A New Capacity Result for the Z-Gaussian Cognitive Interference Channel

Daniela Tuninetti (danielat@uic.edu)

Joint work with: Stefano Rini and Natasha Devroye

Acknowledgment: Insightful discussions with Shlomo Shamai

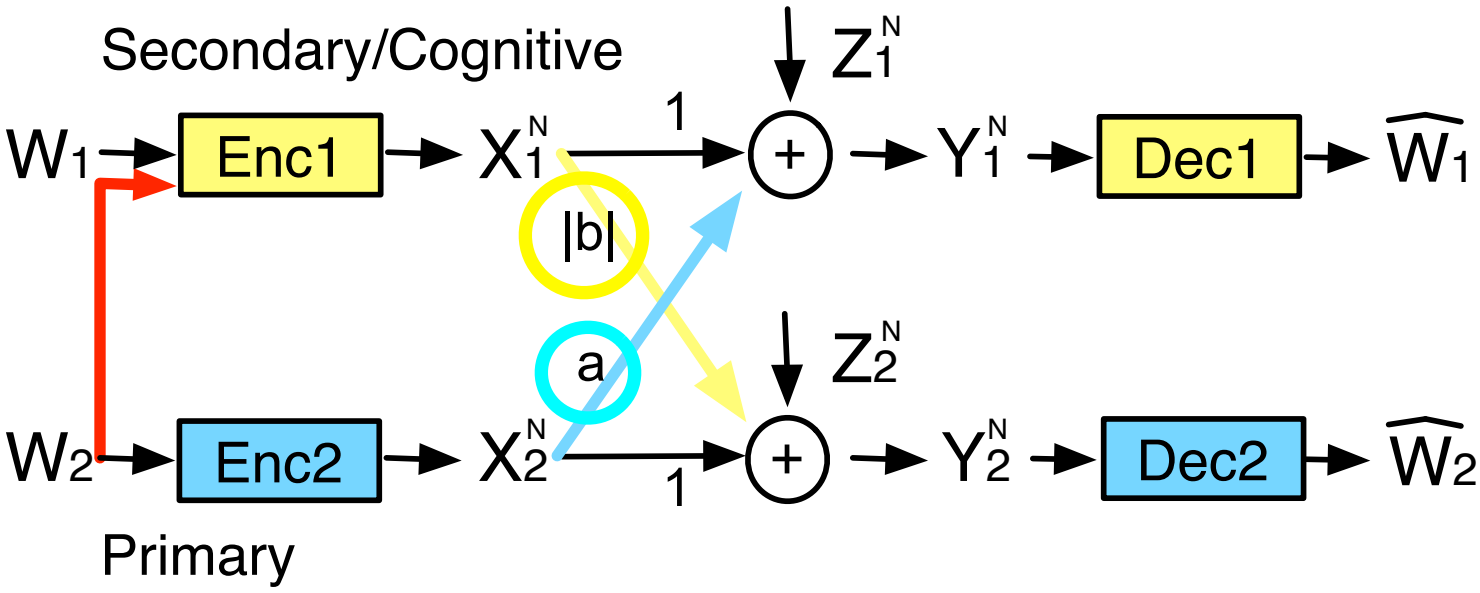
UIC UNIVERSITY OF ILLINOIS AT CHICAGO Department of Electrical and Computer Engineering
COLLEGE OF ENGINEERING

ISIT 2011, St. Petersburg, Russia, August 2011

Outline

1. Cognitive Interference Channel (CIFC):
   a. AWGN CIFC channel model
   b. past work
2. Contributions:
   a. novel outer bound in strong interference
   b. capacity for some Z G-CIFC
   c. extensions to non-Z G-CIFC
3. Conclusion

