Cognitive Networks Achieve Throughput Scaling of a Homogeneous Network

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***Abstract*—Two distinct, but overlapping, networks that operate at the same time, space, and frequency is considered. The first network consists of n randomly distributed *primary users*, which form an ad hoc network. The second network again consists of m randomly distributed ad hoc *secondary users* or *cognitive users*. The primary users have priority access to the spectrum and do not need to change their communication protocol in the presence of the secondary users. The secondary users, however, need to adjust their protocol based on knowledge about the locations of the primary users to bring little loss to the primary network's throughput. By introducing preservation regions around primary receivers, a modified multihop routing protocol is proposed for the cognitive users. Assuming $m = n^{\beta}$ with $\beta > 1$, it is shown that the secondary network achieves almost the same throughput scaling law as a stand-alone network while the primary network throughput is subject to only a vanishingly small fractional loss. Specifically, the primary network achieves the sum throughput of order $n^{1/2}$ and, for any $\delta > 0$, the secondary network achieves the sum throughput of order $m^{1/2-\delta}$ with an arbitrarily small fraction of outage. Thus, almost all secondary source-destination pairs can communicate at a rate of order $m^{-1/2-\delta}$.**

***Index Terms*—Cognitive radio, heterogeneous networks, interference management, routing algorithm, scaling law.**

I. Introduction

IN their pioneering work [1], Gupta and Kumar posed and studied the limits of communication in ad hoc wireless networks. Assuming n nodes are uniformly distributed in a plane and grouped into source-destination (S-D) pairs at random, they showed that one can achieve the sum throughput of $\Omega(\sqrt{n/\log n})$. This is achieved using a multihop transmission scheme in which nodes transmit to one of the nodes in their neighboring cells, requiring full connectivity with at least one node per cell. A trade-off between throughput and delay of fully-connected networks was studied in [2] and was extended in [3] to trade-offs between throughput, delay as well as energy.

The work in [4] has studied relay networks in which a single source transmits its data to the intended destination using the other nodes as relays. Using percolation theory [5], [6], they showed that a constant rate is achievable for a single S-D pair if we allow a small fraction of nodes to be disconnected. This result can be applied to ad hoc networks having multiple S-D pairs and the work in [7] proposed a new multihop routing protocol based on such partial connectivity, that is all S-D pairs perform multihop transmissions based on this partially-connected sub-network. They showed that the proposed multihop routing improves the sum throughput as $\Omega(\sqrt{n})$.

Information-theoretic upper bounds on throughput scaling laws were derived in [8]–[11]. It was shown that the multihop routing using neighbor nodes achieves a throughput scaling close to its upper bound in the power-limited and high attenuation regime. Recently, a hierarchical cooperation scheme was proposed in [12] and was shown to achieve better throughput scaling than the multihop strategy in the interference-limited or low attenuation regime, achieving a scaling very close to their new upper bound. A more general hierarchical cooperation was proposed in [13], which works for an arbitrary node distribution in which a minimum separation between nodes is guaranteed.

The existing literatures have focused on throughput scaling laws of a *single* network. However, the necessity of extending and expanding results to capture *multiple* overlapping networks is becoming apparent. Recent measurements have shown that despite increasing demands for bandwidth, much of the currently licensed spectrum remains unused a surprisingly large portion of the time [14]. In the US, this has led the Federal Communications Commission (FCC) to consider easing the regulations towards *secondary spectrum sharing* through their *Secondary Markets Initiative* [15]. The essence of secondary spectrum sharing involves having primary license holders allow secondary license holders to access the spectrum. Different types of spectrum sharing exist but most agree that the primary users have a higher priority access to the spectrum, while secondary users *opportunistically* use it. These secondary users often require greater sensing abilities and more flexible and diverse communication abilities than legacy primary users. Secondary users are often assumed to be *cognitive radios*, or wireless devices which are able to transmit and receive according to a variety of protocols and are also able to sense and independently

Manuscript received February 27, 2008; revised March 15, 2010; accepted January 31, 2011. Date of current version July 29, 2011. S.-W. Jeon and S.-Y. Chung were supported by the MKE, Korea, under the ITRC support program supervised by the NIPA (NIPA-2010-(C1090-1011-0005)). N. Devroye was supported in part by the NSF under award 1017436. The material in this paper was presented in part at the WiOpt 2009, Seoul, South Korea, June 2009.

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Communicated by S. Ulukus, Associate Editor for Communication Networks.

Digital Object Identifier 10.1109/TIT.2011.2158874

adapt to their environment [16]. These features allow them to behave in a more "intelligent" manner than current wireless devices.

In this paper, we consider *cognitive networks*, which consist of secondary, or cognitive, users who wish to transmit over the spectrum licensed to the primary users. The single-user case in which a single primary and a single cognitive S-D pairs share the spectrum has been considered in the literature, see for example [17]–[20] and the references therein. In [17] the primary and cognitive S-D pairs are modeled as the interference channel with asymmetric side-information. In [19] the communication opportunities are modeled as a two-switch channel. Recently, a single-hop cognitive network was considered in [21], where multiple secondary S-D pairs transmit in the presence of a single primary S-D pair. It was shown that a linear scaling law of the single-hop secondary network is obtained when its operation is constrained to guarantee a particular outage constraint for the primary S-D pair.

We study a more general environment in which a *primary ad hoc network* and a *cognitive ad hoc network* both share the same space, time and frequency dimensions. The primary network consists of n nodes randomly distributed and grouped into S-D pairs at random. The cognitive network consists of m secondary nodes distributed randomly and S-D pairs are again chosen randomly. Our main assumptions are that (1) the primary network continues to operate as if no secondary network were present, (2) the secondary nodes know the locations of the primary nodes, and (3) the secondary network is denser than the primary network. Under these assumptions, we will illustrate routing protocols for the primary and secondary networks that result in *almost the same throughput scaling* as if each were a single network. Note that the constraint that the primary network does *not* alter its protocol because of the secondary network is what makes the problem nontrivial. Indeed, if the primary network were to change its protocol when the secondary network is present, a simple time-sharing scheme is able to achieve the throughput scaling of homogeneous networks for both primary and secondary networks.

For the primary network, we use a routing protocol that is a simple modification of the nearest neighbor multihop schemes in [1], [7]. We propose a novel multihop routing protocol for the secondary network, in which the secondary routes *avoid* passing too close to the primary nodes, reducing the interference to them. We show that when a denser "intelligent" network is layered on top of a sparser oblivious one, then both networks achieve almost the same throughput scalings *simultaneously* as if each were a single network. Specifically, the primary network achieves the sum throughput of $\Omega(n^{1/2})$ in the presence of the secondary network and, for any $\delta > 0$, the secondary network achieves the sum throughput of $\Omega(m^{1/2-\delta})$ with an arbitrarily small fraction of outage in the presence of the primary network. This result may be extended to more than two networks, provided each layered network obeys the same three main assumptions as in the two network case.

This paper is structured as follows. In Section II we outline the system model: we first look at the network geometry, co-existing primary and secondary ad hoc networks, then turn to the information-theoretic achievable rates before stating our assumptions on the primary and secondary network behaviors. In Section III we explain the protocols used for the primary and secondary networks and prove that the claimed throughput scalings may be achieved. In Section IV we briefly discuss the information-theoretic upper bounds and the possibility of alternative routing methods. We conclude in Section V and refer the proofs of the lemmas to the Appendix.

II. System Model

In order to study throughput scaling laws of ad hoc cognitive networks, we must define an underlying network model. We first explain our geometric model, which will be considered in Section III. We then look at the transmission schemes, resulting achievable rates, and assumptions made about the primary and secondary networks.

Throughout this paper, we use $\mathbb{P}(E)$ to denote the probability of an event E and we will be dealing with events which take place with high probability (w.h.p.), or with probability one as the node density tends to infinity.[1] Let $f(n) \geq 0$ and $g(n) \geq 0$ be two functions defined on some subset of the real numbers. We will also use the following order notations [22].

- $f(n) = \mathrm{O}(g(n))$ if there exist $c > 0$ and $n_0 > 0$ such that $f(n) \leq cg(n)$ for all $n \geq n_0$.
- $f(n) = \Omega(g(n))$ if $g(n) = \mathrm{O}(f(n))$.
- $f(n) = \Theta(g(n))$ if $f(n) = \mathrm{O}(g(n))$ and $g(n) = \mathrm{O}(f(n))$.

A. Network Geometry

We consider a planar area in which a network of primary nodes and a network of secondary nodes co-exist. That is, the two networks share the same space, time, code, and frequency dimensions. Fig. 1 illustrates the network model. For the primary network, nodes are distributed according to a Poisson point process (p.p.p.) of density n over a unit square, which are randomly grouped into primary S-D pairs. For the secondary network, nodes are distributed according to a p.p.p. of density m over the same unit square and are also randomly grouped into secondary S-D pairs. The densities of the n primary nodes and m secondary nodes are related according to

$$m = n^{\beta}. \tag{1}$$

Our study is limited to the case where $\beta > 1$, that is the density of the secondary nodes is higher than that of the primary nodes.

