Information theoretic limits of cognitive networks

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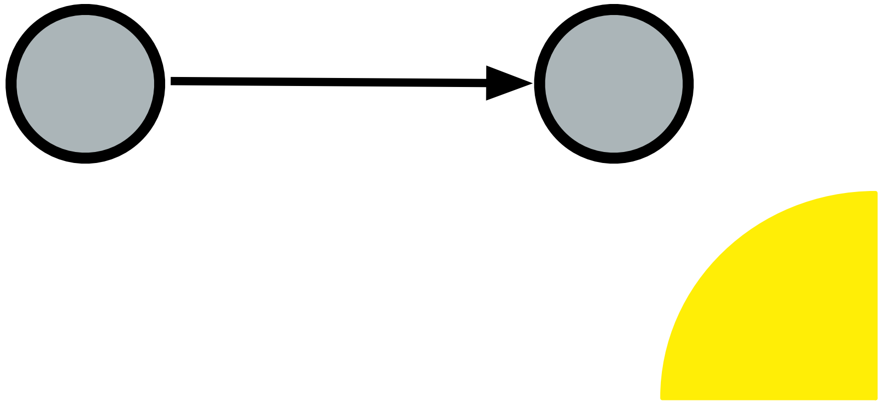
Efficient, reliable communication



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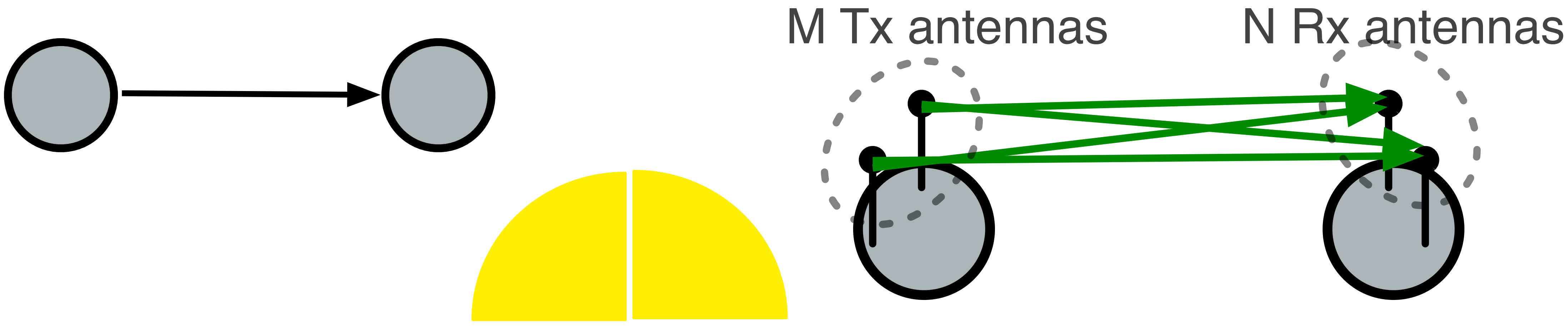

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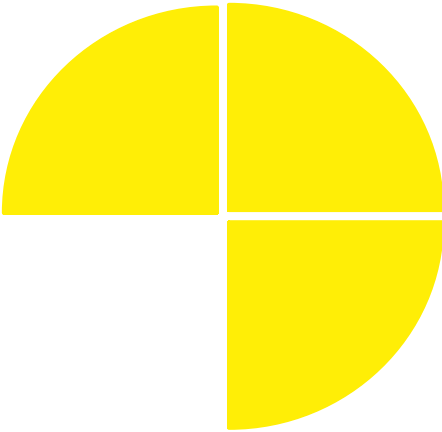

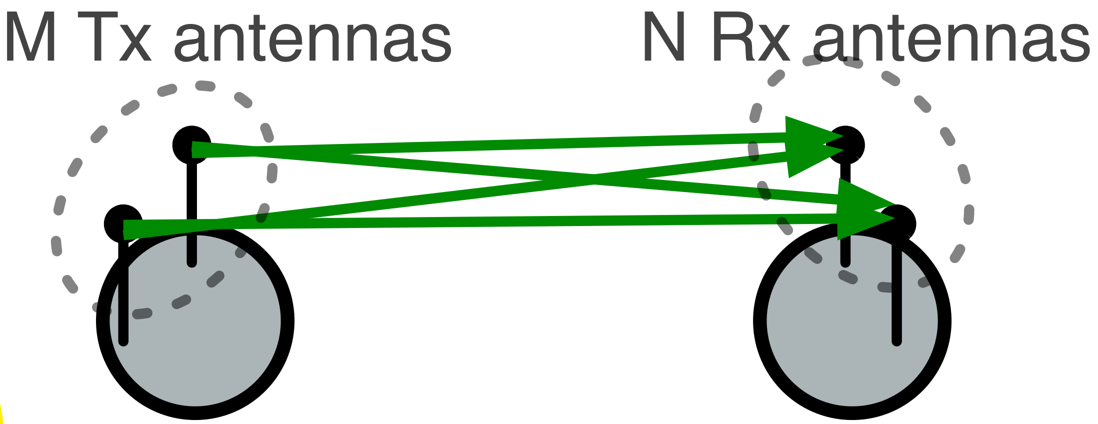

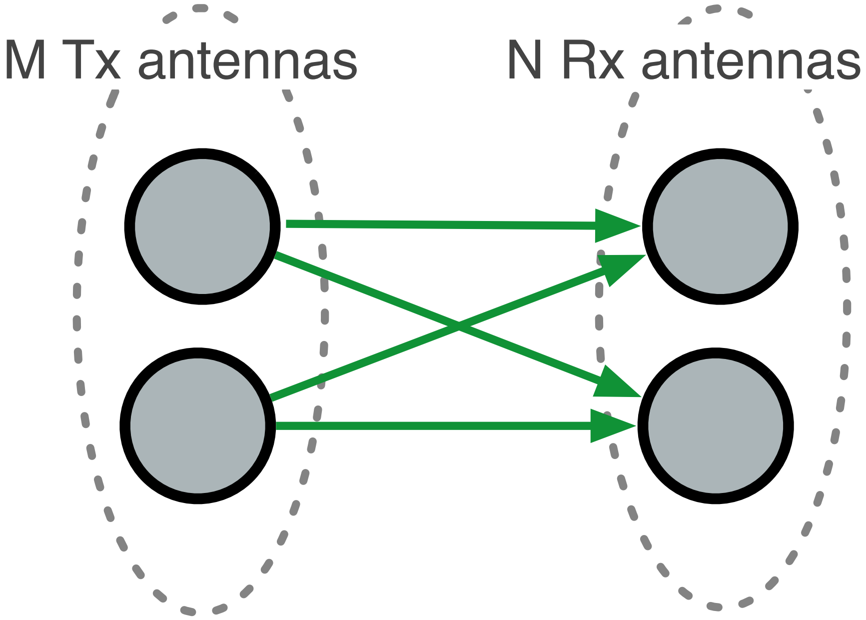

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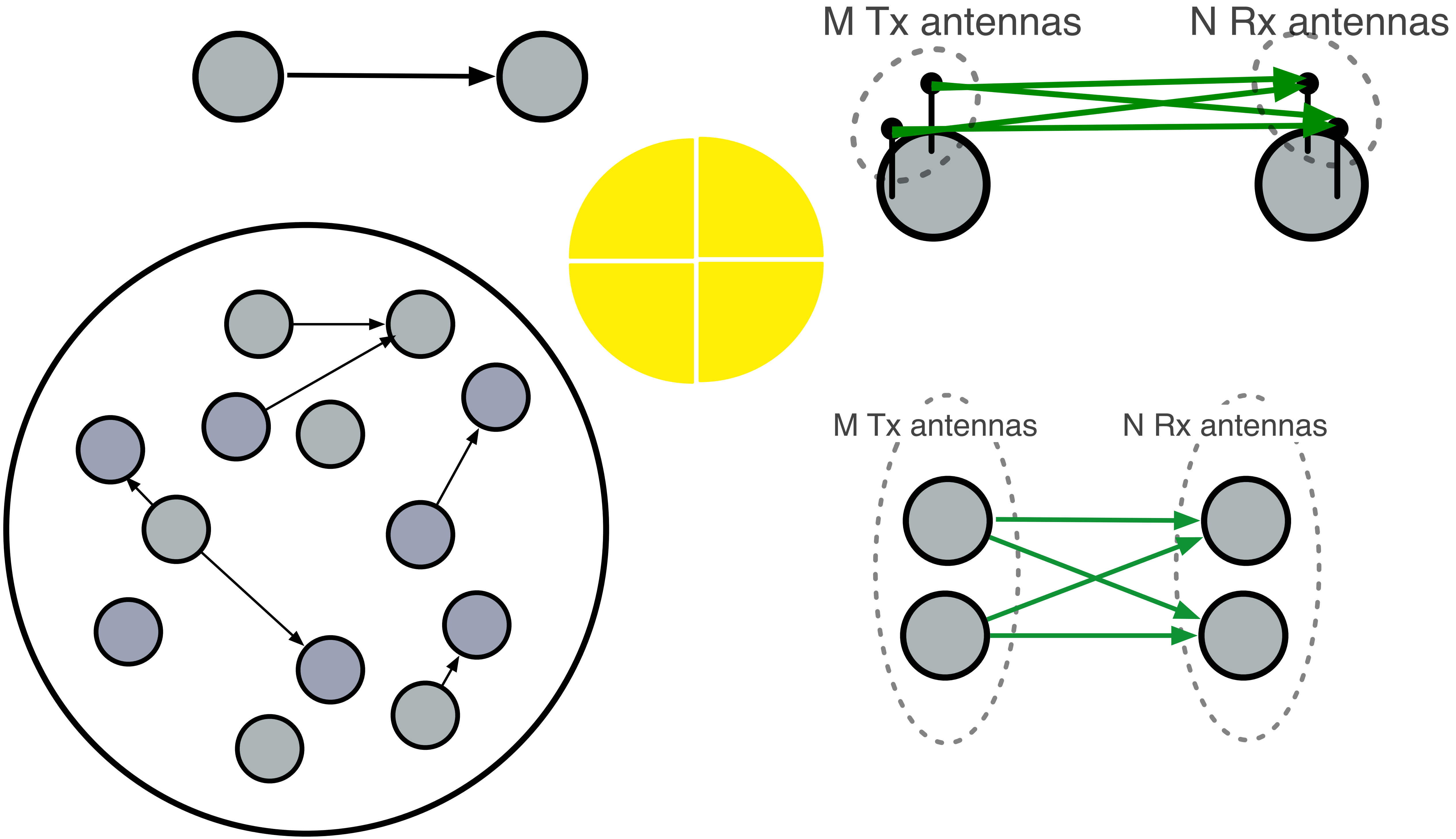
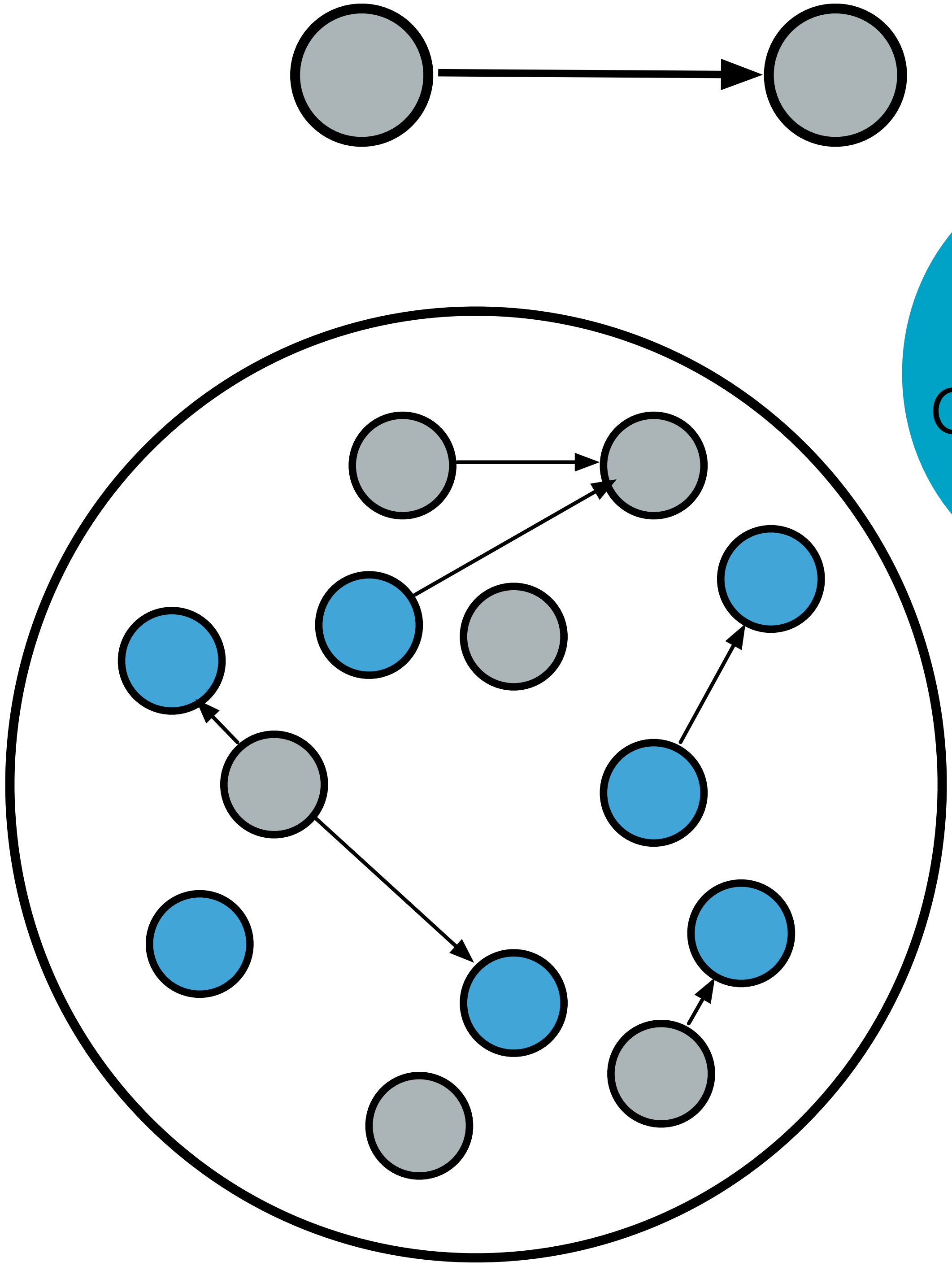
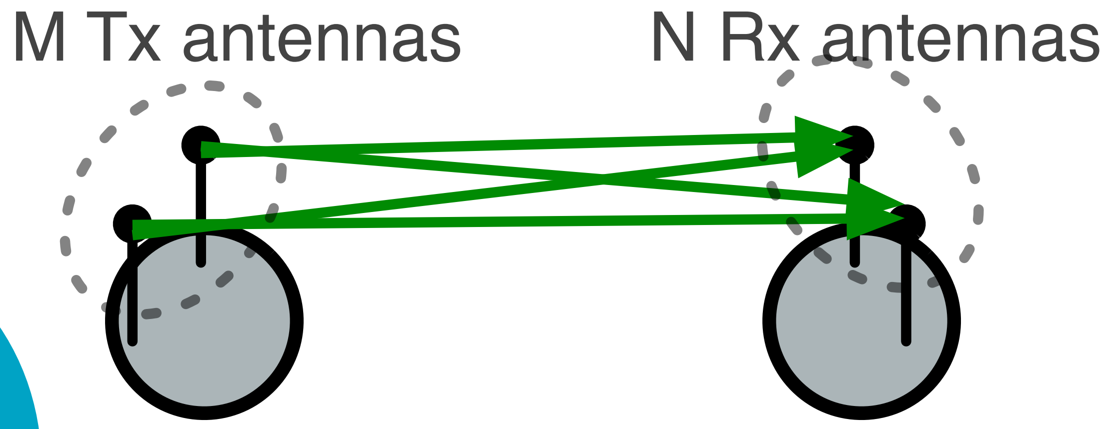
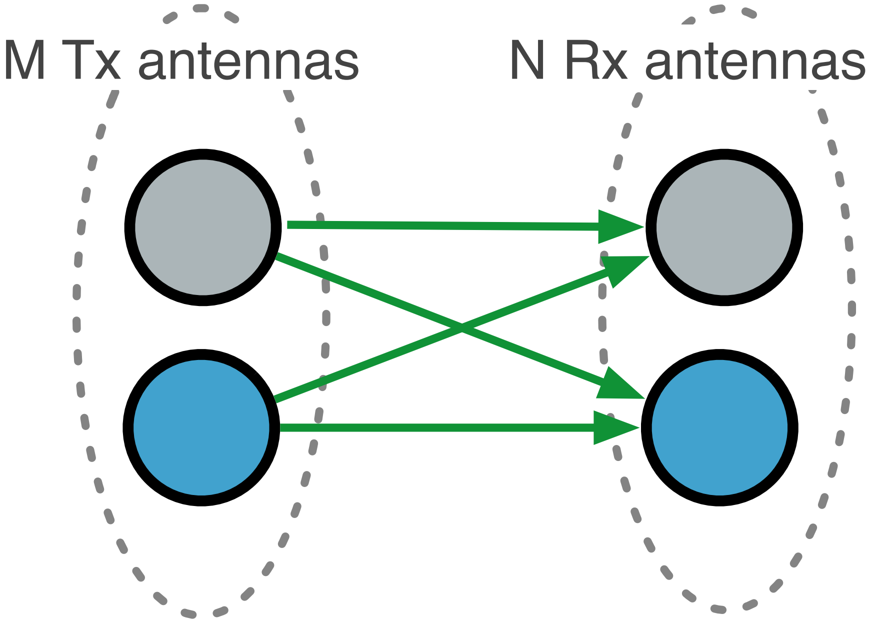


Efficient, reliable communications



With cognition



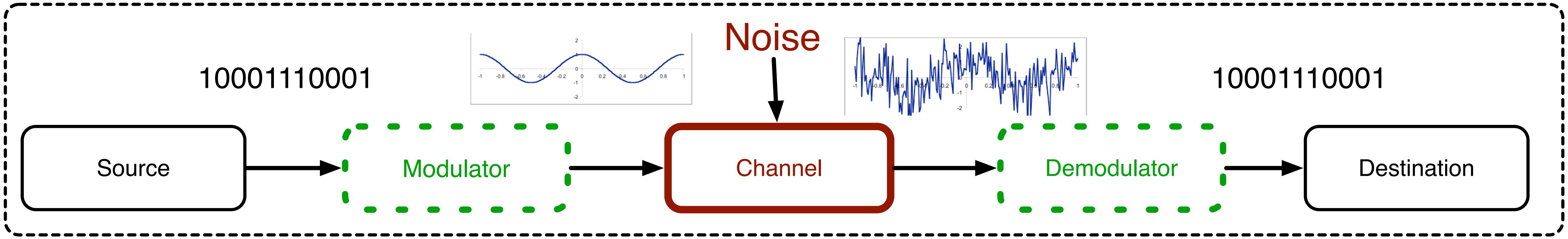


Radio

Software-defined Radio =SDR

Cognitive Radio = CR

Radio



10001110001

Noise

10001110001

Source

Modulator

Channel

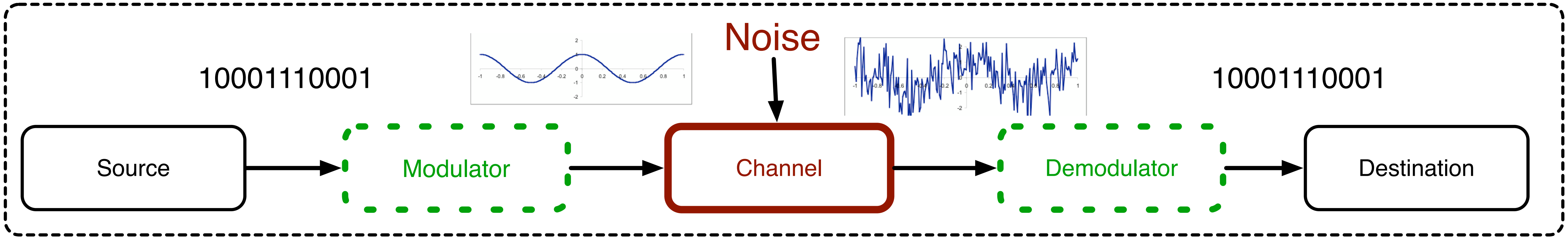
Demodulator

Destination

Software-defined Radio =SDR

Cognitive Radio = CR

Radio



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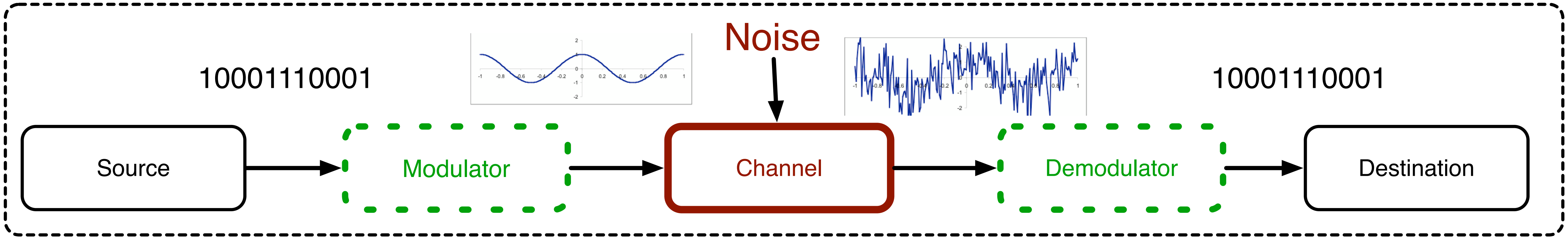

