On the Error Rate of a Communication System Suffering from Additive Radar Interference

Narueporn Nartasilpa, Daniela Tuninetti, Natasha Devroye, Danilo Erricolo
University of Illinois at Chicago, Chicago, IL 60607, USA
Email: {nnarta2,danielat,devroye,derric1}@uic.edu

Abstract—**In the near future, radar and communication systems will share the spectrum. This motivates the study of how the two systems, which have traditionally operated in different bands, may co-exist. This paper investigates the effect of radar interference (unaltered, beyond the communication system designer's control) on an uncoded communication system, using complex-valued modulation schemes when the Maximum-A-Posteriori (MAP) detector is used. For all commonly used higher order modulation schemes, the Symbol Error Rate (SER) exhibits an "error floor" for the radar interference much larger than the signal power, which can be exactly characterized; in this regime the optimal MAP detector behaves like an interference canceller; interestingly, in this regime the channel behaves as a real-valued phase-fading AWGN channel with receiver CSI, thus indicating a loss of one of the two complex dimensions compared to the complex-valued interference-free channel.**

I. Introduction

The shortage of spectrum resources has become a serious problem for the wireless communications industry, which must cope with ever increasing demands for wireless services. Spectrum sharing between radar and communication systems has been proposed to help solve this spectrum scarcity issue [1], [2]. A first step towards understanding how to share the spectrum efficiently is to understand how current unaltered systems affect one another. In this work, we investigate the spectrum sharing problem from the perspective of the communications system that experiences radar interference. We seek to understand how the Symbol Error Rate (SER) of an uncoded communications system using a complex-valued modulation scheme is affected by a specific model of radar interference.

A. Past Work

The effect of radar interference on wireless communications systems has been investigated before. A spectrum sharing algorithm between a MIMO radar and an LTE cellular system was proposed in [3], [4], [5], where it was shown that the loss in radar performance is minimal when one selects the best channel onto which to null-project the radar signal so as to create minimal interference to the LTE system. In [6], a packet scheduling algorithm based on channel sensing was proposed for spectrum allocation for an LTE system during radar intermittent periods causing only a slight performance degradation for the LTE system. While there have been many numerical and experimental studies on the loss of performance of the radar and communications systems once they co-exist [7], [8], [9], [10], [11], [12], [13], there has not been much research on analytical models and analysis using such models to predict the effect of unaltered radar interference on the error rate for the communication systems.

In [14], the author investigated the performance analysis of a BPSK uncoded system under the presence of various interference models and provided simulation results to compare the BER with the baseline interference-free case; it showed, for example, that at relatively high interference power impulsive noise and gated noise are less harmful to the BPSK receiver than complex Gaussian noise; however a framework for the analysis of the performance of a general modulation scheme was not given. In our previous work [15] we investigated the effect of radar interference on an uncoded *real-valued* modulation scheme used on an Additive White Gaussian Noise (AWGN) channel—more details are given in Section III-C—which overlapped in parts with the analysis of [14]. In this work we extend our closed-form analysis of the average Symbol Error Rate (SER) to any *complex-valued* modulation scheme. We explicitly consider commonly used QAM and PSK constellations, but the results and insights are valid in general.

B. Contribution

For an uncoded communication system with AWGN noise as well additive radar interference, we analyze the SER performance for an optimal Maximum-A-Posteriori (MAP) decoder for arbitrary modulation schemes. We model the radar interference, which is beyond our control, as an additive signal of known constant-amplitude and unknown uniformly-distributed (in $[0, 2\pi]$) random phase. Based on the relative value of the radar interference power, measured by the Interference-to-Noise ratio (INR), and the desired signal power, measured by the Signal-to-Noise ratio (SNR), three different regimes for any modulation schemes are identified as follows.

a) Treat Interference as Gaussian Noise (TIN): When INR$\leq$SNR, the radar interference can be treated as Gaussian noise, that is, the optimal MAP decoder is the classical minimum distance decoder as if the overall channel noise were Gaussian. The SER is minimal when INR=0 and increases with INR.

b) Interference Cancellation (IC): For INR$\gg$SNR, the MAP receiver can estimate the radar interference and subtract it from the received signal. The interference can be completely canceled but in doing so, part of the useful signal is also cancelled. This results in an 'error floor' for the SER that does not coincide with the case INR=0. We can exactly

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characterize this 'error floor' as the attainable SER over a narrowband fading channel with multiplicative fading that is perfectly known at the receiver but unknown at the transmitter. Interestingly, our analysis shows that in the limit for infinite INR, a very strong radar interference causes a loss of one of the two complex dimensions compared to the complex-valued interference-free channel.

c) Intermediate regime: When INR$\approx$SNR, the SER attains its highest value; this regime represents the worst operating point from the perspective of a communication system coexisting with a radar system.

