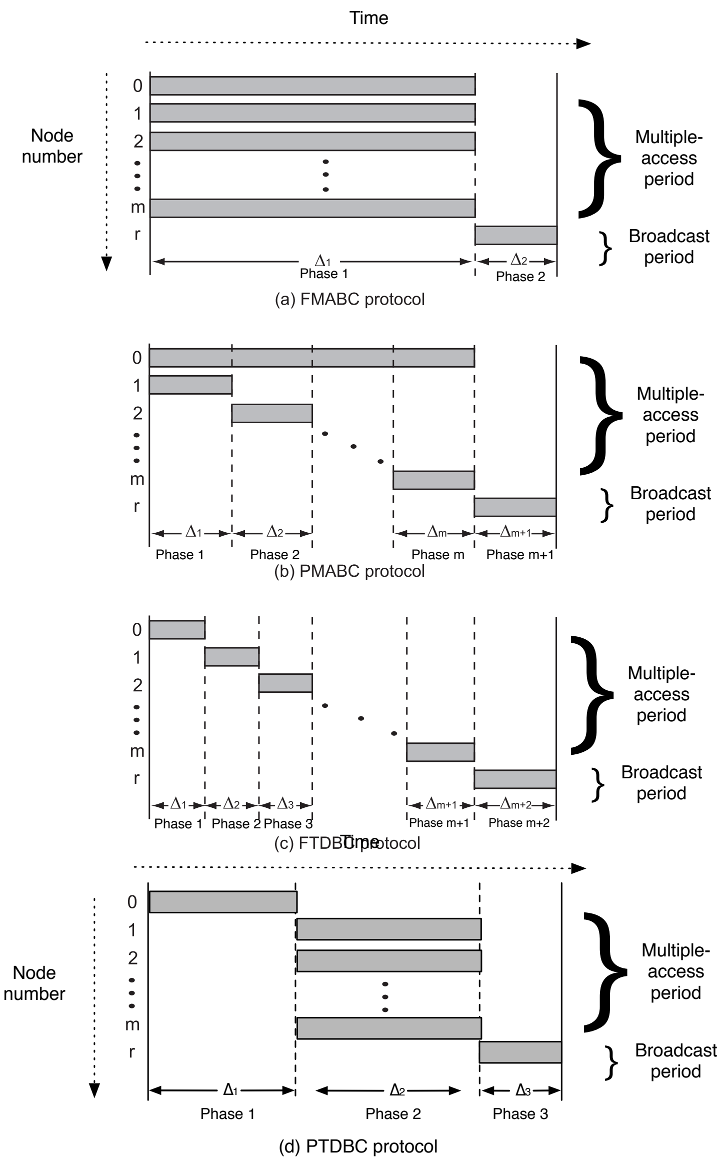
3. Exploiting side-information (random binning)

4. CF-based terminal node cooperation

Per-flow Network coding (N)



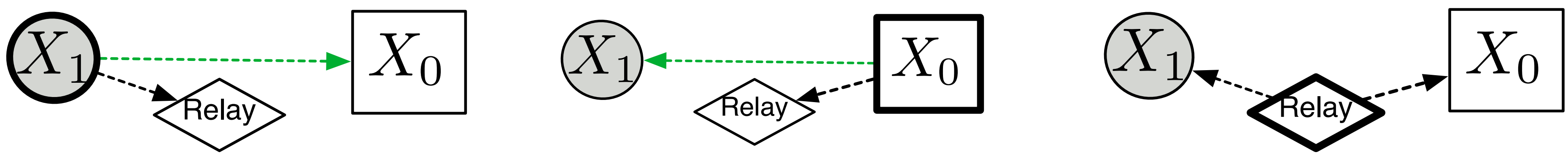




Which protocol is "better"?

1. Extended Marton's region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. CF-based Terminal node cooperation

Random binning (R) for exploiting overheard information

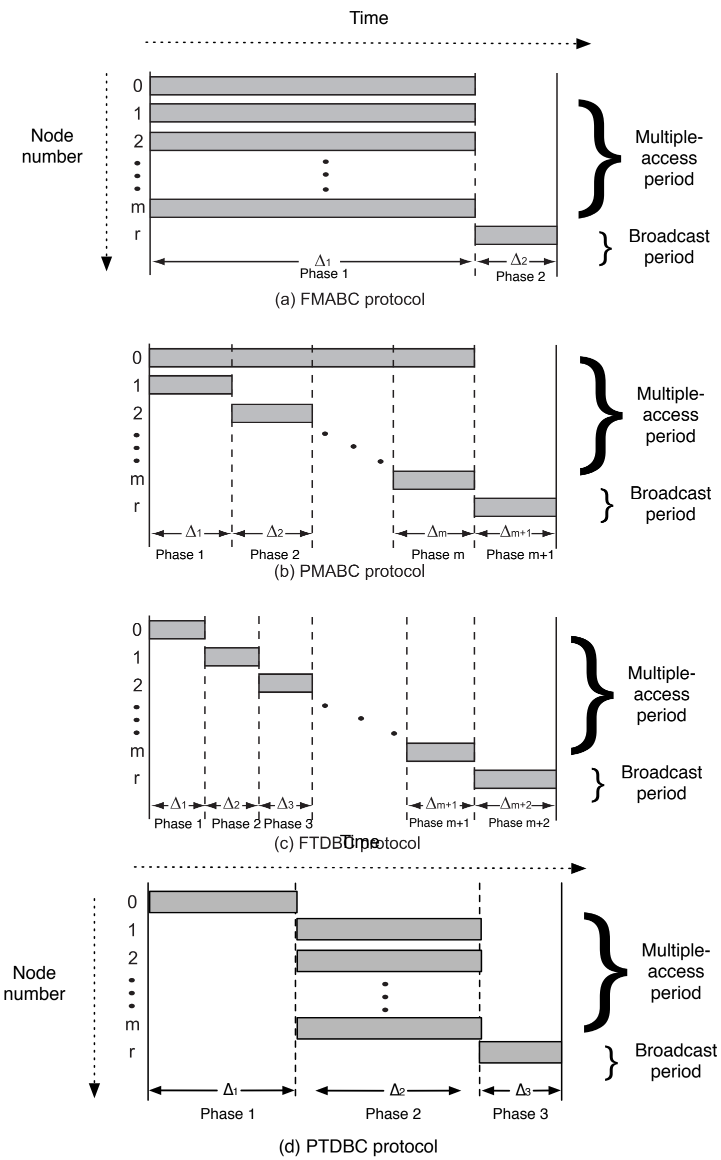


$$R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_r^{(1)})$$

$$R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_0^{(1)}) + \Delta_3 I(X_r^{(3)}; Y_0^{(3)})$$

$$R_{0,1} \leq \Delta_2 I(X_0^{(2)}; Y_r^{(2)})$$

$$R_{0,1} \leq \Delta_2 I(X_0^{(2)}; Y_1^{(2)}) + \Delta_3 I(X_r^{(3)}; Y_1^{(3)})$$

