SYTACom

SYTACom Summer School 2012
June 14-15, 2012
École Polytechnique, Montreal

UIC UNIVERSITY OF ILLINOIS AT CHICAGO Department of Electrical and Computer Engineering
COLLEGE OF ENGINEERING

Structured codes in wireless relay networks

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Joint work with Yiwei Song, *Ph.D. candidate, UIC*
Bobak Nazer, *Assistant Professor, Boston University*
Matt Nokleby + Benhaam Aazhang, *Rice University*

Y. Song, N. Devroye, ``List decoding for nested lattices and applications to relay channels,'' Allerton 2010.

Y. Song, N. Devroye, ``Structured interference-mitigation in two-hop networks,'' ITA 2011.

Y. Song , N. Devroye, ``A lattice Compress-and-Forward strategy for canceling known interference in Gaussian multi-hop channels,'' CISS 2011.

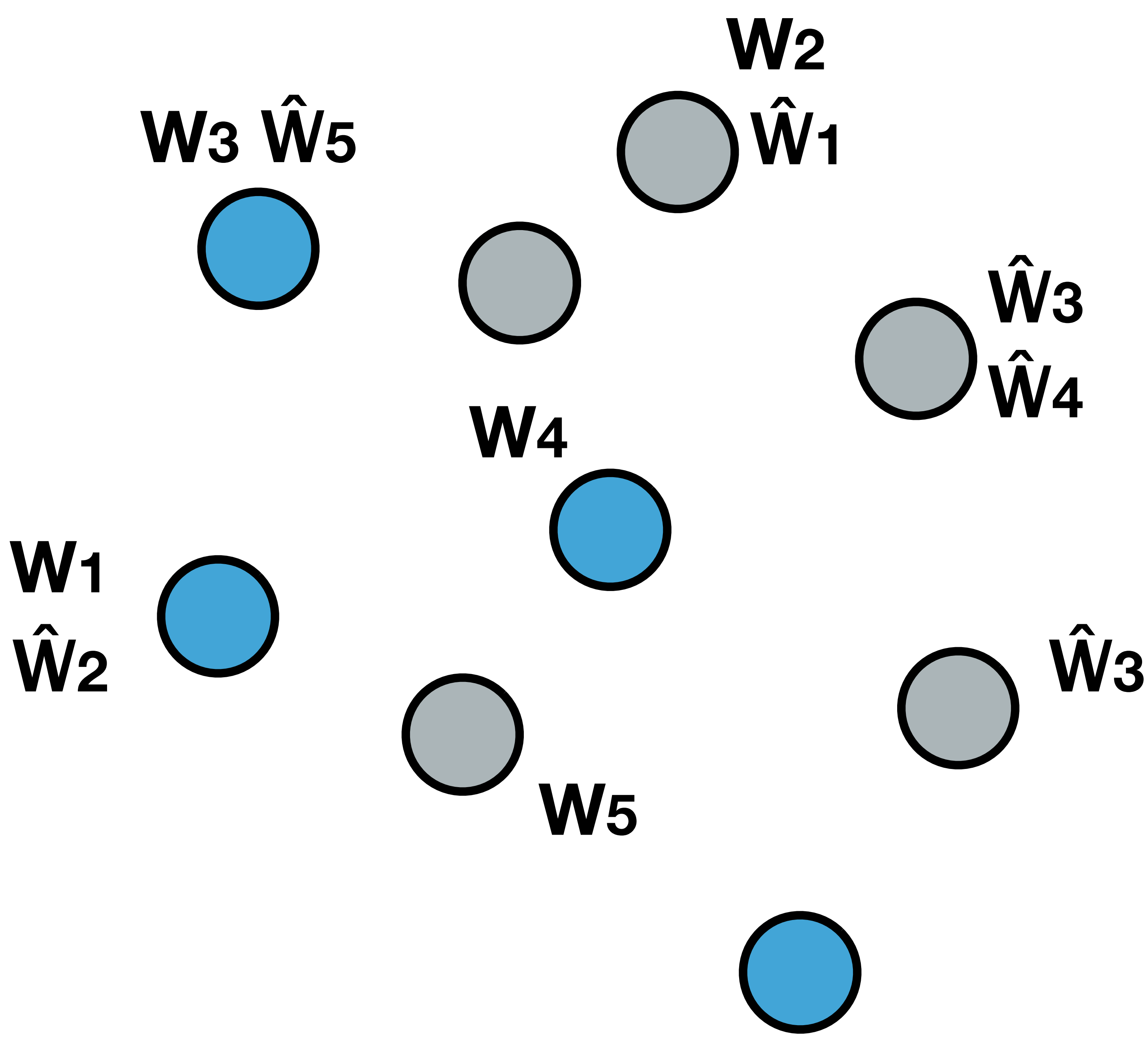
Y. Song, N. Devroye, ``A Lattice Compress-and-Forward Scheme,'' ITW Paraty, 2011.

Y. Song, N. Devroye, and B. Nazer ``Inverse Compute-and-Forward: Extracting Messages from Simultaneously Transmitted Equations,**''** *ISIT* 2011.

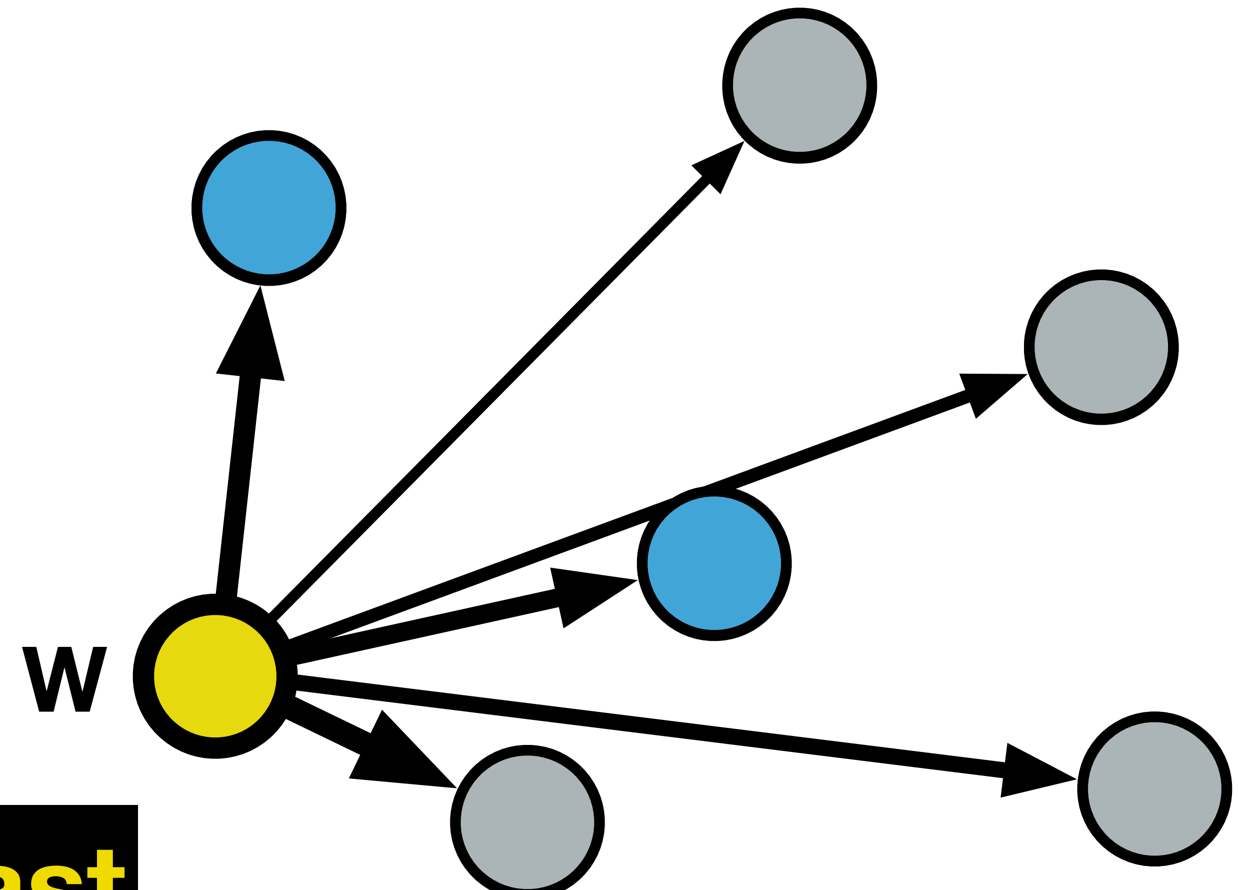
M. Nokleby, B. Nazer, B. Aazhang, and N. Devroye, ``Relays that Cooperate to Compute,**''** to appear in *ISWCS,* 2012.

Y. Song , N. Devroye, ``Lattice codes for relay channels: DF and CF,''' submitted to IEEE Trans. on IT, 2011.

Gaussian (wireless) networks

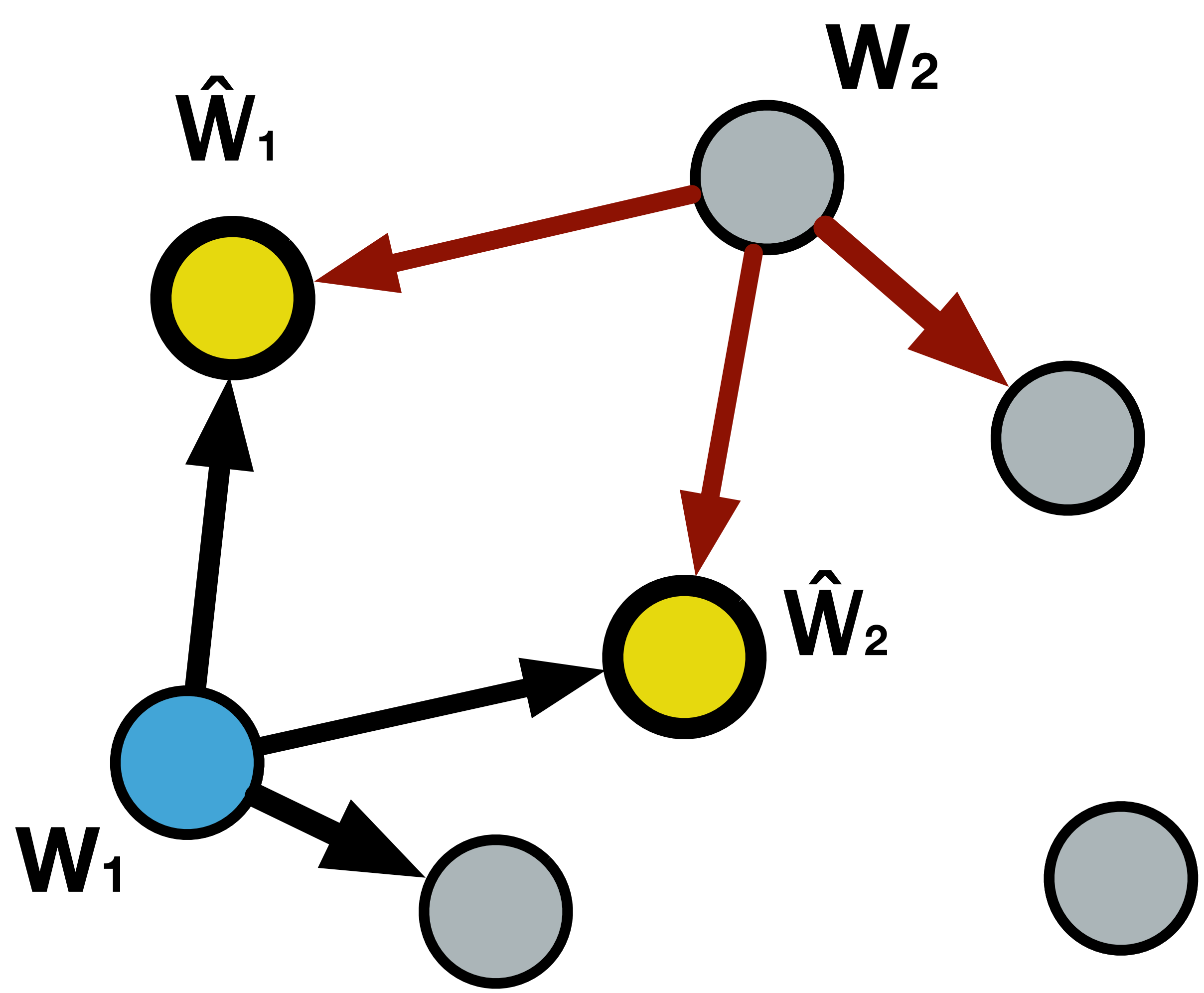


Gaussian (wireless) networks



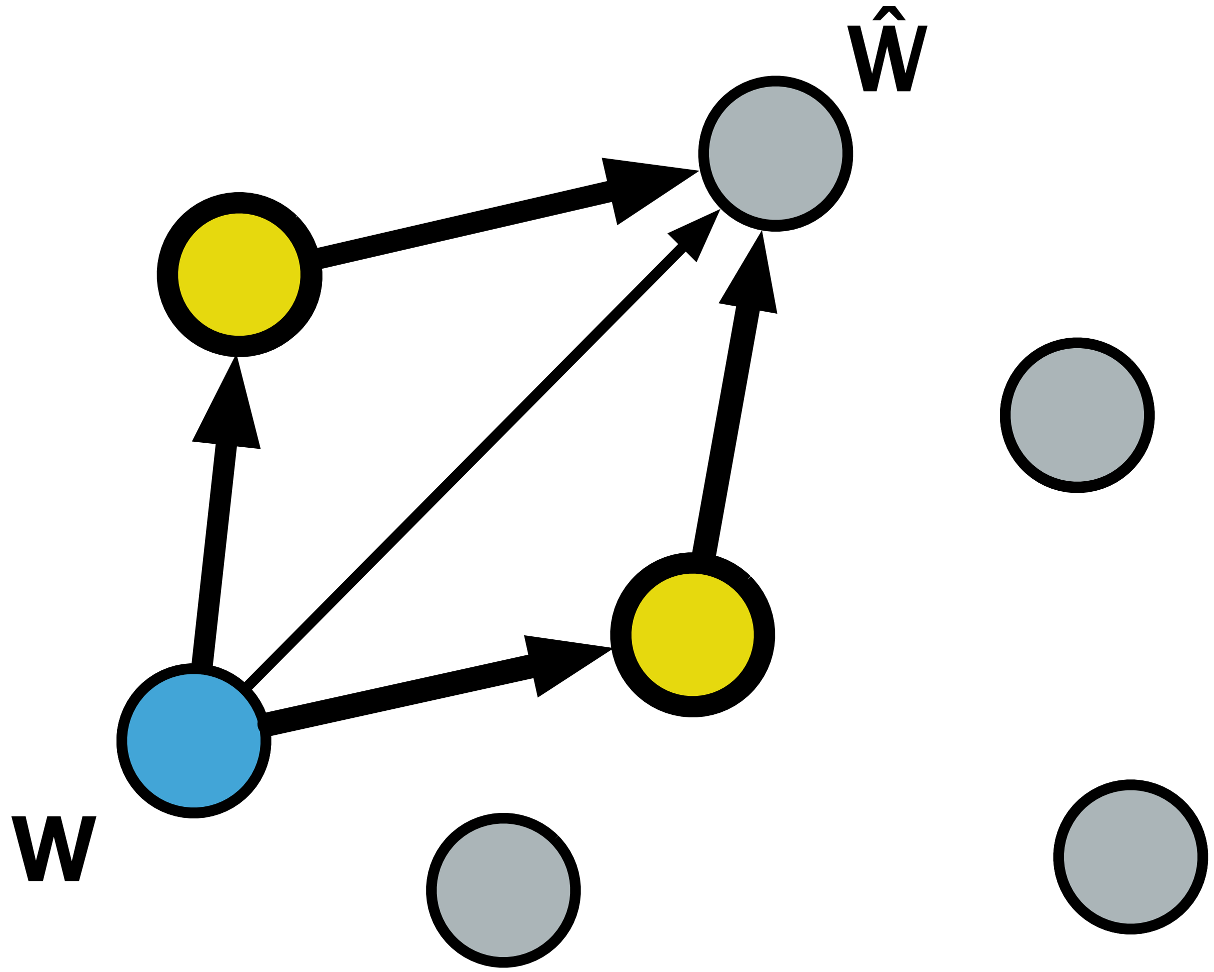
Broadcast

Gaussian (wireless) networks



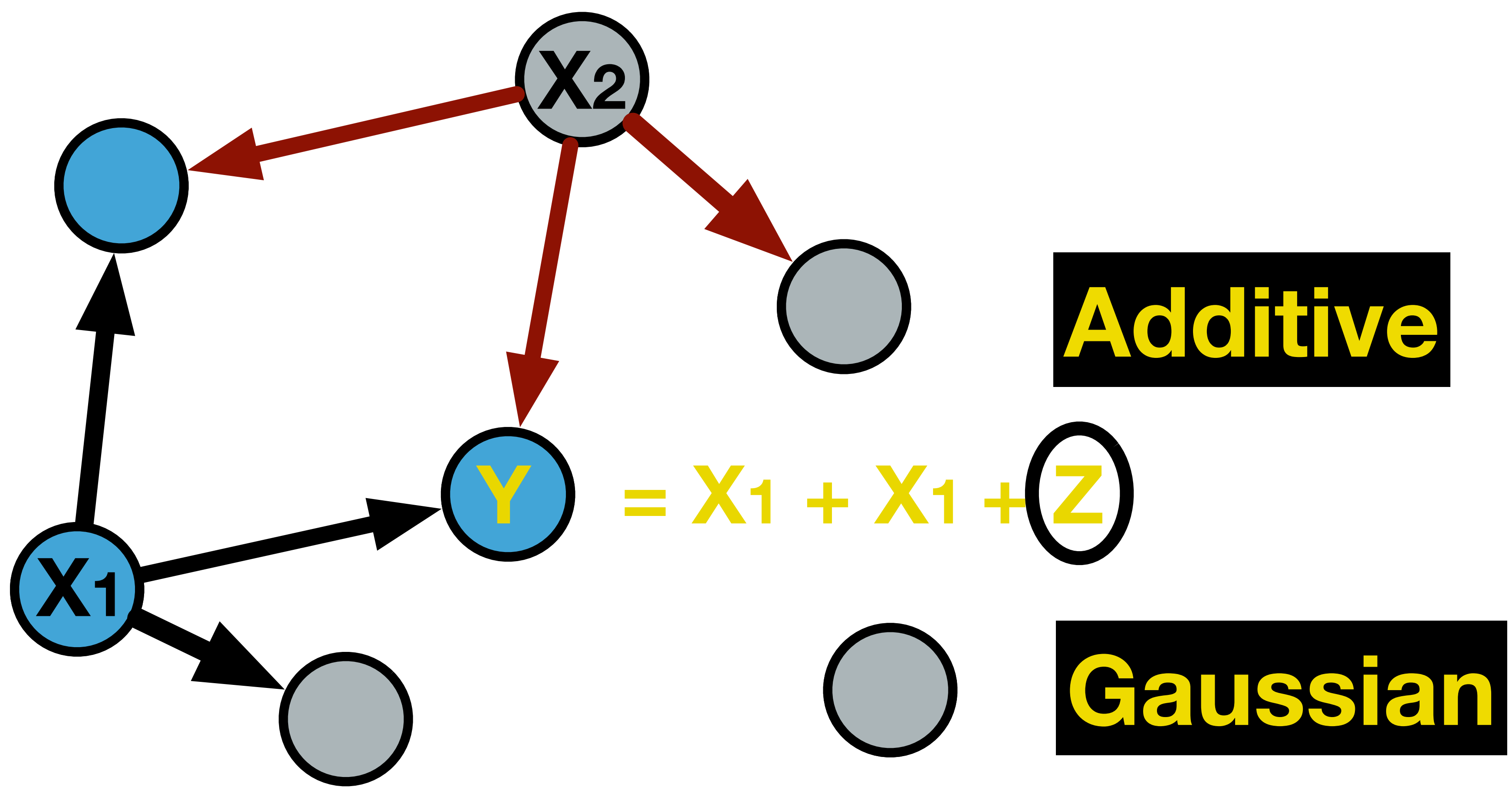
Interference

Gaussian (wireless) networks



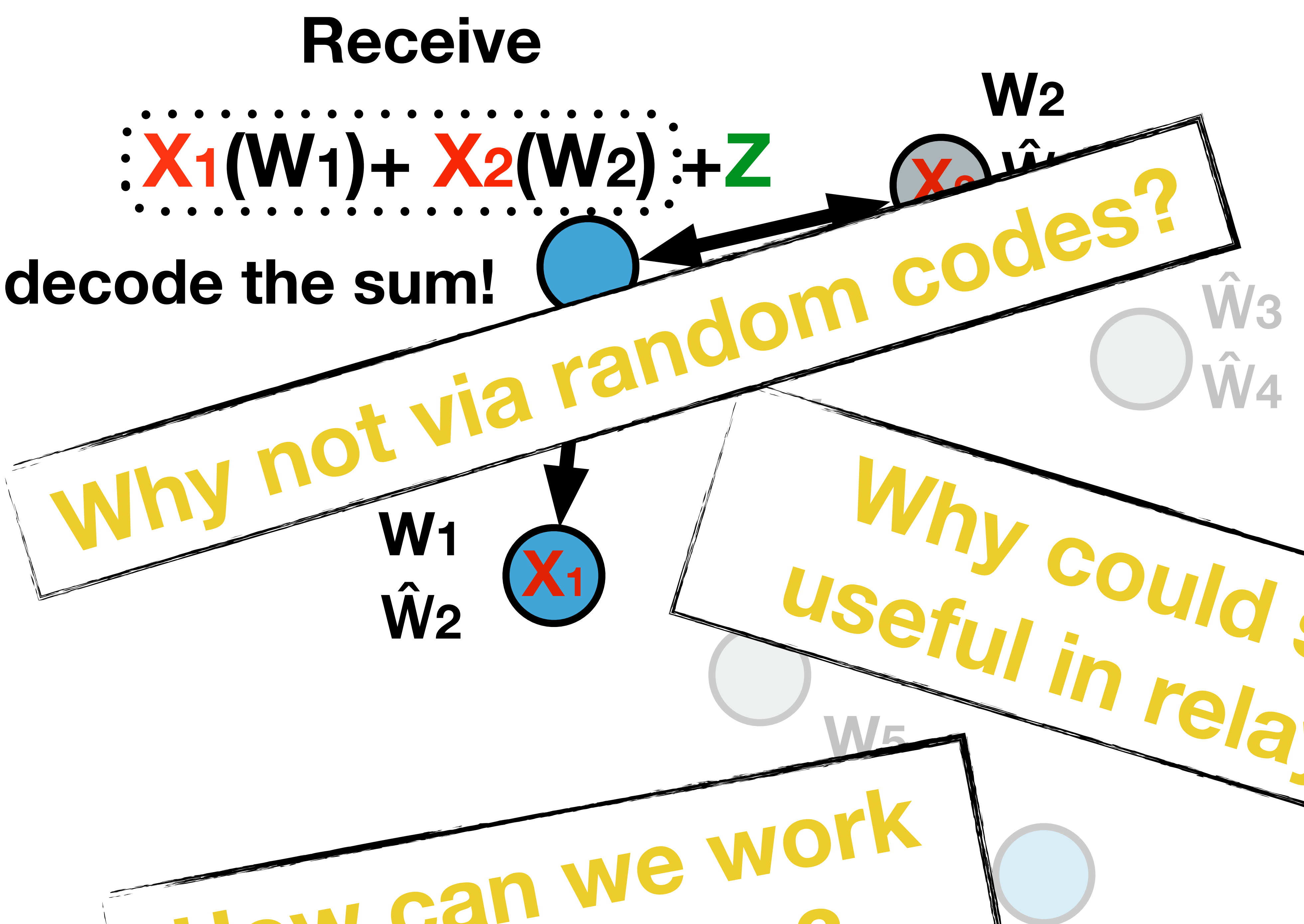
Relaying / cooperation

Gaussian (wireless) networks



Can exploit channel's natural linearity?

Structured codes for Gaussian networks



Why not via random codes?

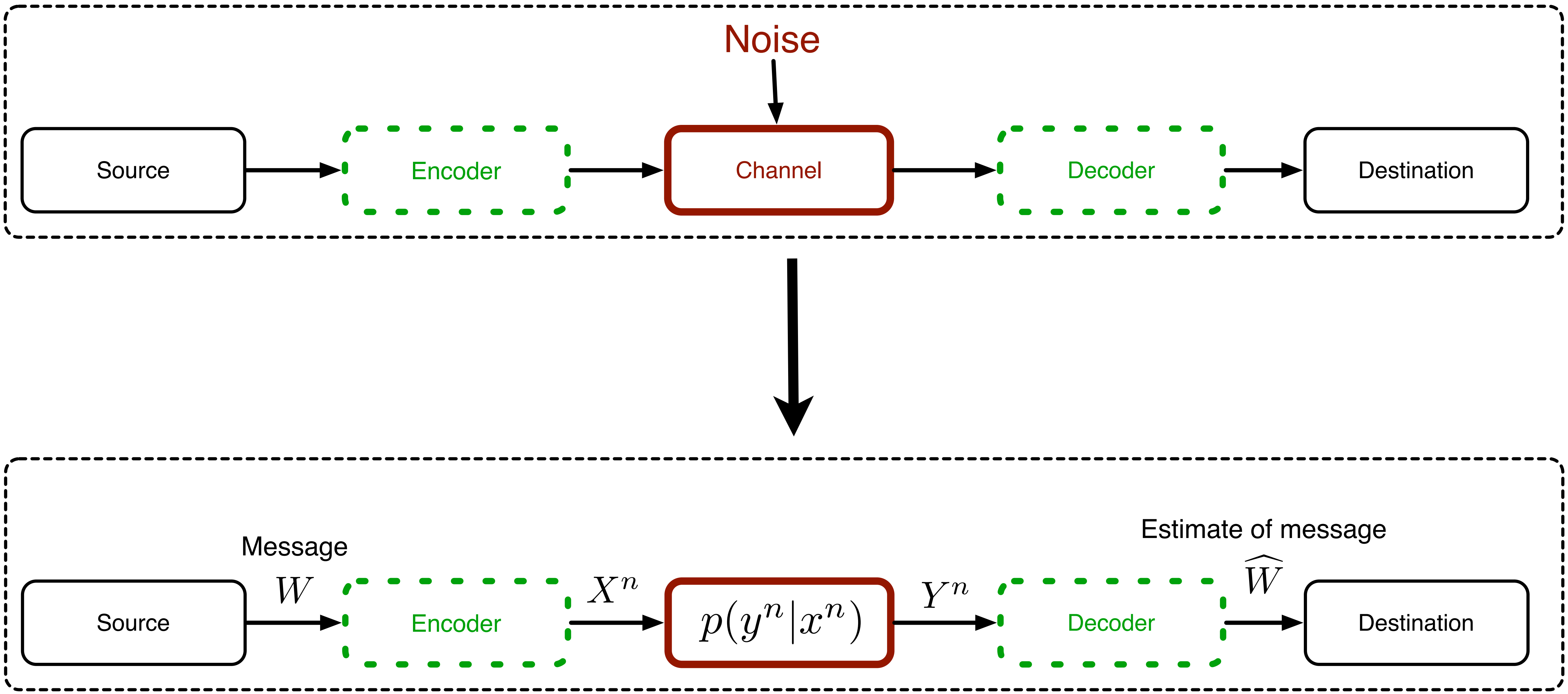
Why could structure be useful in relay networks?

How can we work towards this?

Outline

- **Point to point channels: random codes, lattice (structured) codes**
- Two-way relay channel: the canonical example of structure being useful
- Compute and Forward (+ Inverse Compute and Forward) for relay networks
- Relaying using lattice codes
- Additional lattice examples
- Conclusion

Point-to-point communication system

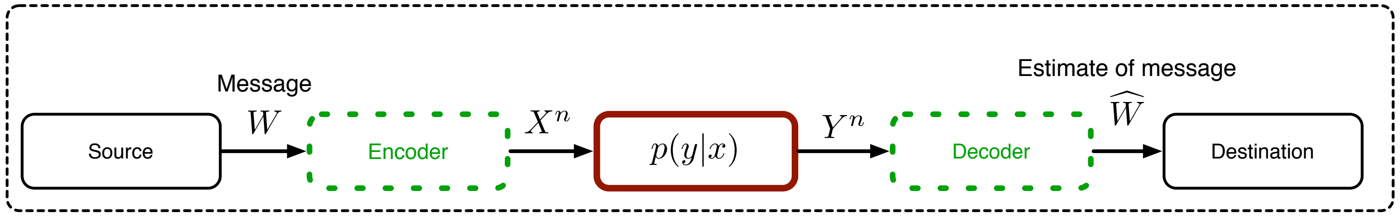


What is the **capacity** of this channel?