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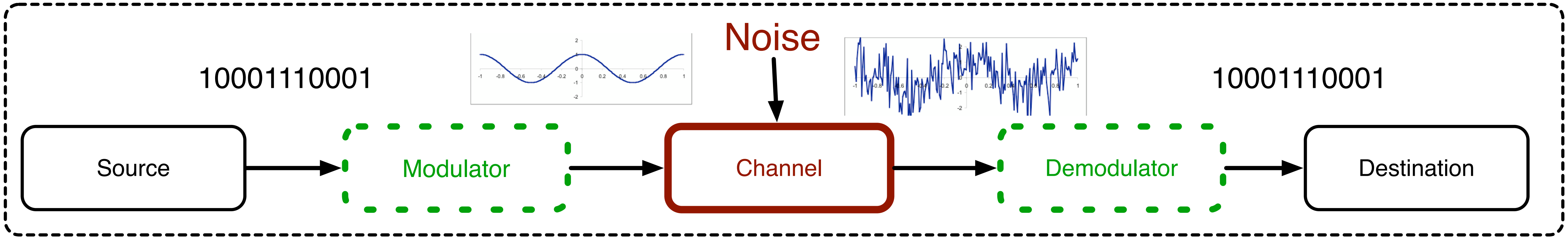
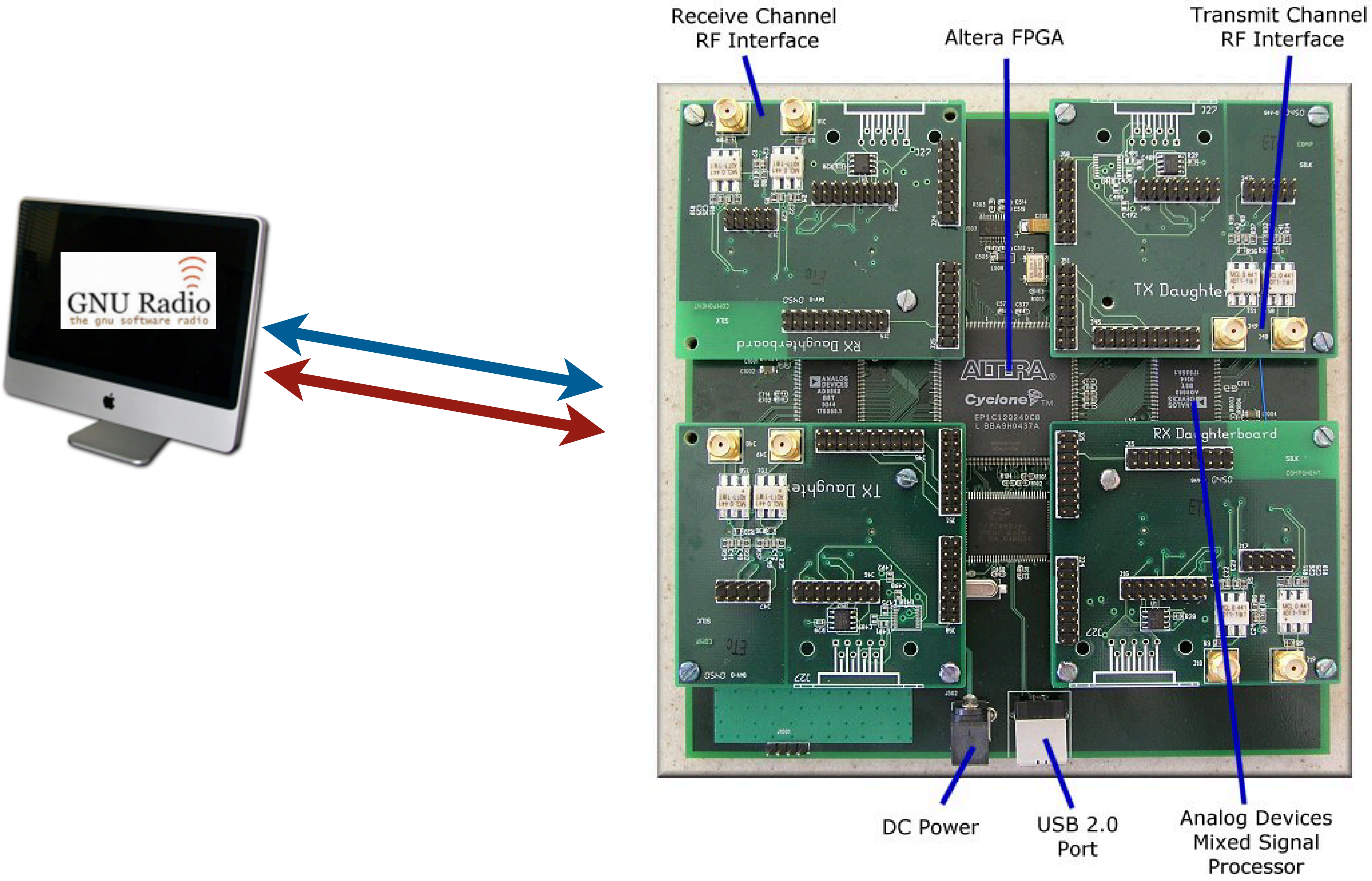


Software-defined Radio =SDR

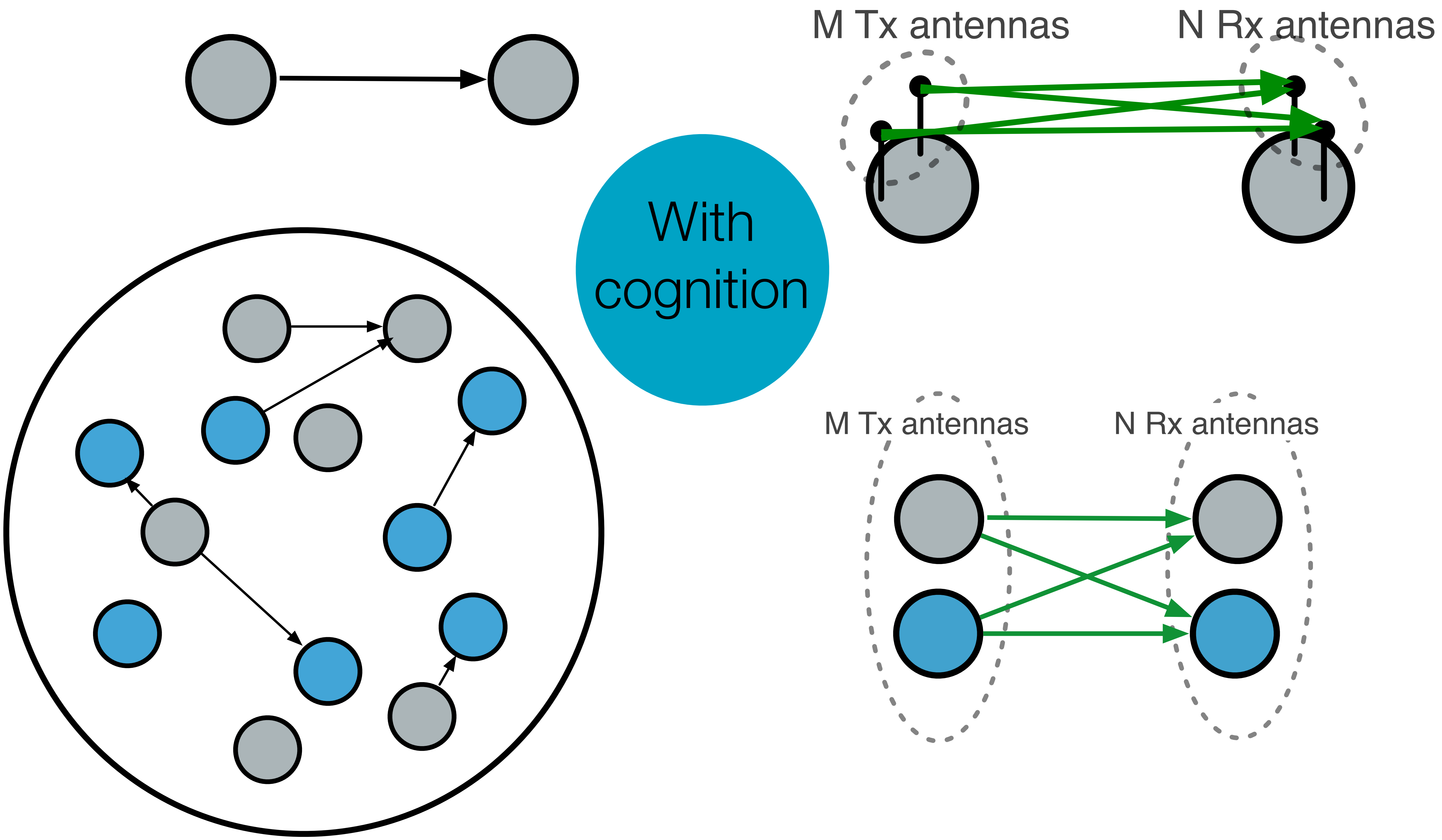
Cognitive Radio = CR

Example: GNU Radio+USRP




Ettus Research
USRP
www.ettus.com

Efficient, reliable communications



With cognition

Capacity regions

↕

Fundamental Limits of Cognitive Networks

UIC University of Illinois at Chicago

Channel capacity

Channel: $p(y|x)$

Channel capacity

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

Channel capacity


X

Channel: p(y|x)

Y

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

Channel capacity

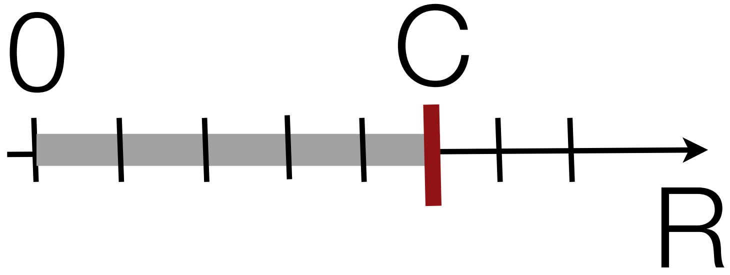
Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

`Highest rate (bits/channel use) that can communicate at reliably`

Mathematical description of capacity

- Can achieve reliable communication for all transmission rates R:

$$R < C$$



Mathematical description of capacity

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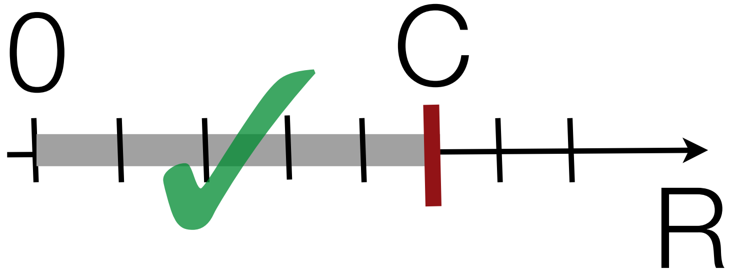

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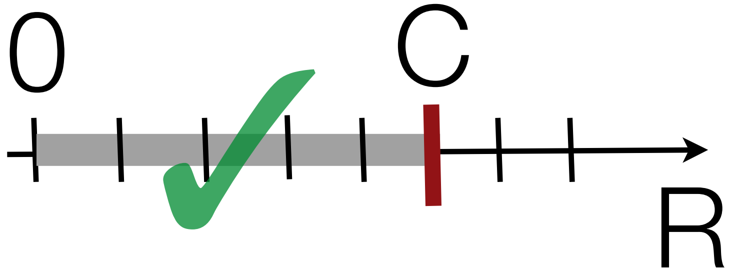
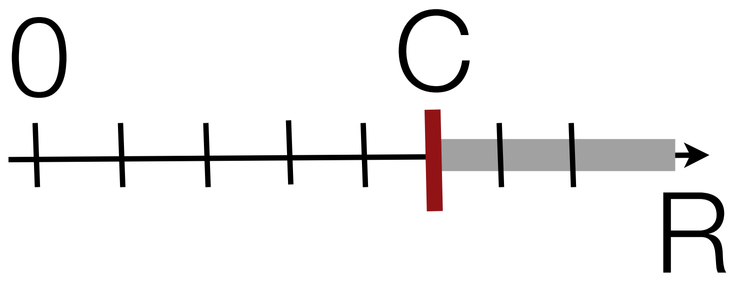


- BUT, probability of decoding error always bounded away from zero if

$R > C$



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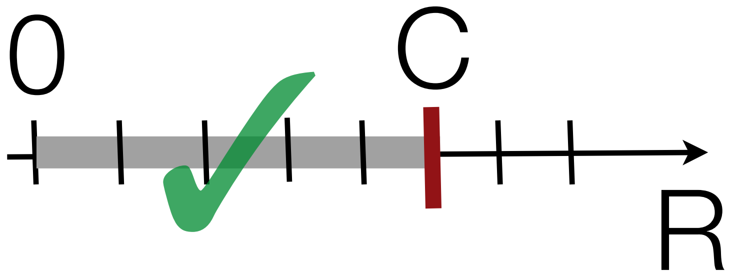

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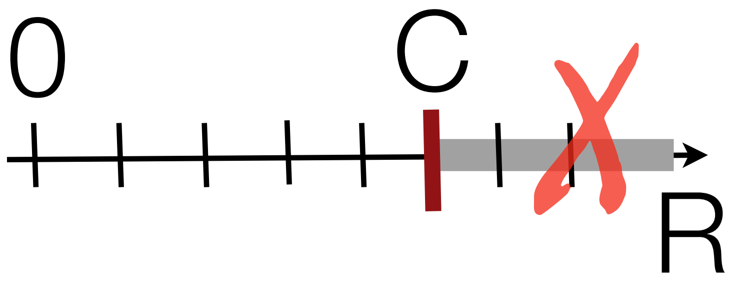
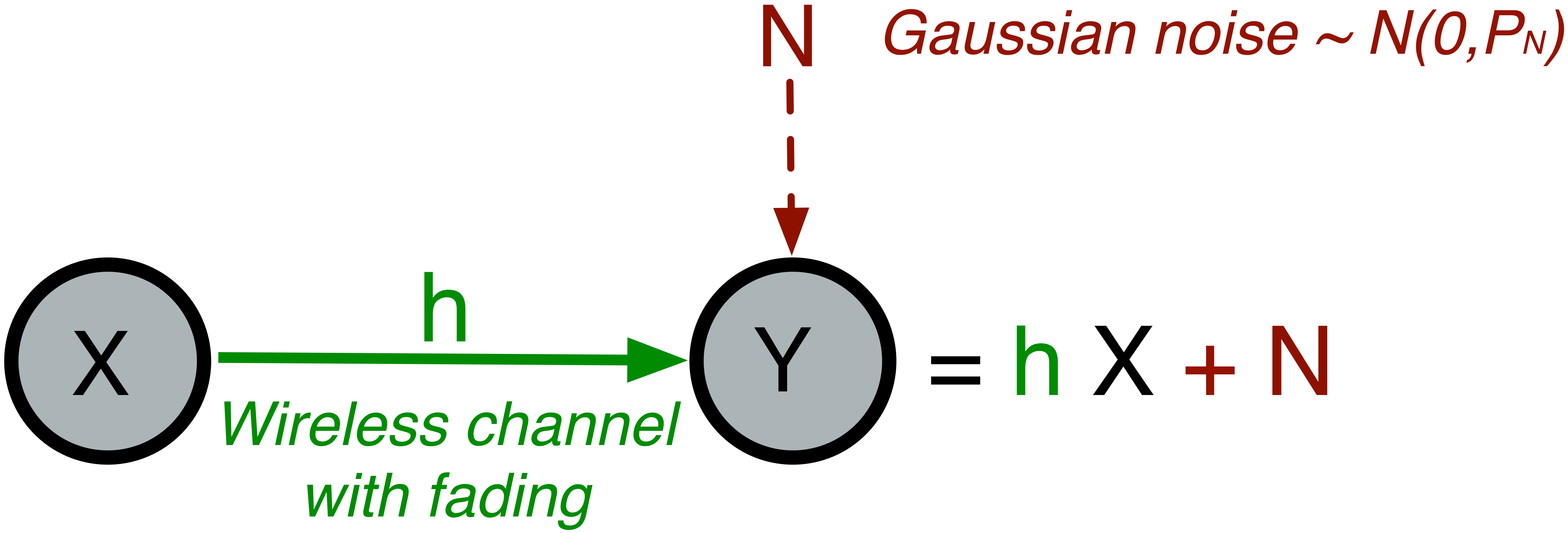
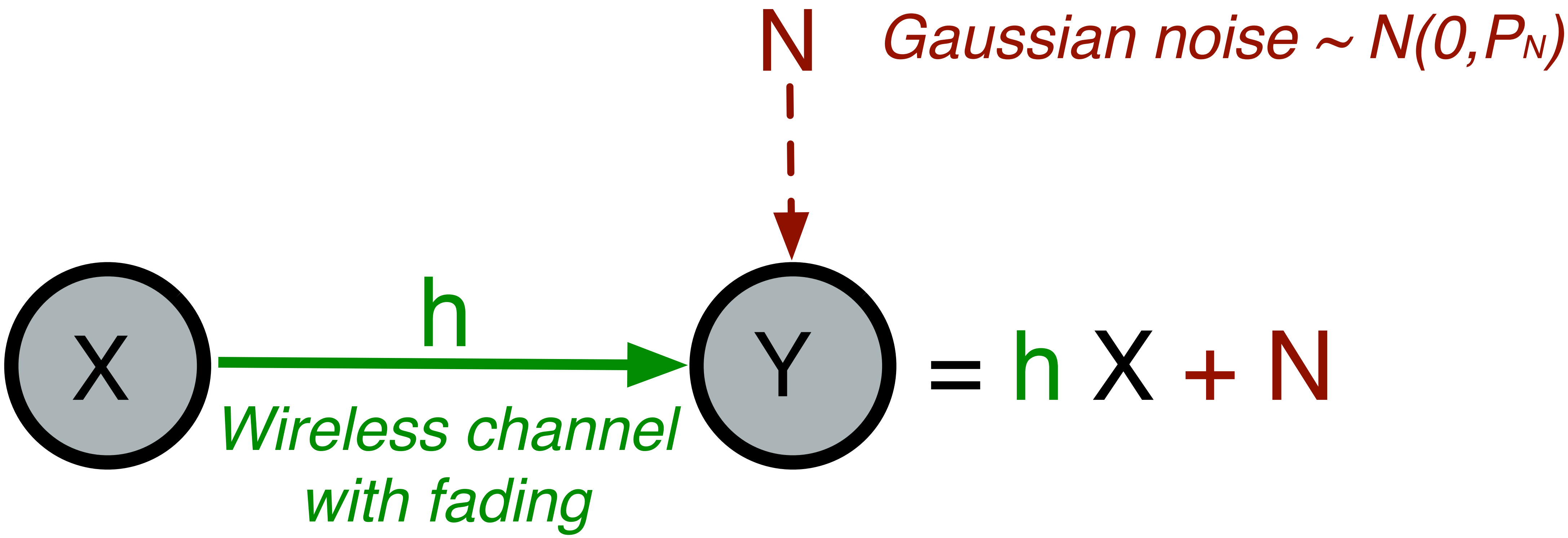


AWGN channel capacity



$= h\,X + N$

AWGN channel capacity



N

Gaussian noise ~ N(0,P_N)

h

X

Y

Wireless channel with fading

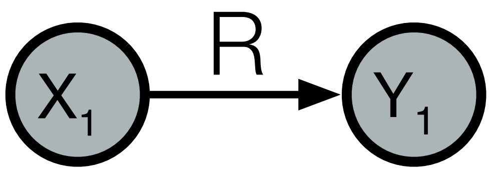
= h X + N

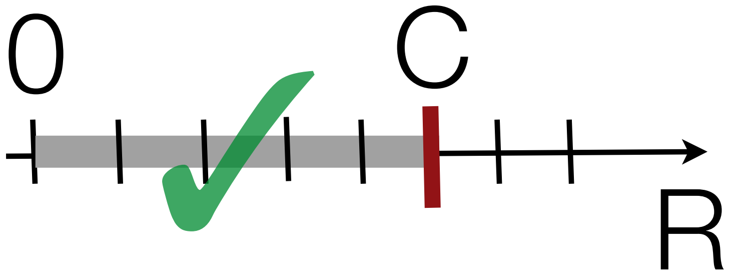
$$C = \frac{1}{2}\log\left(\frac{|h|^2 P + P_N}{P_N}\right)$$

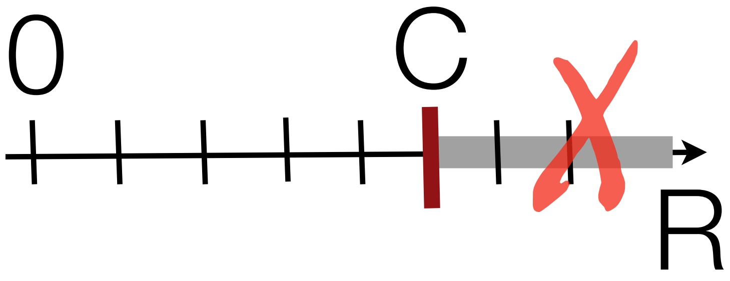
$$= \frac{1}{2}\log\left(1 + SNR\right)\ \ \text{(bits/channel use)}$$

Capacity and capacity regions

- Point to point **capacity**







Capacity and capacity regions

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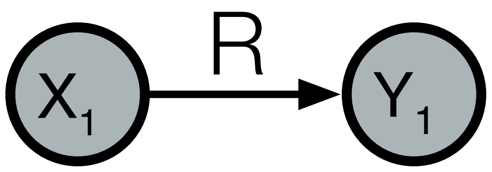
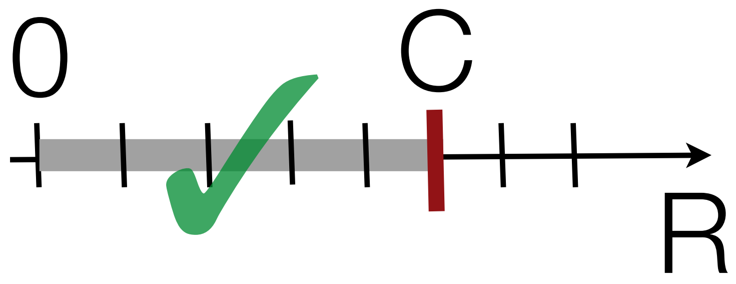
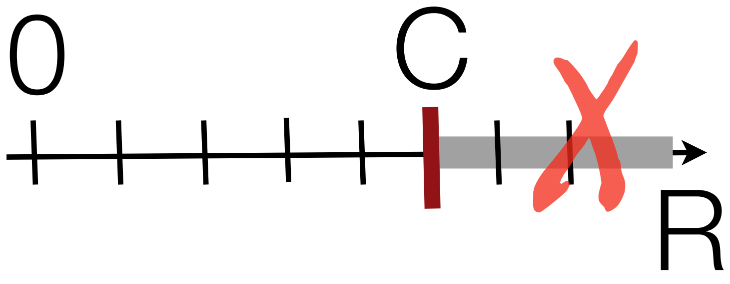
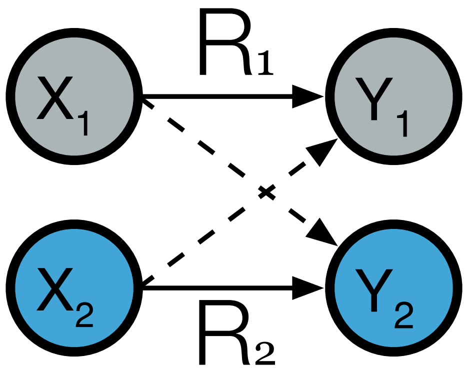
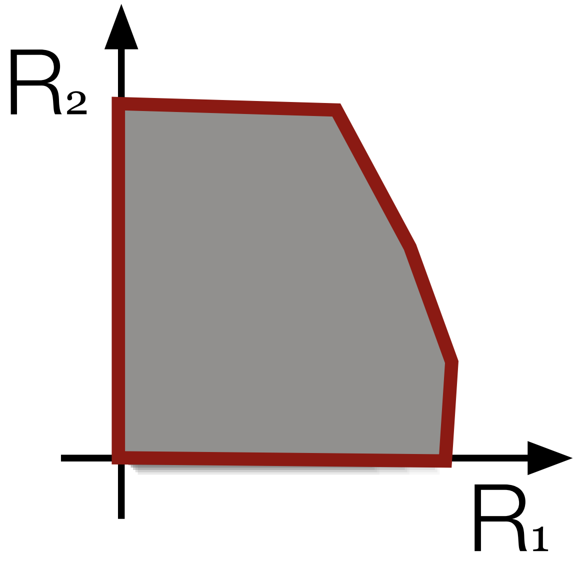


