The IEEE International Symposium on Information Theory 2011

The Capacity of the Interference Channel with a Cognitive Relay in Very Strong Interference

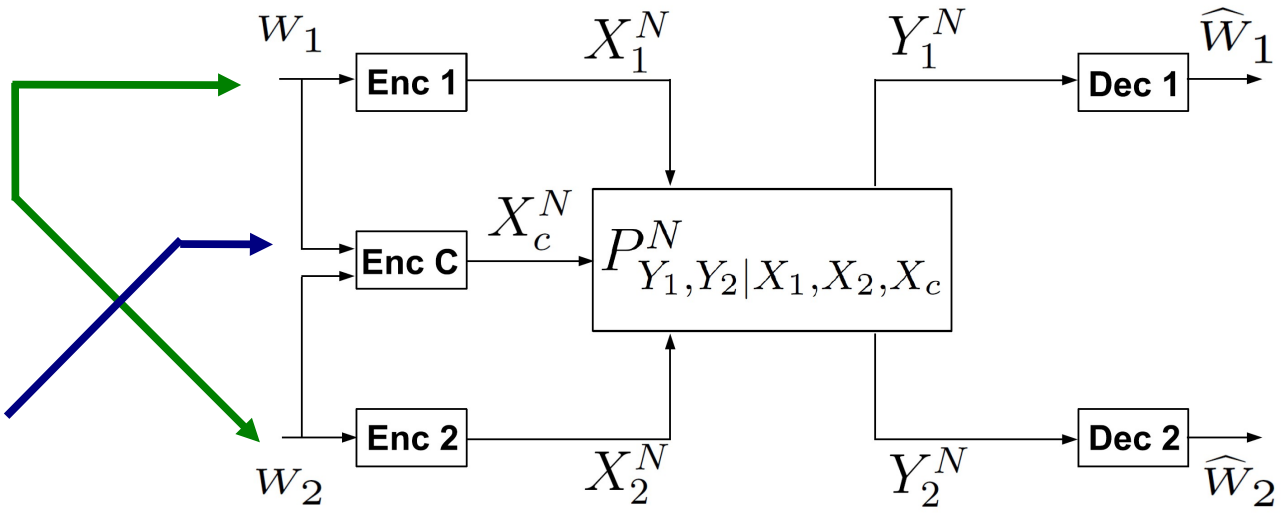
Stefano Rini, Daniela Tuninetti, Natasha Devroye, Andrea Goldsmith

The interference channel with a cognitive relay IFC-CR

Channel Model:

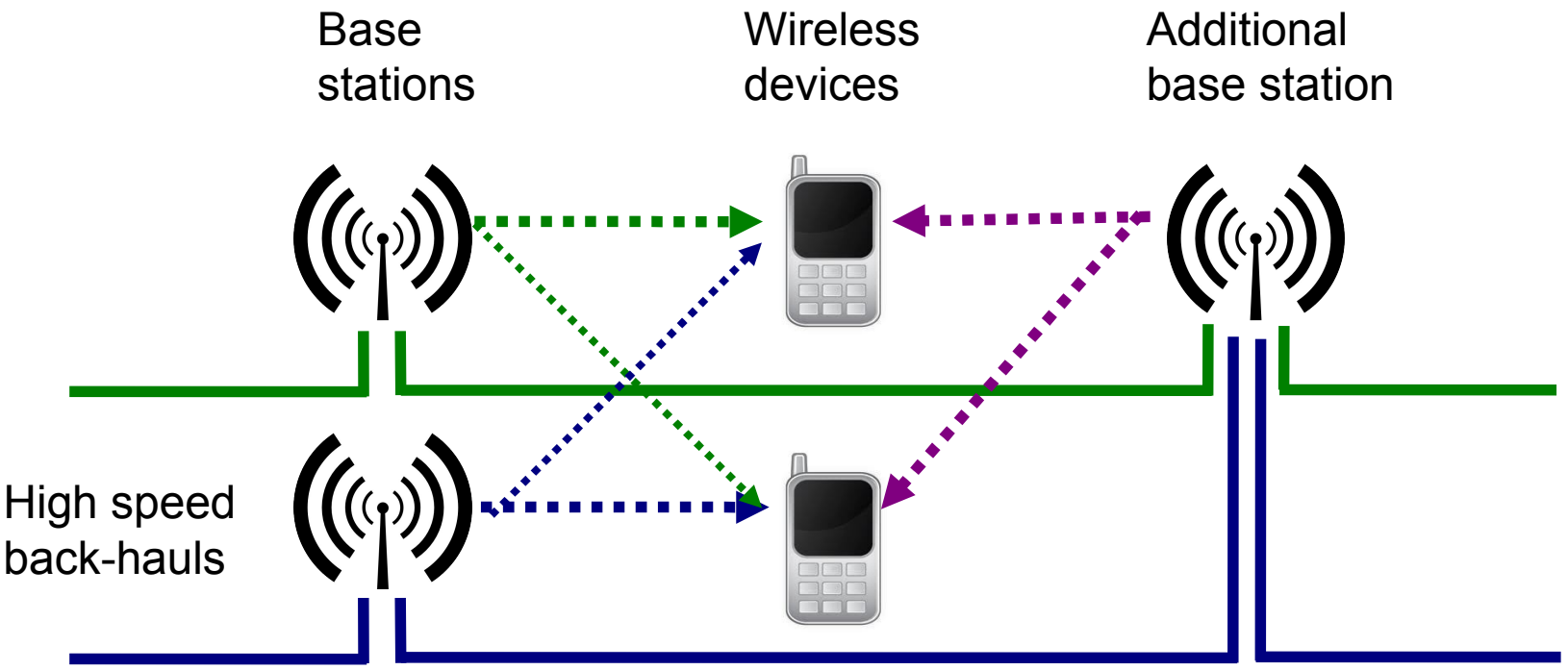
Classical Interference channel:

Cognitive relay:



The interference channel with a cognitive relay IFC-CR

When does it occur in practice?