Intuitively **Formally**

Mathematical description of capacity

- Information channel capacity:



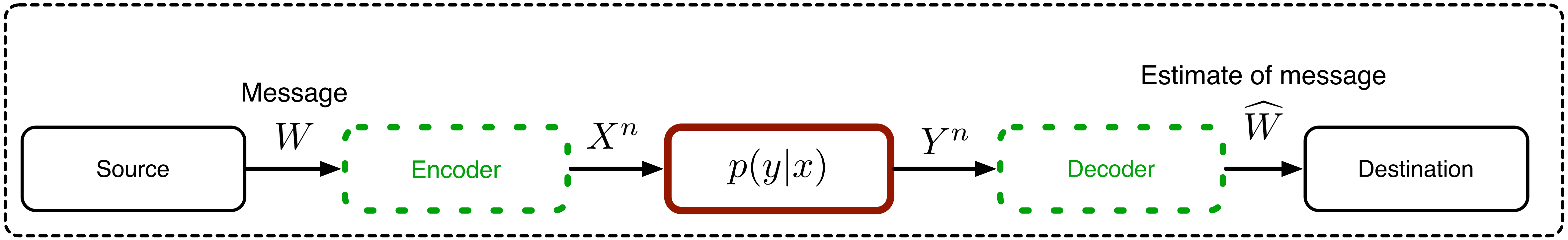
$$C = \max_{p(x)} I(X;Y)$$

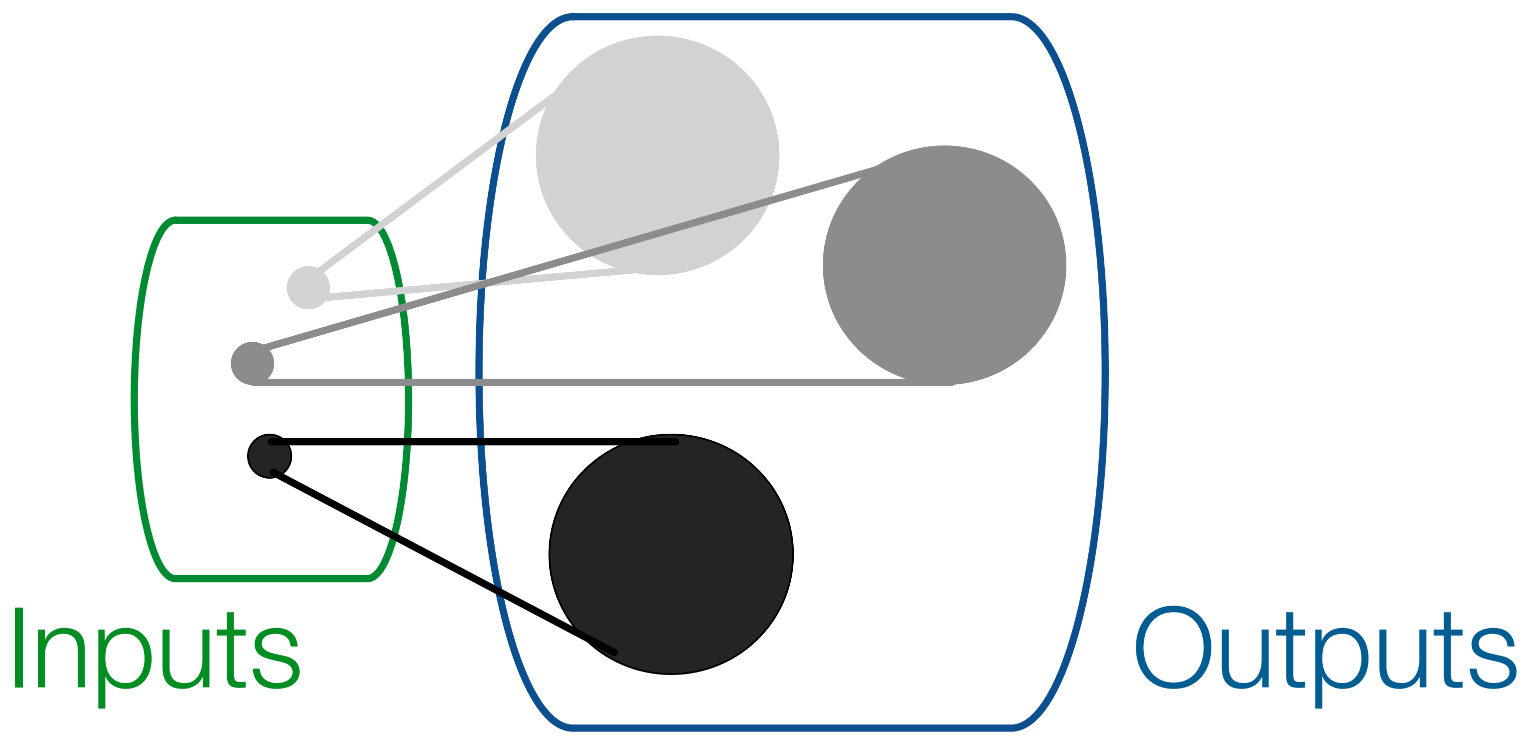
- Operational channel capacity:

```
Highest rate (bits/channel use) that can
communicate at reliably
```

- Channel coding theorem says: information capacity = operational capacity

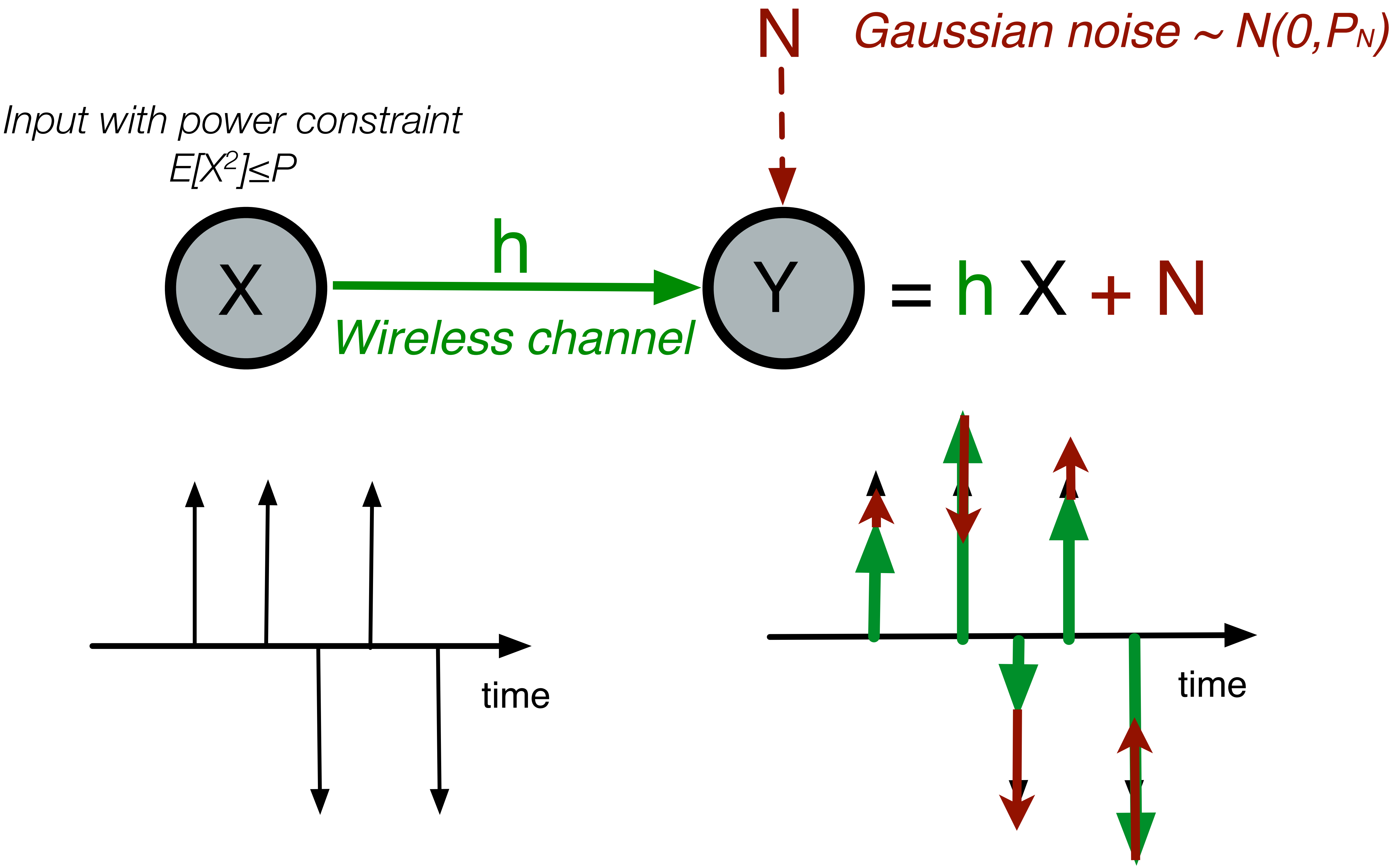
Capacity: key ideas



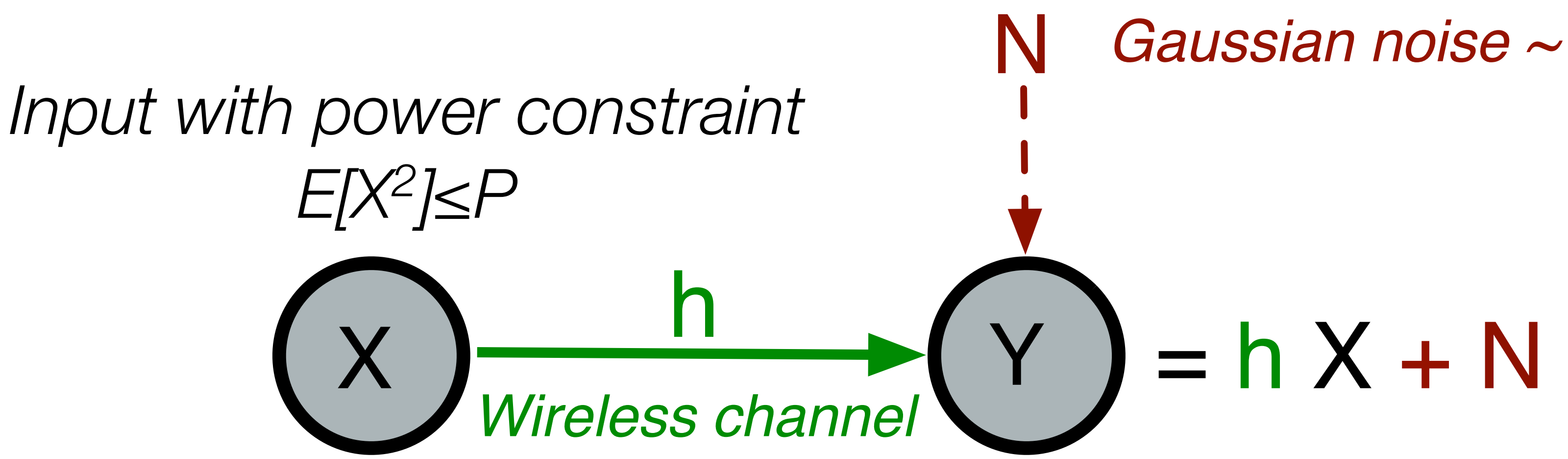


``non-confusable” inputs = channel’s capacity, depends on p(y|x)

Additive white Gaussian noise (AWGN) channel



Additive white Gaussian noise (AWGN) channel



$$C = \max_{p(x)} I(X;Y)$$

$$= \frac{1}{2}\log\left(\frac{|h|^2 P + P_N}{P_N}\right)$$

$$= \frac{1}{2}\log\left(1 + \frac{|h|^2 P}{P_N}\right)$$

$$= \frac{1}{2}\log\left(1 + \mathrm{SNR}\right)$$

May achieve for $p(x) \sim \mathcal{N}(0, P)$

i.i.d. Gaussian codebook!

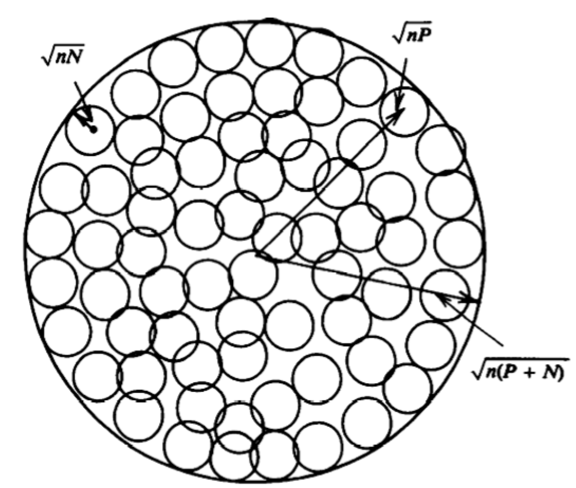


Figure 10.2. Sphere packing for the Gaussian channel.

(Cover and Thomas, *Elements of Information Theory*)

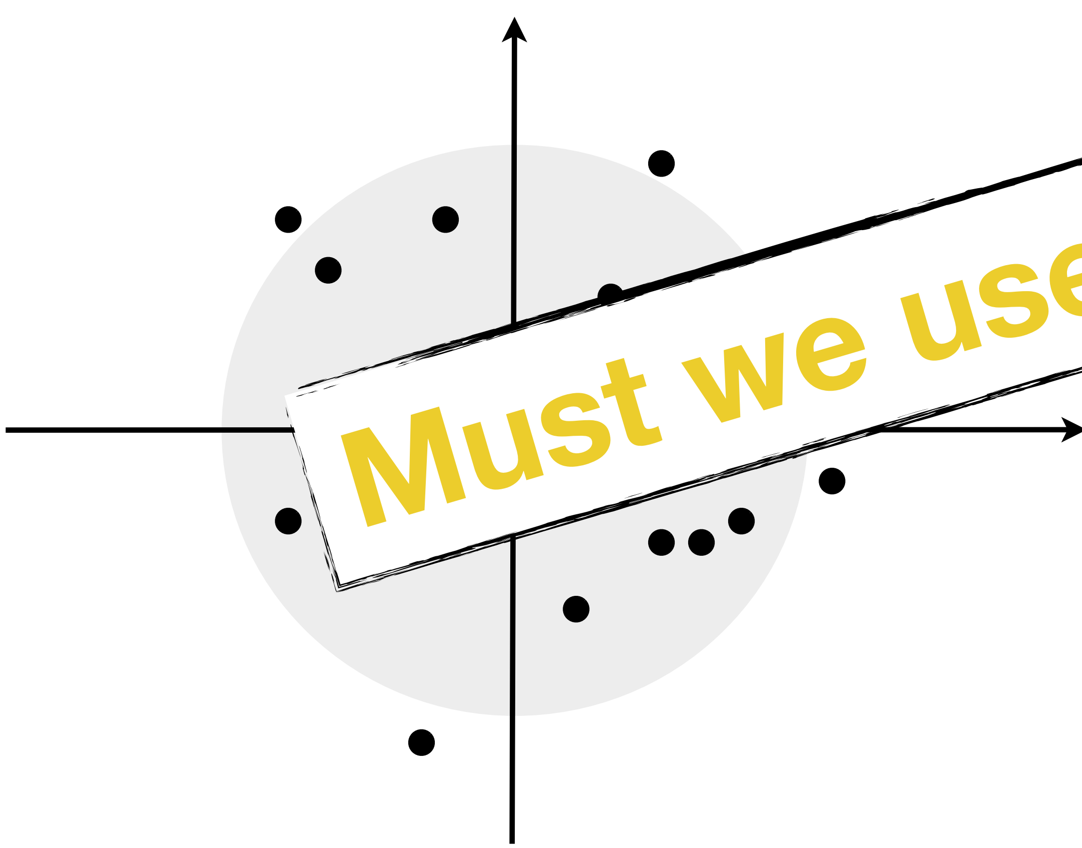
Achieving capacity: random codes

- Generate 2^{nR} (rate R) codewords $X^n = [X_1, X_2, \cdots, X_n]$ independently and element-wise i.i.d. according to

$$p(x^n) = \prod_{i=1}^{n} p_X(x_i)$$

ssian!

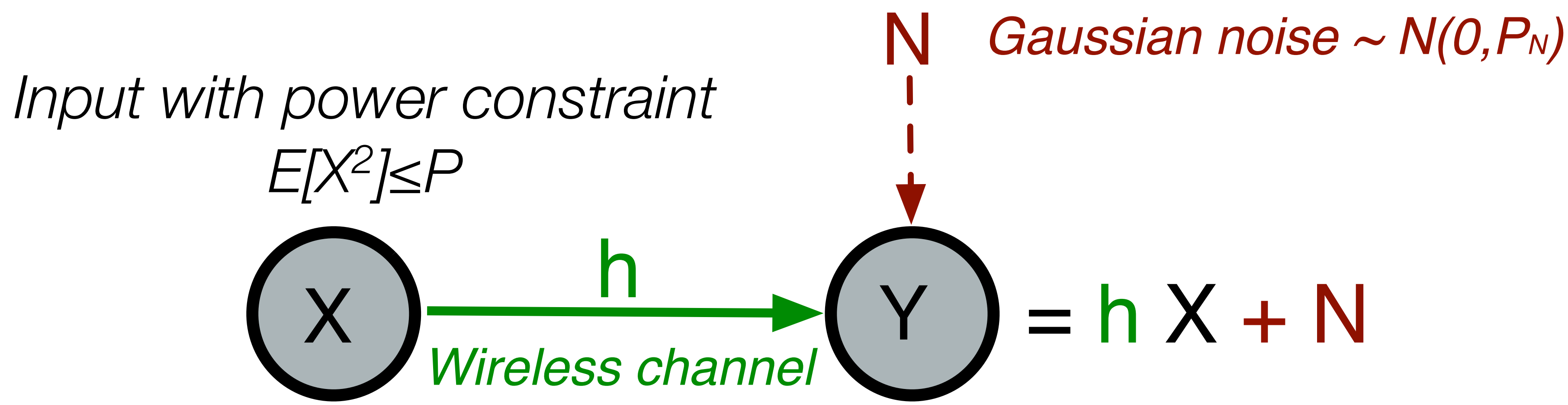
Must we use i.i.d. Gaussian?



n=2

n larger

Achieving capacity: lattice codes



Theorem: Lattice codes achieve the capacity of the AWGN channel

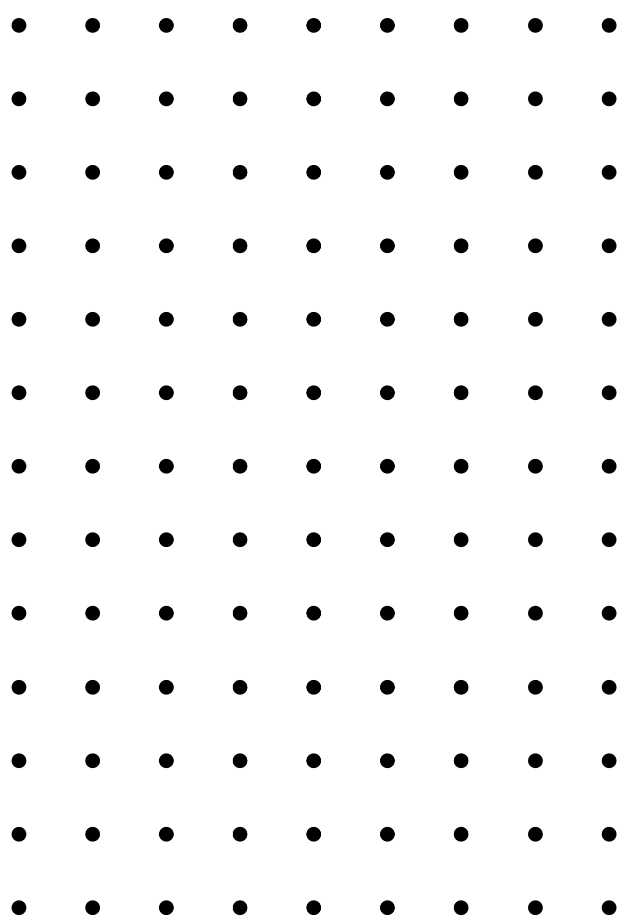
U. Erez and R. Zamir, *Achieving $\frac{1}{2}\log(1+\mathrm{SNR})$ on the AWGN channel with lattice encoding and decoding*, IEEE Transactions on Information Theory, vol. 50, pp. 2293-2314, October 2004.

U. Erez, S. Litsyn, and R. Zamir, *Lattices which are good for (almost) everything*, IEEE Transactions on Information Theory, vol. 51, pp. 3401-3416, October 2005.

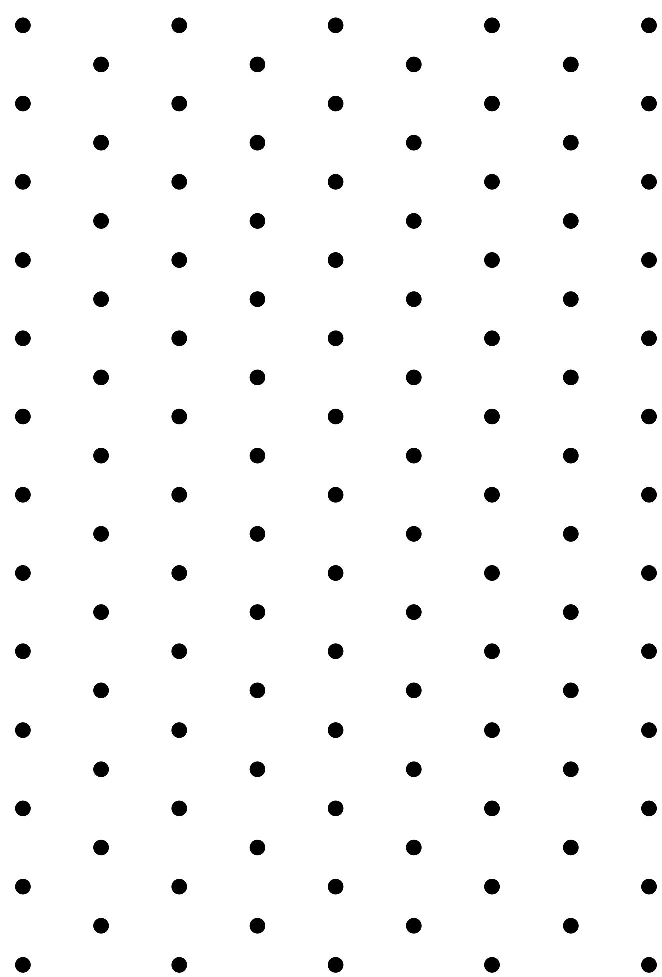
R. Zamir, *Lattices are everywhere*, in Proceedings of the 4th Annual Workshop on Information Theory and its Applications, La Jolla, CA, February 2009.

Lattice code basics

- A lattice Λ is a discrete subgroup of $\mathbb{R}^n$
- $\lambda_1, \lambda_2 \in \Lambda \Rightarrow (\lambda_1 + \lambda_2 \in \Lambda)$ **KEY!** ***(not true of random i.i.d. Gaussian)***

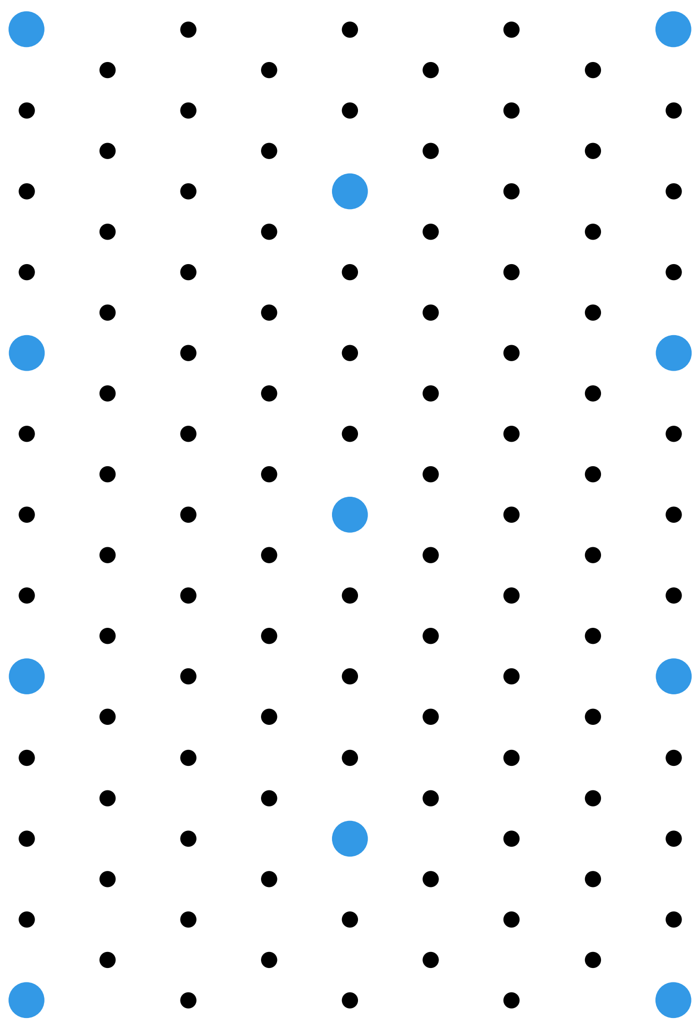


$\mathbb{Z}^n$ is a simple lattice.



$\mathbf{B}\mathbb{Z}^n$

$\mathbf{B} \in \mathbb{R}^n$



Nested lattices $\Lambda \subset \Lambda_{\text{FINE}}$

figures taken with permission and gratitude from [B. Nazer, ISIT 2011 tutorial]

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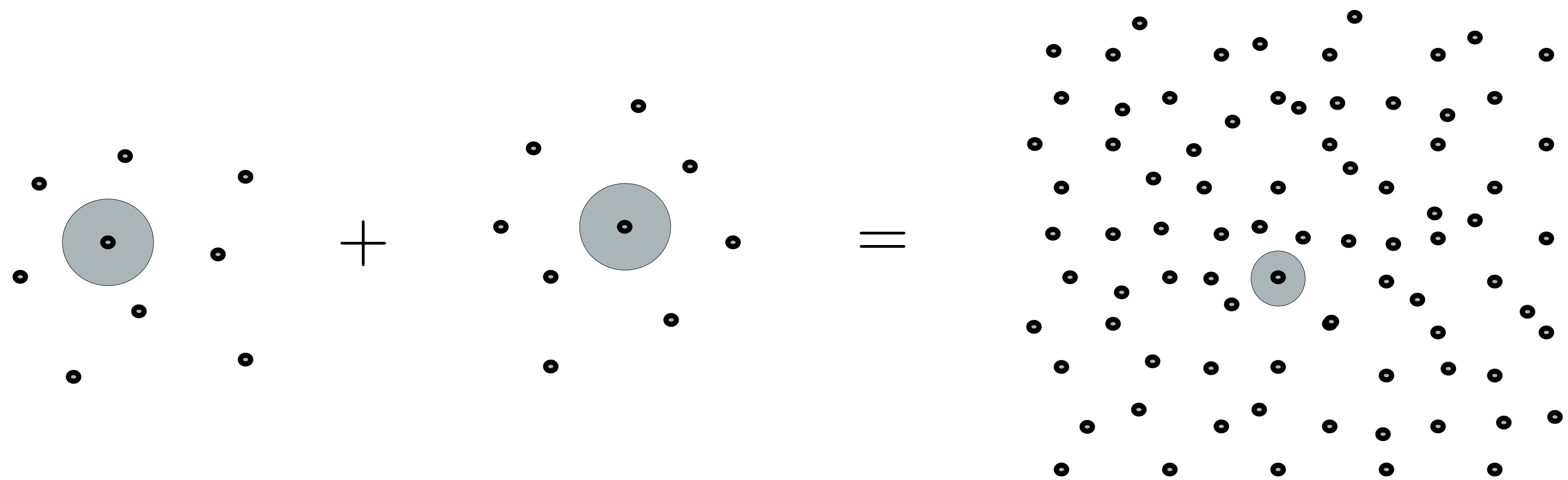
Thursday, June 14, 2012

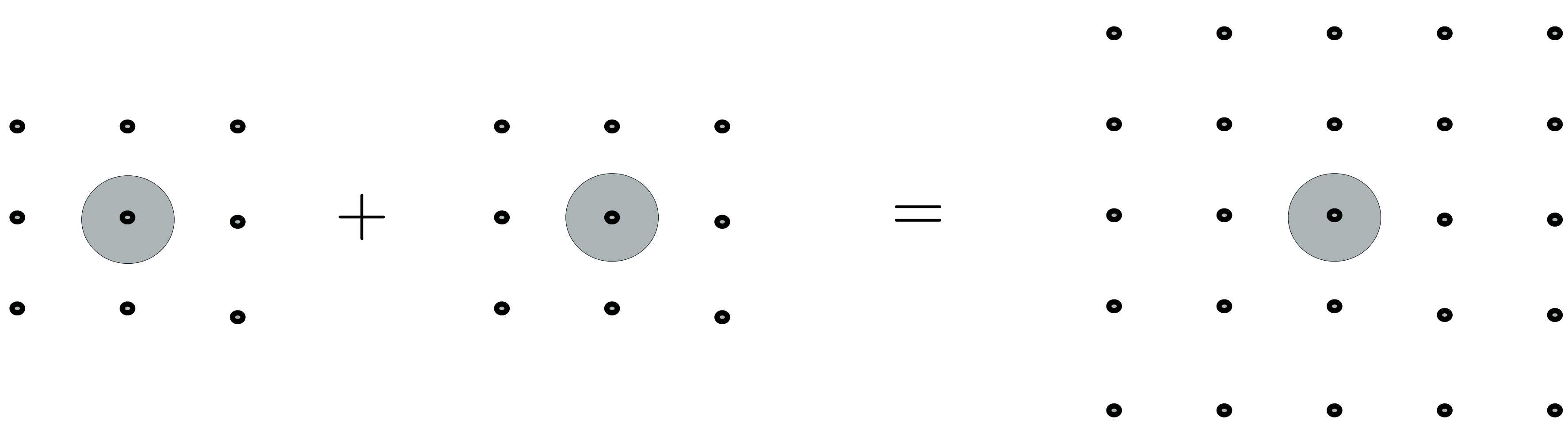
Sums of random codewords



X_1 X_2 $X_1 + X_2$

$\bullet \lambda_1, \lambda_2 \in \Lambda \Rightarrow (\lambda_1 + \lambda_2 \in \Lambda)$ **KEY!** ***(not true of random i.i.d. Gaussian)***

Sums of structured codewords



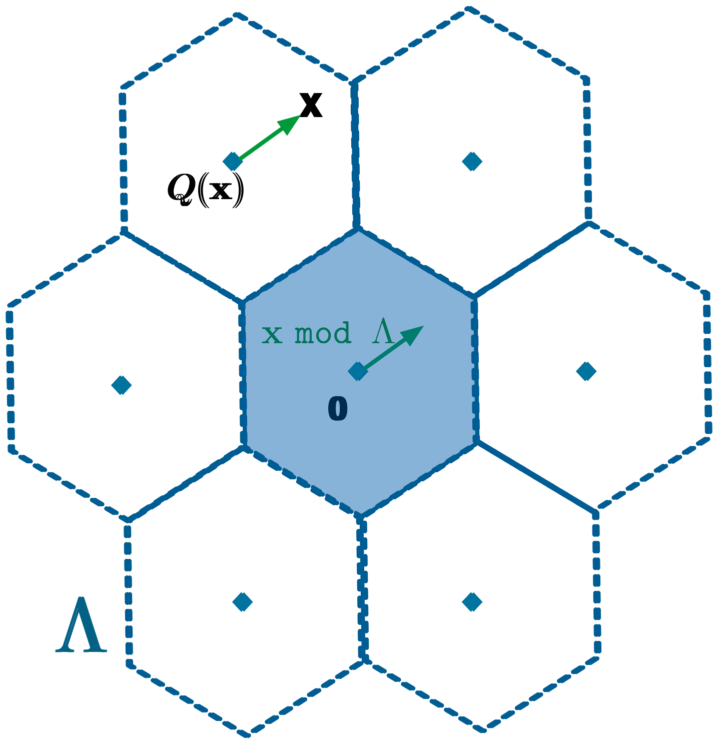
X_1 X_2 $X_1 + X_2$

Lattice notation

- $\Lambda = \{\lambda = G\,\mathbf{i} : \mathbf{i} \in \mathbb{Z}^n\}$, G the generator matrix

- *lattice quantizer* of Λ:

$$Q(\mathbf{X}) = \arg\min_{\lambda \in \Lambda} ||\mathbf{X} - \lambda||$$



- $\mathbf{x} \mod \Lambda := \mathbf{x} - Q(\mathbf{x})$

- *fundamental region* $\mathcal{V} := \{\mathbf{x} : Q(\mathbf{x}) = \mathbf{0}\}$ of volume V

- *second moment per dimension of a uniform distribution over* $\mathcal{V}$:

$$\sigma^2(\Lambda) := \frac{1}{V} \cdot \frac{1}{n} \int_{\mathcal{V}} ||\mathbf{x}||^2 d\mathbf{x}$$

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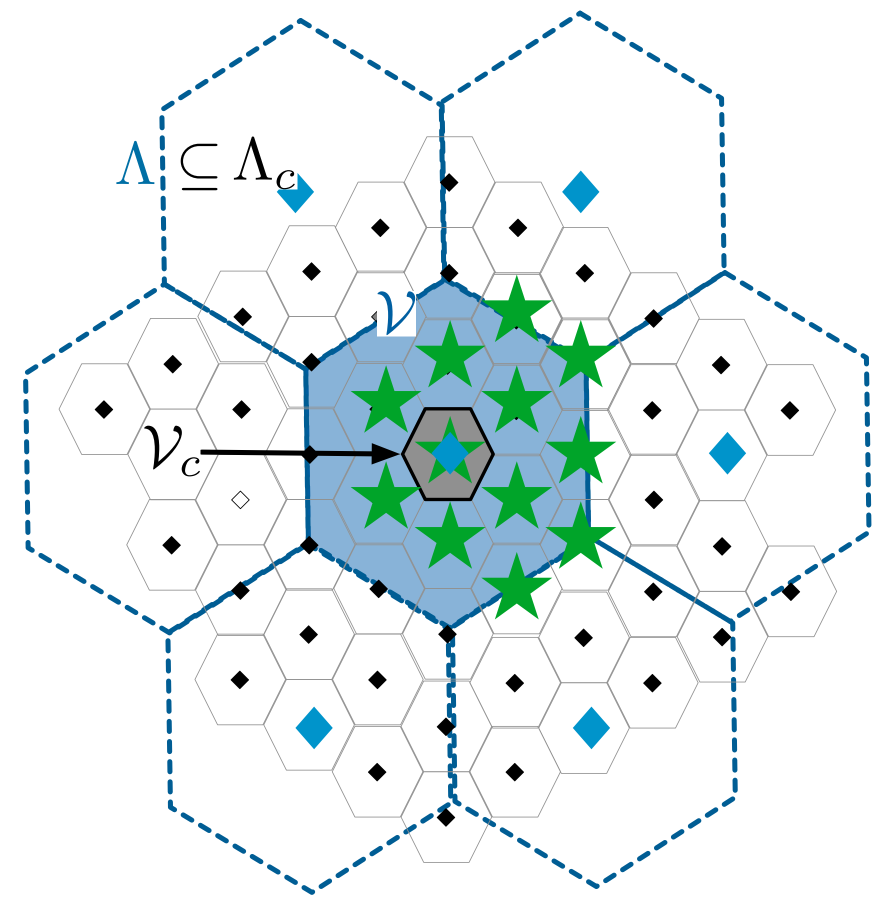
Nested lattice codes

- Nested lattice pair : $\Lambda \subseteq \Lambda_c$

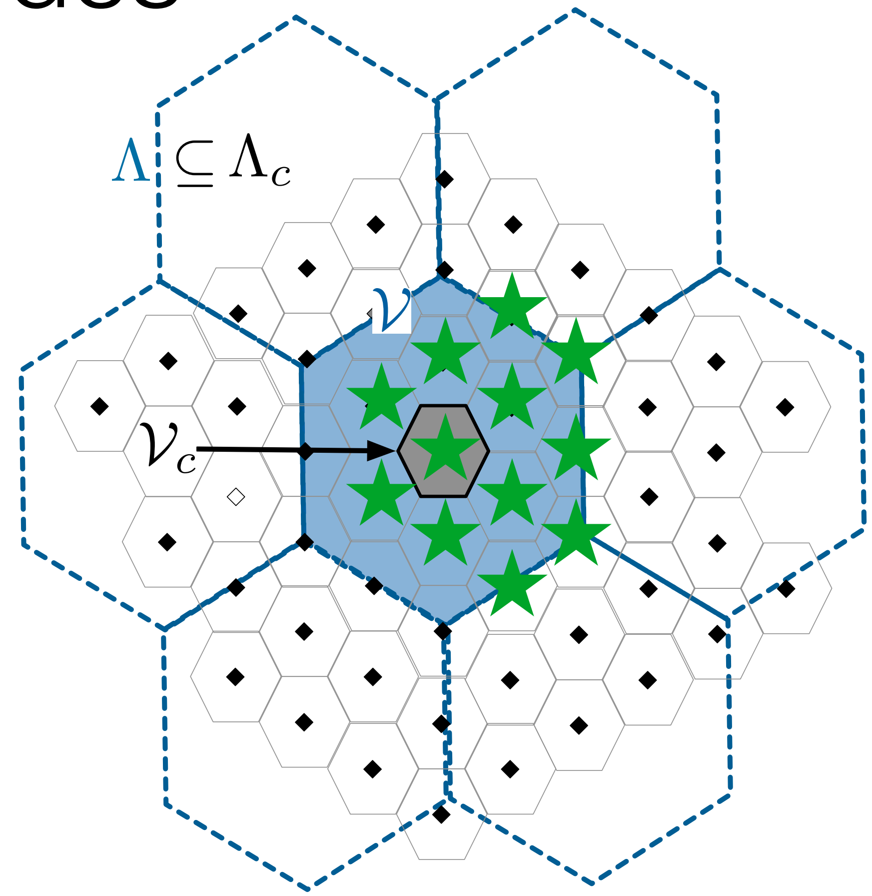
- Lattice in n (blocklength) dimensions

- The code book ★ $\mathcal{C} = \{\Lambda_c \cap \mathcal{V}(\Lambda)\}$ ★ is used to achieve the capacity of AWGN channel
[Erez+Zamir, Trans. IT, 2004]

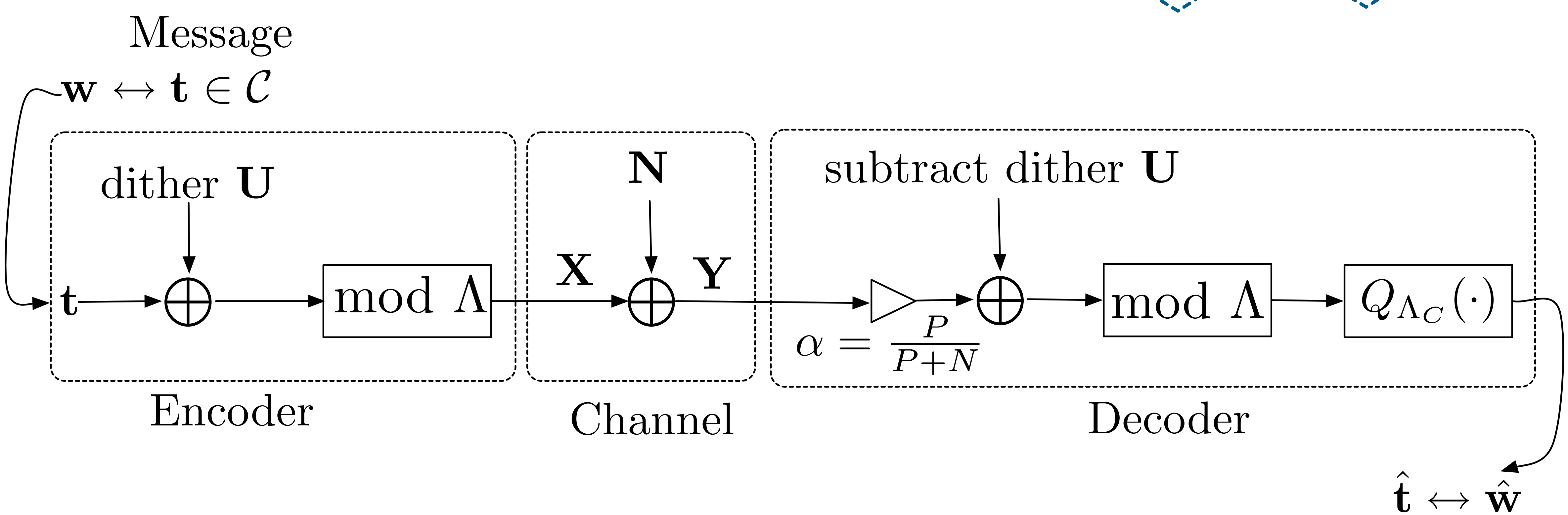


- Coding rate: $R = \frac{1}{n}\log|\mathcal{C}| = \frac{1}{n}\log\frac{V(\Lambda)}{V(\Lambda_c)}$ arbitrary *(# of ★)*

Achieving capacity using lattice codes



Codebook ★ $\mathcal{C} = \{\Lambda_c \cap \mathcal{V}(\Lambda)\}$



Achieves rate $\frac{1}{2}\log\left(1+\frac{P}{N}\right)$

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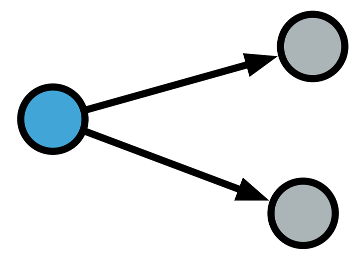
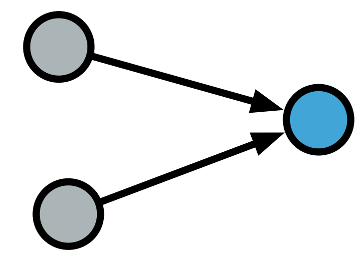
Lattice codes for Gaussian single-hop channels?