These intuitions and the observed three regimes of operation are confirmed to be fundamental by our recent information theoretic analysis [16] of the Shannon's capacity (i.e., largest possible coded rate that can be reliably decoded) for the considered channel model, which exhibit the same exact three regimes of operation.

We also note that the optimal MAP receiver produces very interesting shapes for the decoding region, with elliptical-like shaped that cross multiple times. The exact analysis of the SER for the optimal MAP decoder appears quite involved, but the identified three regimes of operation help in considerably simplifying the closed-form analysis, which is the main contribution of this work.

C. Paper Organization

This paper is organized as follows. Section II presents the system model and the optimal MAP detection scheme. The analysis of the SER performance for any modulation scheme is discussed in Section III. Section IV concludes the paper.

II. System Model and Optimal Detection

A. System Model

Our model assumes the existence of a radar system that transmits a short duty-cycle, periodic, rectangular radar pulse. The effect of such a radar signal (in frequency domain) on a narrowband data communication system at the communication receiver can be modeled by a signal of deterministic envelope and a uniformly distributed phase (see [15] for justification). The discrete-time complex-valued received signal can be expressed as

$$Y = \sqrt{\mathsf{S}}X + \sqrt{\mathsf{I}}e^{j\Theta} + Z, \tag{1}$$

where X is the transmitted symbol from the constellation $\mathcal{X} = \{x_1, ..., x_M\}$ of unit energy and equally likely points, Θ is the random phase of the radar interference uniformly distributed in $[0, 2\pi]$, and Z is a zero-mean unit-variance proper-complex Gaussian noise. The random variables (X, Θ, Z) are mutually independent. Without loss of generality, S denotes the average Signal-to-Noise Ratio (SNR), while I denotes the average Interference-to-Noise Ratio (INR). In the following we assume that the pair (S, I) is fixed and known at the receiver.

Our goal is to evaluate the SER $\Pr[X \neq \widehat{X}(Y)]$, where $\widehat{X}(Y)$ is the estimate at the communication receiver of the transmitted signal X based on the output Y for the AWGN channel with additive radar interference in (1).

B. Optimal MAP Decoder

We denote the channel conditional distribution as

$$f_{Y|X,\Theta}(y|x,\theta) := \frac{1}{\pi}e^{-|y-\sqrt{\mathsf{S}}x-\sqrt{\mathsf{I}}e^{j\theta}|^2}, \ y \in \mathbb{C}. \tag{2}$$

The optimal MAP receiver, when the received signal is $Y = y$, chooses an estimate of the transmit constellation point

$$\begin{aligned}\hat{\ell}^{(\text{map})}(y) &= \arg\max_{\ell\in[1:M]} \mathbb{E}_{\Theta}[f_{Y|X,\Theta}(y|x_\ell,\Theta)] \\ &= \arg\min_{\ell\in[1:M]} \left(|y-\sqrt{\mathsf{S}}x_\ell|^2 - \ln I_0(2\sqrt{\mathsf{I}}|y-\sqrt{\mathsf{S}}x_\ell|)\right), \end{aligned} \tag{3}$$

where the expected value with respect to Θ is because Θ is not assumed known at the receiver, and where I_0 denotes the modified Bessel function of the first kind of order zero, which satisfies $I_0(x) \in [1, e^{|x|}], \ x \in \mathbb{R}$.

C. Approximation of the MAP Decoder at Low INR

At low INR (when $\mathsf{I} \le \mathsf{S}$) we expect that the channel behaves as an AWGN channel because the term $\ln I_0(2\sqrt{\mathsf{I}}|y - \sqrt{\mathsf{S}}x_\ell|)$ in (3) is close to zero. Therefore, for $\mathsf{I} \le \mathsf{S}$ we propose the following "Treat Interference as Gaussian Noise" (TIN) decoder

$$\hat{\ell}^{(\text{tin})}(y) = \arg\min_{\ell\in[1:M]} |y-\sqrt{\mathsf{S}}x_\ell|^2. \tag{4}$$

Note that the decoder in (4) is exactly optimal at $\mathsf{I} = 0$. Our numerical results later on will show that that the TIN decoder in (4) very well captures the behavior of the optimal MAP decoder, as expected, for $\mathsf{I} \le \mathsf{S}$.