The wireless propagation channel typically includes path-loss with distance, shadowing and fading effects. However, in this work we assume the channel gain depends only on the distance between a transmitter (Tx) and its receiver (Rx), and ignore shadowing and fading. Thus, the channel power gain $g(d)$, normalized by a constant, is given by

$$g(d) = d^{-\alpha} \tag{2}$$

where d denotes the distance between a Tx and its Rx and $\alpha > 2$ denotes the path-loss exponent.

[1] For simplicity, we use the notation 'w.h.p.' in the paper to mean an event occurs with high probability as the node densities tend to infinity.

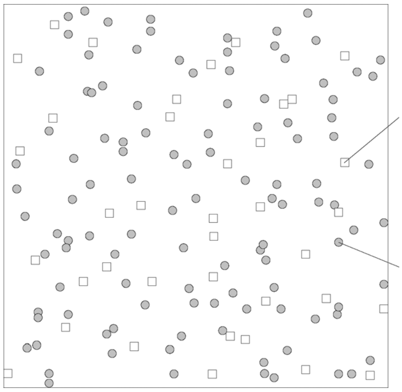

Fig. 1. We consider the geometric network model: the primary nodes as well as the secondary nodes form distinct and co-existing ad hoc networks.

B. Rates and Throughputs Achieved

Each network operates based on slotted transmissions. We assume that the duration of each slot and the coding scheme employed are such that one can achieve the additive white Gaussian noise (AWGN) channel capacity. For a given signal to interference and noise ratio (SINR), this capacity is given by the well known formula $R = \log(1 + \text{SINR})$ bps/Hz assuming the additive interference is also white, Gaussian, and independent from the noise and signal. We assume that primary slots and secondary slots have the same duration and are synchronized with each other. We further assume that all primary and secondary nodes are subject to a transmit power constraint P.

We now characterize the rates achieved by primary and secondary transmission pairs. Suppose that N_p primary pairs and N_s secondary pairs communicate simultaneously. Before proceeding with a detailed description, let us define the notations used in the paper, given by Table I. Then, the ith primary pair can communicate at a rate of

$$R_p^i = \log\left(1 + \frac{P_p^i g\left(\left\|X_{p,\text{tx}}^i - X_{p,\text{rx}}^i\right\|\right)}{N_0 + I_p^i + I_{sp}^i}\right) \tag{3}$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector. Here, I_p^i and I_{sp}^i are given by

$$I_p^i = \sum_{k=1, k\neq i}^{N_p} P_p^k g\left(\left\|X_{p,\text{tx}}^k - X_{p,\text{rx}}^i\right\|\right) \tag{4}$$

and

$$I_{sp}^i = \sum_{k=1}^{N_s} P_s^k g\left(\left\|X_{s,\text{tx}}^k - X_{p,\text{rx}}^i\right\|\right). \tag{5}$$

Similarly, the jth secondary pair can communicate at a rate of

$$R_s^j = \log\left(1 + \frac{P_s^j g\left(\left\|X_{s,\text{tx}}^j - X_{s,\text{rx}}^j\right\|\right)}{N_0 + I_s^j + I_{ps}^j}\right) \tag{6}$$

where I_s^j and I_{ps}^j are given by

$$I_s^j = \sum_{k=1, k\neq j}^{N_s} P_s^k g\left(\left\|X_{s,\text{tx}}^k - X_{s,\text{rx}}^j\right\|\right) \tag{7}$$

and

$$I_{ps}^j = \sum_{k=1}^{N_p} P_p^k g\left(\left\|X_{p,\text{tx}}^k - X_{s,\text{rx}}^j\right\|\right). \tag{8}$$

TABLE I
DEFINITION OF SYMBOLS RELATED TO ACHIEVABLE RATES FOR EACH PRIMARY AND SECONDARY TRANSMIT PAIR

P_p^i	Transmit power of the ith primary pair
P_s^j	Transmit power of the jth secondary pair
N_0	Thermal noise power
$X_{p,\text{tx}}^i$	Tx location of the ith primary pair
$X_{p,\text{rx}}^i$	Rx location of the ith primary pair
$X_{s,\text{tx}}^j$	Tx location of the jth secondary pair
$X_{s,\text{rx}}^j$	Rx location of the jth secondary pair
I_p^i	Interference power from the primary Txs to the Rx of the ith primary pair
I_{sp}^i	Interference power from the secondary Txs to the Rx of the ith primary pair
I_s^j	Interference power from the secondary Txs to the Rx of the jth secondary pair
I_{ps}^j	Interference power from the primary Txs to the Rx of the jth secondary pair
R_p^i	Rate of the ith primary pair
R_s^j	Rate of the jth secondary pair

Throughout the paper, the achievable per-node throughput of the primary and secondary networks are defined as follows.

Definition 1: A throughput of $T_p(n)$ per primary node is said to be achievable w.h.p. if all primary sources can transmit at a rate of $T_p(n)$ (bps/Hz) to their primary destinations w.h.p. in the presence of the secondary network.

Definition 2: Let $\epsilon_s(m) > 0$ denote the outage fraction of the secondary network, which may vary as a function of m. A throughput of $T_s(m)$ per secondary node is said to be $\epsilon_s(m)$-achievable w.h.p. if at least $1 - \epsilon_s(m)$ fraction of secondary sources can transmit at a rate of $T_s(m)$ (bps/Hz) to their secondary destinations w.h.p. in the presence of the primary network.

In Section III, we will propose a secondary routing scheme that makes $\epsilon_s(m) \to 0$ as $m \to \infty$.[2] Thus, although we allow a fraction of secondary S-D pairs to be in outage, almost all secondary S-D pairs will be served at a rate of $T_s(m)$ for sufficiently large m. Let us define $S_p(n)$ as the sum throughput of the primary network, or $T_p(n)$ times the number of primary S-D pairs.[3] Similarly, we define $S_s(m)$ as the sum throughput of the secondary network, or $T_s(m)$ times the number of *served* secondary S-D pairs at a rate of $T_s(m)$. While $T_p(n)$ and $S_p(n)$ represent the per-node and sum throughputs of the primary network *in the presence of the secondary network*, we use the notations $T_{\text{alone}}(n)$ and $S_{\text{alone}}(n)$ to denote the per-node and sum throughputs of the primary network *in the absence of the secondary network*, respectively.

C. Primary and Secondary User Behaviors

As primary and secondary nodes must share the spectrum, the rules or assumptions made about this co-existence are of critical importance to the resulting achievable throughputs and

[2]In this paper, $m \to \infty$ is equivalent to $n \to \infty$ since $m = n^\beta$.

[3]We note that, in general, $S_p(n) \neq \frac{n}{2}T_p(n)$ since the nodes are thrown at random according to a p.p.p. of density n. The actual number of nodes in the network will vary in a particular realization.

scaling laws. Primary networks may be thought of as existing communication systems that operate in licensed bands. These primary users are the license holders, and thus have higher priority access to the spectrum than secondary users. Thus, our first key assumption is that *the primary network does not have to change its protocol due to the secondary network*. In other words, all primary S-D pairs communicate with each other as intended, regardless of the secondary network. The secondary network, which is opportunistic in nature, is responsible for reducing its interference to the primary network to an "acceptable level", while maximizing its own throughput $T_s(m)$. This acceptable level may be defined to be one that does not degrade the throughput scaling of the primary network. More strictly, the secondary network should satisfy w.h.p.

$$\frac{T_p(n)}{T_{\mathrm{alone}}(n)} \geq 1 - \delta_{\mathrm{loss}} \tag{9}$$

during its transmission, where $\delta_{\mathrm{loss}} \in (0,1)$ is the maximum allowable fraction of throughput loss for the primary network. Notice that the above condition guarantees $T_p(n) = \Theta(T_{\mathrm{alone}}(n))$. The secondary network may ensure (9) by adjusting its protocol based on information about the primary network. Thus, our second key assumption is that *the secondary network knows the locations of all primary nodes*. Since the secondary network is denser than the primary network, each secondary node can measure the interference power from its adjacent primary node and send it to a coordinator node. Based on these measured values, the secondary network can establish the locations of the primary nodes.

III. Achievable Throughput Scaling Laws

Since the primary network needs not change its transmission scheme due to the presence of the secondary network, we assume it transmits according to the multihop routing similar to those in [1], [2], which we call Gupta-Kumar (GK) routing. As a primary protocol, we also consider the multihop routing proposed in [7], which we call Franceschetti-Dousse-Tse-Thiran (FDTT) routing. Of greater interest is how the secondary nodes will transmit such that the primary network remains unaffected in terms of throughput scaling.

A. Main Results

The main results of this paper describe achievable throughput scaling laws of the primary and secondary networks. We simply state these results here and derive them in the remainder of this section.