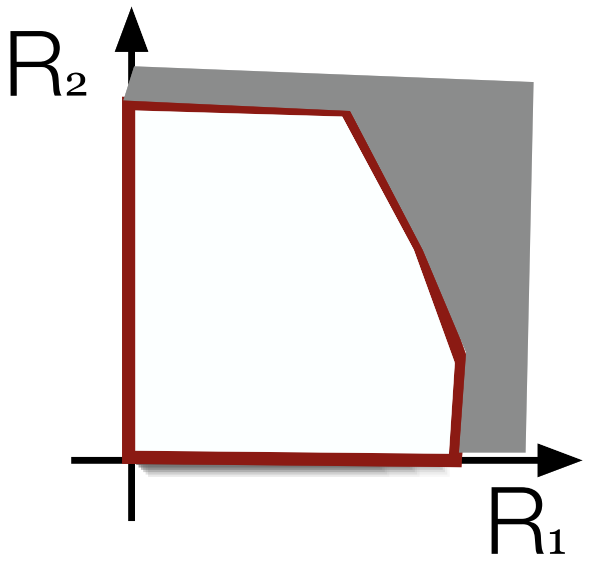




- Multi-user **capacity region**







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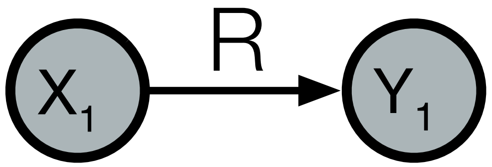

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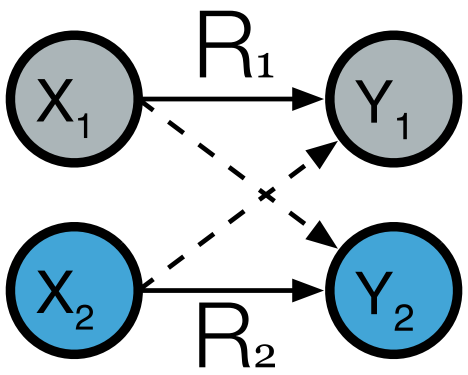

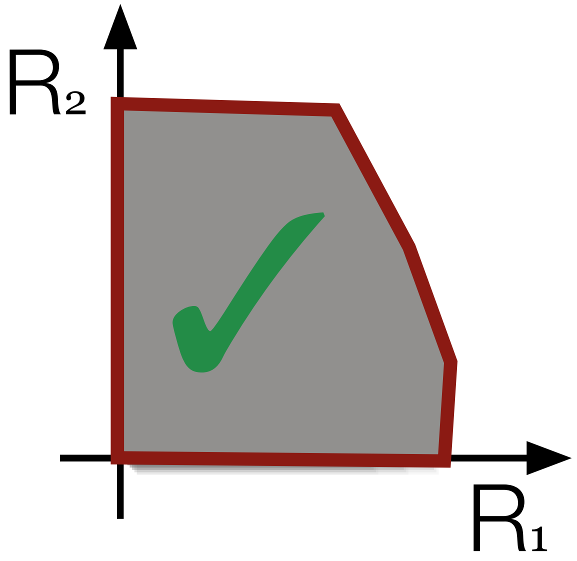

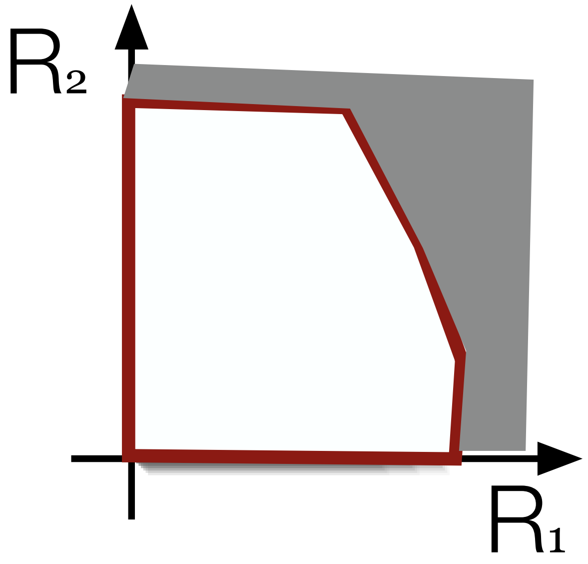

Capacity and capacity regions

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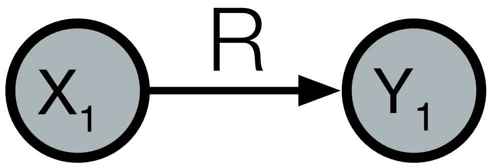

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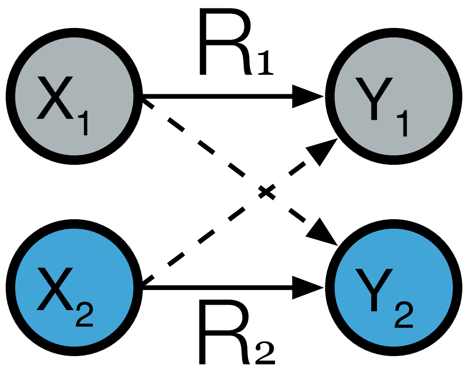
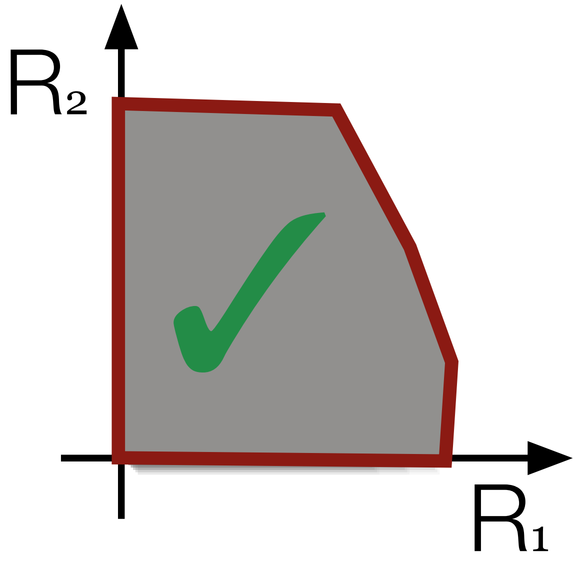
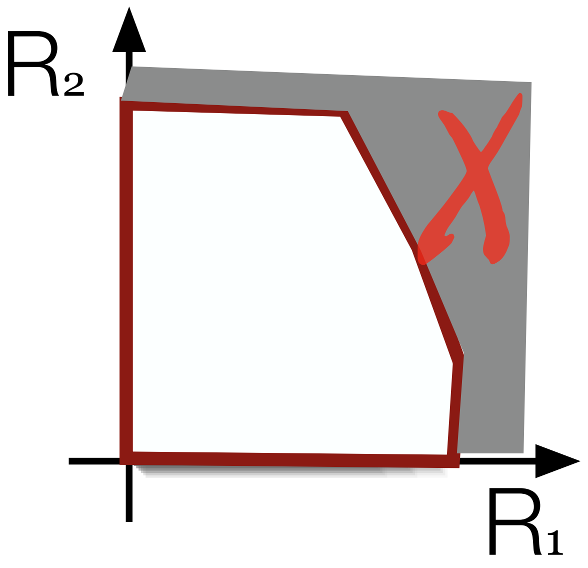
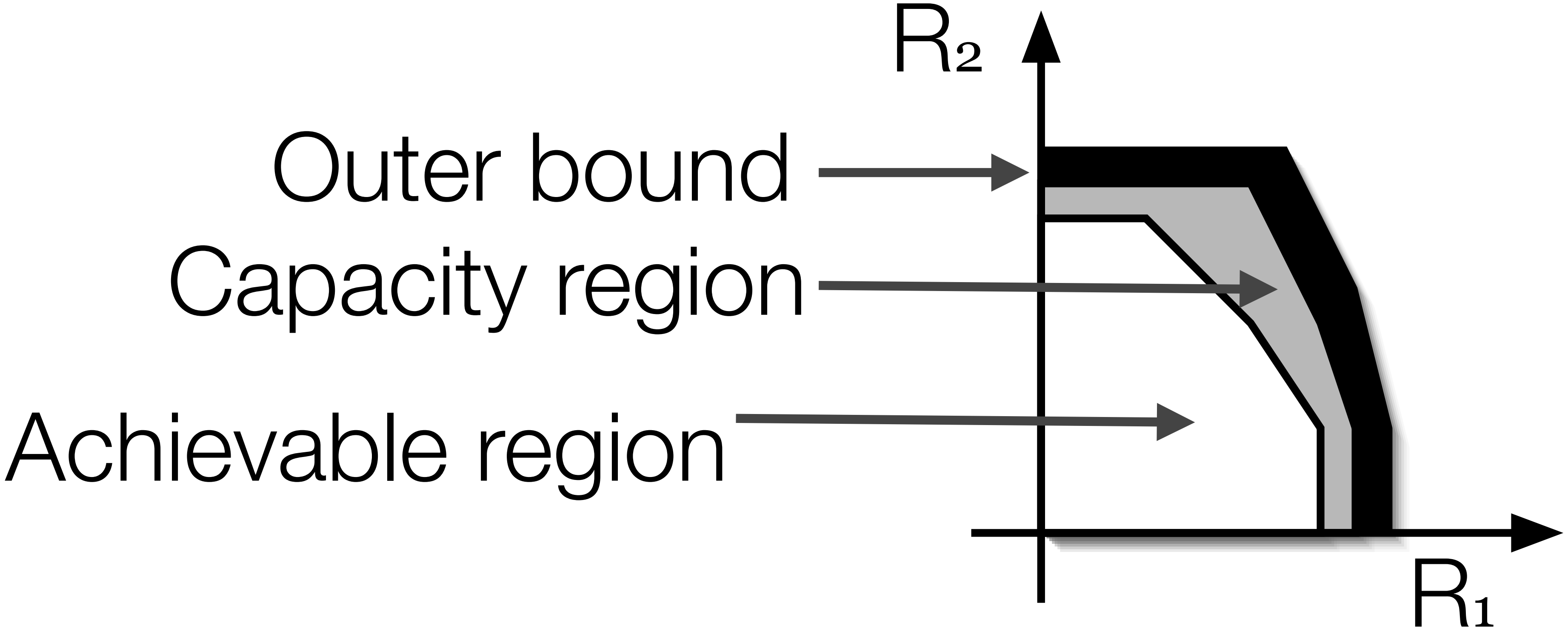






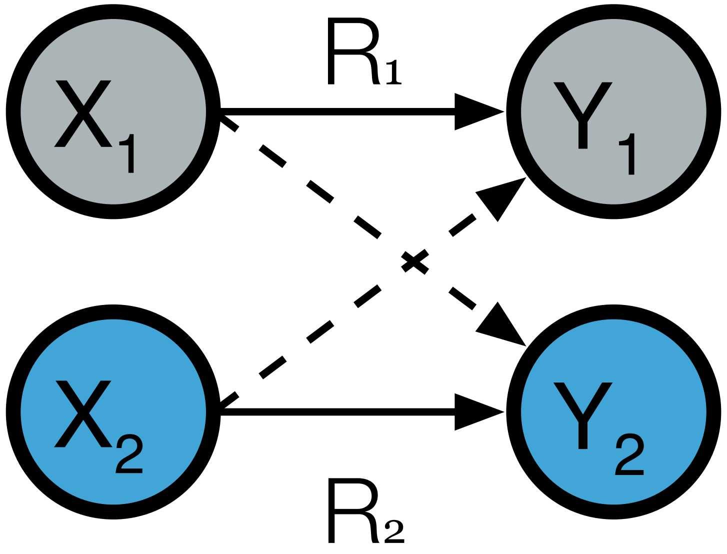
Capacity regions



Outer bound

Capacity region

Achievable region



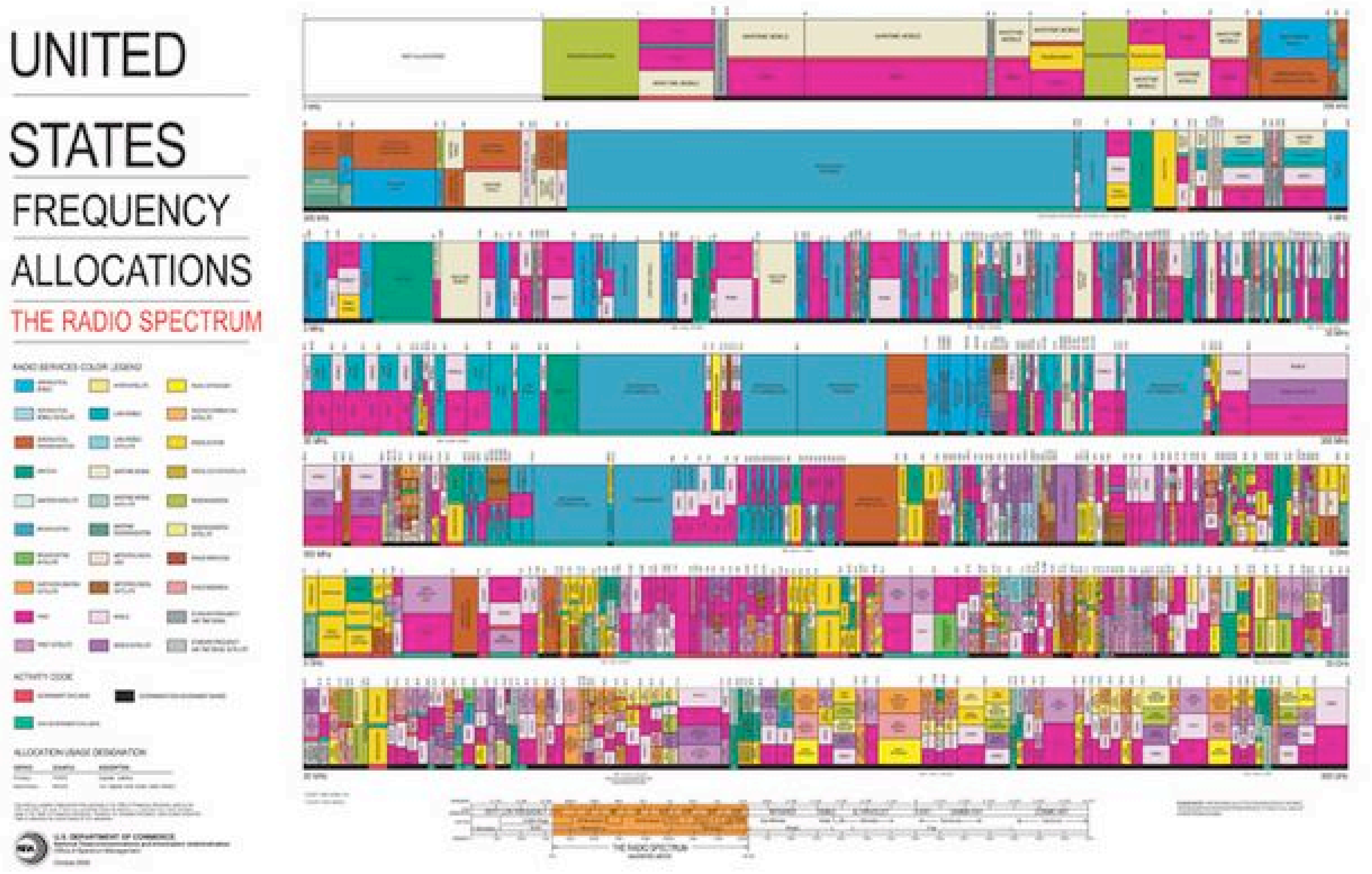
Fundamental Limits of Cognitive Networks

UIC University of Illinois at Chicago

Motivation?

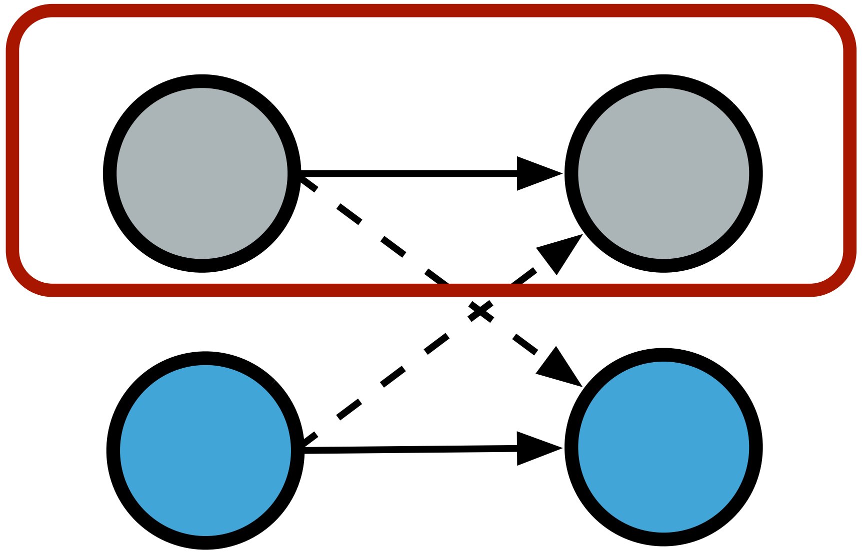
Motivation 1: smart cognitive devices

Motivation 2: spectral efficiency



Spectrum licensing: future

Primary users/ primary license holders



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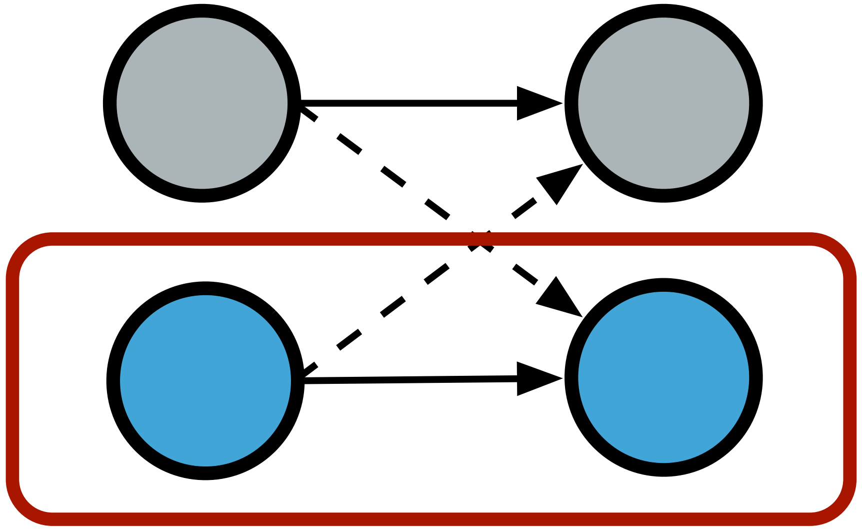
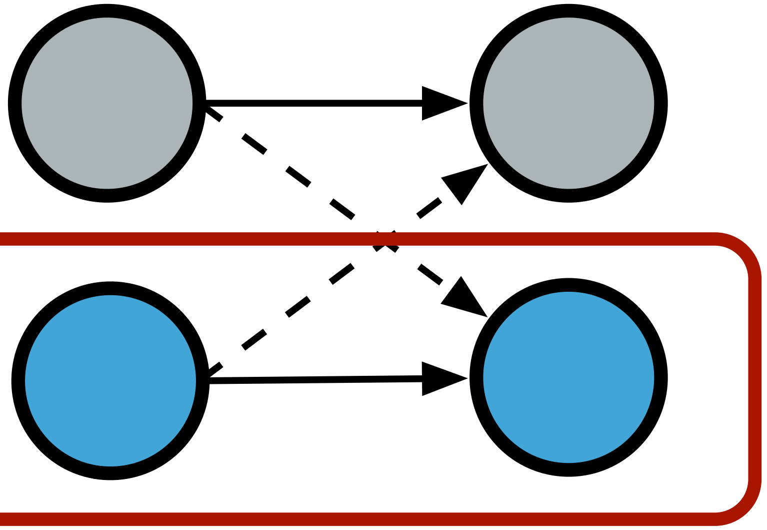