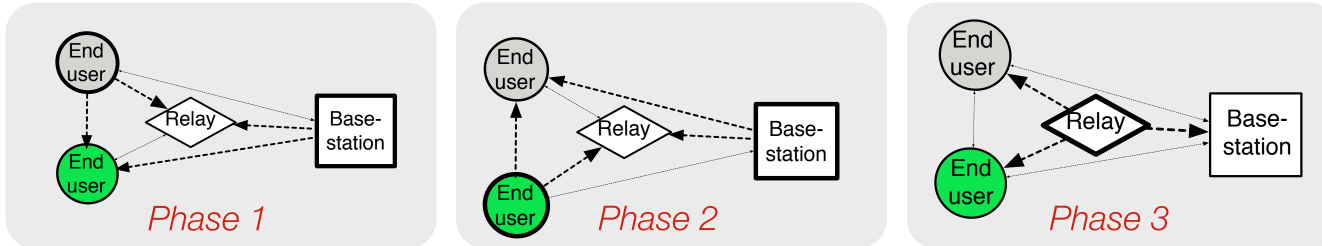


Which protocol is "better"?

1. Extended Marton's region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. CF-based Terminal node cooperation

Cooperation (C) between terminal nodes

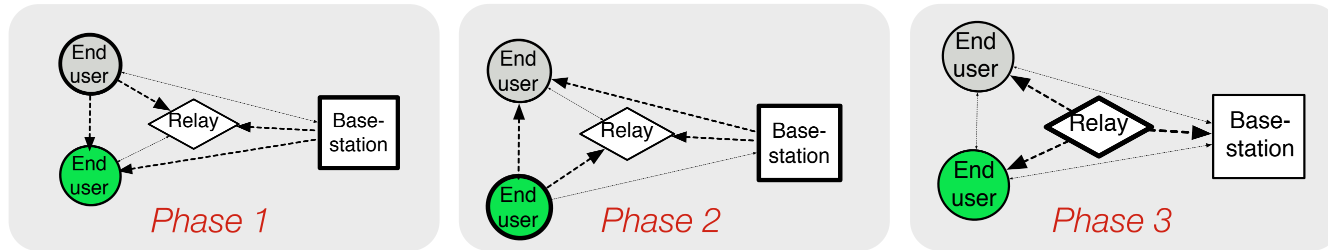
PMABC - NRC



	slot k	slot k+1		
	phase 3	phase 1	phase 2	
Transmit • • •	r : $\mathbf{x}_r^{(3)}(w_{r\|(k)})$	0 : $\mathbf{x}_0^{(1)}(w_{0,1\|(k+1)}, w_{0,2\|(k+1)})$ 1 : $\mathbf{x}_1^{(1)}(w_{1,0\|(k+1)}, w_{1,2\|(k)})$	0 : $\mathbf{x}_0^{(2)}(w_{0,1\|(k+1)}, w_{0,2\|(k+1)})$ 2 : $\mathbf{x}_2^{(2)}(w_{2,0\|(k+1)}, w_{2,1\|(k)})$	• • •
	compress	decode	decode	
Process • • •	1 : $\mathbf{y}_1^{(3)} \rightarrow \hat{\mathbf{y}}_1^{(3)}(w_{1,2\|(k)})$ 2 : $\mathbf{y}_2^{(3)} \rightarrow \hat{\mathbf{y}}_2^{(3)}(w_{2,1\|(k)})$	r : $\mathbf{y}_r^{(1)} \longrightarrow \tilde{w}_{1,0\|(k+1)}$ 2 : $\mathbf{y}_2^{(1)} \rightarrow \tilde{w}_{1,2\|(k)}$ $\mathbf{y}_2^{(3)}, \hat{\mathbf{y}}_1^{(3)}(\tilde{w}_{1,2\|(k)}) \longrightarrow \tilde{w}_{0,2\|(k)}$	r : $\mathbf{y}_r^{(2)} \longrightarrow \tilde{w}_{2,0\|(k+1)}$ $\mathbf{y}_r^{(1)}, \mathbf{y}_r^{(2)} \longrightarrow \tilde{w}_{0,1\|(k+1)}, \tilde{w}_{0,2\|(k+1)}$ 1 : $\mathbf{y}_1^{(2)} \rightarrow \tilde{w}_{2,1\|(k)}$ $\mathbf{y}_1^{(3)}, \hat{\mathbf{y}}_2^{(3)}(\tilde{w}_{2,1\|(k)}) \longrightarrow \tilde{w}_{0,1\|(k)}$	