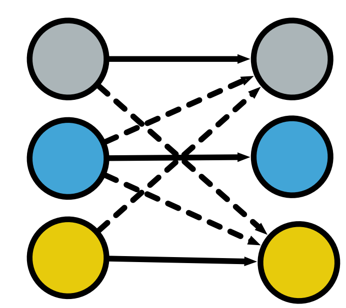
- AWGN channel *[Erez, Zamir, Trans. IT, 2004]*



- AWGN broadcast channel *[Zamir, Shamai, Erez, Trans. IT, 2002]*



- AWGN multiple-access *[Nazer, Gastpar, TransIT 2011]* and ``dirty” multiple-access channels *[Philosof, Khisti, Erez, Zamir, ISIT 2007]*
- Distributed source coding *[Krithivasan, Pradhan, TransIT 2009]*



- AWGN interference channel: interference decoding / interference alignment in K>2 interference channels *[Bresler, Parekh, Tse, TransIT, 2010] [Sridharan, Jafarian, Jafar, Shamai, arXiv 2008]*

What about multi-hop relay networks?

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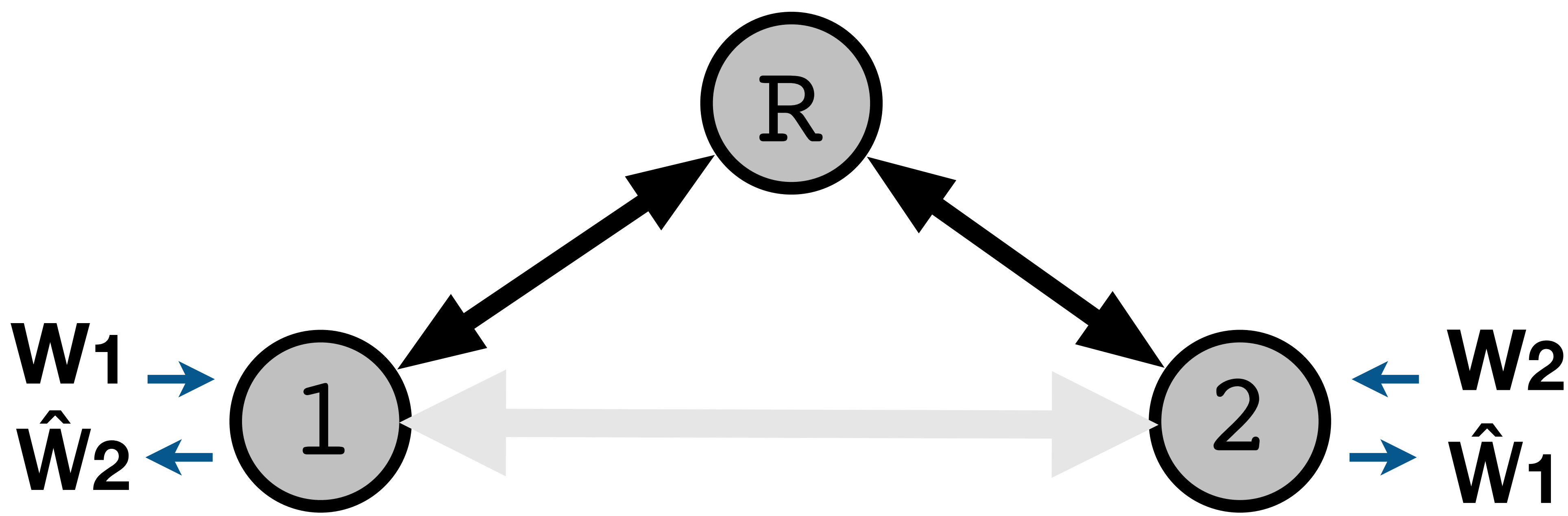
Two-way relay channel

[Wu, Chou, Kung 2004]



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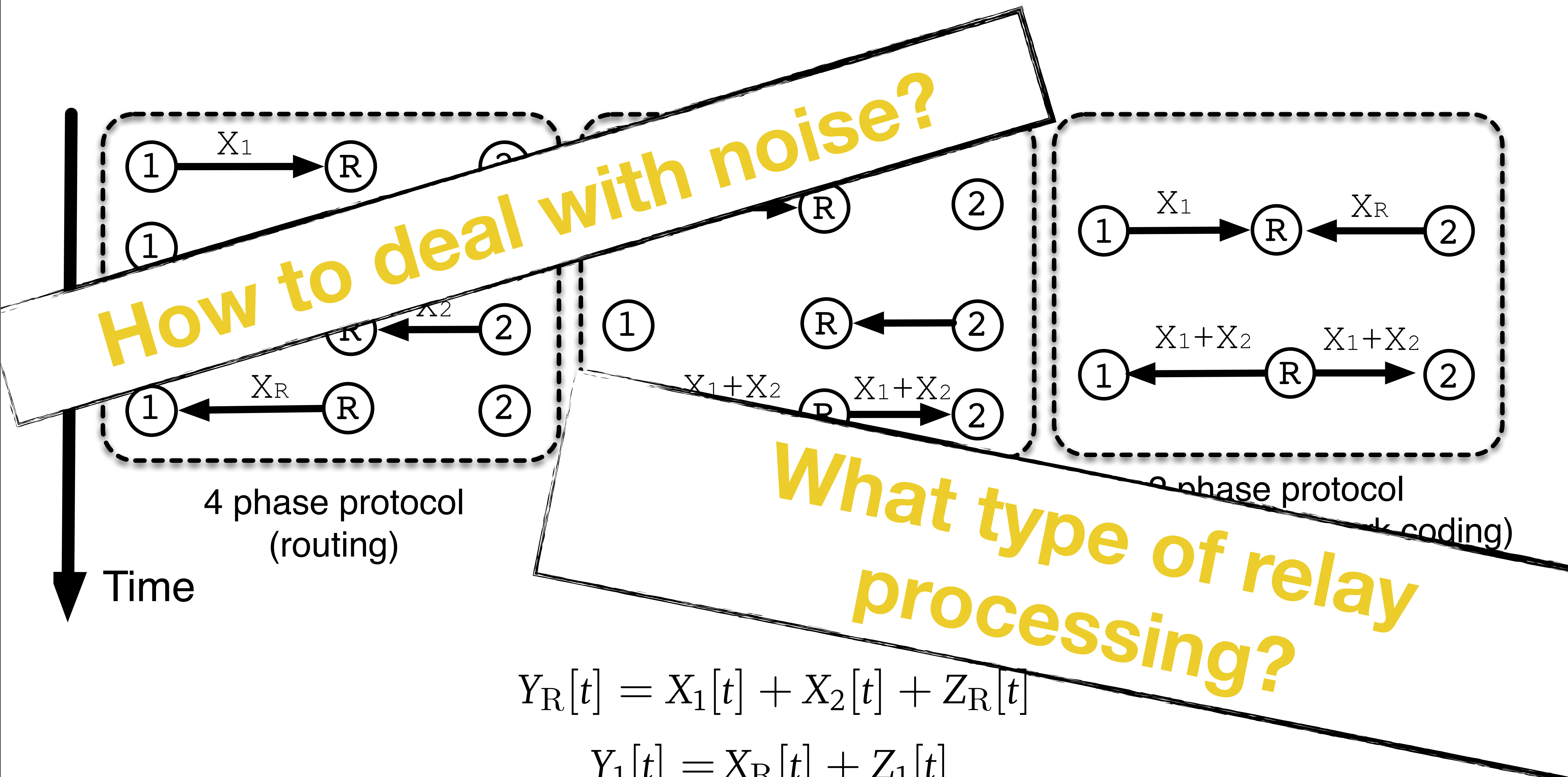
4 possible protocols for half-duplex nodes



4 phase protocol (routing)

Time

$$Y_R[t] = X_1[t] + X_2[t] + Z_R[t]$$

$$Y_1[t] = X_R[t] + Z_1[t]$$

$$Y_2[t] = X_R[t] + Z_2[t]$$

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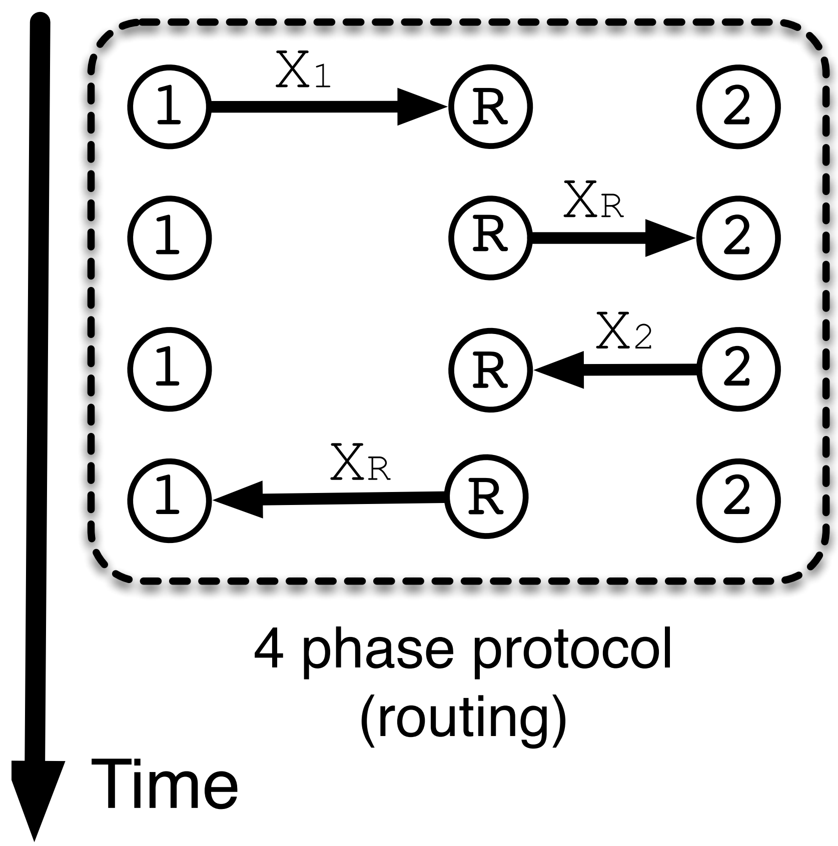
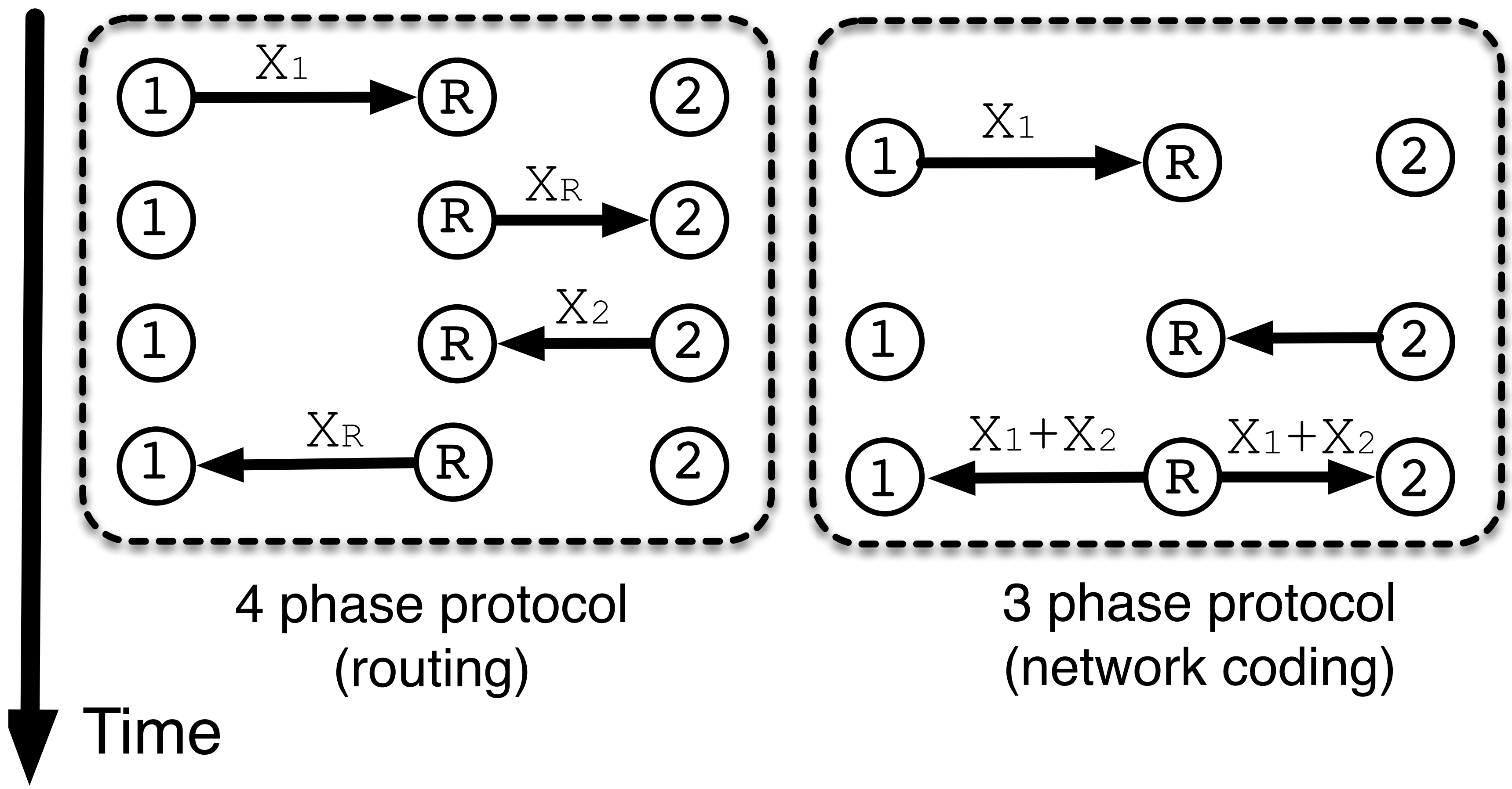
4 phase routing



$$R_{\text{ROUTING}} = \frac{1}{4}\log\left(1+\frac{P}{\sigma^2}\right)$$

3 phase network coding



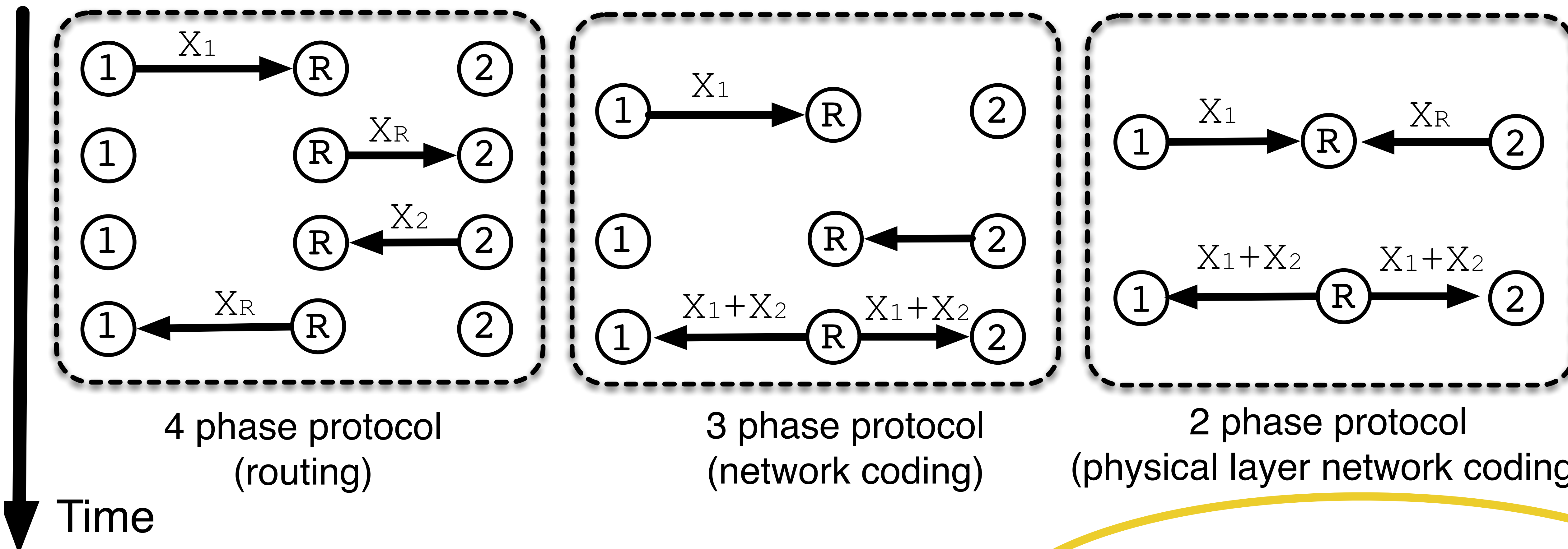
4 phase protocol
(routing)

3 phase protocol
(network coding)

$$R_{\text{ROUTING}} = \frac{1}{4}\log\left(1 + \frac{P}{\sigma^2}\right)$$

$$R_{\text{NETCOD}} = \frac{1}{3}\log\left(1 + \frac{P}{\sigma^2}\right)$$

2 phase physical layer network coding



4 phase protocol
(routing)

3 phase protocol
(network coding)

2 phase protocol
(physical layer network coding)

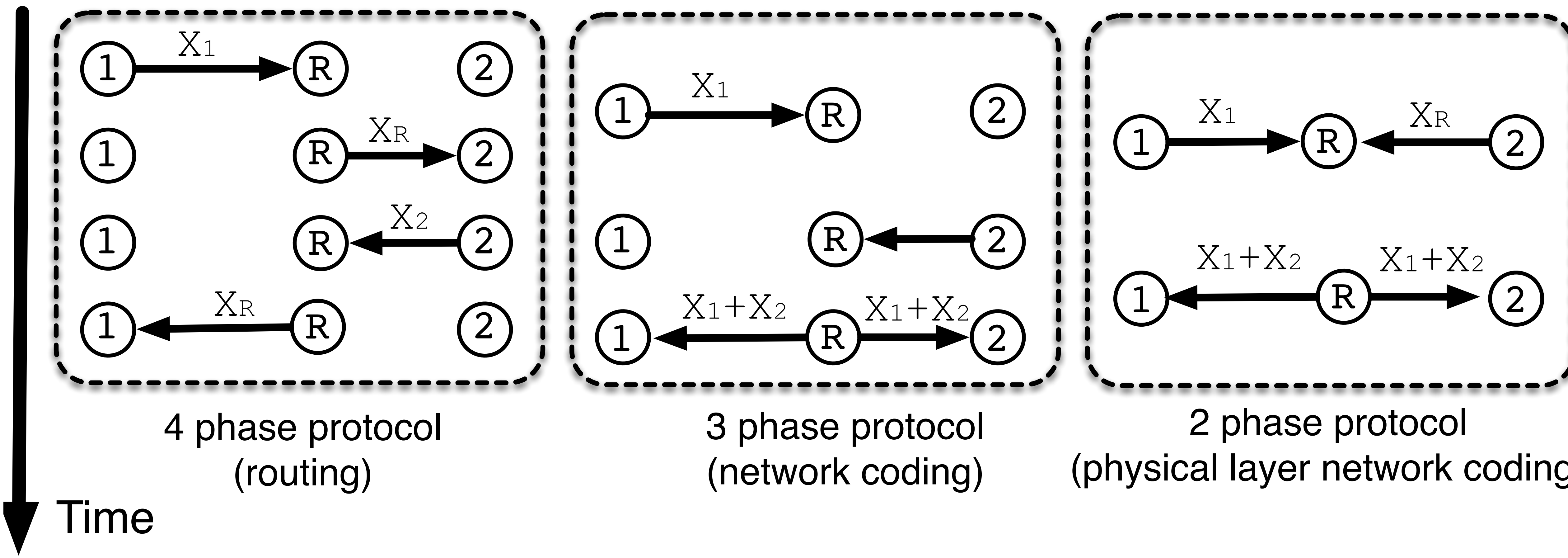
$$R_{\text{LATTICE}} = \frac{1}{2}\log_2\left(\frac{1}{2} + \frac{P}{\sigma^2}\right)$$

$$R_{\text{ROUTING}} = \frac{1}{4}\log\left(1 + \frac{P}{\sigma^2}\right)$$

$$R_{\text{NETCOD}} = \frac{1}{3}\log\left(1 + \frac{P}{\sigma^2}\right)$$

notation taken with permission and gratitude from [B. Nazer, M. Gastpar, Proc. IEEE, 2011]

2 phase physical layer network coding



4 phase protocol (routing)

3 phase protocol (network coding)

2 phase protocol (physical layer network coding)

$$R_{\text{ROUTING}} = \frac{1}{4}\log\left(1 + \frac{P}{\sigma^2}\right) \qquad R_{\text{NETCOD}} = \frac{1}{3}\log\left(1 + \frac{P}{\sigma^2}\right) \qquad R_{\text{LATTICE}} = \frac{1}{2}\log_2\left(\frac{1}{2} + \frac{P}{\sigma^2}\right)$$

$$R_{\text{UPPER}} = \frac{1}{2}\log\left(1 + \frac{P}{\sigma^2}\right)$$

notation taken with permission and gratitude from [B. Nazer, M. Gastpar, Proc. IEEE, 2011]

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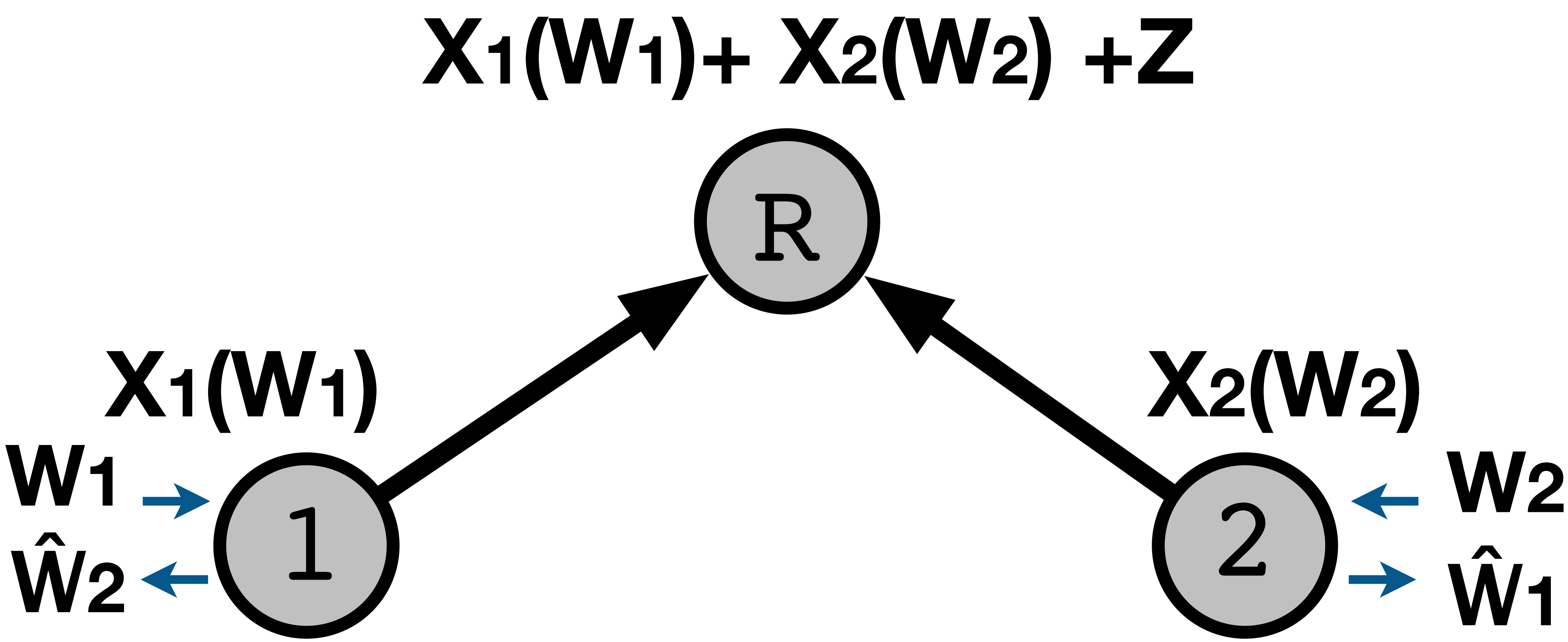
Key idea: structured coding

[Nazer, Gastpar, 2011]
[Nam, Chung, Lee, 2010]
[Wilson, Narayanan, Pfister, Sprintson,2010]



Multiple access channel? $\hat{W}_1, \hat{W}_2$

depends on what the relay needs

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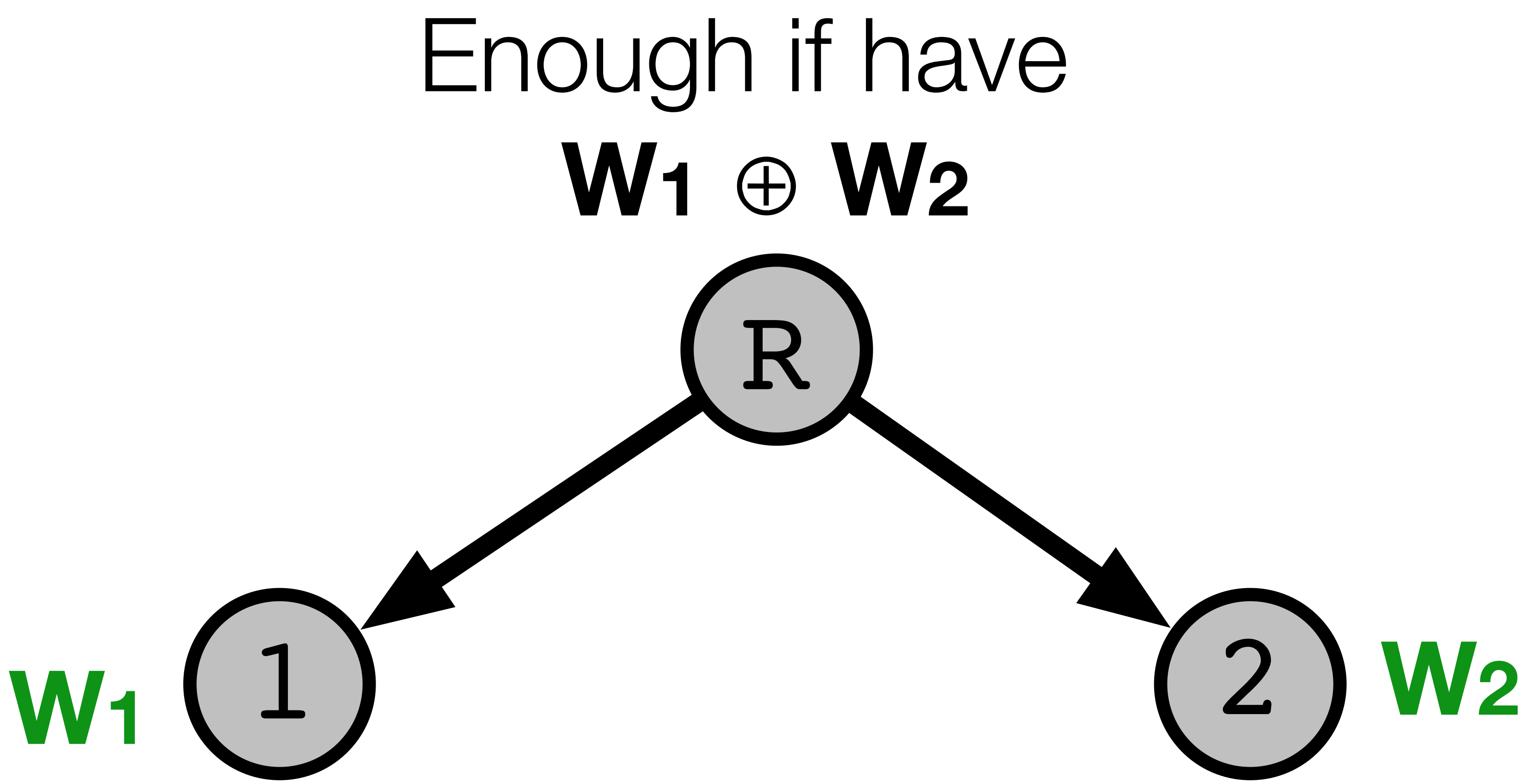
Exploit own-message side-information



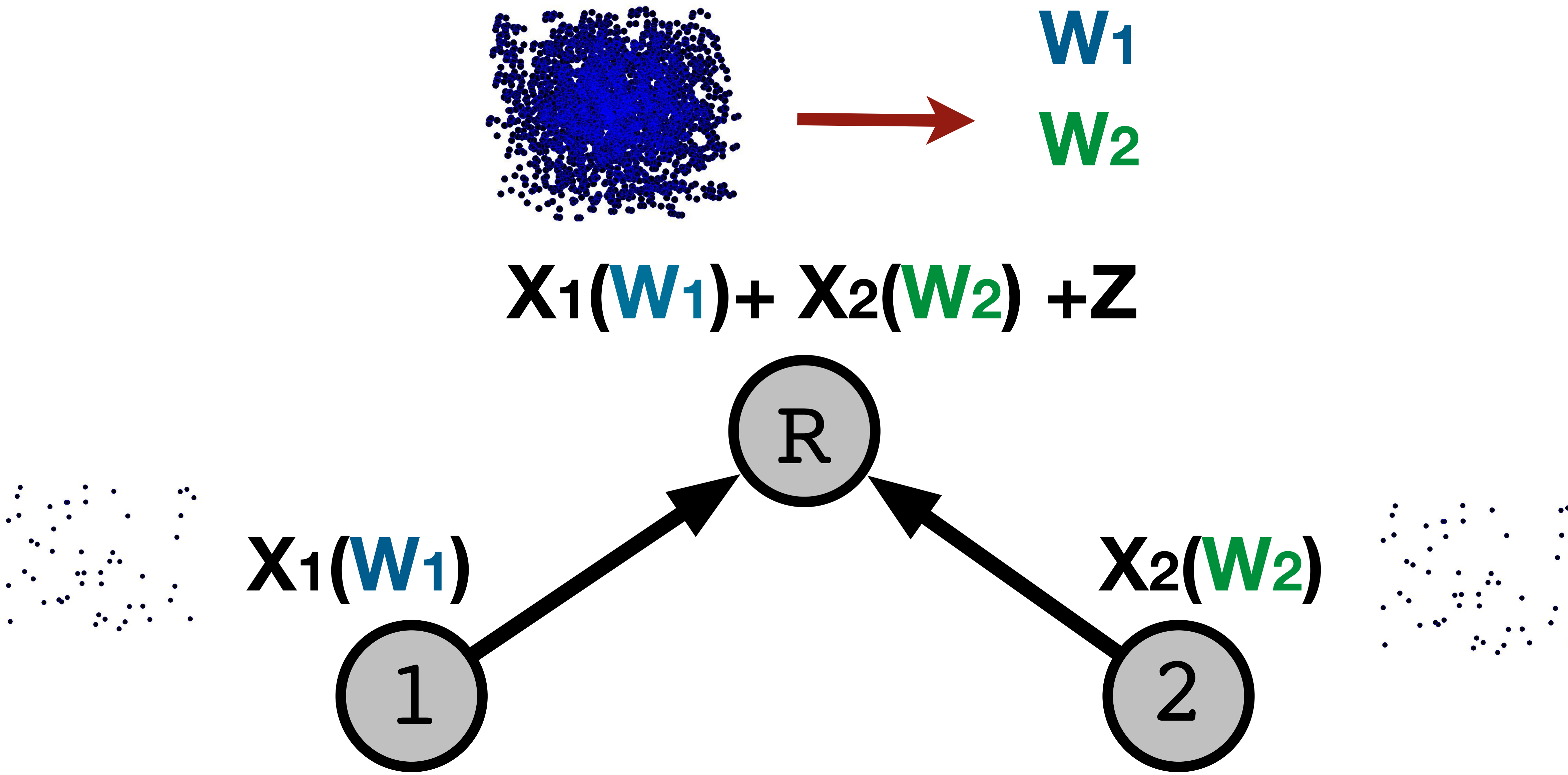
$\hat{W}_2 = W_1 \oplus W_2 \oplus W_1$

$\hat{W}_1 = W_1 \oplus W_2 \oplus W_2$

Don't necessarily need individual messages at relay!