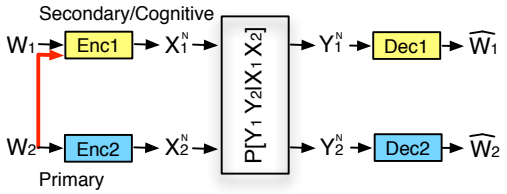
Channel Model



$$Y_1 = X_1 + aX_2 + Z_1,$$
$$Y_2 = |b|X_1 + X_2 + Z_2,$$

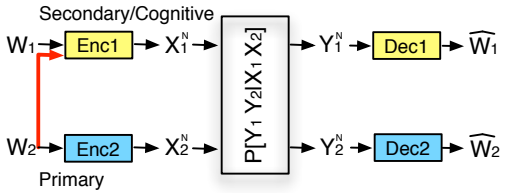
Past Work - inner bounds

the larger, the better



Past Work - inner bounds

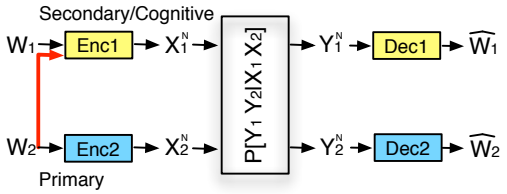
the larger, the better



Maric *et al* ETT08

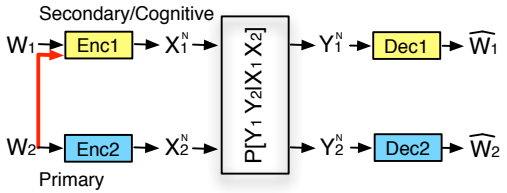
Past Work - inner bounds

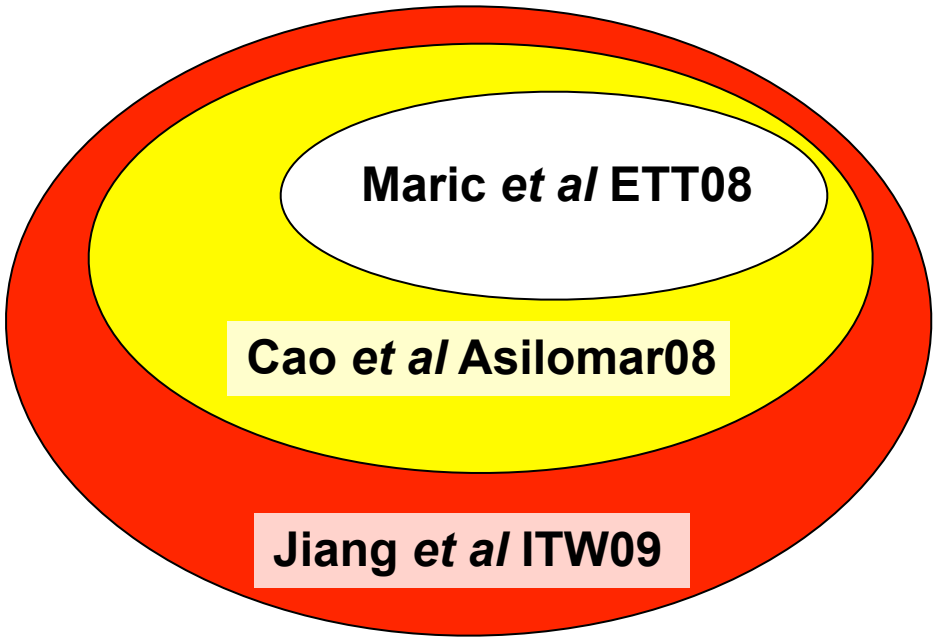
the larger, the better



Past Work - inner bounds

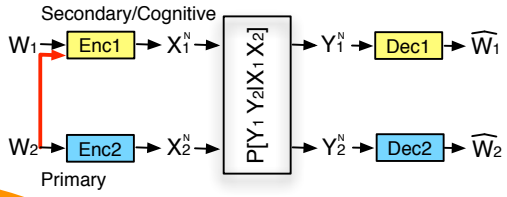
the larger, the better

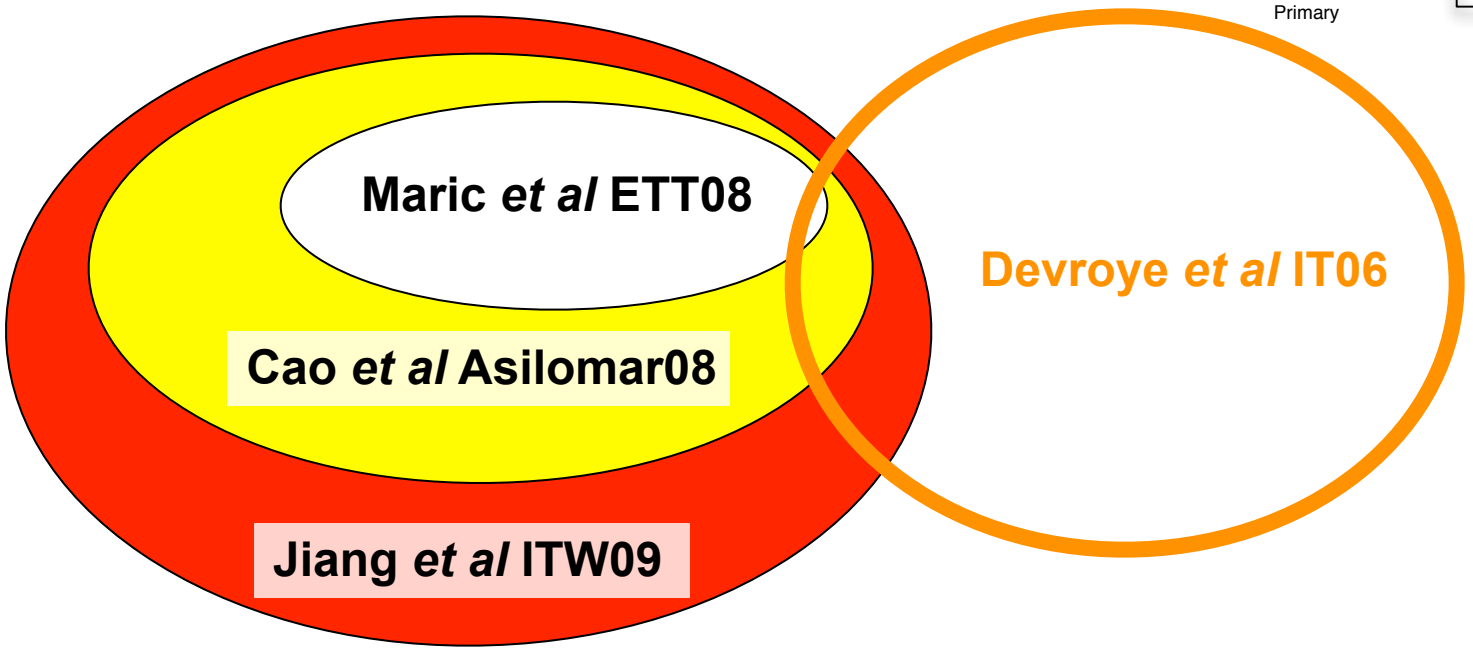