D. Approximation of the MAP Decoder at High INR

At high INR (when $\mathsf{I} \gg \mathsf{S}$) we expect that the communication receiver is able to estimate and thus remove the effect of the radar interference from the received signal. Therefore, for $\mathsf{I} \gg \mathsf{S}$ we propose the following "Interference Cancellation"(IC) decoder

$$\hat{\ell}^{(\text{ic})}(y) = \arg\min_{\ell\in[1:M]} \left(|y-\sqrt{\mathsf{S}}x_\ell| - \sqrt{\mathsf{I}}\right)^2 \tag{5}$$

$$= \arg\min_{\ell\in[1:M]} \left(\frac{|y-\sqrt{\mathsf{S}}x_\ell|^2 - \mathsf{I}}{|y-\sqrt{\mathsf{S}}x_\ell| + \sqrt{\mathsf{I}}}\right)^2. \tag{6}$$

Note that the decoder in (5) is obtained by using the approximation $\ln I_0(z) \approx |z|$ in (3). Moreover, the numerator in (6) can be approximated as

$$\begin{aligned}|y-\sqrt{\mathsf{S}}x_\ell|^2 - \mathsf{I} &= |(\sqrt{\mathsf{S}}x - \sqrt{\mathsf{S}}x_\ell + z) + \sqrt{\mathsf{I}}e^{j\Theta}|^2 - \mathsf{I} \\ &= |\sqrt{\mathsf{S}}x - \sqrt{\mathsf{S}}x_\ell + z|^2 + 2\sqrt{\mathsf{I}}\Re\{e^{-j\Theta}(\sqrt{\mathsf{S}}x - \sqrt{\mathsf{S}}x_\ell + z)\} \\ &= 2\sqrt{\mathsf{I}}\left(1 + O(\sqrt{\mathsf{S}/\mathsf{I}})\right)\cdot y', \end{aligned} \tag{7}$$

where

$$y' := \Re\{e^{-j\Theta}(\sqrt{\mathsf{S}}x - \sqrt{\mathsf{S}}x_\ell + z)\}, \tag{8}$$

and $O(\cdot)$ denotes the big "O" notation, i.e., $f(x) = O(g(x))$ if and only if there exists a positive real number κ and an x_0

such that $|f(x)| \leq \kappa |g(x)|$ for all $x \geq x_0$; and the denominator in (6) can be approximated as

$$\begin{aligned}&\sqrt{|y-\sqrt{\mathsf{S}}x_\ell|^2}+\sqrt{\mathsf{I}}\\&=\sqrt{|\sqrt{\mathsf{S}}x-\sqrt{\mathsf{S}}x_\ell+z|^2+2\sqrt{\mathsf{I}}\ m'\ +\mathsf{I}}+\sqrt{\mathsf{I}}\\&=2\sqrt{\mathsf{I}}\left(1+O(\sqrt{\mathsf{S}/\mathsf{I}})\right).\end{aligned}\tag{9}$$

Therefore, by combining (7) and (9), we see that in the regime $\mathsf{I} \gg \mathsf{S}$ the decoder in (6) is equivalent to optimizing the decoding metric in (8), which can be thought of as corresponding to the following equivalent *real-valued phase-fading AWGN channel*

$$Y_{\text{eq}} := \Re\{e^{-j\Theta}\sqrt{\mathsf{S}}X\} + Z_{\text{eq}},\ Z_{\text{eq}} \sim \mathcal{N}_R(0, 1/2), \tag{10}$$

where the phase-fading $e^{-j\Theta}$ is known at the receiver but not at the transmitter. It is surprising to see that in the regime $\mathsf{I} \gg \mathsf{S}$ essentially one of the two real-valued dimensions of the complex-valued received signal in (1) is completely lost in (10). This is actually a fundamental behavior as proved in our recent information theoretic analysis of the Shannon's capacity for this channel model [16] and can be intuitively seen as follows: a zero-mean unit-variance proper-complex Gaussian input is optimal for the Gaussian channels in (1) with $\mathsf{I} = 0$ and in (10); the capacity of the former (complex-valued channel) is $\log(1 + \mathsf{S})$ while the capacity of the latter (real-valued channel) is only $\frac{1}{2}\log(1 + \mathsf{S})$. This can also be thought of as a loss of half the degrees of freedom.

Our numerical results later on will show that that the IC decoder in (5) very well captures the behavior of the optimal MAP decoder, as expected, for $\mathsf{I} \gg \mathsf{S}$.

III. SER Analysis

We analyze the low INR and high INR regimes separately. We then specialize our results to various modulations.

A. SER Analysis for Arbitrary Constellations at Low INR

The TIN decoder in (4) provides in general an upper bound on the SER with the optimal MAP decoder. The goal is to show that this bound is actually fairly tight for $\mathsf{I} \leq \mathsf{S}$.

