Multi-user information theory and an example: the two-way relay channel

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Monologues vs. Dialogues

Monologue

Dialogue

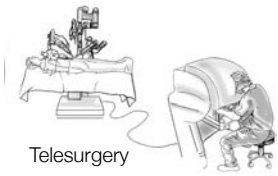
Monologues vs. Dialogues

Monologue = one-way = uni-directional

Dialogue = two-way = bi-directional

Two-way communication applications - wired

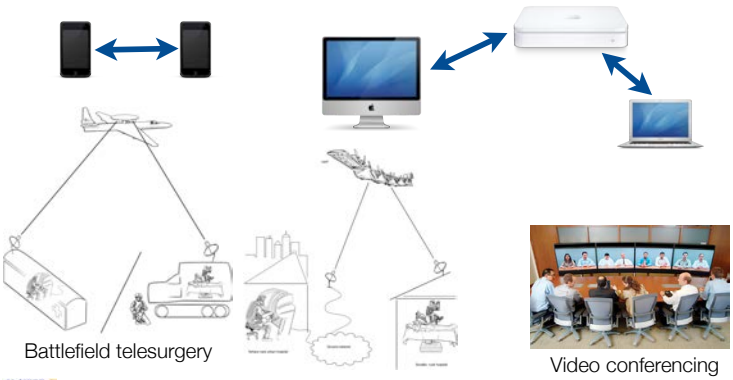
Video conferencing



Telesurgery

Data synchronization

Two-way communication applications - wireless



Battlefield telesurgery

Rural telesurgery

Video conferencing

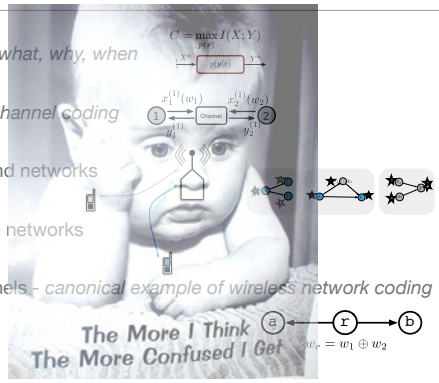
1 Dialogue ≠ or = 2 Monologues ?

It depends....

we will use information theory to find out.

Outline

- Information theory - *what, why, when*
- Two-way channel - *channel coding*
- Wireless channels and networks
- Two-way cellular-like networks
- Two-way relay channels - *canonical example of wireless network coding*



Overall - much is still unknown

Outline

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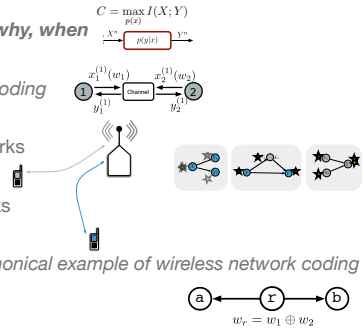


Information theory - *what, why, when*

A Mathematical Theory of Communication. **Bell System Technical Journal,** ***27, 379–423 & 623–656, 1948.***

What is information?

How fast can we communicate?

RANDOMNESS

What is communication?

How much can we compress information?

Information theory's claims to fame

Source coding

- Source = random variable
- Ultimate data compression limit is the source's entropy H

zip

jpg

mp3

Channel coding

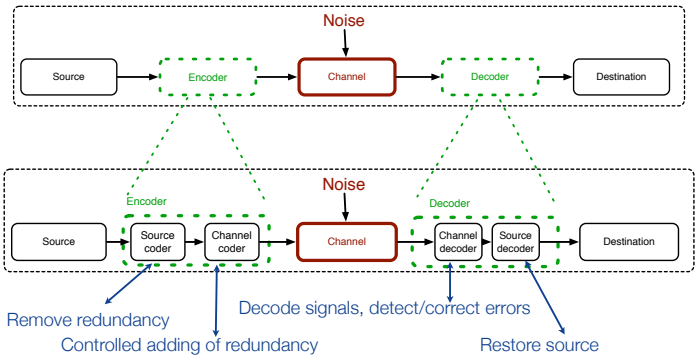
- Channel = conditional distributions
- Ultimate transmission rate is the channel capacity C

Turbo-codes

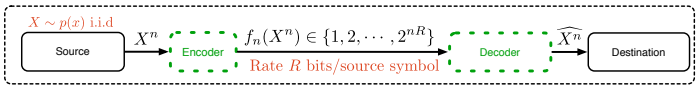
FADING CHANNEL

Reliable communication possible ↔ H<C

Source vs. channel coding



Source coding = data compression

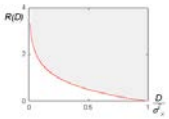


- Lossless: $\widehat{X^n} = X^n$

Minimum R needed is $H(X) = -\sum_x p(x)\log_2(p(x))$

- Lossy: $d(\widehat{X^n}, X^n) \leq D$

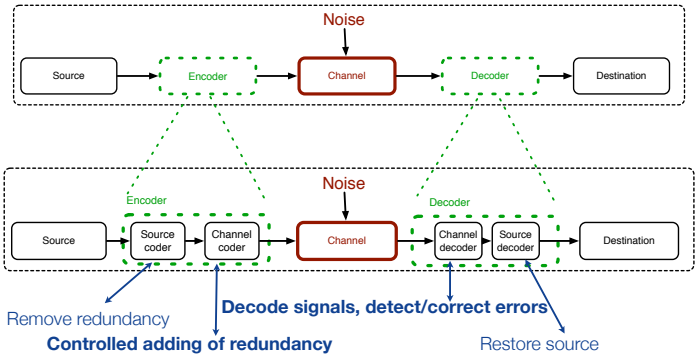
Minimum R needed is described by the Rate-Distortion function $R(D)$



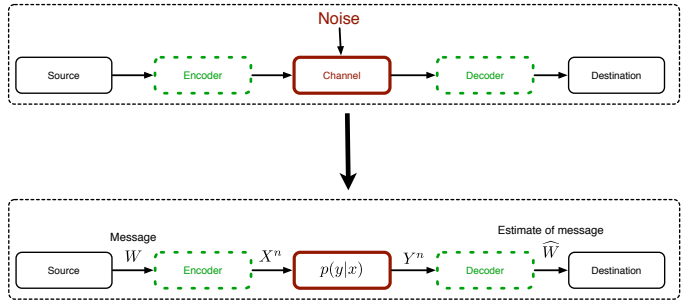
$$R(D) = \min_{p(\hat{x}|x)} I(X;\hat{X})$$

s.t. $d(\widehat{X^n}, X^n) \leq D$

Source vs. channel coding

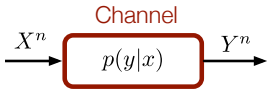


Communication system model



What is the capacity of this channel?

Channel capacity



- Information channel capacity:

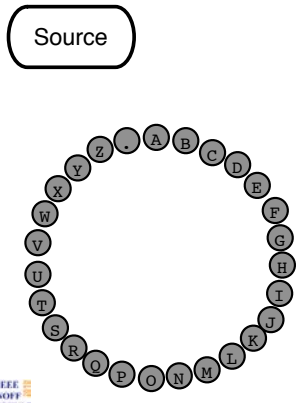
$$C = \max_{p(x)} I(X;Y)$$

- Operational channel capacity:

```
Highest rate (bits/channel use) that can
        communicate at reliably
```

- Channel coding theorem says: information capacity = operational capacity

Channel capacity: a cute example



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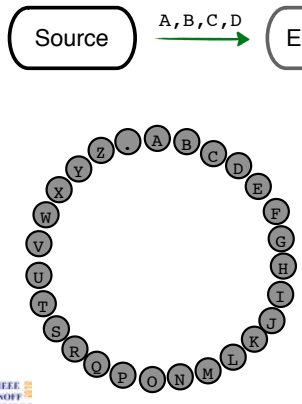
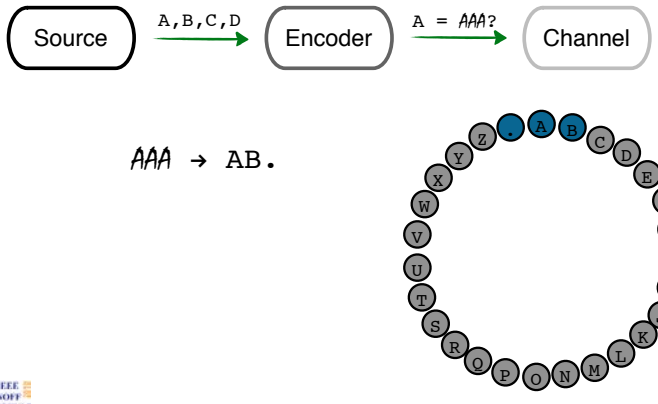


A = A?

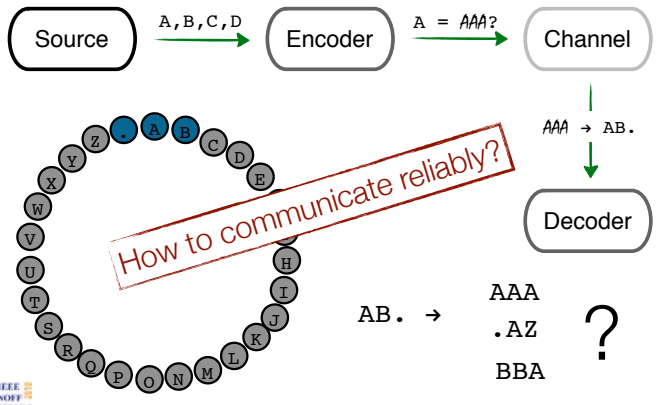
A = AAA?

Channel capacity: a cute example



AAA → AB.

Channel capacity: a cute example



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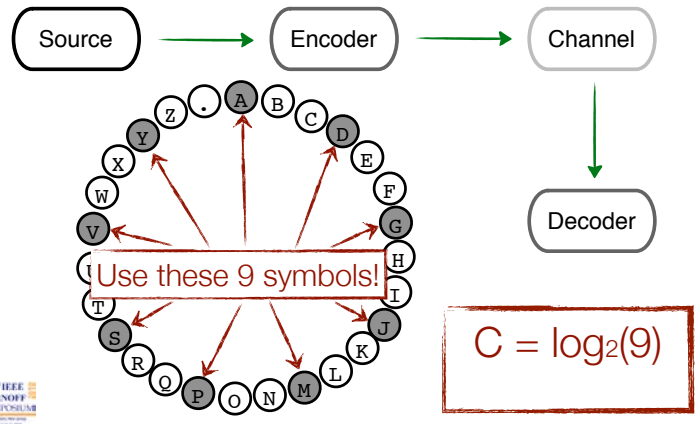
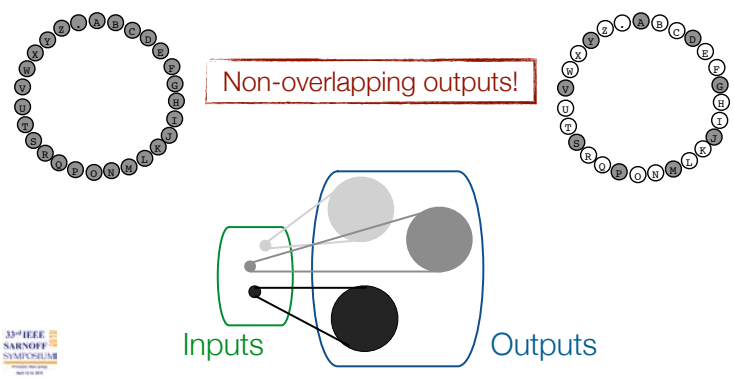


Capacity in general

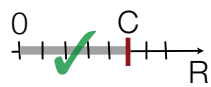
- Main idea was to reduce the rate (from a 27-letter input per channel use to a 9-letter input per channel use) so as to produce



Mathematical description of capacity

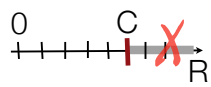
- Can achieve reliable communication for all transmission rates R:

R < C

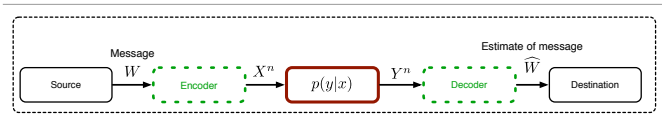


- BUT, probability of decoding error always bounded away from zero if

R > C

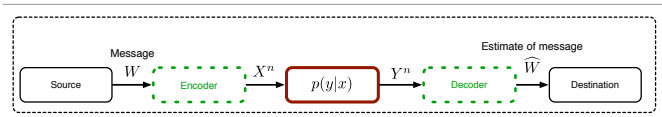


Capacity: key ideas



- choose input set of codewords so they are "non-confusable" at the output
- number of these that we can chose will determine the channel's capacity
- number that we can choose will depend on the distribution p(y|x) which characterizes the channel

One-way channel capacity



$$C = \max_{p(x)} I(X;Y)$$ bits/channel use

"mutual information" between X and Y

$$I(X;Y) = \sum_{x,y} p(x,y) \log\left(\frac{p(x,y)}{p(x)p(y)}\right)$$

A few examples

What is capacity / mutual information?

$$C = \max_{p(x)} \; I(X;Y)$$

Entropy of a random variable

(A) entropy is the measure of **average uncertainty** in the random variable

(B) entropy is the **average number of bits** needed to describe the random variable

(C) entropy is measured in bits?

(D) $H(X) = -\sum_x p(x) \log_2(p(x))$

(E) entropy of a deterministic value is 0

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Entropy of a uniform distribution

- Let X be uniformly distributed over 8 outcomes. What is the entropy of X?

$$H(X) = \sum_{x=1}^{8} p(x)\log_2(p(x)) = -\sum_{x=1}^{8} \tfrac{1}{8}\log_2\left(\tfrac{1}{8}\right) = \log_2(8) = 3 \text{ (bits)}$$

- This is the number of bits needed to describe X!

- By extension, for a discrete random variable taking on K outcomes, the **maximal entropy** is attained by a uniform distribution and is equal to the number of bits needed to describe K:

$$H(X) = \log_2(K)$$

Entropy of a non-uniform distribution

- Suppose X represents the outcome of a horse race with 8 horses, which win with probabilities $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}, \frac{1}{64}\right)$

$$\begin{aligned} H(X) &= -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{8}\log_2\left(\frac{1}{8}\right) - \frac{1}{16}\log_2\left(\frac{1}{16}\right) - 4\frac{1}{64}\log_2\left(\frac{1}{64}\right) \\ &= 2 \text{ (bits)} \end{aligned}$$

- 8 outcomes, 3 bits? But on average can represent with 2 bits!

$$\left(\tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16}, \tfrac{1}{64}, \tfrac{1}{64}, \tfrac{1}{64}, \tfrac{1}{64}\right)$$

$$(000,001,010,011,100,101,110,111)$$

3 bits

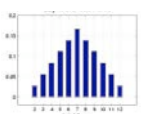
$$(0,10,110,1110,111100,111101,111110,111111)$$

2 bits (on average!)

Entropy of a continuous random variable

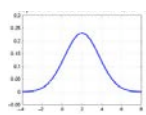
- entropy:

$$H(X) = -\sum_x p(x)\log_2(p(x))$$



- differential entropy:

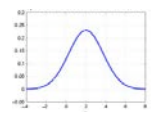
$$h(X) = -\int f(x)\log(x)\,dx$$



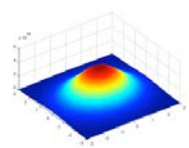
Entropy of a Gaussian random variable

- differential entropies of Gaussian distributions:

$$h\left(\mathcal{N}(0,\sigma^2)\right) = \frac{1}{2}\log\left(2\pi e\sigma^2\right)$$



$$h\left(\mathcal{N}_n(\mu, K)\right) = \frac{1}{2}\log\left((2\pi e)^n|K|\right)$$



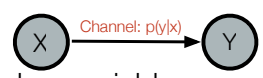
Entropy maximization

- **Uniform** distribution maximizes entropy for a given # outcomes

$$\max_{X:|X|=K} H(X) = \log_2(K)$$

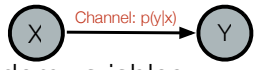
- **Gaussian** maximizes entropy for a given covariance constraint

$$\max_{E[XX^T]=K} h(X) = \frac{1}{2}\log\left((2\pi e)^n|K|\right)$$



Mutual information between 2 random variables:

$$\begin{aligned} I(X;Y) &= \sum p(x,y)\log\left(\frac{p(x,y)}{p(x)p(y)}\right) \\ &= H(X) - H(X|Y) \\ &= H(Y) - H(Y|X) \end{aligned}$$



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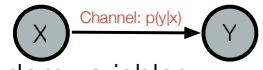
(A) I(X;Y) is the reduction in the uncertainty about X due to knowledge of Y

(B) if X, Y are independent I(X;Y) = 0

(C) if X=Y then I(X;Y) = H(X)

(D) I(X;Y) is non-negative



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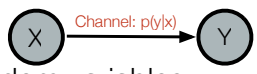

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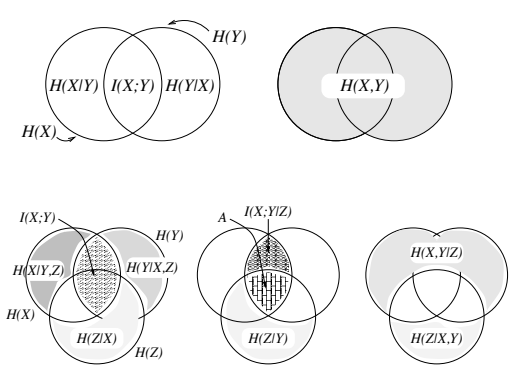
Properties of mutual information



With permission from David J.C. Mackay

Channel capacity

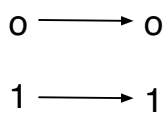
Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

Channel capacity — *Noiseless channel*

Capacity?

1 bit/channel use

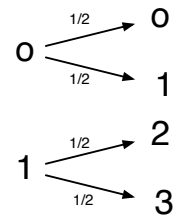


$$
\begin{aligned}
C &= \max_{p(x)} I(X;Y) \\
&= \max_{p(x)} H(X) - H(X|Y) \\
&= \max_{p(x)} H(X) - 0 \\
&= 1
\end{aligned}
$$

Channel capacity — *Non-overlapping outputs*

Capacity?