Past Work - inner bounds

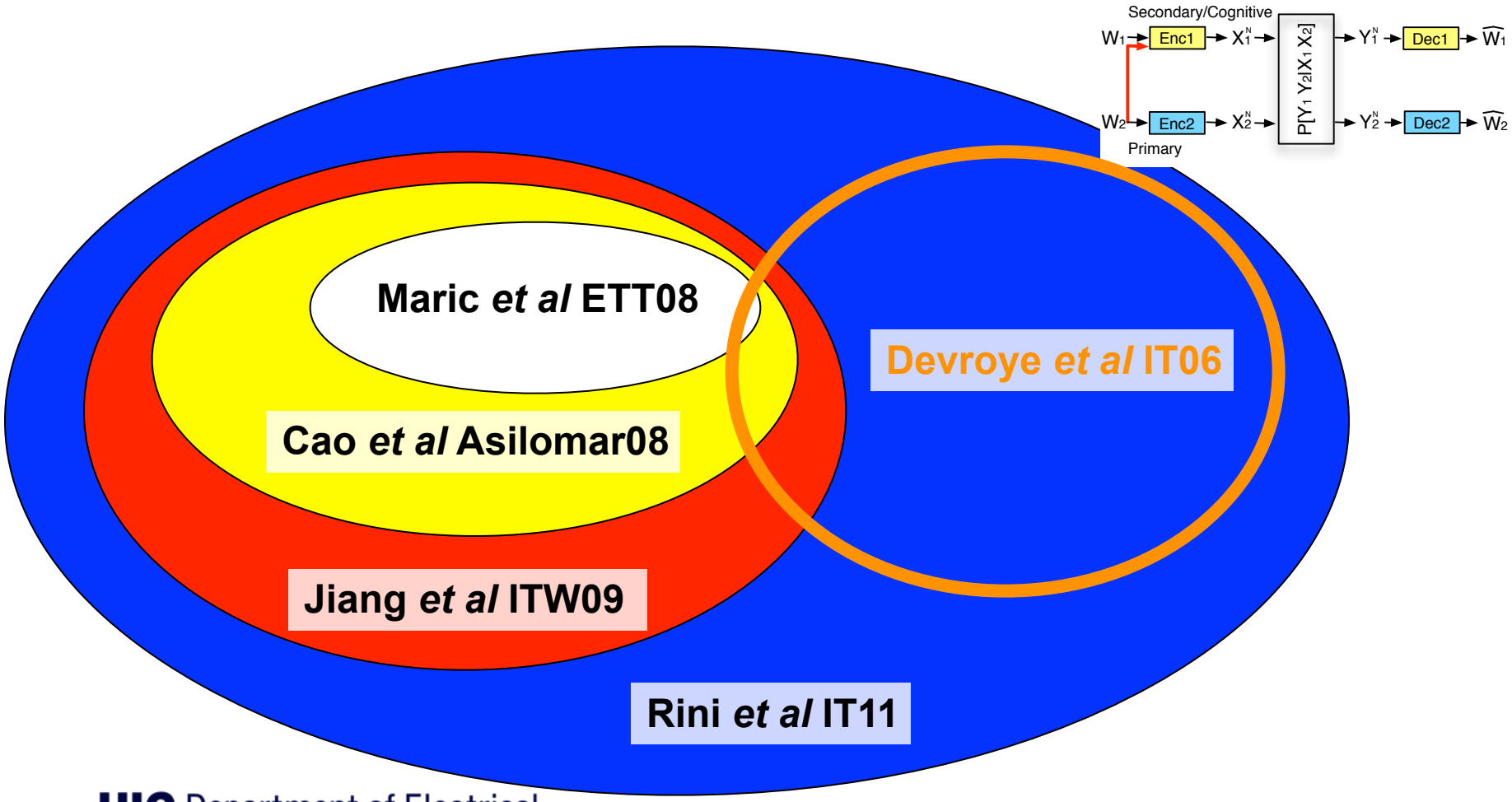
the larger, the better



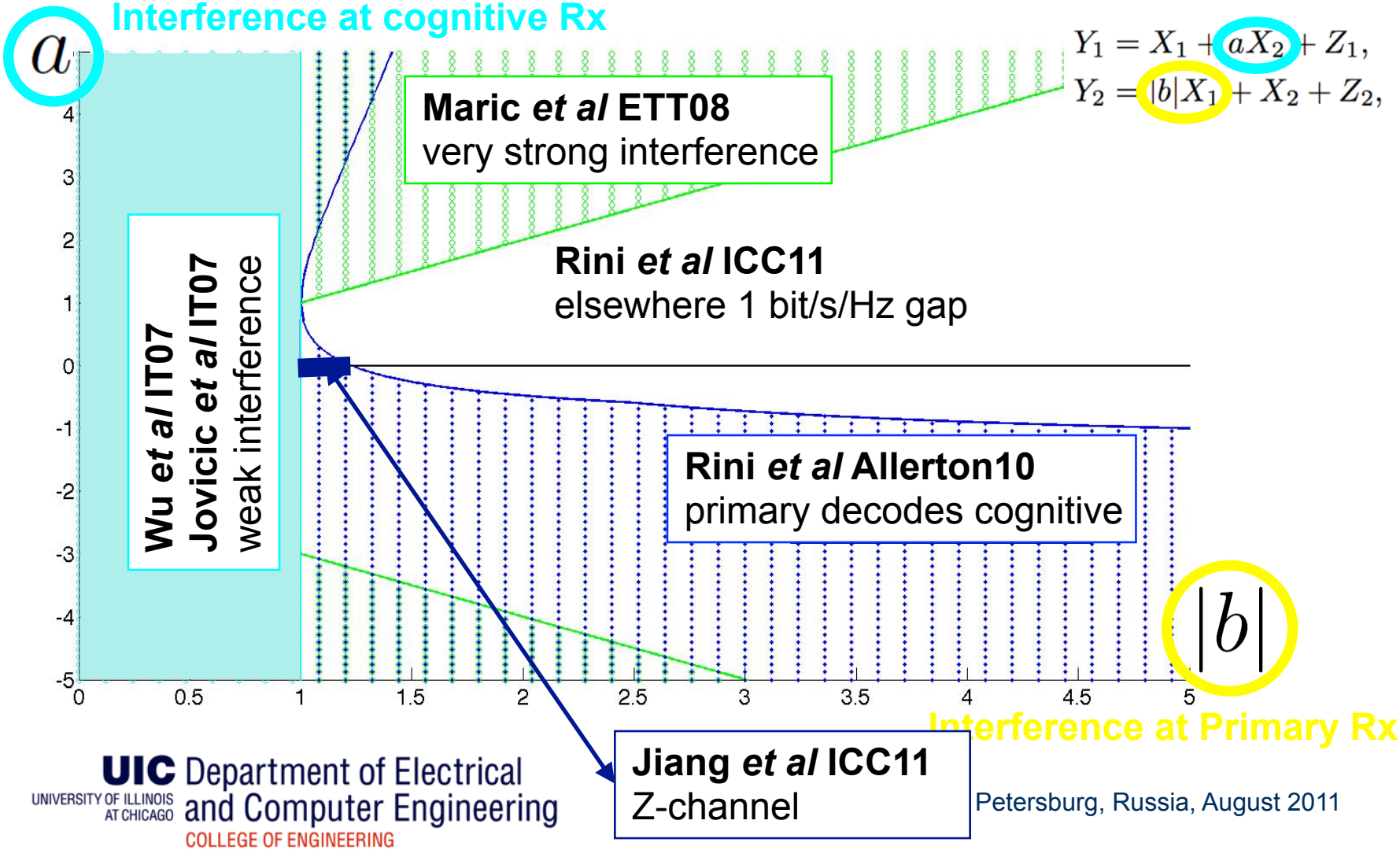


Past Work - inner bounds

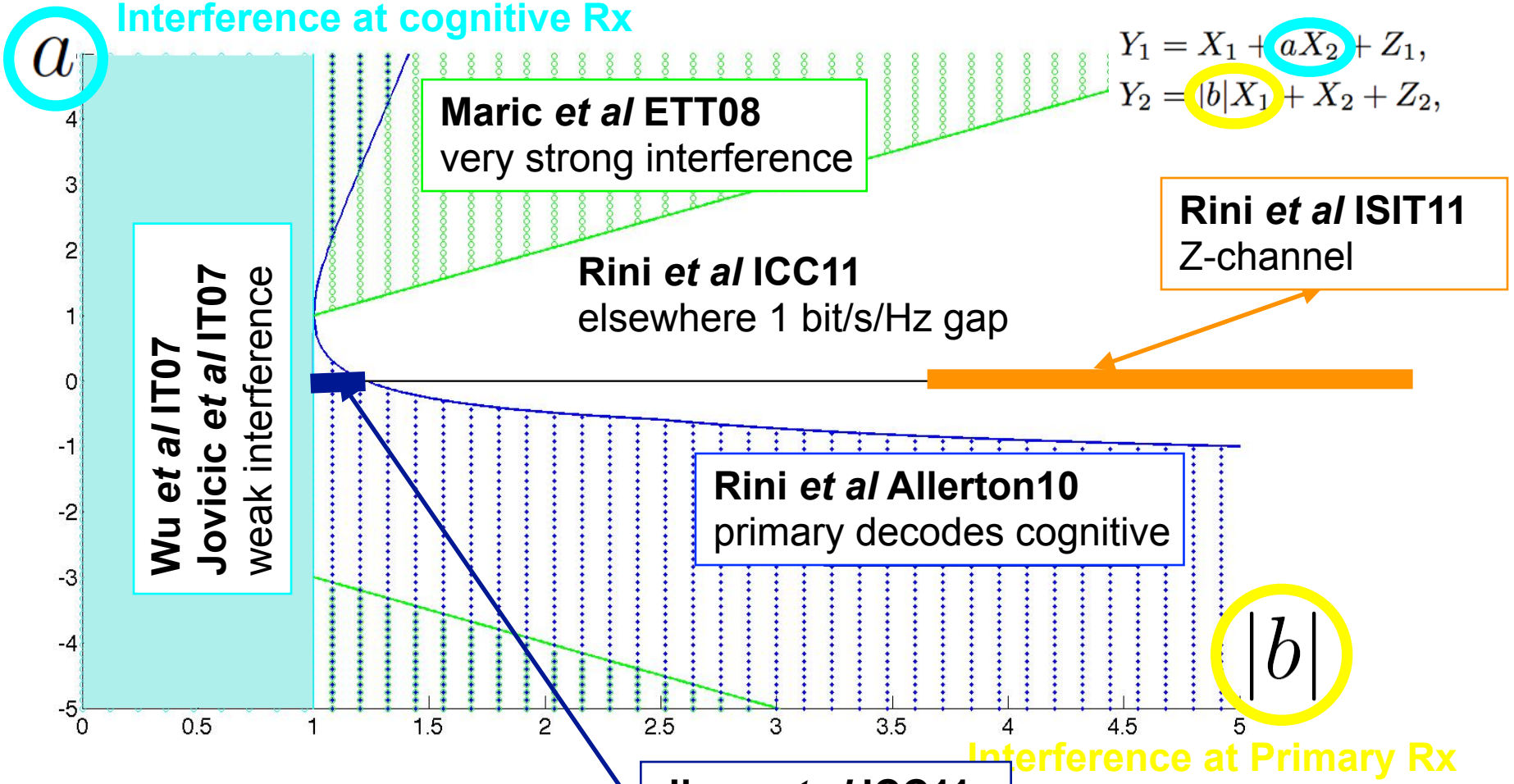
the larger, the better



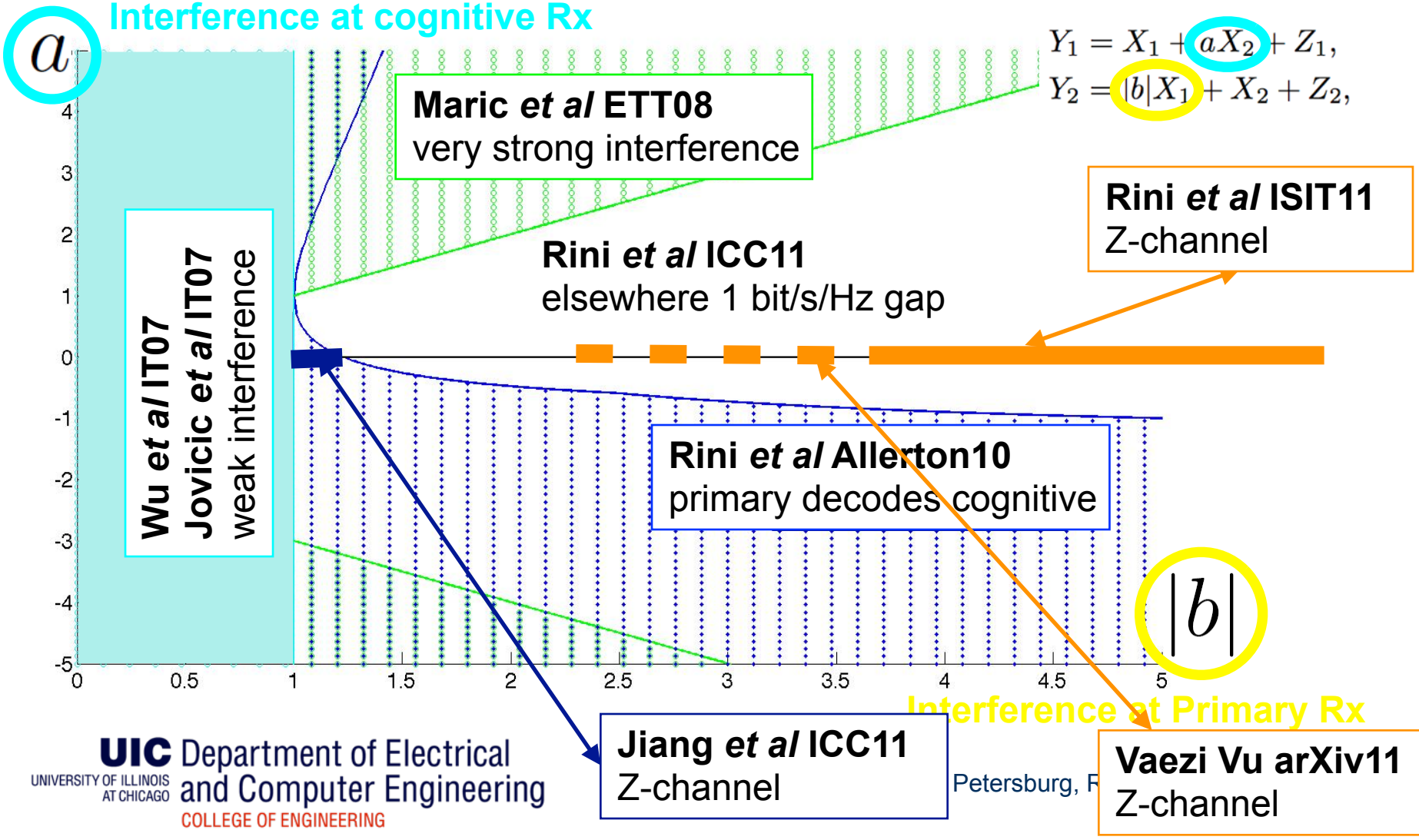