Don't decode what you don't need!

Decode **both** messages



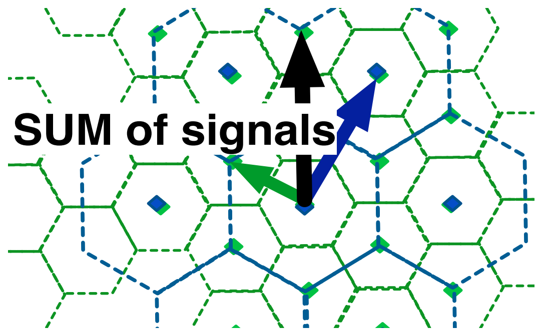
If we use random codes....

Decode **the SUM**

$W_1 \oplus W_2$

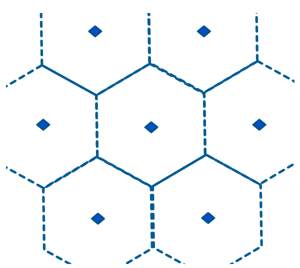
Mapping exists!

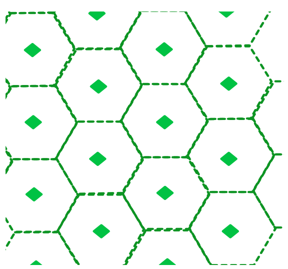
[B. Nazer, M. Gastpar, IT Trans., 2011]



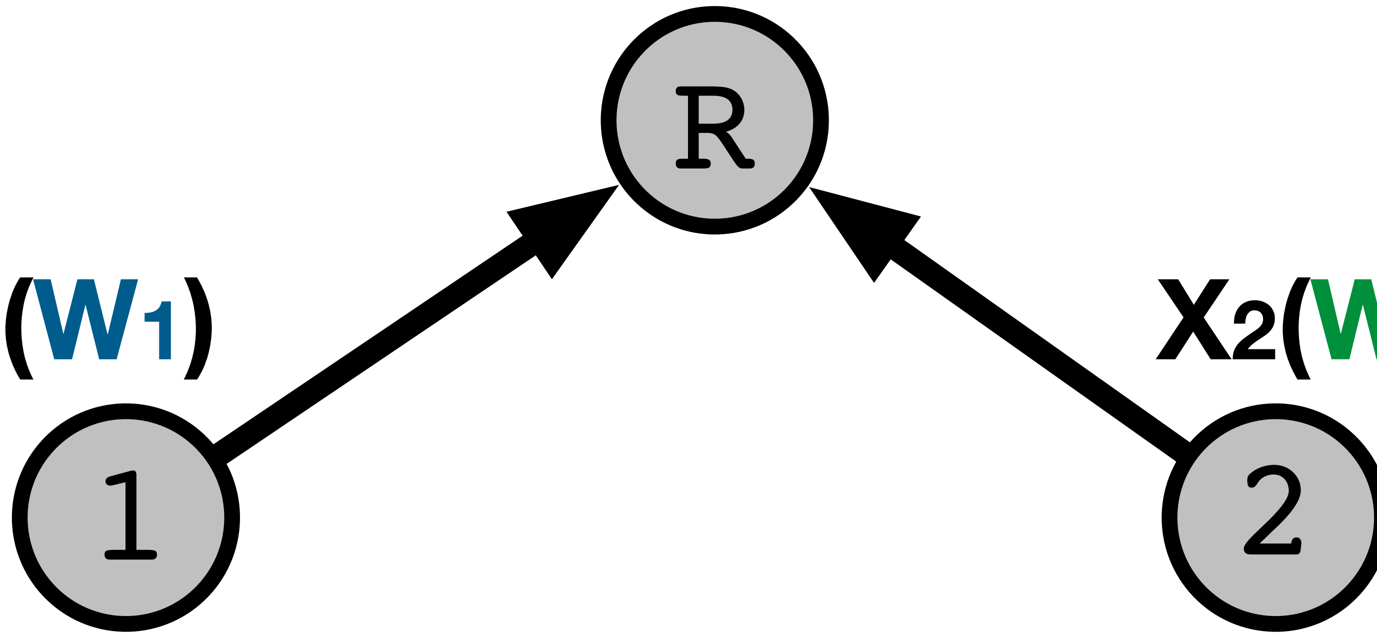
$$X_1(W_1) + X_2(W_2) \bmod \Lambda$$

$$X_1(W_1) + X_2(W_2) + Z$$

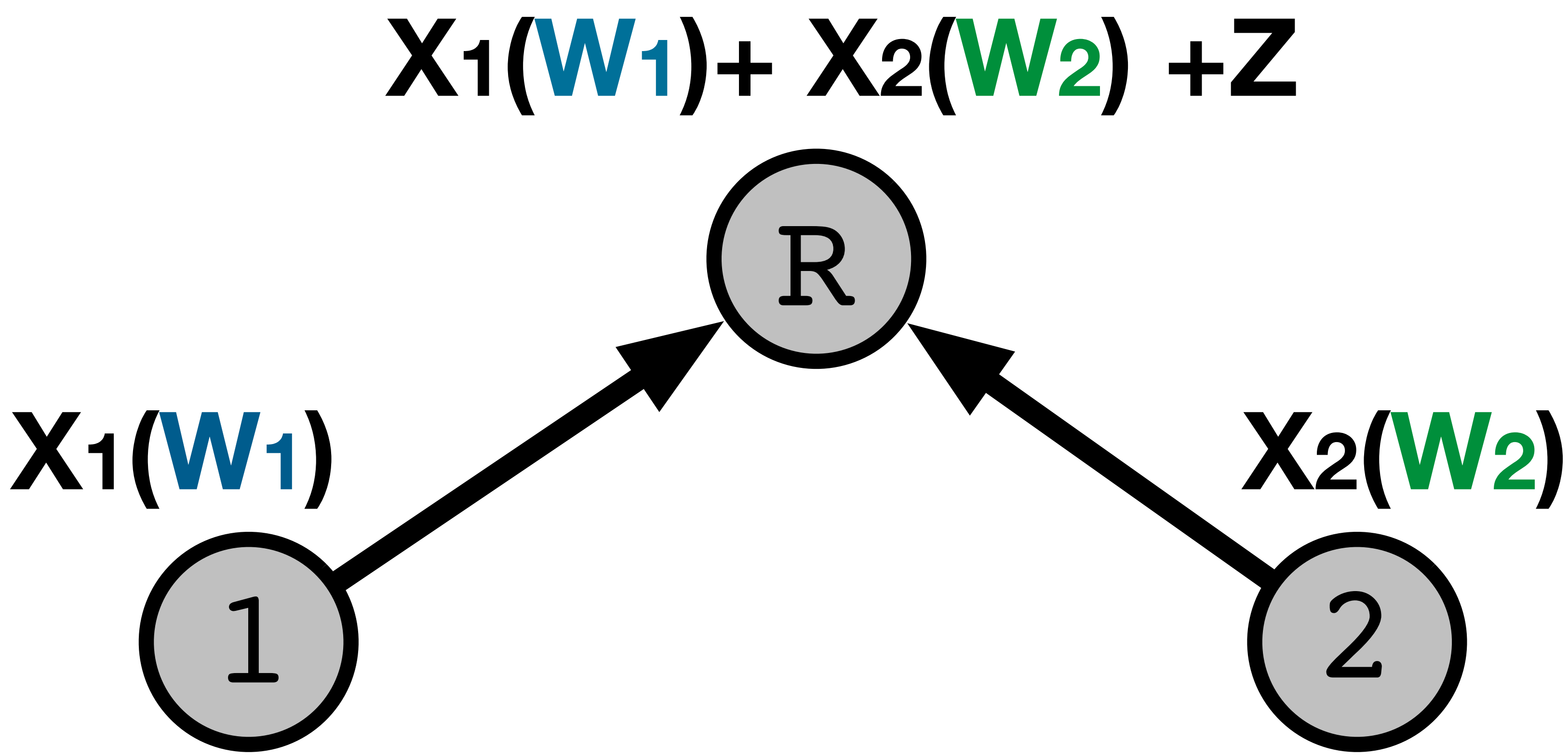




$X_1(W_1)$ $X_2(W_2)$



If we use structured codes....



Random codes

$$R_1 \leq \frac{1}{2}\log\left(1+\frac{P_1}{N_R}\right)$$
$$R_2 \leq \frac{1}{2}\log\left(1+\frac{P_2}{N_R}\right)$$
$$R_1 + R_2 \leq \frac{1}{2}\log\left(1+\frac{P_1+P_2}{N_R}\right)$$

Structured codes

$$R_1 \leq \frac{1}{2}\log\left(\frac{P_1}{P_1+P_2}+\frac{P_1}{N_R}\right)$$
$$R_2 \leq \frac{1}{2}\log\left(\frac{P_2}{P_1+P_2}+\frac{P_2}{N_R}\right).$$

No sum-rate!

Can handle unequal channels and powers!

[Nam, Chung, Lee, 2010]

Gains *(equal power and channel gains)*

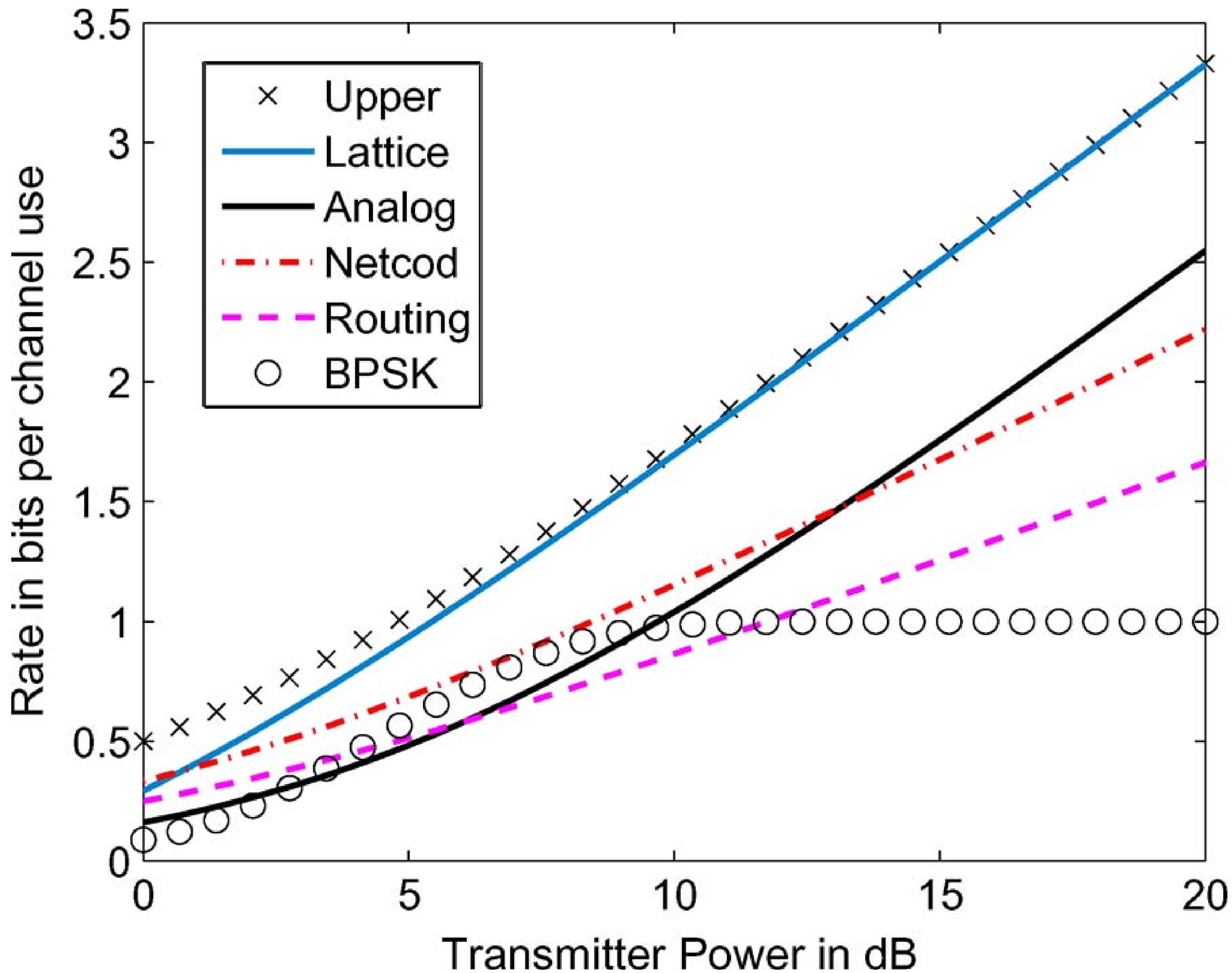
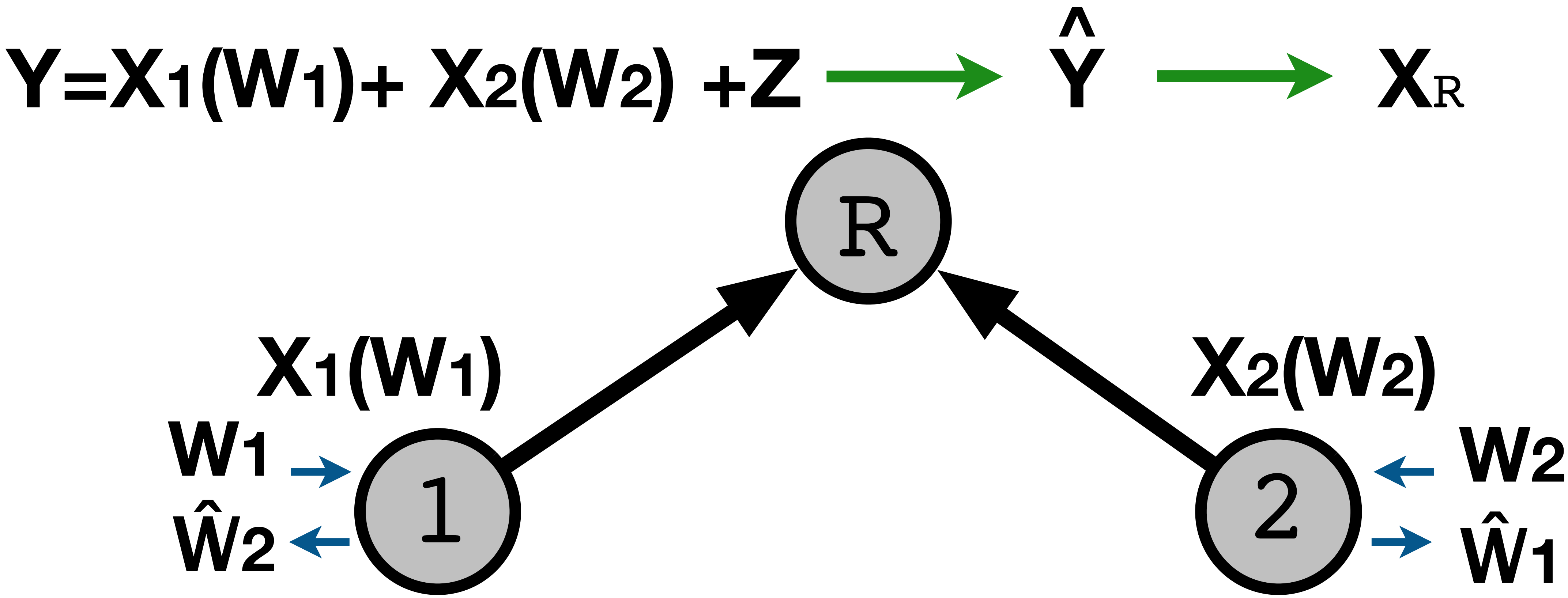


Fig. 11. ***A performance comparison of the schemes for the two-way relay channel discussed in this paper.***

taken with permission and gratitude from
[B. Nazer, M. Gastpar, Proc. of IEEE, Mar. 2011]

Why force the decode?



Why not **compress-and-forward**?

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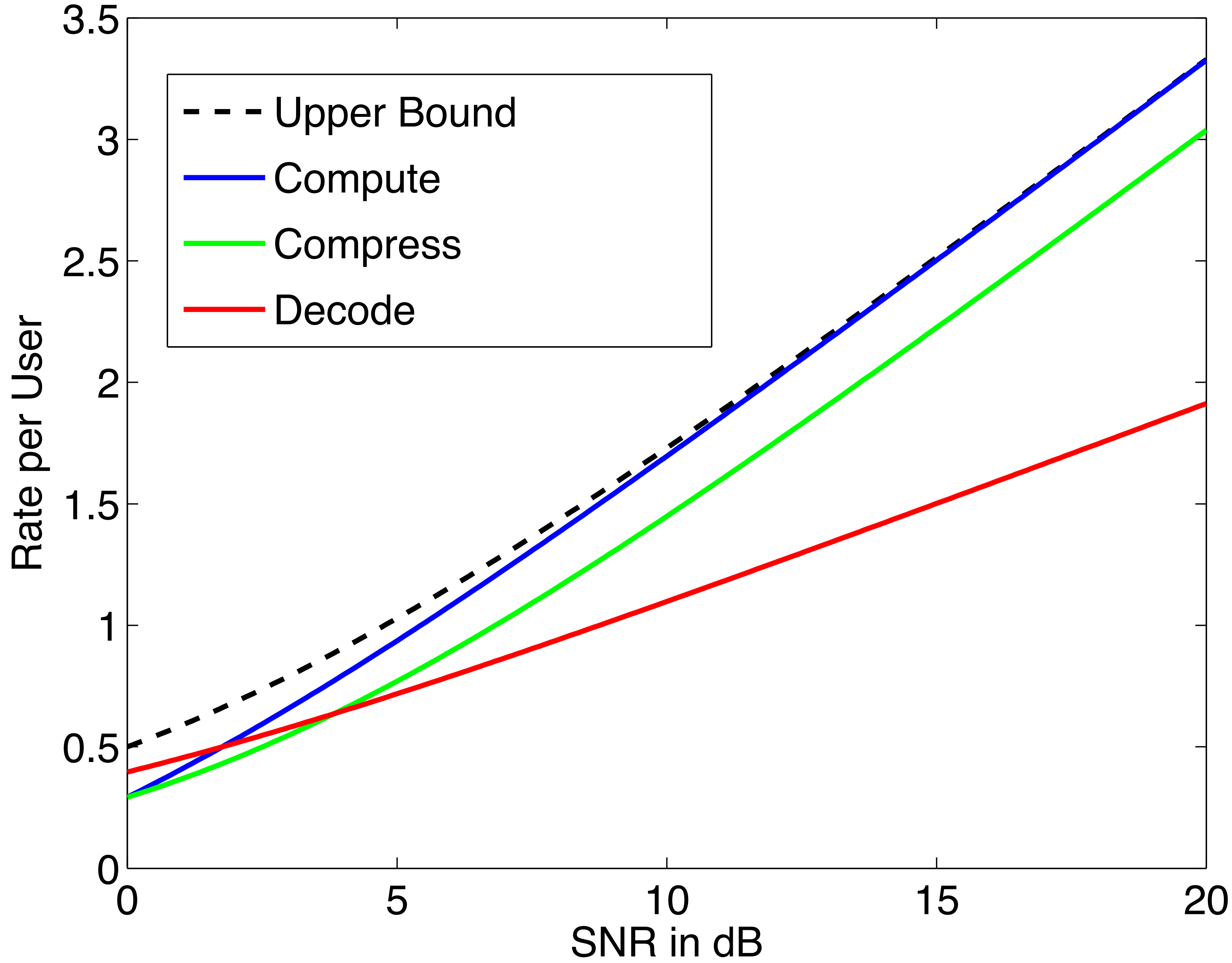
Gains *(equal power and channel gains)*

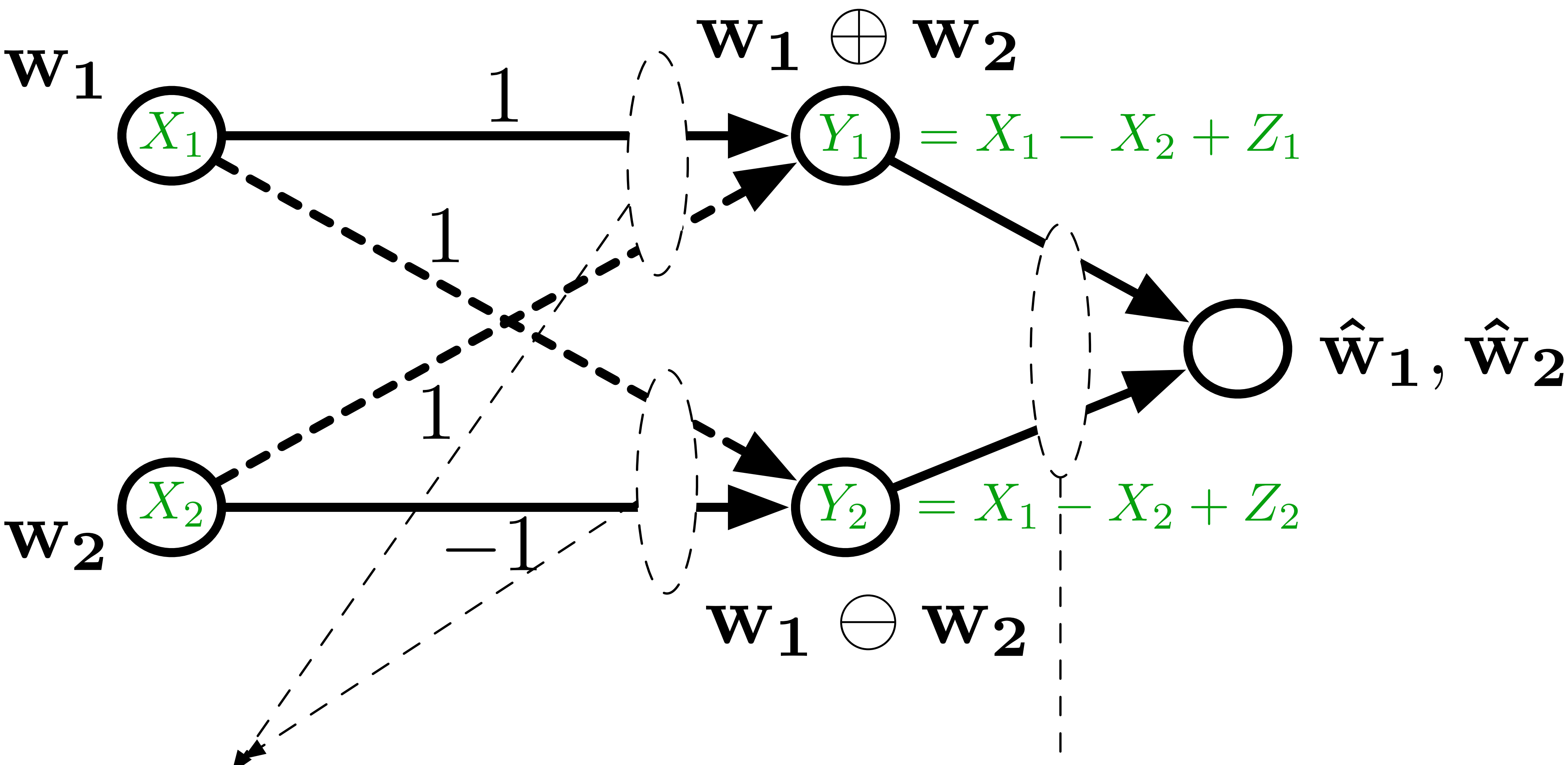


taken with permission and gratitude from
[B. Nazer, M. Gastpar, ISIT Tutorial 2011]

Outline

- Point to point channels: random codes, lattice (structured) codes
- Two-way relay channel: the canonical example of structure being useful
- **Compute and Forward (+ Inverse Compute and Forward) for relay networks**
- Relaying using lattice codes
- Additional lattice examples
- Conclusion

Compute-and-forward



Compute-and-Forward

[B. Nazer, M. Gastpar, IT Trans., 2011]

Inverse of Compute-and-Forward

[Y. Song, N. Devroye, B. Nazer, ISIT 2011]

Naturally "align" desired equations to channel

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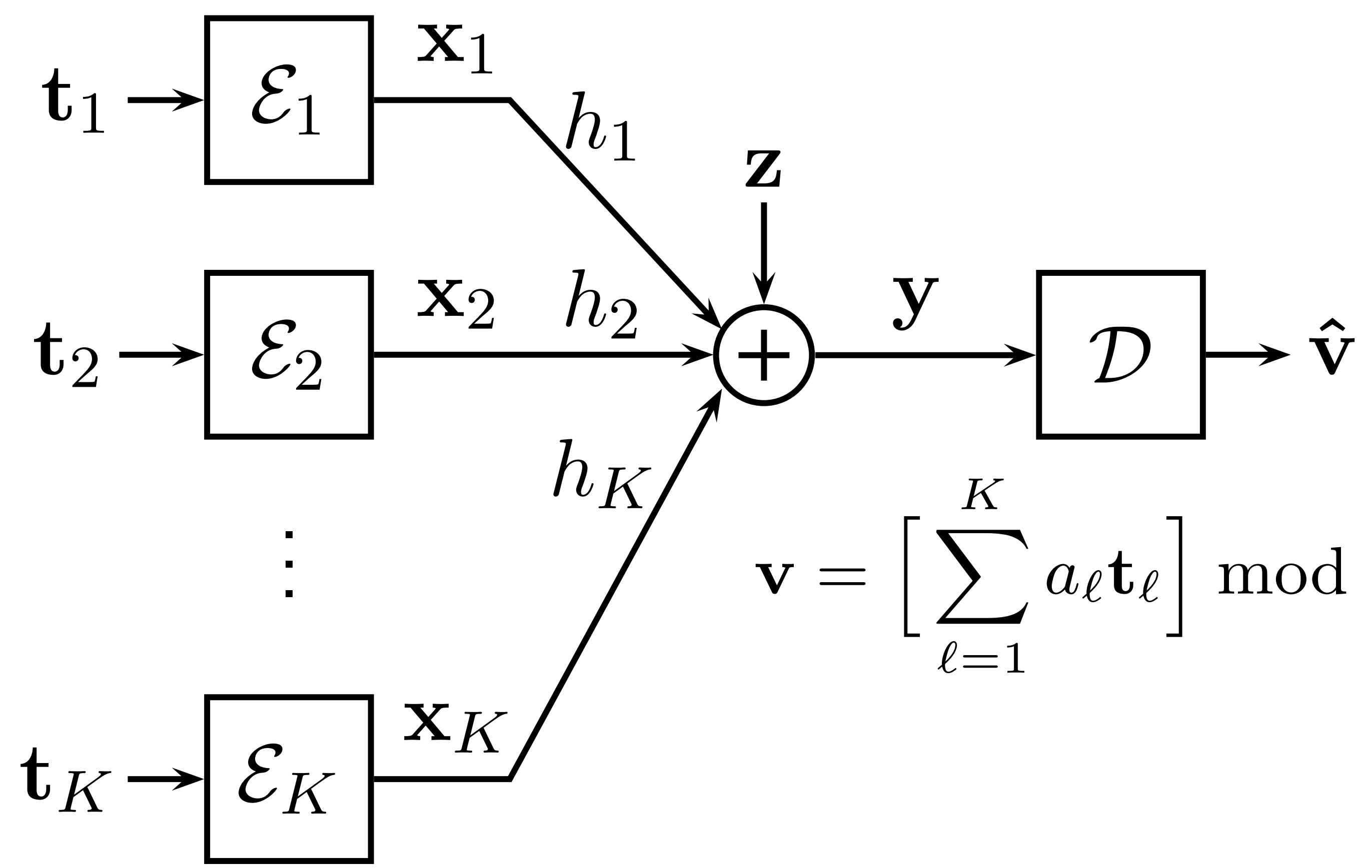
Compute-and-forward

- Txs do not know channels

- Lattice points $\mathbf{t}_i \leftrightarrow$ message $\mathbf{w}_i$ all from same lattice Λ

- transmit dithered codewords $\mathbf{x}_i = [\mathbf{t}_i + \mathbf{U}_i] \mod \Lambda$



$$\mathbf{v} = \left[\sum_{\ell=1}^{K} a_\ell \mathbf{t}_\ell\right] \mod \Lambda$$

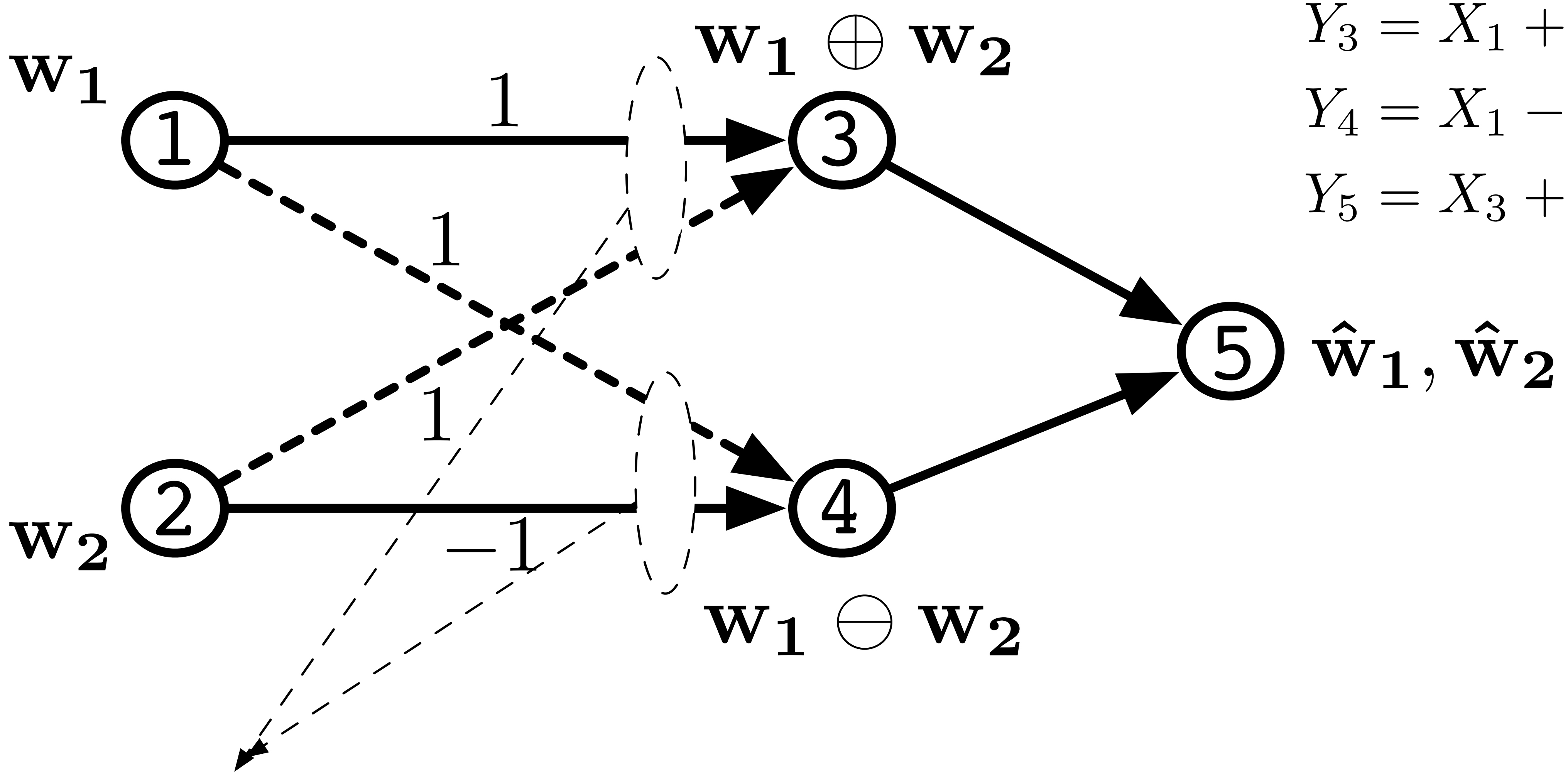
- Rx removes dithers $\mathbf{U}_i$ and recovers the equation $\mathbf{v} = \left[\sum_{l=1}^{K} a_l \mathbf{t}_l\right] \mod \Lambda$, map it back to message space to obtain $\mathbf{u} = \oplus_{l=1}^{N} a_l \mathbf{w}_l$

- equal rate R achievable if

Penalty for not aligning!

$$R < \frac{1}{2}\log\left(\frac{N + P||\mathbf{h}||^2}{N||\mathbf{a}||^2 + P(||\mathbf{h}_k||^2||\mathbf{a}_k||^2 - (\mathbf{h}_k^T\mathbf{a}_k)^2)}\right)$$

Applying this...