1 bit/channel use



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C &= \max_{p(x)} I(X;Y) \\
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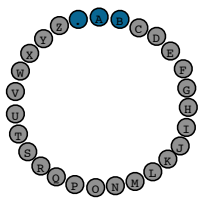
&= 1
\end{aligned}
$$

Channel capacity — *Noisy typewriter*

Capacity?

$\log_2(9)$ bits/channel use

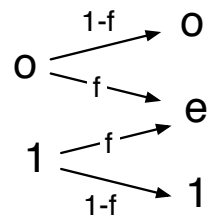


$$
\begin{aligned}
C &= \max_{p(x)} I(X;Y) \\
&= \max_{p(x)} H(X) - H(X|Y) \\
&= \max_{p(x)} H(X) - \log_2(3) \\
&= \log_2(27) - \log_2(3) = \log_2(9)
\end{aligned}
$$

Channel capacity — *Binary erasure channel*

Capacity?

1-f bits/channel use



$$
\begin{aligned}
C &= \max_{p(x)} I(X;Y) \\
&= \max_{p(x)} H(X) - H(X|Y) \\
&= \max_{p(x)} H(X) - \sum p(y) H(X|Y=y) \\
&= \max_{p(x)} H(X) - p\cdot(1-f)\cdot H(X|Y=0) + (1-p)\cdot(1-f)\cdot H(X|Y=1) + p\cdot f\cdot H(X|Y=e) \\
&= \max_{p(x)} H(X) - p\cdot(1-f)\cdot 0 + (1-p)\cdot(1-f)\cdot 0 + f\cdot 1 \\
&= \max_{p(x)} H(X) - f \\
&= 1 - f
\end{aligned}
$$

Discrete memoryless channel capacity

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

"mutual information" between X and Y

$$
I(X;Y) = \sum_{x,y} p(x,y) \log\left(\frac{p(x,y)}{p(x)p(y)}\right)
$$

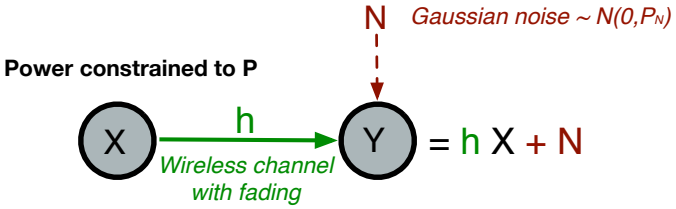
Continuous alphabet channel capacity

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

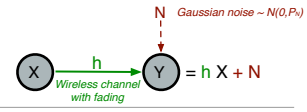
"mutual information" between X and Y

What if
X and Y are not bits, but real numbers?

AWGN channel capacity

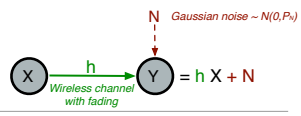


AWGN channel capacity



$$
\begin{aligned}
C &= \max_{p(x):E[XX^T]\le P} I(X;Y) \\
&= \max_{p(x):E[XX^T]\le P} h(X) - h(X|Y) \\
&= \max_{p(x):E[XX^T]\le P} h(Y) - h(Y|X) \\
&= \max_{p(x):E[XX^T]\le P} h(hX+N) - h(hX+N|X) \\
&= \max_{p(x):E[XX^T]\le P} h(hX+N) - h(N) \\
&= \frac{1}{2}\log(2\pi e(|h|^2P+P_N)) - \frac{1}{2}\log(2\pi e P_N) \\
&= \frac{1}{2}\log\left(\frac{|h|^2P+P_N}{P_N}\right)
\end{aligned}
$$

AWGN channel capacity



$$
\begin{aligned}
C &= \max_{p(x):E[XX^T]\le P} I(X;Y) \\
&= \max_{p(x):E[XX^T]\le P} h(X) - h(X|Y) \\
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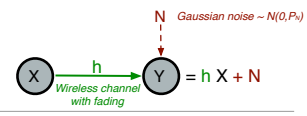
&= \frac{1}{2}\log\left(\frac{|h|^2P+P_N}{P_N}\right)
\end{aligned}
$$

signal power at Rx

noise power at Rx

AWGN channel capacity



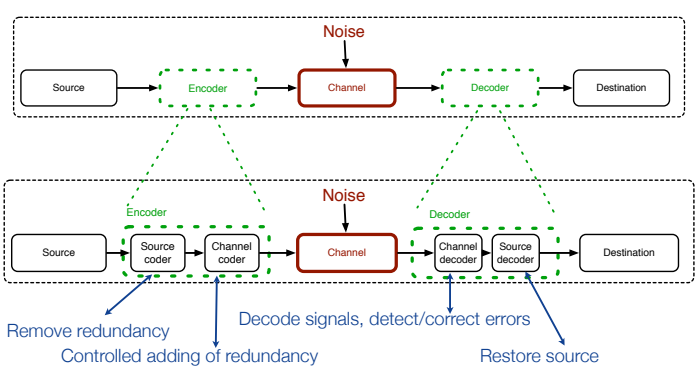
$$
\begin{aligned}
C &= \frac{1}{2}\log\left(\frac{|h|^2P+P_N}{P_N}\right) \\
&= \frac{1}{2}\log\left(1+SNR\right) \ \text{(bits/channel use)}
\end{aligned}
$$

What about bits/second and bandwidth of the channel?

$$C = W\log_2\left(1+\frac{P}{WN_0}\right) \ \text{(bits/second)}$$

[Bandwidth W, h=1, spectral density $N_0/2$]

Source vs. channel coding



Use?

Forney, G.D. and Costello, D.J., ``Channel Coding: The Road to Channel Capacity,'' Proceedings of the IEEE, Volume 95, Issue 6, pp.1150-1177, June 2007.

Claude Shannon — *Born on the planet Earth (Sol III) in the year 1916 A.D. Generally regarded as the father of the Information Age, he formulated the notion of channel capacity in 1948 A.D. Within several decades, mathematicians and engineers had devised practical ways to communicate reliably at data rates within 1% of the Shannon limit . . .*

Encyclopedia Galactica, 166th ed.

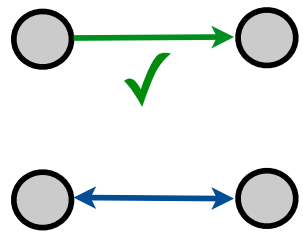
- algebraic codes
- convolutional codes
- iterative codes (LDPC, turbo)

-Robert J. McEliece, Allerton 2000

Use?

- Benchmark for performance of practical systems
- Guideline in designing systems - what's worth shooting for?
- Theoretical insights can lead to practical insights

So now what?

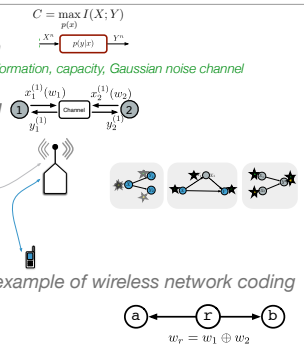


Unsolved

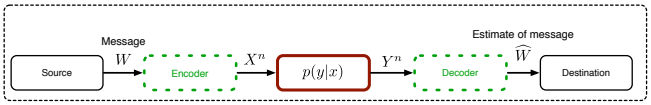
Fundamental

Outline

- Information theory - *what, why, when*
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel
- **Two-way channel - *channel coding***
- Wireless channels and networks
- Two-way cellular-like networks
- Two-way relay channels - *canonical example of wireless network coding*



One-way channel capacity



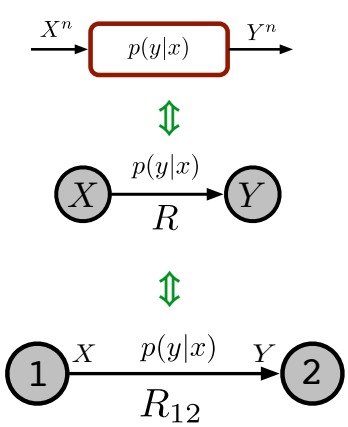
$$C = \max_{p(x)} I(X;Y)$$ bits/channel use

"mutual information" between X and Y

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

symmetric in its arguments!

One-way channel capacity - notation



Two-way channel (historical)

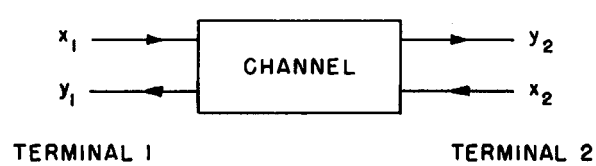
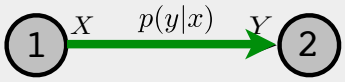


FIGURE 1

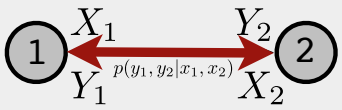
[Shannon '61]

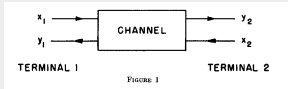
Two-way channel capacity

One-way



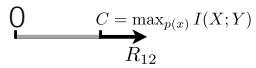
Two-way

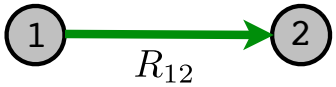




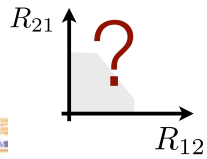
Two-way channel capacity region

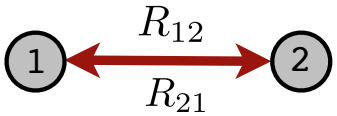
One-way ***Capacity*** ✓





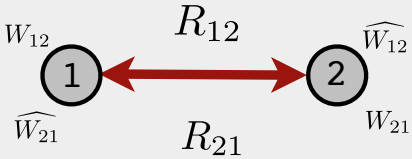
Two-way
Capacity Region





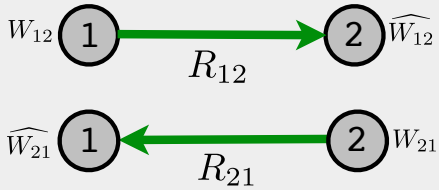
When is

Two-way



equal to

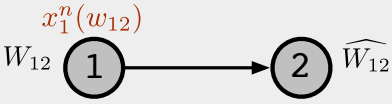
Two one-ways



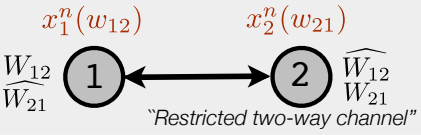
?

Models for two-way adaptation

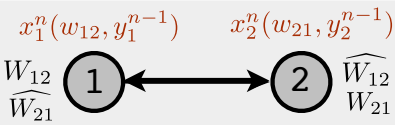
One-way: no adaptation possible



Two-way: no adaptation

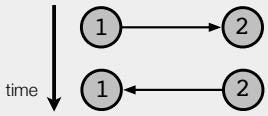


Two-way: full adaptation

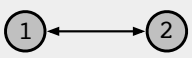


Duplex

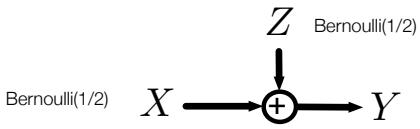
Two-way: half duplex



Two-way: full duplex



(Aside: I(X;Y) and I(X;Y|Z))

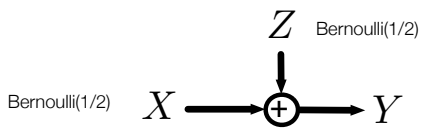


X \ Y	Z = 0	Z = 1
0	0	1
1	1	2

$$I(X;Y) = \sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y)\log\frac{p(x,y)}{p(x)p(y)} = E_{p(x,y)}\log\frac{p(X,Y)}{p(X)p(Y)}$$

$$I(X;Y|Z) = \sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}}\sum_{z\in\mathcal{Z}} p(x,y,z)\log\frac{p(x,y|z)}{p(x|z)p(y|z)} = E_{p(x,y,z)}\log\frac{p(X,Y|Z)}{p(X|Z)p(Y|Z)}$$

(Aside: I(X;Y) and I(X;Y|Z))

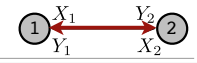


X \ Y \ Z	0	1
0	0	1
1	1	2

$$I(X;Y) = H(Y) - H(Y|X)$$
$$= \log_2(4)/4 + \log_2(2)/2 + \log_2(4)/4 - \log_2(2)$$
$$= 0.5 \text{ bits}$$

$$I(X;Y|Z) = H(Y|Z) - H(Y|X,Z)$$
$$= \log_2(2) - 0$$
$$= 1 \text{ bit}$$

General results



Inner bound

$$R_1 \le I(X_1; Y_2|X_2)$$
$$R_2 \le I(X_2; Y_1|X_1)$$

where X_1 and X_2 follow the joint distribution $p(x_1, x_2) = p(x_1)p(x_2)$.

Not in general equal!

Outer bound

$$R_1 \le I(X_1; Y_2|X_2)$$
$$R_2 \le I(X_2; Y_1|X_1)$$

where the joint distribution of random variables X_1 and X_2 is $p(x_1, x_2)$.

[Shannon '61]

When is capacity known

- Parallel two-way channel
- Mod-2 adder
- Two-way restricted channel
- Two-way "push-to-talk" channel
- Two-way Gaussian noise channel (full & half duplex, restricted & unrestricted)

When is capacity unknown

- General unrestricted discrete memoryless channels
- Binary multiplier channel (BMC)

Capacity: two parallel channels

[Shannon '61]

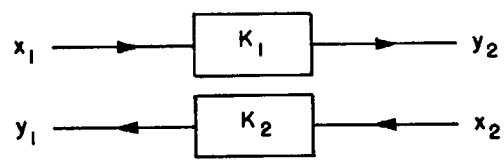
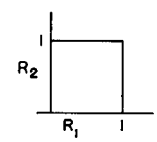


FIGURE 2

$$R_{12} \le C_{K_1}$$
$$R_{21} \le C_{K_2}$$



Capacity: binary mod-2 adder channel

[Shannon '61]

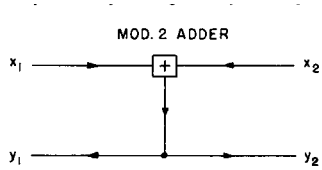


FIGURE 4

$$y_1 = y_2 = x_1 + x_2 \pmod 2.$$

How to achieve capacity region?

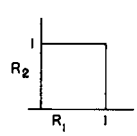


FIGURE 3

Achieving mod 2 adder channel capacity

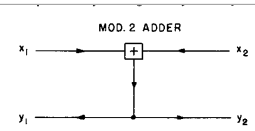


FIGURE 4

$$y_1 = y_2 = x_1 + x_2 \pmod 2.$$

Receiver 1:

$$\hat{x_2} = y_1 \oplus x_1$$
$$= x_1 \oplus x_2 \oplus x_1$$
$$= x_2$$

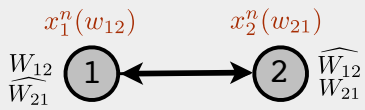
Y_1=Y_2 \ X_2	0	1
X_1 0	0	1
1	1	0

Receiver 2:

$$\hat{x_1} = y_2 \oplus x_2$$
$$= x_1 \oplus x_2 \oplus x_2$$
$$= x_1$$

EXPLOIT TWO-WAY!

Capacity: restricted channel



Capacity region:

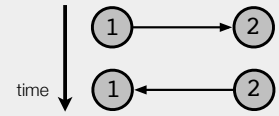
$$R_1 \le I(X_1; Y_2|X_2)$$
$$R_2 \le I(X_2; Y_1|X_1)$$

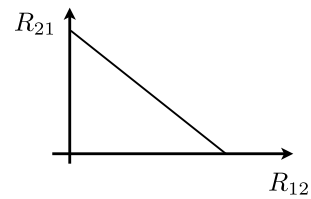
where X_1 and X_2 follow the joint distribution $p(x_1, x_2) = p(x_1)p(x_2)$.

[Shannon '61]

Capacity: ``push-to-talk" channel

Two-way: half duplex





Capacity: Gaussian noise channel

$$Y_1 = aX_1 + bX_2 + N_1 \sim \mathcal{N}(0, \sigma_1^2)$$
$$Y_2 = cX_1 + dX_2 + N_2 \sim \mathcal{N}(0, \sigma_2^2)$$

Capacity region:

$$R_1 \le (1/2)\log(1 + c^2P_1/\sigma_2^2)$$
$$R_2 \le (1/2)\log(1 + b^2P_2/\sigma_1^2)$$

No dependence on ``a" or ``d"

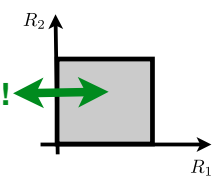
[Han '84]

Capacity: Gaussian noise channel

$$Y_1 = aX_1 + bX_2 + N_1$$
$$Y_2 = cX_1 + dX_2 + N_2$$

$$R_1 \le (1/2)\log(1 + c^2P_1/\sigma_2^2)$$
$$R_2 \le (1/2)\log(1 + b^2P_2/\sigma_1^2)$$

- **TWO PARALLEL CHANNELS!!**
- Achieved by Gaussian inputs
- ``Feedback" does not help here



When is capacity known

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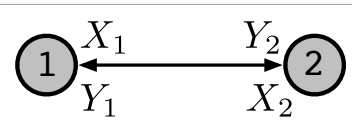
- Two-way Gaussian noise channel (full & half duplex, restricted & unrestricted)

When is capacity unknown

- General unrestricted discrete memoryless channels
- Binary multiplier channel (BMC)

Capacity unknown: Binary Multiplier Channel



$Y_1=Y_2$	X_2: 0	X_2: 1
X_1: 0	0	0
X_1: 1	0	1

$$y_1 = y_2 = x_1x_2, \text{ where } x_1, x_2 \in \{0, 1\}.$$

lower/upper bounds	Rate (bits)
Shannon's lower bound	0.61695
Hagelbarger's lower bound	0.593
Schalkwijk's lower bound	0.63056
Shannon's upper bound	0.69424
Zheng's upper bound	0.6390

C. E. Shannon, *Two-way communication channels*, in Proc. 4th Berkeley Symp. Math. Satist. Probab., vol. 1, 1961, pp. 611-644.
J.P. M. Schalkwijk, *The binary multiplying channel - a coding scheme that operates beyond the Shannon inner bound*, IEEE Trans. Inform. Theory, vol. IT-28(1), pp. 107-110, jan 1982.
J.P. M. Schalkwijk, *On an extension of an achievable rate region for the binary multiplying channel*, IEEE Trans. Inform. Theory, vol. IT-29, pp. 445-448, May 1983.
J.P. M. Schalkwijk, *Extending the achievable rate region of the binary multiplying channel*, In Proc. Int. Symp. on Information Theory, page 302, Budapest, Hungary, Jun 1991.
Z. Zheng, T. Berger,J.P. M. Schalkwijk, *New outer bounds to capacity region of two-way channels*, IEEE Trans. Inform. Theory, vol. IT-32, pp. 383-386, May. 1986.