Past Work - AWGN capacity



Past Work - AWGN capacity



Past Work - AWGN capacity



Contributions

1. New strong interference regime outer bound based on G-BC-DMS
2. Capacity for the Z G-CIFC
3. Extensions to non-Z G-CIFC

Outer Bound in Strong Interf.

$$Y_1 = X_1 + aX_2 + Z_1,$$
$$Y_2 = |b|X_1 + X_2 + Z_2,$$

Maric *et al* for |b|>1:

$Y_2 = |b|\, X_1(W_1,W_2) + X_2(W_2) + Z_2$

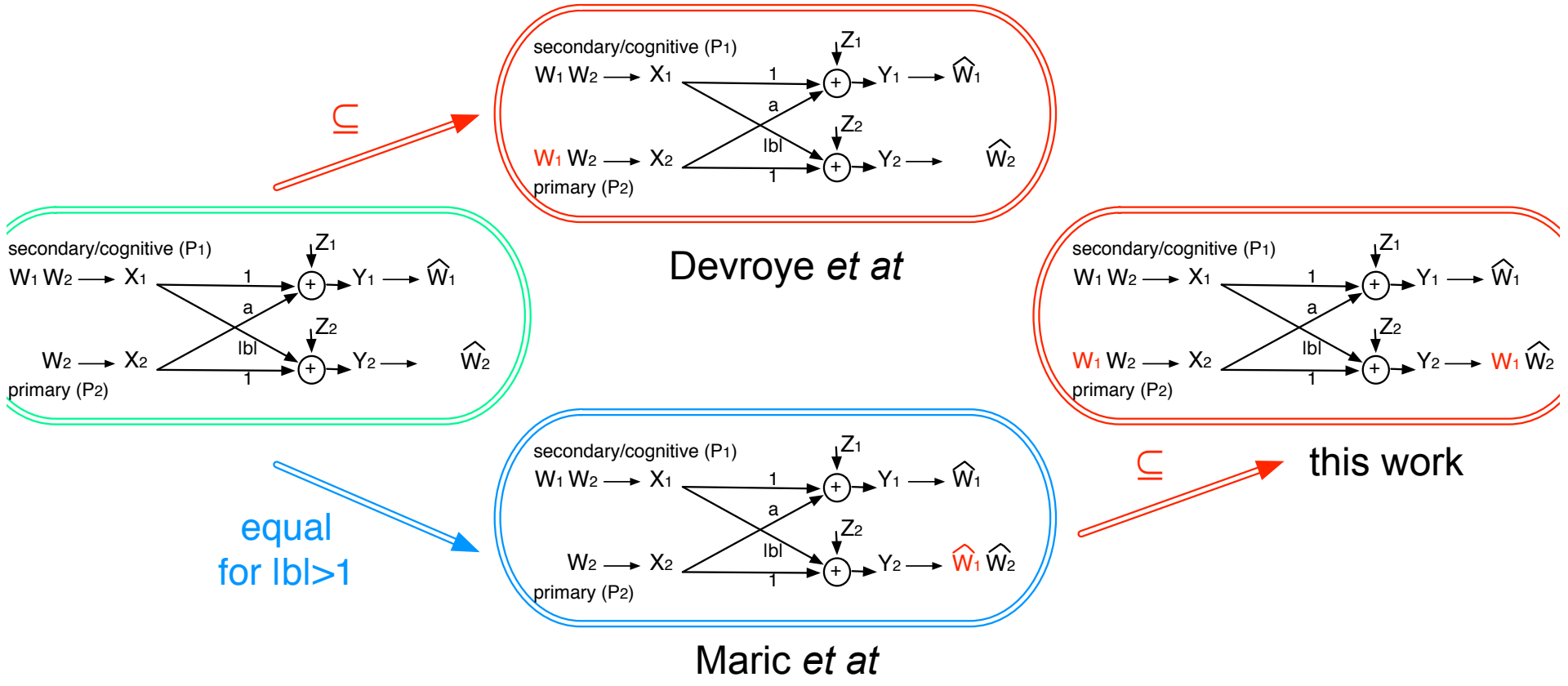
→ W_2 → $X_2(W_2)$

→ $(Y_2\text{-}X_2)/|b| + a\, X_2$

$= X_1(W_1,W_2) + a\, X_2(W_2) + Z_2/|b|$

`less noisy' than Y_1 → W_1

Outer Bound in Strong Interf.