The interference channel with a cognitive relay IFC-CR

Why do we study it?

There are many channel models out there,
but we want to answer the following questions:

- How do we **manage the interference** at the two decoders simultaneously?
- How do we **implement cooperation** with the two users simultaneously?
- Are there **novel coding strategies** to be learned from this model?

R.T.D.:"Outer Bounds for the Interference Channel with a Cognitive Relay," ITW Dublin,2010

The interference channel with a cognitive relay IFC-CR

Some literature:

- Sahin & Erkip intrduce the model

O. Sahin and E. Erkip,
"Achievable Rates for the Gaussian Interference Relay Channel,"

- Jiang et al. propose an achievable scheme

J. Jiang, I. Maric, A. Goldsmith, and S. Cui,
"Achievable Rate Regions for Broadcast Channels With Cognitive Relays,"

- Sridharan al. propose outer bounds for the gaussian case

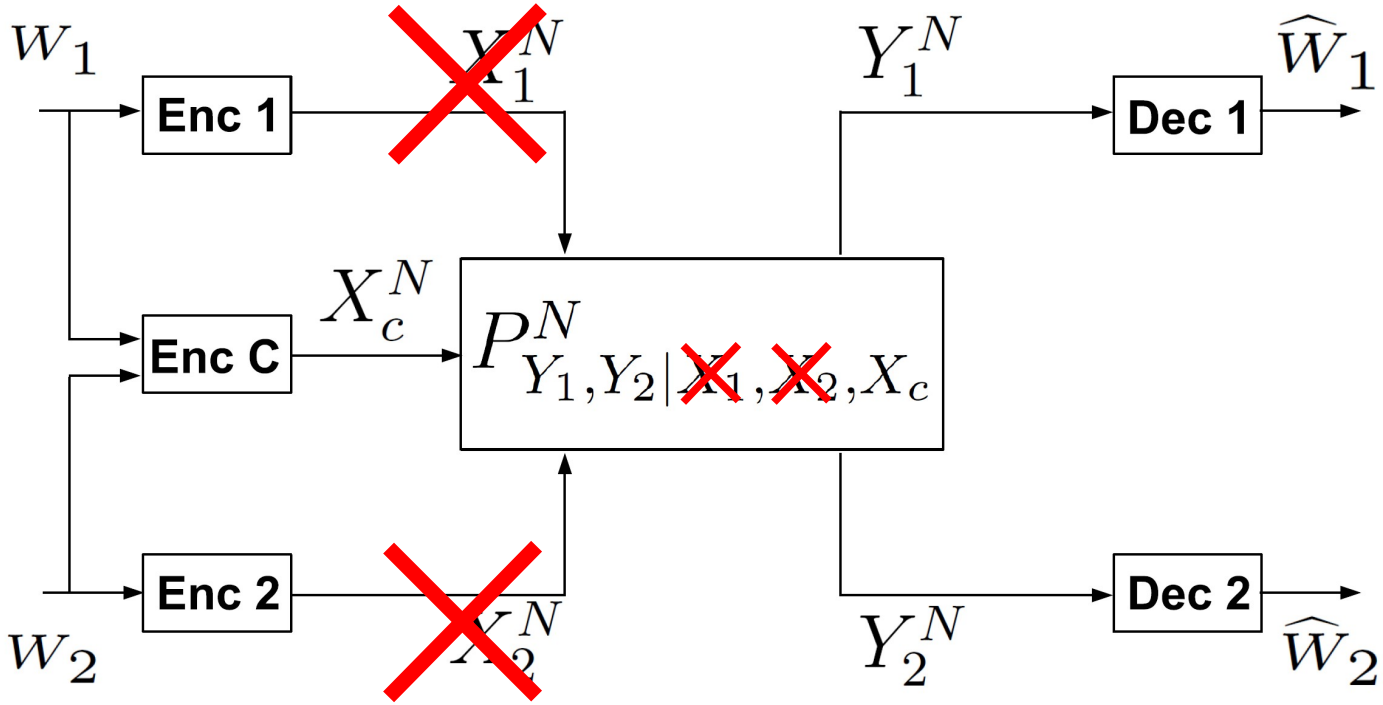
S. Sridharan, S. Vishwanath, S. Jafar, and S. Shamai,
"On the capacity of cognitive relay assisted Gaussian interference channel,"