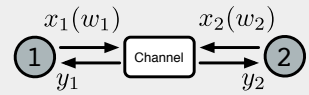
General techniques for two-way channels

Adaptation

Adaptive codewords

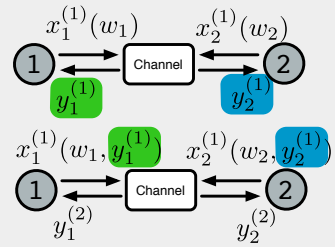
Single-letter

$C = f(X_1, X_2)$



Two-letter

$C = f(X_1^{(1)}, X_1^{(2)}, X_2^{(1)}, X_2^{(2)})$

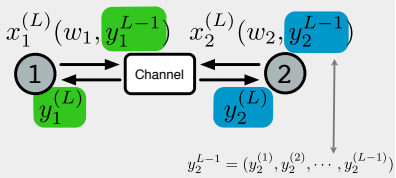


G. Kramer, "Capacity results for the discrete memoryless network," *IEEE Trans. Inf. Theory*, vol. 49, no. 1, pp. 4–21, Jan. 2003.

S. M. S. Tatikonda, "The capacity of channels with feedback," *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 323–349, Jan. 2009.

Adaptive codewords

L-letter



The space over which we can code (x's) is enormous!

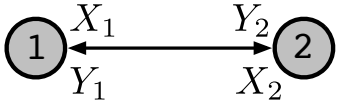
G. Kramer, "Capacity results for the discrete memoryless network," *IEEE Trans. Inf. Theory*, vol. 49, no. 1, pp. 4–21, Jan. 2003.

S. M. S. Tatikonda, "The capacity of channels with feedback," *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 323–349, Jan. 2009.

Non-adaptive codewords:

$X_1 \in \{0, 1\}$

$Y_1 = X_1 X_2$



$Y_2 = Y_1 \mod 2$

$X_2 \in \{0, 1, 2\}$

	Code of user 1		Code of user 2
$u_{11} = 0$	0 0 0	$u_{21} = 0$	1 0 2
$u_{11} = 1$	1 1 1	$u_{21} = 1$	0 2 0

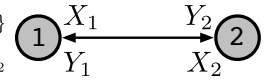
L=N=3 channel uses

G. Kramer, "Directed information for channels with feedback," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 1998.

Adaptive codewords:

$X_1 \in \{0, 1\}$

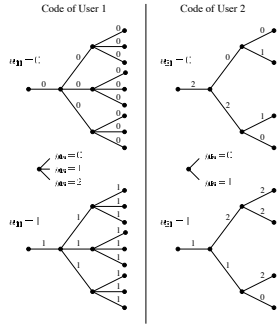
$Y_1 = X_1 X_2$



$Y_2 = Y_1 \mod 2$

$X_2 \in \{0, 1, 2\}$

Adaptive codewords $\mathbf{A}_1^3$



Adaptive codewords $\mathbf{A}_2^3$

L=N=3 channel uses

G. Kramer, "Directed information for channels with feedback," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 1998.

Can we take this adaptation into account?

CAUSAL adaptation

From mutual information to directed information

Need to extend the symmetric $I(X^N;Y^N)$ to account for causally adaptive codewords

$X^N := (X_1, X_2, \cdots, X_N)$
$Y^N := (Y_1, Y_2, \cdots, Y_N)$

Marko/Massey's ``Directed Information"

$$I(X^N \to Y^N) := \sum_{n=1}^{N} I(X^n; Y_n | Y^{n-1})$$

Kramer's ``Causally conditioned Directed Information"

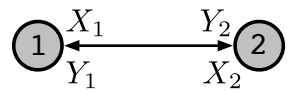
$$I(X^N \to Y^N \| Z^N) := \sum_{n=1}^{N} I(X^n; Y_n | Y^{n-1} Z^n)$$

G. Kramer, "Directed information for channels with feedback," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 1998.

H. Marko, "The bidirectional communication theory- a generalization of information theory," *IEEE Trans. Commun.*, vol. COM-21, no. 12, pp. 1345–1351, Dec. 1973.

J. Massey, "Causality, feedback and directed information," in *Proc. IEEE Int. Symp. Inform. Theory and its Applications*, Honolulu, Nov. 1990.

Aside - could this be what we need?



$$I(X^N;Y^N) = I(Y^N;X^N)$$

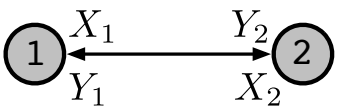
BUT

$$I(X^n \to Y^N) \neq I(Y^n \to X^N)$$

Usage: capacity of two-way channels

$X_1 \in \{0,1\}$

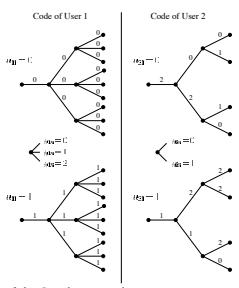
$Y_1 = X_1 X_2$



$Y_2 = Y_1 \mod 2$

$X_2 \in \{0,1,2\}$

Adaptive codewords $\mathbf{A}_1^3$

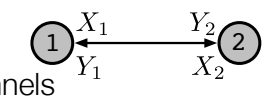


Adaptive codewords $\mathbf{A}_2^3$

L=N=3 channel uses

G. Kramer, "Directed information for channels with feedback," Ph.D. dissertation, Swiss Federal Institute of Technology Zurich, 1998.

Usage: capacity of two-way channels



The capacity region of two-way channel is given by the limit as $L \to \infty$ of the regions

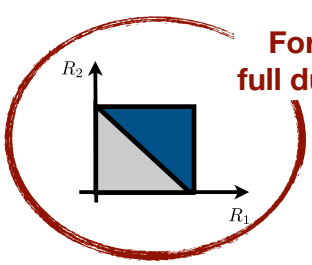
$$R_1 = I(\mathbf{A_1^L} \to Y_2^L \| X_2^L)$$
$$R_2 = I(\mathbf{A_2^L} \to Y_1^L \| X_1^L)$$

where $\mathbf{A_1^L}, \mathbf{A_2^L}$ are adaptive independent codewords.

Difficulty: space of codewords hard to compute!

Take away points - AWGN two-way channel

- If have half-duplex constraint and memoryless channels, time-share
- If have full-duplex - obtain two parallel clean channels

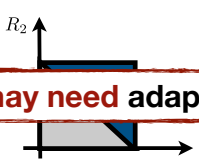


For applications - full duplex gains a lot!

Take away points - Discrete memoryless two-way channel

- If have half-duplex constraint (``push-to-talk"), time-share
- If have parallel two-way channels, mod-2 adder
- If have restricted channel $R_1 \leq I(X_1;Y_2|X_2)$, $R_2 \leq I(X_2;Y_1|X_1)$

where X_1 and X_2 follow the joint distribution $p(x_1,x_2) = p(x_1)p(x_2)$.

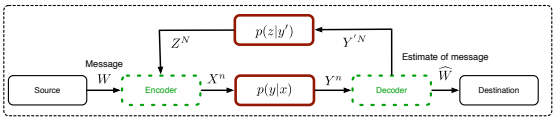


In general may need adaptive codewords

In general OPEN PROBLEM

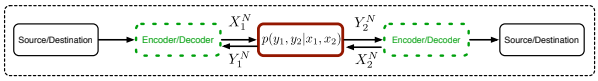
Relationship to feedback channels

- Feedback channel



Information is still one-way!

- Two-way channel



Set rate in ← direction to 0

Properly choose $p(y_1, y_2|x_1, x_2)$

Two-way source coding = what skipping

- Two-way (lossy) source coding using block-coding and protocols of K rounds

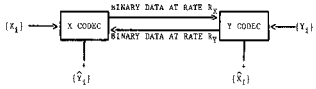
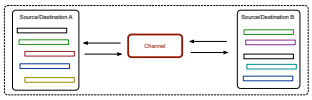


Fig. 1. Two-way source coding scheme.

A. Kaspi, "Two-Way Source Coding with a Fidelity Criterion," *IEEE Trans. Inform. Theory*, vol. IT-31, no. 6, pp. 735–740, November 1985.

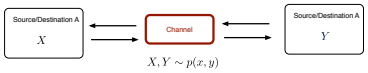
N. Martar. Two-way successively refined joint source-channel coding. *IEEE Trans. Inf. Theory*, 52(4):1483–1494, 2006.

- Set reconciliation framework



Y. Minsky, A. Trachtenberg, and R. Zippel, "Set reconciliation with nearly optimal communication complexity," *IEEE Transactions on Information Theory*, vol. 49, no. 9, pp. 2213–2218, 2003.

- Interactive communication framework / communication complexity

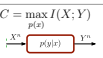


A. Orlitsky, "Worst-case interactive communication I: Two messages are almost optimal," *IEEE Trans. Inform. Theory*, vol. 36, pp. 1111–1126, Sept. 1990.

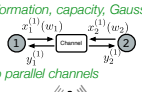
A. Orlitsky, "Worst-case interactive communication II: Two messages are not optimal," *IEEE Trans. Inform. Theory*, vol. 37, pp. 468–480, Apr. 1991.

Outline

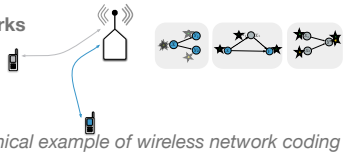
- Information theory - *what, why, when*
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel



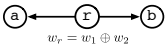
- Two-way channel - *channel coding*
  Adaptive codewords, capacity in Gaussian noise = two parallel channels



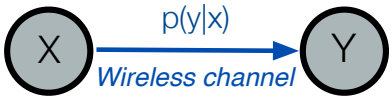
- **Wireless channels and networks**



- Two-way cellular-like networks
- Two-way relay channels - *canonical example of wireless network coding*



Point-to-point



- Channel capacity ✓
- How to approach it for memoryless Gaussian noise channels ✓

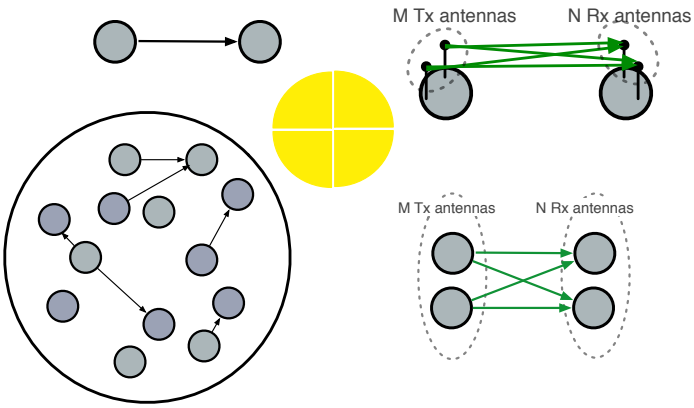
Is that the end of the story?

NO! Motivation 1: two-way channels

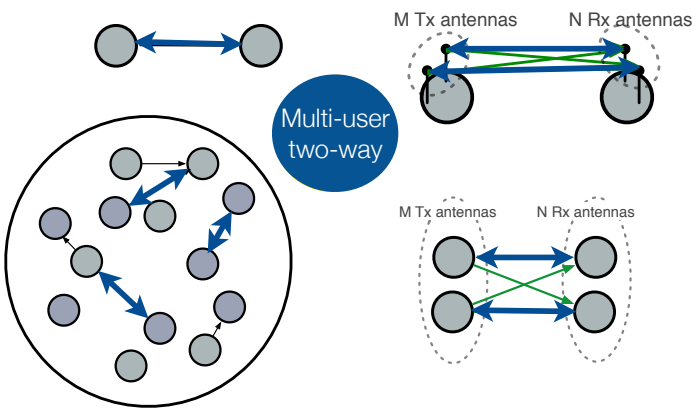
Unsolved

Fundamental

NO! Motivation 2 - networks

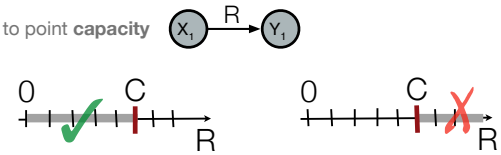


Two-way networks!

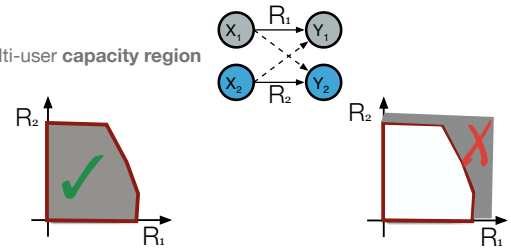


Capacity and capacity regions

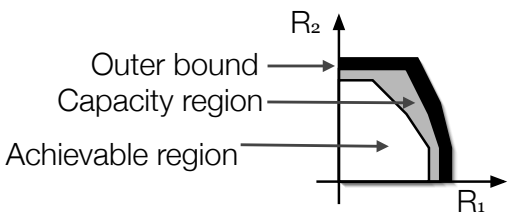
- Point to point **capacity**

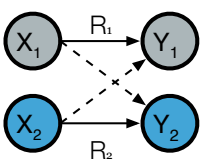


- Multi-user **capacity region**

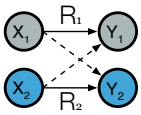


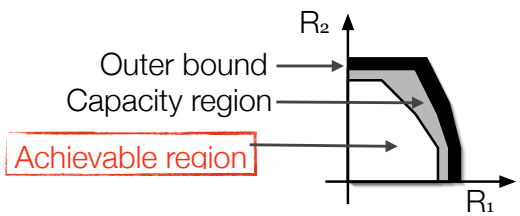
Capacity regions





Achievable rate region

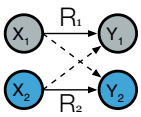


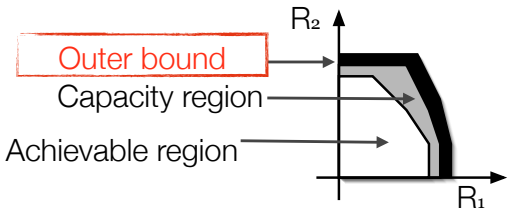


- Propose a coding scheme (random codes!)
- Prove that as long as ⇒ holds, reliable communication possible

$$R_1 \le I(X_1;Y|X_2)$$
$$R_2 \le I(X_2;Y|X_1)$$
$$R_1 + R_2 \le I(X_1,X_2;Y)$$

Outer bound

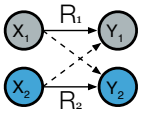


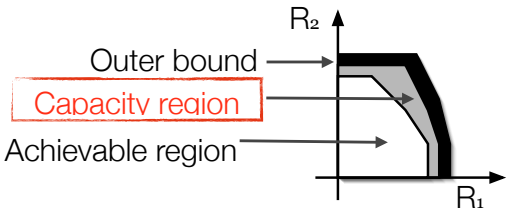


$$R_1 \le I(X_1;Y|X_2)$$
$$R_2 \le I(X_2;Y|X_1)$$
$$R_1 + R_2 \le I(X_1,X_2;Y)$$

- Prove that error is bounded away from 0 when ⇑ not satisfied
- Find a more capable channel whose capacity is known
- Be creative!