With TIN, the probability of correct detection given that the symbol x_ℓ was transmitted is

$$\begin{aligned}P^{(\text{tin})}_{\text{c}|x_\ell} &= \mathbb{P}\left[|Y-\sqrt{\mathsf{S}}x_\ell|^2 \leq \min_{k:k\neq\ell}|Y-\sqrt{\mathsf{S}}x_k|^2 \ \middle|\ X=x_\ell\right]\\&=\mathbb{P}\left[\Re\{(\sqrt{\mathsf{I}}e^{j\Theta}+Z)e^{-j\angle x_k-x_l}\}\leq\frac{\sqrt{\mathsf{S}}d_{k,\ell}}{2},\forall k\neq\ell\right],\end{aligned}\tag{11}$$

where the Euclidean distance between x_ℓ and x_k is

$$d_{k,\ell} := |x_k - x_\ell|. \tag{12}$$

For later use, the minimum distance is indicated as

$$d_{\min} := \min_{k\neq\ell} d_{k,\ell}. \tag{13}$$

Therefore the SER of the TIN decoder, based on (11), for equally likely input symbols is given by

$$P^{(\text{map})}_{\text{e}} \leq P^{(\text{tin})}_{\text{e}} = 1 - \frac{1}{M}\sum_{\ell\in[1:M]} P^{(\text{tin})}_{\text{c}|x_\ell} \tag{14}$$

$$\leq (M-1)\mathbb{P}\left[\Re\{\sqrt{\mathsf{I}}e^{j\Theta}+Z\} > \min_{k\neq\ell}\frac{\sqrt{\mathsf{S}}d_{\ell,k}}{2}\right] \tag{15}$$

$$= (M-1)\mathbb{E}_\Theta\left[Q\left(\sqrt{\frac{\mathsf{S}d^2_{\min}}{2}}-\sqrt{2\mathsf{I}}\cos(\Theta)\right)\right], \tag{16}$$

where (15) is obtained from the union bound and only depends on the cardinality and minimum distance of the constellation. Further simplifications of (14)-(16) will be given later on for specific modulation schemes.

B. SER Analysis for Arbitrary Constellations at High INR

The IC decoder in (5) provides in general an upper bound on the SER with the optimal MAP decoder. The goal is to show that this bound is actually fairly tight for $\mathsf{I} \gg \mathsf{S}$.

Define

$$r_\ell := \Re\{e^{-j\Theta}x_\ell\},\ \ell \in [1:M]. \tag{17}$$

Consider that the symbol r_ℓ was transmitted on the equivalent AWGN channel in (10); the probability of correct detection is

$$\begin{aligned}P^{(\text{eq})}_{\text{c}|x_\ell} &= \mathbb{P}\left[|Y_{\text{eq}}-\sqrt{\mathsf{S}}r_\ell|^2 \leq \min_{k:k\neq\ell}|Y_{\text{eq}}-\sqrt{\mathsf{S}}r_k|^2 \ \middle|\ X=x_\ell\right]\\&=\mathbb{P}\left[Z_{\text{eq}}\ \text{sign}(r_k-r_\ell)\leq\frac{\sqrt{\mathsf{S}}|r_k-r_\ell|}{2},\ \forall k\neq\ell\right],\\&=1-\mathbb{E}_\Theta\left[Q\left(\Delta^+_\ell(\Theta)\right)+\ Q\left(\Delta^-_\ell(\Theta)\right)\right]\end{aligned}\tag{18}$$

$$\Delta^+_\ell(\Theta) := \min_{k\neq\ell:\text{sign}(r_k-r_\ell)>0}\sqrt{\frac{\mathsf{S}}{2}}\left|r_k-r_\ell\right|, \tag{19}$$

$$\Delta^-_\ell(\Theta) := \min_{k\neq\ell:\text{sign}(r_k-r_\ell)<0}\sqrt{\frac{\mathsf{S}}{2}}\left|r_k-r_\ell\right|, \tag{20}$$

with the convention that $\Delta^\pm_\ell(\Theta)$ is infinite if no index k can be found to satisfy the corresponding condition and where r_ℓ was defined in (17). Thus the SER of the IC decoder is

$$\begin{aligned}\lim_{\mathsf{I}\to\infty}P^{(\text{map})}_{\text{e}} &\leq \lim_{\mathsf{I}\to\infty}P^{(\text{ic})}_{\text{e}} = \frac{1}{M}\sum_{\ell\in[1:M]}P^{(\text{eq})}_{\text{e}|x_\ell}\\&=\frac{1}{M}\sum_{\ell\in[1:M]}\ \mathbb{E}_\Theta\left[Q\left(\Delta^+_\ell(\Theta)\right)+Q\left(\Delta^-_\ell(\Theta)\right)\right].\end{aligned}\tag{21}$$

Further simplifications of (21) do not seem possible in general.