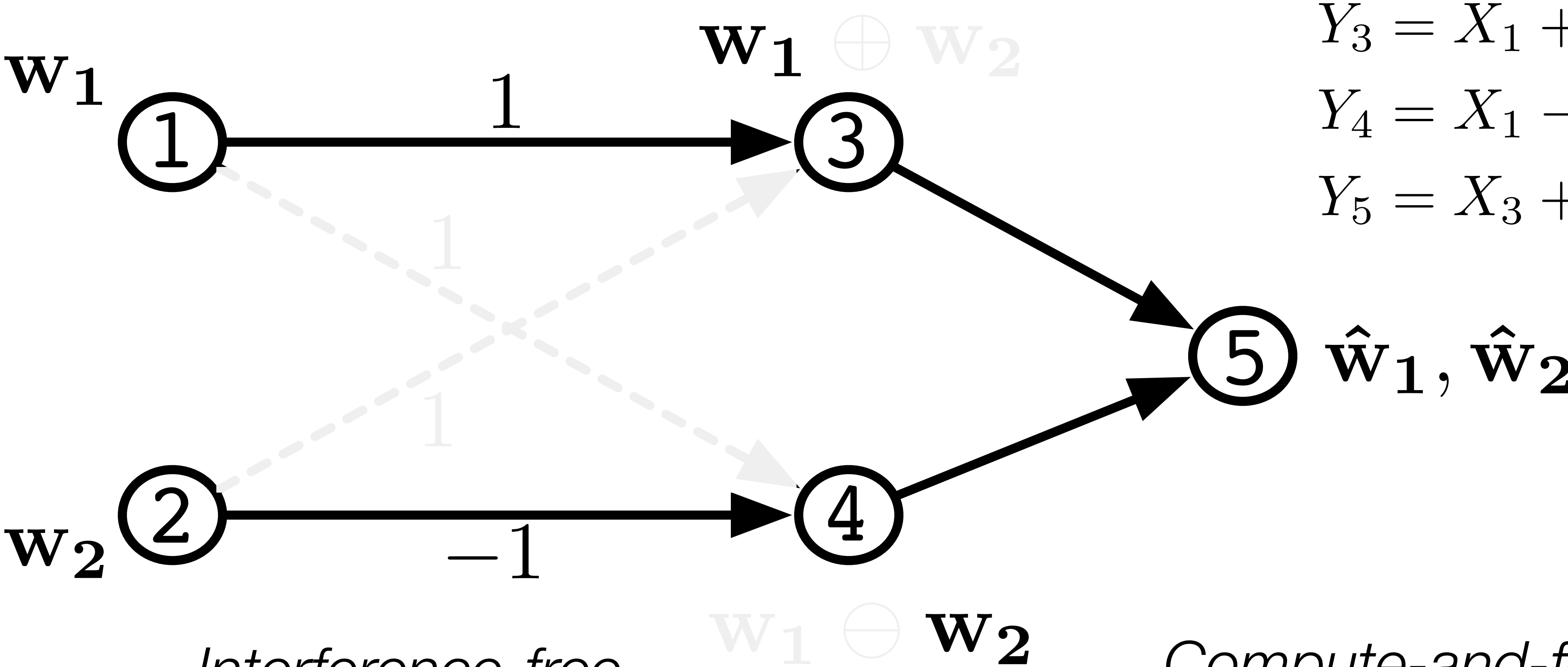
$$Y_3 = X_1 + X_2 + Z_3,$$
$$Y_4 = X_1 - X_2 + Z_4,$$
$$Y_5 = X_3 + X_4 + Z_5,$$

Compute-and-Forward

[B. Nazer, M. Gastpar, IT Trans., 2011]

$$R_1 < \frac{1}{2}\log\left(\frac{1}{2} + S\right)$$
$$R_2 < \frac{1}{2}\log\left(\frac{1}{2} + S\right)$$

If no interference links....



$$Y_3 = X_1 + X_2 + Z_3,$$
$$Y_4 = X_1 - X_2 + Z_4,$$
$$Y_5 = X_3 + X_4 + Z_5,$$

Interference-free

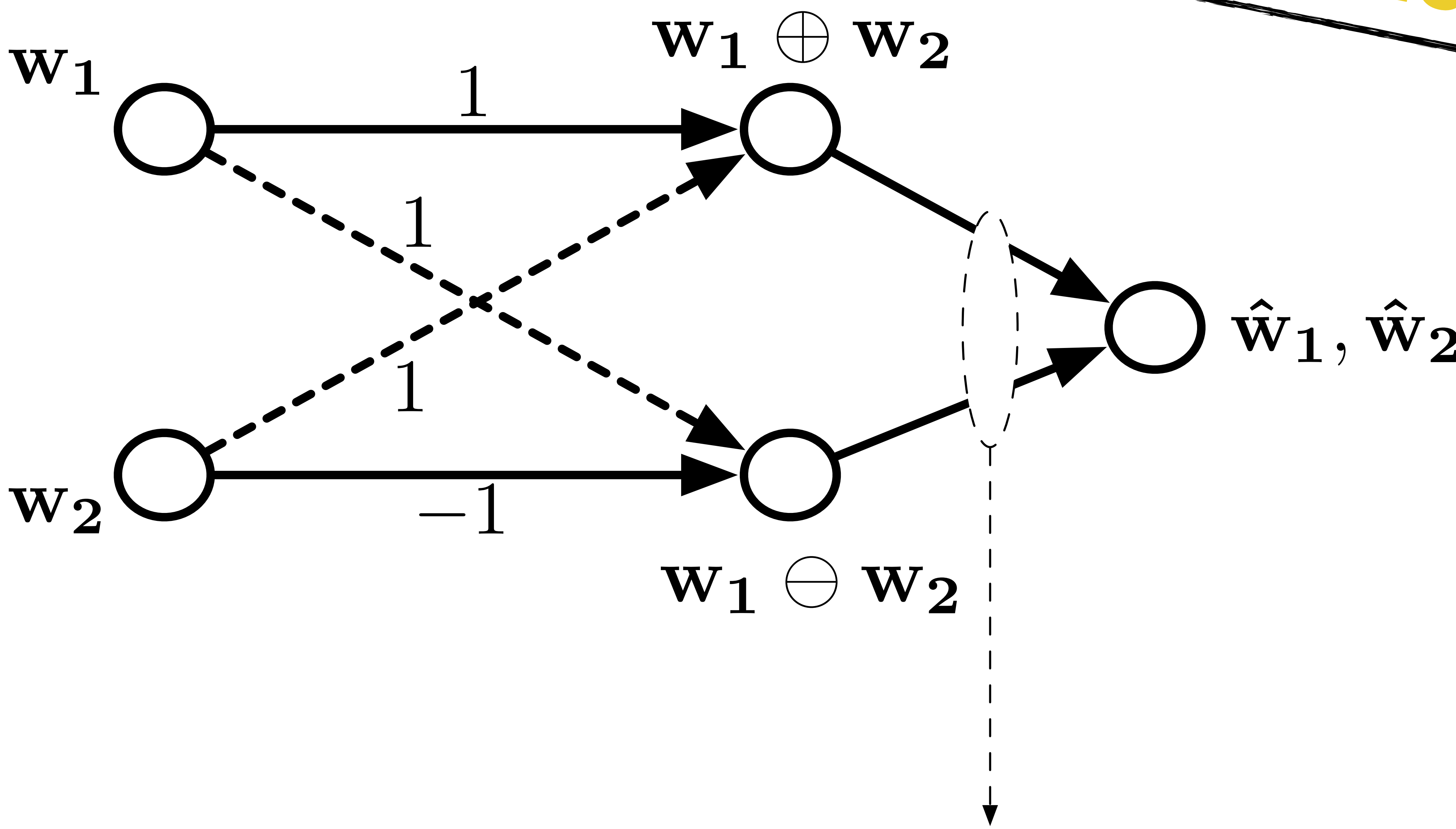
$$R_1 < \frac{1}{2}\log(1+S)$$
$$R_2 \leq \frac{1}{2}\log(1+S)$$

Compute-and-forward

$$R_1 < \frac{1}{2}\log\left(\frac{1}{2}+S\right)$$
$$R_2 < \frac{1}{2}\log\left(\frac{1}{2}+S\right)$$

Inverse Compute-and-Forward

Extracting messages from equations over the air?



Inverse of Compute-and-Forward

[Y. Song, N. Devroye, B. Nazer, ISIT 2011]

Inverse Compute-and

Approach 1: allowable equations (ind. codewords at Txs)

$$\mathbf{u_1} = a_{11}\mathbf{w_1} \oplus a_{12}\mathbf{w_2}$$

$\Rightarrow X_1$

Power S_1

$= X_1 + X_2 + Z$

$Z \sim \mathcal{N}(0,1)$

Pow

$\widehat{\mathbf{w_1}}, \widehat{\mathbf{w_2}}$

$, \hat{\mathbf{w}}_2) \neq (\mathbf{w}_1, \mathbf{w}_2)) < \epsilon$.

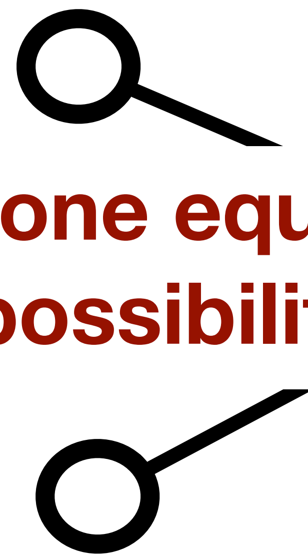
Approach 2: MAC with common messages (correlated codewords at Txs)

(zero-pad)

$\mathbf{w_1}$ rate R_1 $\mathbf{w_2}$ rate R_2

Approach 1: allowable equations

$$\mathbf{u_1} = a_{11}\mathbf{w_1} \oplus a_{12}\mathbf{w_2}$$



Key idea: if one equation is fixed, limits the number of possibilities of the other!

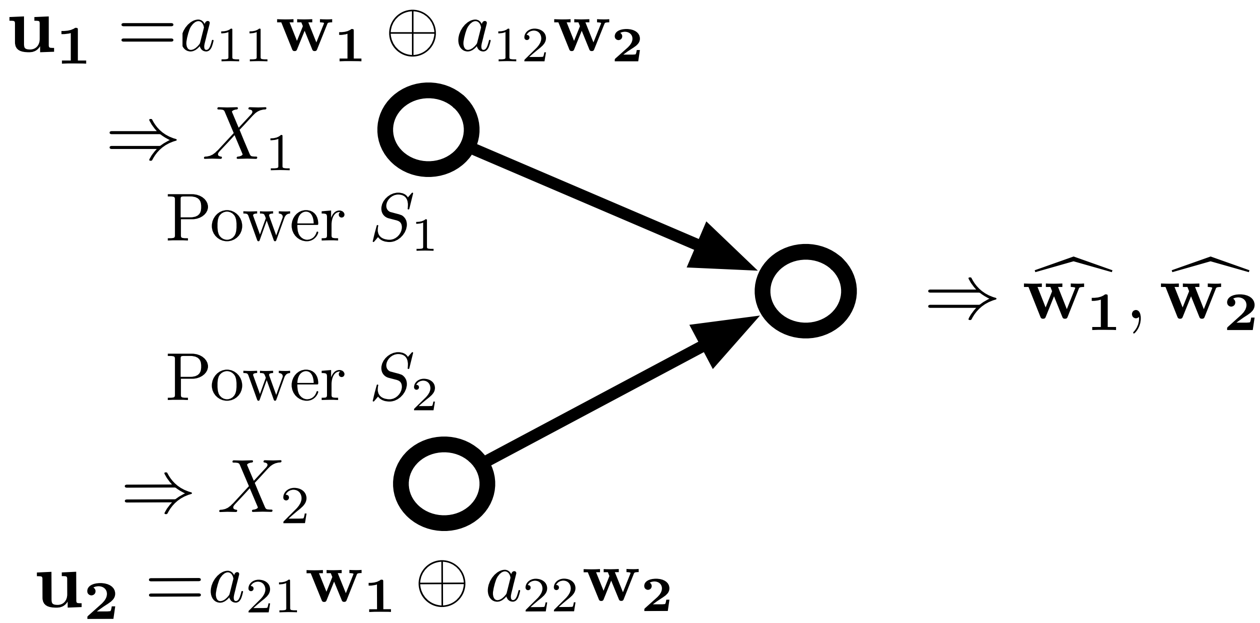
$$\mathbf{u_2} = a_{21}\mathbf{w_1} \oplus a_{22}\mathbf{w_2}$$

$\mathbf{w_1}$ rate R_1

$\mathbf{w_2}$ rate R_2

$\rightarrow$ $|\mathbf{u_1}| = |\mathbf{u_2}| = 2^{nR_{MAX}}$ $\rightarrow$ $2R_{MAX} \geq R_1 + R_2$

Approach 1: allowable equations

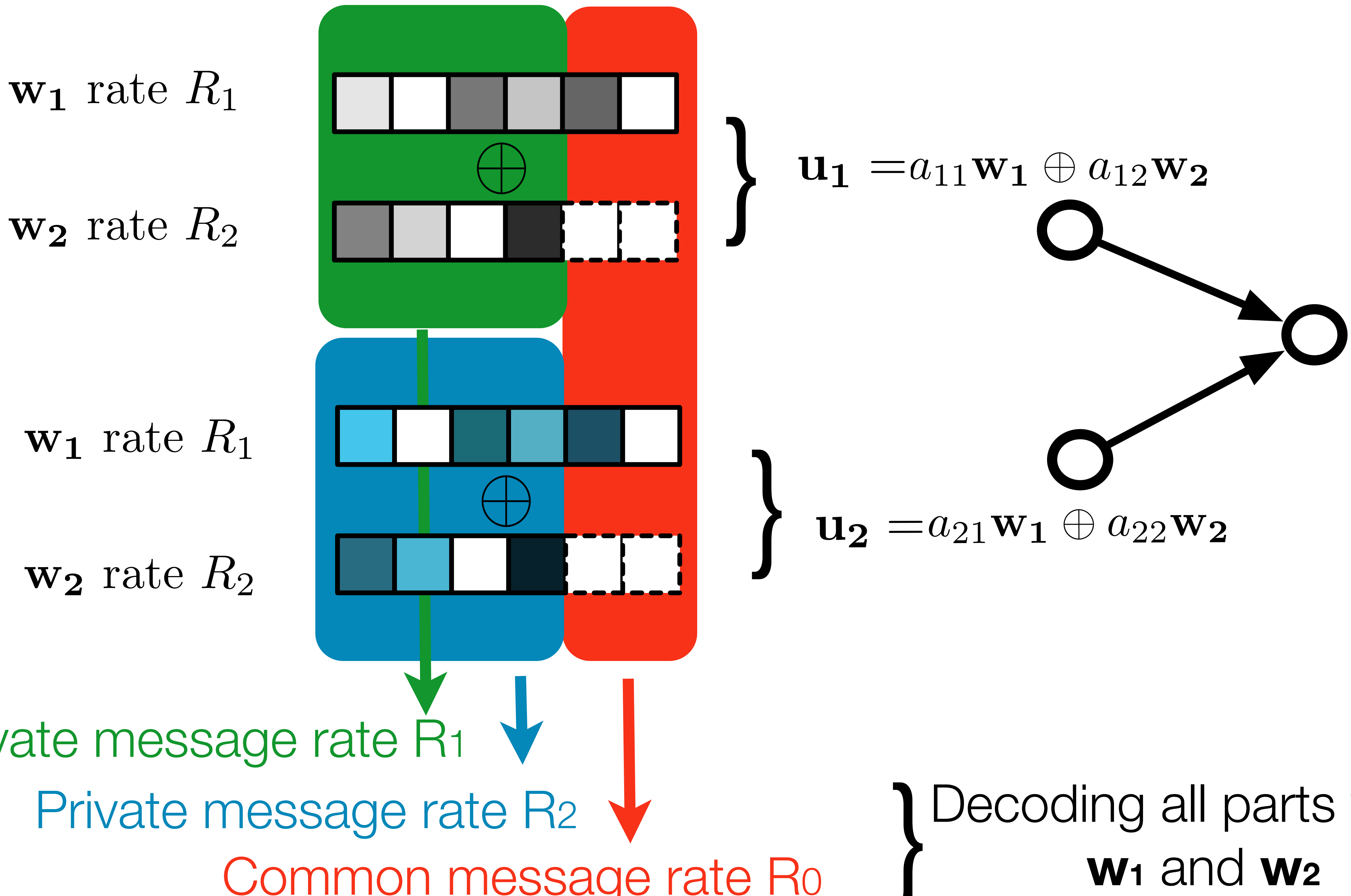


$$\min(R_1, R_2) < \min(C(S_1), C(S_2))$$
$$R_1 + R_2 < C(S_1 + S_2).$$

Can handle special cases like coefficients =0

[Slepian, Wolf, 1973], [Han, 1979]

Approach 2: MAC with common messages

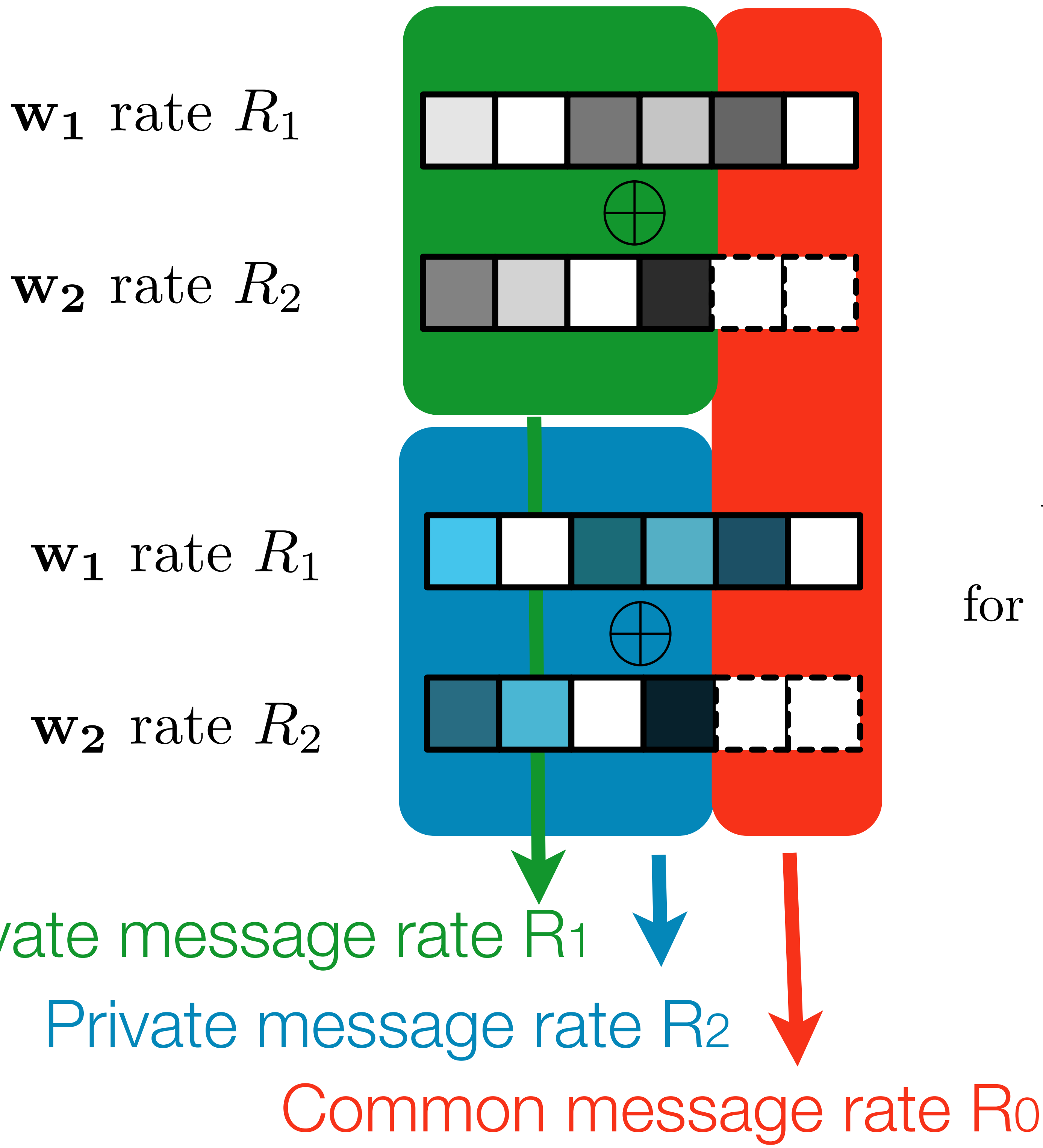


$$\mathbf{u_1} = a_{11}\mathbf{w_1} \oplus a_{12}\mathbf{w_2}$$

$$\mathbf{u_2} = a_{21}\mathbf{w_1} \oplus a_{22}\mathbf{w_2}$$

Decoding all parts yields **$\mathbf{w_1}$** and **$\mathbf{w_2}$**

Approach 2: MAC with common messages

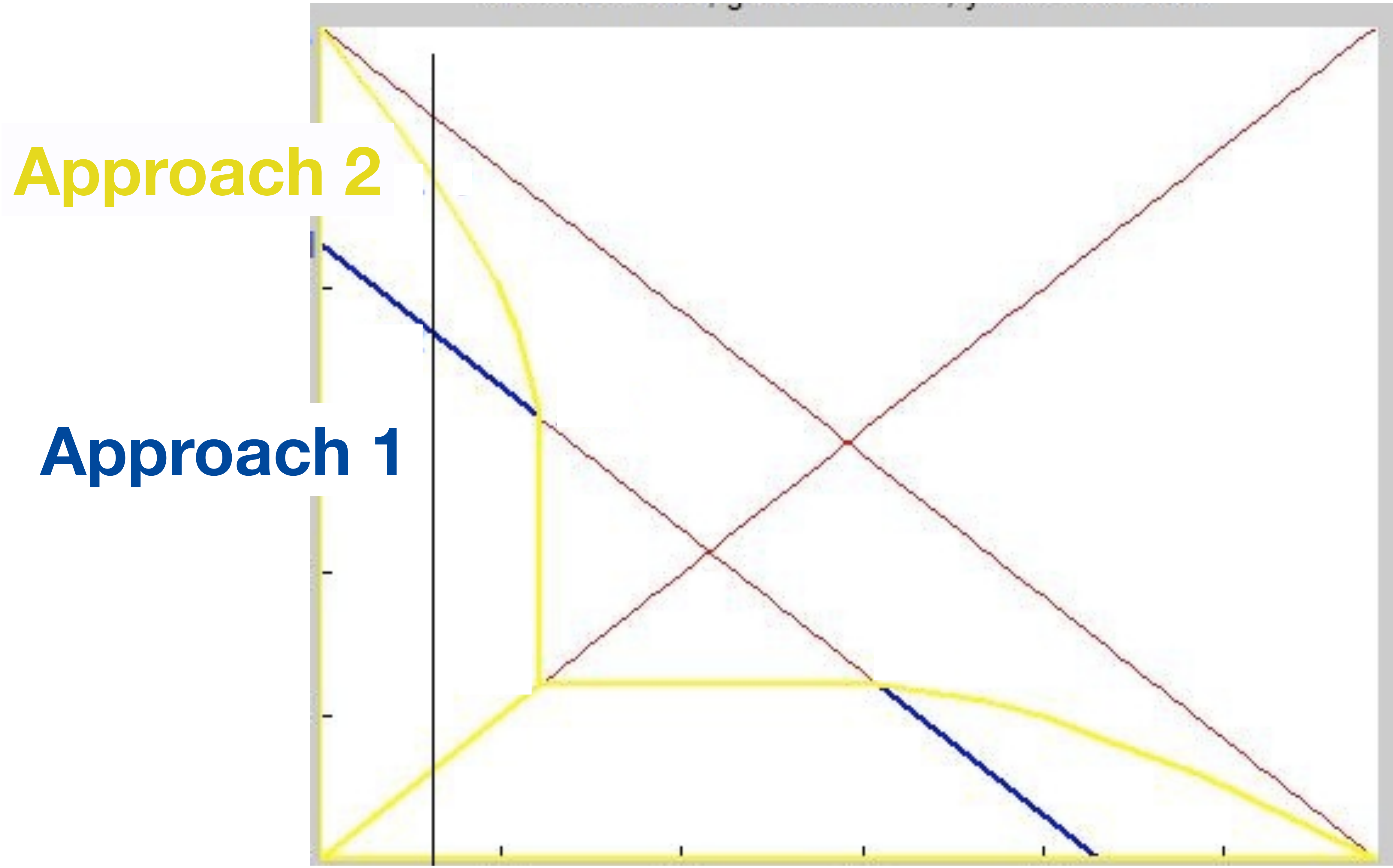


$$
\begin{aligned}
R_1 &< I(X_1;Y|X_2,V)\\
R_2 &< I(X_2;Y|X_1,V)\\
R_1+R_2 &< I(X_1,X_2;Y|V)\\
R_0+R_1+R_2 &< I(X_1,X_2;Y)
\end{aligned}
$$

for some $p_V(v)p_{X_1|V}(x_1|v)p_{X_2|V}(x_2|v)$.

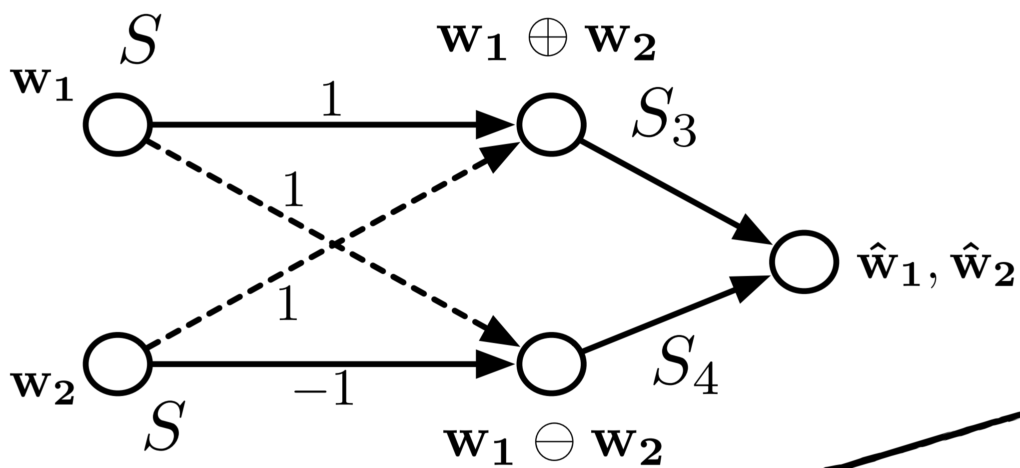
} Decoding all parts yields **w1** and **w2**

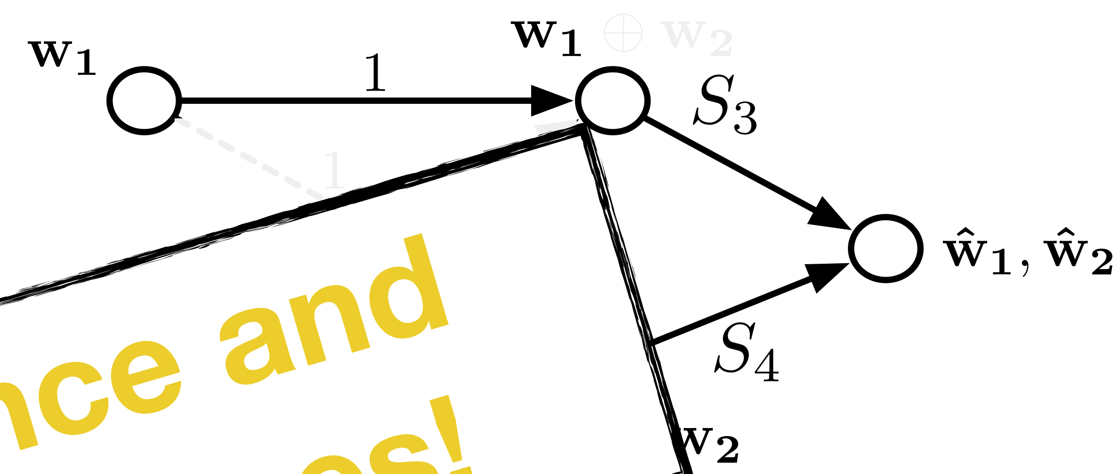
Inverse Compute-and-Forward regions



CF+ICF

Interference-free





Exploit interference and correlation it produces!

$$\min(R_1, R_2) < \min\left(\frac{1}{2}\log(1+S_3), \frac{1}{2}\log(1+S_4)\right)$$

$$R_1 + R_2 < \frac{1}{2}\log(1+S_3+S_4).$$

$$R_1 < \min\left\{\frac{1}{2}\log(1+S), \frac{1}{2}\log(1+S_3)\right\}$$

$$R_2 < \min\left\{\frac{1}{2}\log(1+S), \frac{1}{2}\log(1+S_4)\right\}$$

$$R_1 + R_2 < \frac{1}{2}\log(1+S_3+S_4).$$

strictly contains at high SNR

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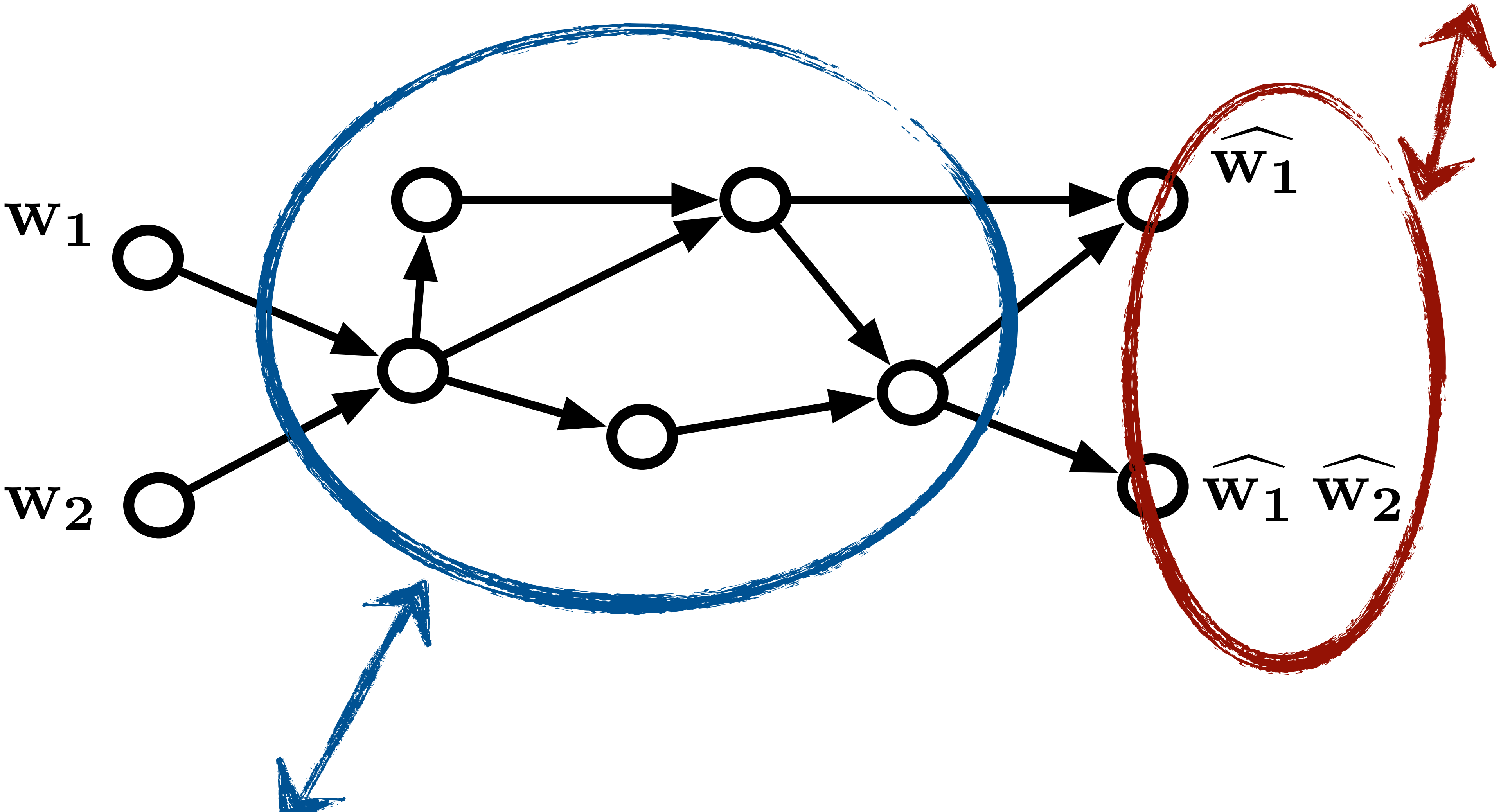
In general

Combine for a unified rate region!