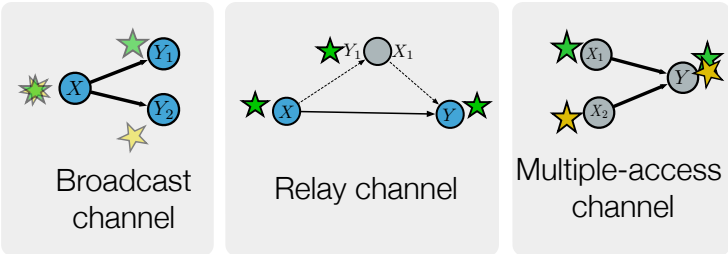
Capacity regions





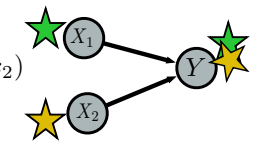
- Limit of communication, NOT how to achieve it in practice necessarily!
- However, benchmark and guidance in practical designs

Three key multi-user channels



Multiple-access channel (MAC)

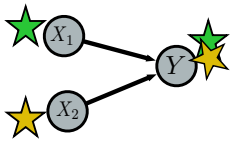
$$p(x_1, x_2) = p(x_1)p(x_2)$$



$$p(y|x_1, x_2)$$

- Introduced by Shannon in 1961
- Capacity known for discrete and Gaussian noise channels
  - Capacity [Ahlswede '71, Liao '72]
  - MIMO [Telatar '99]
  - Fading [Gallager '94, Shamai+Wyner '97, Tse+Hanly '98]

Multiple-access channel (MAC)

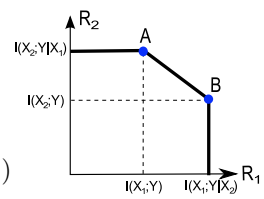


- Capacity region is the closure of the convex hull of all rate pairs (R1, R2) satisfying

$$R_1 \le I(X_1; Y|X_2)$$
$$R_2 \le I(X_2; Y|X_1)$$
$$R_1 + R_2 \le I(X_1, X_2; Y)$$

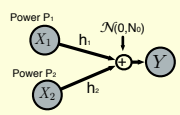
for some distribution

$$p(x_1, x_2, y) = p(x_1)p(x_2)p(y|x_1, x_2)$$



Gaussian MAC

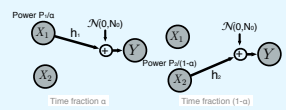
Optimal multiple access



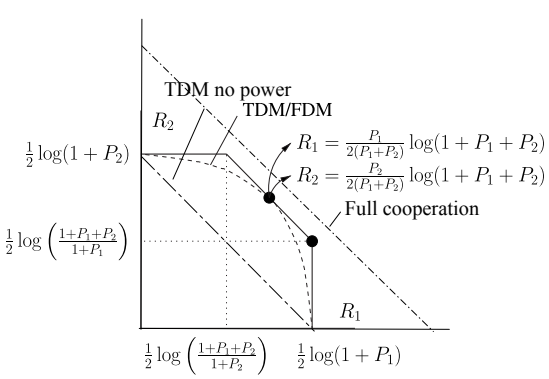
$$R_1 < \frac{1}{2}\log_2(1 + P_1)$$
$$R_2 < \frac{1}{2}\log_2(1 + P_2)$$
$$R_1 + R_2 < \frac{1}{2}\log_2(1 + P_1 + P_2)$$

TDM/FDM multiple access

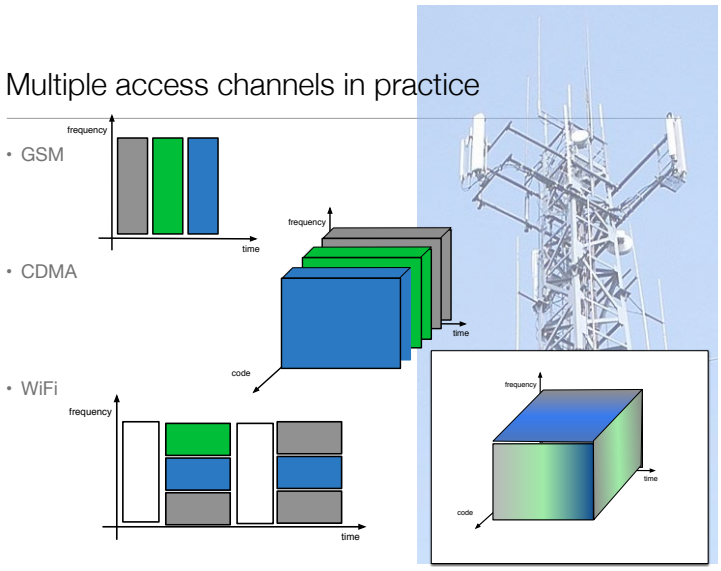
$$R_1 = \frac{\alpha}{2}\log\left(1 + \frac{P_1}{\alpha}\right) \qquad R_1 \le \frac{\alpha}{2}\log_2(1 + P_1)$$
$$R_2 = \frac{1-\alpha}{2}\log\left(1 + \frac{P_2}{1-\alpha}\right) \qquad R_2 \le \frac{1-\alpha}{2}\log_2(1 + P_2)$$



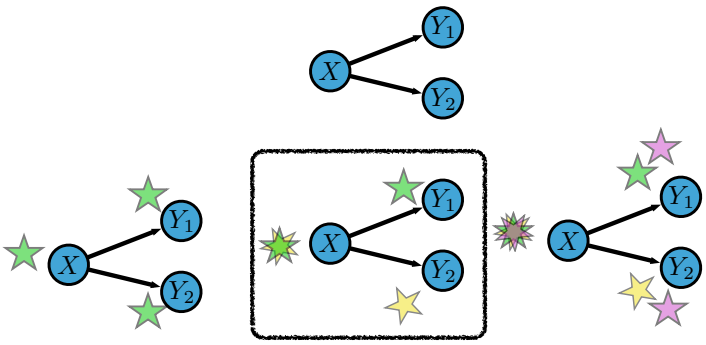
Multiple access channel



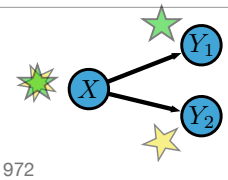
Multiple access channels in practice



Broadcast channel



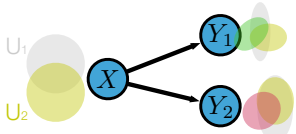
Broadcast channel



$p(y_1, y_2|x)$

- Introduced by Cover in 1972
- Capacity known special cases:
  - Degraded broadcast channels [Bergmans '73'74, Gallager '74]
  - General BC with degraded message sets [Korner + Marton '77]
  - Gaussian MIMO broadcast channel [Weingarten, Steinberg, Shamai '06]
- Best achievable rate region [Marton '79]

Best achievable rate region: Marton's region



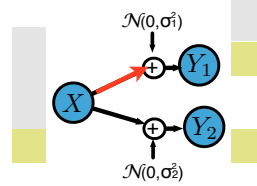
- Capacity region is the closure of the convex hull of all rate pairs (R_1, R_2) satisfying

$$
\begin{aligned}
&0 \le R_1 \le I(U_1;Y_1)\\
&0 \le R_2 \le I(U_2;Y_2)\\
&R_1 + R_2 \le I(U_1;Y_1) + I(U_2;Y_2) - I(U_1;U_2)
\end{aligned}
$$

for some distribution

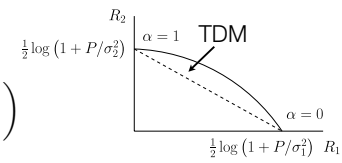
$$p(u_1, u_2, x, y_1, y_2) = p(u_1, u_2)p(x|u_1, u_2)p(y_1, y_2|x)$$

Gaussian broadcast channel

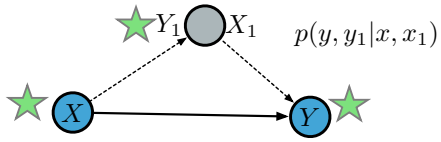


$$R_1 < \frac{1}{2}\log_2\left(1 + \frac{(1-\alpha)P}{\sigma_1^2}\right)$$

$$R_2 < \frac{1}{2}\log_2\left(1 + \frac{\alpha P}{(1-\alpha)P + \sigma_2^2}\right)$$



Relay channel

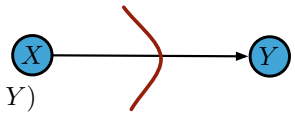


$p(y, y_1|x, x_1)$

- Introduced by Van der Meulen in 1968
- Capacity known special cases:
  - Physically degraded relay channels [Cover, El Gamal '79]
- 3 types of forwarding:
  - Decode+forward (DF), Compress+forward (CF), Amplify+forward (AF)

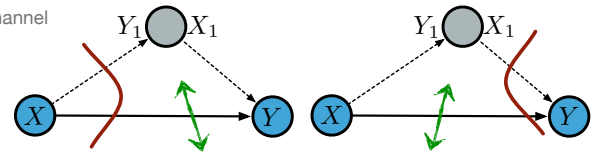
Relay channel: outer bounds

- Cut set outer bound intuitive + useful!
- Point-to-point



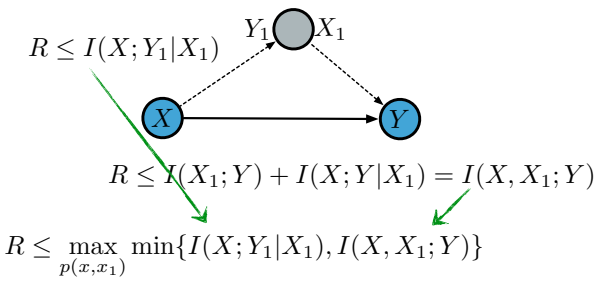
$$R_{out} = \cup_{p(x)} I(X;Y)$$

- Relay channel



$$R_{out} = \cup_{p(x,x_1)} \min\{I(X;Y_1,Y|X_1), I(X,X_1;Y)\}$$

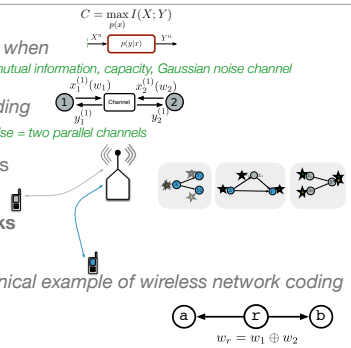
Relay channel: Decode + forward (DF)

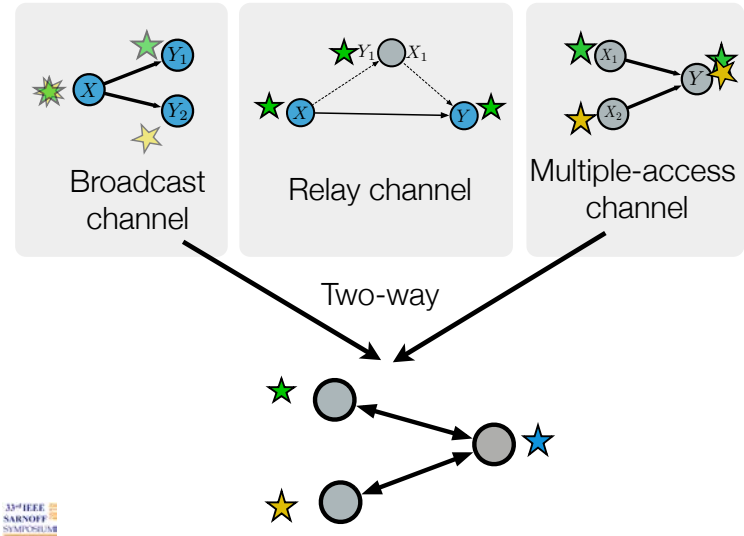


- Exploit broadcast transmission of source
- Source + relay transmit simultaneously (full duplex)
- Create joint codebooks at source + relay

Outline

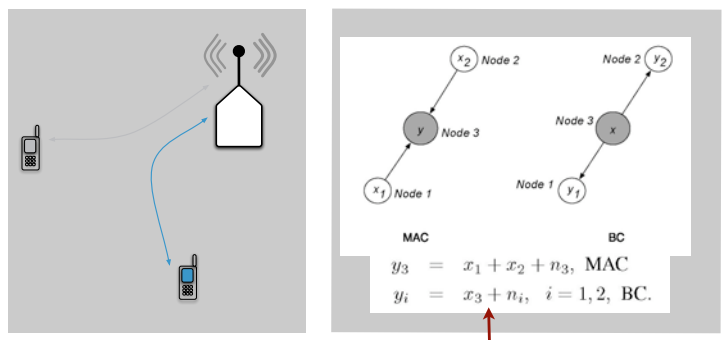
- Information theory - *what, why, when*
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel
- Two-way channel - *channel coding*
  Adaptive codewords, capacity in Gaussian noise = two parallel channels
- Wireless channels and networks
  MAC, relay, BC channels
- **Two-way cellular-like networks**
- Two-way relay channels - *canonical example of wireless network coding*





Two-way MAC/BC channels

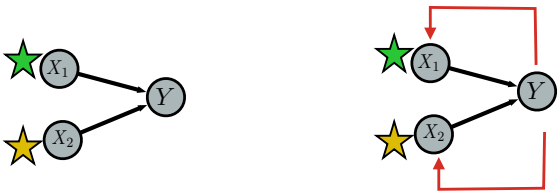
- account for feedback / data transmission tradeoff in a MAC/BC channel



Model, results and image from

D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
D. Dash and A. Sabharwal, "An outer bound for a multiuser two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Monticello, Sep. 2006.

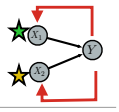
MAC with feedback



No feedback

With feedback

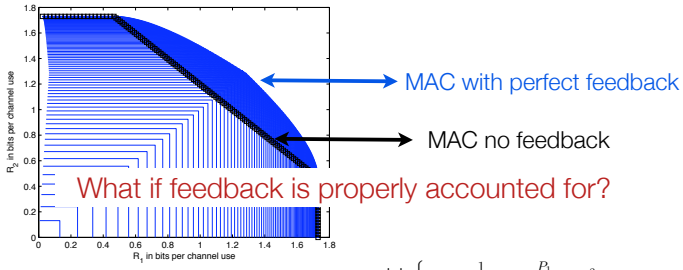
MAC with feedback results



- Discrete Memoryless MAC
  - capacity region with feedback not known
  - with perfect feedback [Cover+Leung '81], [Bross+Lapidoth '05]
  - with generalized feedback [Carleial '82], [King '78]
  - noisy feedback bounds [Gastpar+Kramer '06]
- Gaussian MAC
  - perfect feedback - capacity is known [Ozarow '84]
  - noisy feedback bounds [Gastpar+Kramer '06]

D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
D. Dash and A. Sabharwal, "An outer bound for a multiuser two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Monticello, Sep. 2006.

MAC with feedback visualization



$$C_{fb} = \bigcup_{0\le\rho\le 1}\Big\{ R_1 \le \frac{1}{2}\log(1+\frac{P_1}{N}(1-\rho^2)) \quad R_2 \le \frac{1}{2}\log(1+\frac{P_2}{N}(1-\rho^2)) \quad R_1+R_2 \le \frac{1}{2}\log(1+\frac{P_1+P_2+2\rho\sqrt{P_1P_2}}{N})\Big\}$$

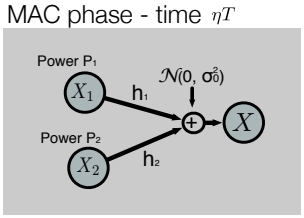
Feedback allows the sources to correlate

D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
D. Dash and A. Sabharwal, "An outer bound for a multiuser two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Monticello, Sep. 2006.

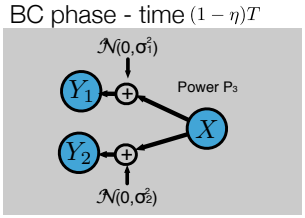
Two-way framework

- half-duplex nodes

MAC phase - time ηT



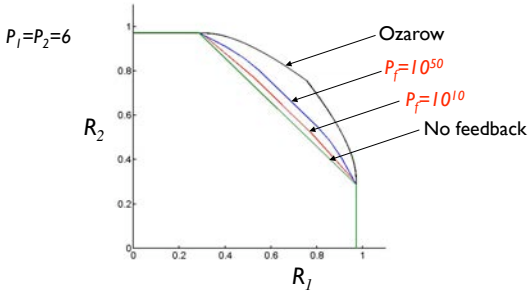
BC phase - time $(1-\eta)T$



- what if BC phase used only for feedback?
- power Pf = P3 is used to feedback

D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
D. Dash and A. Sabharwal, "An outer bound for a multiuser two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Monticello, Sep. 2006.

How large does feedback power need to be?

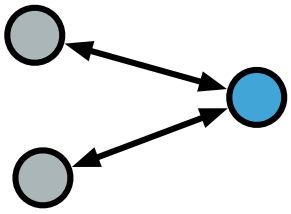


- $R_1 = R_2 = R$ for $P_1 = P_2 = P$
- What is minimum P_f to have a rate gain ?
- Cooperative info = $\log(1+\alpha P) \approx (1-\eta)\log(1+P_f)$

D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
D. Dash and A. Sabharwal, "An outer bound for a multiuser two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Monticello, Sep. 2006.

Two-way is necessary!

- If have data in both directions, then the joint encoding of up+down links outperforms separate treatment of up+down links!
- combine feedback and new information!



D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
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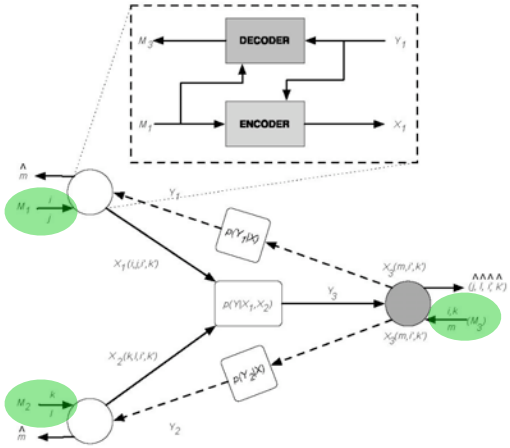

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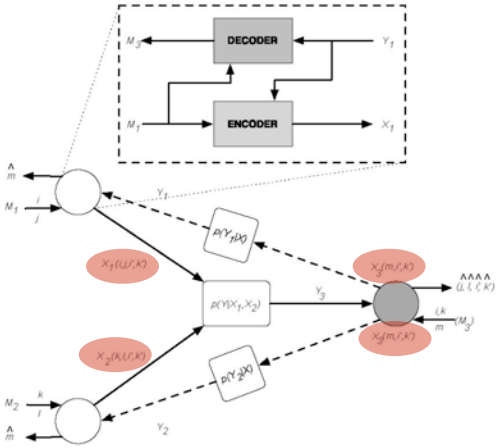

D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
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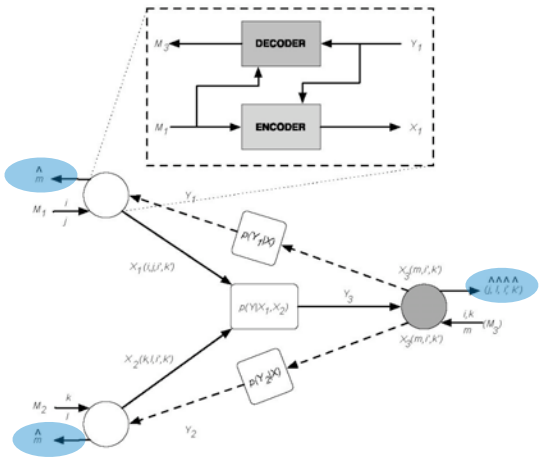
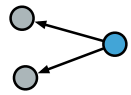
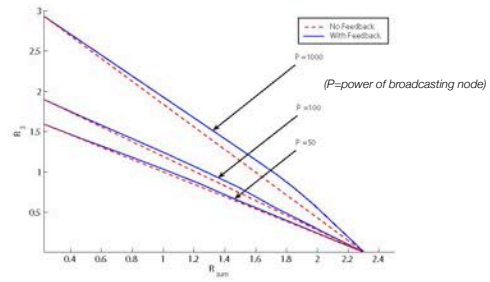
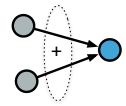


D. Dash, A. Khoshnevis, and A. Sabharwal, "An achievable rate region for a multiuser half-duplex two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2006, pp. 707–711.
D. Dash and A. Sabharwal, "An outer bound for a multiuser two-way channel," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Monticello, Sep. 2006.