Secondary users

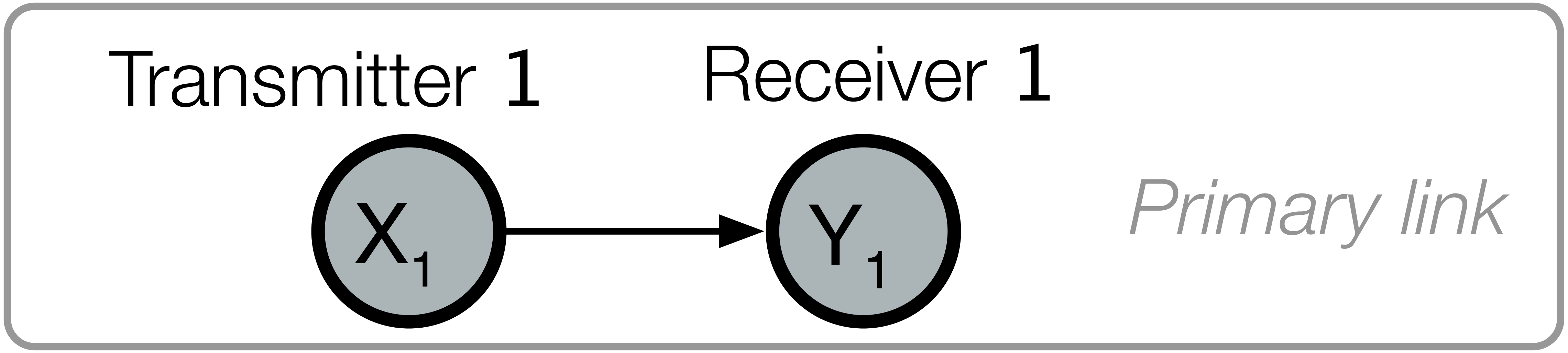
Spectrum licensing: future

Primary users/ primary license holders



Secondary users ↔ Cognitive radios

Secondary spectrum usage



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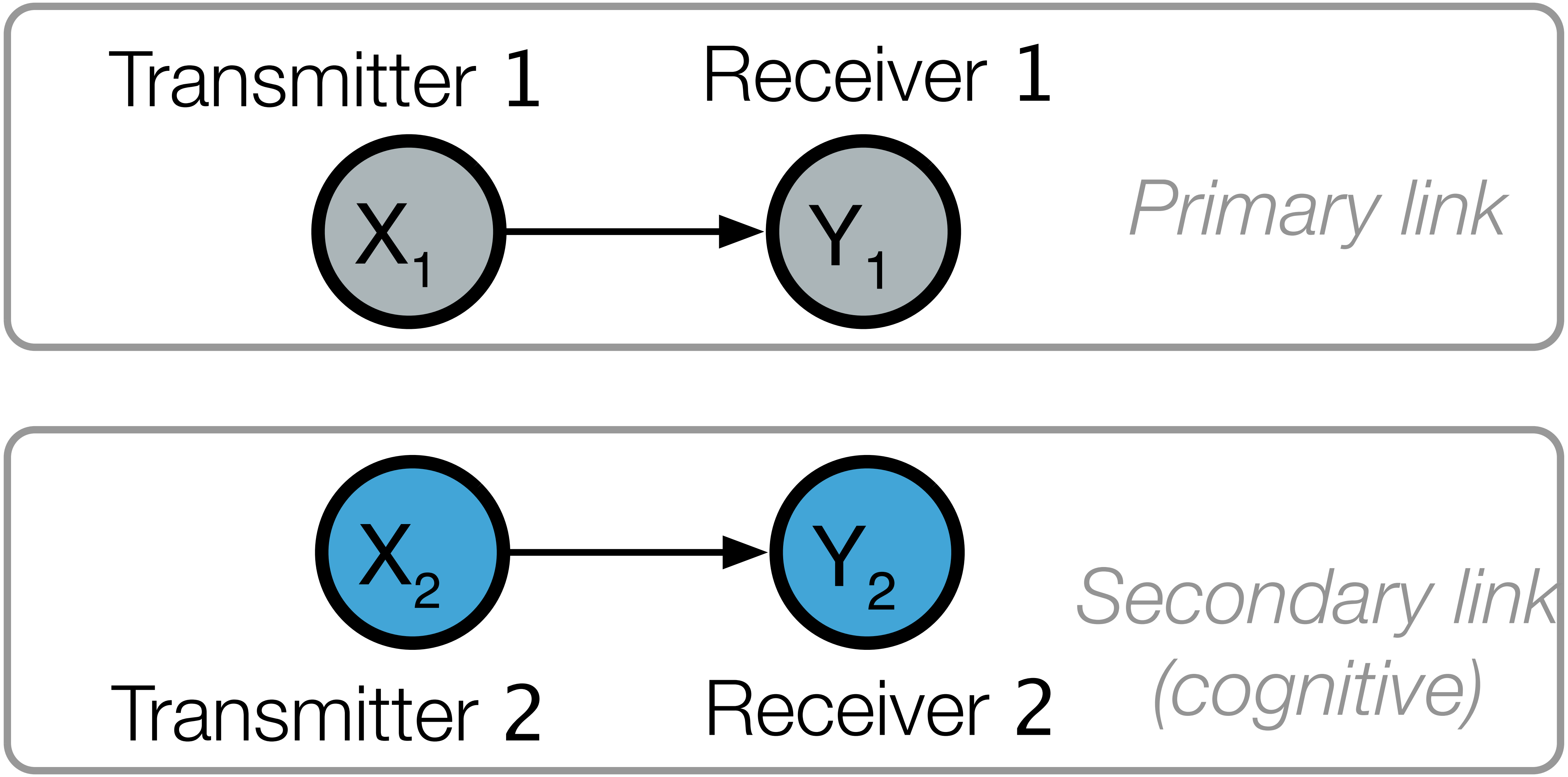


Transmitter 1

Receiver 1

X_1 → Y_1

Primary link

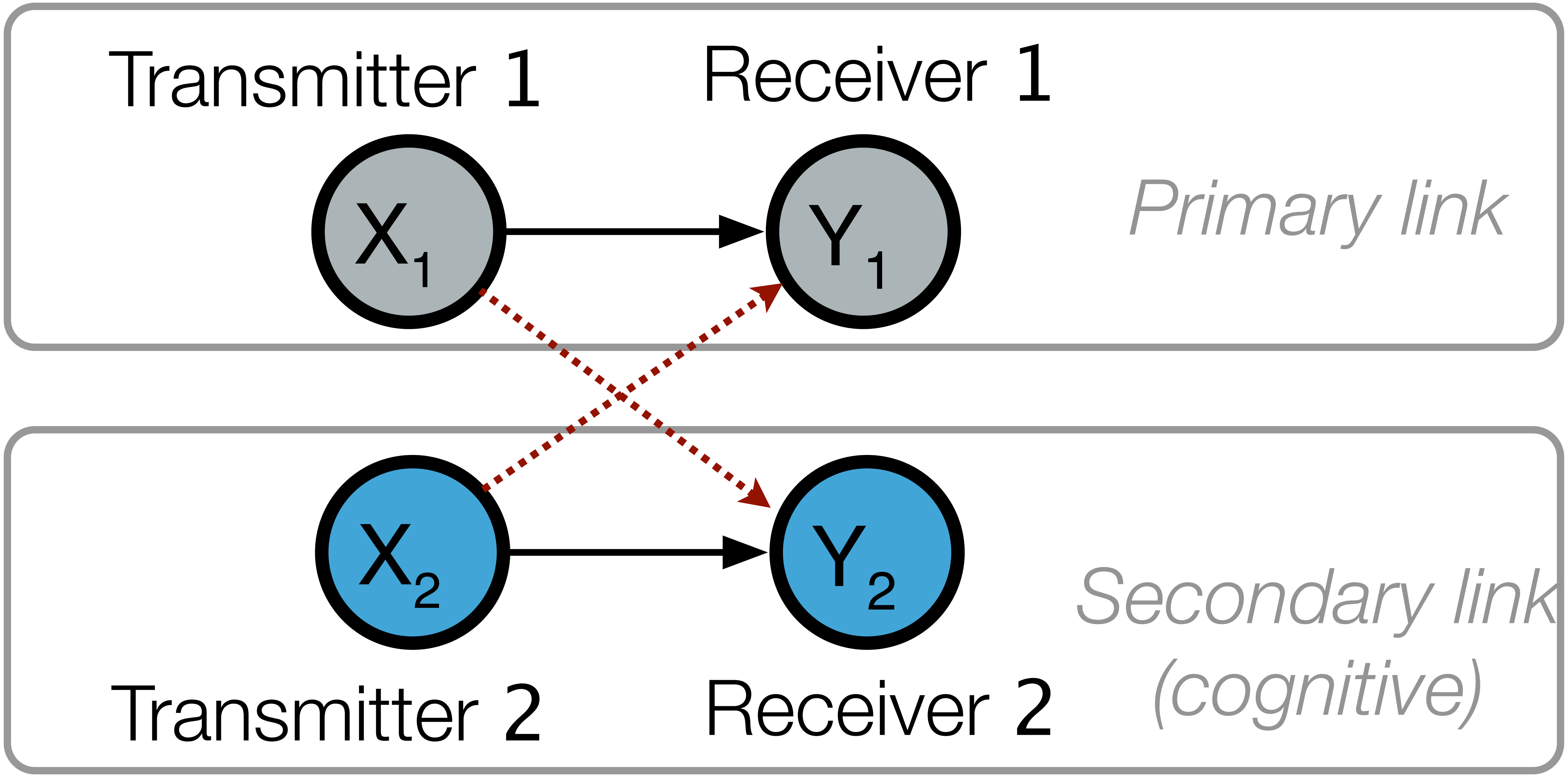
X_2 → Y_2

Transmitter 2

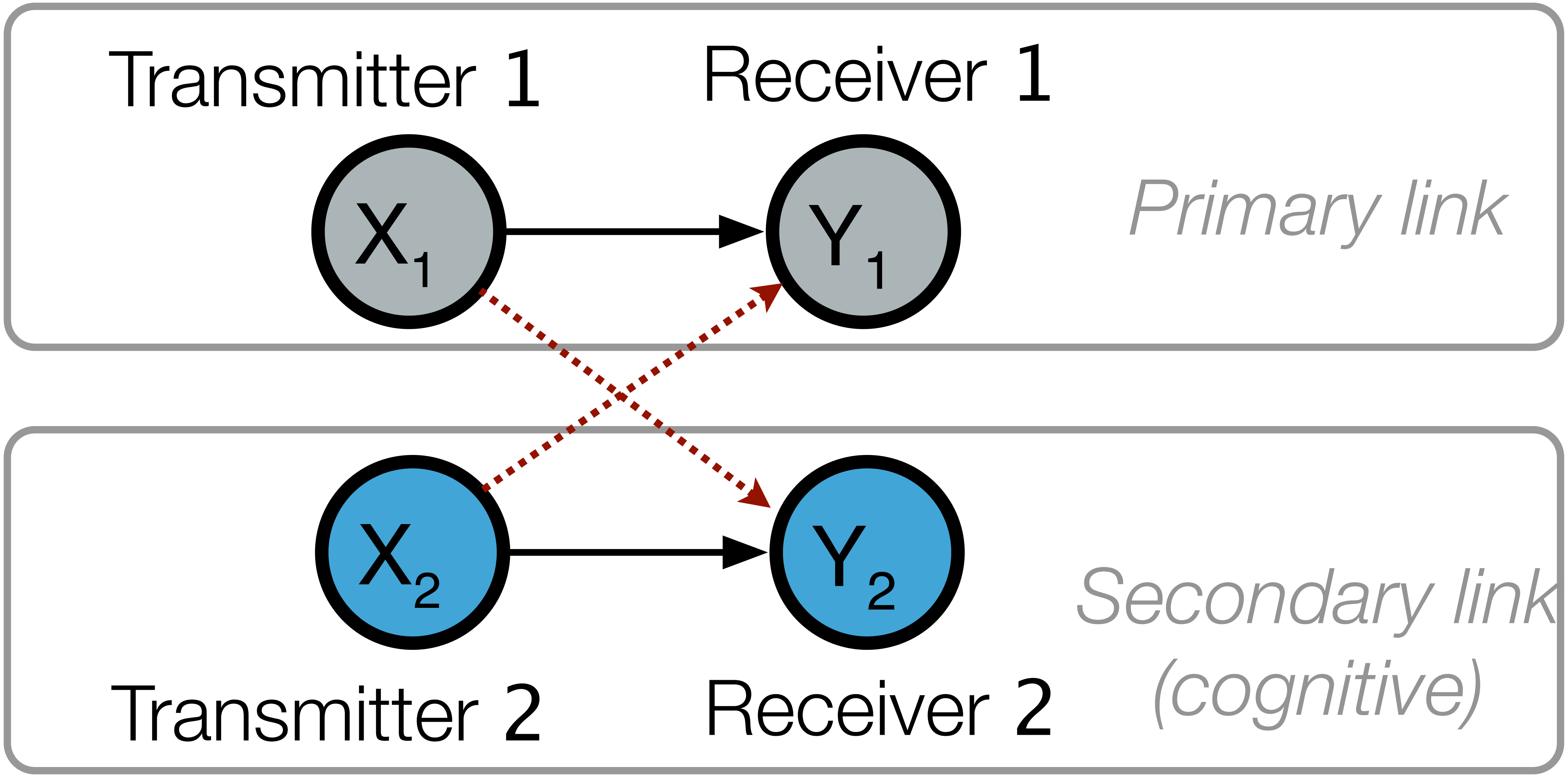
Receiver 2

Secondary link (cognitive)

Secondary spectrum usage

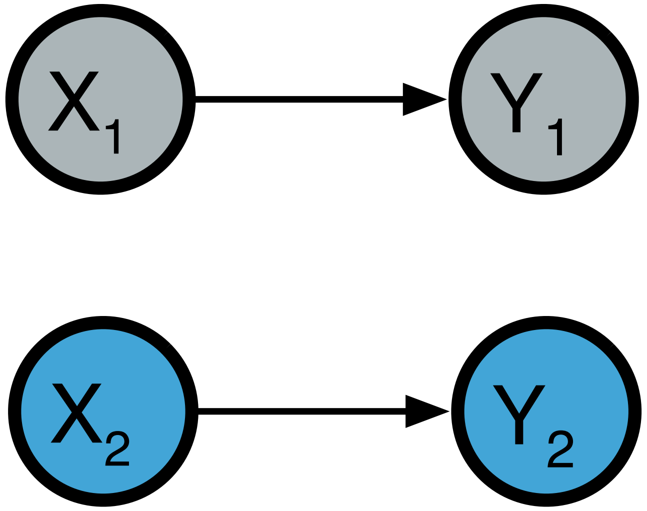


Secondary spectrum usage



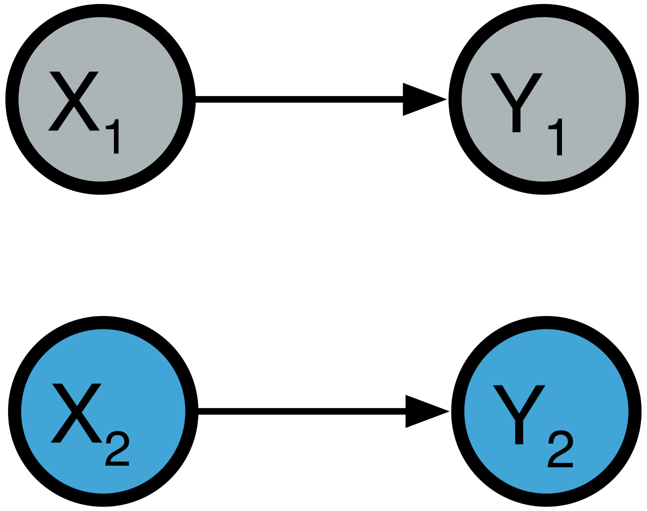
What can the cognitive link do?

Cognition



- Assumptions on primary/secondary models will dictate behavior + performance

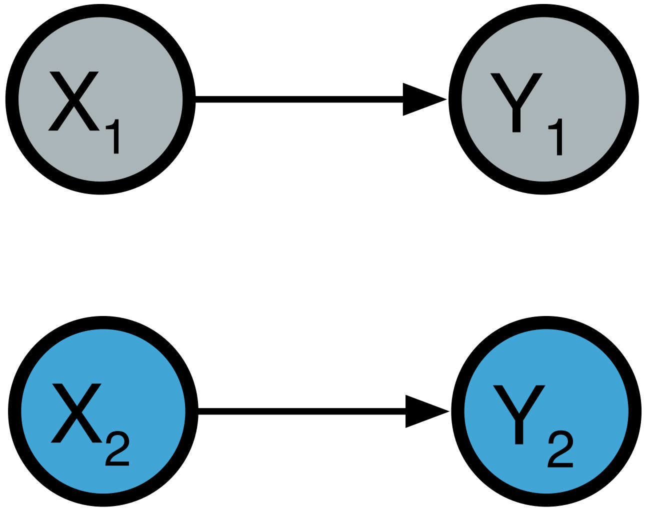
Cognition



- Assumptions on primary/secondary models will dictate behavior + performance

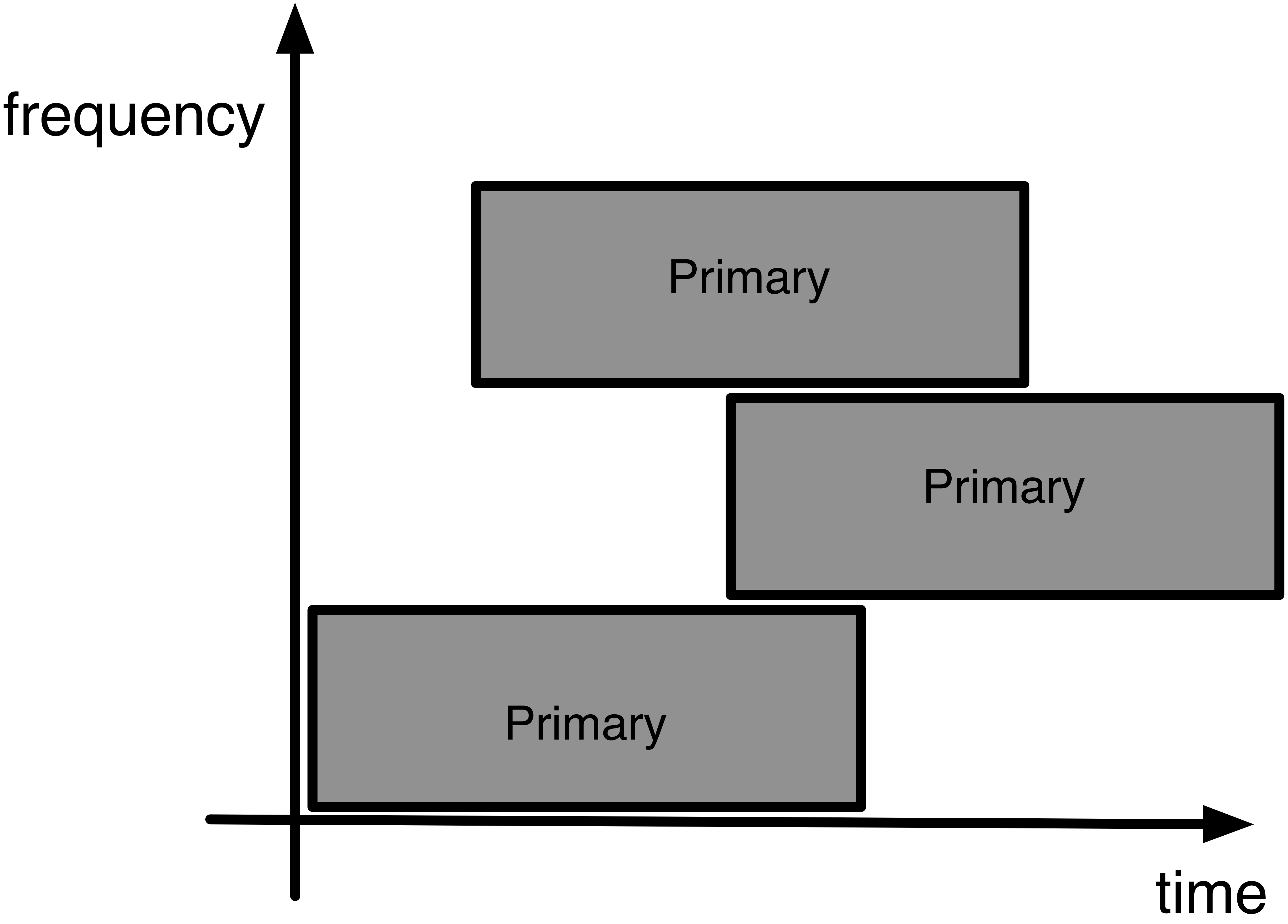
- Cognition boils down to **side-information** and how to use it

Cognition



- Assumptions on primary/secondary models will dictate behavior + performance
- Cognition boils down to **side-information** and how to use it
- Use information theory to tell us which techniques are most promising