- Rini et al. propose outer bounds for the general case

S. Rini, D. Tuninetti, and N. Devroye,
"Outer Bounds for the Interference Channel with a Cognitive Relay,"

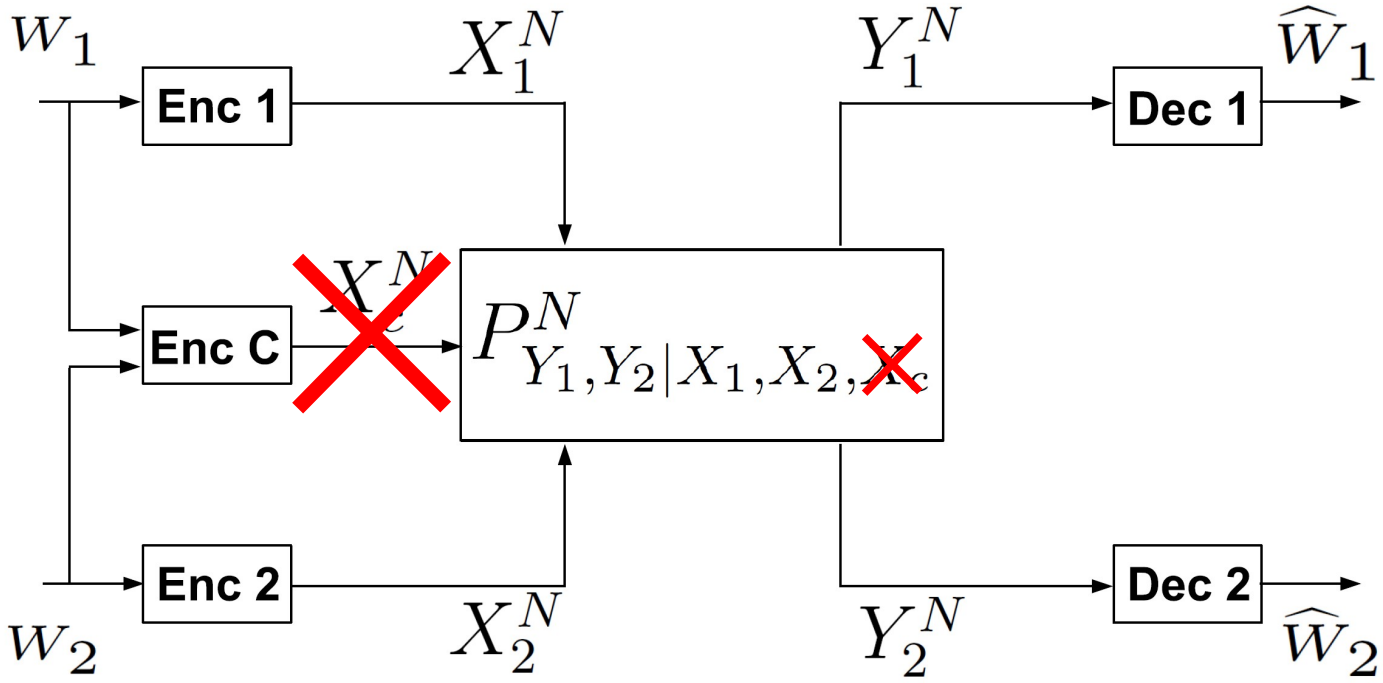
The interference channel with a cognitive relay IFC-CR

Broadcast channel:



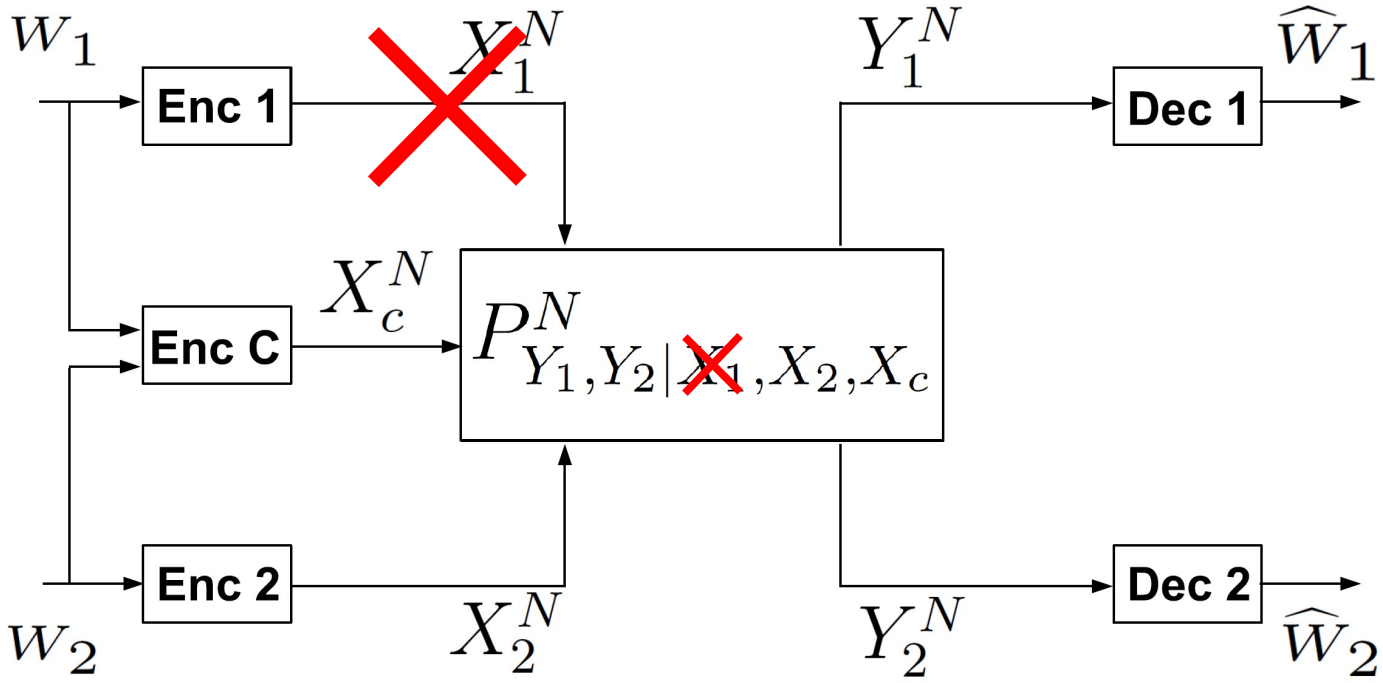
The interference channel with a cognitive relay IFC-CR