Inverse
Compute and forward to extract messages



Compute and forward to decode $a\mathbf{w_1} \oplus b\mathbf{w_2}$

Outline

- Point to point channels: random codes, lattice (structured) codes
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- Additional lattice examples
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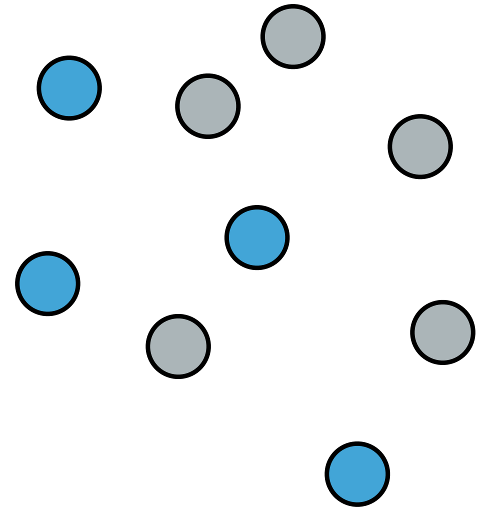
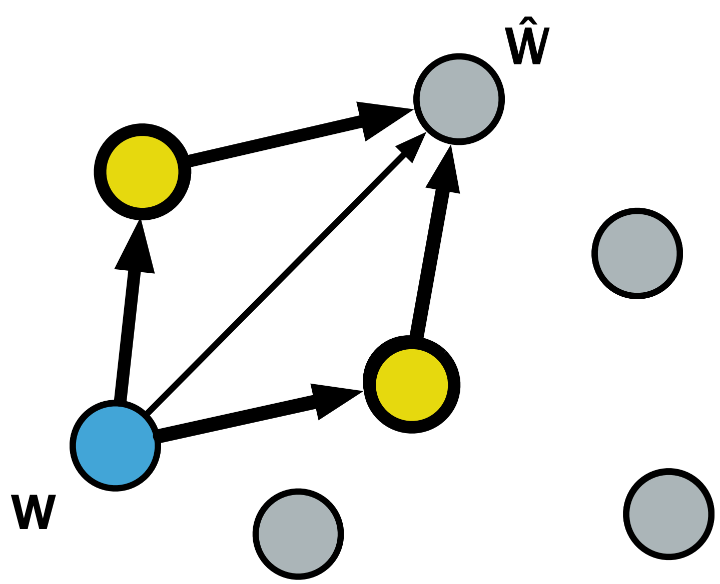
Random codes for Gaussian networks



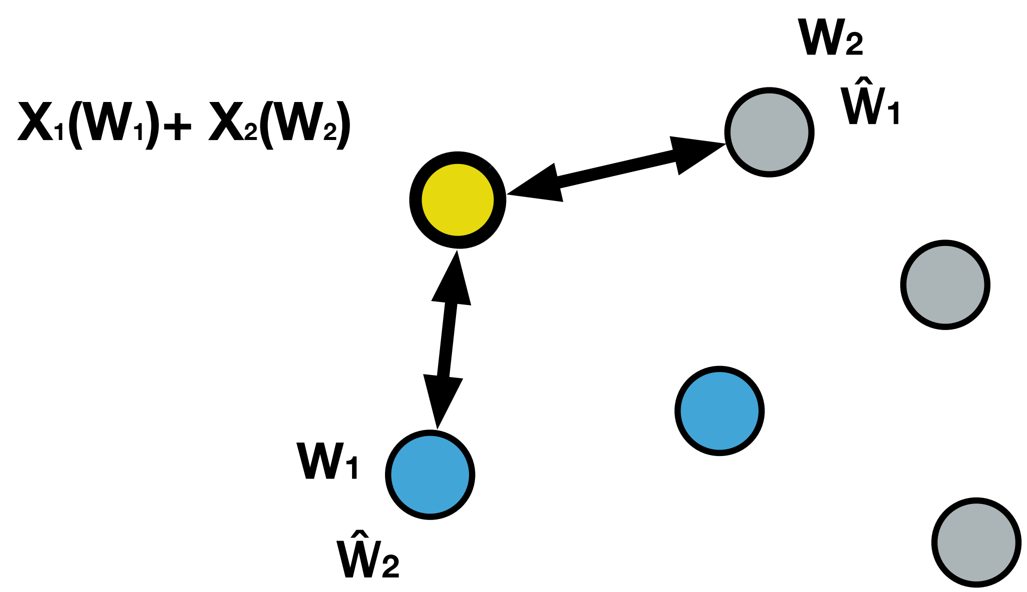
Structured codes for Gaussian networks

- **have:** cooperation



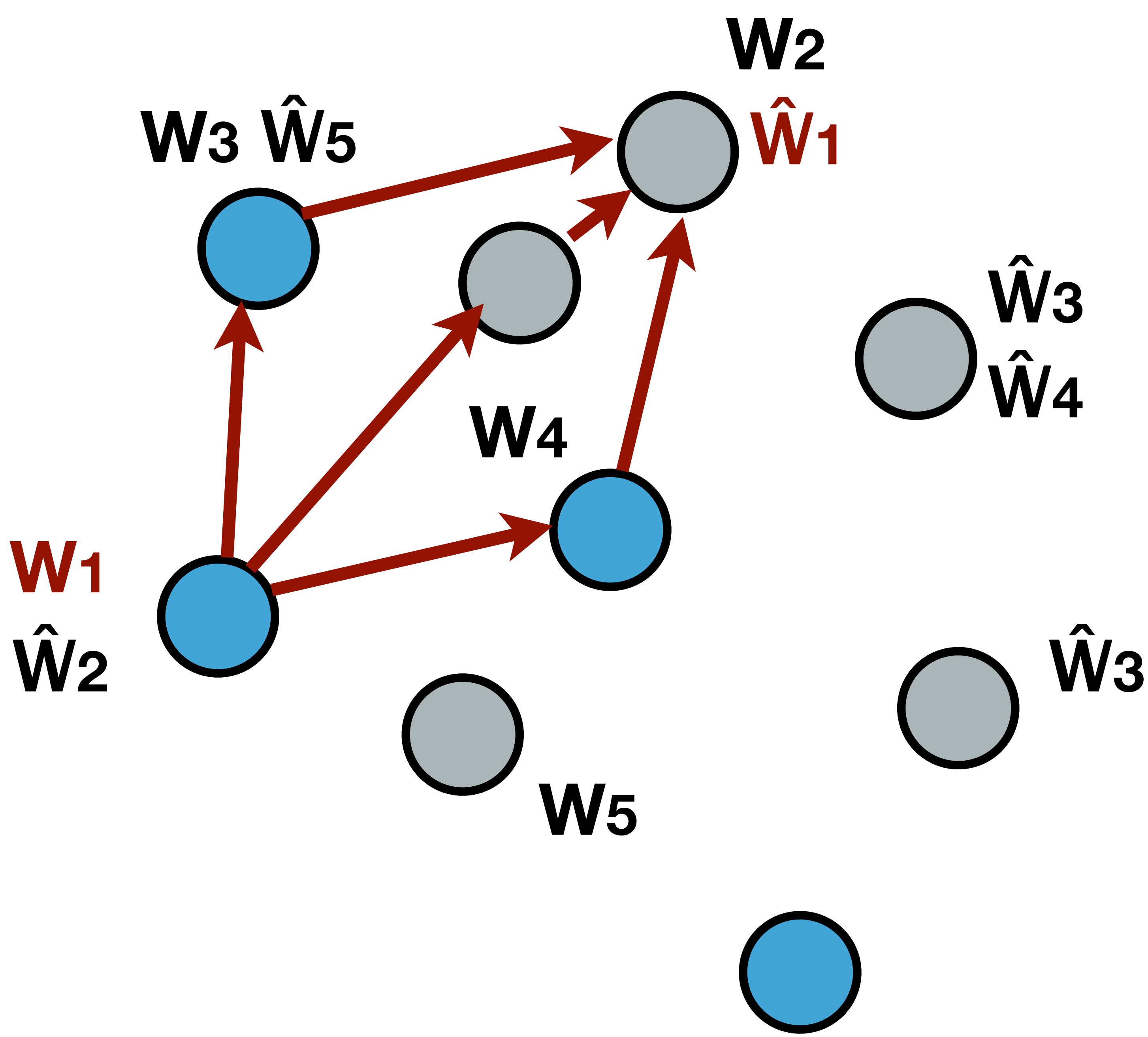
- **missing:** “decode the sum”

- **have:** “decode the sum”
- **have:** “extract from the sum”

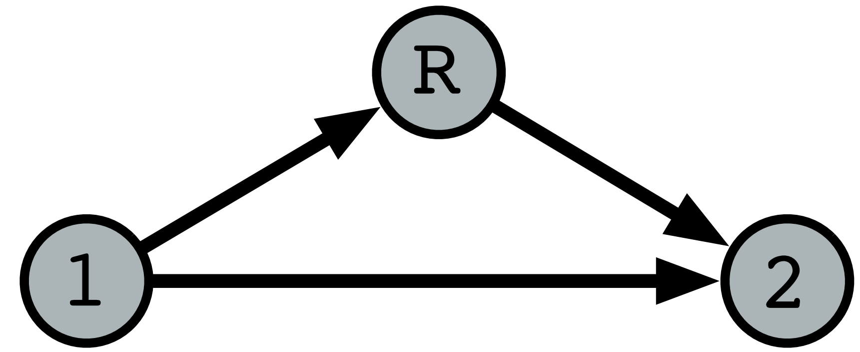


- **missing:** cooperation

Cooperation in wireless relay networks



General relay network theorems



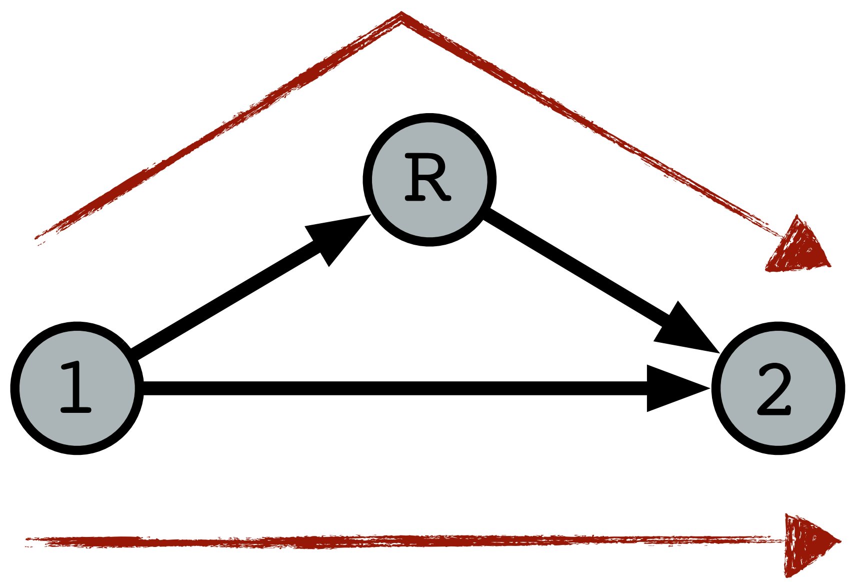
- AWGN relay channel DF and CF schemes first considered in *[Cover, El Gamal, 1979]*
  - DF extension to arbitrary # of relays and sources in *[Xie, Kumar, 2004]*
  - CF extension / generalization to arbitrary # of relays and sources in *[K*

All based on RANDOM coding

  - Qu… and sources in *[Avestimehr, Diggavi, Tse, 2011] (finite gap)*
  - Noisy network coding *[Lim, Kim, El Gamal, Chung, 2010] (finite gap)*
- Lattice-based schemes?
  - Quantize-map-forward extended to lattice codes in *[Ozgur, Diggavi, 2011]*
  - Compute-and-forward framework *[Nazer, Gastpar, TransIT, 2011], [Niesen, Whiting, 2011]*

Lattice codes missing in?

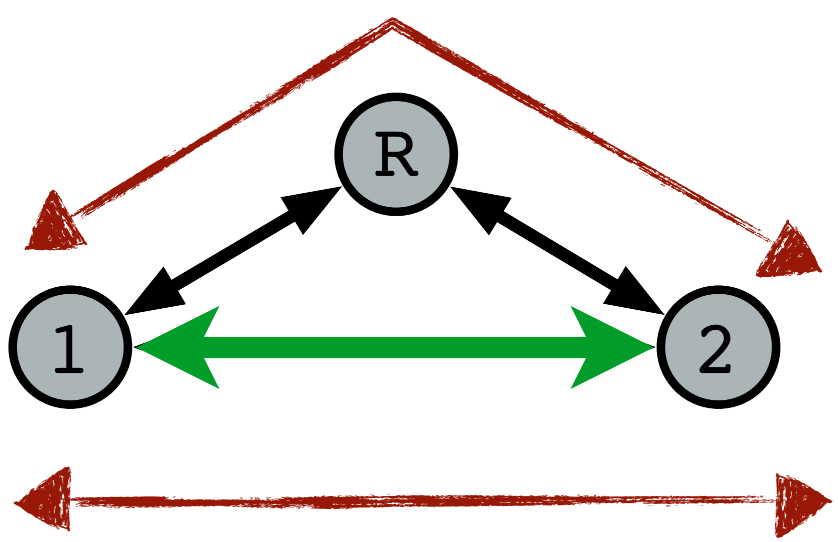
- AWGN relay channel ?



``Cooperation”

Various links carry same message!

- Two-way relay channel in **presence of direct links?**



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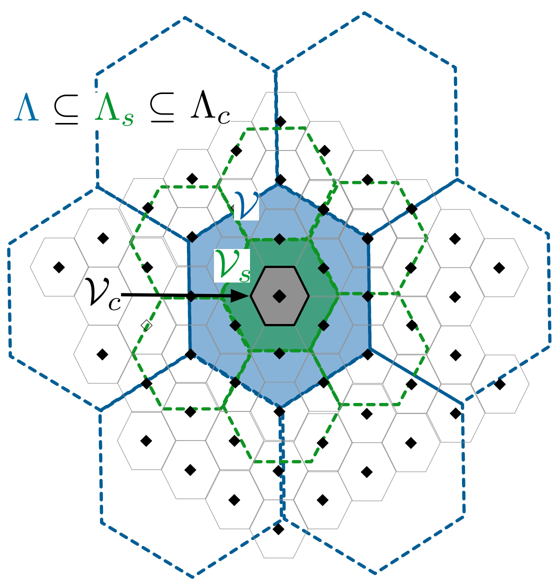
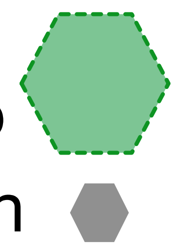
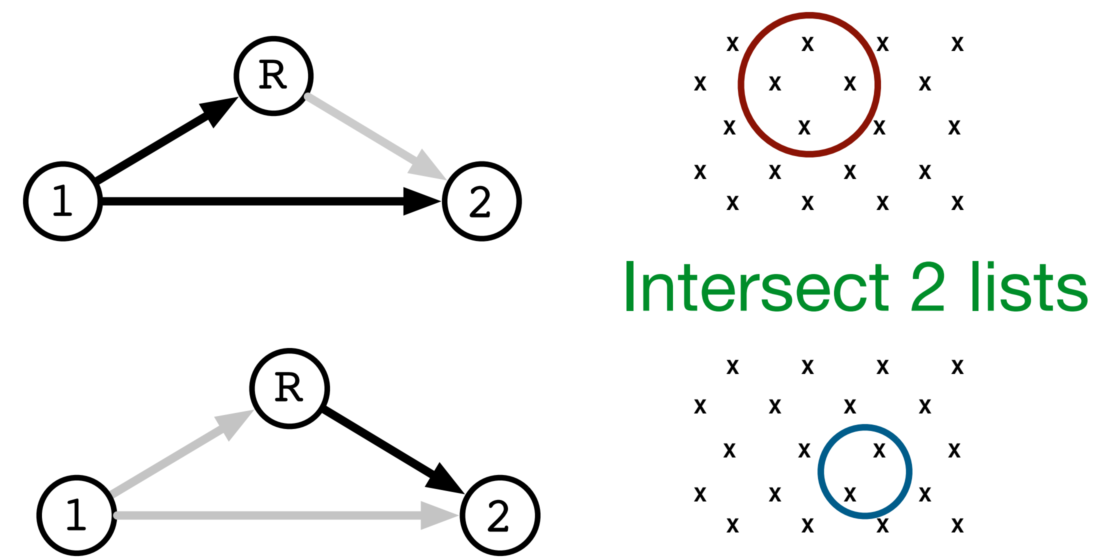
Enabling lattice "Cooperation"

Lattice list decoder

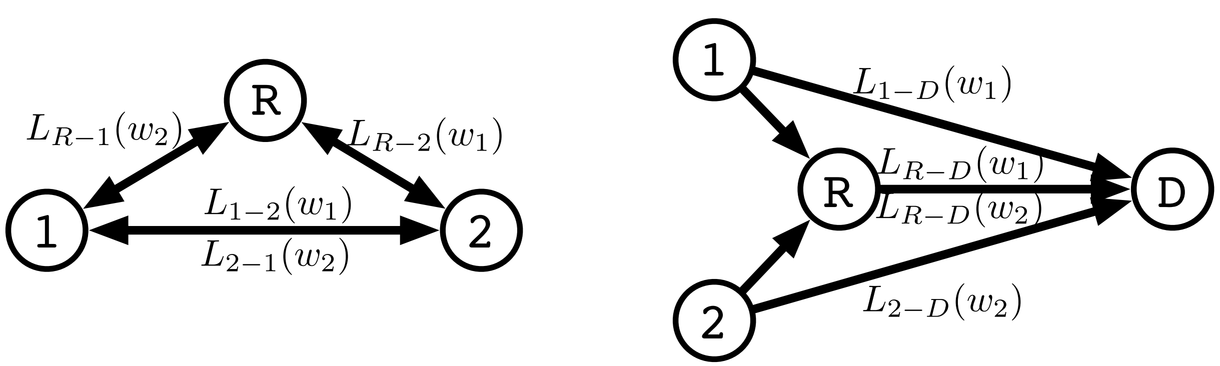


decode to 
rather than

Lattices achieve DF rate

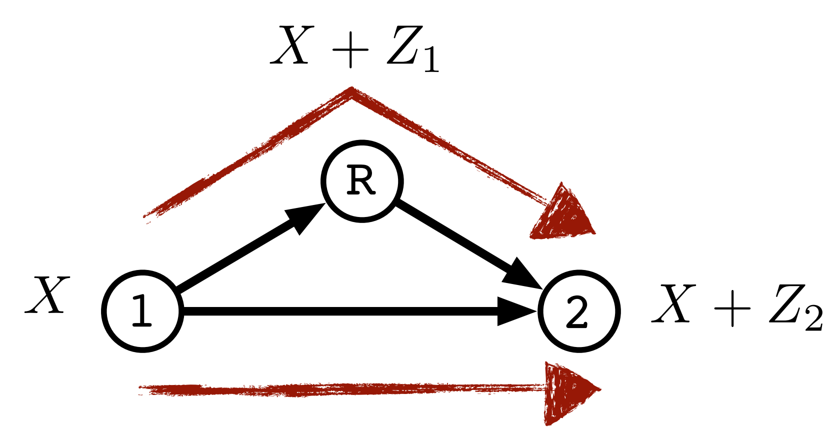


Lattices in multi-source networks



combine "decode sum"
and direct-link cooperation

Lattices achieve CF rate



lattices good source and channel
codes, special Wyner-Ziv

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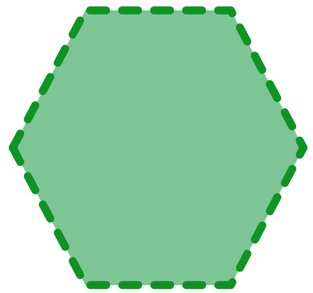
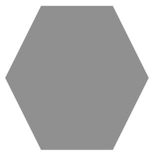
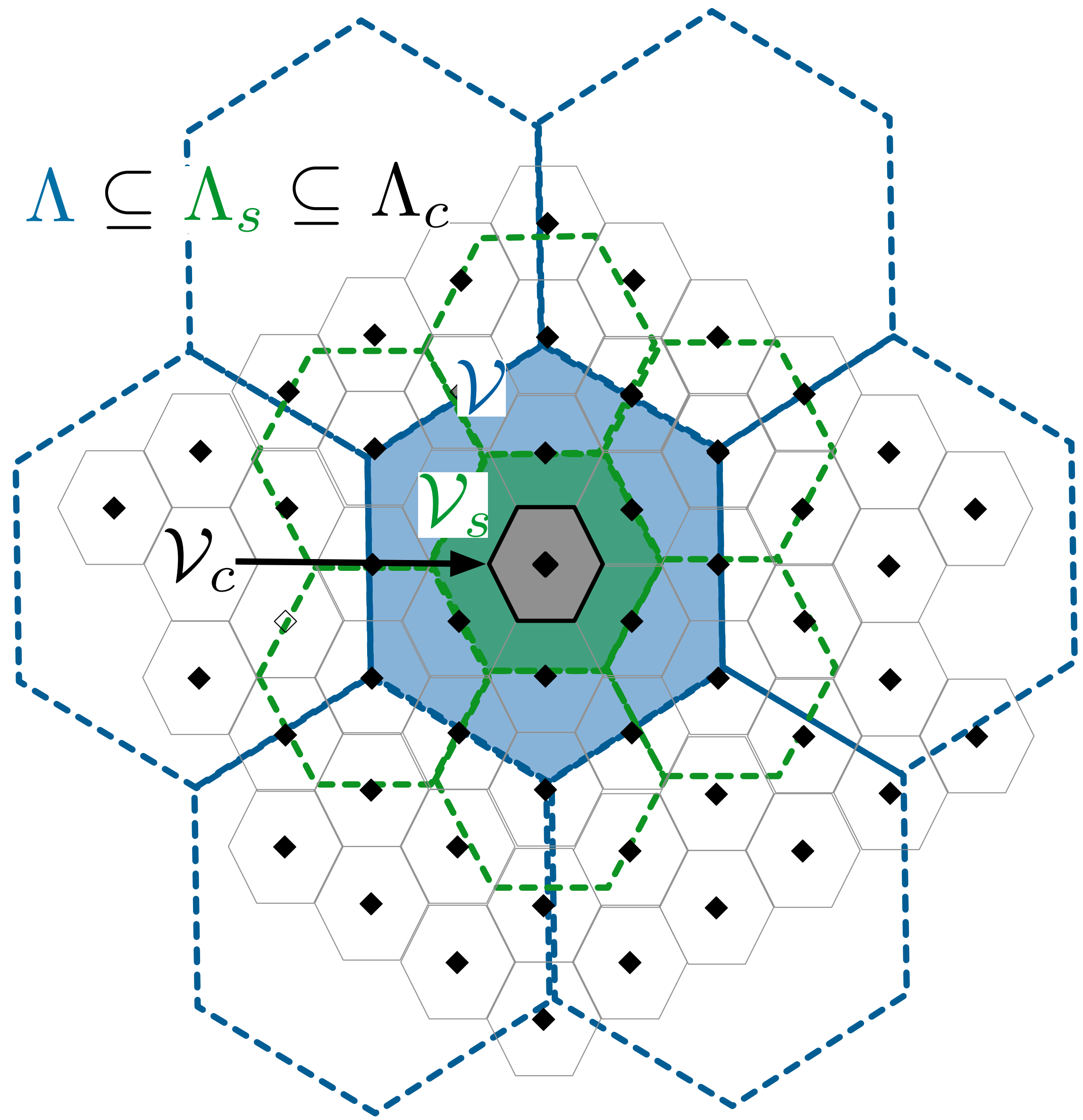
Lattice list decoder

- IDEA: decode to  rather than to 



- results in a **list** of codewords
- require **correct** codeword to be in list
- how many (lower bound) are in list?
- Theorem: $2^{n(R-C(P/N))}$

[Y. Song, N. Devroye, submitted to IT Trans., 2011]

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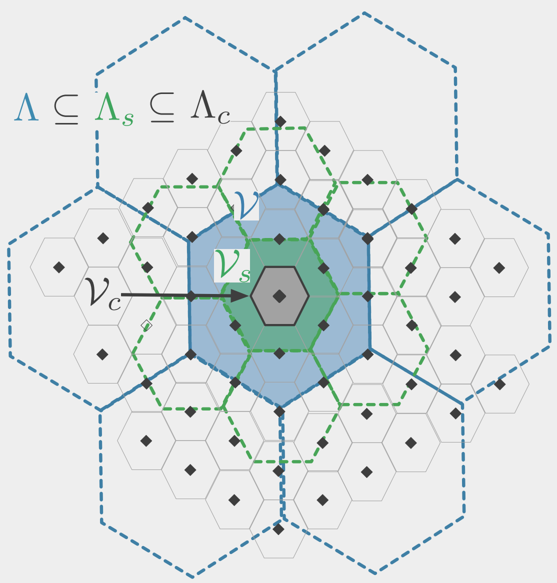
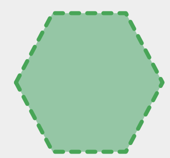
Enabling lattice "Cooperation"

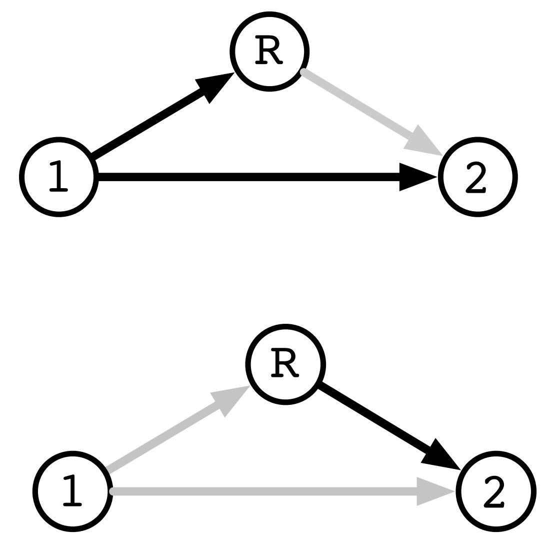
Lattice list decoder

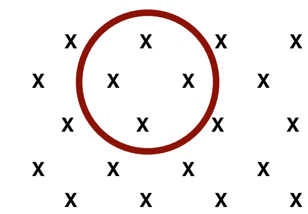


decode to 

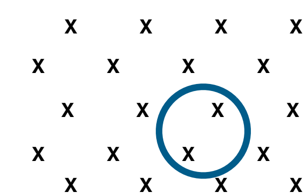
rather than 

Lattices achieve DF rate

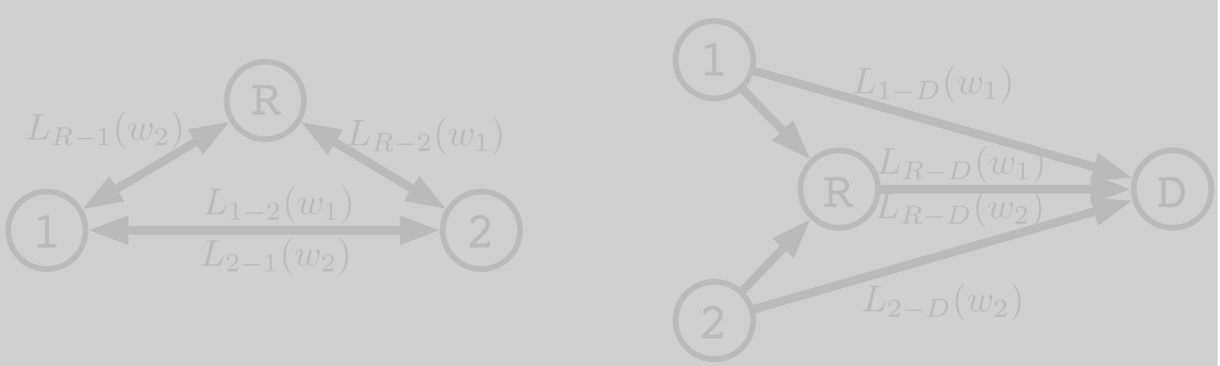




Intersect 2 lists

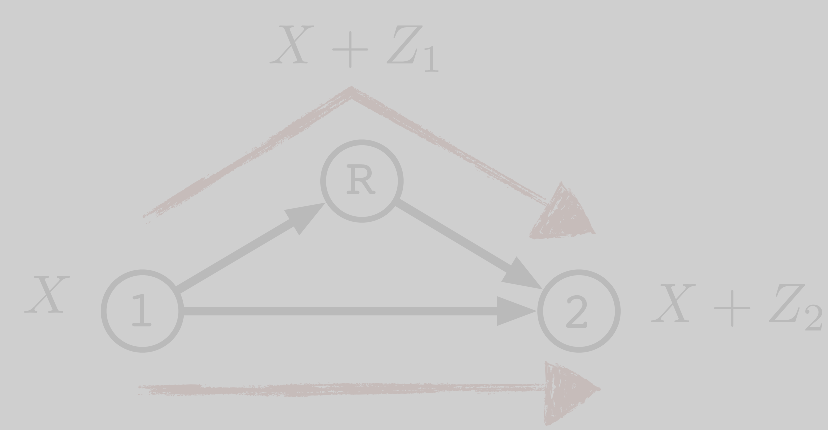


Lattices in multi-source networks



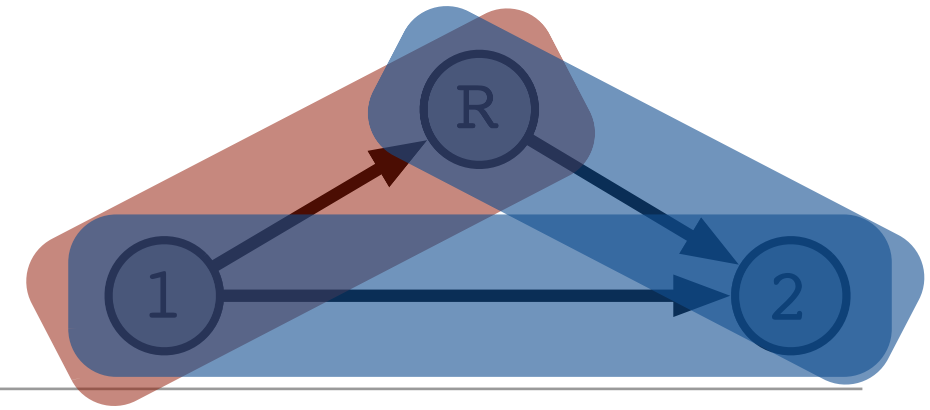
combine "decode sum"
and direct-link cooperation

Lattices achieve CF rate



lattices good source and channel
codes, special Wyner-Ziv

Decode and forward relaying



$$R_{DF} = \max_{p(x_1, x_R)} \{\min\{I(X_1; Y_R | X_R), I(X_1, X_R; Y_2)\}\}$$

- Irregular Markov Encoding with Successive Decoding
  [Cover, El Gamal 1979]
- Regular Encoding with Backward Decoding
  [Willems 1992]
- Regular Encoding with Sliding Window Decoding
  [Xie, Kumar 2002]
- Nice survey
  [Kramer, Gastpar, Gupta 2005]

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Single source: lattice DF

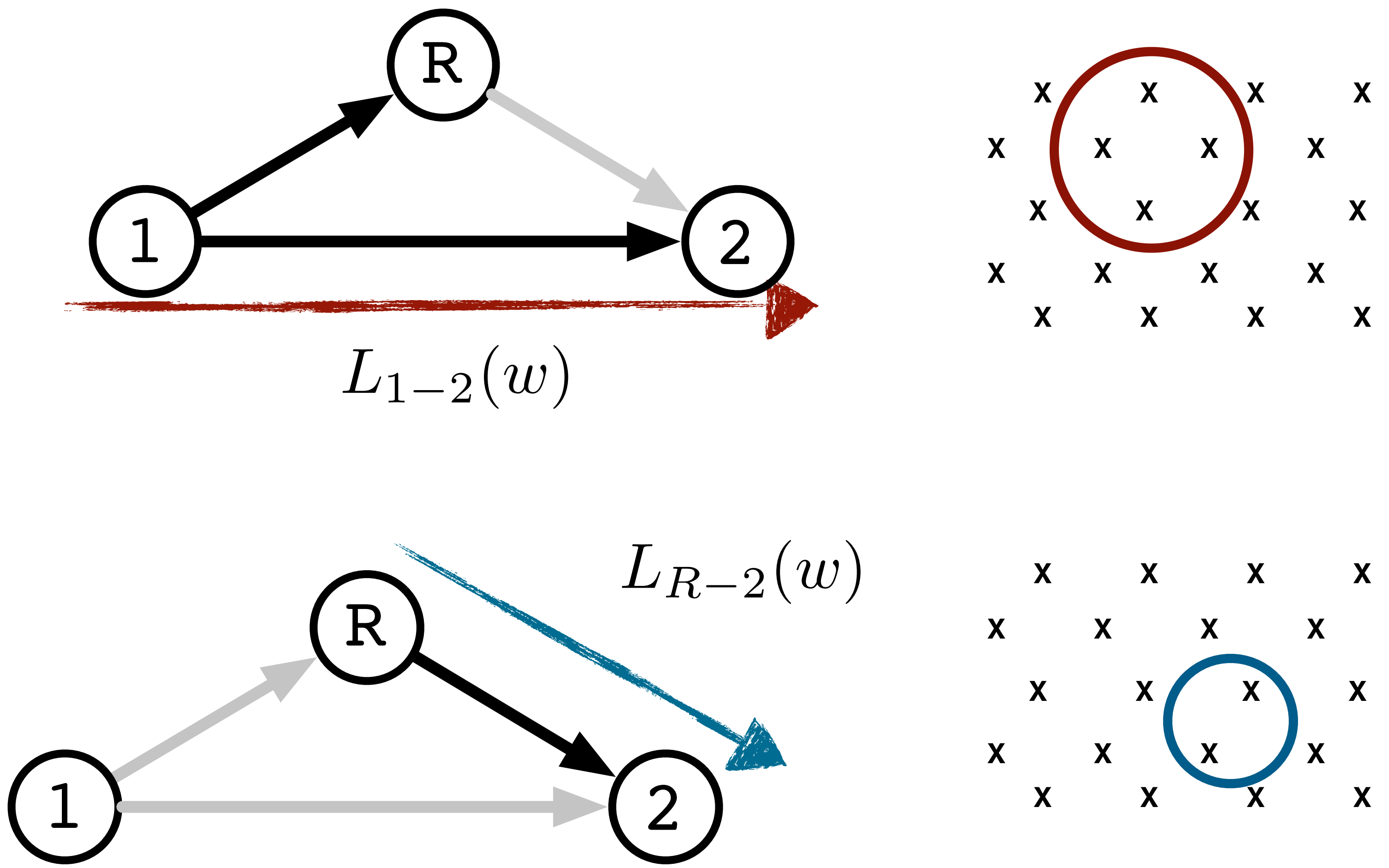
Lattices achieve the DF rate for the relay channel. The following Decode-and-Forward rates can be achieved using nested lattice codes for the Gaussian relay channel:

$$R < \max_{\alpha \in [0,1]} \min \left\{ \frac{1}{2} \log \left(1 + \frac{\alpha P}{N_R} \right), \frac{1}{2} \log \left(1 + \frac{P + P_R + 2\sqrt{\bar{\alpha} P P_R}}{N_2} \right) \right\}.$$

Achieved using NESTED LATTICE CODES!