1. White spaces



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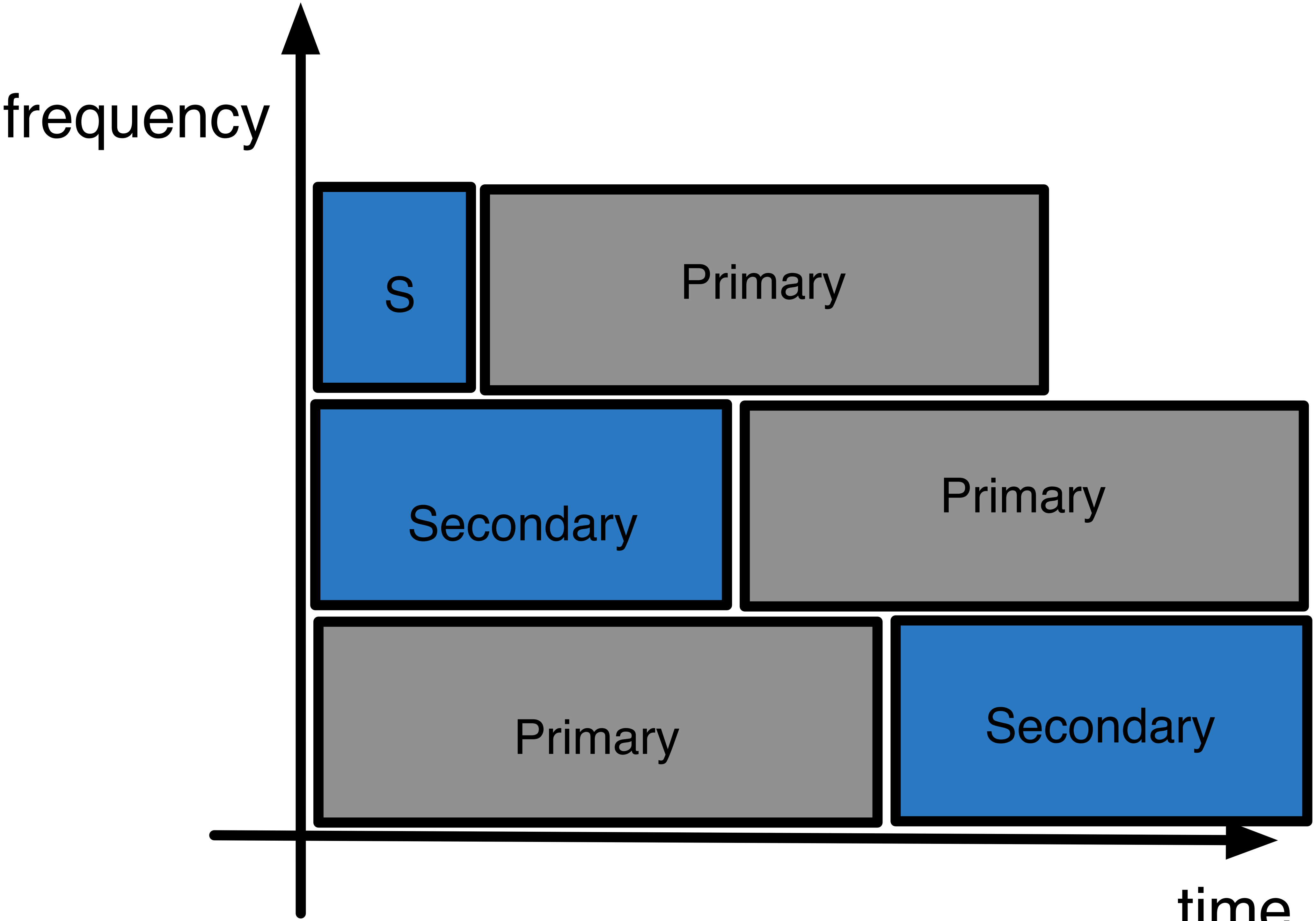
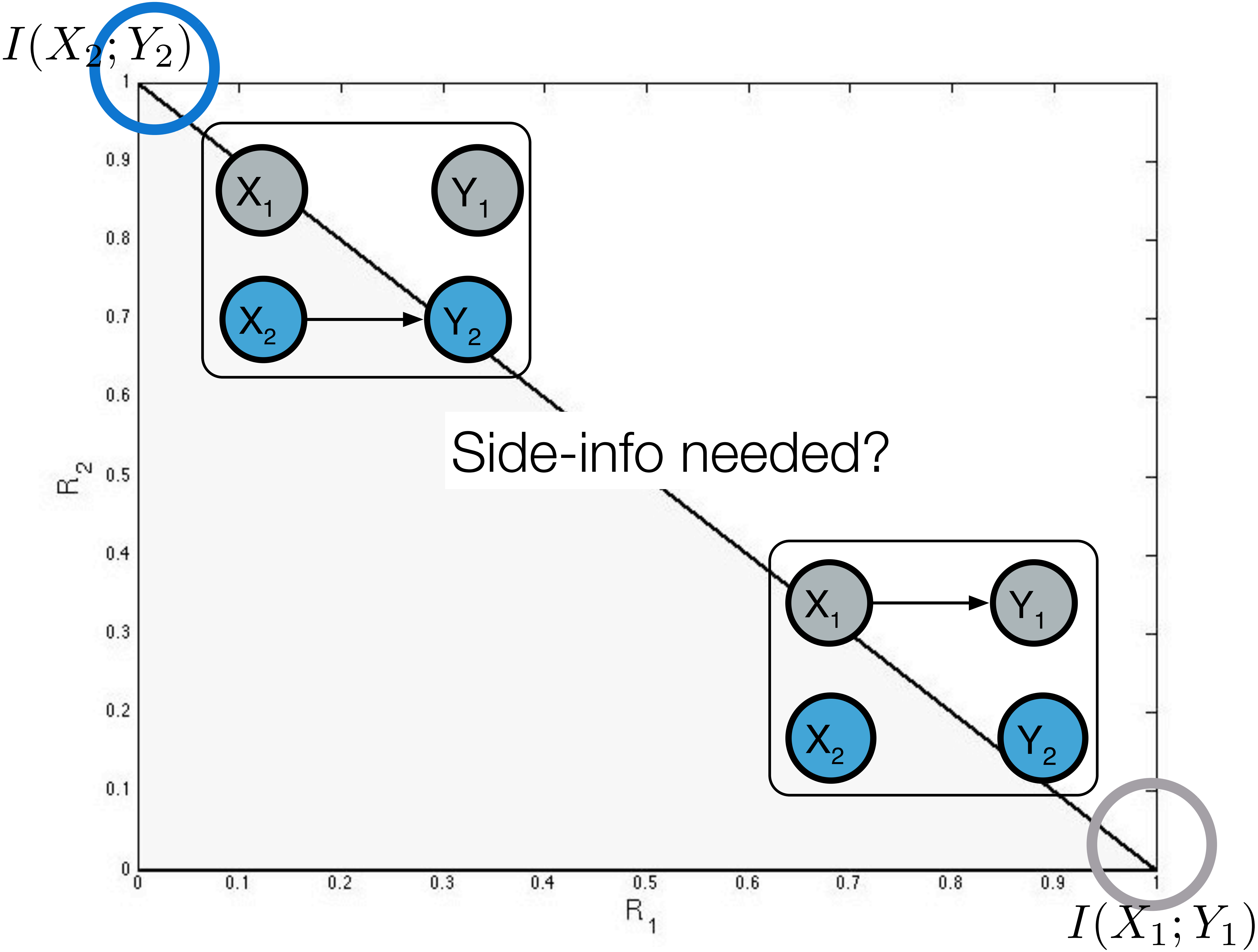
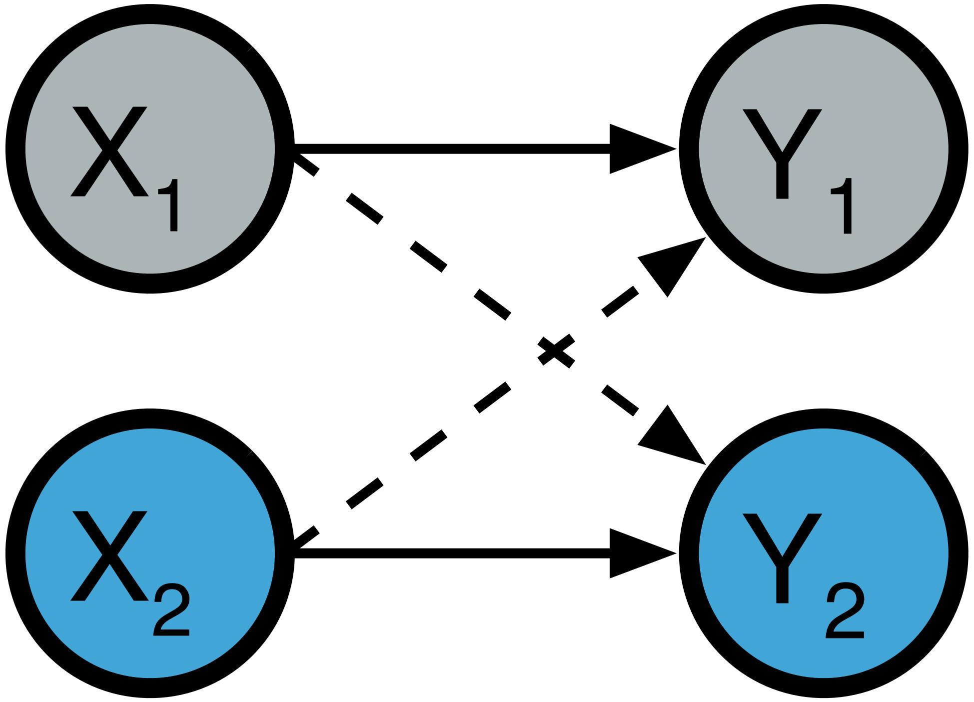




Side-info needed?

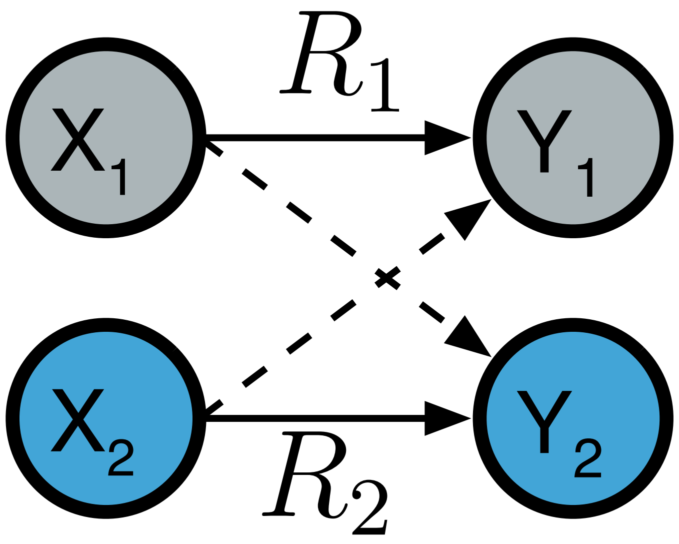
2. Just transmit



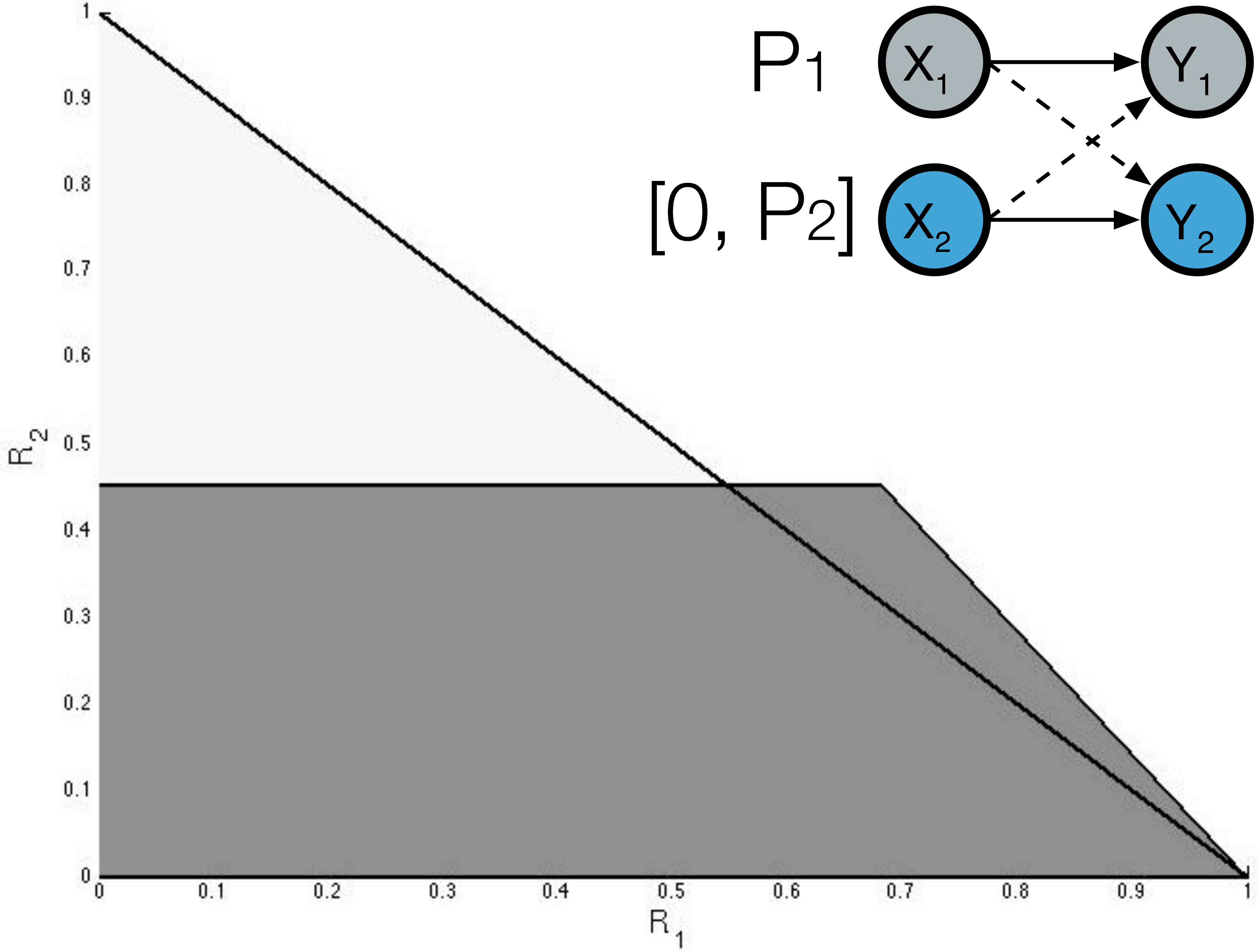
Interfere with each other!

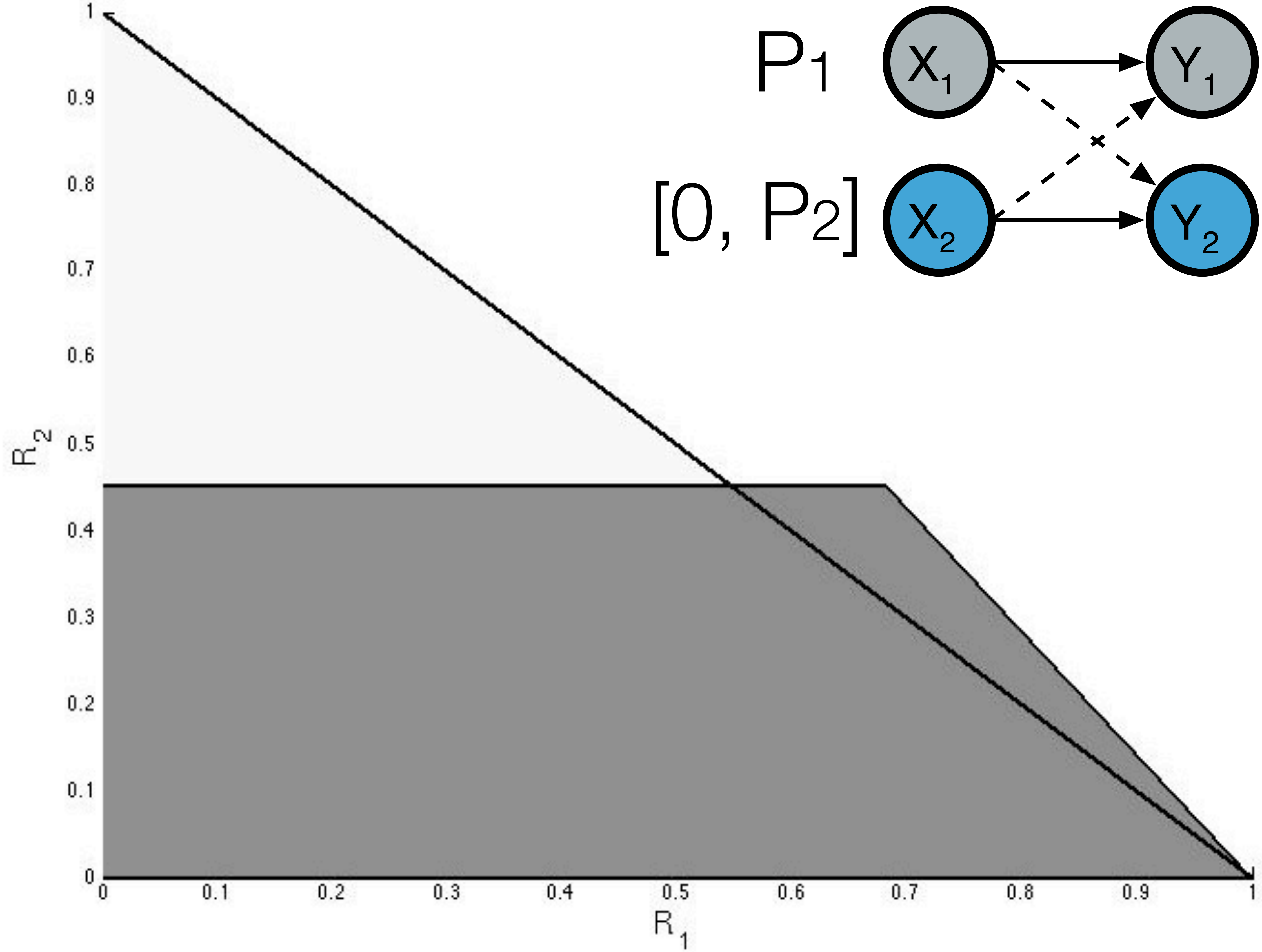
2. Just transmit

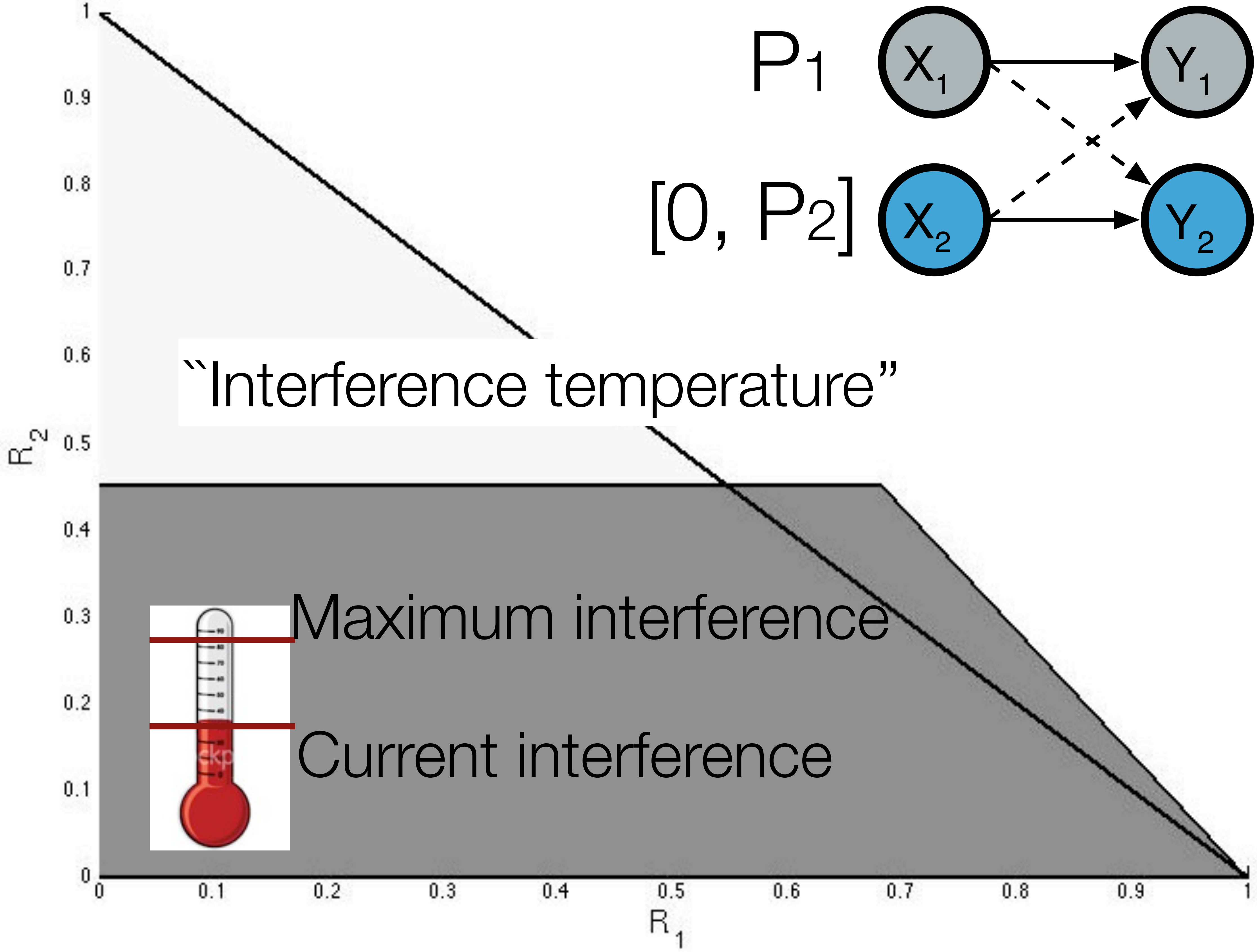
$$R_1 \leq \frac{1}{2}\log_2\left(1 + \frac{\text{Power of signal 1}}{\text{Interference from signal 2} + \text{ Noise}}\right)$$



$$R_2 \leq \frac{1}{2}\log_2\left(1 + \frac{\text{Power of signal 2}}{\text{Interference from signal 1} + \text{ Noise}}\right)$$


P$_1$
[0, P$_2$]
X$_1$
Y$_1$
X$_2$
Y$_2$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R$_2$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
R$_1$

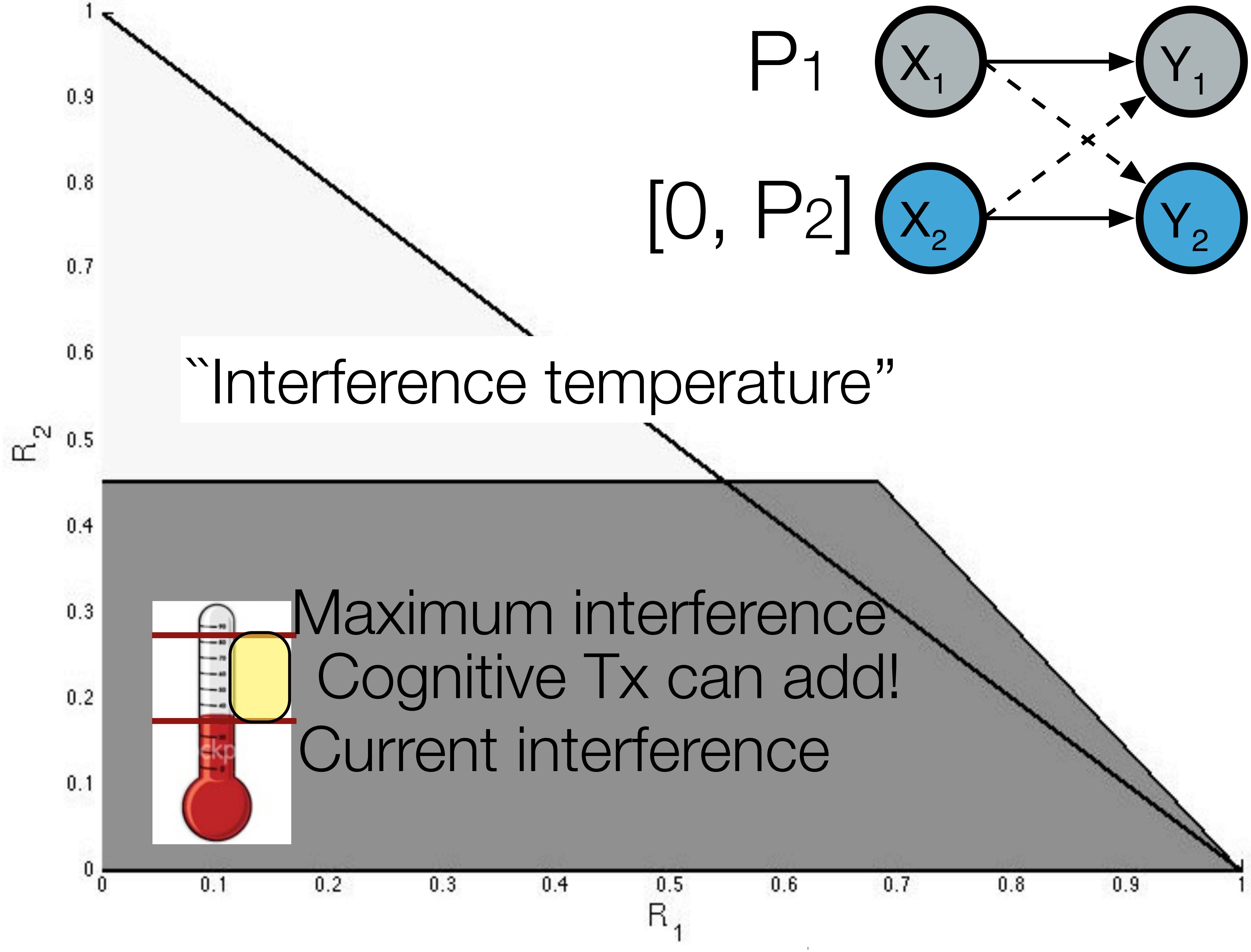

P1
[0, P2]
X1
X2
Y1
Y2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
R1



“Interference temperature”