Interference channel:



The interference channel with a cognitive relay IFC-CR

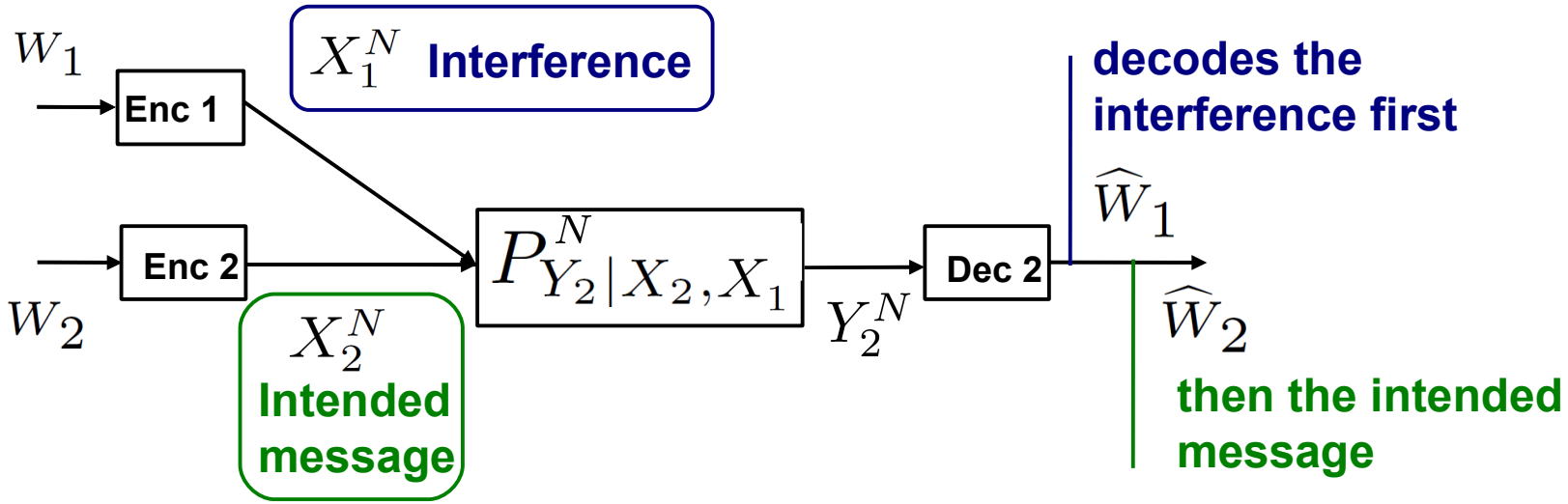
Cognitive interference channel:



Strong/very strong interference regime

Strong interference regime at Rx 2

For all these channel models, there is a regime where one decoder can decode the message of the other user from it's channel output



This regime is usually referred to as “strong interference” regime

Strong/very strong interference regime

Strong interference regime at RX 2

this condition is somewhat intuitive:

Assume that decoder 2 has decoded the intended message X_2

and assume that decoder 1 is provided with the interference X_2,

then decoder 2 can **still** decode X_1 more easily than decoder 1

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2)$$

Strong/very strong interference regime

Further conditions:

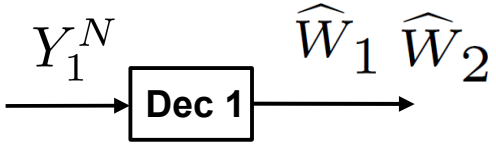
To achieve capacity in the strong interference at one decoder, one usually needs a further conditions:

- Strong interference regime at both decoders:
  that is when both decoders are in strong interference
- Very strong interference at RX 2
  or the 'weak' decoder can decode both messages better than what the 'strong' one does.

Strong/very strong interference regime

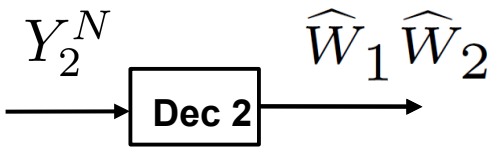
Very strong interference regime at RX 2

'weak' receiver



$$I(X_1;Y_1|X_2) \quad I(X_1,X_2;Y_1)$$

$$|\wedge \qquad + \qquad \vee|$$



$$I(X_1;Y_2|X_2) \quad I(X_1,X_2;Y_2)$$

'strong' receiver

Strong at RX 2

very strong at RX 2

Strong/very strong interference regime

Strong/very strong interference regime at RX 1/RX 2

In this regime it is usually possible to derive an outer bound using the condition above.