For any $\delta_{\mathrm{loss}} \in (0,1)$, the primary network can achieve the following per-node and sum throughputs w.h.p.

$$T_p(n) = (1-\delta_{\mathrm{loss}})T_{\mathrm{alone}}(n) \tag{10}$$

and

$$S_p(n) = (1-\delta_{\mathrm{loss}})S_{\mathrm{alone}}(n) \tag{11}$$

where

$$T_{\mathrm{alone}}(n) = \begin{cases} \Omega\left(\frac{1}{\sqrt{n\log n}}\right), & \text{for } GK \text{ routing} \\ \Omega\left(\frac{1}{\sqrt{n}}\right), & \text{for FDTT routing} \end{cases} \tag{12}$$

and $S_{\mathrm{alone}}(n) = \Omega(nT_{\mathrm{alone}}(n))$. The following per-node and sum throughputs are $\epsilon_s(m)$-achievable w.h.p. for the secondary network:

$$T_s(m) = \begin{cases} \Omega\left(\frac{1}{\sqrt{m\log m}}\right), & \text{if } \beta > 2 \\ \Omega\left(\frac{1}{\sqrt{m}(\log m)^{3/2}}\right), & \text{if } 1 < \beta \leq 2 \end{cases} \tag{13}$$

and

$$S_s(m) = \Omega(mT_s(m)) \tag{14}$$

where

$$\epsilon_s(m) = \begin{cases} \mathrm{O}\left(\frac{\sqrt{\log m}}{m^{1/2-1/\beta}}\right), & \text{if } \beta > 2 \\ \mathrm{O}\left(\frac{(\log m)^2}{m^{1-1/\beta}}\right), & \text{if } 1 < \beta \leq 2 \end{cases} \tag{15}$$

which converges to zero as $m \to \infty$.

This result is of particular interest as it shows that the primary network can not only operate at the same scaling law as if the secondary network were absent, but the secondary network can also achieve almost the same scaling law obtained by the multihop routing as if the primary network did not exist. That is, compared to the stand-alone sum throughput of $\Omega(m^{1/2})$, the secondary network achieves $\Omega(m^{1/2-\delta})$ for any $\delta > 0$ with an arbitrarily small fraction of outage. Thus, almost all secondary S-D pairs can communicate at a rate of $\Omega(m^{-1/2-\delta})$ in the limit of large m.

In the remainder of this section, we first outline the operation of the primary network and then focus on the design of a secondary network protocol under the given primary protocol. We analyze achievable throughputs of the primary and secondary networks, which will determine the throughput scaling of both co-existing networks. Throughout this work, we place the proofs of technical lemmas in the Appendix and show the main proofs in the text.

B. Network Protocols

We assume the primary network communicates according to the GK routing protocol. We will explain the FDTT routing in Section IV-B, which can be extended from the results of the GK routing. The challenge is thus to prove that the secondary nodes can exchange information in such a way that satisfies $T_p(n) \geq (1-\delta_{\mathrm{loss}})T_{\mathrm{alone}}(n)$ w.h.p. We first outline a primary network protocol, and then design a secondary network protocol which operates in the presence of the primary network.

1) Primary Network Protocol: We assume that the primary network delivers data using the multihop routing, in a manner similar to [1], [2]. The basic multihop protocol is as follows:

- Divide the unit area into square cells of area a.
- A 9-time division multiple access (TDMA) scheme is used, in which each cell is activated during one out of 9 slots.
- Define the horizontal data path (HDP) and the vertical data path (VDP) of a S-D pair as the horizontal line and the vertical line connecting a source to its destination, respectively. Each source transmits data to its destination by first hopping to the adjacent cells on its HDP and then on its VDP.

- When a cell becomes active, it delivers its traffic. Specifically, a Tx node in the active cell transmits a packet to a node in an adjacent cell (or in the same cell). A simple round-robin scheme is used for all Tx nodes in the same cell.
- At each transmission, a Tx node transmits with power Pd^{α}, where d denotes the distance between the Tx and its Rx.

This protocol requires full connectivity, meaning that each cell should have at least one node. Let a_p denote the area of a primary cell. The following lemma indicates how to determine a_p satisfying this requirement.

Lemma 1: The following facts hold:

a) The number of primary nodes in a unit area is within $((1-\epsilon)n, (1+\epsilon)n)$ w.h.p., where $\epsilon > 0$ is an arbitrarily small constant.

b) Suppose $a_p = \frac{2\log n}{n}$. Then, each primary cell has at least one primary node w.h.p.

Proof: The proof is in the Appendix. ■

Based on Lemma 1, we set $a_p = \frac{2\log n}{n}$. Under the given primary protocol, $T_{\text{alone}}(n) = \Omega(1/\sqrt{n\log n})$ and $S_{\text{alone}}(n) = \Omega(\sqrt{n/\log n})$ are achievable w.h.p. when the secondary network is absent or silent.

Results similar to Lemma 1 can be found in [1], [2], where their proposed schemes also achieve the same $T_{\text{alone}}(n)$ and $S_{\text{alone}}(n)$. Note that the Gupta-Kumar's model [1], [2] assumes uniformly distributed nodes in the network and a constant rate between Tx and Rx if SINR is higher than a certain level. Although we assume that the network is constructed according to a p.p.p. (rather than uniform) and that the information-theoretic rate $\log(1+\text{SINR})$ is achievable (rather than a constant rate), the above primary network protocol provides the same throughput scaling as that under the Gupta-Kumar's model.

2) Secondary Network Protocol: Since the secondary nodes know the primary nodes' locations, an intuitive idea is to have the secondary network operate in a multihop fashion in which they circumvent each primary node in order to reduce the effect of secondary transmissions to the primary nodes.

Around each primary node we define its *preservation region:* a square containing 9 secondary cells, with the primary node at the center cell. The secondary nodes, when determining their routing paths, need to avoid these preservation regions. Our protocol for the secondary ad hoc network is the same as the basic multihop protocol except that

- The secondary cell size is $a_s = \frac{2\log m}{m}$.
- At each transmission a secondary node transmits its packet *three* times repeatedly (rather than once) using three slots.
- The secondary paths avoid the preservation regions (see Fig. 2). That is, if the HDP or VDP of a secondary S-D pair is blocked by a preservation region, this data path circumvents the preservation region by using its adjacent cells. If a secondary source (or its destination) belongs to preservation regions or its data path is disconnected by preservation regions, the corresponding S-D pair is not served.
- At each transmission, a Tx node transmits with power $\delta_P P d^{\alpha}$, where d denotes the distance between the Tx and its Rx and $\delta_P \in (0,1]$.

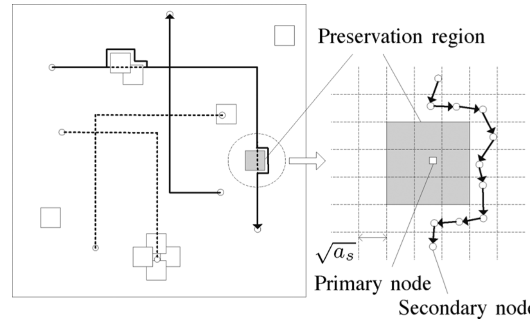

Fig. 2. Examples of secondary data paths: a secondary S-D pair goes around if it is blocked by a preservation region. If a source (or its destination) is in a preservation region or its data path is disconnected by preservation regions, the corresponding S-D pair is not served.

Since a_s converges to zero as $m \to \infty$, there exists m_0 such that the power constraint is satisfied for any $\delta_P \in (0,1]$ if $m \geq m_0$. We will show in Lemma 2 that adjusting δ_P induces a trade-off between the rates of the primary and secondary networks that satisfies the condition (9) while the scaling laws of both networks are unchanged.

Unlike the primary protocol, each secondary cell transmits a secondary packet three times repeatedly when it is activated. As we will show later, the repeated secondary transmissions can guarantee the secondary receivers a certain minimum distance from all primary interferers for at least one packet, thus guaranteeing the secondary network a nontrivial rate. Therefore, the duration of the secondary 9-TDMA scheme is three times longer than that of the primary 9-TDMA. The main difference between this scheme and the previous multihop routing schemes is that the secondary multihop paths must circumvent the preservation regions and that a portion of secondary S-D pairs is not served. But this portion will be negligible as $m \to \infty$. By re-routing the secondary nodes' transmission around the primary nodes' preservation regions, we can guarantee the primary nodes a nontrivial rate.

Similar to Lemma 1, we can also prove that the total number of secondary nodes is within $((1-\epsilon)m, (1+\epsilon)m)$ w.h.p. and that each secondary cell has at least one secondary node w.h.p.

C. *Throughput Analysis*

In this subsection, we analyze the per-node and sum throughputs of each network under the given protocols and derive throughput scaling laws with respect to the node densities.

1) Primary Network Throughputs: Let us consider the primary network in the presence of the secondary network. We first show that each primary cell can sustain a constant aggregate rate (Lemma 2), which may be used in conjunction with the number of data paths each primary cell must transmit (Lemma 3) to obtain the per-node and sum throughputs in Theorem 1.