Maximum interference

Current interference

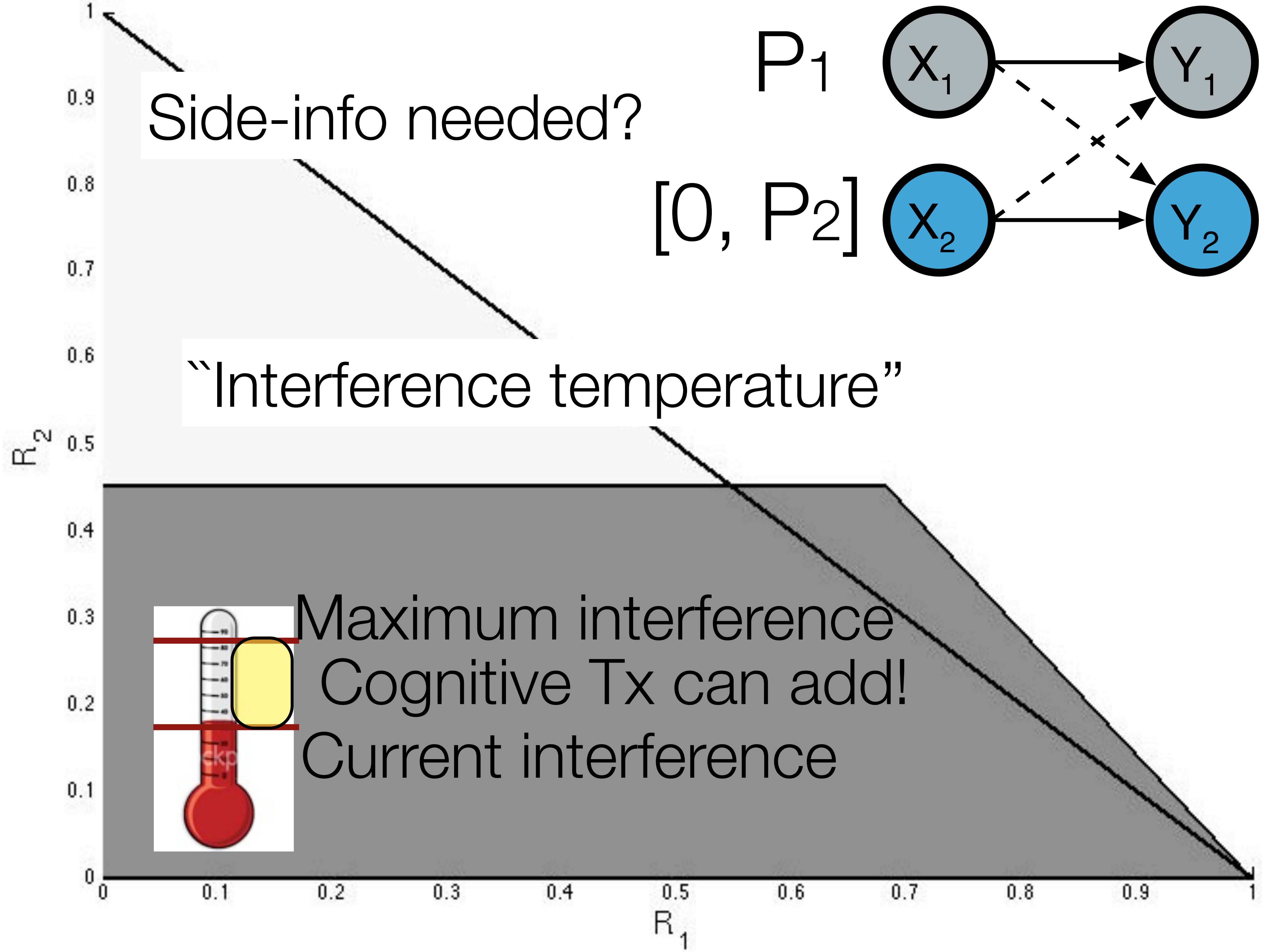


“Interference temperature”

Maximum interference

Cognitive Tx can add!

Current interference



Side-info needed?

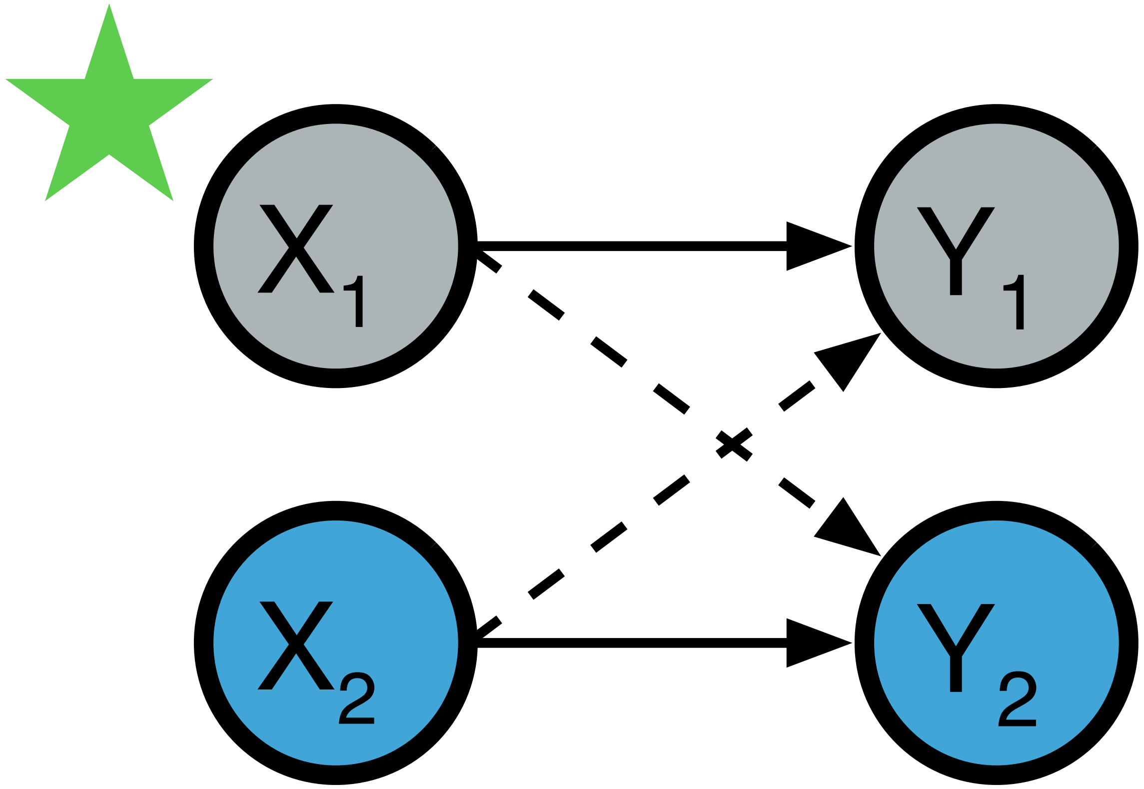
"Interference temperature"

Maximum interference

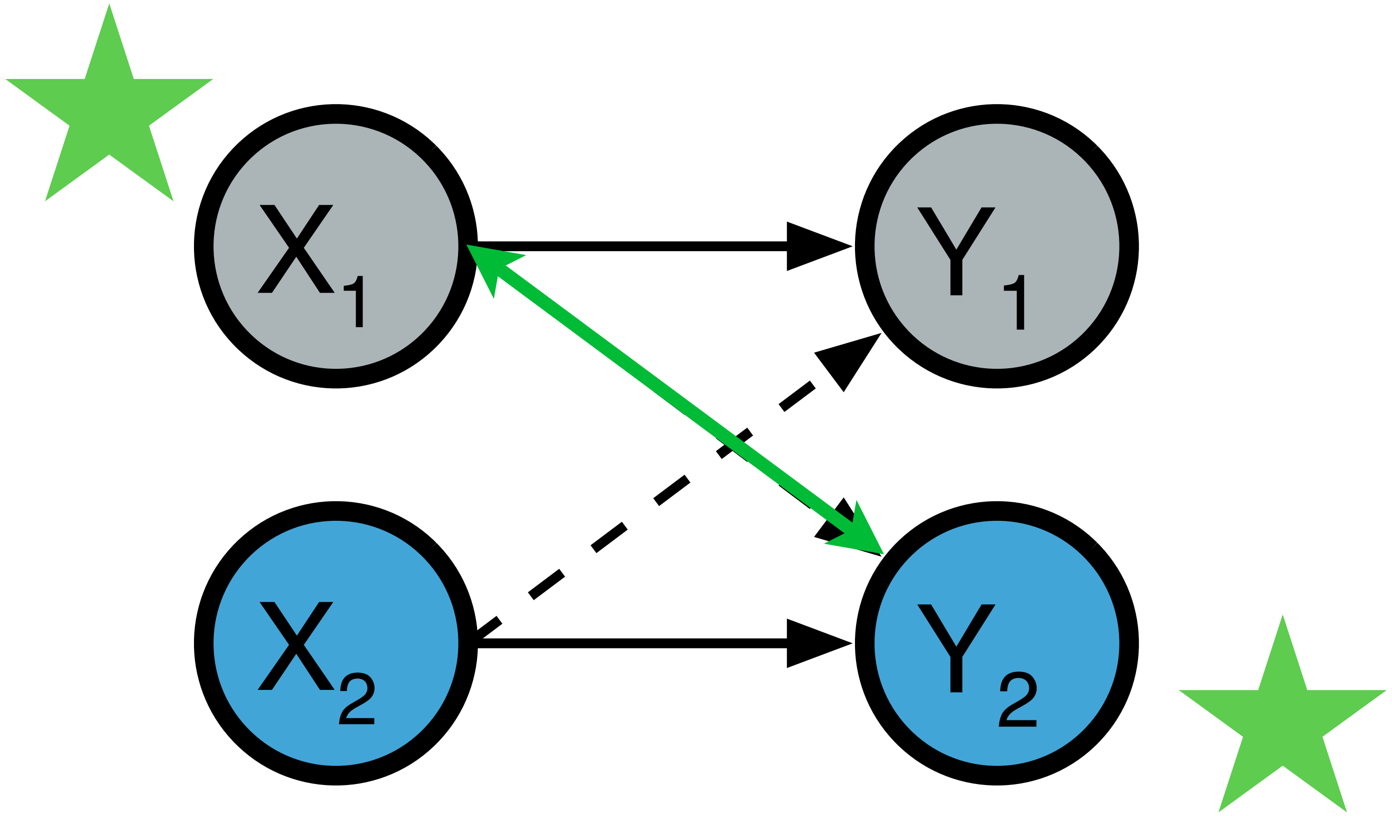
Cognitive Tx can add!