Alternative lattice-based DF scheme in [Nokleby, Aazhang, 2011, 2012]

Central idea behind using lists



- **view cooperation between links as intersection of independent lists**

$$L_{1-2}(w) \cap L_{R-2}(w) \Rightarrow \text{UNIQUE } w$$

- **mimic all Block Markov steps for achievability**

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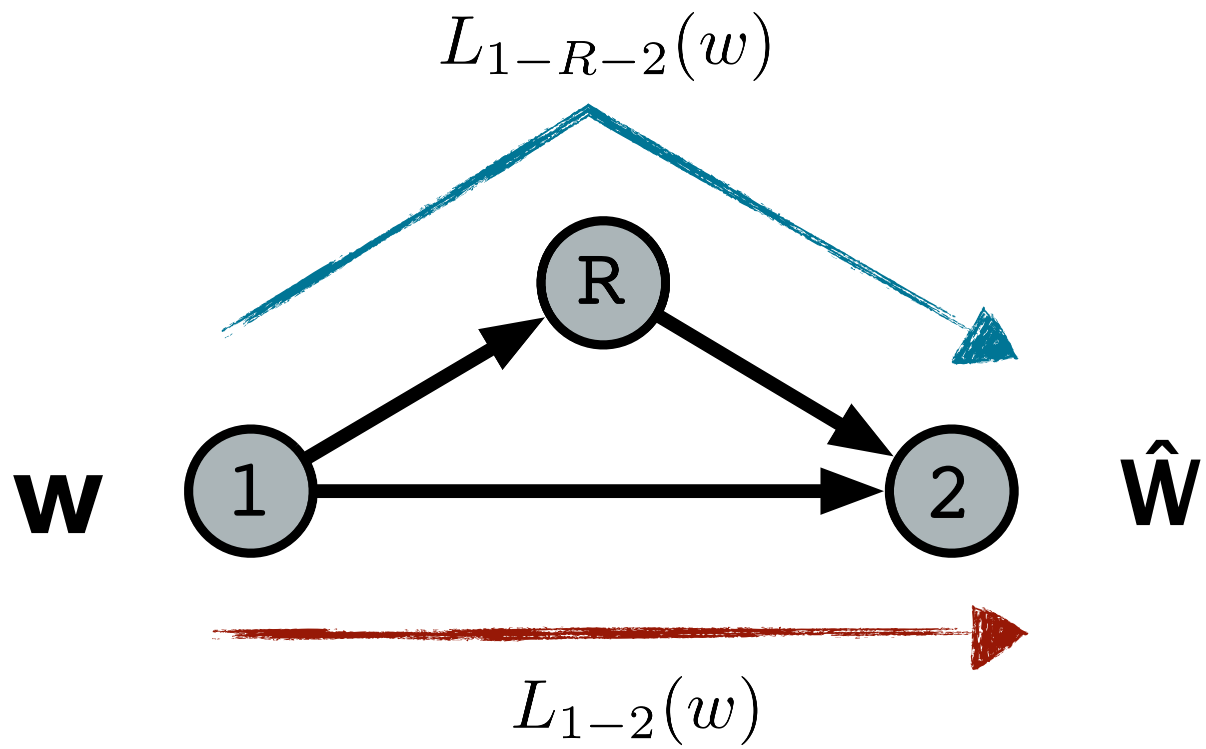
An aside.....



- **ideally would want this list, rather than forcing a decode.....**

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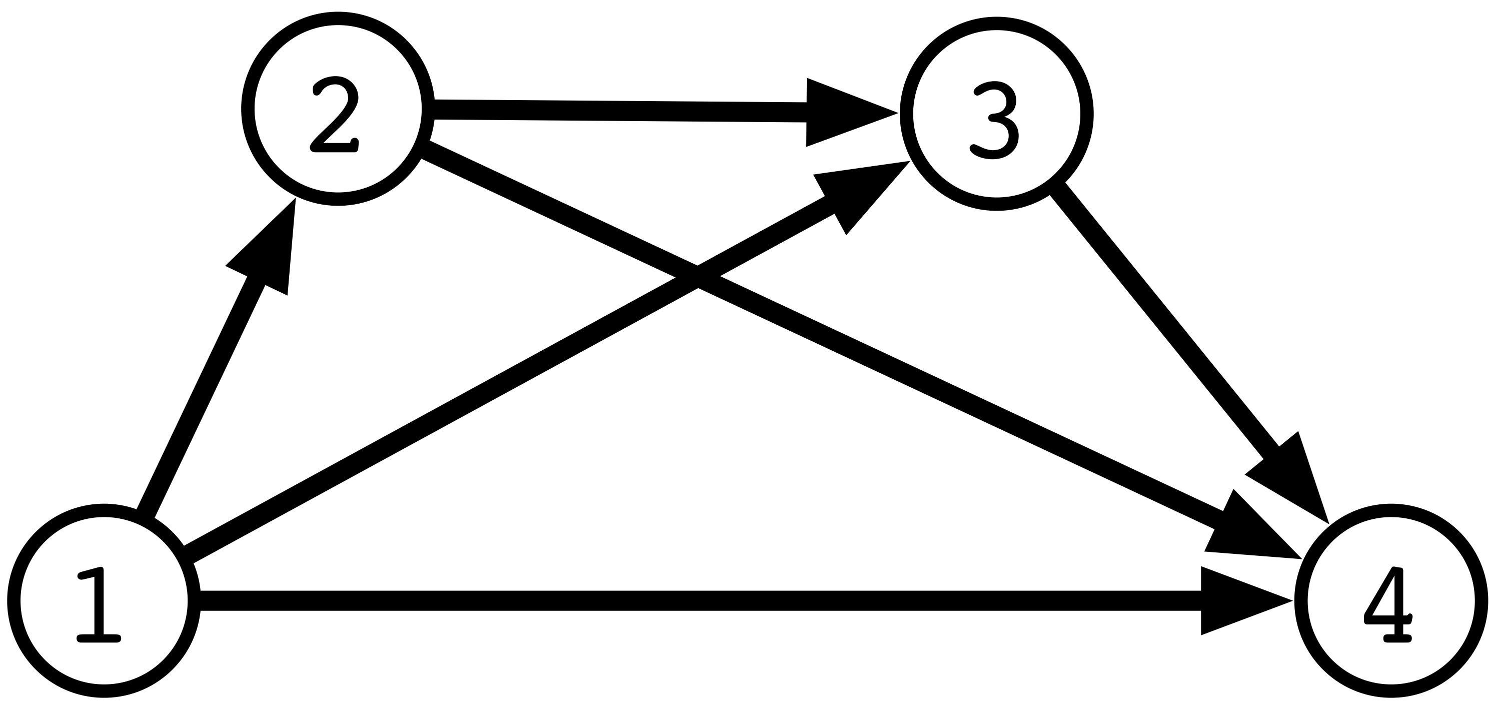
Single-source, multiple relay

[Xie, Kumar 2004]

[Kramer, Gastpar, Gupta 2005]

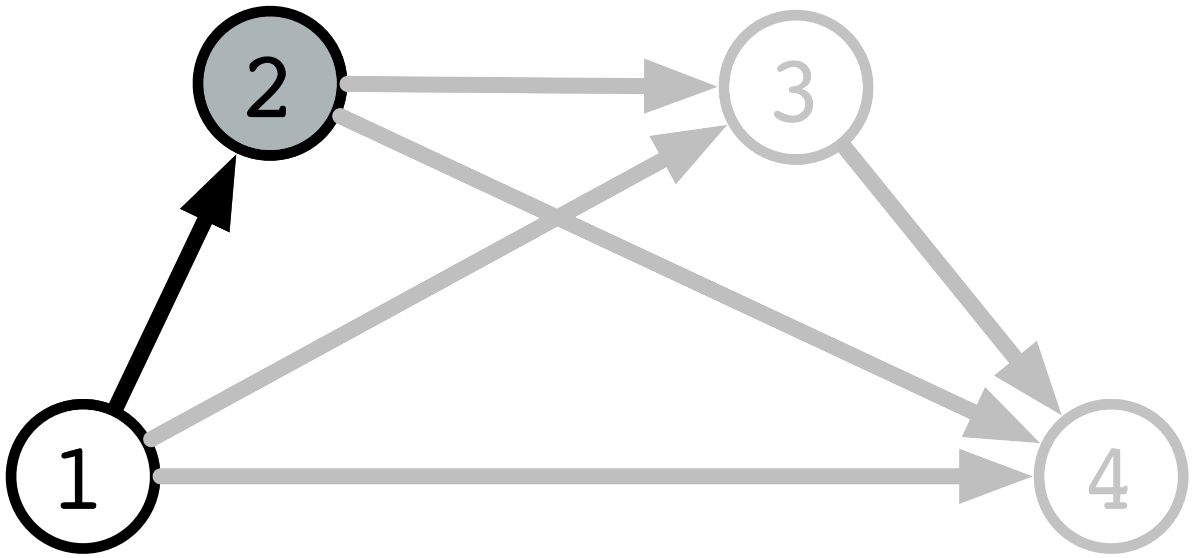
$$Y_2 = X_1 + Z_2, \quad Z_2 \sim \mathcal{N}(0, N_2)$$

$$Y_3 = X_1 + X_2 + Z_3, \quad Z_3 \sim \mathcal{N}(0, N_3)$$

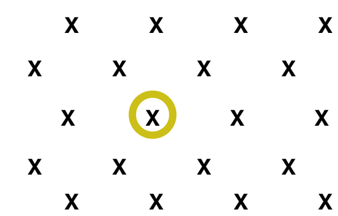


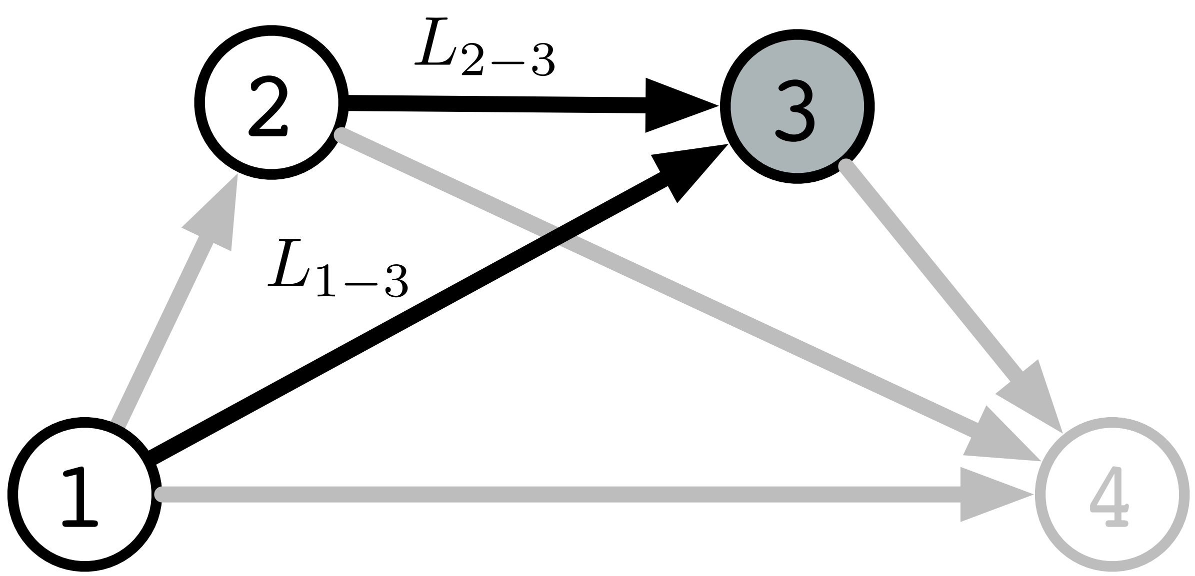
$$Y_4 = X_1 + X_2 + X_3 + Z_4, \quad Z_4 \sim \mathcal{N}(0, N_4)$$

NESTED LATTICE CODES can mimic the regular encoding / sliding window decoding DF rate

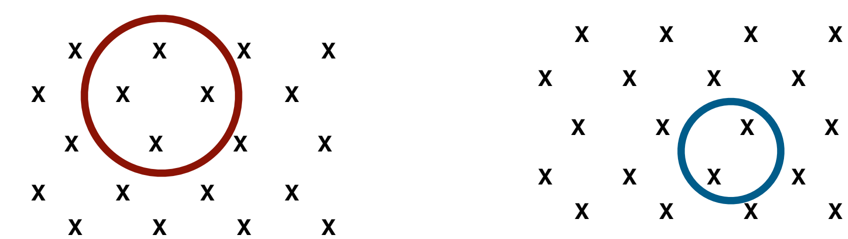


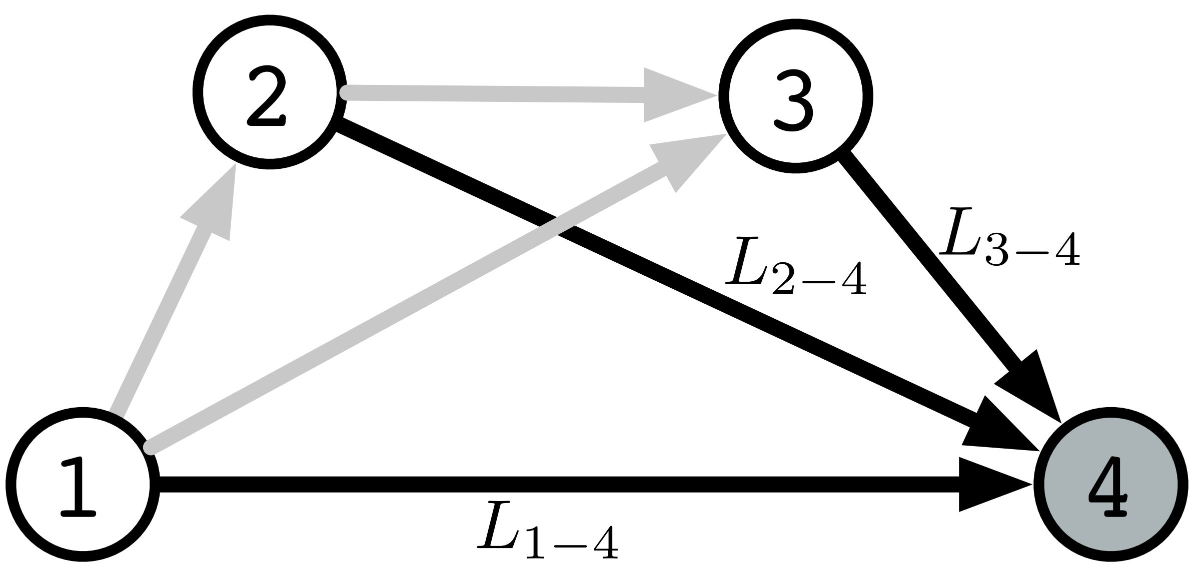
- Unique decoding



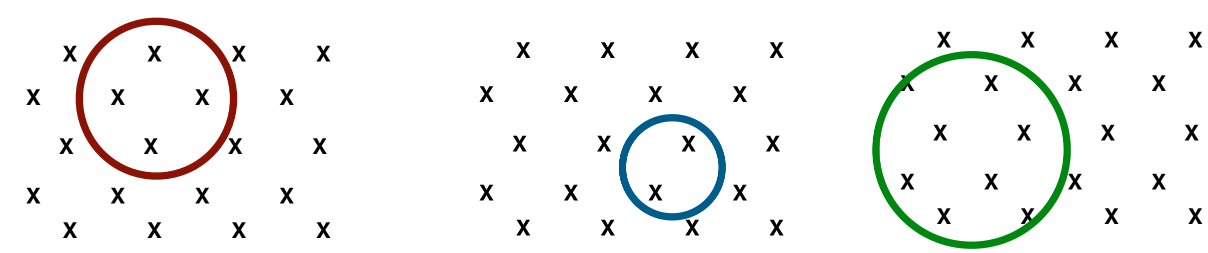


- Intersection 2 lists



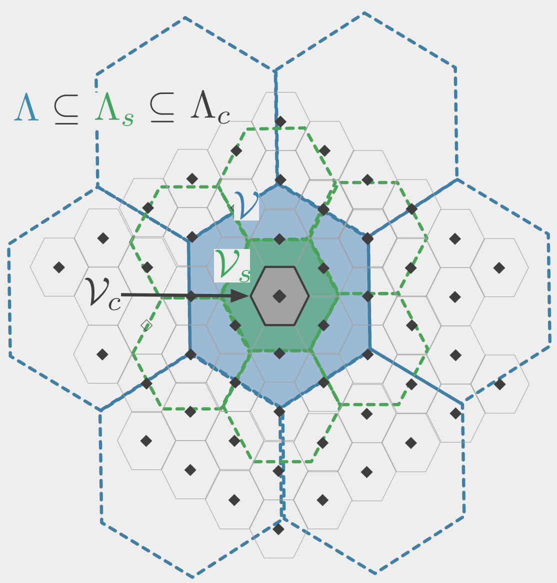


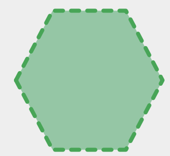
- Intersection 3 lists



Enabling lattice ``Cooperation”

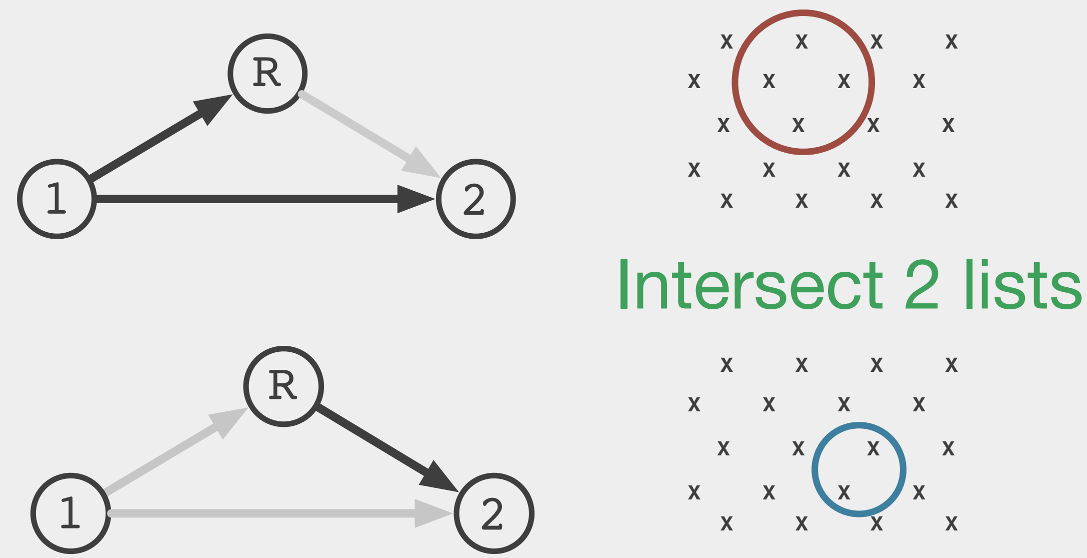
Lattice list decoder



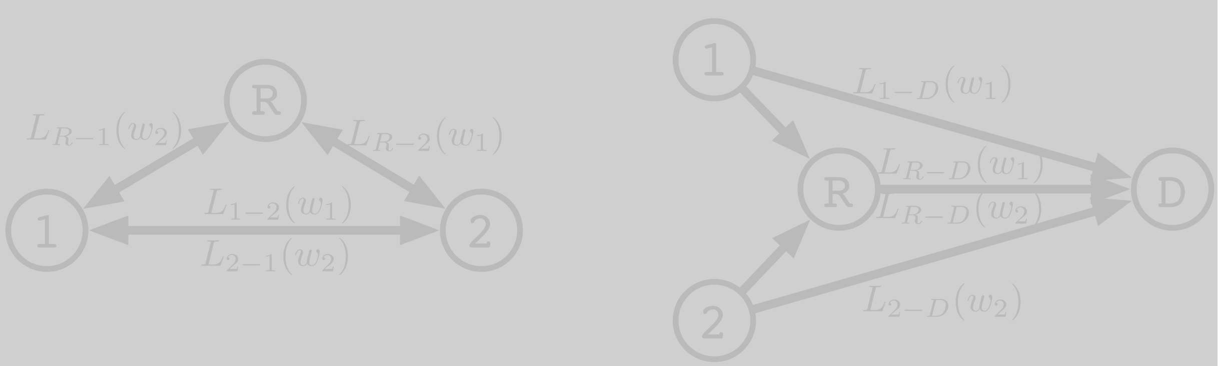
decode to 

rather than 

Lattices achieve DF rate

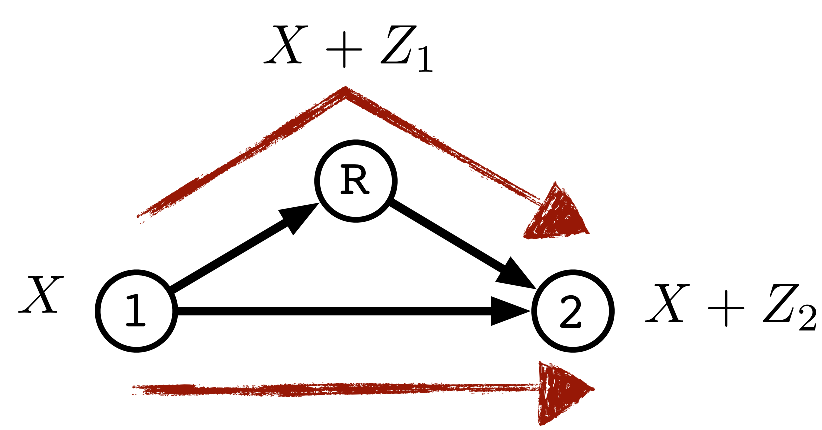


Lattices in multi-source networks



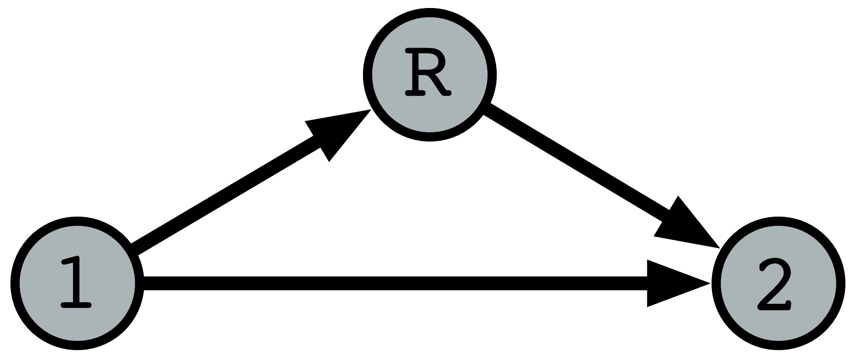
combine ``decode sum”
and direct-link cooperation

Lattices achieve CF rate

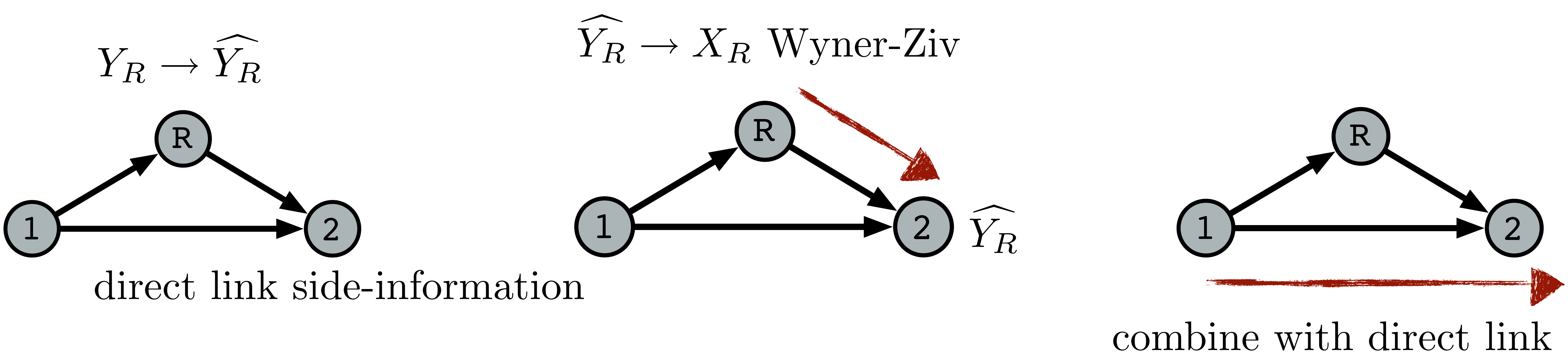


lattices good source and channel
codes, special Wyner-Ziv

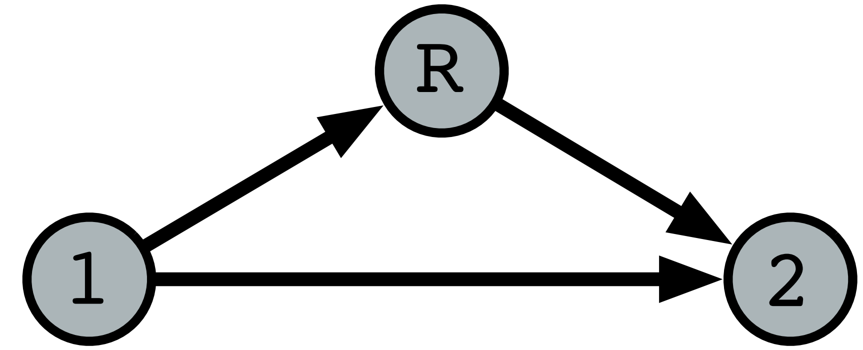
Compress and forward (CF)



- DF limited by need to decode at relay
- CF is NOT limited in this fashion



A Lattice CF scheme



Theorem. For the three user Gaussian relay channel described by the input/output equations $Y_R = X_1 + N_R$ at the relay's receiver and $Y_2 = X_1 + X_R + N_2$ at the destination, with corresponding input and noise powers P_1, P_R, N_R, N_2, the following rate may be achieved using lattice codes in a lattice Compress-and-Forward fashion:

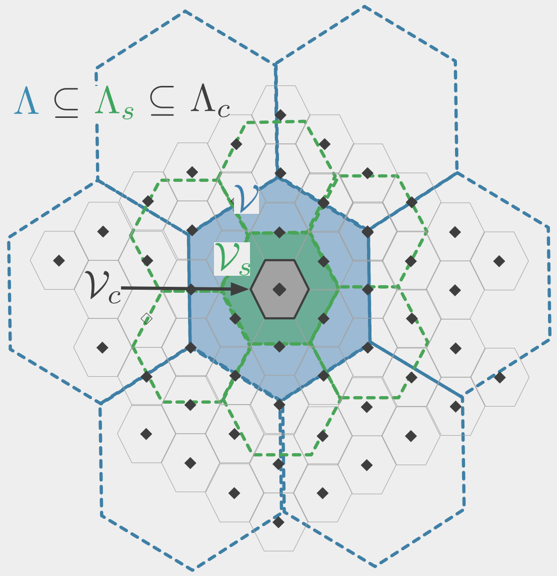
$$R < \frac{1}{2}\log\left(1 + \frac{P_1}{N_2} + \frac{P_1 P_R}{P_1 N_R + P_1 N_2 + P_R N_R + N_R N_2}\right).$$

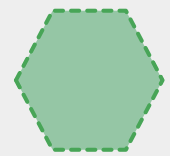
same as that achieved by Gaussian codes in the CF scheme of [Cover, El Gamal, 1979]

uses lattice version of Gaussian Wyner-Ziv [Zamir, Shamai, Erez 2002]

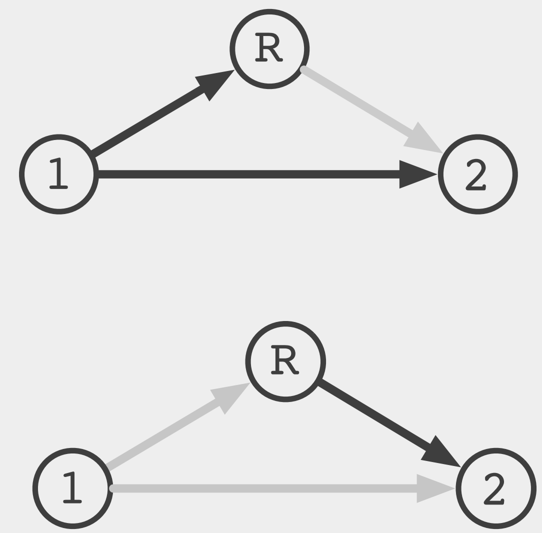
Enabling lattice ``Cooperation"

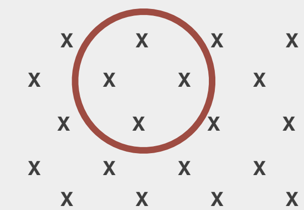
Lattice list decoder



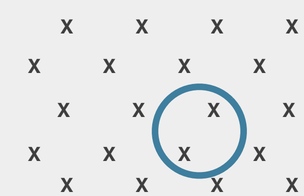
decode to 
rather than 

Lattices achieve DF rate

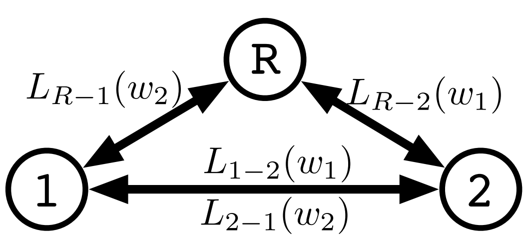


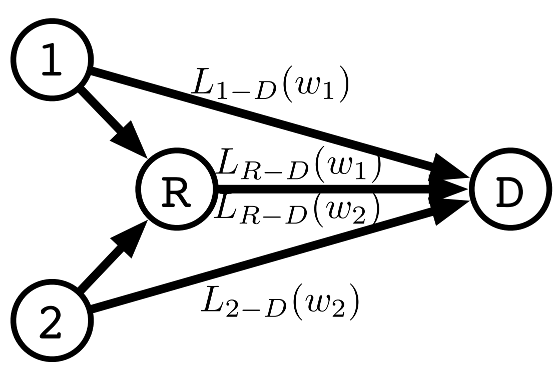


Intersect 2 lists



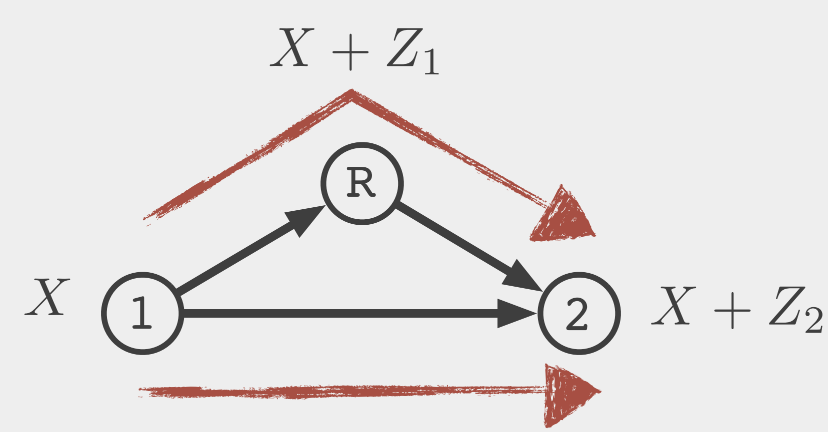
Lattices in multi-source networks





combine ``decode sum"
and direct-link cooperation

Lattices achieve CF rate



lattices good source and channel
codes, special Wyner-Ziv

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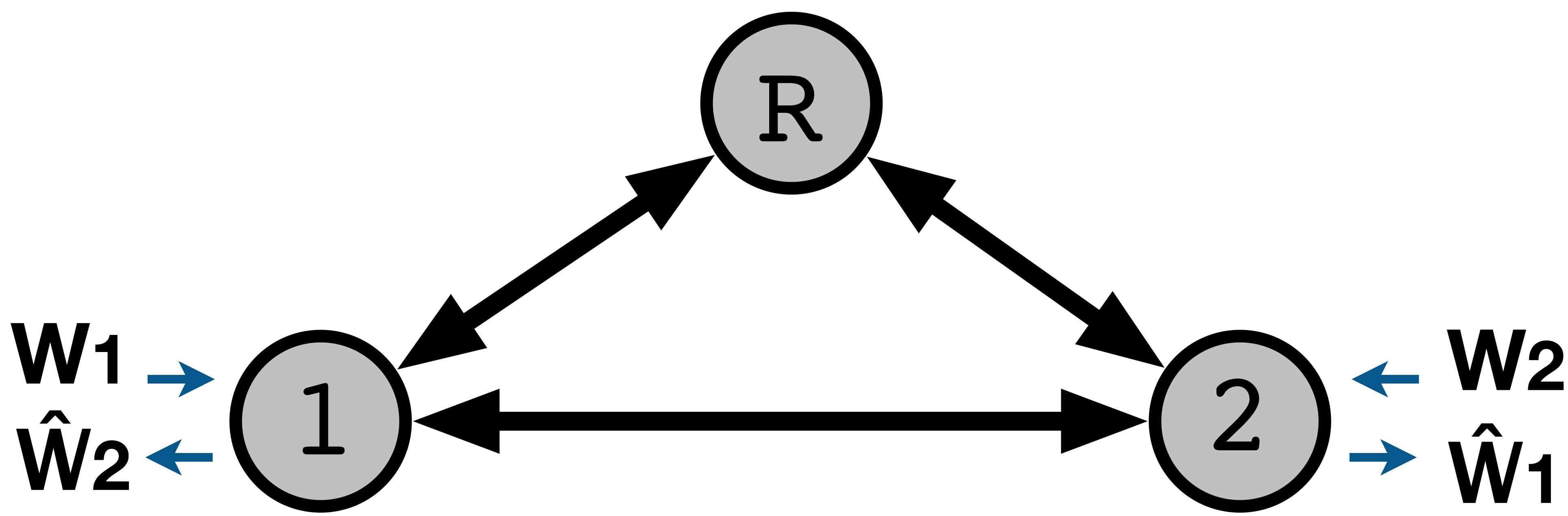
Two-way relay channel (with direct links)