Two-way gains seen



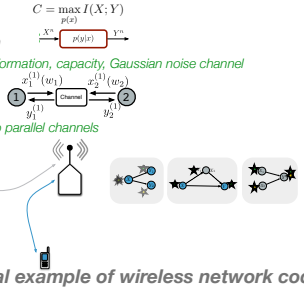


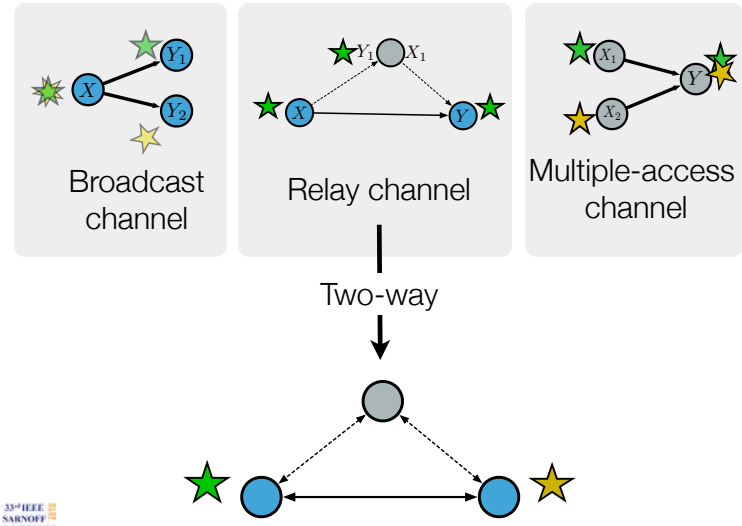


Outline

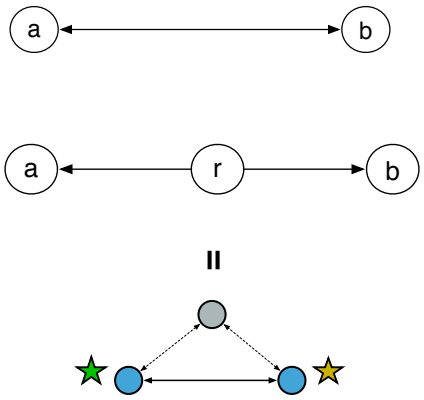
- Information theory - *what, why, when*
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel
- Two-way channel - *channel coding*
  Adaptive codewords, capacity in Gaussian noise = two parallel channels
- Wireless channels and networks
  MAC, relay, BC channels
- Two-way cellular-like networks
  Tradeoff between forward information and feedback
- **Two-way relay channels - *canonical example of wireless network coding***
  - *Single flow, single relay*
  - *Multiple flows, single relay*
  - *Single flow, multiple relays*

$$C = \max_{p(x)} I(X;Y)$$

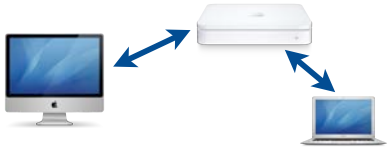


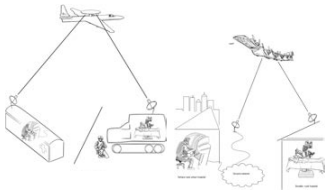


Two-way relay channel

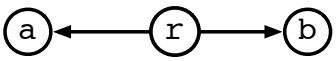


Motivation





Telesurgery



(and extensions)

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D. Gunduz, A. J. Goldsmith, and H. Poor, "MIMO two-way relay channel: Diversity-multiplexing tradeoff analysis," in *Proc. Asilomar Conf. Signals, Systems and Computers*, Pacific Grove, Oct. 2008.
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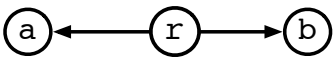

Early work

$$w_r = w_1 \oplus w_2$$

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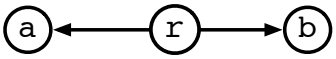

Full-duplex

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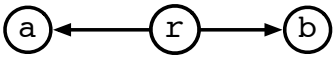

Half-duplex

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M. P. Wilson, K. Narayanan, H. Pfister, and A. Sprintson, "Joint physical layer coding and network coding for bi-directional relaying," 2008. [Online]. Available: http://arxiv.org/abs/0805.0012
R. Wyrembelski, T. Oechtering, I. Bjelakovic, C. Schnurr, and H. Boche, "Capacity of gaussian MIMO bidirectional broadcast channels," in *Proc. IEEE Int. Symp. Inf. Theory*, Toronto, July 2008, pp. 584–588.
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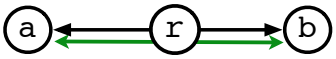

Direct link

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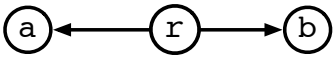

All through the relay

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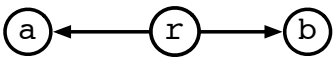

Relaying type

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Decode and forward

Compress and forward

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Lattice codes

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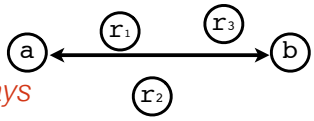

Multiple relays

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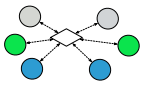
Multiple terminals



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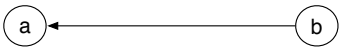
Two-way relay channel: half-duplex

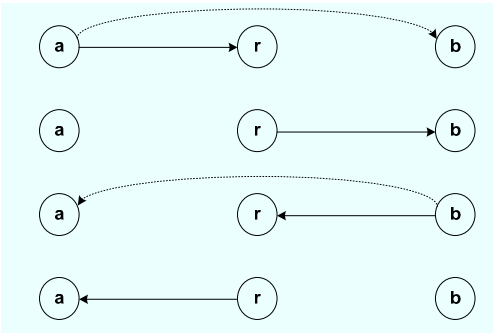
Nodes can either transmit

or receive



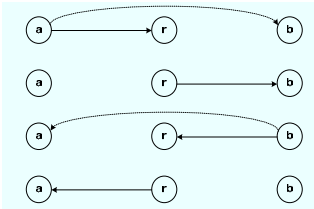
but not both.

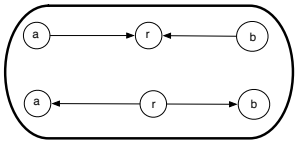
Temporal "phases": who transmits when



Are 4 phases needed? **NO!**

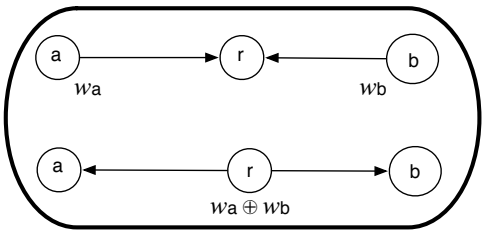
Better protocol





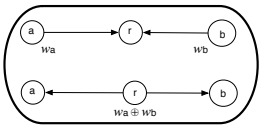
BETTER: 2 phases!

Message-level network coding



Message level network coding

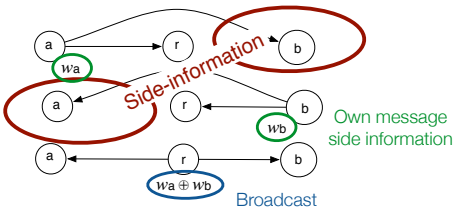
One possible approach we could take:



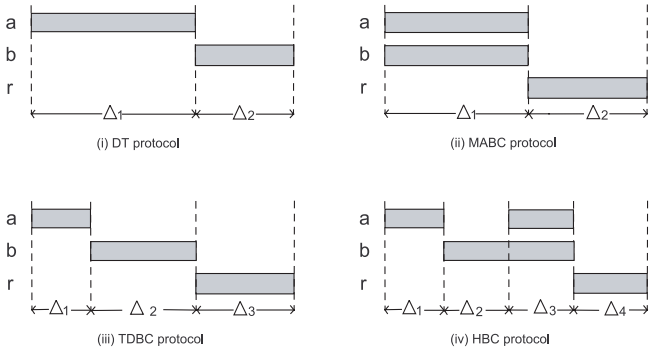
- Transmission Strategy:
  - Phase 1: Node a sends w_a.
  - Phase 2: Node b sends w_b.
  - Phase 3: Node r (the relay) sends $w_a \oplus w_b$
- Decoding
  - Node a computes $w_a \oplus (w_a \oplus w_b) = w_b$, and
  - Node b computes $w_b \oplus (w_a \oplus w_b) = w_a$.
- Is this the best strategy?

Key exploits

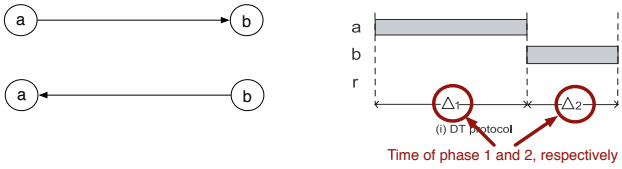
- ``own message side information" at nodes used to cancel out own message
- ``overheard side information" available to nodes when not transmitting
- broadcast nature of wireless channels: relay broadcasts one thing, both nodes hear it.



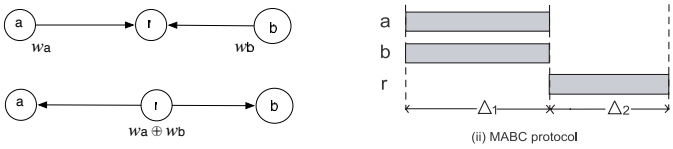
Four possible protocols



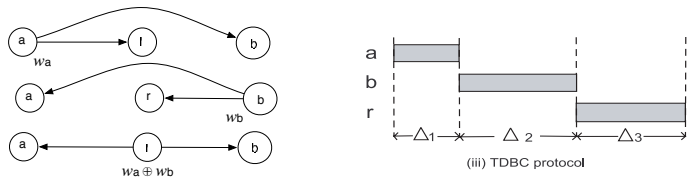
(i) DT: Direct Transmission



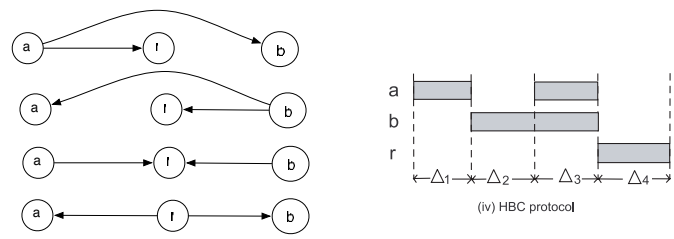
(ii) MABC: Multiple Access Broadcast Channel



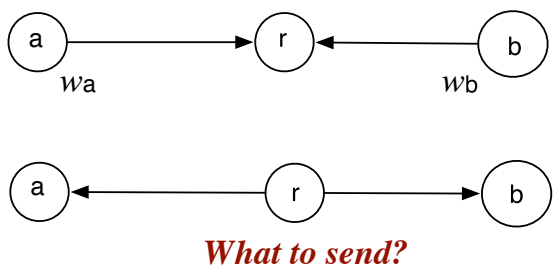
(iii) TDBC: Time Division Broadcast Channel



(iv) HBC: Hybrid Broadcast

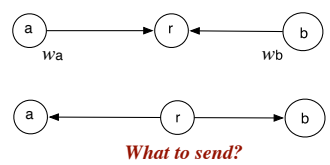


Relaying schemes



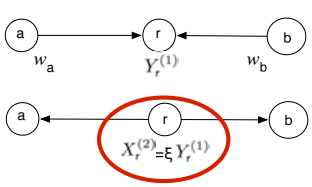
Relaying schemes

- *Amplify and Forward* (AF)
- *Decode and Forward* (DF)
- *Compress and Forward* (CF)
- *Mixed Forward*



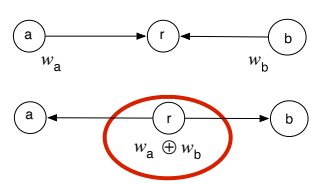
Amplify and forward (AF)

- The relay sends a scaled version of the signal it receives.
- Very little computation is needed.



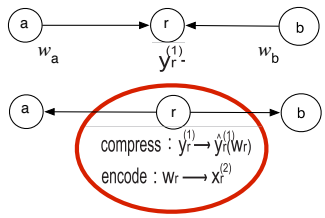
Decode and forward (DF)

- The relay decodes both w_a and w_b.
- Much computation, and transmitter codebooks are needed at the relay.



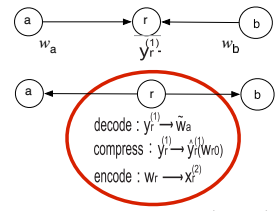
Compress and forward (CF)

- The relay compresses/quantizes the received signal.
- Less computation than DF and transmitter codebooks are not needed at the relay.



Mixed Forward (MF)

- The relay decodes w_a and compresses w_b, combines them into a new message w_r according to a bijective function, which it encodes and transmits.



Bijective function B forms $w_r = B(\tilde{w}_a, w_{r0})$

Comparison of protocols

Protocol	Side information	Phase	Interference
MABC	not present	2	present
TDBC	present	3	not present
HBC	present	4	present
Relaying	**Complexity**	**Noise**	**Relay needs**
AF	very low	carried	nothing
DF	high	eliminated	full codebooks
CF	low	distortion	$p(y_r)$
Mixed	moderate	partially carried	a codebook, $p(y_r)$

Achievable rate regions: one example

- **Theorem 1**: The capacity region of the half-duplex bi-directional relay channel with the MABC protocol is the union of

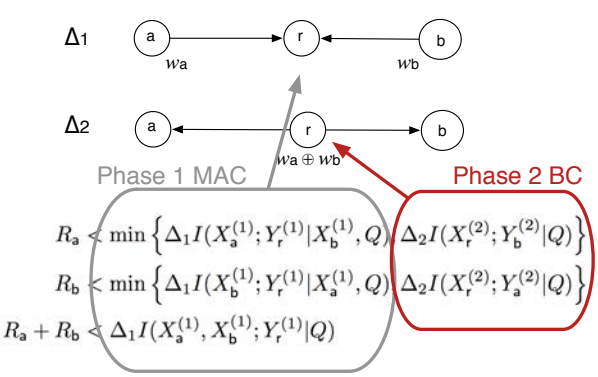
$$R_a < \min\left\{\Delta_1 I(X_a^{(1)}; Y_r^{(1)}|X_b^{(1)}, Q), \Delta_2 I(X_r^{(2)}; Y_b^{(2)}|Q)\right\}$$

$$R_b < \min\left\{\Delta_1 I(X_b^{(1)}; Y_r^{(1)}|X_a^{(1)}, Q), \Delta_2 I(X_r^{(2)}; Y_a^{(2)}|Q)\right\}$$

$$R_a + R_b < \Delta_1 I(X_a^{(1)}, X_b^{(1)}; Y_r^{(1)}|Q)$$

over all joint distributions $p(q)p^{(1)}(x_a|q)p^{(1)}(x_b|q)p^{(2)}(x_r|q)$ with $|\mathcal{Q}| \leq 5$.

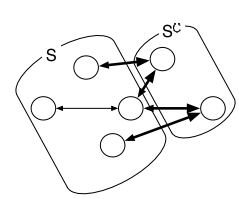
Achievable rate regions: an example



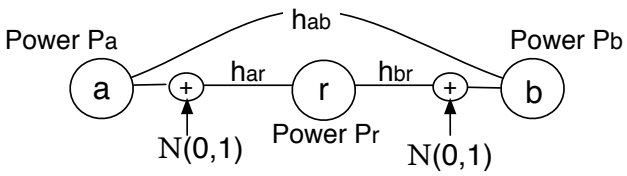
Outer bounds: cut-set bound

If the rates $\{R^{(ij)}\}$ are achievable with a protocol P and $R_\Sigma(S \to S^c)$ denotes the total rate of independent information sent from set S to set S^c then for all sets S:

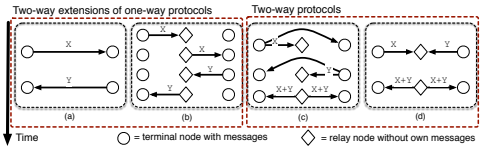
$$R_\Sigma(S \to S^c) \leq \sum_i \Delta_i I(X_{(S)}^{(i)}; Y_{(S_c)}^{(i)}|X_{(S^c)}^{(i)}, Q).$$

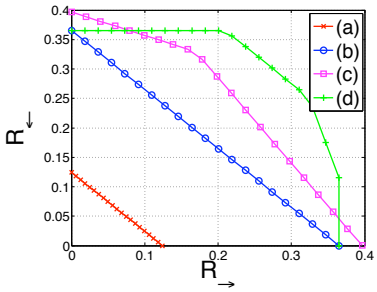


Simulations for the Gaussian noise channel

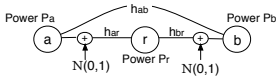


Gaussian simulations

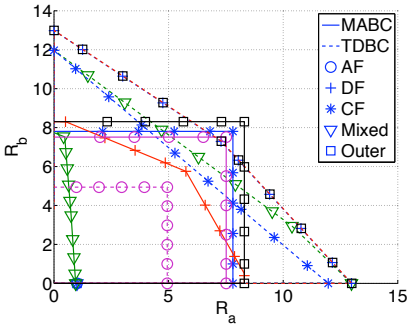




Gaussian simulations



$$h_{\text{ar}} = h_{\text{br}} = 1,\ h_{\text{ab}} = 0.2,\ N = 1,\ \text{and}\ P = 50\ \text{dB}.$$



Recent developments

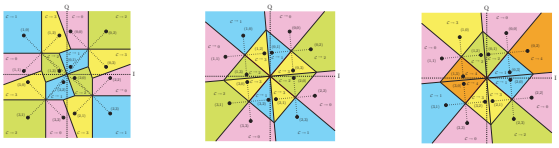
- Capacity is known to within a constant # of bits in Gaussian noise

  Capacity of the Gaussian two-way Relay channel to within 1/2 bit, W. Nam, S.-Y. Chung and W.H. Lee. Submitted to IEEE Trans. Inf, Theory.

  Capacity of the Two Way Relay Channel within a Constant Gap, S. Avestimehr, A. Sezgin, and D. Tse, European Transactions on Telecommunications, to appear.

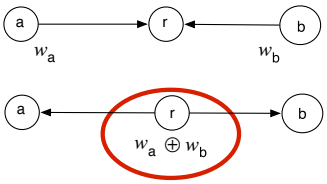
- Constellation design for two-way relaying is considered in

  ``Optimized constellations for two-way wireless relaying with physical network coding,'' Toshiaki Koike-Akino, Petar Popovski, and Vahid Tarokh, *IEEE Journal on Selected Areas in Communications* , vol. 27, no. 5, pp. 773-787, June 2009.