Capacity G-BC-DMS

Capacity G-BC-DMS

1. $C_{general\text{-}BC\text{-}DMS}$ [Korner and Marton IT`77]

Capacity G-BC-DMS

1. $C_{\text{general-BC-DMS}}$ [Korner and Marton IT`77]
2. $C_{\text{MIMO-G-BC-DMS}}$ [Weingarten *et al* ISIT`06] as $C_{\text{MIMO-G-BC-PR}} \cap C_{\text{G-p2p-2msg}}$

Capacity G-BC-DMS

1. $C_{\text{general-BC-DMS}}$ [Korner and Marton IT`77]
2. $C_{\text{MIMO-G-BC-DMS}}$ [Weingarten *et al* ISIT`06] as $C_{\text{MIMO-G-BC-PR}} \cap C_{\text{G-p2p-2msg}}$
3. Here we computed $C_{\text{MISO-G-BC-DMS}}$ with the "extrema inequality" [Liu *et at* IT`07]

Capacity G-BC-DMS

1. $C_{\text{general-BC-DMS}}$ [Korner and Marton IT`77]
2. $C_{\text{MIMO-G-BC-DMS}}$ [Weingarten *et al* ISIT`06] as $C_{\text{MIMO-G-BC-PR}} \cap C_{\text{G-p2p-2msg}}$
3. Here we computed $C_{\text{MISO-G-BC-DMS}}$ with the "extrema inequality" [Liu *et at* IT`07]
4. BC-DMS can be tighter than SI

Capacity G-BC-DMS

1. $C_{\text{general-BC-DMS}}$ [Korner and Marton IT`77]
2. $C_{\text{MIMO-G-BC-DMS}}$ [Weingarten *et al* ISIT`06] as $C_{\text{MIMO-G-BC-PR}} \cap C_{\text{G-p2p-2msg}}$
3. Here we computed $C_{\text{MISO-G-BC-DMS}}$ with the "extrema inequality" [Liu *et at* IT`07]
4. BC-DMS can be tighter than SI
5. We <u>outer bound</u> $C_{\text{MISO-G-BC-DMS}}$ to have one parameter only, i.e., why our result is looser than [Vaezi *et al* arXiv`11].

Capacity for Z G-CIGC

Capacity for Z G-CIGC

1. Achievability uses superposition coding [Korner and Marton IT`77]:

Capacity for Z G-CIGC

1. Achievability uses superposition coding [Korner and Marton IT`77]:
   a. U_i iid $N(0,v_i)$ carries W_i, i=1,2

Capacity for Z G-CIGC

1. Achievability uses superposition coding [Korner and Marton IT`77]:
   a. U_i iid $N(0,v_i)$ carries W_i, $i=1,2$
   b. primary: $X_2(W_2) = U_2 : v_2<P_2$

Capacity for Z G-CIGC

1. Achievability uses superposition coding [Korner and Marton IT`77]:
   a. U_i iid $N(0,v_i)$ carries W_i, $i=1,2$
   b. primary: $X_2(W_2) = U_2 : v_2<P_2$
   c. cognitive: $X_1(W_1,W_2) = U_1 + U_2 : v_1+v_2<P_1$

Capacity for Z G-CIGC

1. Achievability uses superposition coding [Korner and Marton IT`77]:
   a. U_i iid $N(0,v_i)$ carries W_i, $i=1,2$
   b. primary: $X_2(W_2) = U_2 : v_2<P_2$
   c. cognitive: $X_1(W_1,W_2) = U_1 + U_2 : v_1+v_2<P_1$
   d. primary: decodes (U_1,U_2) from Y_2

Capacity for Z G-CIGC

1. Achievability uses superposition coding [Korner and Marton IT`77]:
   a. U_i iid $N(0,v_i)$ carries W_i, $i=1,2$
   b. primary: $X_2(W_2) = U_2 : v_2<P_2$
   c. cognitive: $X_1(W_1,W_2) = U_1 + U_2 : v_1+v_2<P_1$
   d. primary: decodes (U_1,U_2) from Y_2
   e. cognitive: decodes U_1 from Y_1

Capacity for Z G-CIGC - cont.

1. Our outer bound on $C_{\text{MISO-G-BC-DMS}}$ has R_1, R_2, R_1+R_2 bounds.
2. For $\boxed{|b| \geq \sqrt{1+P_2(1+P_1)} + \sqrt{P_1 P_2}}$ the sum-rate bound is redundant and equality of inner and outer bound follows.
3. Had we been more careful in outer bounding $C_{\text{MISO-G-BC-DMS}}$, we could have shown capacity for $\boxed{|b| \geq \sqrt{1+P_2}}$ as in [Vaezi *et al* arXiv`11].