This holds for the:

- the more capable broadcast channel
- the interference channel in strong/very strong interference
- the cognitive interference channel in strong/very strong interference

In this paper

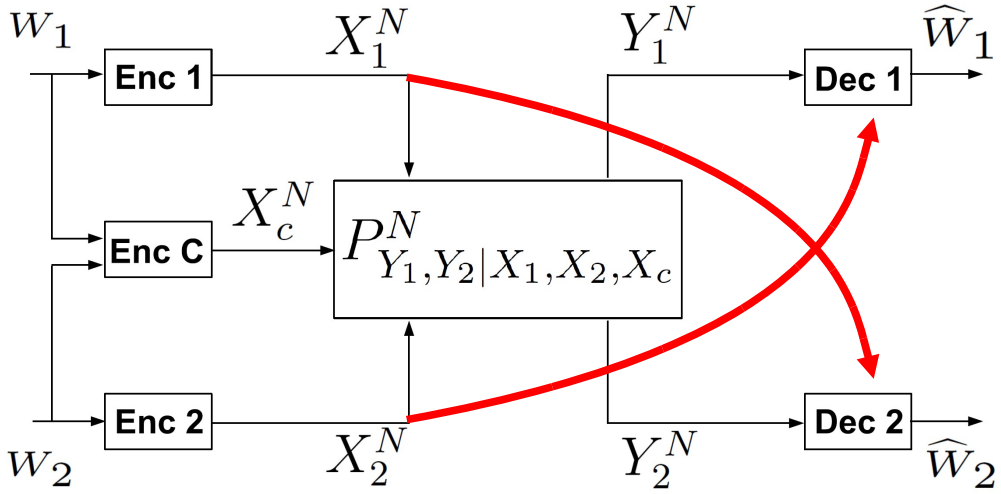
Our contribution:

- we obtain two new capacity results for the IFC-CR based on these ideas
- these are the first capacity results for this channel!
- look at the Gaussian case, where we can plot these results

Strong interference at RX 1 outer bound

Single rate bounds:

$$R_1 \leq I(Y_1; X_1, X_c | X_2, Q)$$



Provide interference as side information

$$R_2 \leq I(Y_2; X_2, X_c | X_1, Q)$$

Strong interference at RX 1 outer bound

Sum rate bounds:

Use an genie and impose the strong interference condition

$$\begin{aligned} R_1 + R_2 &\leq I(Y_1; V, U_1, X_1) + I(Y_2; U_2, X_2, X_c | V, U_1, X_1) \\ &\leq I(Y_1; X_1, X_2, X_c) \end{aligned}$$

using the fact that

$$I(Y_2; X_2, X_c | X_1) \leq I(Y_1; X_2, X_c | X_1)$$

implies

$$I(Y_2; X_2, X_c | X_1, U) \leq I(Y_1; X_2, X_c | X_1, U)$$

as in the more capable broadcast channel.

Strong interference at RX 1 outer bound

Strong interference at Rx 1 outer bound:

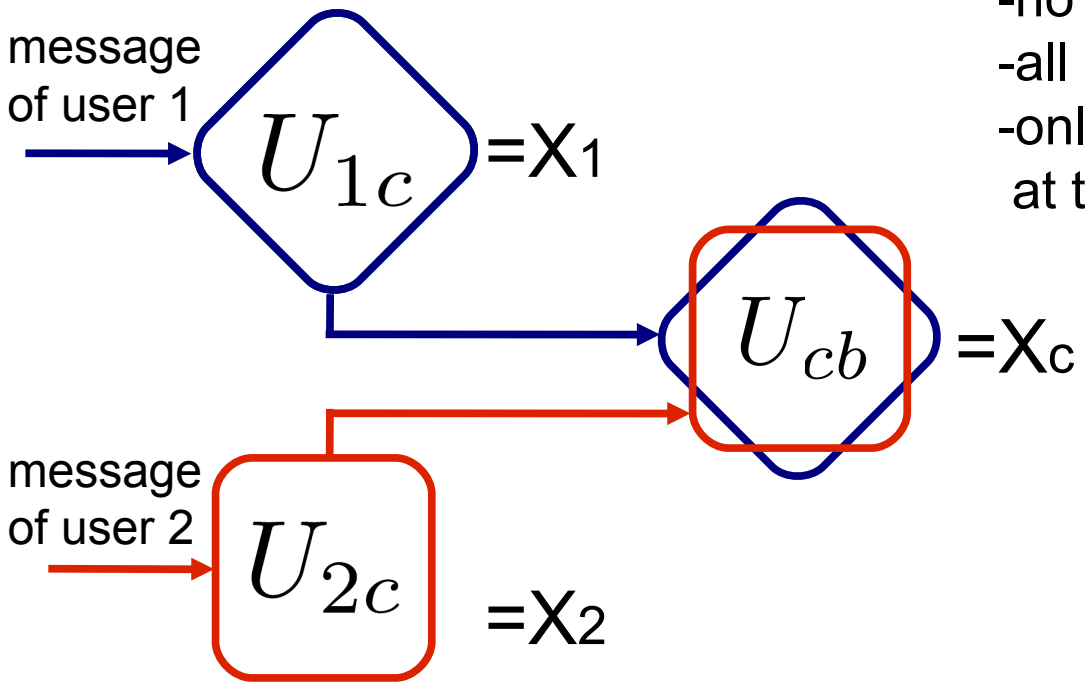
The capacity region is contained in

$$R_1 \leq I(Y_1; X_1, X_c | X_2, Q),$$
$$R_2 \leq I(Y_2; X_2, X_c | X_1, Q),$$
$$R_1 + R_2 \leq I(Y_1; X_1, X_2, X_c | Q),$$

when

$$I(Y_2; X_2, X_c | X_1) \leq I(Y_1; X_2, X_c | X_1)$$

Achievability of the outer bound



-no rate split
-all common messages
-only superposition coding
at the cognitive relay