Cooperation (C) between terminal nodes

PMABC - NRC

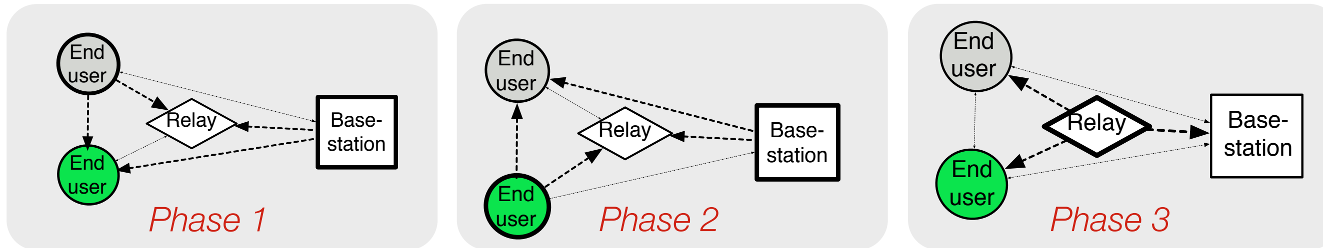


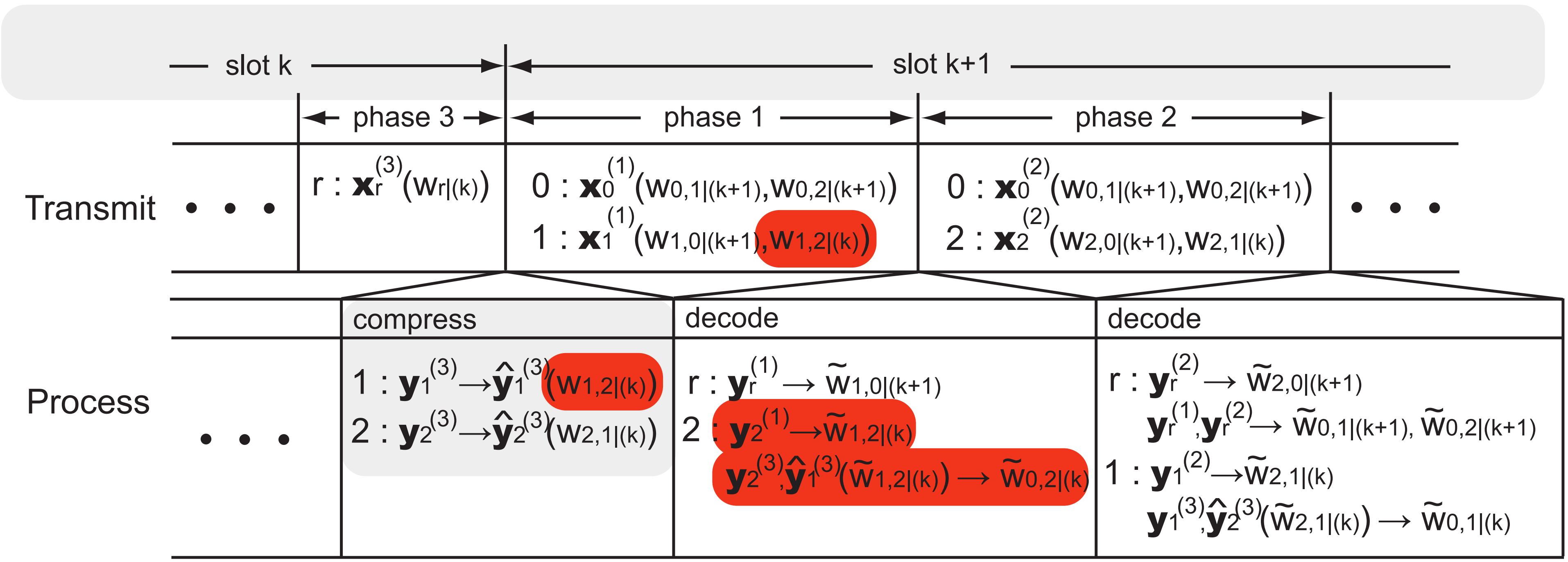
	slot k	slot k+1		
	phase 3	phase 1	phase 2	
Transmit • • •	r : $\mathbf{x}_r^{(3)}(w_{r\|(k)})$	0 : $\mathbf{x}_0^{(1)}(w_{0,1\|(k+1)}, w_{0,2\|(k+1)})$ 1 : $\mathbf{x}_1^{(1)}(w_{1,0\|(k+1)}, w_{1,2\|(k)})$	0 : $\mathbf{x}_0^{(2)}(w_{0,1\|(k+1)}, w_{0,2\|(k+1)})$ 2 : $\mathbf{x}_2^{(2)}(w_{2,0\|(k+1)}, w_{2,1\|(k)})$	• • •

	compress	decode	decode
Process • • •	1 : $\mathbf{y}_1^{(3)} \rightarrow \hat{\mathbf{y}}_1^{(3)}(w_{1,2\|(k)})$ 2 : $\mathbf{y}_2^{(3)} \rightarrow \hat{\mathbf{y}}_2^{(3)}(w_{2,1\|(k)})$	r : $\mathbf{y}_r^{(1)} \longrightarrow \tilde{w}_{1,0\|(k+1)}$ 2 : $\mathbf{y}_2^{(1)} \rightarrow \tilde{w}_{1,2\|(k)}$ $\mathbf{y}_2^{(3)}, \hat{\mathbf{y}}_1^{(3)}(\tilde{w}_{1,2\|(k)}) \longrightarrow \tilde{w}_{0,2\|(k)}$	r : $\mathbf{y}_r^{(2)} \longrightarrow \tilde{w}_{2,0\|(k+1)}$ $\mathbf{y}_r^{(1)}, \mathbf{y}_r^{(2)} \longrightarrow \tilde{w}_{0,1\|(k+1)}, \tilde{w}_{0,2\|(k+1)}$ 1 : $\mathbf{y}_1^{(2)} \rightarrow \tilde{w}_{2,1\|(k)}$ $\mathbf{y}_1^{(3)}, \hat{\mathbf{y}}_2^{(3)}(\tilde{w}_{2,1\|(k)}) \longrightarrow \tilde{w}_{0,1\|(k)}$