Current interference

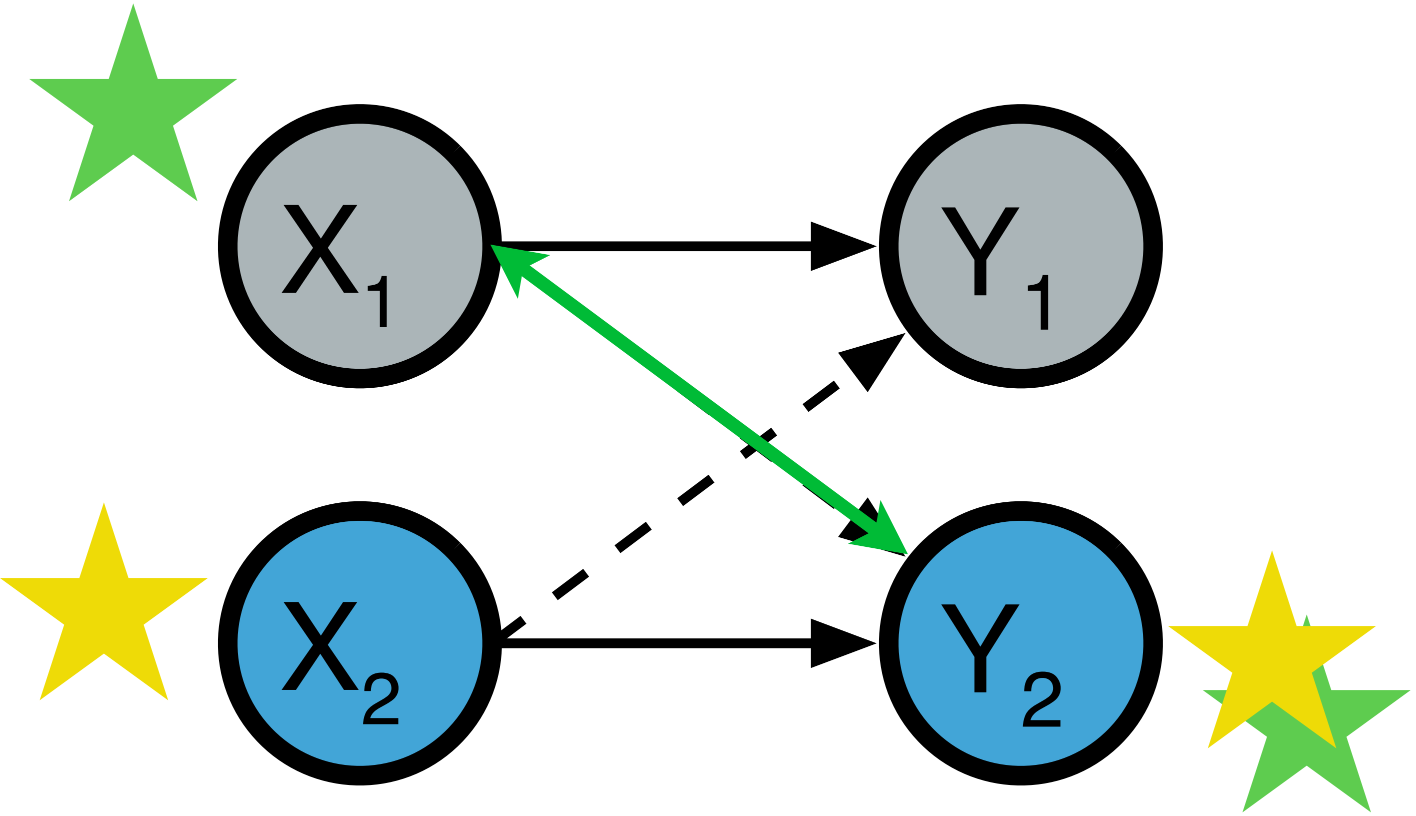
3. Opportunistic “cognitive” decoding



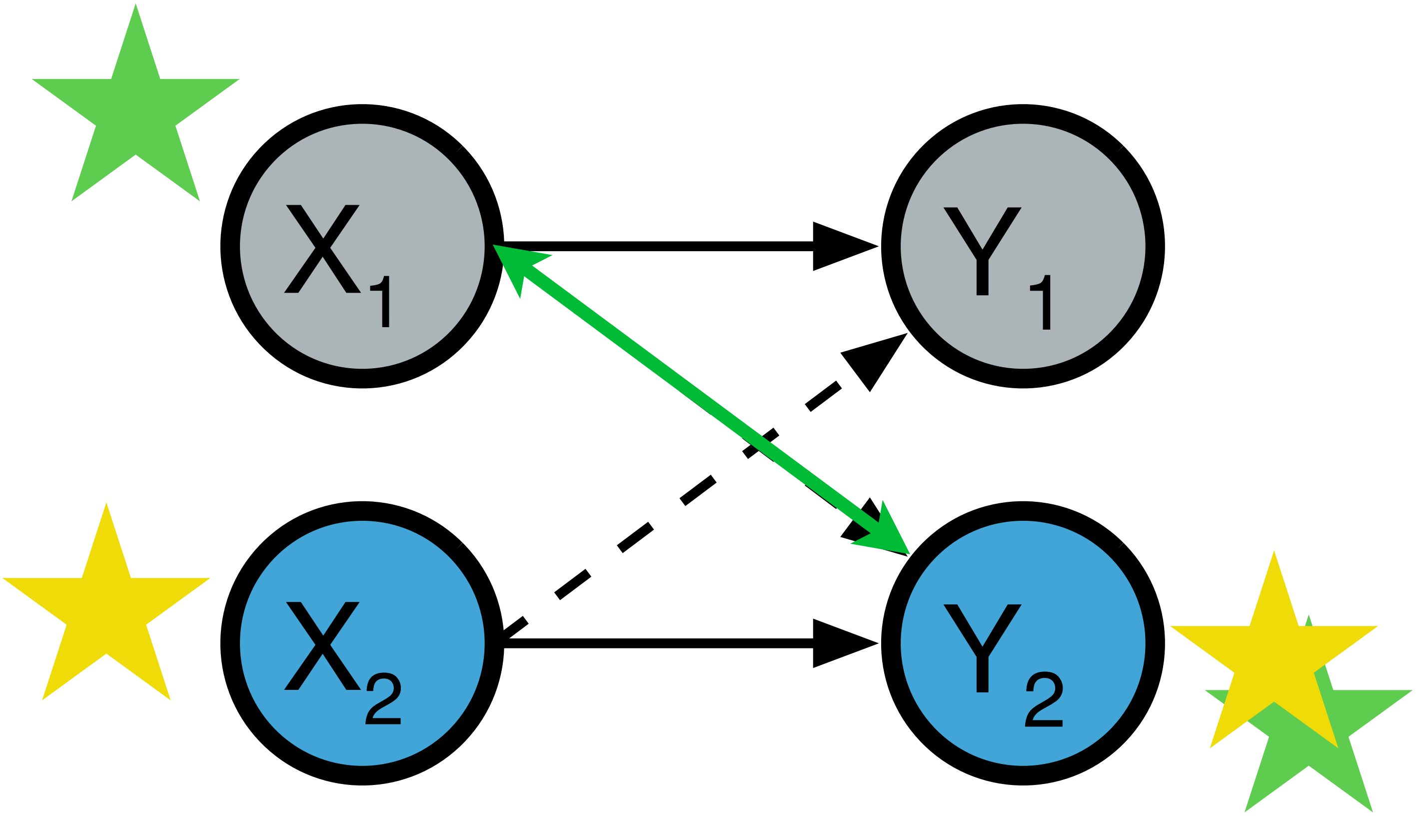
3. Opportunistic “cognitive” decoding



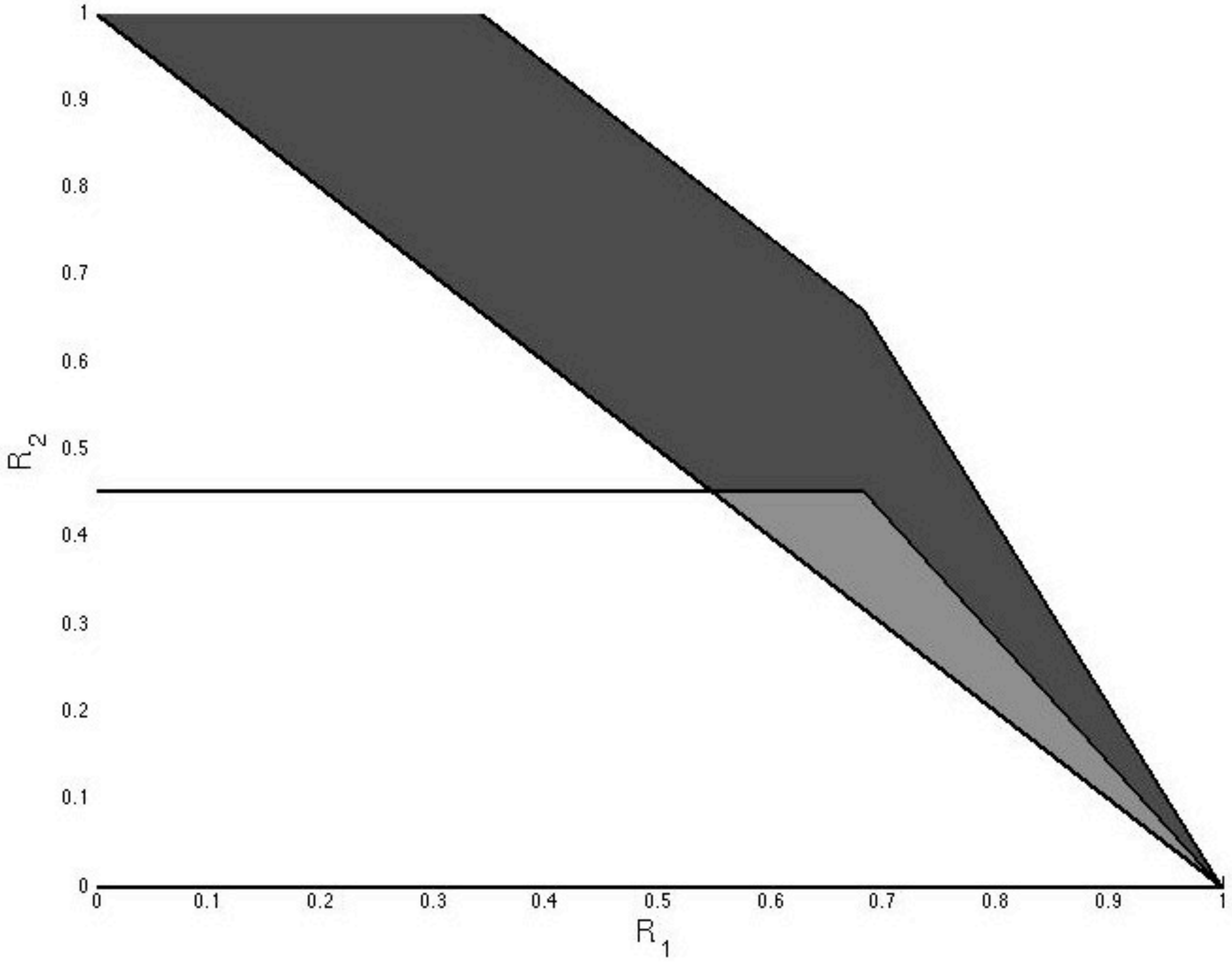
3. Opportunistic “cognitive” decoding



3. Opportunistic “cognitive” decoding

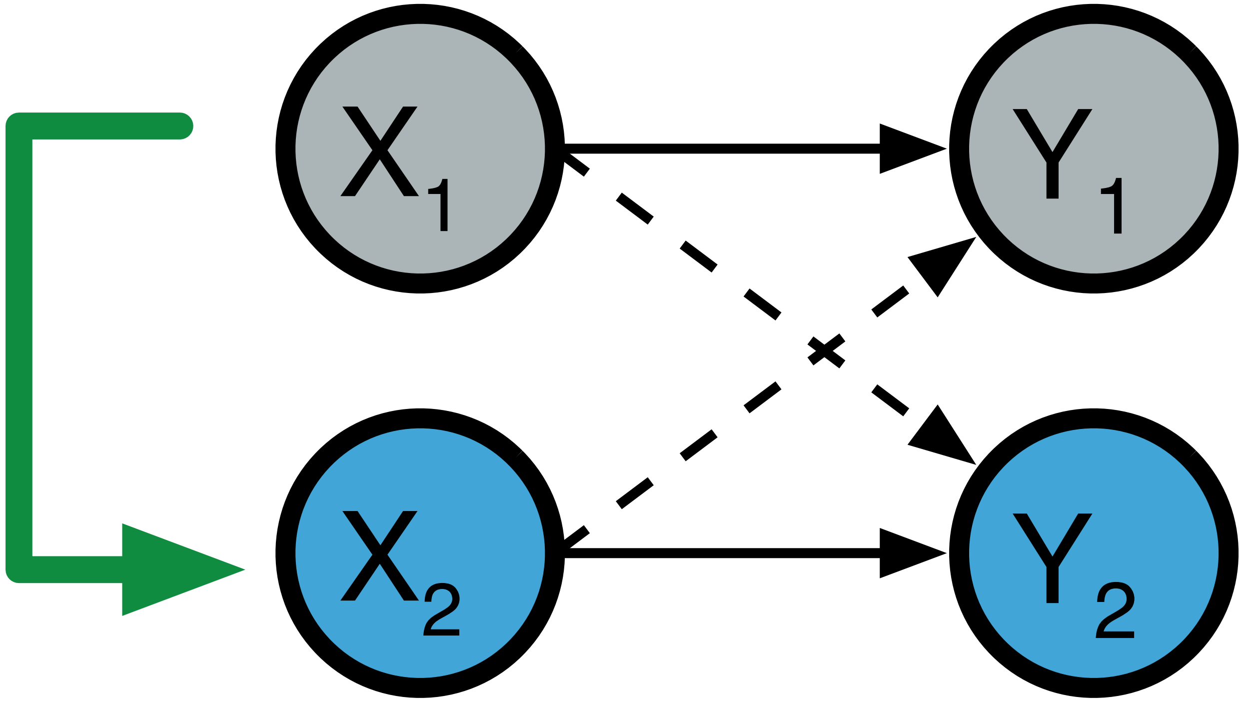


Side-info needed?



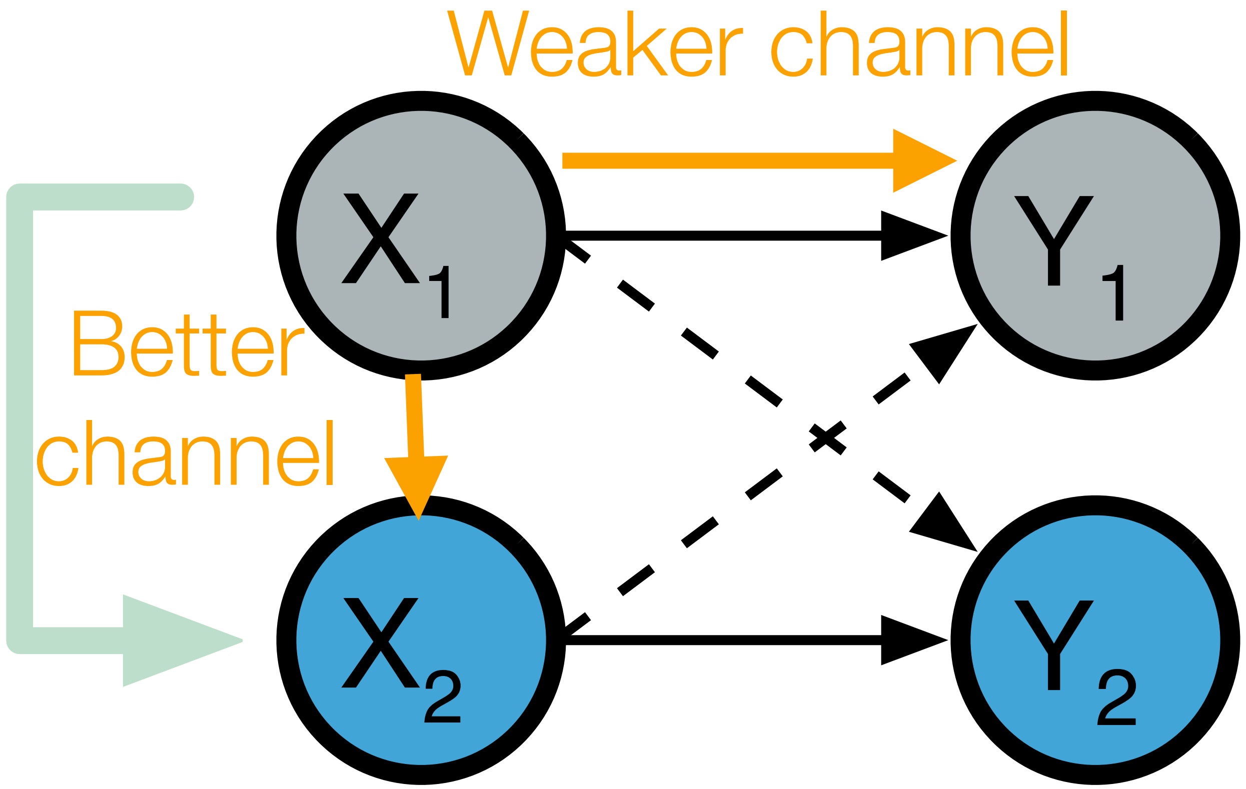
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
R_1

4. Simultaneous Cognitive Transmission



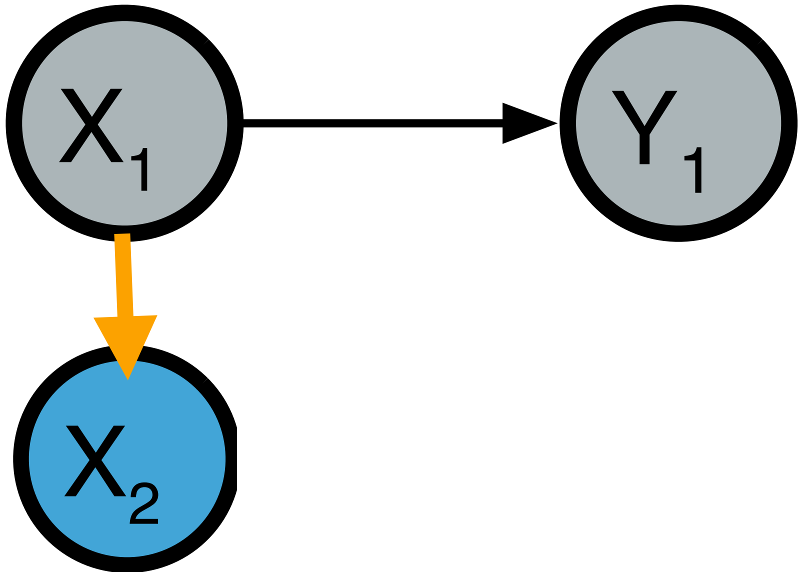
Assumption: Tx 2 knows message encoded by X_1 a-priori

4. Simultaneous Cognitive Transmission

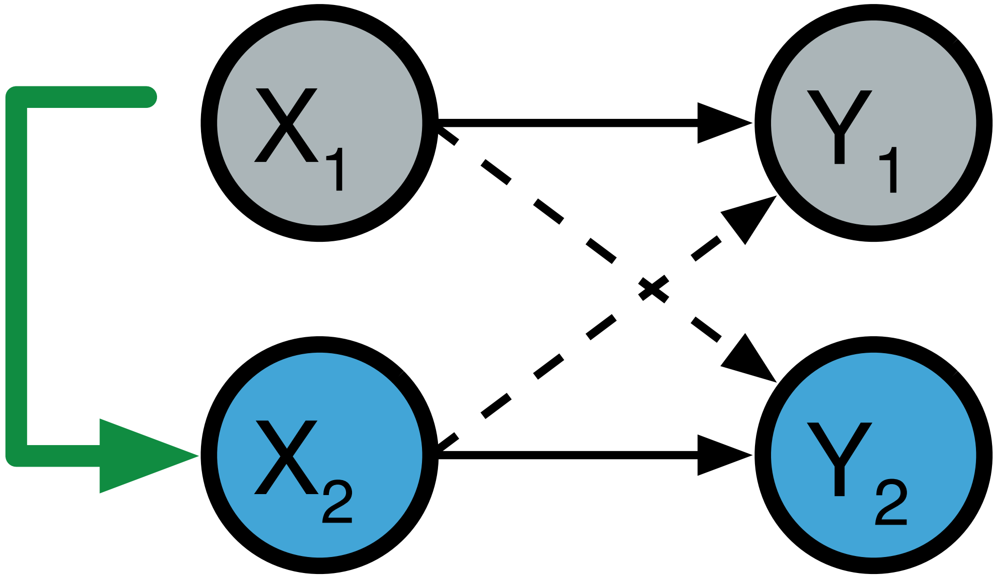


Cognitive Tx may obtain primary's message in a fraction of the time

4. Simultaneous Cognitive Transmission

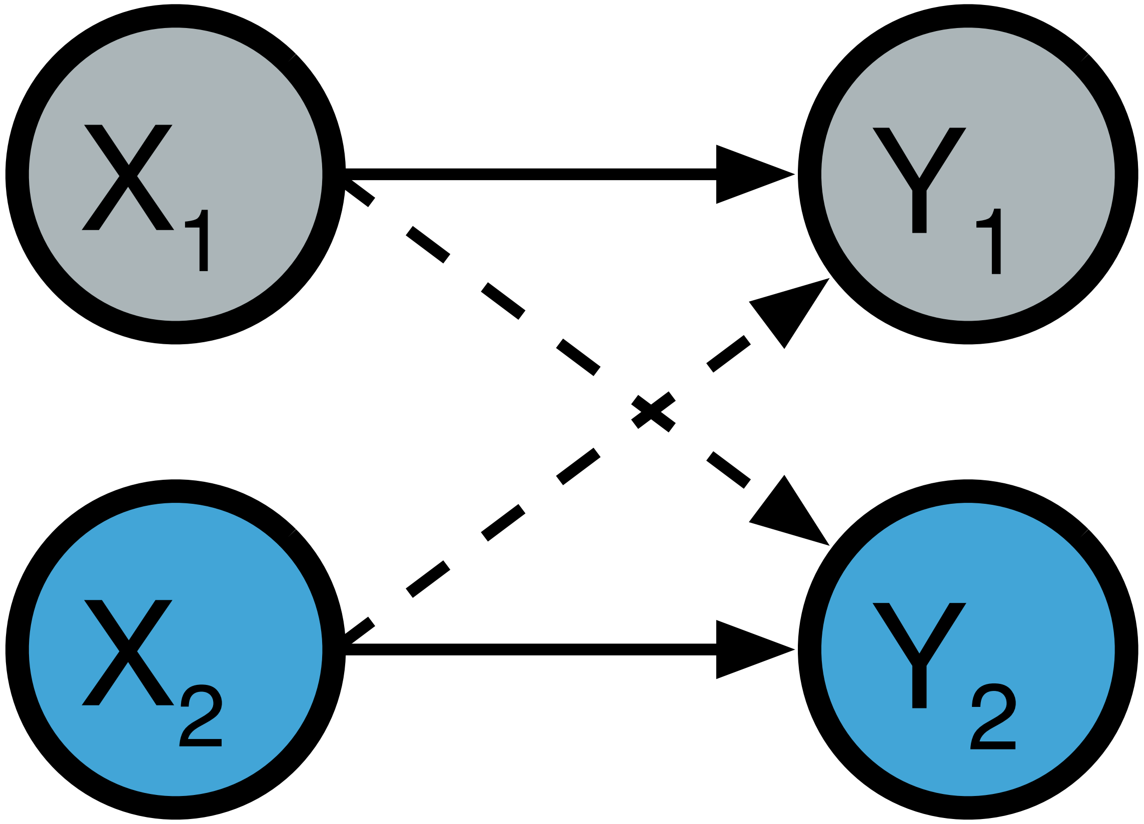


Primary transmission



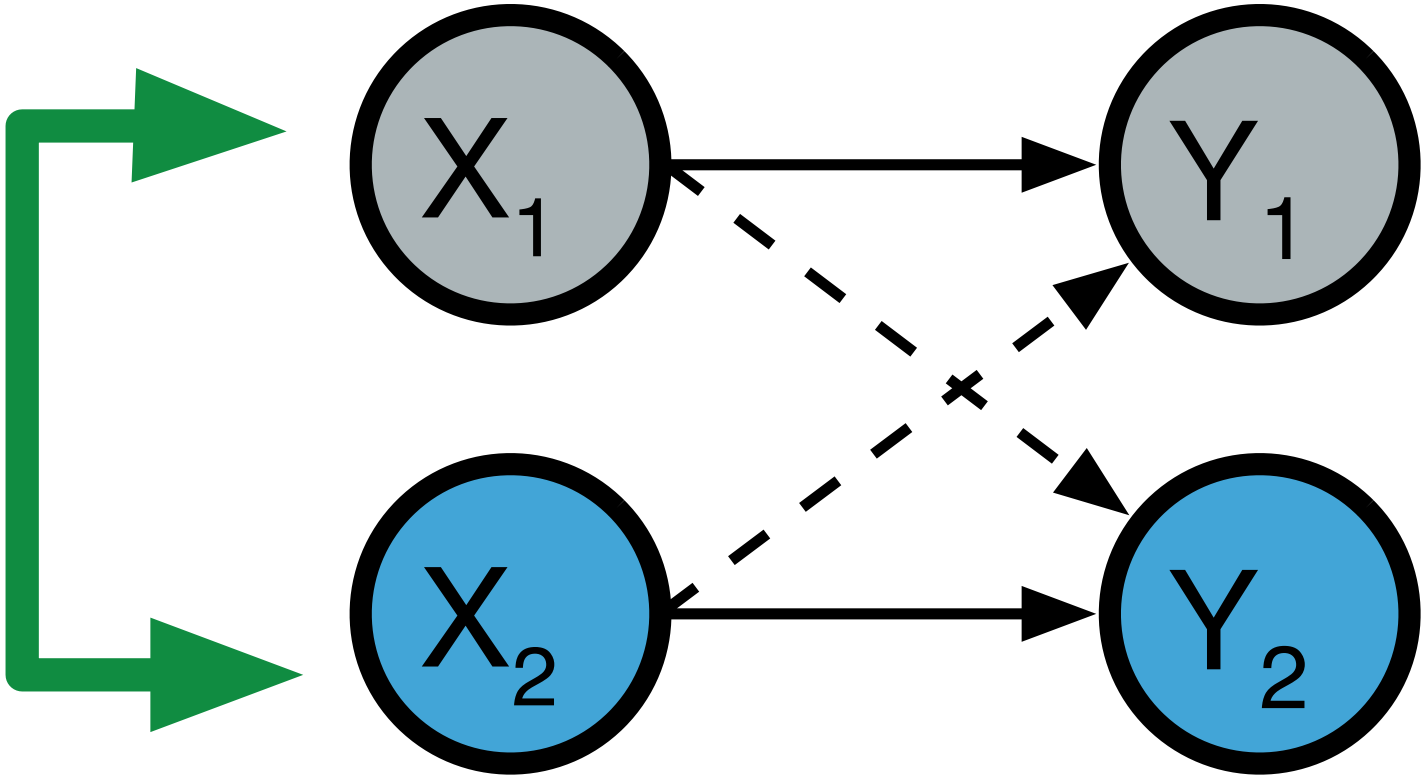
Re-transmission

Cognitive Tx may overhear primary's message



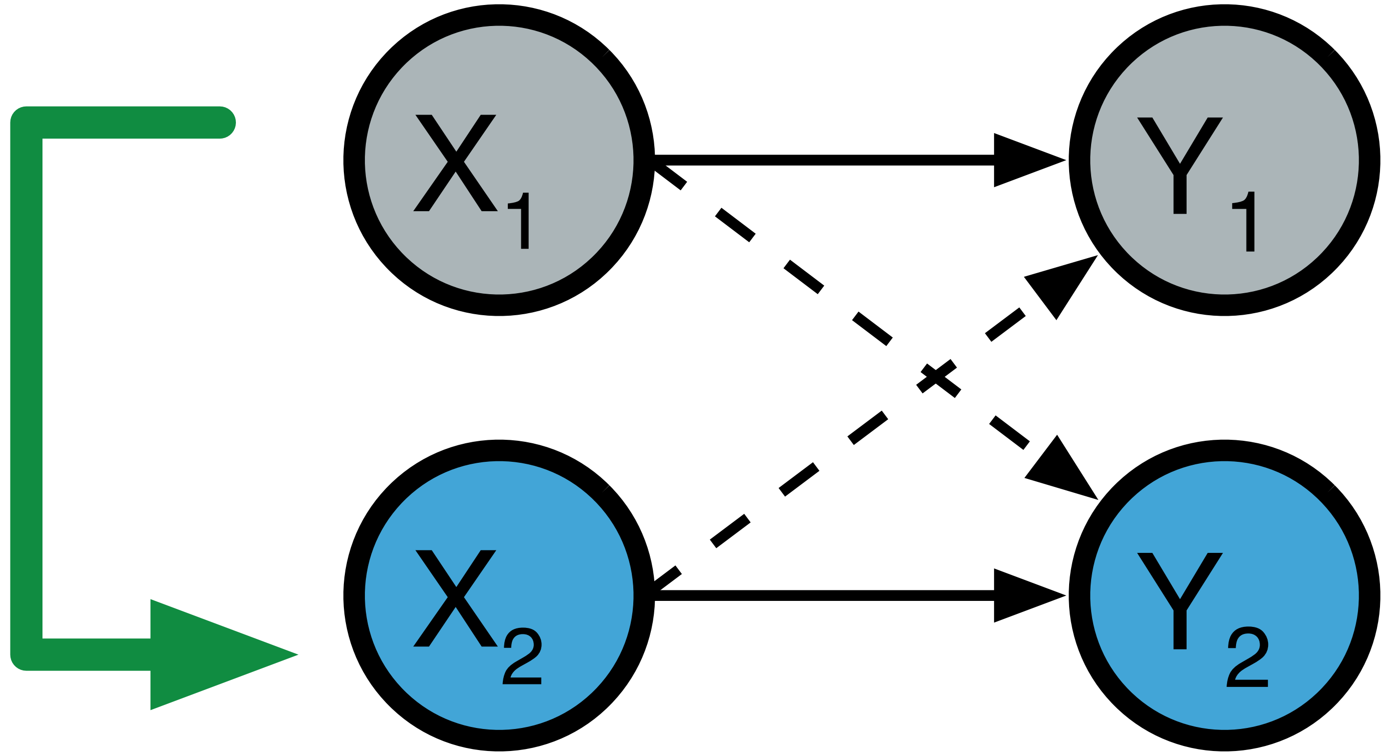
“Competitive”

Interference channel



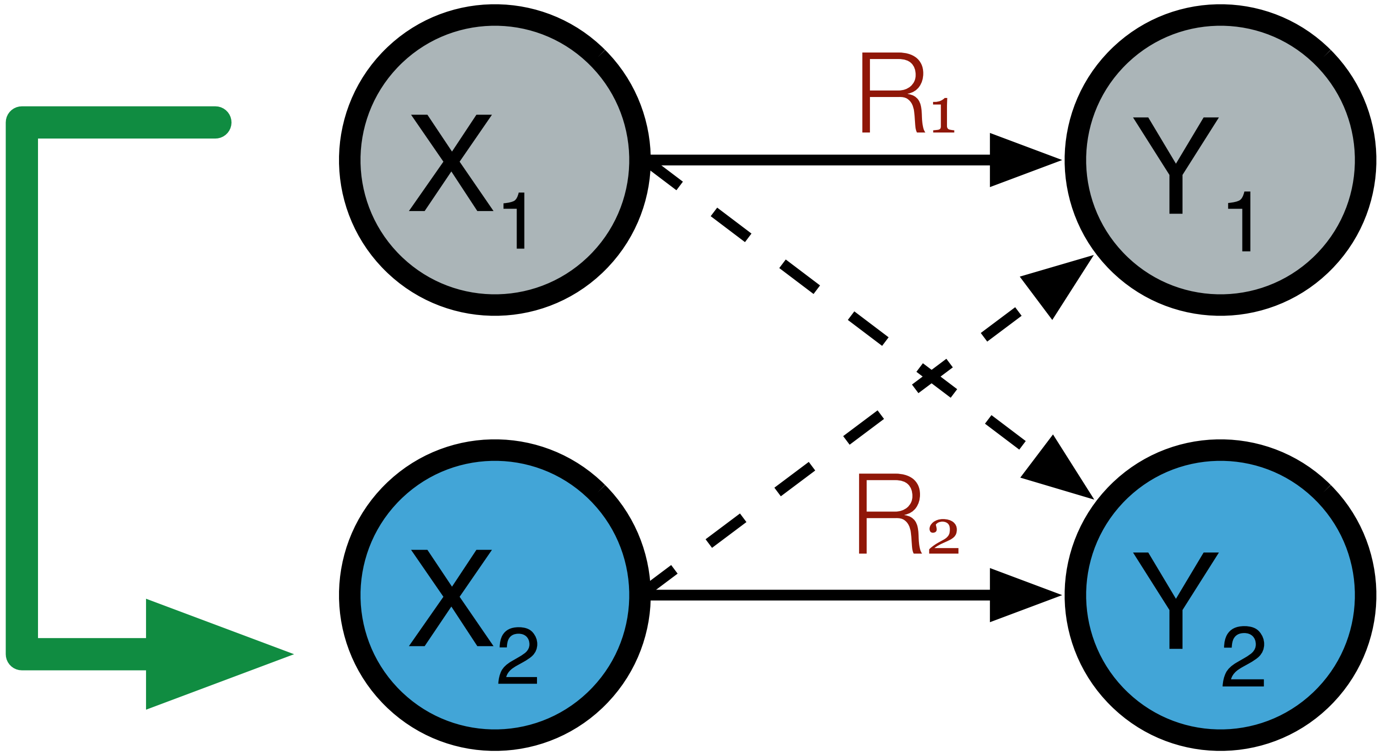
“Cooperative”