Achievability of the outer bound

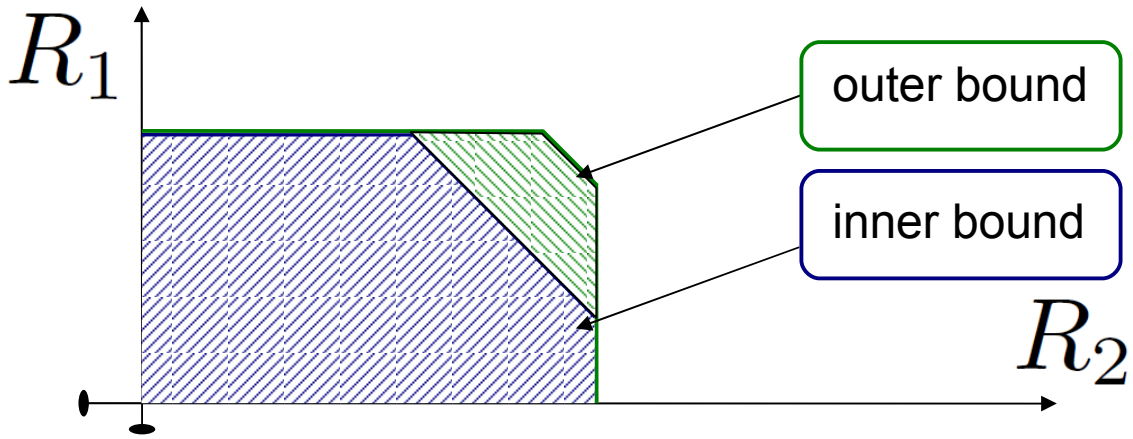
Achievable region

$$
\begin{aligned}
R_1 &\leq I(Y_1; X_1, X_c | X_2, Q), \\
R_2 &\leq I(Y_2; X_2, X_c | X_1, Q), \\
R_1 + R_2 &\leq I(Y_1; X_1, X_2, X_c | Q), \\
R_1 + R_2 &\leq I(Y_2; X_1, X_2, X_c | Q),
\end{aligned}
$$

Achievability of the outer bound

The inner and outer bound differ only in one sum rate outer bound.
The two region coincide if:

- we can drop one sum rate from the inner bound
  - very strong interference condition
- we can add one sum rate at the outer bound
  - strong interference at both RXs



Capacity in very strong interference at RX 1

When:

$$I(Y_2; X_2, X_c|X_1) \leq I(Y_1; X_2, X_c|X_1)$$

the channel is in strong interference at RX1, and when

$$I(Y_1; X_1, X_2, X_c) \leq I(Y_2; X_1, X_2, X_c)$$

we can drop the sum rate

$$R_1 + R_2 \leq I(Y_2; X_1, X_2, X_c|Q),$$

In the inner bound

CAPACITY !

Capacity in strong Interference at both RXs

When

$$I(Y_2; X_2, X_c|X_1) \leq I(Y_1; X_2, X_c|X_1)$$

the channel is in strong interference at RX1, and when

$$I(Y_1; X_1, X_c|X_2) \leq I(Y_2; X_1, X_c|X_2)$$

the channel is in strong interference at RX2, so we add the bound

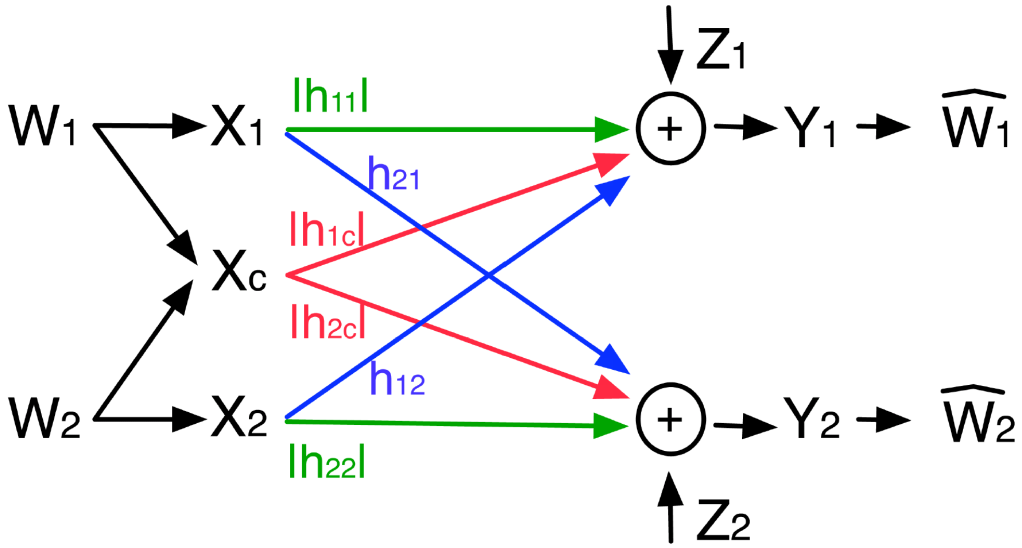
$$R_1 + R_2 \leq I(Y_2; X_1, X_2, X_c|Q),$$

In the outer bound.