Cooperation (C) between terminal nodes

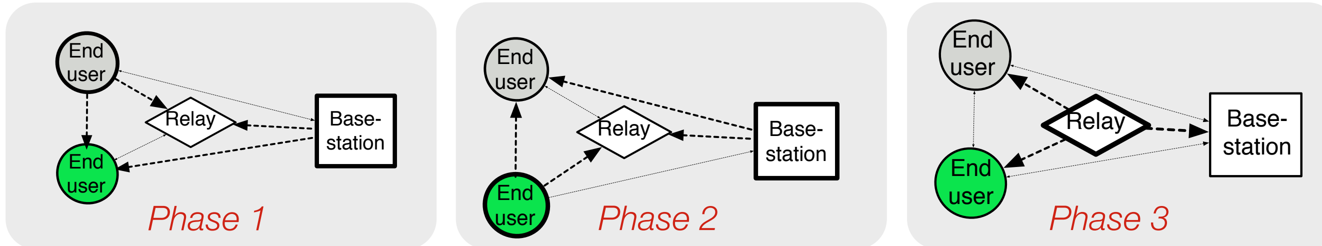
PMABC - NRC





Cooperation (C) between terminal nodes

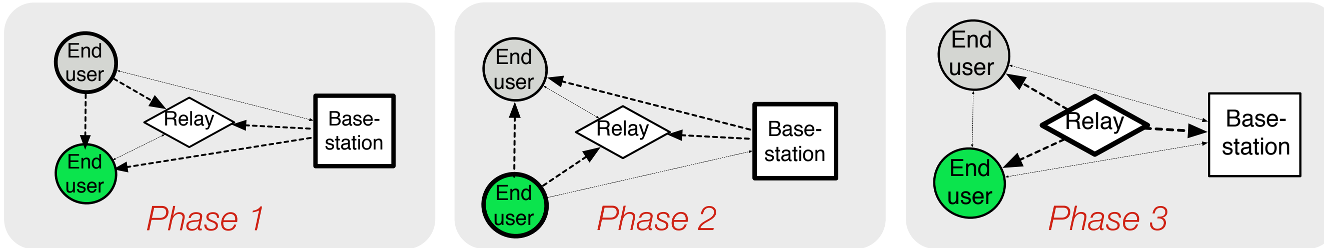
PMABC - NRC



		slot k	slot k+1	slot k+1	
		phase 3	phase 1	phase 2	
Transmit	• • •	$r : \mathbf{x}_r^{(3)}(w_{r\|(k)})$	$0 : \mathbf{x}_0^{(1)}(w_{0,1\|(k+1)}, w_{0,2\|(k+1)})$ $1 : \mathbf{x}_1^{(1)}(w_{1,0\|(k+1)}, w_{1,2\|(k)})$	$0 : \mathbf{x}_0^{(2)}(w_{0,1\|(k+1)}, w_{0,2\|(k+1)})$ $2 : \mathbf{x}_2^{(2)}(w_{2,0\|(k+1)}, w_{2,1\|(k)})$	• • •
		compress	decode	decode	
Process	• • •	$1 : \mathbf{y}_1^{(3)} \to \hat{\mathbf{y}}_1^{(3)}(w_{1,2\|(k)})$ $2 : \mathbf{y}_2^{(3)} \to \hat{\mathbf{y}}_2^{(3)}(w_{2,1\|(k)})$	$r : \mathbf{y}_r^{(1)} \to \tilde{w}_{1,0\|(k+1)}$ $2 : \mathbf{y}_2^{(1)} \to \tilde{w}_{1,2\|(k)}$ $\mathbf{y}_2^{(3)}, \hat{\mathbf{y}}_1^{(3)}(\tilde{w}_{1,2\|(k)}) \to \tilde{w}_{0,2\|(k)}$	$r : \mathbf{y}_r^{(2)} \to \tilde{w}_{2,0\|(k+1)}$ $\mathbf{y}_r^{(1)}, \mathbf{y}_r^{(2)} \to \tilde{w}_{0,1\|(k+1)}, \tilde{w}_{0,2\|(k+1)}$ $1 : \mathbf{y}_1^{(2)} \to \tilde{w}_{2,1\|(k)}$ $\mathbf{y}_1^{(3)}, \hat{\mathbf{y}}_2^{(3)}(\tilde{w}_{2,1\|(k)}) \to \tilde{w}_{0,1\|(k)}$	

Cooperation (C) between terminal nodes

PMABC - NRC



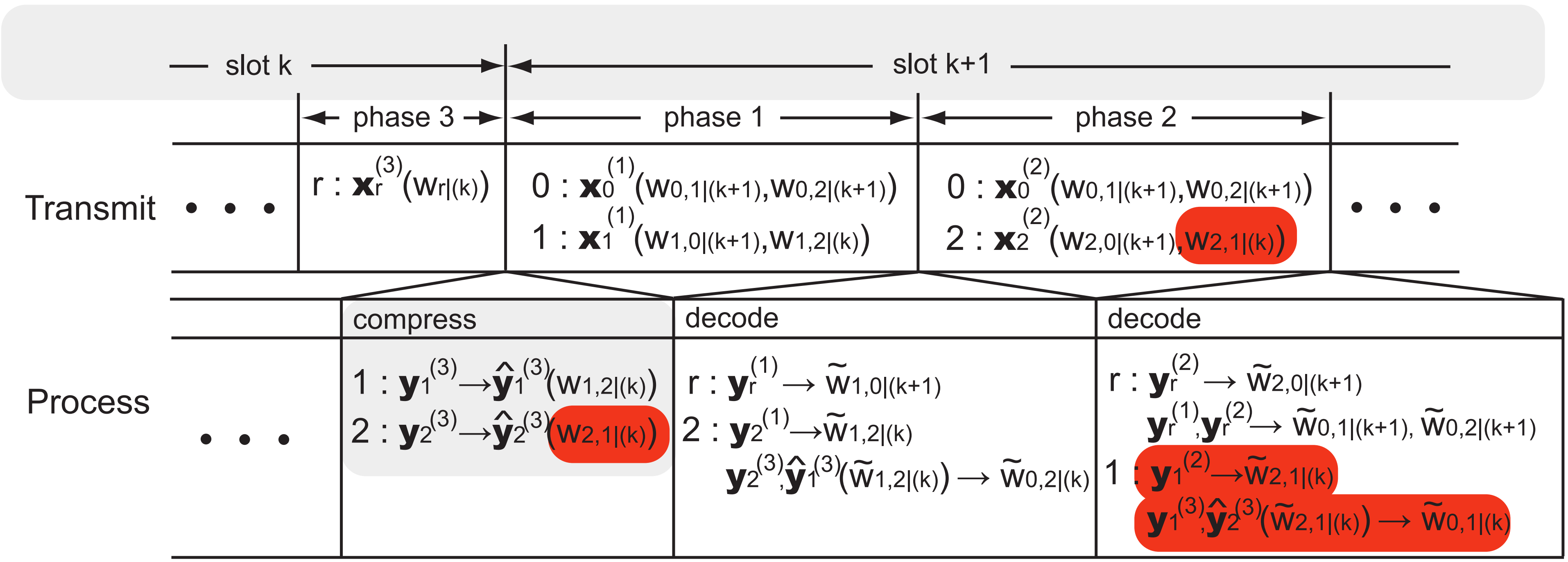


TABLE I

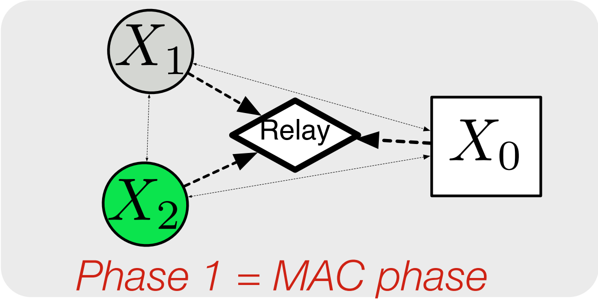
PROTOCOLS AND CODING SCHEMES

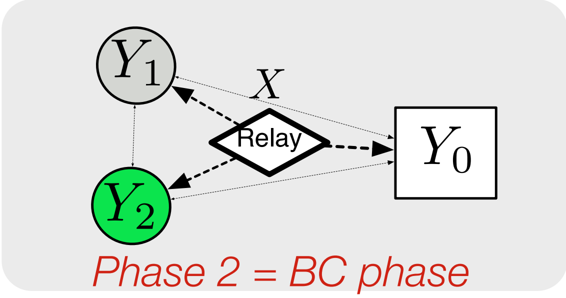
Protocol	Multiple Access	Marton's Broadcast	Network coding	Random binning	User cooperation
Simplest	–	–	–	–	–
FMABC	X	X	–	–	–
FMABC-N	X	X	X	–	–
PMABC	X	X	–	–	–
PMABC-NR	X	X	X	X	–
PMABC-NRC	X	X	X	X	X
FTDBC	–	X	–	–	–
FTDBC-NR	–	X	X	X	–
FTDBC-NRC	–	X	X	X	X
PTDBC	X	X	–	–	–
PTDBC-NR	X	X	X	X	–

N = Network coding
R = Random binning
C = Cooperation between terminals

Outer bounds - half-duplex cut-set

FMABC





$$R_{0,1} + R_{0,2} \leq \Delta_1 I(X_0^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_2^{(1)})$$

$$R_{1,0} + R_{2,0} \leq \Delta_2 I(X_r^{(2)}; Y_0^{(2)})$$

$$R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_r^{(1)} | X_2^{(1)}, X_0^{(1)})$$