C. SER Analysis for M-PAM from [15]

In the next subsections we will analyze the SER for some commonly used *complex-valued* constellations. Before we do so, we first summarize the results for real-valued constellations, and of PAM in particular, from our past work [15] as it will form the basis of comparison for this study.

Low INR regime. For $\mathsf{I} < \mathsf{S}$ the TIN decoder in (14) gives

$$P_{\text{e,PAM}}^{(\text{map})} \le P_{\text{e,PAM}}^{(\text{tin})} = \mathbb{E}_\Theta \left[A_{M,\mathsf{S}}(\Theta)\right]$$
$$= 2\left(1-\frac{1}{M}\right)\mathbb{E}_\Theta\left[Q\left(\sqrt{\frac{6\mathsf{S}}{M^2-1}}-\sqrt{2\mathsf{I}}\cos(\Theta)\right)\right], \quad (22)$$

where the function $A_{M,\mathsf{S}}(\Theta)$ is defined in (24). Notice that the expression in (22) is that of the SER of an M-PAM constellation except for the extra term $\sqrt{2\mathsf{I}}\cos(\Theta)$ that accounts for the radar interference, as seen in (16).

High INR regime. For $\mathsf{I} \gg \mathsf{S}$ the IC decoder in (21) gives

$$\lim_{\mathsf{I}\to\infty} P_{\text{e,PAM}}^{(\text{map})} \le \lim_{\mathsf{I}\to\infty} P_{\text{e,PAM}}^{(\text{ic})} = P_{\text{e,PAM}}^{(\text{eq})}$$
$$= 2\left(1-\frac{1}{M}\right)\mathbb{E}_\Theta\left[Q\left(\sqrt{\frac{6\mathsf{S}}{M^2-1}\cos^2(\Theta)}\right)\right], \quad (23)$$

which is the (exact) SER for the fading channel in (10) since in the case of real-valued modulation it reduces to $Y_{\text{eq}} = \cos(\Theta)\sqrt{\mathsf{S}}X + Z_{\text{eq}},\ Z_{\text{eq}} \sim \mathcal{N}_R(0, 1/2)$.

In [15], we reported the optimal decoding regions and the bit error rate for the BPSK case. In the next section, for sake of comparing with complex-valued constellations, we will report the optimal decoding regions and the SER for the 16-PAM.

D. SER Analysis for square M-QAM

In this section we analyze the SER for square M-QAM, where M is the square of an integer number, which is the Cartesian product of two $\sqrt{M}$-PAM with half the energy.

Low INR regime. A square M-QAM can be seen as one $\sqrt{M}$-PAM with half the energy along the real-axis (i.e., the SER would be $A_{\sqrt{M},\mathsf{S}/2}(\Theta)$ defined next in (24)) and another $\sqrt{M}$-PAM with half the energy along the imaginary-axis (i.e., the SER would be $B_{\sqrt{M},\mathsf{S}/2}(\Theta)$ defined next in (25)). The functionals $A_{\sqrt{M},\mathsf{S}/2}(\Theta)$ and $B_{\sqrt{M},\mathsf{S}/2}(\Theta)$ we just mentioned are the SER of a PAM conditioned on Θ and obtained as generalizations of the expression in (22) as:

$$A_{M,\mathsf{S}}(\Theta) := 2\frac{M-1}{M}\left(\frac{1}{2}Q\left(\sqrt{\frac{6\ \mathsf{S}}{M^2-1}}+\sqrt{2\mathsf{I}}\cos(\Theta)\right)\right.$$
$$\left.+\frac{1}{2}Q\left(\sqrt{\frac{6\ \mathsf{S}}{M^2-1}}-\sqrt{2\mathsf{I}}\cos(\Theta)\right)\right), \quad (24)$$
$$B_{M,\mathsf{S}}(\Theta) := 2\frac{M-1}{M}\left(\frac{1}{2}Q\left(\sqrt{\frac{6\ \mathsf{S}}{M^2-1}}+\sqrt{2\mathsf{I}}\sin(\Theta)\right)\right.$$
$$\left.+\frac{1}{2}Q\left(\sqrt{\frac{6\ \mathsf{S}}{M^2-1}}-\sqrt{2\mathsf{I}}\sin(\Theta)\right)\right). \quad (25)$$

Therefore, conditioned on Θ, the probability of a correct detection for this square M-QAM is the product of correct decision probabilities for two PAM systems given by

$$P_{\text{c,QAM}|\Theta} = \left(1 - A_{\sqrt{M},\mathsf{S}/2}(\Theta)\right)\left(1 - B_{\sqrt{M},\mathsf{S}/2}(\Theta)\right). \quad (26)$$