Let $R_p(n)$ and $R_{\text{alone}}(n)$ denote the achievable aggregate rate of each primary cell in the presence and in the absence of the secondary network, respectively. We define

$$I = P2^{\alpha/2+3}\sum_{t=1}^{\infty} t(3t-2)^{-\alpha} \tag{16}$$

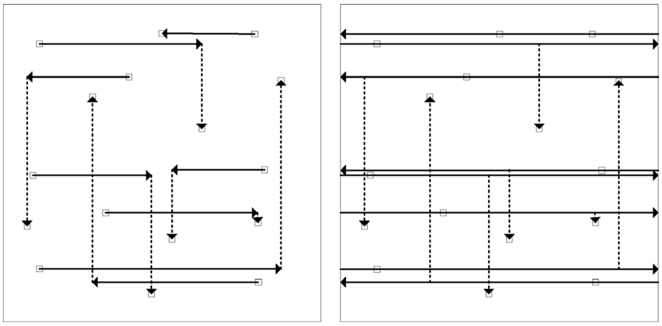

Fig. 3. Examples of original HDPs (left) and their extended HDPs (right) of the primary S-D pairs.

having a finite value for $\alpha > 2$, which will be used to derive an upper bound on the interference power of the primary and secondary networks. Then the following lemma holds.

Lemma 2: If $\delta_P \in (0, \min\{\delta_{P,\max}, 1\}]$, then

$$\lim_{n\to\infty} \frac{R_p(n)}{R_{\text{alone}}(n)} \geq 1 - \delta_{\text{loss}} \tag{17}$$

where $\delta_{P,\max} = \frac{P}{I}((1 + P/(N_0 + x_{\min}))^{1-\delta_{\text{loss}}} - 1)^{-1} - \frac{N_0+x_{\min}}{I} > 0$ and I is given by (16). Here $x_{\min}$ is a solution of $\min_{x\in[0,I]} \log(1 + \frac{P}{N_0+x+\delta_P I})/\log(1 + \frac{P}{N_0+x})$. Moreover, $R_{\text{alone}}(n)$ is lower bounded by K_p, where $K_p = \frac{1}{9}\log(1 + \frac{P}{N_0+I})$ is a constant independent of n.

Proof: The proof is in the Appendix. ■

The essence of the proof of Lemma 2 lies in showing that the secondary nodes, even as $m \to \infty$, do not cause the aggregate rate of each primary cell to decay with m. This is done by introducing the preservation regions, which guarantee the minimum distance of $\sqrt{a_s}$ from all secondary Txs to the primary Rxs. This lemma will be used to show that (9) can be satisfied w.h.p. if $\delta_P \in (0, \min\{\delta_{P,\max}, 1\}]$ in Theorem 1.

The next lemma determines the number of data paths that each cell should carry. To obtain an upper bound, we extend each HDP to the entire horizontal line and all cells through which this horizontal line passes should deliver the corresponding data of HDP (see Fig. 3). Similarly, we extend each VDP to the entire vertical line. We define this entire horizontal and vertical line as an *extended* HDP and an *extended* VDP, respectively. Throughout the rest of the paper, our analysis deals with extended HDPs and VDPs instead of original HDPs and VDPs. Since we are adding hops to our routing scheme, the extended traffic gives a lower bound on the achievable throughput.

Lemma 3: Each primary cell needs to carry at most $4\sqrt{2n\log n}$ data paths w.h.p.

Proof: The proof is in the Appendix. ■

Lemma 3 shows how the number of data paths varies with the node density n. Lemmas 1 to 3 will be used to prove the main theorem, stated next.

Theorem 1: For any $\delta_{\text{loss}} \in (0,1)$, by setting $\delta_P \in (0, \min\{\delta_{P,\max}, 1\}]$, the primary network can achieve $T_p(n) = (1-\delta_{\text{loss}})T_{\text{alone}}(n)$ and $S_p(n) = (1-\delta_{\text{loss}})S_{\text{alone}}(n)$ w.h.p., where

$$T_{\text{alone}}(n) = \frac{K_p}{4\sqrt{2}} \frac{1}{\sqrt{n\log n}} \tag{18}$$

and

$$S_{\text{alone}}(n) = \frac{K_p(1-\epsilon)}{8\sqrt{2}} \sqrt{\frac{n}{\log n}}. \tag{19}$$

The definitions of $\delta_{P,\max}$ and K_p are given in Lemma 2.

Proof: First consider the stand-alone throughput of the primary network. Since each primary cell can sustain a rate of K_p (Lemma 2), each primary S-D pair can achieve a rate of at least K_p divided by the maximum number of data paths per primary cell. The number of data paths is upper bounded by $4\sqrt{2n\log n}$ w.h.p. (Lemma 3). Therefore, $T_{\text{alone}}(n)$ is lower bounded by $\frac{K_p}{4\sqrt{2n\log n}}$ w.h.p. Now the whole network contains at least $(1-\epsilon)\frac{n}{2}$ primary S-D pairs w.h.p. (Lemma 1). Therefore, $S_{\text{alone}}(n)$ is lower bounded by $(1-\epsilon)\frac{n}{2}T_{\text{alone}}(n)$ w.h.p.

Finally Lemma 2 shows that, for any $\delta_{\text{loss}} \in (0,1)$, if we set $\delta_P \in (0, \min\{\delta_{P,\max}, 1\}]$, then $R_p(n) = (1-\delta_{\text{loss}})R_{\text{alone}}(n)$ is achievable in the limit of large n. Since the number of primary data paths carried by each primary cell and the total number of primary S-D pairs are not changed due to the secondary network, $T_p(n) = (1-\delta_{\text{loss}})T_{\text{alone}}(n)$ and $S_p(n) = (1-\delta_{\text{loss}})S_{\text{alone}}(n)$ are also achievable w.h.p., which completes the proof. ■

2) Secondary Network Throughputs: Let us now consider the per-node and sum throughputs of the secondary network in the presence of the primary network. The main difference between the primary and secondary transmission schemes arises from the presence of the preservation regions. Recall that the secondary nodes wish to transmit according to a multihop protocol, but their path may be blocked by a preservation region. In this case, they must circumvent the preservation region, or possibly the cluster of preservation regions.[4] As we will see later, circumventing these preservation regions (clusters) only degrades the secondary network's throughput scaling by at most $\Theta(\log n)$ due to the relative primary and secondary node densities: the secondary nodes increase at the rate $m = n^\beta$ and $\beta > 1$.

Formally, we define the *distance* of two preservation regions as the minimum distance of any two points in each preservation region. A set of preservation regions is said to form a *cluster* if for each of the preservation regions in the cluster there exists another preservation region in the same cluster at a distance less than $2\sqrt{a_s}$. Hence, if there is no cluster that includes two preservation regions, then the re-routed paths from each of these two preservation regions will be delivered by two disjoint sets of secondary cells. We also define the *effective area* of a cluster as the total area of all secondary cells in the cluster and at a distance less than $2\sqrt{a_s}$ from the cluster. Fig. 4 illustrates an example of a cluster of preservation regions and the corresponding effective area. From the definition, we know that the effective area of a cluster consisting of N preservation regions is less than or equal to $49a_sN$.

[4]Since the primary nodes are distributed according to a p.p.p., clustering of preservation regions may occur.

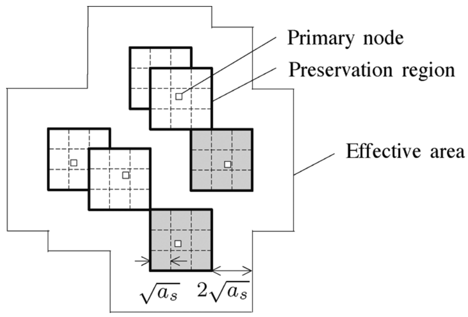

Fig. 4. Example of a cluster of preservation regions and the corresponding effective area, where the distance of the two shaded preservation regions is equal to $\sqrt{a_s}$.

Lemma 4: Any cluster of preservation regions has at most $4\log n$ preservation regions w.h.p.

Proof: The proof is in the Appendix. ■

This lemma is needed to ensure that the secondary network remains connected, to bound the number of data paths that pass through secondary cells, and to prove the next lemma. As mentioned earlier, whenever a secondary source or destination lies within a preservation region or there is no possible data path, this pair is not served. The next lemma shows that the fraction of these unserved secondary S-D pairs is arbitrarily small w.h.p.

Lemma 5: The fraction of unserved secondary S-D pairs is upper bounded by $\epsilon_{s,1}(m) = \Theta(\frac{(\log m)^2}{m^{1-1/\beta}})$ w.h.p., which converges to zero as $m \to \infty$.

Proof: The proof is in the Appendix. ■

Next, Lemma 6 shows that, in the presence of the primary network, each secondary cell may sustain a constant aggregate rate.

Lemma 6: Each secondary cell can sustain traffic at a rate of K_s independent of m. Here K_s is given by $\frac{1}{27}\log(1+\frac{\delta_P P}{N_0+(1+\delta_P)I+2^{3\alpha/2}P})$ and I is given by (16).