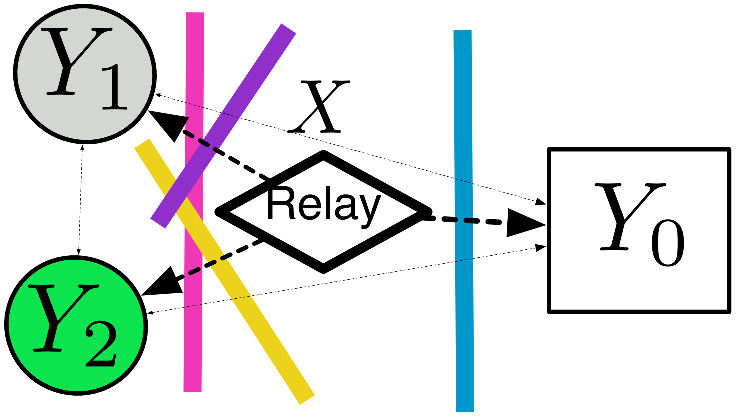
$$R_{2,0} \leq \Delta_1 I(X_2^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_0^{(1)})$$

$$R_{1,0} + R_{2,0} \leq \Delta_1 I(X_1^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_0^{(1)})$$

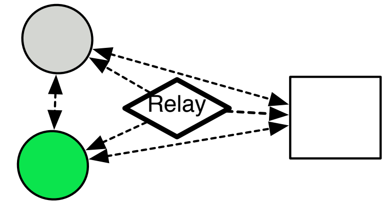
$$R_{0,1} \leq \Delta_2 I(X_r^{(2)}; Y_1^{(2)})$$

$$R_{0,2} \leq \Delta_2 I(X_r^{(2)}; Y_2^{(2)})$$

$$R_{0,1} + R_{0,2} \leq \Delta_2 I(X_r^{(2)}; Y_1^{(2)}, Y_2^{(2)})$$



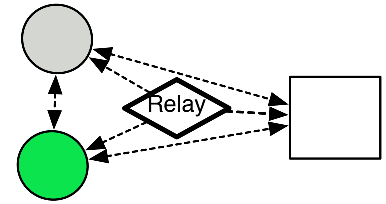
Simulations in Gaussian noise



$$\mathbf{Y}[k] = \mathbf{H}\mathbf{X}[k] + \mathbf{Z}[k]$$

$$\mathbf{H_1} = \begin{bmatrix} 0 & 0.3 & 0.05 & 1 \\ 0.3 & 0 & 1.5 & 1 \\ 0.05 & 1.5 & 0 & 0.2 \\ 1 & 1 & 0.2 & 0 \end{bmatrix} \quad \mathbf{H_2} = \begin{bmatrix} 0 & 0.9 & 0.4 & 1 \\ 0 & 0 & 0.02 & 1 \\ 0 & 0.02 & 0 & 0.5 \\ 1 & 1 & 0.5 & 0 \end{bmatrix}.$$

Simulations in Gaussian noise



$$\mathbf{Y}[k] = \mathbf{H}\mathbf{X}[k] + \mathbf{Z}[k]$$

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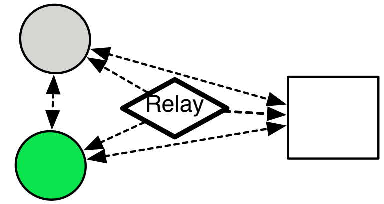
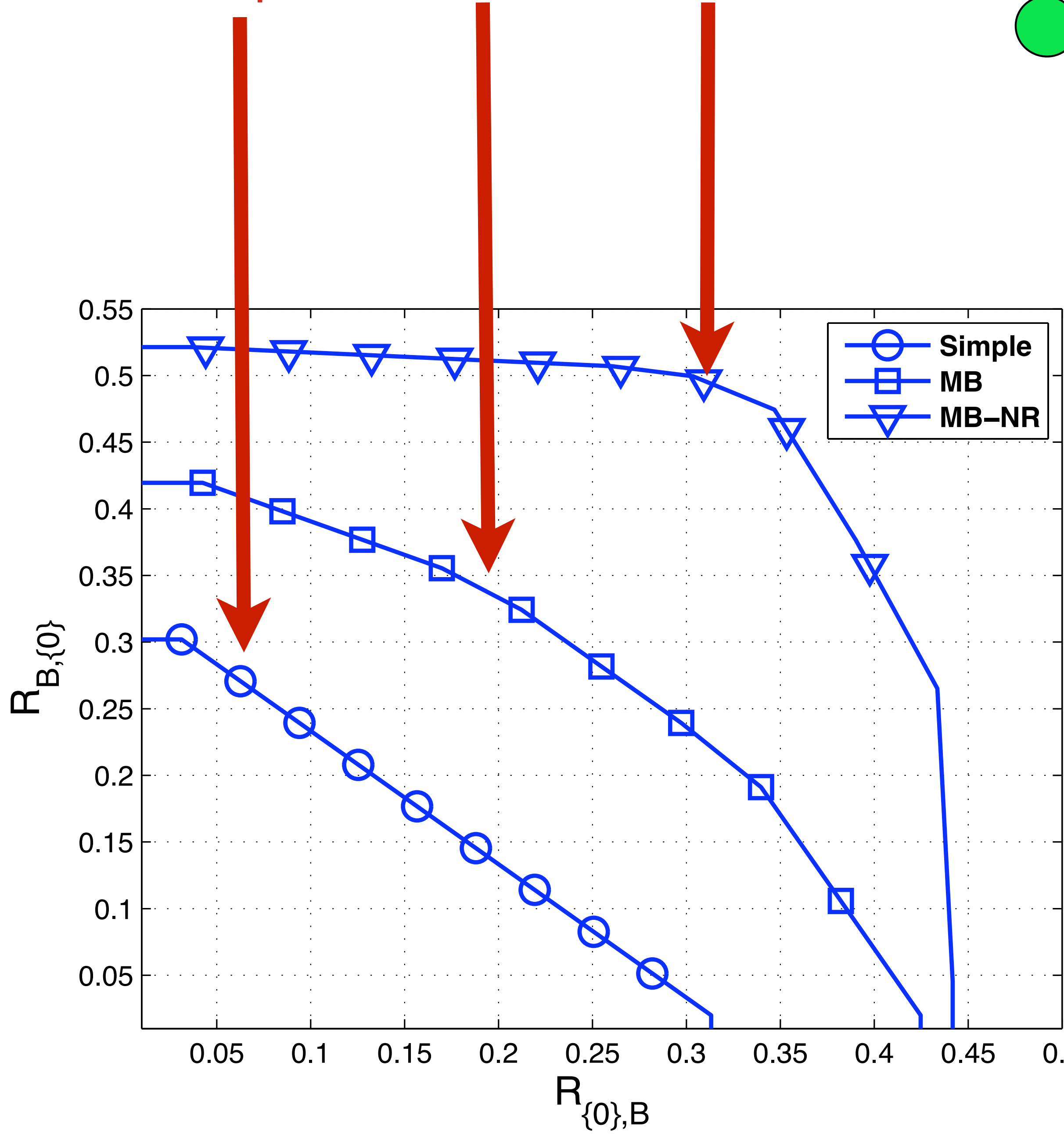
Evaluate expressions assuming Gaussian input distributions and optimize over:

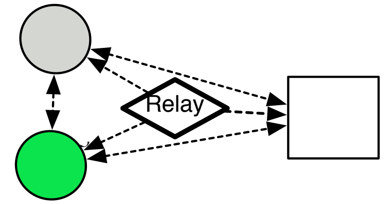
- phase durations
- correlation matrices of Marton binning RVs subject to power constraints
- compression parameters

Simple - MB - MB+NR





Network coding + random binning



FMABC

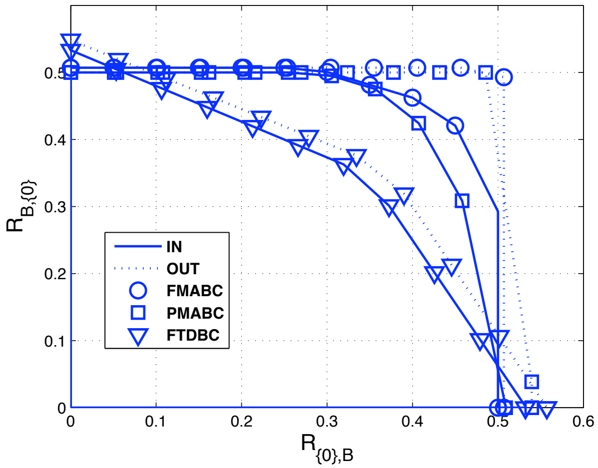


Fig. 7. Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_1$.

FMABC/
PMABC

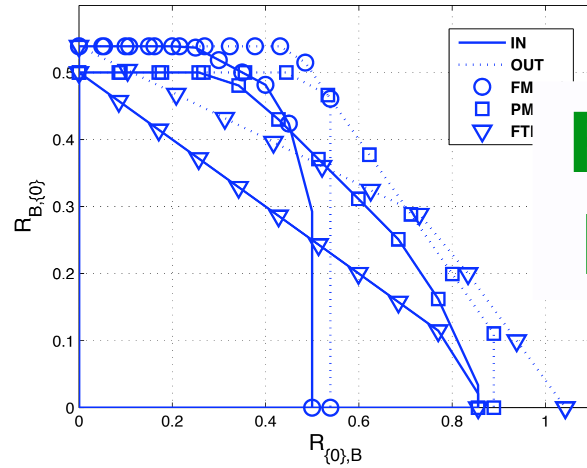


Fig. 8. Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_2$.

PMABC

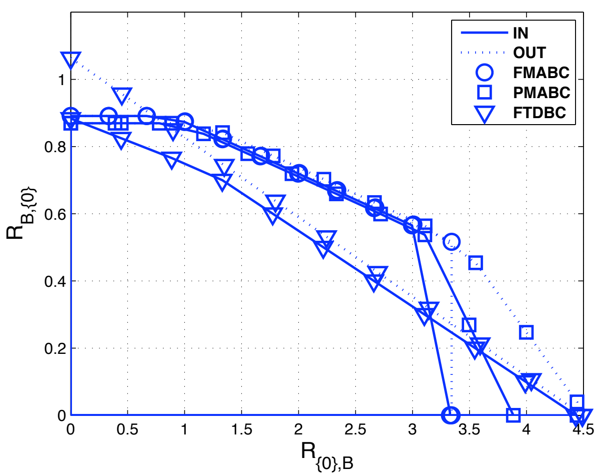


Fig. 9. Comparison with $P_0 = P_r = 20, P_1 = P_2 = 0$ dB, $H = H_1$.

FTDBC

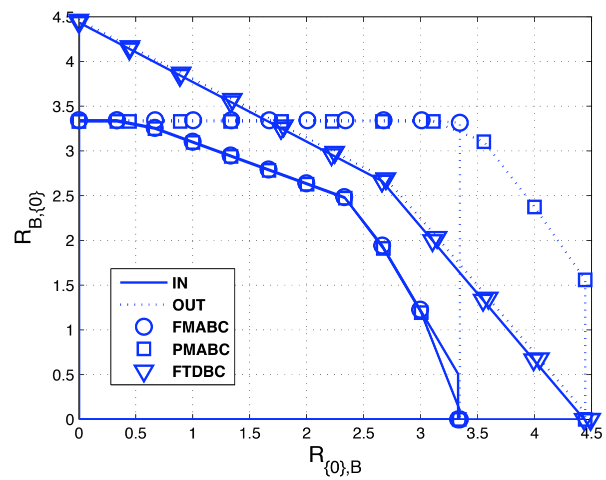
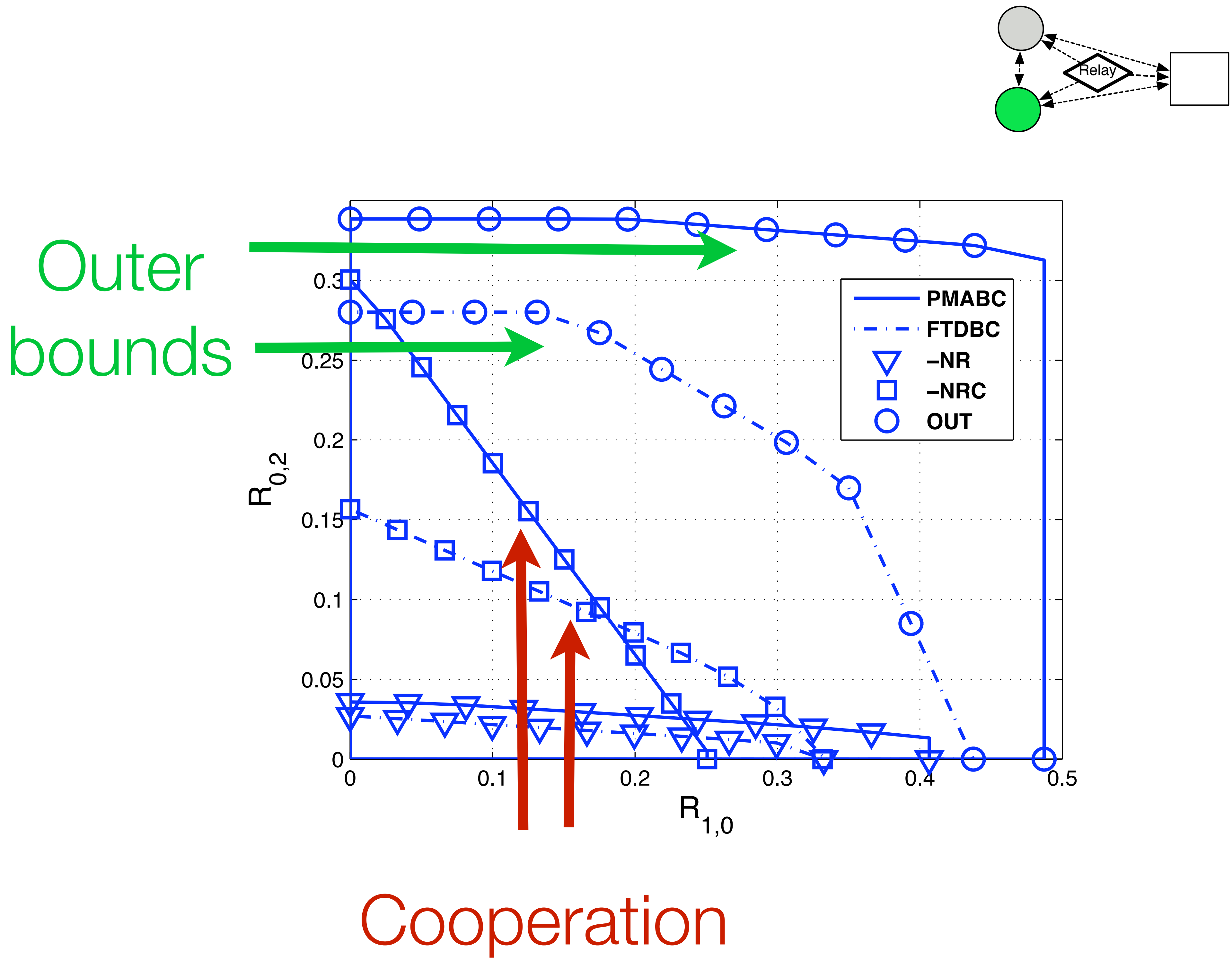