Relation to network coding?

- bit-level / packet level network coding ➔ **Decode and Forward (DF)**

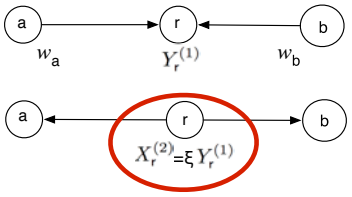


- excellent systems-level demonstration of 2-way relaying gains (all layers, actual testbed)

S. Katti, H. Rahul, W. Hu, D. Katabi, M. Medard, and J. Crowcroft, "XORs in the air: Practical wireless network coding," in *ACM SIGCOMM*, Pisa, Sep. 2006.

Relation to network coding?

- physical / analog network coding ➔ similar to **Amplify and Forward (AF)**

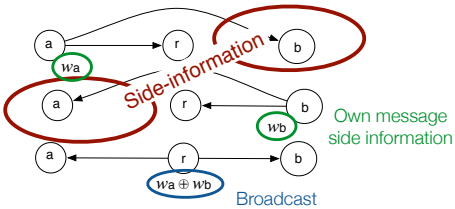


- excellent systems-level demonstration of analog network coding (all layers, actual testbed)

] S. Katti, S. Gollakota, and D. Katabi, "Embracing wireless interference: Analog network coding," in *ACM SIGCOMM*, Kyoto, Aug. 2007.

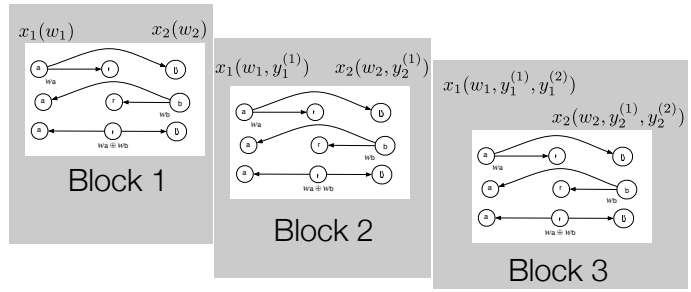
Key exploits

- ``own message side information" at nodes used to cancel out own message
- ``overheard side information" available to nodes when not transmitting
- broadcast nature of wireless channels: relay broadcasts one thing, both nodes hear it.



Open questions

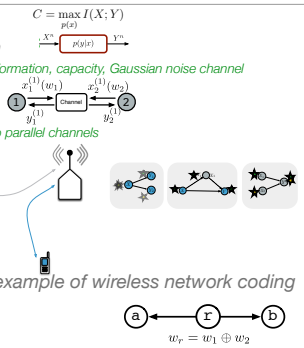
- codeword adaptation?



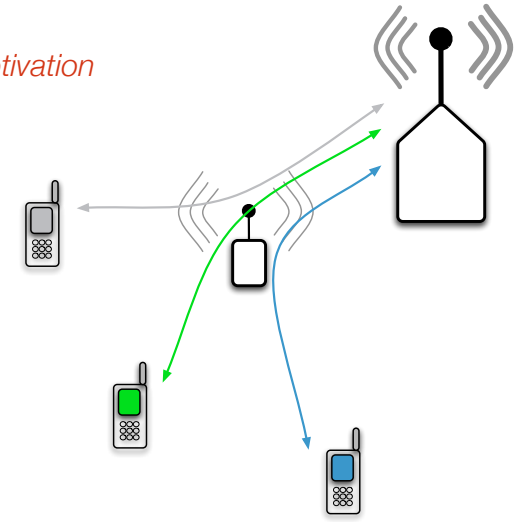
- **Will it be used beyond academic demonstrations?**

Outline

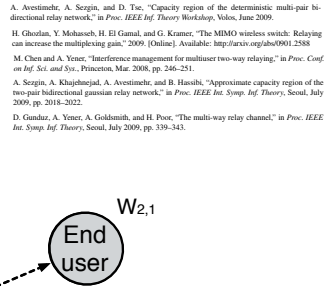
- Information theory - *what, why, when*
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel
- Two-way channel - *channel coding*
  Adaptive codewords, capacity in Gaussian noise = two parallel channels
- Wireless channels and networks
  MAC, relay, BC channels
- Two-way cellular-like networks
  Tradeoff between forward information and feedback
- Two-way relay channels - *canonical example of wireless network coding*
  - *Single flow, single relay*
  - ***Multiple flows, single relay***
  - *Single flow, multiple relays*



Motivation



Multiple terminals



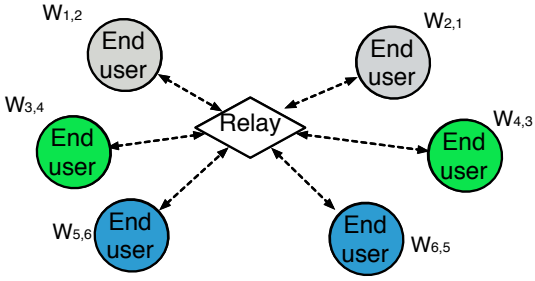
A. Avestimehr, A. Sezgin, and D. Tse, "Capacity region of the deterministic multi-pair bi-directional relay network," in *Proc. IEEE Inf. Theory Workshop*, Volos, June 2009.

H. Ghozlan, Y. Mohasseb, H. El Gamal, and G. Kramer, "The MIMO wireless switch: Relaying can increase the multiplexing gain," 2009. [Online]. Available: http://arxiv.org/abs/0901.2588

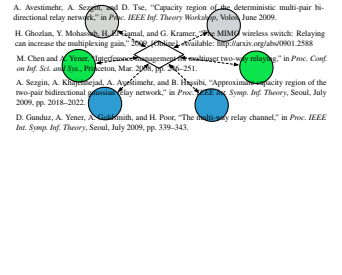
M. Chen and A. Yener, "Interference management for multiuser two-way relaying," in *Proc. Conf. on Inf. Sci. and Sys.*, Princeton, Mar. 2008, pp. 246–251.

A. Sezgin, A. Khajehnejad, A. Avestimehr, and B. Hassibi, "Approximate capacity region of the two-pair bidirectional gaussian relay network," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 2018–2022.

D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 339–343.



Multiple terminals



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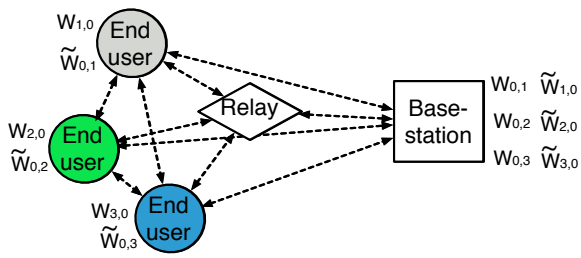

Arbitrary (m) number of end users

Multiple terminals

A. Avestimehr, A. Sezgin, and D. Tse, "Capacity region of the deterministic multi-pair bi-directional relay network," in *Proc. IEEE Inf. Theory Workshop*, Volos, June 2009.

H. Ghozlan, Y. Mohasseb, H. El Gamal, and G. Kramer, "The MIMO wireless switch: Relaying can increase the multiplexing gain," 2009. [Online]. Available: http://arxiv.org/abs/0901.2588

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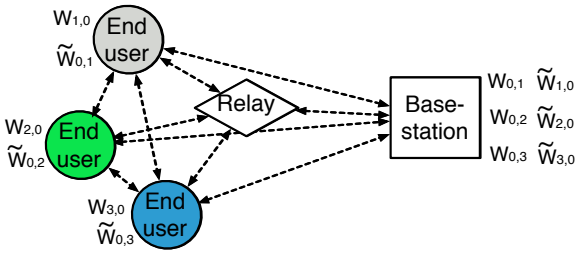
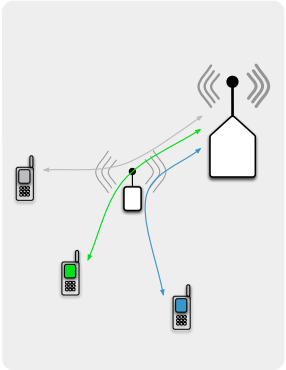
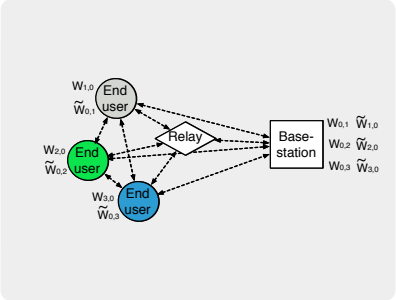
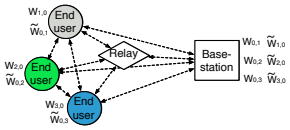


Motivation





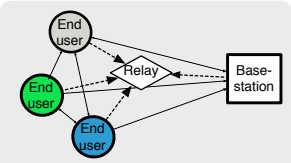
To be presented at ISIT 2010: S. Kim, B. Smida, and N. Devroye, "Multi-pair bi-directional relay networks," in *Proc. IEEE Int. Symp. Inf. Theory*, Austin, TX, Jun. 2010.



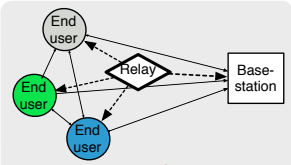
Half-duplex nodes

- Decode + forward relay
- Compress + forward end user cooperation
- Per-flow network coding of messages at relay

"Protocols" = time "phases"

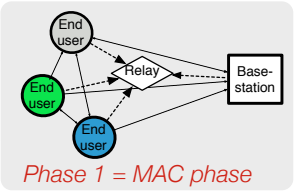


Phase 1 = MAC phase

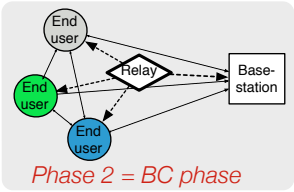


Phase 2 = BC phase

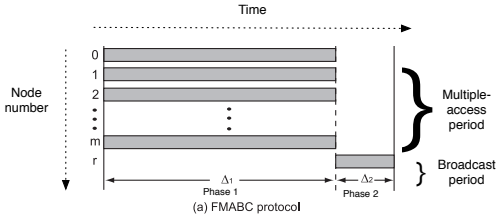
Protocol 1: FMABC (Full MAC then BC)



Phase 1 = MAC phase

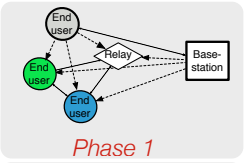


Phase 2 = BC phase

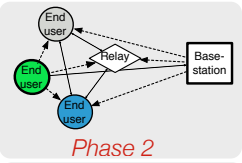


(a) FMABC protocol

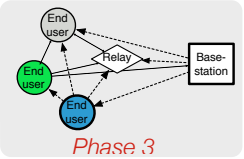
Protocol 2: PMABC (Partial MAC then BC)



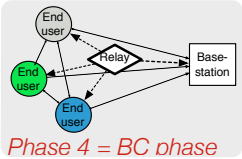
Phase 1



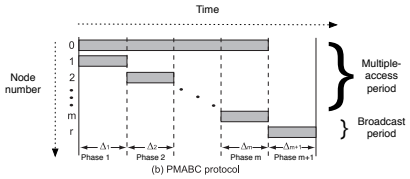
Phase 2



Phase 3

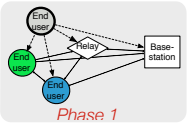


Phase 4 = BC phase

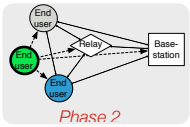


(b) PMABC protocol

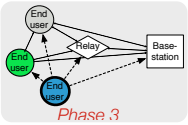
Protocol 3: FTDBC (Full Time Division then BC)



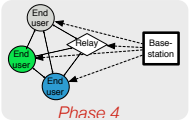
Phase 1



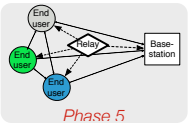
Phase 2



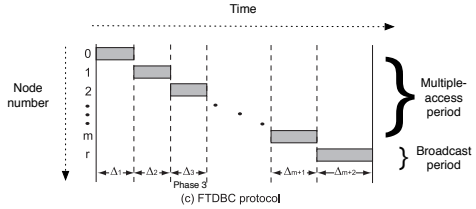
Phase 3



Phase 4

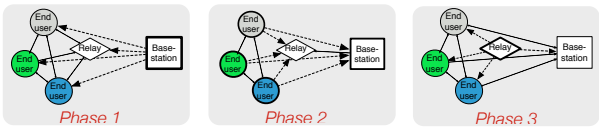


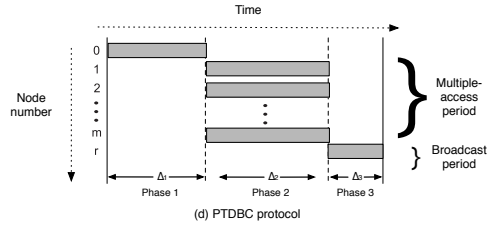
Phase 5

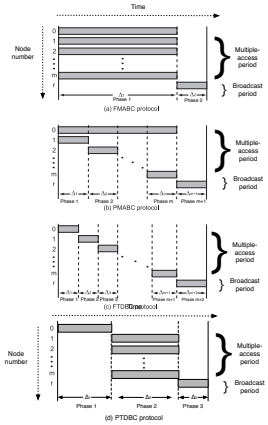


(c) FTDBC protocol

Protocol 4: PTDBC (Partial Time Division then BC)



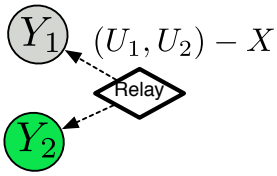




Which protocol is "better"?

1. Extended Marton's region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. Terminal node cooperation

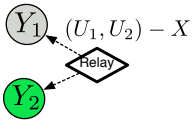
Marton's region



$$R_1 \le I(U_1; Y_1)$$
$$R_2 \le I(U_2; Y_2)$$
$$R_1 + R_2 \le I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2)$$

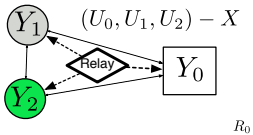
over all joint distributions $p(u_1, u_2, x)$

Extended Marton's



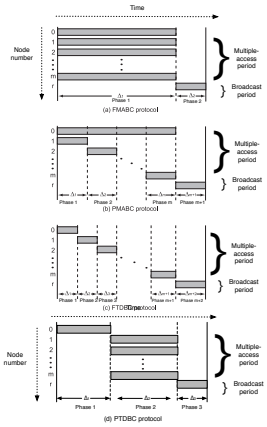
$$R_1 \le I(U_1; Y_1)$$
$$R_2 \le I(U_2; Y_2)$$
$$R_1 + R_2 \le I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2)$$

over all joint distributions $p(u_1, u_2, x)$



$$R_0 \le I(U_0; Y_0)$$
$$R_1 \le I(U_1; Y_1)$$
$$R_2 \le I(U_2; Y_2)$$
$$R_0 + R_1 \le I(U_0; Y_0) + I(U_1; Y_1) - I(U_0; U_1)$$
$$R_0 + R_2 \le I(U_0; Y_0) + I(U_2; Y_2) - I(U_0; U_2)$$
$$R_1 + R_2 \le I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2)$$
$$R_0 + R_1 + R_2 \le I(U_0; Y_0) + I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_0) - I(U_2; U_1, U_0)$$

over all joint distributions $p(u_0, u_1, u_2, x)$

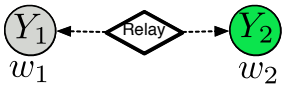


Which protocol is "better"?

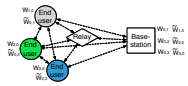
1. Extended Marton's region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. Terminal node cooperation

Per-flow Network coding (N)

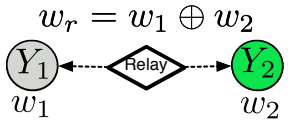
$$w_r = w_1 \oplus w_2$$



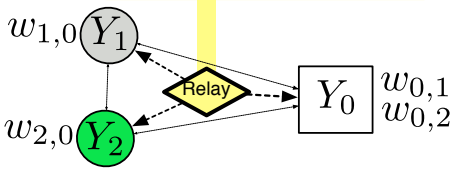
Per-flow Network coding (N)



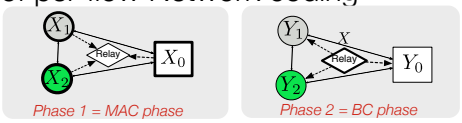
$$w_r = w_1 \oplus w_2$$



$$w_{r,1} = w_{1,0} \oplus w_{0,1}$$
$$w_{r,2} = w_{2,0} \oplus w_{0,2}$$
$$x_r(w_{r,1}, w_{r,2})$$