Proof: The proof is in the Appendix. ■

The main challenge in proving Lemma 6 is the presence of the primary Txs. Since the primary node density is smaller than the secondary node density, the primary cells are relatively further away from each other, thus requiring higher power to communicate. Although the relatively higher power could be a potential problem because the secondary nodes repeat their transmissions for three slots, the interfering primary transmission occurs at a certain minimum distance away from the secondary Rx on one of these slots. Hence, this allows us to bound the interference of the more powerful primary nodes without changing the scaling laws. From Lemma 2, the value of δ_P, which is a normalized transmit power of the secondary Txs, should be smaller than $\min\{\delta_{P,\max},1\}$ in order to satisfy (9). We also notice that the range of δ_P does not affect the throughput scalings of the secondary network.

Let us define the secondary cells that border the preservation regions as *loaded* cells and the other cells as *regular* cells. The loaded cells will be required to carry not only their own traffic, but also re-routed traffic around the preservation regions and, as a result, could deliver more data than the regular cells. The next lemma bounds the number of data paths that each regular cell and each loaded cell must transport. As the number of data paths each cell could carry was essentially the limiting factor in the sum throughput of the primary network, the following lemma is of crucial importance for the secondary sum throughput scaling law.

Lemma 7: Each regular secondary cell needs to carry at most $4\sqrt{2m\log m}$ data paths and each loaded secondary cell carries at most $4(12/\beta\log m+1)\sqrt{2m\log m}$ data paths w.h.p.

Proof: The proof is in the Appendix. ■

As it will be shown later, for $1<\beta\le 2$ the loaded cells are the bottleneck of the overall throughput. But even in this case, at most $\Theta(\log m)$ throughput degradation occurs. For $\beta>2$, since the secondary network is much denser than the primary network, the fraction of secondary data paths needing to be re-routed diminishes to zero as the node densities increase. Thus, in the limit, almost all secondary cells behave as regular cells.

Finally, we can use the previous lemmas to obtain the per-node and sum throughputs of the secondary network in the following theorem.

Theorem 2: For any $\delta_{\text{loss}} \in (0,1)$, by setting $\delta_P \in (0,\min\{\delta_{P,\max},1\}]$, the following per-node and sum throughputs are $\epsilon_s(m)$-achievable w.h.p. for the secondary network:

$$T_s(m) = \begin{cases} \frac{K_s}{4\sqrt{2}}\frac{1}{\sqrt{m\log m}}, & \text{if } \beta>2 \\ \frac{K_s}{4\sqrt{2}(12/\beta\log m+1)}\frac{1}{\sqrt{m\log m}}, & \text{if } 1<\beta\le 2 \end{cases} \tag{20}$$

and

$$S_s(m) = \begin{cases} \frac{K_s(1-\epsilon)(1-\epsilon_s(m))}{8\sqrt{2}}\sqrt{\frac{m}{\log m}}, & \text{if } \beta>2 \\ \frac{K_s(1-\epsilon)(1-\epsilon_s(m))}{8\sqrt{2}(12/\beta\log m+1)}\sqrt{\frac{m}{\log m}}, & \text{if } 1<\beta\le 2 \end{cases} \tag{21}$$

where

$$\epsilon_s(m) = \begin{cases} O\left(\frac{\sqrt{\log m}}{m^{1/2-1/\beta}}\right), & \text{if } \beta>2 \\ O\left(\frac{(\log m)^2}{m^{1-1/\beta}}\right), & \text{if } 1<\beta\le 2 \end{cases} \tag{22}$$

which converges to zero as $m\to\infty$. The definitions of $\delta_{P,\max}$ and K_s are given in Lemmas 2 and 6, respectively.

Proof: Note that by setting $\delta_P \in (0,\min\{\delta_{P,\max},1\}]$, the secondary network satisfies (9) during its transmission. Let us first consider $\beta>2$. Let m_h (similarly, m_v) denote the number of secondary S-D pairs whose original or re-routed HDPs (VDPs) pass through loaded cells. Suppose the following two cases where the projections of two preservation regions on the y-axis are at a distance greater than or equal to $2\sqrt{a_s}$ (Fig. 5(a)) and less than $2\sqrt{a_s}$ (Fig. 5(b)), respectively. For the first case, all extended HDPs in the area of $1\times 10\sqrt{a_s}$ will pass through the loaded cells generated by two preservation regions. But for the second case, the number of extended HDPs passing through the loaded cells is less than the previous case w.h.p. because the corresponding area is smaller than $1\times 10\sqrt{a_s}$. Thus, assuming that projections of all preservation regions on the y-axis are at a distance of at least $2\sqrt{a_s}$ from each other gives an upper bound on m_h. In this worst-case scenario, all sources located in the area of $1\times 5(1+\epsilon)n\sqrt{a_s}$ generate

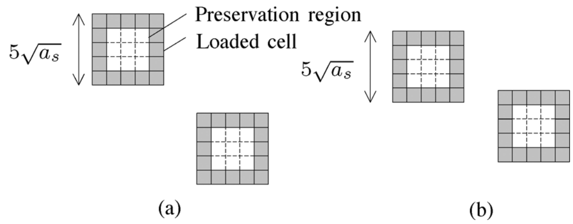

Fig. 5. Upper bound on the number of secondary S-D pairs whose extended HDPs pass through the loaded cells.

extended HDPs w.h.p., which must pass through the loaded cells, where we use the fact that the number of preservation regions is upper bounded by $(1+\epsilon)n$ w.h.p. By assuming that all nodes are sources, the resulting upper bound follows $\mathrm{Poisson}(\lambda = 5(1+\epsilon)nm\sqrt{a_s})$. Similarly, an upper bound on m_v follows $\mathrm{Poisson}(\lambda = 5(1+\epsilon)nm\sqrt{a_s})$. From Lemma 8

$$\mathbb{P}\left(m_h \geq 10(1+\epsilon)nm\sqrt{a_s}\right) \leq e^{-5(1+\epsilon)nm\sqrt{a_s}}\left(\frac{e}{2}\right)^{10(1+\epsilon)nm\sqrt{a_s}}. \quad (23)$$

Then

$$\begin{aligned}\mathbb{P}&\left(m_h + m_v \geq 20(1+\epsilon)nm\sqrt{a_s}\right)\\ &\leq \mathbb{P}\left((m_h \geq 10(1+\epsilon)nm\sqrt{a_s}) \cup (m_v \geq 10(1+\epsilon)nm\sqrt{a_s})\right)\\ &\leq 2e^{-5(1+\epsilon)nm\sqrt{a_s}}\left(\frac{e}{2}\right)^{10(1+\epsilon)nm\sqrt{a_s}} \end{aligned} \quad (24)$$

which converges to zero as $n \to \infty$. Hence, we obtain w.h.p.

$$m_h + m_v \leq \epsilon_{s,2}(m)(1-\epsilon)\frac{m}{2} \quad (25)$$

where $\epsilon_{s,2}(m) = 40\sqrt{2}\frac{1+\epsilon}{1-\epsilon}\frac{\sqrt{\log m}}{m^{1/2-1/\beta}}$. In conclusion, the fraction of S-D pairs whose data paths pass through the loaded cells is upper bounded by $\epsilon_{s,2}(m)$ w.h.p., which tends to zero as $m \to \infty$. This indicates that almost all data paths will pass through regular cells rather than loaded cells. If we treat the S-D pairs passing through the loaded cells and the S-D pairs not served as outages, $\epsilon_s(m)$ is obviously upper bounded w.h.p. by

$$\epsilon_s(m) \leq \epsilon_{s,1}(m) + \epsilon_{s,2}(m) = \Theta\left(\frac{\sqrt{\log m}}{m^{1/2-1/\beta}}\right) \quad (26)$$

where we use the fact that the fraction of S-D pairs not served is upper bounded by $\epsilon_{s,1}(m)$ w.h.p. (Lemma 5). Then the achievable per-node throughput is determined by the rate of S-D pairs passing only the regular cells. Since each secondary cell can sustain a constant rate of K_s w.h.p. (Lemma 6), from the result of Lemma 7, each served secondary S-D pair that passes only through regular cells can achieve a rate of at least $\frac{K_s}{4\sqrt{2m\log m}}$ w.h.p. Therefore, $T_s(m)$ is lower bounded by $\frac{K_s}{4\sqrt{2}}\frac{1}{\sqrt{m\log m}}$ w.h.p.

Let us now consider the case where $1 < \beta \leq 2$. Unlike the previous case, most served S-D pairs in this case pass through loaded cells, which will become bottlenecks. By assuming that all served S-D pairs pass through loaded cells, we obtain a lower bound on $T_s(m)$ with $\epsilon_s(m) \leq \epsilon_{s,1}(m) = \Theta(\frac{(\log m)^2}{m^{1-1/\beta}})$, which is an upper bound on the fraction of unserved S-D pairs. Therefore, based on Lemmas 6 and 7, $T_s(m)$ is lower bounded by $\frac{K_s}{4(12/\beta\log m+1)\sqrt{2m\log m}}$ w.h.p.