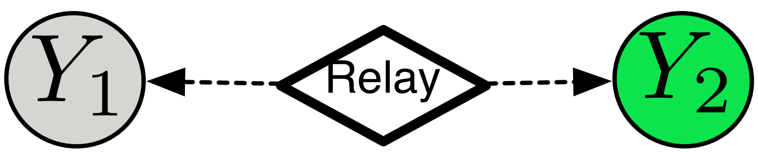
Fig. 10. Comparison with $P_0 = P_1 = P_2 = P_r = 20$ dB, $H = H_1$.


Relay
Outer bounds
Cooperation
PMABC
FTDBC
–NR
–NRC
OUT
$R_{0,2}$
$R_{1,0}$
0
0.05
0.1
0.15
0.2
0.25
0.3
0
0.1
0.2
0.3
0.4
0.5

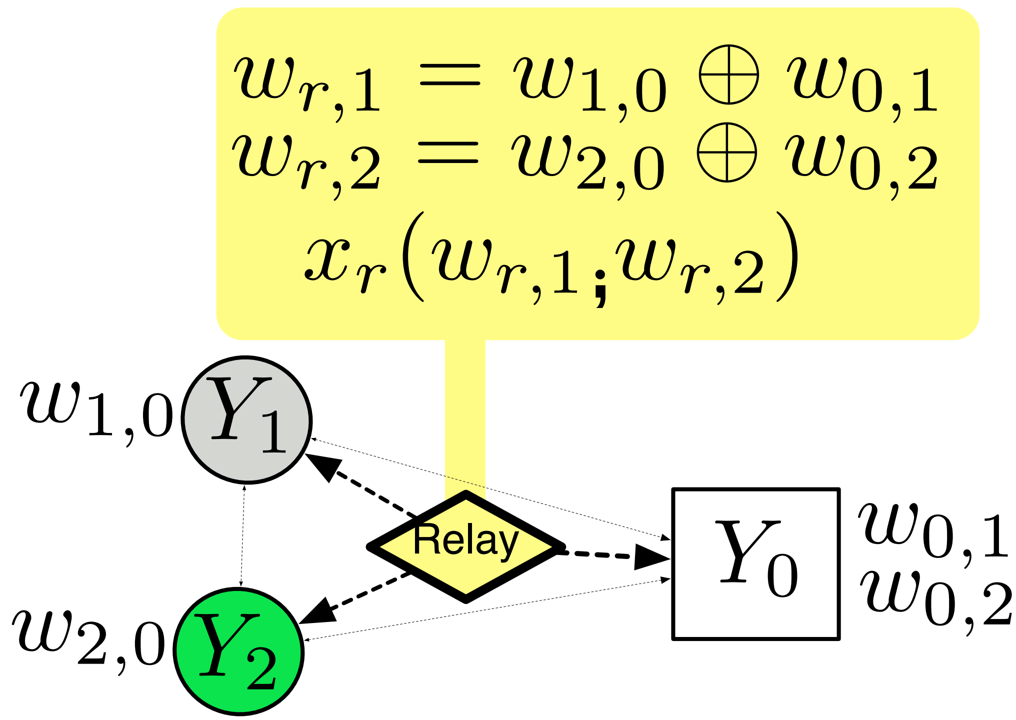
Multi-flow take-away points

- Most schemes use *per-flow network coding*

$$w_r = w_1 \oplus w_2$$

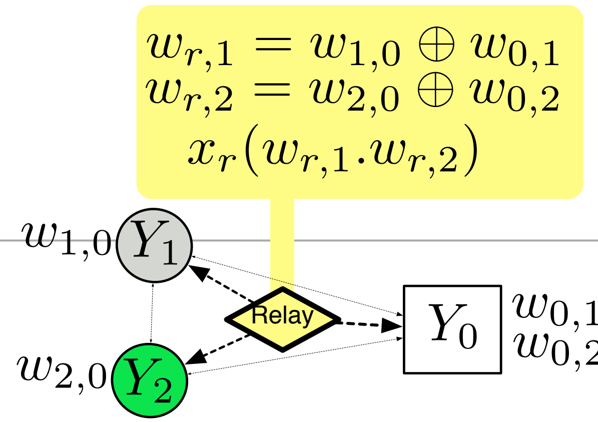


One flow



Multiple flows

Multi-flow take-away points



- Most schemes use *per-flow network coding*

- Significantly more complex: protocols and opportunities abound. Only starting to understand when to do what.

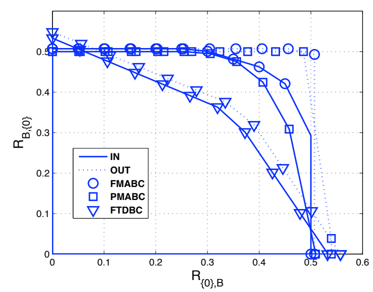
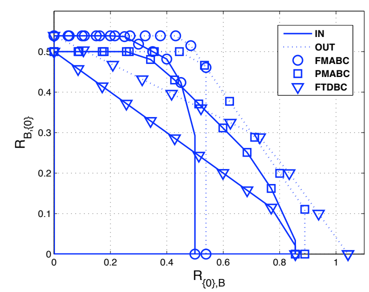


Fig. 7. Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_1$.