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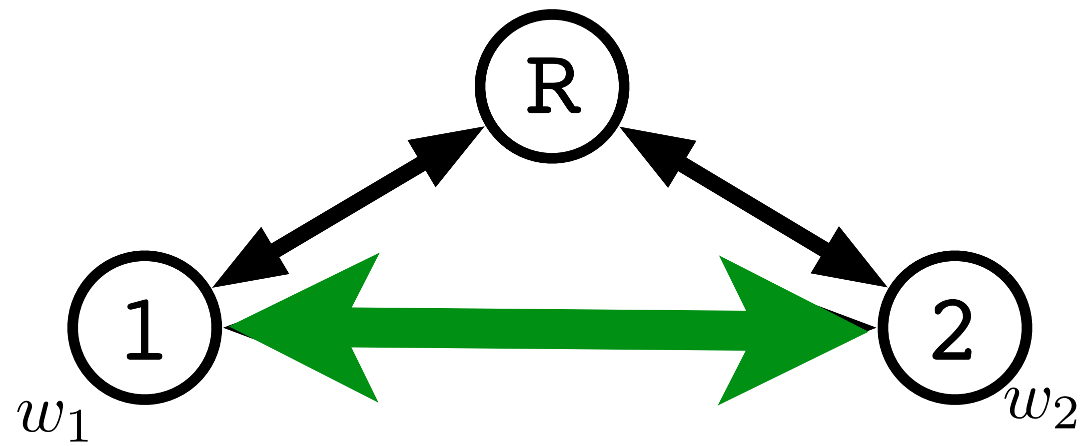
Two-way relay channel (with direct links)

$$Y_R = X_1 + X_2 + Z_R, \quad Z_R \sim \mathcal{N}(0, N_R)$$



$$Y_1 = X_1 + X_2 + X_R + Z_1 \qquad Y_2 = X_1 + X_2 + X_R + Z_2$$

$$Z_1 \sim \mathcal{N}(0, N_1) \qquad Z_2 \sim \mathcal{N}(0, N_2)$$

- we derive a new achievable rate region using **nested lattices**, **with direct link**
- this region attains **constant gaps** for certain degraded channels

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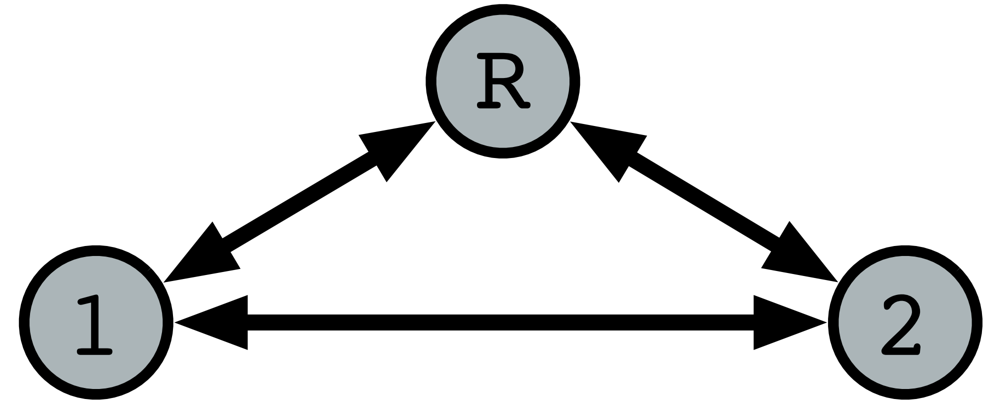


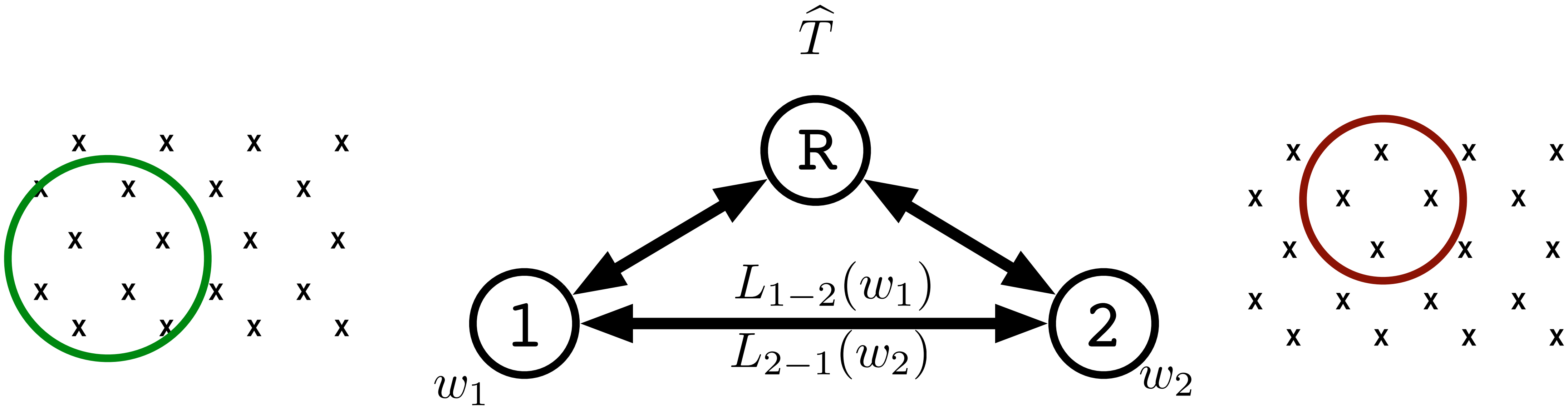
Rate region

- Theorem: For the two-way relay channel with direct links, we may achieve:

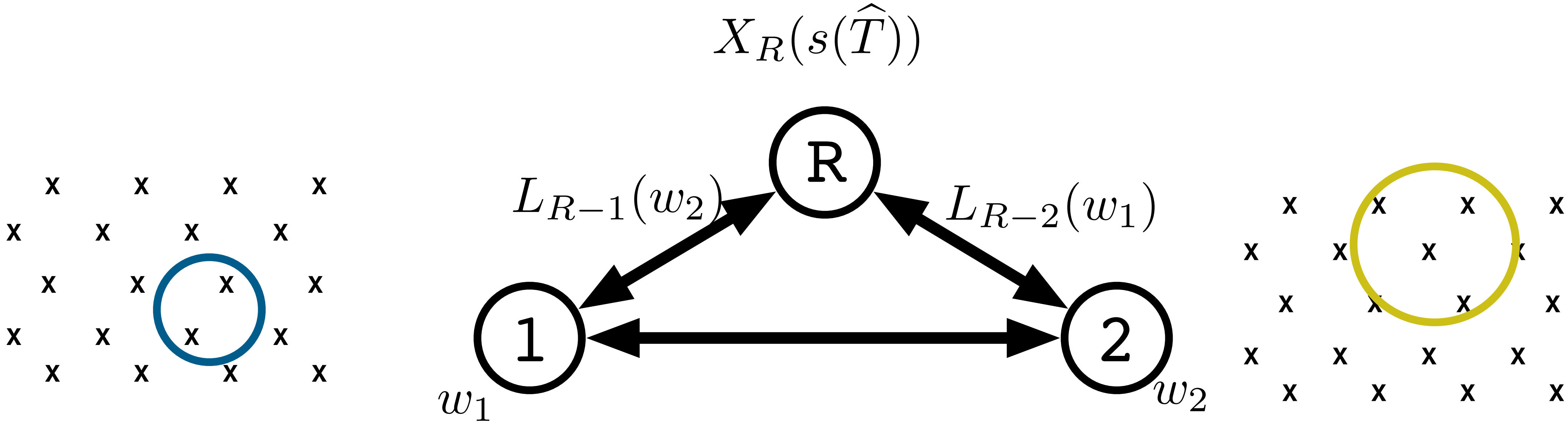
$$R_1 \leq \min\left(\left[\frac{1}{2}\log\left(\frac{P_1}{P_1+P_2}+\frac{P_1}{N_R}\right)\right]^+, \frac{1}{2}\log\left(1+\frac{P_1+P_R}{N_2}\right)\right)$$
$$R_2 \leq \min\left(\left[\frac{1}{2}\log\left(\frac{P_2}{P_1+P_2}+\frac{P_2}{N_R}\right)\right]^+, \frac{1}{2}\log\left(1+\frac{P_2+P_R}{N_1}\right)\right)$$

- eliminates “MAC”-like constraints at relay *[Xie, CWIT, 2007]*

[Y. Song, N. Devroye, submitted to IT Trans., 2011]



intersect 2 lists **intersect 2 lists**



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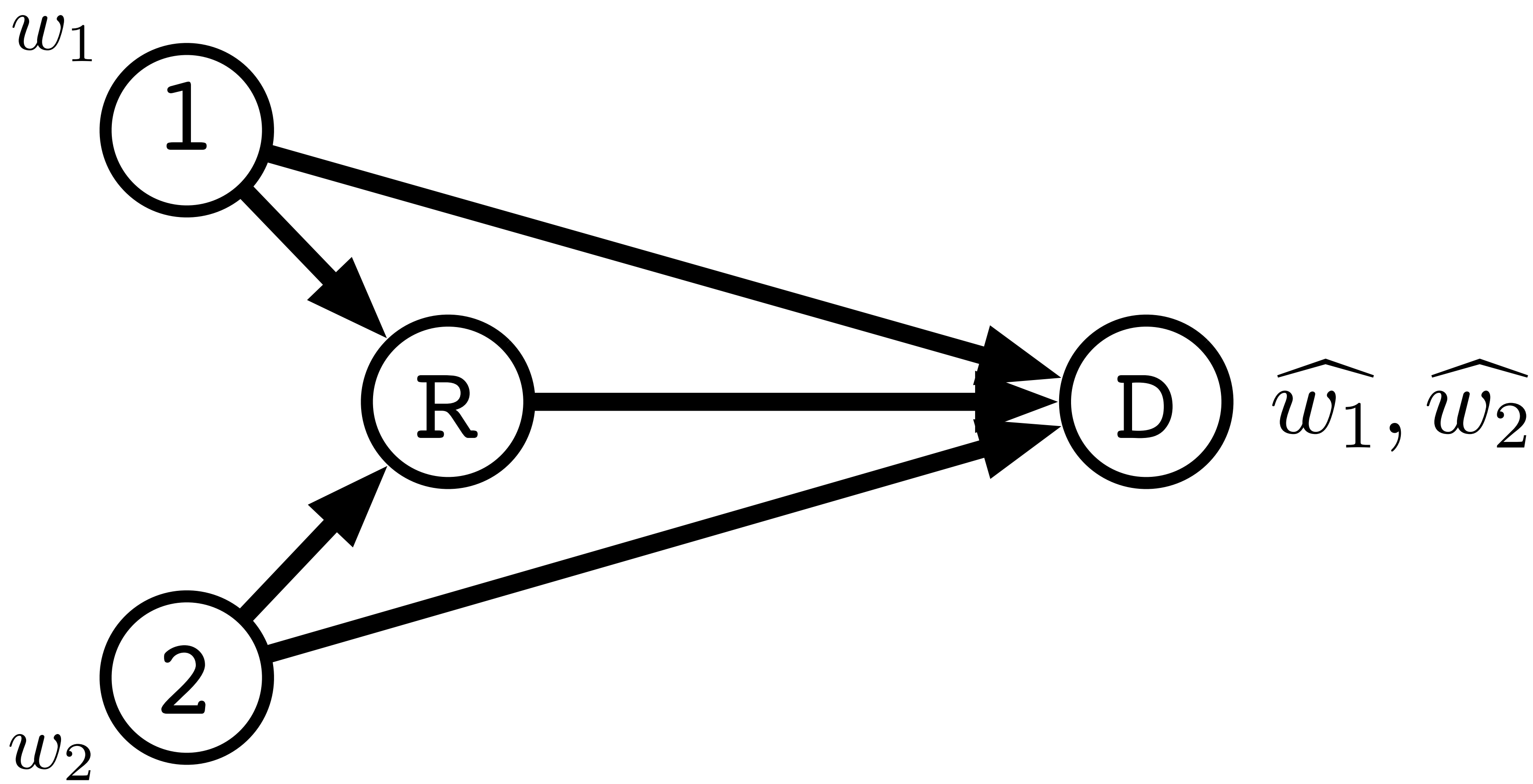
Lattices for the multiple-access relay channel



$$Y_R = X_1 + X_2 + Z_R, \;\; Z_R \sim \mathcal{N}(0, N_R)$$

$$Y_D = X_1 + X_2 + X_R + Z_D, \;\; Z_D \sim \mathcal{N}(0, N_D)$$

Key idea: decode+forward sum at the relay

Lattices for the multiple-access relay channel

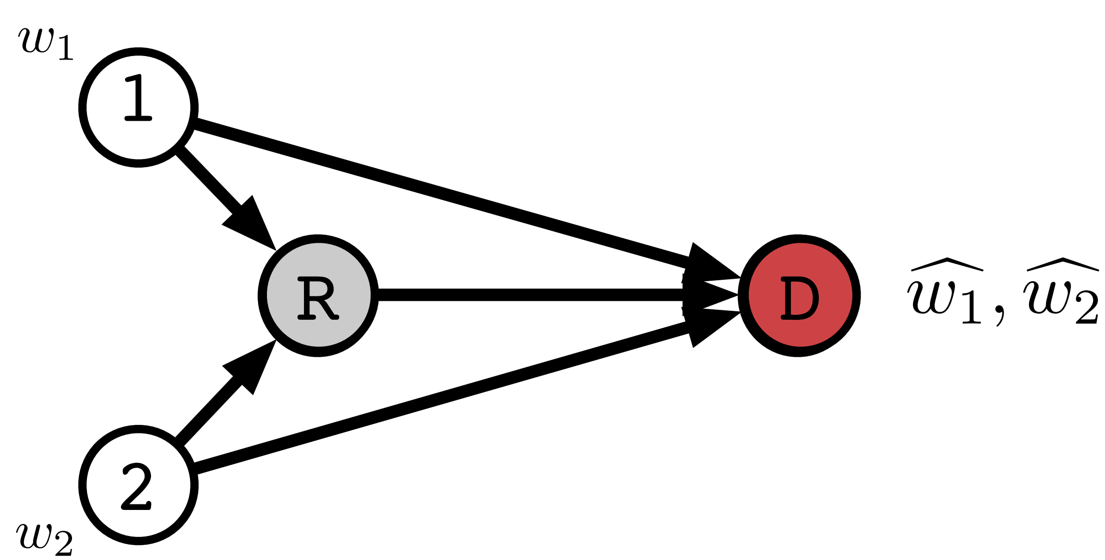
Theorem: The following rates are achievable for the AWGN multiple access relay channel:

$$R_1 < \min\left(\left[\frac{1}{2}\log\left(\frac{P_1}{P_1+P_2}+\frac{P_1}{N_R}\right)\right]^+, \frac{1}{2}\log\left(1+\frac{P_1+P_R}{N_D}\right)\right)$$

$$R_2 < \min\left(\left[\frac{1}{2}\log\left(\frac{P_2}{P_1+P_2}+\frac{P_2}{N_R}\right)\right]^+, \frac{1}{2}\log\left(1+\frac{P_2+P_R}{N_D}\right)\right)$$

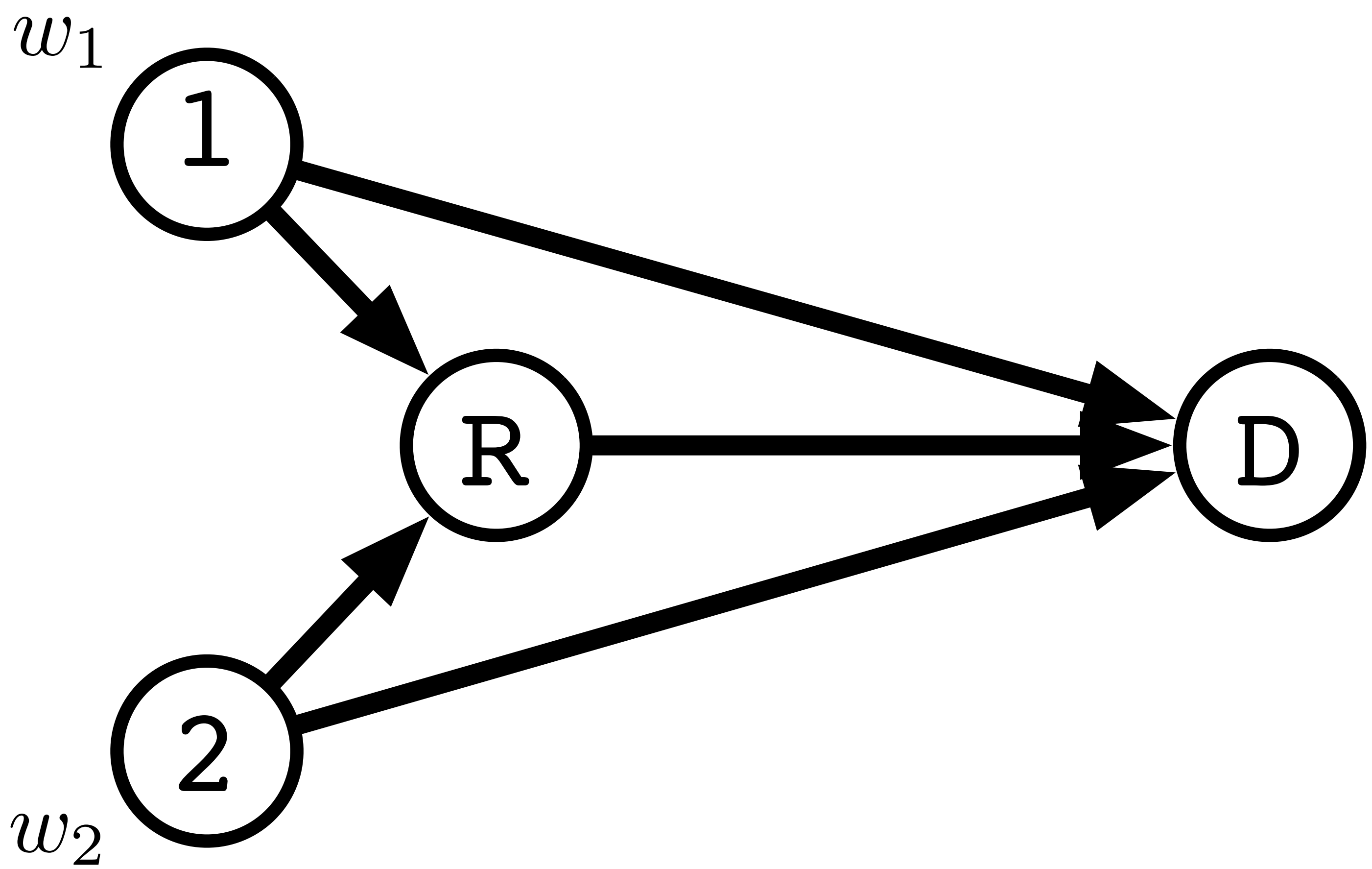
$$R_1+R_2 < \frac{1}{2}\log\left(1+\frac{P_1+P_2+P_R}{N_D}\right).$$

missing sum-rate constraint at relay!



Key idea: decode+forward sum at the relay

Lattices for the Compute-and-Forward MARC



instead of

$\widehat{w_1}, \widehat{w_2}$

want

$\widehat{w_1 \oplus w_2}$

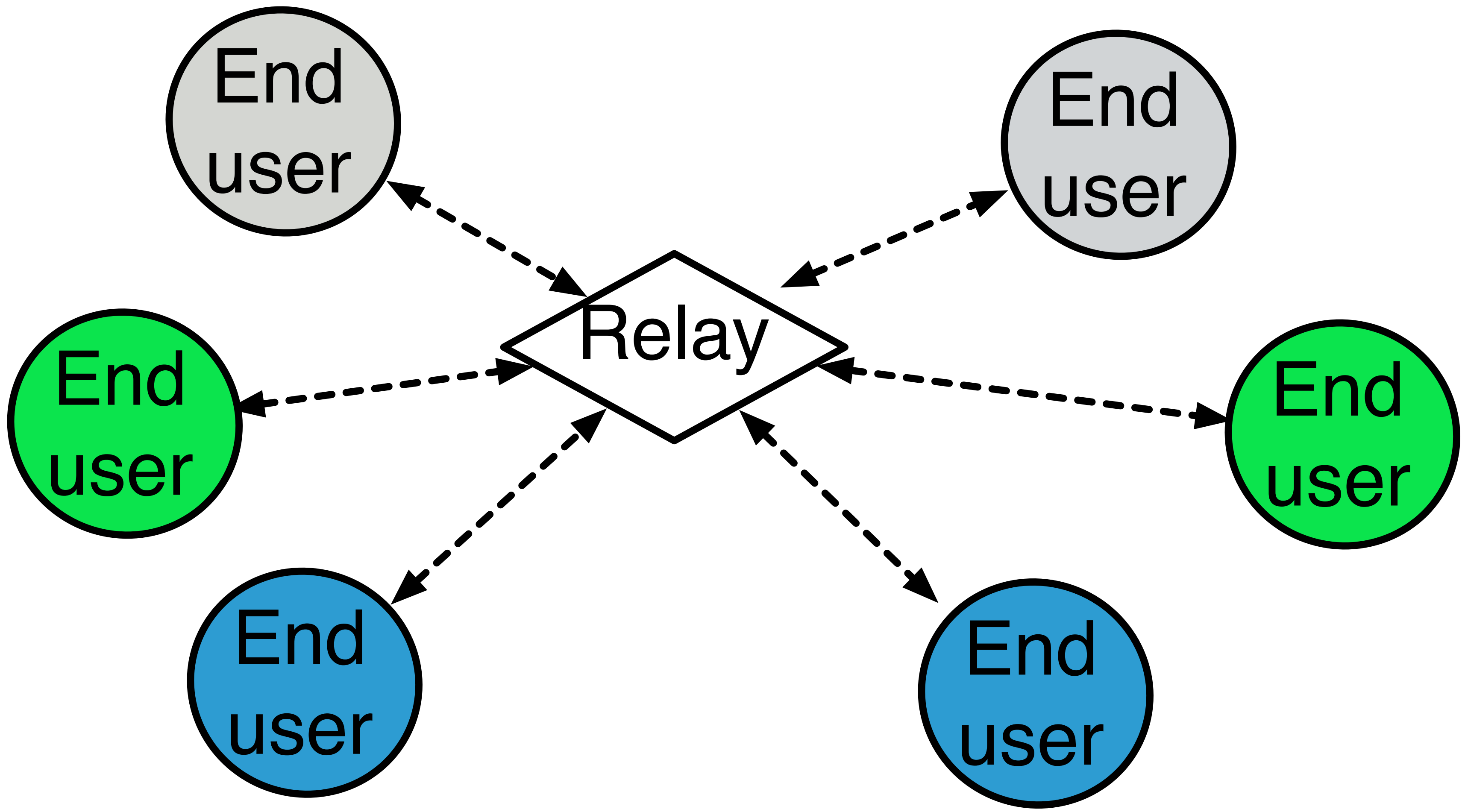
$$Y_R = X_1 + X_2 + Z_R, \ \ Z_R \sim \mathcal{N}(0, N_R)$$

$$Y_D = X_1 + X_2 + X_R + Z_D, \ \ Z_D \sim \mathcal{N}(0, N_D)$$

Key idea: decode+forward real sum at the relay

[M. Nokleby, B. Nazer, B. Aazhang, N. Devroye, to appear in ISWCS 2012]

Lattices for multi-pair two-way relay network



Key idea: successive decoding of pairwise lattice sums

A. Avestimehr, A. Sezgin, and D. Tse, "Capacity region of the deterministic multi-pair bi-directional relay network," in *Proc. IEEE Inf. Theory Workshop*, Volos, June 2009.

H. Ghozlan, Y. Mohasseb, H. El Gamal, and G. Kramer, "The MIMO wireless switch: Relaying can increase the multiplexing gain," 2009. [Online]. Available: http://arxiv.org/abs/0901.2588

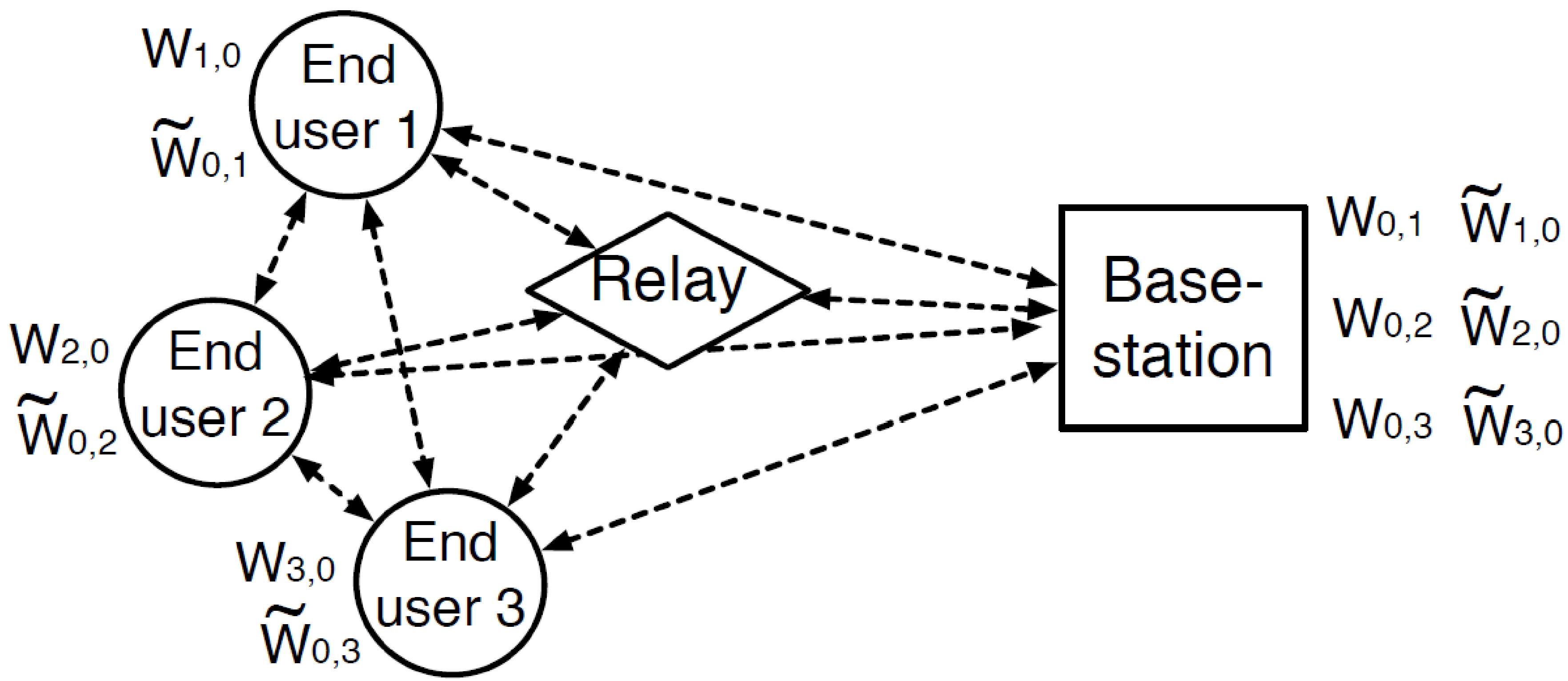
A. Sezgin, A. Khajehnejad, A. Avestimehr, and B. Hassibi, "Approximate capacity region of the two-pair bidirectional gaussian relay network," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 2018–2022.

M. Chen and A. Yener, "Power allocation for F/TDMA multiuser two-way relay networks," *IEEE Trans. Wireless Comm.*, vol. 9, no. 2, pp. 546–551, 2010.

D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 339–343.

D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," http://arxiv.org/abs/1004.2434/.

Lattices for the multi-pair two-way relay network



Key idea: successive decoding of pairwise lattice sums

[S.-J. Kim, B. Smida, and N. Devroye, ISIT 2011]

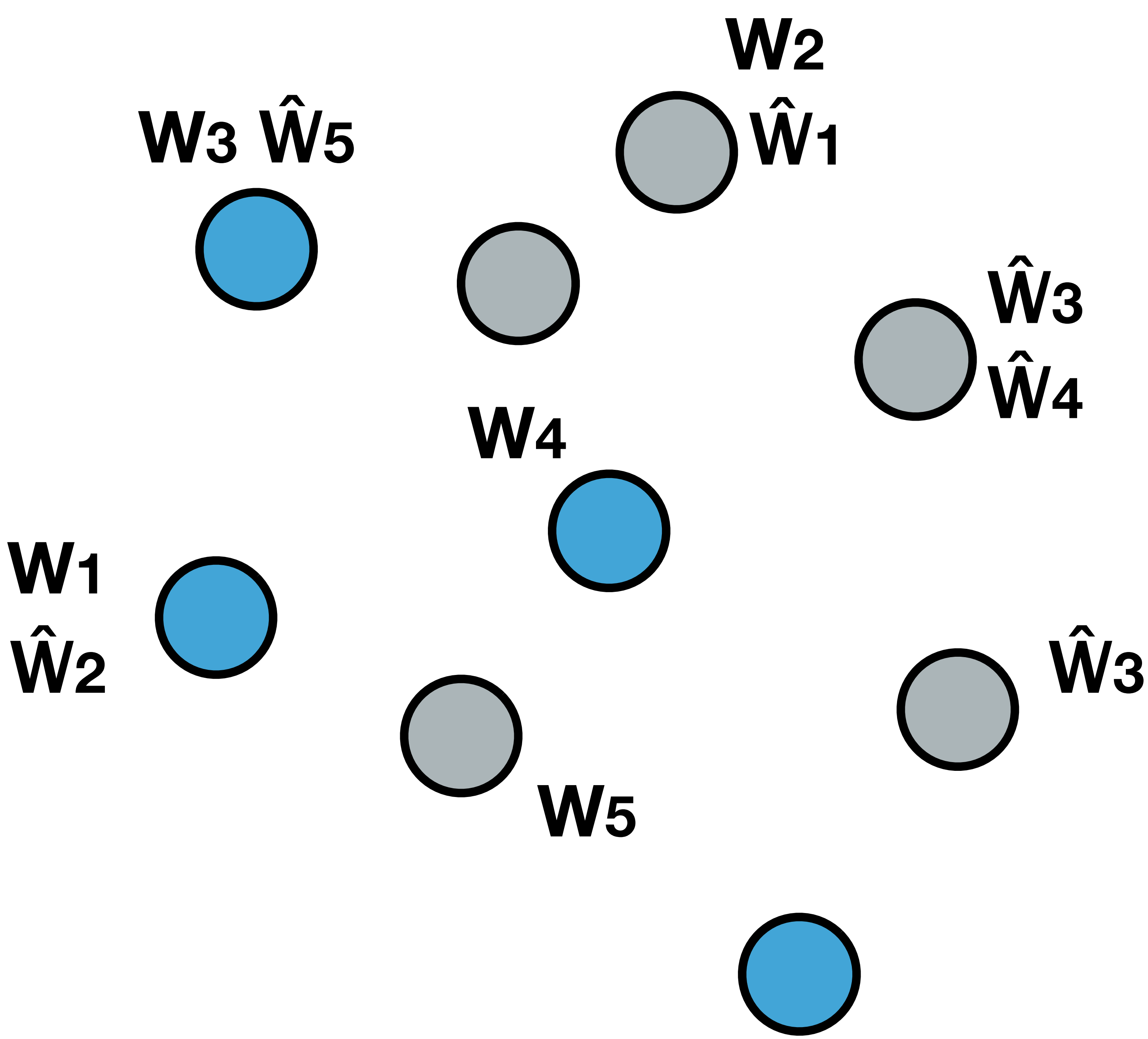
Much much more!

- Matt Nokleby and Benhaam Aazhang's recent work explores combining cooperation and compute-and-forward

- Uri Erez + Ram Zamir's work, website, slides and "Lattices are Everywhere" have excellent surveys

- Bobak Nazer and Michael Gastpar's survey article "Reliable Physical Layer Network Coding" in *Proc. of IEEE,* 2011 has many references!

Conclusion: lattice codes to



Compute and forward

Inverse compute and forward

Relay

Exploit linearity!

Conclusion

- can random codes be replaced by structured codes in Gaussian networks?

- how to combine different techniques in a comprehensive but manageable fashion?

- is structure necessary and how well can compress-and-forward do?

Questions?

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http://www.ece.uic.edu/Devroye