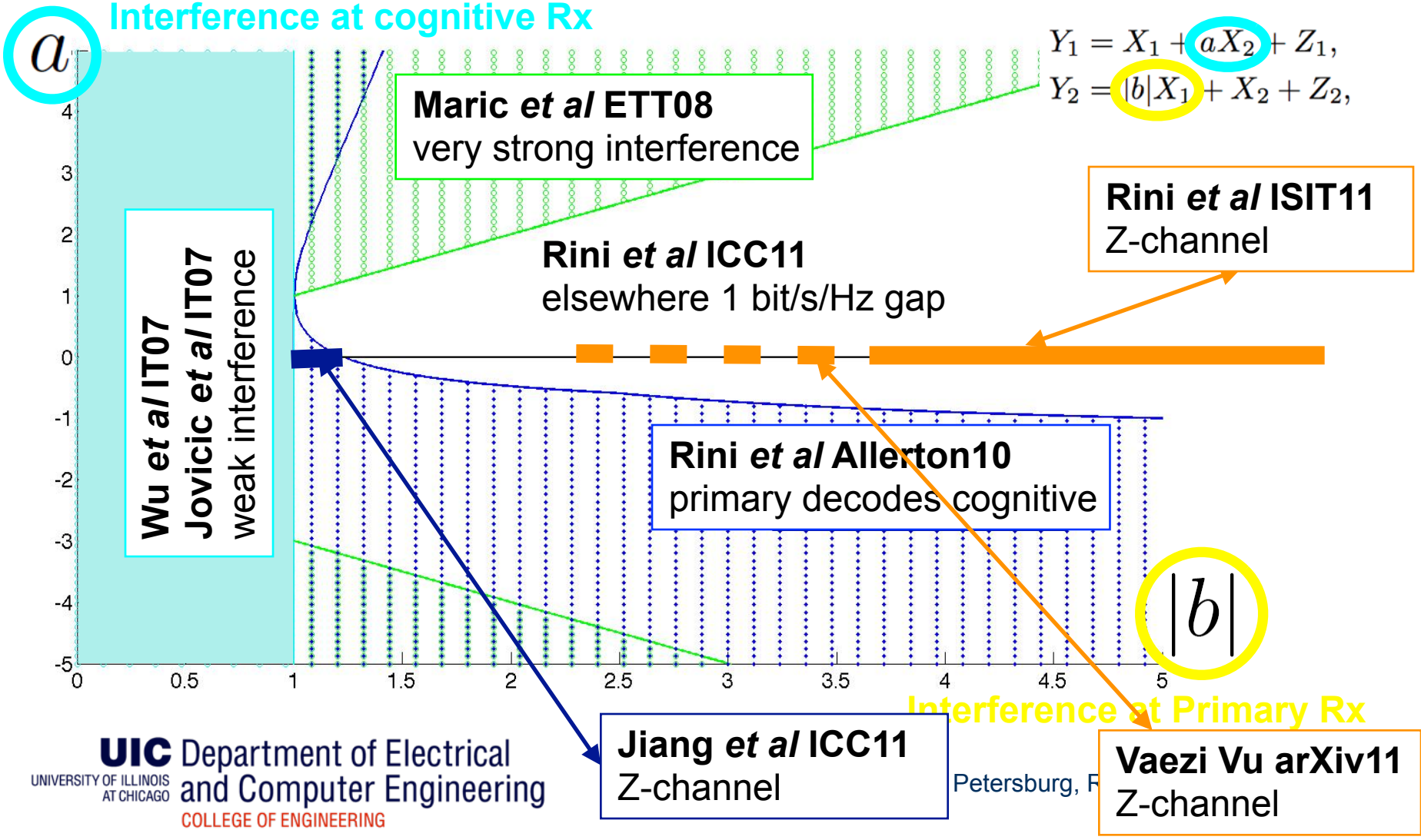
Extension to non-Z G-CIGC

1. Our technique extend to non-Z channels. To prove capacity: must identify the set of MISO-G-BC-DMS for which $X_2(W_2)$ is optimal -- rather than $X_2(W_1,W_2)$.
2. We show in the paper a non-Z channel (with non-zero *a*) for which our G-BC-DMS is tight up to Matlab numerical precision.

Conclusion

1. G-BC-DMS outer bound for CIFC is strong interference
2. Capacity for the Z CIFC for sufficiently strong interference -- can be tightened with less crude bounding
3. Open problem: there are still parameters for which capacity is known to within 1 bit only.

Conclusion


Interference at cognitive Rx
a
$Y_1 = X_1 + aX_2 + Z_1,$
$Y_2 = |b|X_1 + X_2 + Z_2,$
Maric et al ETT08
very strong interference
Rini et al ICC11
elsewhere 1 bit/s/Hz gap
Rini et al ISIT11
Z-channel
Wu et al IT07
Jovicic et al IT07
weak interference
Rini et al Allerton10
primary decodes cognitive
$|b|$
Interference at Primary Rx
Jiang et al ICC11
Z-channel
Vaezi Vu arXiv11
Z-channel
4
3
2
1
0
-1
-2
-3
-4
-5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Details in:

1. IEEE Info.Theory Trans. July 2011:
   a. largest inner bound region for general CIFC (IZS 3/10)
   b. capacity for the *semi-deterministic* CIFC (ICC 6/11)
   c. capacity for the *better cognitive decoding regime* (largest class of memoryless DMCs for which capacity is known)
2. IEEE Info.Theory Trans., revised Aug 2011:
   a. capacity for the G-CIFC in the *primary decodes cognitive regime* (Allerton 9/10)
   b. capacity for the *Z G-CIFC with strong interference* (ISIT 8/11)
   c. capacity to within 1 bits/s/Hz for any G-CIFC (ICC 6/11, improvement on gap of ITW 1/10, based on gDoF in ITW 10/9)