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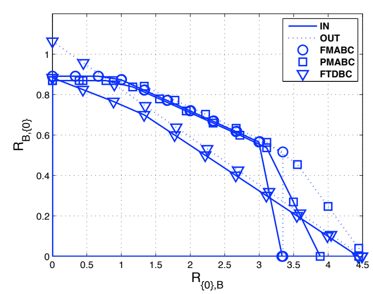

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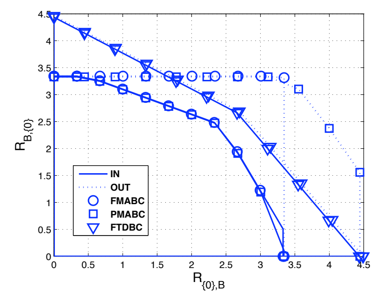
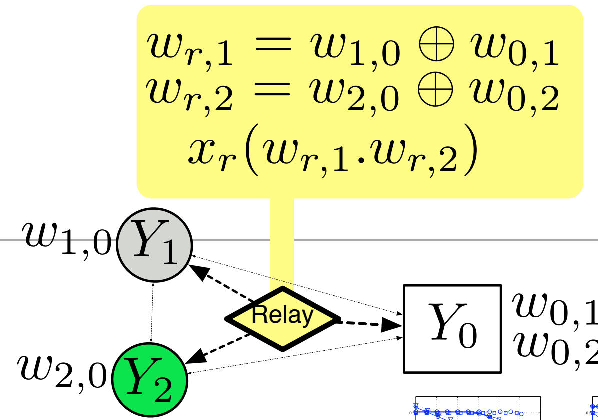
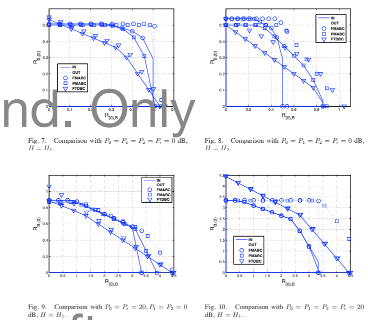


Fig. 10. Comparison with $P_0 = P_1 = P_2 = P_r = 20$ dB, $H = H_1$.

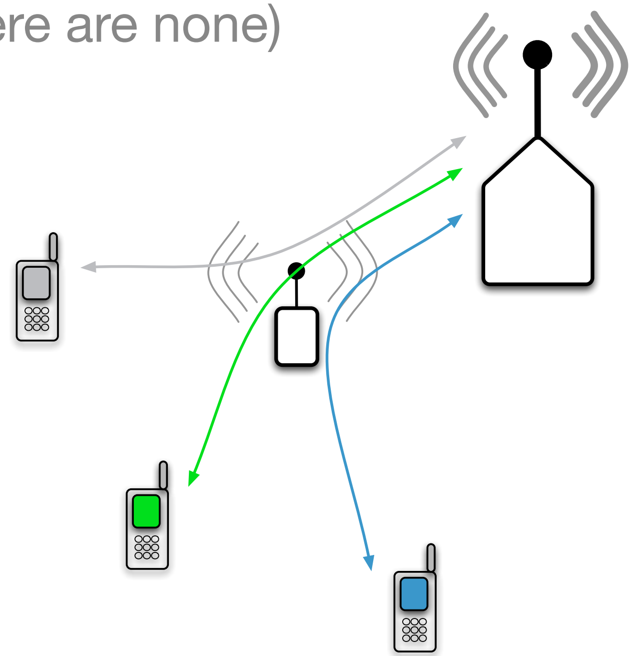
Multi-flow take-away points



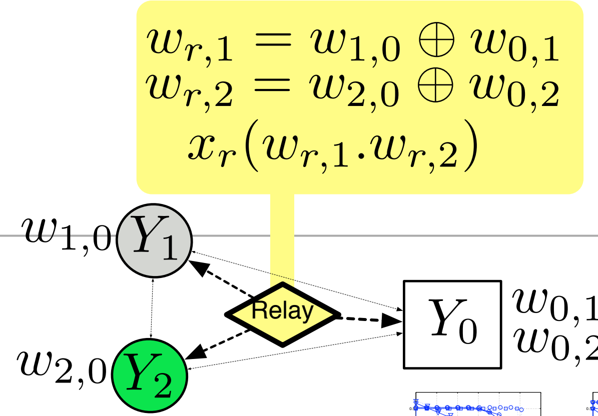
- Most schemes use *per-flow network coding*



- Significantly more complex: protocols and opportunities abound. Only starting to understand when to do what.
- Due to practical relevance - crucial to develop insight and thereafter demonstrations (there are none)



Multi-flow take-away points



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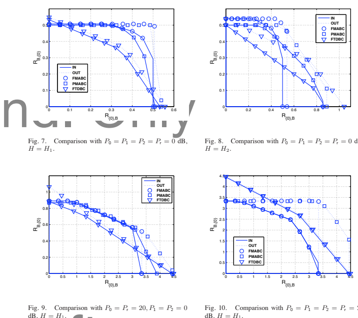

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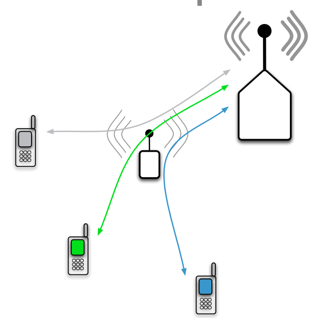
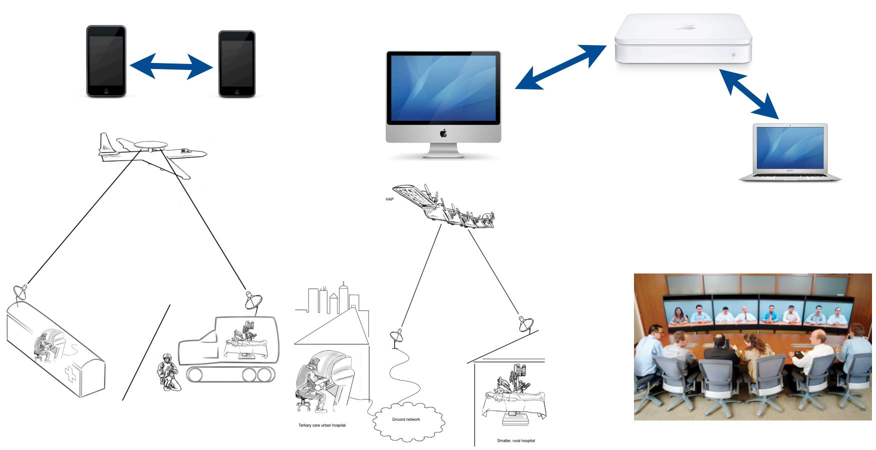


- Two-way nature must be explicitly accounted for (side-information, ability to network code and broadcast) in order to see gains.

Future areas of two-way channels

- one-way information theory ``fairly” well understood
- advances in processing power
- never ending desire for bandwidth and limited wireless spectrum



→ Two-way wireless networks

→ When is two-way processing needed?

Questions?

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http://www.ece.uic.edu/~devroye