Example of per-flow Network coding



FMABC

$$R_{1,0} \le \Delta_1 I(X_1^{(1)}; Y_r^{(1)} | X_0^{(1)}, X_2^{(1)})$$
$$R_{2,0} \le \Delta_1 I(X_2^{(1)}; Y_r^{(1)} | X_0^{(1)}, X_1^{(1)})$$
$$R_{0,1} + R_{0,2} \le \Delta_1 I(X_0^{(1)}; Y_1^{(1)} | X_1^{(1)}, X_2^{(1)})$$
$$R_{0,1} + R_{1,0} + R_{1,0} \le \Delta_1 I(X_0^{(1)}, X_1^{(1)}; Y_r^{(1)} | X_2^{(1)})$$
$$R_{0,1} + R_{0,2} + R_{2,0} \le \Delta_1 I(X_0^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_1^{(1)})$$
$$R_{1,0} + R_{2,0} \le \Delta_1 I(X_1^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_0^{(1)})$$
$$R_{0,1} + R_{0,2} + R_{1,0} + R_{2,0} \le \Delta_1 I(X_0^{(1)}, X_1^{(1)}, X_2^{(1)}; Y_r^{(1)})$$

$$\equiv$$

FMABC-N

$$R_{1,0} \le \Delta_1 I(X_1^{(1)}; Y_r^{(1)} | X_0^{(1)}, X_2^{(1)})$$
$$R_{2,0} \le \Delta_1 I(X_2^{(1)}; Y_r^{(1)} | X_0^{(1)}, X_1^{(1)})$$
$$R_{0,1} + R_{0,2} \le \Delta_1 I(X_0^{(1)}; Y_1^{(1)} | X_1^{(1)}, X_2^{(1)})$$
$$R_{0,1} + R_{0,2} + R_{1,0} \le \Delta_1 I(X_0^{(1)}, X_1^{(1)}; Y_r^{(1)} | X_2^{(1)})$$
$$R_{0,1} + R_{0,2} + R_{2,0} \le \Delta_1 I(X_0^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_1^{(1)})$$
$$R_{1,0} + R_{2,0} \le \Delta_1 I(X_1^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_0^{(1)})$$
$$R_{0,1} + R_{0,2} + R_{1,0} + R_{2,0} \le \Delta_1 I(X_0^{(1)}, X_1^{(1)}, X_2^{(1)}; Y_r^{(1)})$$

$$R_{0,1} \le \Delta_2 I(U_1^{(2)}; Y_1^{(2)})$$
$$R_{0,2} \le \Delta_2 I(U_2^{(2)}; Y_2^{(2)})$$
$$R_{1,0} + R_{2,0} \le \Delta_2 I(U_0^{(2)}; Y_0^{(2)})$$
$$R_{1,0} + R_{2,0} + R_{0,1} \le \Delta_2 (I(U_0^{(2)}; Y_0^{(2)}) + I(U_1^{(2)}; Y_1^{(2)}) - I(U_0^{(2)}; U_1^{(2)}))$$
$$R_{1,0} + R_{2,0} + R_{0,2} \le \Delta_2 (I(U_0^{(2)}; Y_0^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_0^{(2)}; U_2^{(2)}))$$
$$R_{0,1} + R_{0,2} \le \Delta_2 (I(U_1^{(2)}; Y_1^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_1^{(2)}; U_2^{(2)}))$$
$$R_{1,0} + R_{2,0} + R_{0,1} + R_{0,2} \le \Delta_2 (I(U_0^{(2)}; Y_0^{(2)}) + I(U_1^{(2)}; Y_1^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_1^{(2)}; U_0^{(2)}) - I(U_2^{(2)}; U_1^{(2)}, U_0^{(2)}))$$

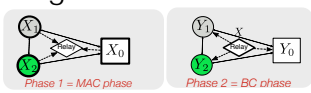
$$R_{0,1} \le \Delta_2 I(U_1^{(2)}; Y_1^{(2)})$$
$$R_{0,2} \le \Delta_2 I(U_2^{(2)}; Y_2^{(2)})$$
$$R_{1,0} \le \Delta_2 I(U_1^{(2)}; Y_0^{(2)}, U_2^{(2)})$$
$$R_{2,0} \le \Delta_2 I(U_2^{(2)}; Y_0^{(2)}, U_1^{(2)})$$
$$R_{1,0} + R_{2,0} \le \Delta_2 I(U_1^{(2)}, U_2^{(2)}; Y_0^{(2)})$$
$$R_{0,1} + R_{0,2} \le \Delta_2 (I(U_1^{(2)}; Y_1^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_1^{(2)}; U_2^{(2)}))$$

Phase 2 = BC phase

Example of per-flow Network coding

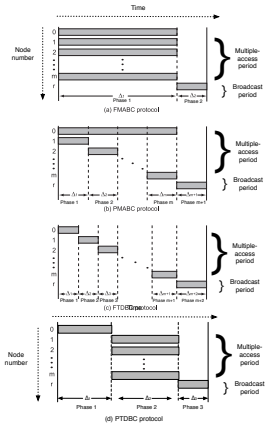
FMABC

$$R_{0,1} \le \Delta_2 I(U_1^{(2)}; Y_1^{(2)})$$
$$R_{0,2} \le \Delta_2 I(U_2^{(2)}; Y_2^{(2)})$$
$$R_{1,0} + R_{2,0} \le \Delta_2 I(U_0^{(2)}; Y_0^{(2)})$$
$$R_{1,0} + R_{2,0} + R_{0,1} \le \Delta_2 (I(U_0^{(2)}; Y_0^{(2)}) + I(U_1^{(2)}; Y_1^{(2)}) - I(U_0^{(2)}; U_1^{(2)}))$$
$$R_{1,0} + R_{2,0} + R_{0,2} \le \Delta_2 (I(U_0^{(2)}; Y_0^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_0^{(2)}; U_2^{(2)}))$$
$$R_{0,1} + R_{0,2} \le \Delta_2 (I(U_1^{(2)}; Y_1^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_1^{(2)}; U_2^{(2)}))$$
$$R_{1,0} + R_{2,0} + R_{0,1} + R_{0,2} \le \Delta_2 (I(U_0^{(2)}; Y_0^{(2)}) + I(U_1^{(2)}; Y_1^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_1^{(2)}; U_0^{(2)}) - I(U_2^{(2)}; U_1^{(2)}, U_0^{(2)}))$$



FMABC-N

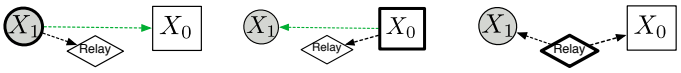
$$R_{0,1} \le \Delta_2 I(U_1^{(2)}; Y_1^{(2)})$$
$$R_{0,2} \le \Delta_2 I(U_2^{(2)}; Y_2^{(2)})$$
$$R_{1,0} \le \Delta_2 I(U_1^{(2)}; Y_0^{(2)}, U_2^{(2)})$$
$$R_{2,0} \le \Delta_2 I(U_2^{(2)}; Y_0^{(2)}, U_1^{(2)})$$
$$R_{1,0} + R_{2,0} \le \Delta_2 I(U_1^{(2)}, U_2^{(2)}; Y_0^{(2)})$$
$$R_{0,1} + R_{0,2} \le \Delta_2 (I(U_1^{(2)}; Y_1^{(2)}) + I(U_2^{(2)}; Y_2^{(2)}) - I(U_1^{(2)}; U_2^{(2)}))$$



Which protocol is "better"?

1. Extended Marton's region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. Terminal node cooperation

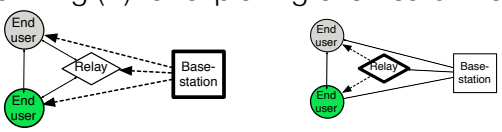
Random binning (R) for exploiting overheard information

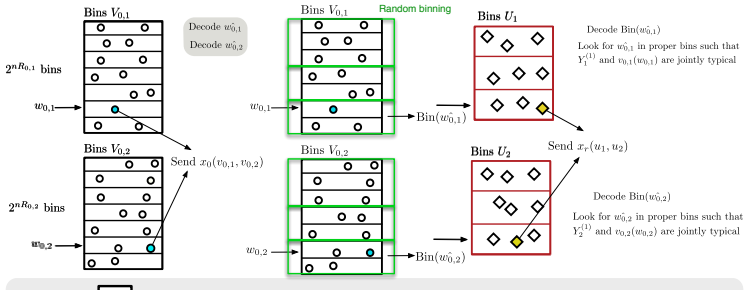


$$R_{1,0} \le \Delta_1 I(X_1^{(1)}; Y_r^{(1)})$$
$$R_{1,0} \le \Delta_1 I(X_1^{(1)}; Y_0^{(1)}) + \Delta_3 I(X_r^{(3)}; Y_0^{(3)})$$
$$R_{0,1} \le \Delta_2 I(X_0^{(2)}; Y_r^{(2)})$$
$$R_{0,1} \le \Delta_2 I(X_0^{(2)}; Y_1^{(2)}) + \Delta_3 I(X_r^{(3)}; Y_1^{(3)})$$

Random binning with network coding

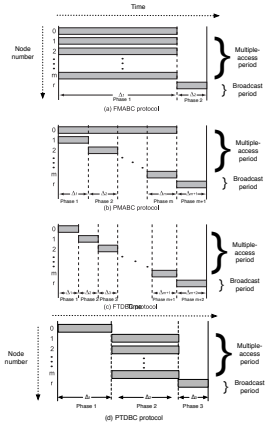
Random binning (R) for exploiting overheard information

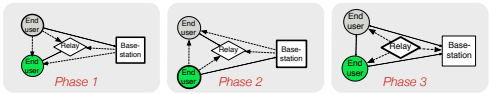


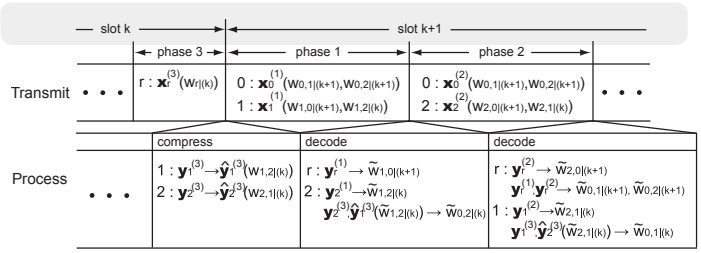


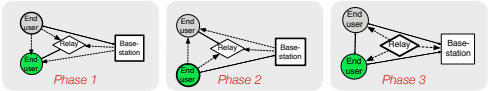
Where Base-station

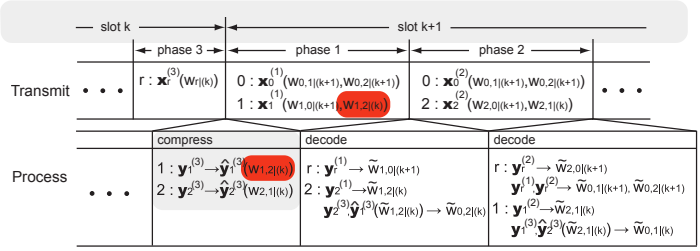
Random binning and Marton's binning

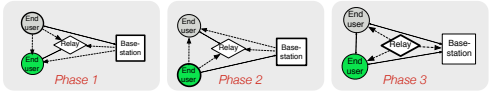


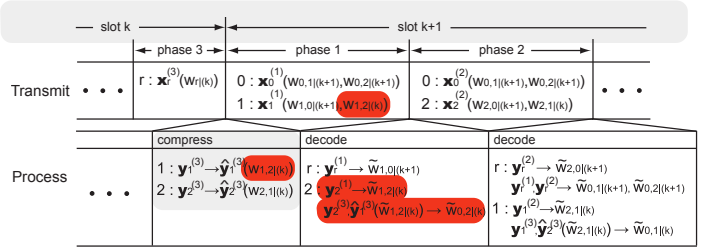


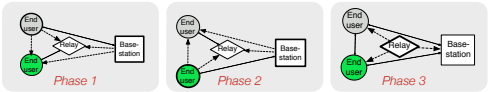


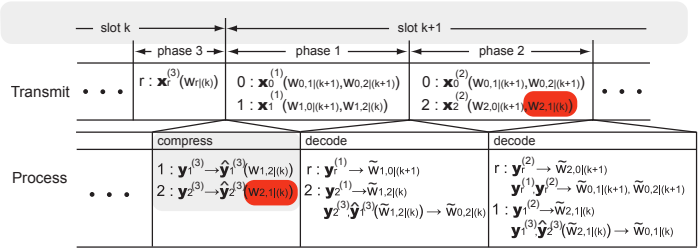


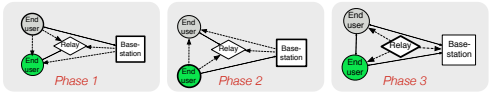


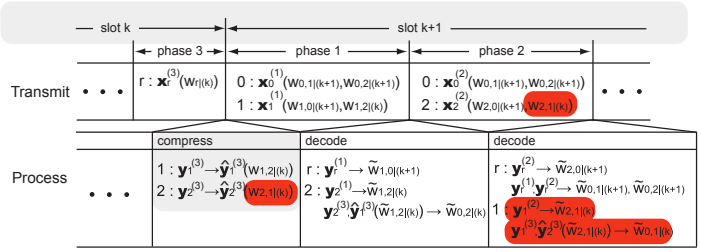












(a) FMABC protocol

(b) PMABC protocol

(c) FTDBC protocol

(d) PTDBC protocol

Which protocol is “better”?

1. Extended Marton’s region for broadcasting
2. Per-flow network coding
3. Random-binning to exploit side-information
4. Terminal node cooperation

Cooperation (C) between terminal nodes

PMABC - NRC

		slot k	slot k+1		
		phase 3	phase 1	phase 2	
Transmit	• • •	r : $\mathbf{x}_r^{(3)}(w_{r\|(k)})$	0 : $\mathbf{x}_0^{(1)}(w_{0,1\|(k+1)},w_{0,2\|(k+1)})$ 1 : $\mathbf{x}_1^{(1)}(w_{1,0\|(k+1)},w_{1,2\|(k)})$	0 : $\mathbf{x}_0^{(2)}(w_{0,1\|(k+1)},w_{0,2\|(k+1)})$ 2 : $\mathbf{x}_2^{(2)}(w_{2,0\|(k+1)},w_{2,1\|(k)})$	• • •
		compress	decode	decode	
Process	• • •	1 : $\mathbf{y}_1^{(3)}\rightarrow\hat{\mathbf{y}}_1^{(3)}(w_{1,2\|(k)})$ 2 : $\mathbf{y}_2^{(3)}\rightarrow\hat{\mathbf{y}}_2^{(3)}(w_{2,1\|(k)})$	r : $\mathbf{y}_r^{(1)}\longrightarrow\tilde{w}_{1,0\|(k+1)}$ 2 : $\mathbf{y}_2^{(1)}\rightarrow\tilde{w}_{1,2\|(k)}$ $\mathbf{y}_2^{(3)},\hat{\mathbf{y}}_1^{(3)}(\tilde{w}_{1,2\|(k)})\longrightarrow\tilde{w}_{0,2\|(k)}$	r : $\mathbf{y}_r^{(2)}\longrightarrow\tilde{w}_{2,0\|(k+1)}$ $\mathbf{y}_r^{(1)},\mathbf{y}_r^{(2)}\longrightarrow\tilde{w}_{0,1\|(k+1)},\tilde{w}_{0,2\|(k+1)}$ 1 : $\mathbf{y}_1^{(2)}\rightarrow\tilde{w}_{2,1\|(k)}$ $\mathbf{y}_1^{(3)},\hat{\mathbf{y}}_2^{(3)}(\tilde{w}_{2,1\|(k)})\longrightarrow\tilde{w}_{0,1\|(k)}$	

Cooperation (C) between terminal nodes

PMABC - NRC

		slot k	slot k+1		
		phase 3	phase 1	phase 2	
Transmit	• • •	r : $\mathbf{x}_r^{(3)}(w_{r\|(k)})$	0 : $\mathbf{x}_0^{(1)}(w_{0,1\|(k+1)},w_{0,2\|(k+1)})$ 1 : $\mathbf{x}_1^{(1)}(w_{1,0\|(k+1)},w_{1,2\|(k)})$	0 : $\mathbf{x}_0^{(2)}(w_{0,1\|(k+1)},w_{0,2\|(k+1)})$ 2 : $\mathbf{x}_2^{(2)}(w_{2,0\|(k+1)},w_{2,1\|(k)})$	• • •
		compress	decode	decode	
Process	• • •	1 : $\mathbf{y}_1^{(3)}\rightarrow\hat{\mathbf{y}}_1^{(3)}(w_{1,2\|(k)})$ 2 : $\mathbf{y}_2^{(3)}\rightarrow\hat{\mathbf{y}}_2^{(3)}(w_{2,1\|(k)})$	r : $\mathbf{y}_r^{(1)}\longrightarrow\tilde{w}_{1,0\|(k+1)}$ 2 : $\mathbf{y}_2^{(1)}\rightarrow\tilde{w}_{1,2\|(k)}$ $\mathbf{y}_2^{(3)},\hat{\mathbf{y}}_1^{(3)}(\tilde{w}_{1,2\|(k)})\longrightarrow\tilde{w}_{0,2\|(k)}$	r : $\mathbf{y}_r^{(2)}\longrightarrow\tilde{w}_{2,0\|(k+1)}$ $\mathbf{y}_r^{(1)},\mathbf{y}_r^{(2)}\longrightarrow\tilde{w}_{0,1\|(k+1)},\tilde{w}_{0,2\|(k+1)}$ 1 : $\mathbf{y}_1^{(2)}\rightarrow\tilde{w}_{2,1\|(k)}$ $\mathbf{y}_1^{(3)},\hat{\mathbf{y}}_2^{(3)}(\tilde{w}_{2,1\|(k)})\longrightarrow\tilde{w}_{0,1\|(k)}$	

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PMABC - NRC

		slot k	slot k+1		
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PMABC - NRC

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Process	• • •	1 : $\mathbf{y}_1^{(3)}\rightarrow\hat{\mathbf{y}}_1^{(3)}(w_{1,2\|(k)})$ 2 : $\mathbf{y}_2^{(3)}\rightarrow\hat{\mathbf{y}}_2^{(3)}(w_{2,1\|(k)})$	r : $\mathbf{y}_r^{(1)}\longrightarrow\tilde{w}_{1,0\|(k+1)}$ 2 : $\mathbf{y}_2^{(1)}\rightarrow\tilde{w}_{1,2\|(k)}$ $\mathbf{y}_2^{(3)},\hat{\mathbf{y}}_1^{(3)}(\tilde{w}_{1,2\|(k)})\longrightarrow\tilde{w}_{0,2\|(k)}$	r : $\mathbf{y}_r^{(2)}\longrightarrow\tilde{w}_{2,0\|(k+1)}$ $\mathbf{y}_r^{(1)},\mathbf{y}_r^{(2)}\longrightarrow\tilde{w}_{0,1\|(k+1)},\tilde{w}_{0,2\|(k+1)}$ 1 : $\mathbf{y}_1^{(2)}\rightarrow\tilde{w}_{2,1\|(k)}$ $\mathbf{y}_1^{(3)},\hat{\mathbf{y}}_2^{(3)}(\tilde{w}_{2,1\|(k)})\longrightarrow\tilde{w}_{0,1\|(k)}$	

Cooperation (C) between terminal nodes

PMABC - NRC

		slot k	slot k+1		
		phase 3	phase 1	phase 2	
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TABLE I

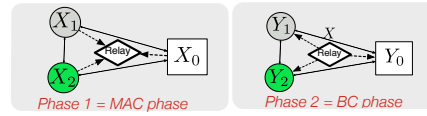
PROTOCOLS AND CODING SCHEMES

Protocol	Multiple Access	Marton's Broadcast	Network coding	Random binning	User cooperation
Simplest	–	–	–	–	–
FMABC	X	X	–	–	–
FMABC-N	X	X	X	–	–
PMABC	X	X	–	–	–
PMABC-NR	X	X	X	X	–
PMABC-NRC	X	X	X	X	X
FTDBC	–	X	–	–	–
FTDBC-NR	–	X	X	X	–
FTDBC-NRC	–	X	X	X	X
PTDBC	X	X	–	–	–
PTDBC-NR	X	X	X	X	–