By averaging the expression in (26) over Θ we get

$$P_{\text{e,QAM}}^{(\text{map})} \le P_{\text{e,QAM}}^{(\text{tin})} = \mathbb{E}\left[1 - P_{\text{c,QAM}|\Theta}\right]$$
$$= 4\left(1-\frac{1}{\sqrt{M}}\right)\mathbb{E}_\Theta\left[Q\left(\sqrt{\frac{3\ \mathsf{S}}{M-1}}-\sqrt{2\mathsf{I}}\cos(\Theta)\right)\right]$$
$$- \mathbb{E}_\Theta\left[A_{\sqrt{M},\mathsf{S}/2}(\Theta)\cdot B_{\sqrt{M},\mathsf{S}/2}(\Theta)\right]. \quad (27)$$

As we shall see next, the expression in (27) follows extremely closely the SER of the optimal MAP decoder when $\mathsf{I} < \mathsf{S}$.

High INR regime. It does not seem to be possible to express in simple terms the functions $\Delta_\ell^{\pm}(\Theta)$ in (19)-(20) for the square QAM, but they can be very simply evaluated numerically.

E. SER Analysis for M-PSK

For a unit-energy M-PSK, let $x_\ell := e^{j\phi_\ell}$ and $\phi_\ell := \frac{2\pi}{M}\ell$ for $\ell \in [1 : M]$. Then we can express

$$x_k - x_\ell = 2e^{j\frac{\pi+\phi_k+\phi_\ell}{2}}\sin\left(\frac{\phi_k-\phi_\ell}{2}\right). \quad (28)$$

Low INR regime. For $\mathsf{I} < \mathsf{S}$, by the symmetry of the constellation, we use (16) to get

$$P_{\text{e,PSK}}^{(\text{map})} \le P_{\text{e,PSK}}^{(\text{tin})} = 1 - P_{\text{c,PSK}|x_M}^{(\text{tin})}$$
$$\le (M-1)\mathbb{E}_\Theta\left[Q\left(\sqrt{2\mathsf{S}}\sin\left(\frac{\pi}{M}\right)-\sqrt{2\mathsf{I}}\cos(\Theta)\right)\right], \quad (29)$$

where (29) is the union bound. For $M \ge 4$ and $\mathsf{S} \ge 5$ dB we can tightly approximate $P_{\text{e,PSK}}^{(\text{tin})}$ by replacing the factor $M-1$ in (29) with 2, the number of nearest neighbors.

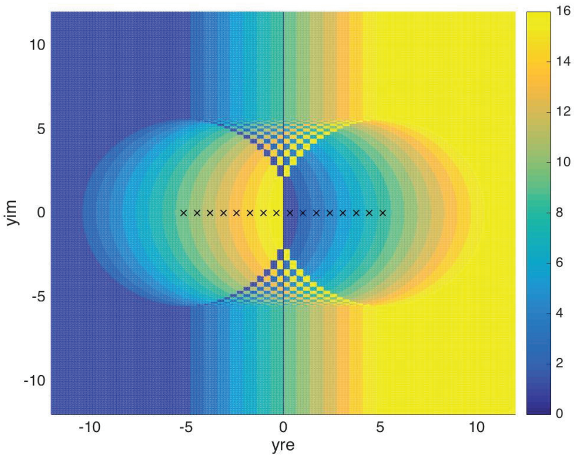
High INR regime. For $\mathsf{I} \gg \mathsf{S}$, by the symmetry of the constellation, we use an equivalent form of (18) to get

$$\lim_{\mathsf{I}\to\infty} P_{\text{e,PSK}}^{(\text{map})} \le \lim_{\mathsf{I}\to\infty} P_{\text{e,PSK}}^{(\text{ic})} = P_{\text{e,PSK}}^{(\text{eq})} = P_{\text{e,PSK}|x_M}^{(\text{eq})}$$
$$= \mathbb{P}\left[\left(Z_{\text{eq}} + v_k(\Theta)\right)v_k(\Theta) < 0, \text{for some } k < M\right], \quad (30)$$
$$v_k(\Theta) := r_M - r_k = \sqrt{\mathsf{S}}\sin\left(\frac{\phi_k}{2}\right)\sin\left(\frac{\phi_k}{2}-\Theta\right), \quad (31)$$

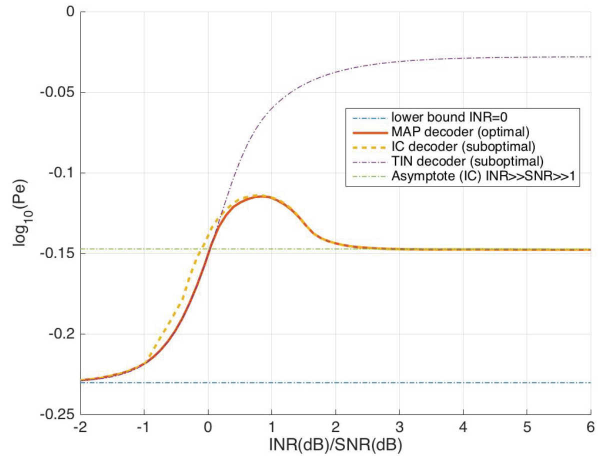
where r_k was defined in (17). It does not seem to be possible to express in simple terms the probability in (30), but this can be done numerically.