Since there are at least $(1-\epsilon)(1-\epsilon_s(m))\frac{m}{2}$ nonoutage S-D pairs, $S_s(m)$ is lower bounded by $(1-\epsilon)(1-\epsilon_s(m))\frac{m}{2}T_s(m)$ w.h.p., which completes the proof. ■

IV. Discussions

In this section, we study information-theoretic upper bounds and compare them with the achievable lower bounds derived in the previous section. We also consider throughput scaling laws when the primary network uses the FDTT routing in [7] and alternative routing methods when different assumptions about the primary and secondary networks are given.

A. Cut-Set Upper Bound

In Section III, we show that the secondary network can achieve $T_s(m) = \Omega(m^{-1/2-\delta_l})$ and $S_s(m) = \Omega(m^{1/2-\delta_l})$ for any $\delta_l > 0$ with an arbitrarily small fraction of outage. Then the question is how far this achievable scaling law is from the optimal one. To partially answer this question, we briefly introduce information-theoretic upper bounds developed for single networks. The cut-set upper bound in [12] shows that for the power-limited regime, i.e., the power constraint of the secondary nodes scales as $m^{-\alpha/2}$, and $\alpha \geq 3$, the stand-alone sum throughput of the secondary network is upper bounded by $S_{\mathrm{alone}}(m) = \mathrm{O}(m^{1/2+\delta_u})$ w.h.p. for any $\delta_u > 0$. From the fact that $S_s(m) \leq S_{\mathrm{alone}}(m)$, we know $S_s(m) = \mathrm{O}(m^{1/2+\delta_u})$ w.h.p. Notice that this cut-set upper bound holds for any arbitrary outage $\epsilon_s(m) \in [0,1)$. Also, for a given $\epsilon_s(m) \in [0,1)$, $T_s(m)$ is upper bounded by $\frac{S_s(m)}{(1-\epsilon_s(m))(1-\epsilon)\frac{m}{2}} = \mathrm{O}(\frac{S_s(m)}{m})$ w.h.p. In summary, we obtain $T_s(m) = \mathrm{O}(m^{-1/2+\delta_u})$ and $S_s(m) = \mathrm{O}(m^{1/2+\delta_u})$ w.h.p. for any $\epsilon_s(m) \in [0,1)$. Therefore, the multiplicative gap $m^{\delta_u+\delta_l}$ between the upper and lower bounds has an arbitrarily small exponent, i.e., $\delta_u + \delta_l$.

B. FDTT Routing

As mentioned before, the multihop routing in [7] can be adopted as a primary protocol, which provides the sum throughput of $\Omega(\sqrt{n})$. The key observation is that the construction of multihop data paths with a hop distance of $\mathrm{O}(1/\sqrt{n})$ is possible, which consists of the "highway" for multihop transmission. During Phase 1, each source directly transmits its packet to the closest node on the highway and, during Phase 2, the packet is delivered to the node on the highway closest to the destination by multihop transmissions using the nodes on the highway. Finally, during Phase 3, the destination directly receives the packet from the closet node on the highway.

Assuming that the primary network operates based on the FDTT routing, we can derive throughput scaling laws of the secondary network. As the same manner in Section III, we assume that the transmit power of each primary Tx scales according to its hop distance so that each primary Rx will receive the intended signal with a constant power. Note that since the hop distance for Phase 1 (or 3) is given by $\mathrm{O}(\log n/\sqrt{n})$, the maximum transmit power of Phase 1 (or 3) is greater than that of the GK routing. For the GK routing, the hop distance is given

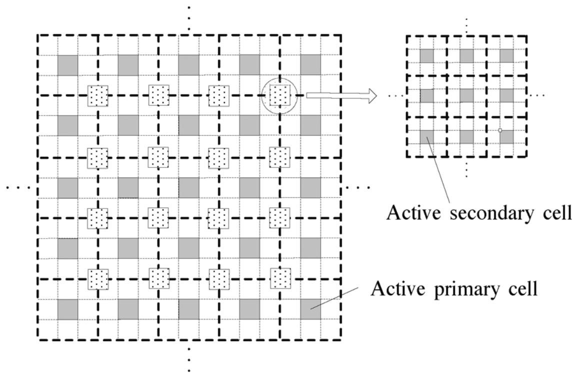

Fig. 6. Alternative secondary protocol with different information about the primary network: the secondary network operates based on 81-TDMA.

by $O(\sqrt{\log n/n})$. The maximum transmit power of Phase 2, on the other hand, is smaller than that of the GK routing because the hop distance is given by $O(1/\sqrt{n})$. Therefore, we can apply the previous secondary routing protocol during Phase 2 of the primary FDTT routing, which will cause less interference to the secondary network. Based on the analysis used for the GK routing, we derive similar results in Theorems 1 and 2 except now we have $T_{\text{alone}}(n) = \Omega(1/\sqrt{n})$ and $S_{\text{alone}}(n) = \Omega(\sqrt{n})$.

C. Alternative Routing

The main assumption defining the role of the primary and secondary networks is that the primary network operates as if no secondary network were present and the secondary network adjusts its protocol based on the locations of the primary nodes. If the secondary network knows when the primary nodes are activated in addition to their locations, then 81-TDMA between the secondary cells in Fig. 6 can achieve $T_s(m) = \Omega(1/\sqrt{m\log m})$ and $S_s(m) = \Omega(\sqrt{m/\log m})$ w.h.p. Specifically, each group of the secondary cells can be activated based on the 9-TDMA (dotted region) and within each group secondary cells operate 9-TDMA. Furthermore if the primary network can change its protocol in the presence of the secondary network, then a simple time-sharing, which allocates $1-\delta_{\text{loss}}$ and δ_{loss} fractions of time for the primary and secondary networks respectively, will satisfy (9) while providing a stand-alone throughput scaling law for the secondary network.

V. Conclusion

In this paper, we studied two co-existing ad hoc networks with different priorities (a primary and a secondary network) and analyzed their simultaneous throughput scalings. It was shown that each network can achieve almost the same throughput scaling as when the other network is absent. Although we allow outage for the secondary S-D pairs, the fraction of pairs in outage converges to zero as the node densities increase. Furthermore, these scalings can be achieved by adjusting the secondary protocol while keeping that of the primary network unchanged. In essence, the primary network is unaware of the presence of the secondary network. To achieve this result, the secondary nodes need knowledge of the locations of the primary nodes and the secondary nodes need to be denser than the primary. For $0 < \beta \le 1$ where the primary network is denser than the secondary network, on the other hand, it seems to be more challenging to achieve similar throughput scaling results while keeping the primary unchanged, as there are many primary nodes around each secondary node. Our result may be extended to more than two networks, provided each layered network obeys the same three main assumptions as in the two network case.

Appendix

Before proving our lemmas, we recall the following useful lemma from [7].

Lemma 8 (Franceschetti, Dousse, Tse, and Thiran): Let X be a Poisson random variable with parameter λ. Then

$$\mathbb{P}(X \ge x) \le \frac{e^{-\lambda}(e\lambda)^x}{x^x}, \quad \text{for } x > \lambda. \tag{27}$$

Proof: We refer readers to the paper [7]. ■

Proof of Lemma 1: Let X_1 denote the number of primary nodes in a unit area. For part (a), we wish to show that $\mathbb{P}(|X_1 - n| \ge \epsilon n) \to 0$ as $n \to \infty$. Noting that X_1 is a Poisson random variable with mean n and standard deviation $\sqrt{n}$, we use Chebyshev's inequality to see that

$$\mathbb{P}\left(|X_1 - n| \ge (\epsilon\sqrt{n})\sqrt{n}\right) \le \frac{1}{(\epsilon\sqrt{n})^2}.$$

Clearly, as n tends to infinity we can make this quantity arbitrarily small.

For part (b), let X_2 denote the number of primary nodes in a primary cell. Then $\mathbb{P}(X_2 = 0)$ is given by

$$\mathbb{P}(X_2 = 0) = \left.\frac{e^{-2\log n}(2\log n)^k}{k!}\right|_{k=0} = \frac{1}{n^2}. \tag{28}$$

Therefore, the probability that there is at least one cell having no node is upper bounded by $n\mathbb{P}(X_2 = 0)$, where the union bound and the fact that there are at most n primary cells are used. Since $\frac{1}{n} \to 0$ as $n \to \infty$, (b) holds w.h.p., which completes the proof.