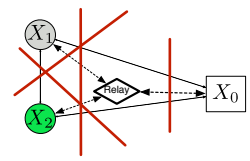
N = Network coding
R = Random binning
C = Cooperation between terminals

Outer bounds - half-duplex cut-set

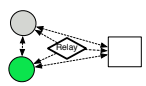
FMABC



$$R_{0,1} + R_{0,2} \leq \Delta_1 I(X_0^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_2^{(1)})$$
$$R_{1,0} + R_{2,0} \leq \Delta_2 I(X_r^{(2)}; Y_0^{(2)})$$
$$R_{1,0} \leq \Delta_1 I(X_1^{(1)}; Y_r^{(1)} | X_2^{(1)}, X_0^{(1)})$$
$$R_{2,0} \leq \Delta_1 I(X_2^{(1)}; Y_r^{(1)} | X_1^{(1)}, X_0^{(1)})$$
$$R_{1,0} + R_{2,0} \leq \Delta_1 I(X_1^{(1)}, X_2^{(1)}; Y_r^{(1)} | X_0^{(1)})$$
$$R_{0,1} \leq \Delta_2 I(X_r^{(2)}; Y_1^{(2)})$$
$$R_{0,2} \leq \Delta_2 I(X_r^{(2)}; Y_2^{(2)})$$
$$R_{0,1} + R_{0,2} \leq \Delta_2 I(X_r^{(2)}; Y_1^{(2)}, Y_2^{(2)})$$



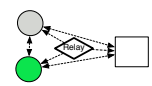
Simulations in Gaussian noise



$$\mathbf{Y}[k] = \mathbf{H}\mathbf{X}[k] + \mathbf{Z}[k]$$

$$\mathbf{H_1} = \begin{bmatrix} 0 & 0.3 & 0.05 & 1 \\ 0.3 & 0 & 1.5 & 1 \\ 0.05 & 1.5 & 0 & 0.2 \\ 1 & 1 & 0.2 & 0 \end{bmatrix} \quad \mathbf{H_2} = \begin{bmatrix} 0 & 0.9 & 0.4 & 1 \\ 0 & 0 & 0.02 & 1 \\ 0 & 0.02 & 0 & 0.5 \\ 1 & 1 & 0.5 & 0 \end{bmatrix}.$$

Simulations in Gaussian noise



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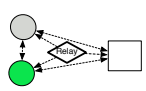
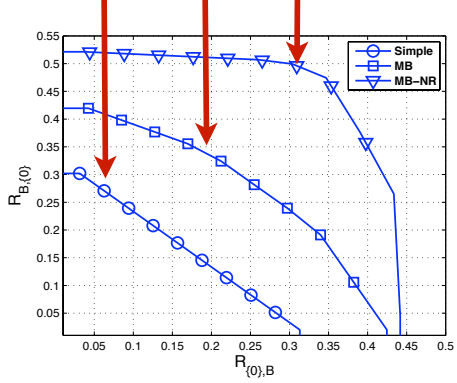
Evaluate and optimize over

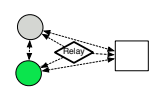
- phase durations
- correlation matrices of Marton binning RVs subject to power constraints
- compression parameters

Simple - MB - MB+NR





Network coding + random binning



FMABC

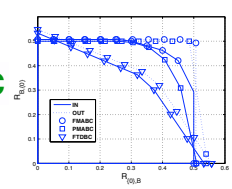


Fig. 7. Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_1$.

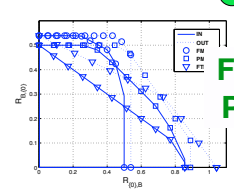


Fig. 8. Comparison with $P_0 = P_1 = P_2 = P_r = 0$ dB, $H = H_2$.

FMABC/ PMABC

PMABC

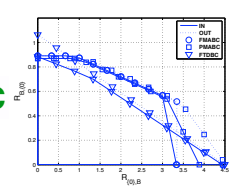


Fig. 9. Comparison with $P_0 = P_r = 20$, $P_1 = P_2 = 0$ dB, $H = H_1$.

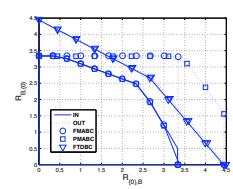
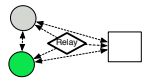
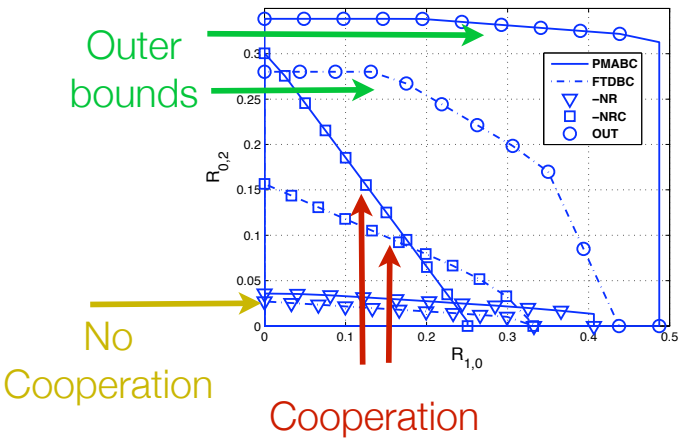


Fig. 10. Comparison with $P_0 = P_1 = P_2 = P_r = 20$ dB, $H = H_1$.

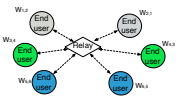
FTDBC





Other recent multi-flow developments

- In A. Sezgin, A. Avestimehr, M. Khajehnejad, and B. Hassibi, "Divide-and-conquer: Approaching the capacity of the two-pair bidirectional Gaussian relay network," *Arxiv preprint arXiv:1001.4271*, 2010. no base-station or direct link is assumed. Capacity is obtained in high-SNR linear deterministic model; capacity to within 2 bits/sec/Hz for general Gaussian channel.

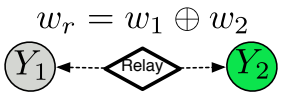


- In D. Gunduz, A. Yener, A. Goldsmith, and H. Poor, "The multi-way relay channel," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, July 2009, pp. 339–343. a generalized multi-way cluster model is explored; bounds on the symmetric rates are obtained.
- In M. Chen and A. Yener, "Interference management for multiuser two-way relaying," in *Proc. Conf. on Inf. Sci. and Sys.*, Princeton, Mar. 2008, pp. 246–251. CDMA up/down-links are assumed.
- In H. Ghozlan, Y. Mohasseb, H. El Gamal, and G. Kramer, "The MIMO wireless switch: Relaying can increase the multiplexing gain," 2009. [Online]. Available: http://arxiv.org/abs/0901.2588 the relay has multiple antennas and the delay-limited diversity-multiplexing tradeoff is explored, showing that the multiplexing gain may be improved (over fully connected interference case).

Multi-flow take-away points

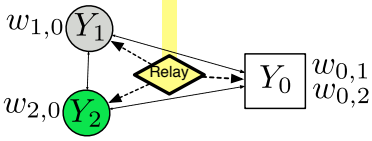
- Most schemes use *per-flow network coding*

$$w_r = w_1 \oplus w_2$$



One flow

$$w_{r,1} = w_{1,0} \oplus w_{0,1}$$
$$w_{r,2} = w_{2,0} \oplus w_{0,2}$$
$$x_r(w_{r,1}, w_{r,2})$$



Multiple flows

Multi-flow take-away points

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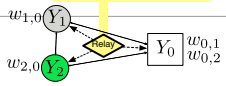
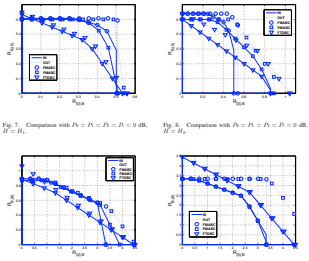


- Most schemes use *per-flow network coding*
- Significantly more complex: protocols and opportunities abound. Only starting to understand when to do what.



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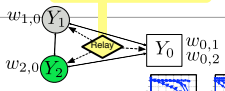

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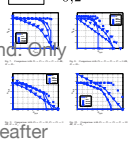
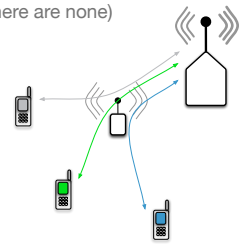


- Due to practical relevance - crucial to develop insight and thereafter demonstrations (there are none)



Multi-flow take-away points

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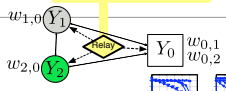

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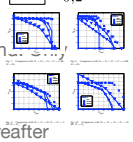


- Due to practical relevance - crucial to develop insight and thereafter demonstrations (there are none)

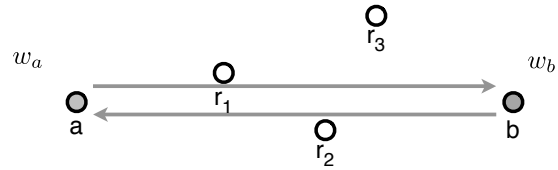
- Two-way nature must be explicitly accounted for (side-information, ability to network code and broadcast) in order to see gains.

Outline

- Information theory - *what, why, when*
  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel
- Two-way channel - *channel coding*
  Adaptive codewords, capacity in Gaussian noise = two parallel channels
- Wireless channels and networks
  MAC, relay, BC channels
- Two-way cellular-like networks
  Tradeoff between forward information and feedback
- Two-way relay channels - *canonical example of wireless network coding*
  - *Single flow, single relay*
  - *Multiple flows, single relay*
  - ***Single flow, multiple relays***

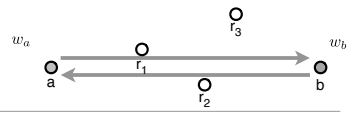
$w_r = w_1 \oplus w_2$

Multiple relays



S. Kim, N. Devroye, and V. Tarokh, "A class of bi-directional multi-relay protocols," in *Proc. IEEE Int. Symp. Inf. Theory*, Seoul, Jun. 2009, pp. 349–353.

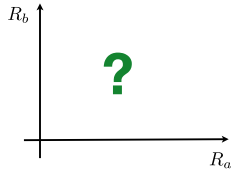
Assumptions



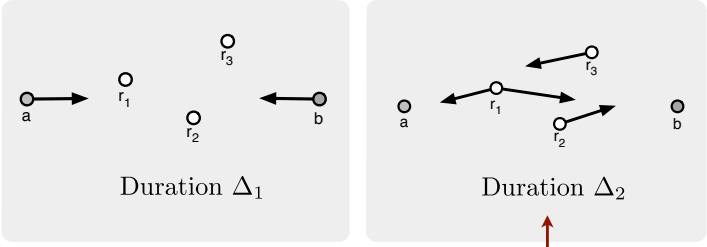
- half-duplex nodes --> protocols in time

- Decode and Forward (DF) and Amplify and Forward (AF) only

- What rates are achievable, what protocols are best?



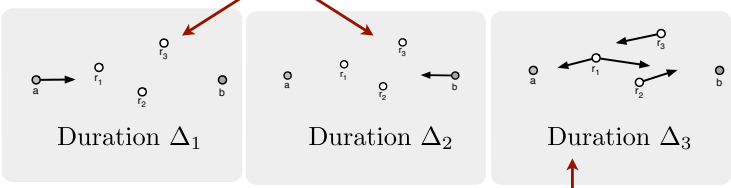
Protocol 1: (m,2) MABC



If r_i decodes w_a and w_b : $x_{ri}(w_a,w_b)$
If r_i decodes w_a only : $x_{ri}(w_a)$
If r_i decodes w_b only : $x_{ri}(w_b)$
Otherwise, r_i is silent.♪

Protocol 2: (m,3) TDBC

Allows for direct transmissions from a to b (side-information)



If r_i decodes w_a and w_b : $x_{ri}(w_a,w_b)$
If r_i decodes w_a only : $x_{ri}(w_a)$
If r_i decodes w_b only : $x_{ri}(w_b)$
Otherwise, r_i is silent.♪

Protocol 3: (m,m+2) Multi-hop Multi-relay

(m,2m+2) MHMR

Better!

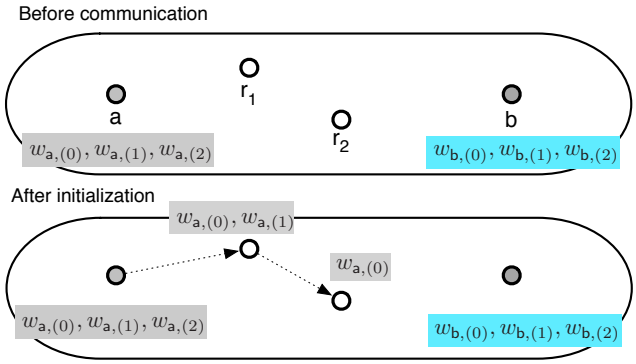
Naive *- does not combine messages in 2 directions!*

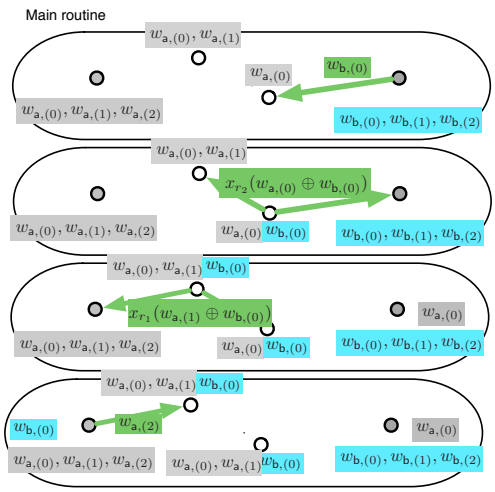
Messages split

$$w_a \longrightarrow w_{a,(0)}, w_{a,(1)}, \cdots w_{a,(K)}$$

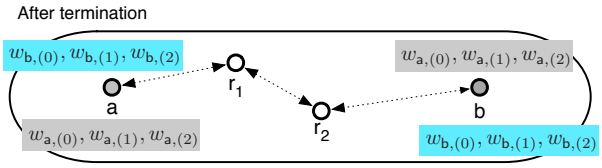
$$w_b \longrightarrow w_{b,(0)}, w_{b,(1)}, \cdots w_{b,(K)}$$

Initialization

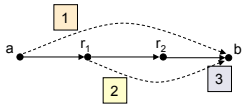




After termination



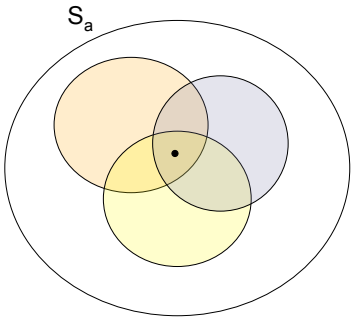
Exploit overheard information



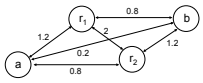
: ambiguity set of a → b

: ambiguity set of r_1 → b

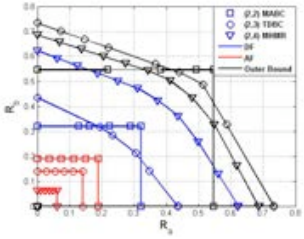
: ambiguity set of r_2 → b



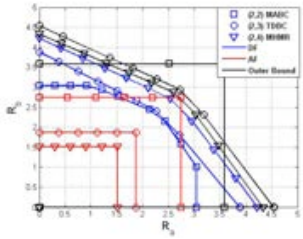
Performance in Gaussian noise



0 dB



20 dB

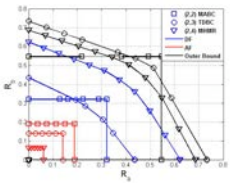


- In low SNR : (2,4) DF MHMR
- In high SNR : (2,4) DF MHMR or (2,2) AF MABC

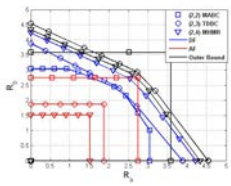
Multi-relay take-aways

- numerical results indicate that (m,m+2) MHMR protocol with information flowing in 2 directions yields the largest rates

0 dB



20 dB



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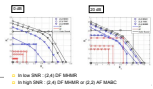
Multi-relay take aways



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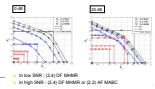

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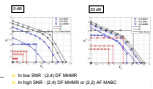
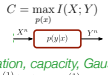


- numerical results indicate that (m,m+2) MHMR protocol with information flowing in 2 directions yields the largest rates
- for low # of hops, or at high SNR, AF relaying may do well, but rapidly degrades as # hops increases or SNR decreases
- fundamentally unsolved, in part due to complexity of space.
- unique two-way characteristics exploited: own message side information to network code (so far only at the bit level rather than the signal level)

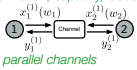
Outline

- Information theory - *what, why, when*

  $C = \max_{p(x)} I(X;Y)$

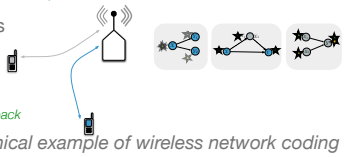
  

  Source coding, channel coding, entropy and mutual information, capacity, Gaussian noise channel
- Two-way channel - *channel coding*

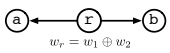
  

  Adaptive codewords, capacity in Gaussian noise = two parallel channels
- Wireless channels and networks

  MAC, relay, BC channels

  
- Two-way cellular-like networks

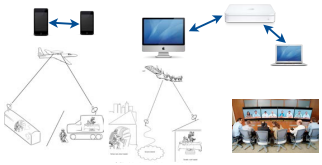
  Tradeoff between forward information and feedback
- Two-way relay channels - *canonical example of wireless network coding*
  - *Single flow, single relay*
  - *Multiple flows, single relay*
  - *Single flow, multiple relays*

  $w_r = w_1 \oplus w_2$

Future areas of two-way channels

- one-way information theory ``fairly" well understood
- advances in processing power
- never ending desire for bandwidth and limited wireless spectrum



→ **Two-way wireless networks**

Questions?

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