F. Numerical Results

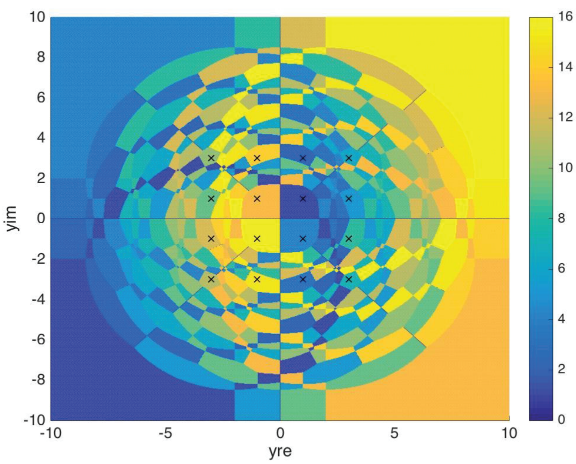
We now specialized the obtained bounds for low and high INR regimes, for constellations with $M = 16$ points. Fig. 1 reports the optimal MAP decoding region (left hand side) and the SER (right hand side) for PAM (1st row), QAM (2nd row) and PSK (3rd row). The $\times$ symbols on the left hand side figures indicate the position of the constellation points. The points are numbered 1 (whose decoding region is in dark blue) through 16 (whose decoding region is in yellow). In Fig. 1(a), for PAM, 1 is the left most point and 16 the right most point. In Fig. 1(c), for QAM, 1 is the bottom left point and the other points have an increasing number going upward then move to the next column on the right starting from the bottom again. In Fig. 1(e), for PSK, 1 is the point on the positive real axis and the other points have an increasing number moving counterclockwise.



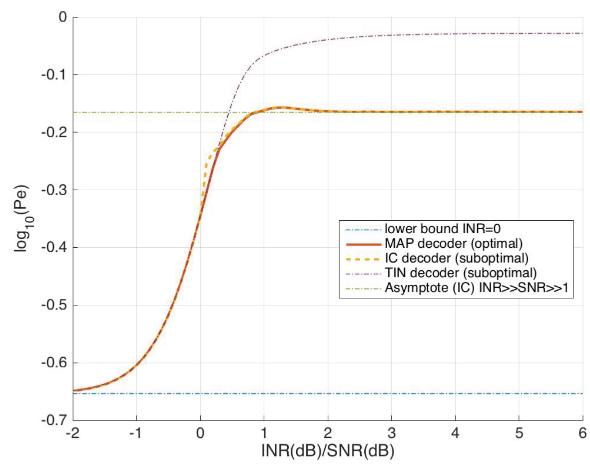
(a) Optimal decoding regions 16-PAM.



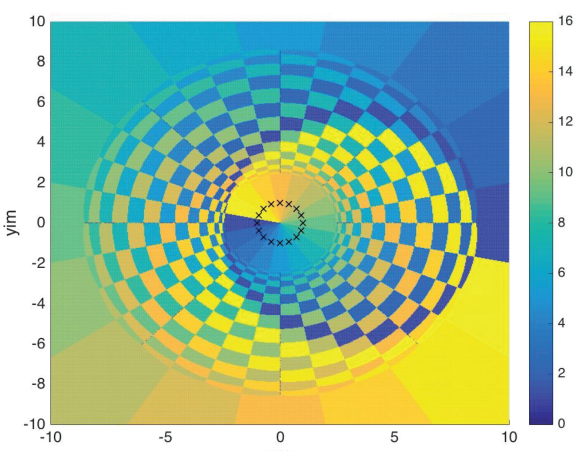
(b) SER for 16-PAM.