Proof of Lemma 2: Suppose that at a given moment, there are $N_p(n)$ active primary cells and $N_s(n)$ active secondary cells, including the ith active primary cell. Then, the rate of the ith active primary cell is given by

$$R_p^i(n) = \frac{1}{9}\log\left(1 + \frac{P_p^i g\left(\|X_{p,\text{tx}}^i - X_{p,\text{rx}}^i\|\right)}{N_0 + I_p^i(n) + I_{sp}^i(n)}\right) \tag{29}$$

where $\frac{1}{9}$ indicates the loss in rate due to the 9-TDMA transmission of primary cells. The rate of the ith active primary cell in the absence of the secondary network is given by $R_{\text{alone}}^i(n) = R_p^i(n)$ by setting $I_{sp}^i(n) = 0$. Fig. 7 illustrates the worst case interference from the secondary interferers to the Rx of the ith active primary cell, where the dotted region denotes the preservation region around the primary Rx and the shaded cells denote the active secondary cells based on the 9-TDMA. Because of the preservation region, the minimum distance of $\sqrt{a_s}$ can be guaranteed from all secondary transmitting interferers to the primary

Rx. Thus, there exist 8 secondary interferers at a distance of at least $\sqrt{a_s}$, and 16 secondary interferers at a distance of at least $4\sqrt{a_s}$, and so on. Then, $I_{sp}^i(n)$ is upper bounded by

$$\begin{aligned} I_{sp}^i(n) &= \sum_{k=1}^{N_s(n)} P_s^k g\left(\left\|X_{s,\text{tx}}^k - X_{p,\text{rx}}^i\right\|\right) \\ &< \delta_P P(\sqrt{2a_s})^\alpha \sum_{t=1}^{\infty} 8t\left((3t-2)\sqrt{a_s}\right)^{-\alpha} \\ &= \delta_P I \end{aligned} \tag{30}$$

where we use the fact that $P_s^k \le \delta_P P(\sqrt{2a_s})^\alpha$. Similarly, there exist 8 primary interferers at a distance of at least $\sqrt{a_p}$, and 16 primary interferers at a distance of at least $4\sqrt{a_p}$, and so on. Then

$$\begin{aligned} I_p^i(n) &= \sum_{k=1,k\neq i}^{N_p(n)} P_p^k g\left(\left\|X_{p,\text{tx}}^k - X_{p,\text{rx}}^i\right\|\right) \\ &< P2^{\alpha/2+3}\sum_{t=1}^{\infty} t(3t-2)^{-\alpha} = I \end{aligned} \tag{31}$$

where we use the fact that $P_p^k \le P(\sqrt{2a_p})^\alpha$. Then

$$\begin{aligned} \lim_{n\to\infty}\frac{R_p^i(n)}{R_{\text{alone}}^i(n)} &\ge \lim_{n\to\infty}\frac{\log\left(1+\frac{P}{N_0+I_p^i(n)+\delta_P I}\right)}{\log\left(1+\frac{P}{N_0+I_p^i(n)}\right)} \\ &\ge \min_{x\in[0,I]}\frac{\log\left(1+\frac{P}{N_0+x+\delta_P I}\right)}{\log\left(1+\frac{P}{N_0+x}\right)}. \end{aligned} \tag{32}$$

Hence, from the definition of $x_{\min}$, we obtain

$$\lim_{n\to\infty}\frac{R_p^i(n)}{R_{\text{alone}}^i(n)} \ge \frac{\log\left(1+\frac{P}{N_0+x_{\min}+\delta_P I}\right)}{\log\left(1+\frac{P}{N_0+x_{\min}}\right)}. \tag{33}$$

Notice that $\delta_{P,\max}$ is the value of δ_P such that the right-hand side of (33) is equal to $1-\delta_{\text{loss}}$. One can easily know that $\delta_{P,\max} > 0$ since $\delta_{\text{loss}} > 0$. Thus, $\lim_{n\to\infty}\frac{R_p^i(n)}{R_{\text{alone}}^i(n)} \ge 1-\delta_{\text{loss}}$ if we set $\delta_P \in (0, \min\{\delta_{P,\max}, 1\}]$. Because the above inequalities holds for any i, we obtain $\lim_{n\to\infty}\frac{R_p(n)}{R_{\text{alone}}(n)} \ge 1-\delta_{\text{loss}}$ if we set $\delta_P \in (0, \min\{\delta_{P,\max}, 1\}]$. Lastly, we obtain

$$R_{\text{alone}}(n) > \frac{1}{9}\log\left(1+\frac{P}{N_0+I}\right) = K_p. \tag{34}$$

Therefore, Lemma 2 holds.

Proof of Lemma 3: Let n_h denote the number of extended HDPs that should be delivered by a primary cell. Similarly, n_v denotes the number of extended VDPs that should be delivered by a primary cell. When HDPs are extended, the extended HDPs of all primary sources located in the area of $1\times\sqrt{a_p}$ should be handled by the primary cell. By assuming that all primary

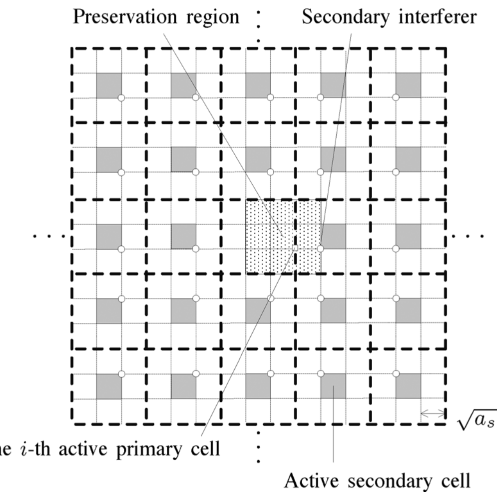

Fig. 7. Amount of interference from the secondary interferers to the Rx of the ith primary pair, where the shaded cells indicate the active secondary cells based on the 9-TDMA.

nodes are sources, the resulting upper bound on n_h follows $\text{Poisson}(\lambda = n\sqrt{a_p})$. Using Lemma 8, we obtain

$$\begin{aligned} \mathbb{P}(n_h \ge 2n\sqrt{a_p}) &\le \left.\frac{e^{-n\sqrt{a_p}}(en\sqrt{a_p})^x}{x^x}\right|_{x=2n\sqrt{a_p}} \\ &= e^{-n\sqrt{a_p}}\left(\frac{e}{2}\right)^{2n\sqrt{a_p}}. \end{aligned} \tag{35}$$

Similarly, the extended VDPs of all primary destinations located in the area of $\sqrt{a_p}\times 1$ should be also handled by the primary cell. By assuming that all primary nodes are destinations, we obtain

$$\mathbb{P}(n_v \ge 2n\sqrt{a_p}) \le e^{-n\sqrt{a_p}}\left(\frac{e}{2}\right)^{2n\sqrt{a_p}}. \tag{36}$$

From (35) and (36), we obtain

$$\begin{aligned} &\mathbb{P}(n_h + n_v \ge 4n\sqrt{a_p}) \\ &\quad \le \mathbb{P}\left((n_h \ge 2n\sqrt{a_p}) \cup (n_v \ge 2n\sqrt{a_p})\right) \\ &\quad \le 2e^{-n\sqrt{a_p}}\left(\frac{e}{2}\right)^{2n\sqrt{a_p}} \end{aligned} \tag{37}$$

where the last inequality comes from the union bound.

Therefore, the probability that there is at least one primary cell supporting more than $4n\sqrt{a_p}$ extended data paths is upper bounded by $2ne^{-n\sqrt{a_p}}(\frac{e}{2})^{2n\sqrt{a_p}}$, where the union bound and the fact that there are at most n primary cells are used. Since $2ne^{-n\sqrt{a_p}}(\frac{e}{2})^{2n\sqrt{a_p}} \to 0$ as $n\to\infty$, each primary cell should deliver the corresponding data of at most $4n\sqrt{a_p}$ extended data paths w.h.p., where $a_p = \frac{2\log n}{n}$. Note that the above bounds also hold for the original data paths, which completes the proof.

Proof of Lemma 4: Divide the network into $\frac{n}{2\log n}$ disjoint regions with area $\frac{2\log n}{n}$ each. Let X_i denote the number of primary nodes in the ith disjoint region. From the union bound and Lemma 8, we obtain

$$\begin{aligned}\mathbb{P}\left(X_i < 4\log n \text{ for all } i \in \left\{1,\dots,\frac{n}{2\log n}\right\}\right) &\\ \geq 1-\sum_i \mathbb{P}(X_i \geq 4\log n) &\\ \geq 1-\frac{n}{2\log n}e^{-2\log n}\left(\frac{e}{2}\right)^{4\log n} & \quad (38)\end{aligned}$$

which converges to one as $n \to \infty$. Hence, the number of primary nodes in any disjoint region is less than $4\log n$ w.h.p. Suppose that there exists a cluster with more than or equal to $4\log n$ preservation regions. Then we can find a sub-cluster of $4\log n$ preservation regions in the original cluster, whose effective area is less than or equal to $(49a_s)4\log n$. Hence, we can set disjoint regions with area $\frac{2\log n}{n}$ each such that one of the disjoint regions contains the sub-cluster because $(49a_s)4\log n < \frac{2\log n}{n}$ in the limit of large n. This means that there would exist a region with area $\frac{2\log n}{n}$ containing at least $4\log n$ primary nodes, a contradiction because such an event occurs with probability zero. In conclusion, every cluster consists of at most $4\log n$ preservation regions w.h.p., which completes the proof.