CAPACITY !

The Gaussian IFC-CR

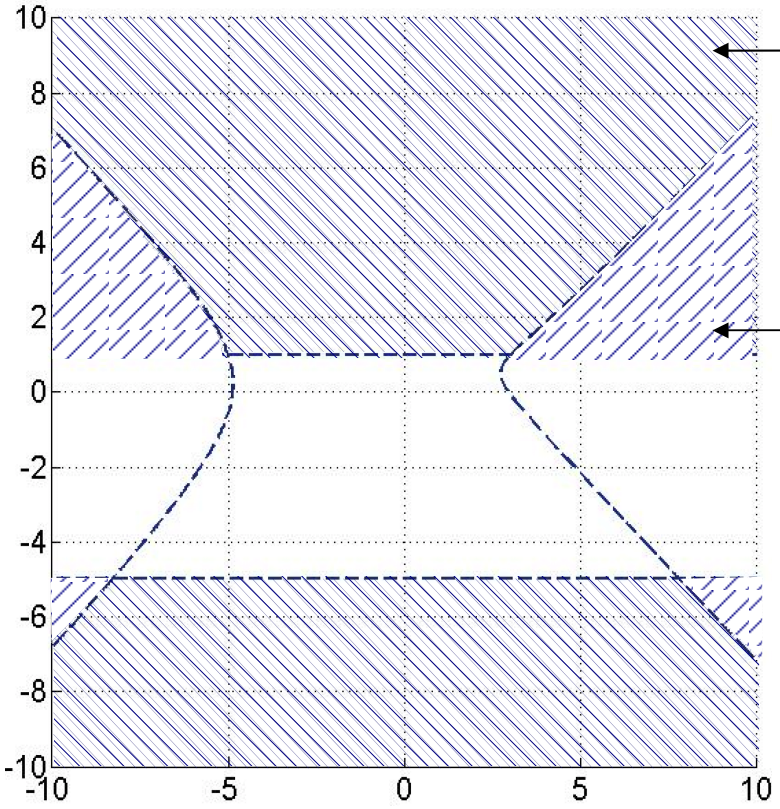
Let's visualize these results for the Gaussian case:



So we can see how the strong-very strong interference regimes play out.