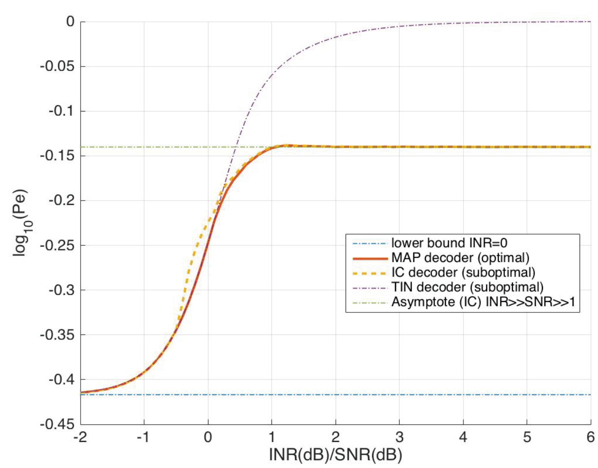
(c) Optimal decoding regions 16-QAM.



(d) SER for 16-QAM.



(e) Optimal decoding regions 16-PSK.



(f) SER for 16-PSK.

Fig. 1: Left hand side: Decoding regions for $\mathsf{S} = 10$ dB and $\mathsf{I} = 15$ dB. Right hand side: SER for $\mathsf{S} = 10$ dB vs. I in dB.

The optimal MAP decoding regions are for $\mathsf{S} = 10$ dB and $\mathsf{I} = 15$ dB, but similar behaviors are observed for all $\mathsf{I} \gtrapprox \mathsf{S}$ (for $\mathsf{I} \lessapprox \mathsf{S}$ the decoding regions are as for the AWGN case and based on minimum Euclidean distance). It is interesting to see that the decoding regions at high INR spread to not only the nearest surrounding neighbors but also other regions nearby. Note that elliptical-like areas grow larger as I increases. These "artsy" shapes look like layers of elliptical-like rings stacked slightly to the side or on top of one another. This can be understood from (5) for the high INR regime; the IC decoder prefers point x_1 to x_2 if $\left(|y - \sqrt{\mathsf{S}}x_1| - \sqrt{\mathsf{I}}\right)^2 < \left(|y - \sqrt{\mathsf{S}}x_2| - \sqrt{\mathsf{I}}\right)^2$; if $|y - \sqrt{\mathsf{S}}x_1|^2 > \mathsf{I}$ (i.e., y is 'far' from x_1) and $|y - \sqrt{\mathsf{S}}x_2|^2 < \mathsf{I}$ (i.e., y is 'close' to x_2) then the IC decoder prefers point x_1 to x_2 if $|y - \sqrt{\mathsf{S}}x_1| + |y - \sqrt{\mathsf{S}}x_2| < 2\sqrt{\mathsf{I}}$, which defines the inside of an ellipse and together with the other two conditions identify a 'slice' of an ellipse.

The SER curves are for a fixed $\mathsf{S} = 10$ dB and plotted against INR normalized by the SNR (all in dB). We plot the optimal SER (solid line), the SER curves for the TIN and IC decoders (dashed lines), and two asymptotes for $\mathsf{I} = 0$ and for $\mathsf{I} \to \infty$ (dash-dotted horizontal lines). The SER is always minimal at $\mathsf{I} = 0$, where it corresponds to the case of AWGN channel only without radar interference; it is maximal for $\mathsf{I}_{\text{dB}}/\mathsf{S}_{\text{dB}} \approx 1$; then it decreases and flattens out to the (exact) asymptote given by our high INR analysis for $\mathsf{I} \to \infty$; this last regime appears to kick in at $\mathsf{I}_{\text{dB}}/\mathsf{S}_{\text{dB}} \approx 2$. Note that at low INR the TIN decoder provides an excellent approximation for the performance of the optimal MAP decoder, while the IC decoder does not perform well in general. The opposite holds at high INR. In general, and quite surprisingly, the IC decoder is actually not much off compared to the optimal MAP decoder, even at low INR.

IV. Conclusions

In this paper, we studied the SER performance of uncoded complex-valued modulation schemes at a communication receiver in a complex-valued Gaussian channel with additive constant-amplitude and random-phase radar-induced interference. We derived the SER expressions for arbitrary constellations for the optimal MAP decoder that tends to the "Treat Interference as Gaussian Noise" (TIN) decoder when the radar interference power, measured by INR, is smaller than the communication signal power, measured by SNR, while it tends to the "Interference Cancellation" (IC) decoder when the INR of the radar signal is much larger than the SNR of the intended signal. For all commonly used higher order modulation schemes, the SER exhibits an "error floor" for large INR, which can be exactly characterized. Interestingly, when INR is much greater than SNR, the channel behaves as a real-valued phase-fading AWGN channel indicating a loss of half the degree of freedom.

Future work includes the investigation of optimal constellation design when the interference power increases much higher than the signal power, as well as extensions to other channel models, such as OFDM channels, fading channels, MIMO channels, channel-coded systems and multi-user channels.

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