Proof of Lemma 5: Let $A_{p,1}$ denote the area of all preservation regions, $A_{p,2}$ denote the area of all disjoint regions due to the preservation regions except the biggest region, and $A_p = A_{p,1}+A_{p,2}$. Define m_p as the number of secondary nodes in the area of A_p that follows $\text{Poisson}(\lambda = mA_p)$. The number of secondary S-D pairs not served is clearly upper bounded by m_p. From Lemma 8, we obtain

$$\mathbb{P}(m_p \geq 2mA_p) = e^{-mA_p}\left(\frac{e}{2}\right)^{2mA_p}. \quad (39)$$

An upper bound on $A_{p,1}$ is obtained if we assume none of the preservation regions overlap. Thus, as each preservation region has an area of $9a_s$ and there are at most $(1+\epsilon)n$ such regions w.h.p., we obtain w.h.p.

$$A_{p,1} \leq 9(1+\epsilon)na_s. \quad (40)$$

To derive an upper bound on $A_{p,2}$, we assume all preservation regions form clusters having $4\log n$ preservation regions each (Lemma 4) as shown in Fig. 8(a), where the shaded regions denote $A_{p,2}$. Then the maximum disjoint area that is generated by a cluster of $4\log n$ preservation regions is given in Fig. 8(b) since a circle maximizes the area of a region for a given perimeter. Because each preservation region contributes a length of at most $6\sqrt{a_s}$ to the circumference of this circle, the radius is upper bounded by $\frac{48\log n\sqrt{a_s}}{\pi}$. Thus, $A_{p,2}$ is upper bounded w.h.p. by

$$\begin{aligned}A_{p,2} &< \frac{(1+\epsilon)n}{4\log n}\frac{\pi}{4}\left(\frac{48\log n\sqrt{a_s}}{\pi}\right)^2\\ &= \frac{144(1+\epsilon)}{\pi}n\log na_s \quad (41)\end{aligned}$$

where we use the fact that the total number of clusters having $4\log n$ preservation regions in each cluster is upper bounded by $\frac{(1+\epsilon)n}{4\log n}$ w.h.p. From (40) and (41), A_p is upper bounded by

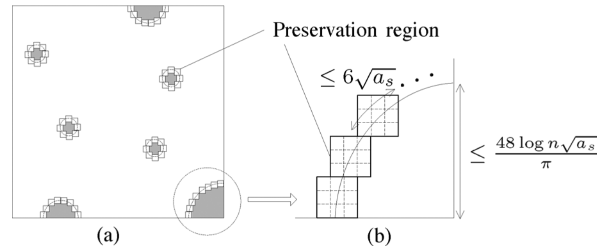

Fig. 8. Given that the size of any cluster of preservation regions is limited to $4\log n$, this figure illustrates the worst-case scenario for the number of secondary S-D pairs that are not served when their data pathes are disconnected by the preservation regions.

$18\beta(1+\epsilon)\frac{\pi+16\log n}{\pi}n^{1-\beta}\log n$ w.h.p. By substituting A_p for its upper bound in (39), we obtain

$$\begin{aligned}\mathbb{P}\left(m_p \geq 36\beta(1+\epsilon)\frac{\pi+16\log n}{\pi}n\log n\right) &\\ \leq e^{-18\beta(1+\epsilon)\frac{\pi+16\log n}{\pi}n\log n}\left(\frac{e}{2}\right)^{36\beta(1+\epsilon)\frac{\pi+16\log n}{\pi}n\log n} & \quad (42)\end{aligned}$$

which converges to zero as $n \to \infty$. Thus, we obtain w.h.p.

$$m_p < \epsilon_{s,1}(m)(1-\epsilon)\frac{m}{2} \quad (43)$$

where $\epsilon_{s,1}(m) = 72\frac{1+\epsilon}{1-\epsilon}\frac{\pi+16/\beta\log m}{\pi}\frac{\log m}{m^{1-1/\beta}}$. Since the total number of secondary S-D pairs is lower bounded by $(1-\epsilon)\frac{m}{2}$ w.h.p., the fraction of unserved S-D pairs is upper bounded by $\epsilon_{s,1}(m)$ w.h.p., which completes the proof.

Proof of Lemma 6: Since the same secondary packet is transmitted three times, the minimum distance of $\frac{\sqrt{a_p}}{2}$ from all primary interferers to the secondary Rx can be guaranteed for one out of three transmissions. Then the interference from primary interferers of that packet is upper bounded by

$$\begin{aligned}I_{ps} &< P(\sqrt{2a_p})^\alpha\sum_{t=1}^{\infty}8k((3t-2)\sqrt{a_p})^{-\alpha}\\ &\quad + P(\sqrt{2a_p})^\alpha\left(\frac{\sqrt{a_p}}{2}\right)^{-\alpha}\\ &= I + 2^{3\alpha/2}P \quad (44)\end{aligned}$$

where we use the same technique as in Lemma 2. Similarly, I_s is upper bounded by $\delta_P I$. Thus, the rate of each secondary cell is lower bounded by

$$\frac{1}{27}\log\left(1+\frac{\delta_P P}{N_0+(1+\delta_P)I+2^{3\alpha/2}P}\right) = K_s \quad (45)$$

where $\frac{1}{27}$ indicates the rate loss due to the 9-TDMA and repeated (three times) transmissions of the same secondary packet. Therefore, Lemma 6 holds.

Proof of Lemma 7: Let $m_{h,1}$ and $m_{h,2}$ denote the number of extended HDPs including re-routed paths that should be delivered by a secondary regular cell and by a secondary loaded cell, respectively. Similarly, we can define $m_{v,1}$ and $m_{v,2}$ for extended VDPs.

Let us first consider a regular cell. This regular cell delivers the corresponding data of extended HDPs passing through it. Then all extended HDPs of the secondary sources located in the

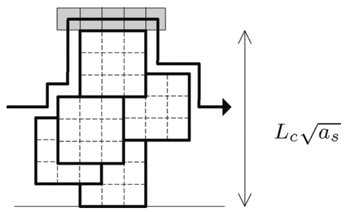

Fig. 9. Upper bound on the number of re-routed HDPs passing through the shaded cells.

area of $1 \times \sqrt{a_s}$ should be handled by the regular cell, where we ignore the effect of S-D pairs not served, which yields an upper bound on the total number of HDPs. By assuming that all secondary nodes are sources, the resulting upper bound on $m_{h,1}$ follows $\text{Poisson}(\lambda = m\sqrt{a_s})$. From Lemma 8, we obtain

$$\mathbb{P}(m_{h,1} \geq 2m\sqrt{a_s}) \leq e^{-m\sqrt{a_s}} \left(\frac{e}{2}\right)^{2m\sqrt{a_s}}. \tag{46}$$

We obtain the same bound for $m_{v,1}$ by assuming that all secondary nodes are destinations and then

$$\begin{aligned}
&\mathbb{P}(m_{h,1} + m_{v,1} \geq 4m\sqrt{a_s}) \\
&\quad \leq \mathbb{P}\left((m_{h,1} \geq 2m\sqrt{a_s}) \cup (m_{v,2} \geq 2m\sqrt{a_s})\right) \\
&\quad \leq 2e^{-m\sqrt{a_s}} \left(\frac{e}{2}\right)^{2m\sqrt{a_s}}.
\end{aligned} \tag{47}$$

From the union bound and the fact that there are at most m secondary cells, each regular cell should deliver the corresponding data of at most $4m\sqrt{a_s}$ extended data paths w.h.p., where we use the fact that $2me^{-m\sqrt{a_s}}(\frac{e}{2})^{2m\sqrt{a_s}} \to 0$ as $m \to \infty$.

Let us now consider a loaded cell. Unlike in the primary data path which has no obstacles, a secondary data path should circumvent any preservation regions which lie on its path. Therefore, the loaded cells should deliver more data paths than the regular cells w.h.p. Suppose a cluster of preservation regions located on the boundary of the network in Fig. 9, whose projection on y-axis has a length of $L_c\sqrt{a_s}$. Then consider an upper bound on L_c such that all extended HDPs of the secondary sources located in the area of $1 \times L_c\sqrt{a_s}$ are re-routed through the shaded cells, where we ignore the effect of S-D pairs not served (which yields an upper bound on the total number of extended HDPs). From Lemma 4 and the fact that the length of each preservation region is $3\sqrt{a_s}$, we obtain $L_c \leq \frac{12}{\beta}\log m$ w.h.p. The other loaded cells will deliver less HDPs than the shaded cells w.h.p. Therefore, by assuming that all secondary nodes are sources, the resulting upper bound on $m_{h,2}$ follows $\text{Poisson}(\lambda = m(\frac{12}{\beta}\log m + 1)\sqrt{a_s})$. Note that the upper bound on $m_{h,2}$ is the same as the upper bound on $m_{h,1}$ except for a factor of $\frac{12}{\beta}\log m + 1$, where $\frac{12}{\beta}\log m$ comes from the re-routed HDPs and 1 comes from the original HDPs. Therefore, we can apply the same analysis used in the regular case. In conclusion, each loaded cell should deliver the corresponding data of at most $4m(\frac{12}{\beta}\log m + 1)\sqrt{a_s}$ extended data paths w.h.p. Since the above bounds also hold for the original data paths, Lemma 7 holds.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for many valuable suggestions that have improved the presentation of this paper.

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