The Gaussian IFC-CR

h_{12}



h_{21}

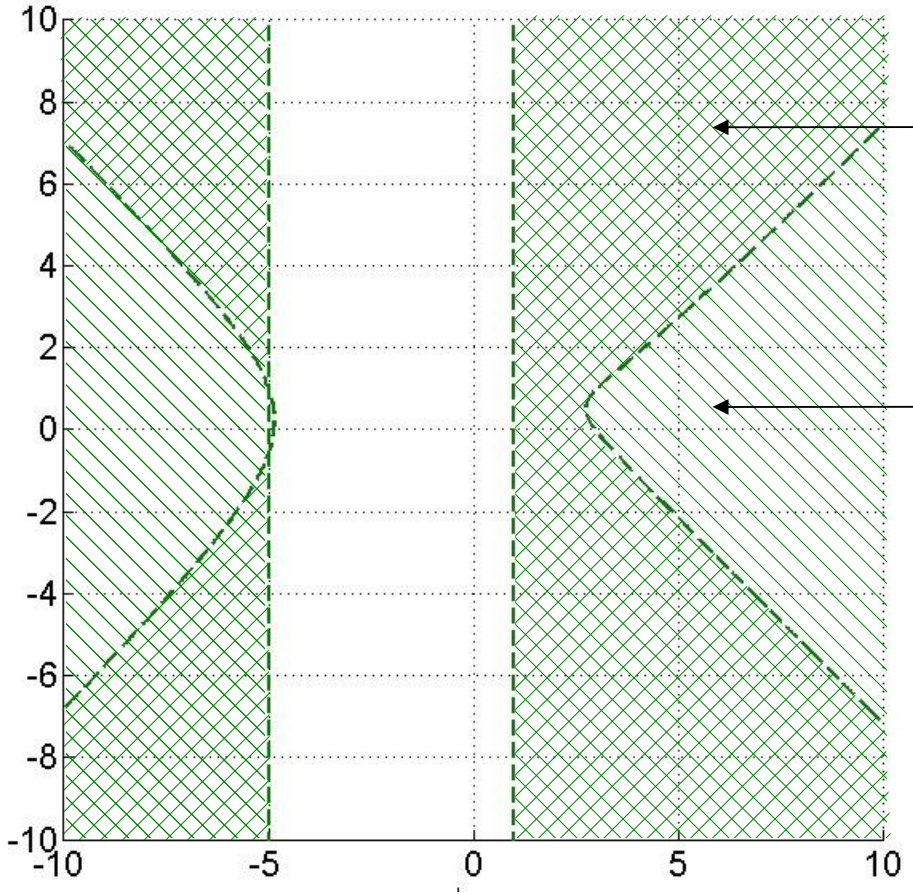
Strong inteference at Rx 1

Very strong interference at Rx 1

$h_{11} = h_{22} = 1,$
$h_{1c} = h_{2c} = 2.$

The Gaussian IFC-CR

h_{12}



very strong Interference at Rx 2

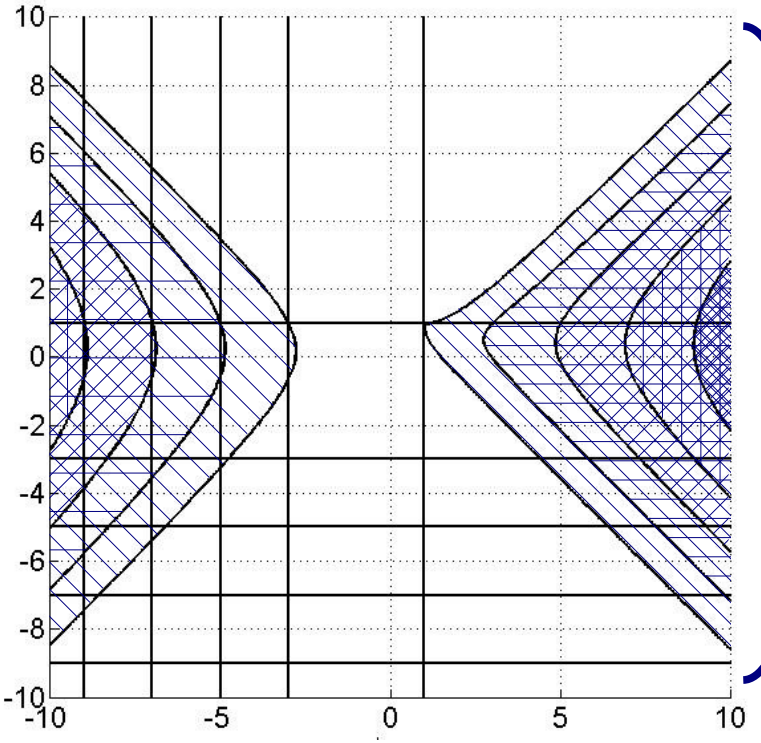
Strong interference at Rx 2

$h_{11} = h_{22} = 1,$
$h_{1c} = h_{2c} = 2.$

h_{21}

The Gaussian IFC-CR

h_{12}

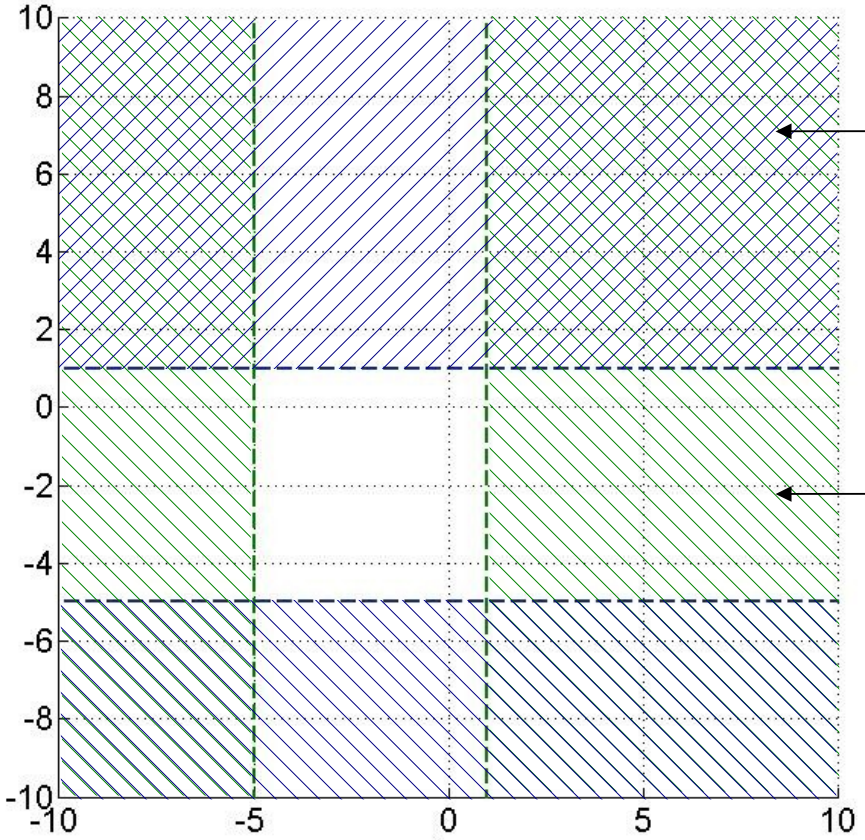


$$I(Y_1; X_1, X_2, X_c) = I(Y_2; X_1, X_2, X_c)$$

h_{21}

The Gaussian IFC-CR

h_{12}



Strong interference at Rx 1

Strong interference at Rx 2

h_{21}

$h_{11} = h_{22} = 1,$
$h_{1c} = h_{2c} = 2.$

Conclusion

What did we do:

derive two new capacity results for the interference channel with a cognitive relay:

- very strong interference channel an RX 1/RX 2
- strong interference at both decoders

Thank you!

Any question ?

Look for the journal version online!

On the Capacity of the Interference Channel with a Cognitive Relay