2 Tx antenna
Broadcast channel



“Cognitive”

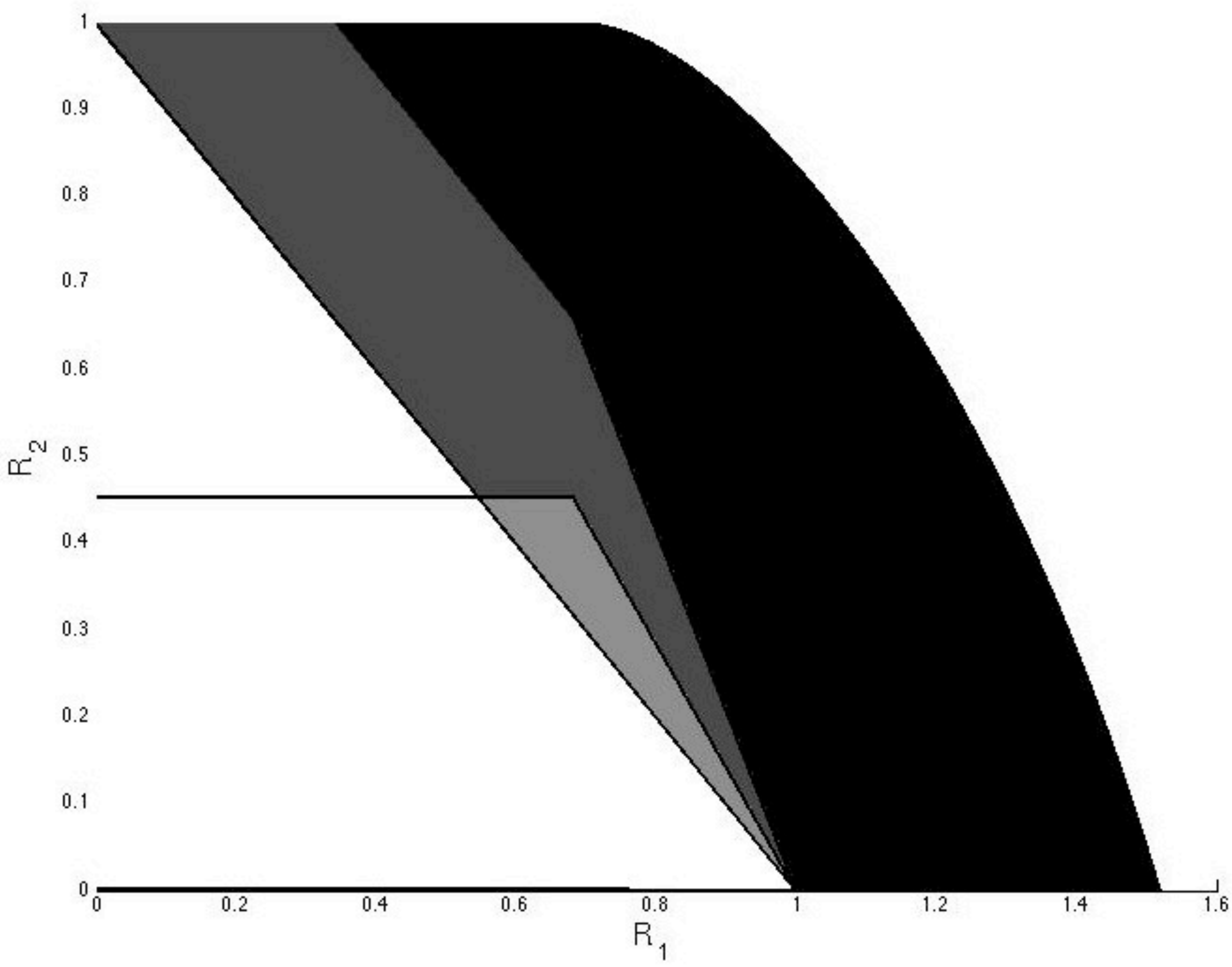
Cognitive channel



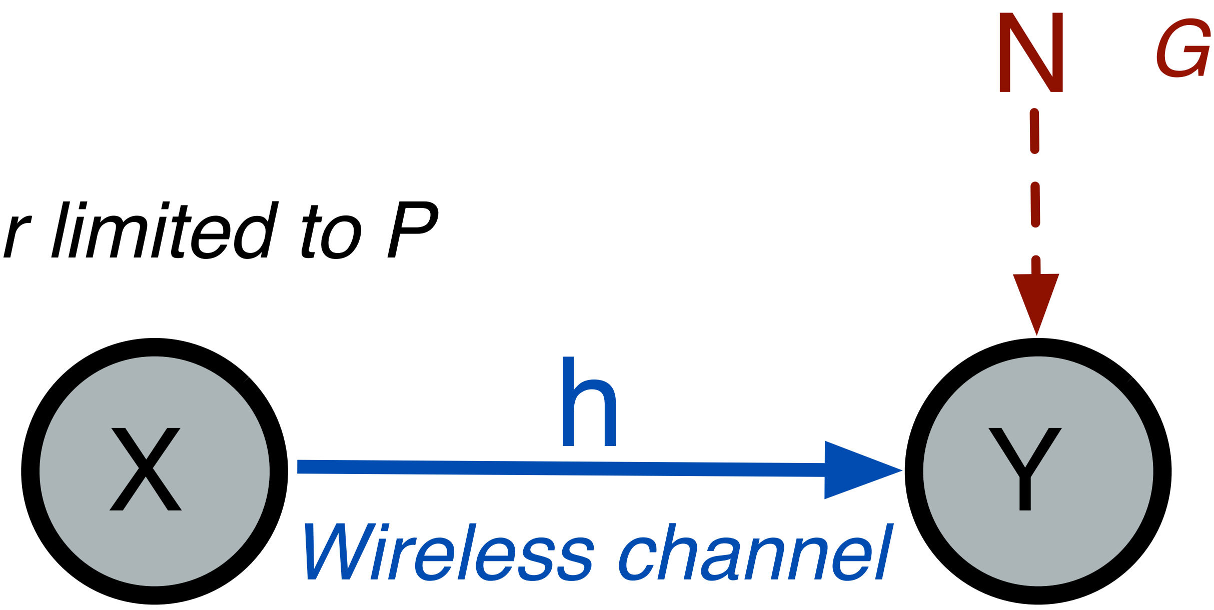
“Cognitive”

Cognitive channel

What rates (R_1, R_2) are achievable?


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1

Gaussian noise channel capacity



N *Gaussian noise ~ $N(0,P_N)$*

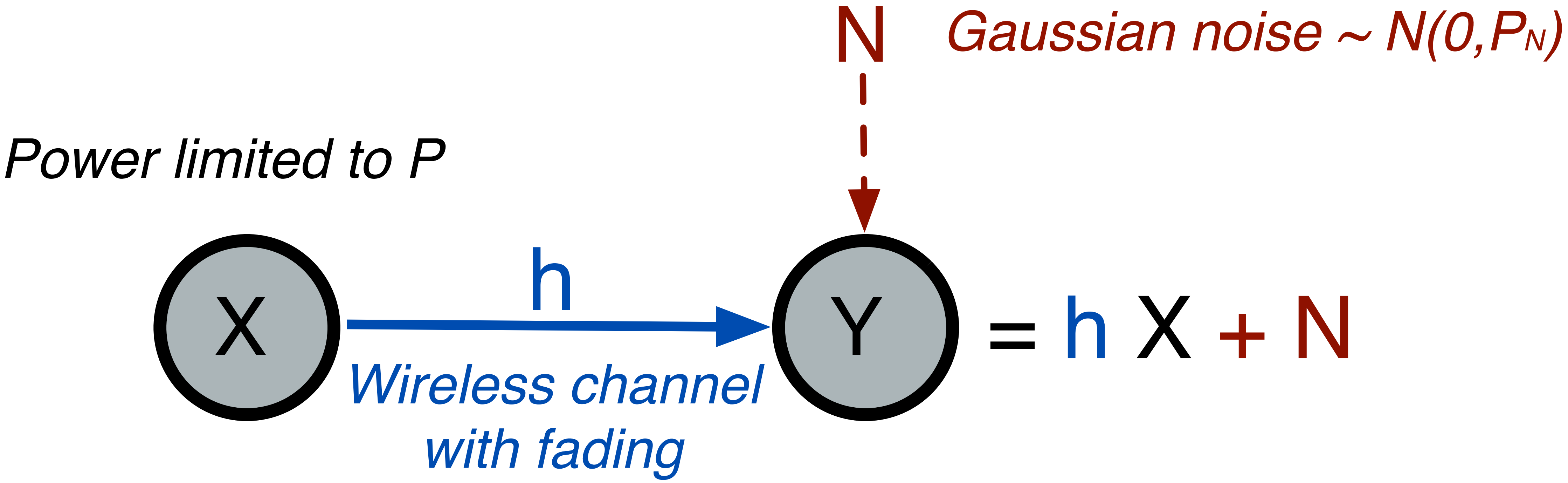
Power limited to P

h

X $Y = hX + N$

Wireless channel with fading

Gaussian noise channel capacity



N *Gaussian noise ~ N(0,P_N)*

Power limited to P

X

h

Wireless channel with fading

Y = h X + N

Capacity

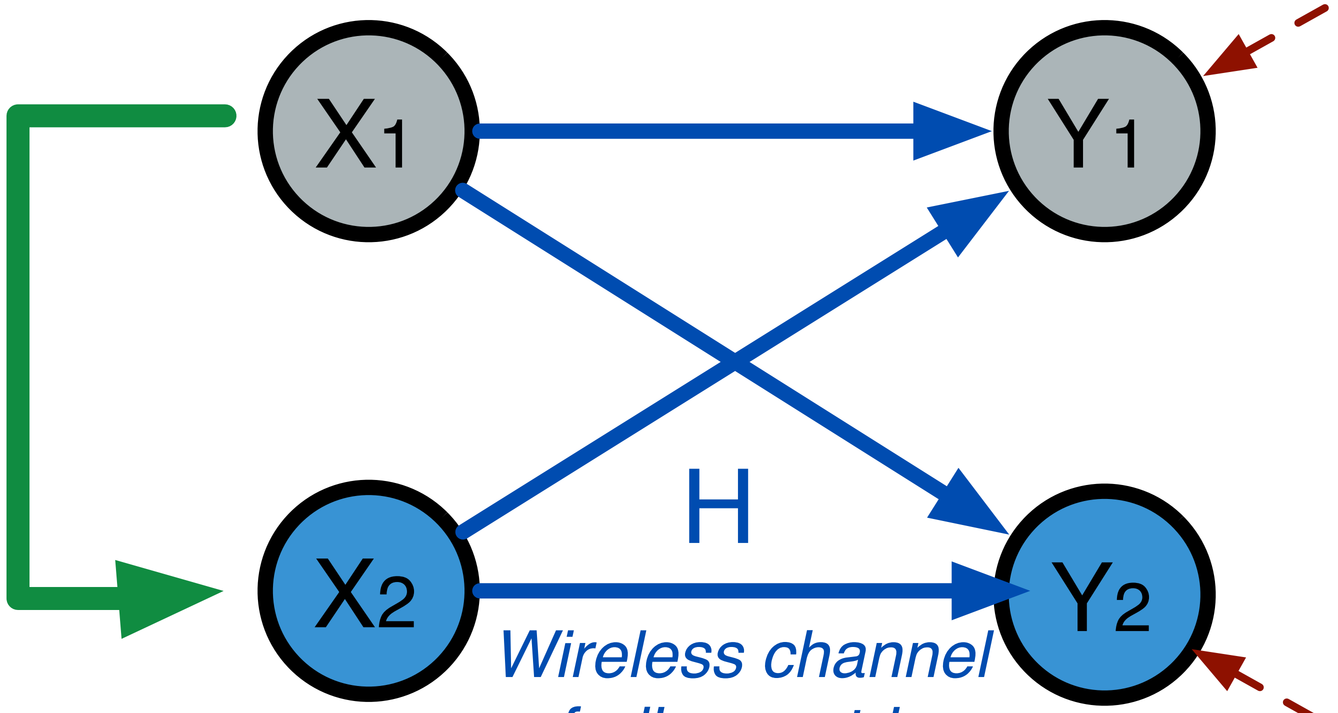
$$C = \max_{p(x):E[|X|^2]\leq P} I(X;Y)$$

$$= \frac{1}{2}\log_2\left(\frac{|h|^2 P + P_N}{P_N}\right)$$

$$= \frac{1}{2}\log_2\left(1+\mathrm{SNR}\right)$$

(bits/channel use)

Gaussian noise channel capacity



Power limited to P_1

N_1 *Gaussian noise ~ $N(0,P_{N1})$*

X_1

Y_1

$$\mathbf{Y} = H\,\mathbf{X} + \mathbf{N}$$

H

X_2

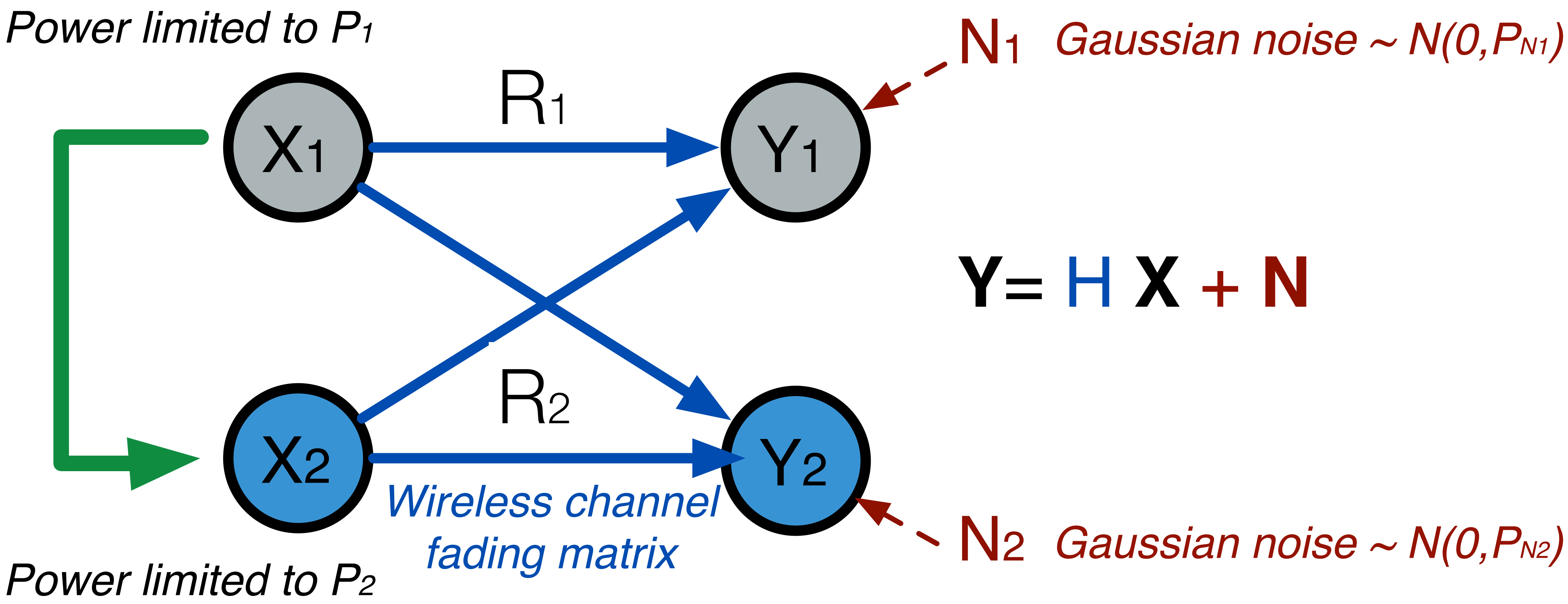
Y_2

Wireless channel fading matrix

N_2 *Gaussian noise ~ $N(0,P_{N2})$*

Power limited to P_2

Gaussian noise channel capacity



Power limited to P_1

N_1 *Gaussian noise ~ $N(0,P_{N1})$*

X_1

R_1

Y_1

$$\mathbf{Y} = H\,\mathbf{X} + \mathbf{N}$$

X_2

R_2

Y_2

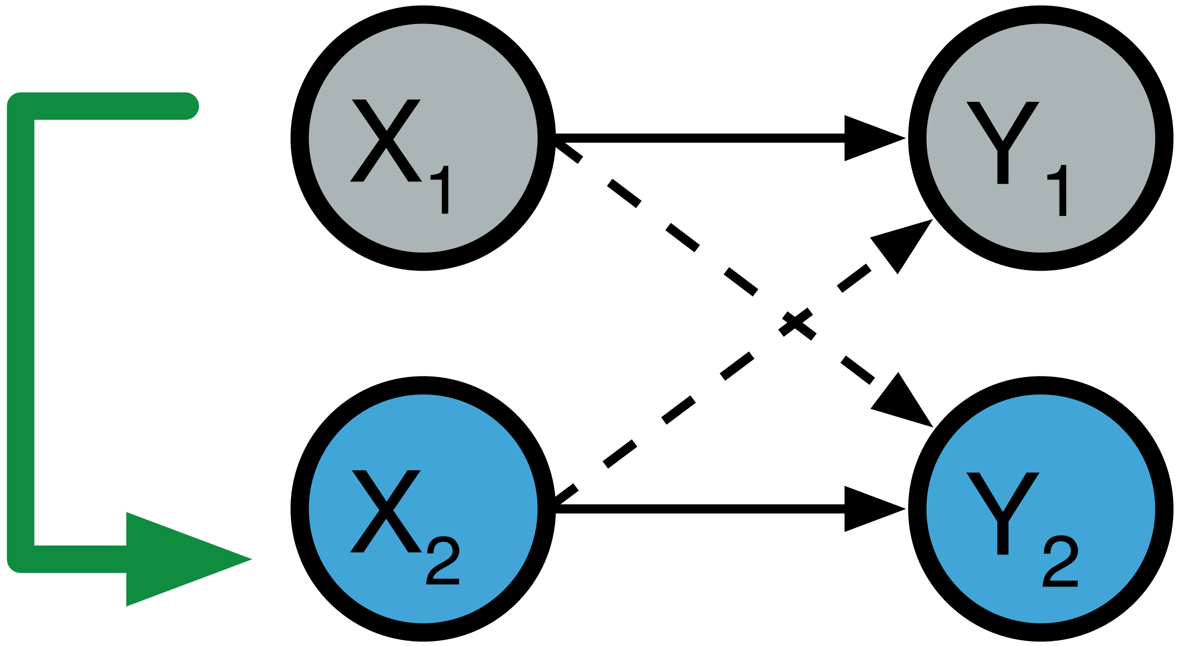
Wireless channel fading matrix

N_2 *Gaussian noise ~ $N(0,P_{N2})$*

Power limited to P_2

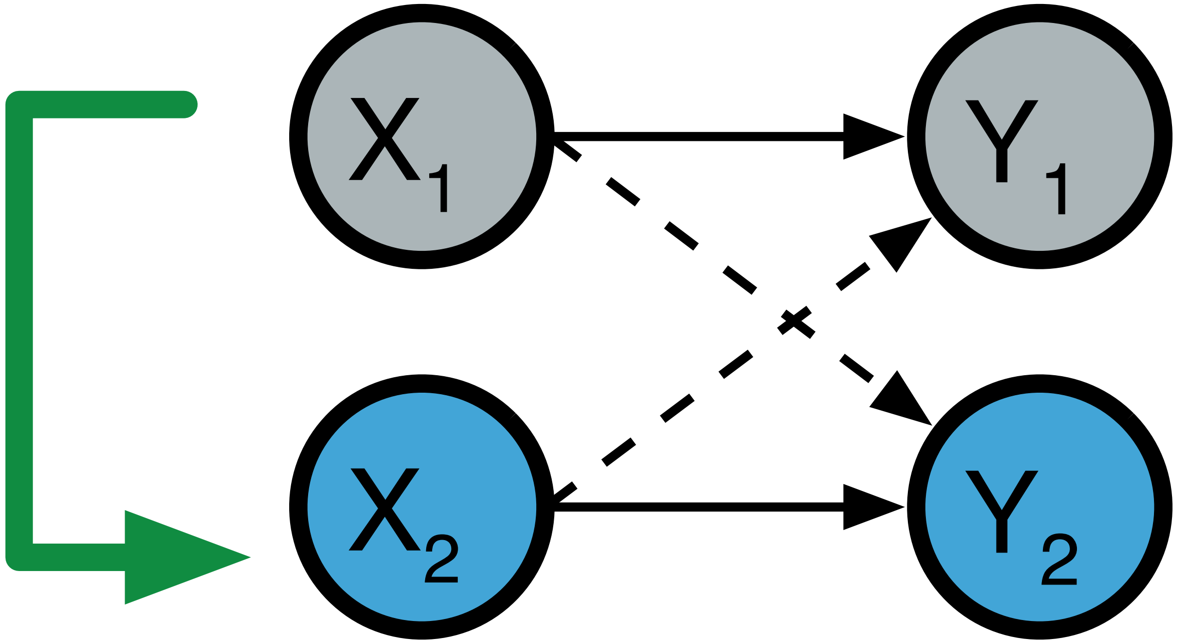
What rates are achievable?

Intuition



A priori message knowledge

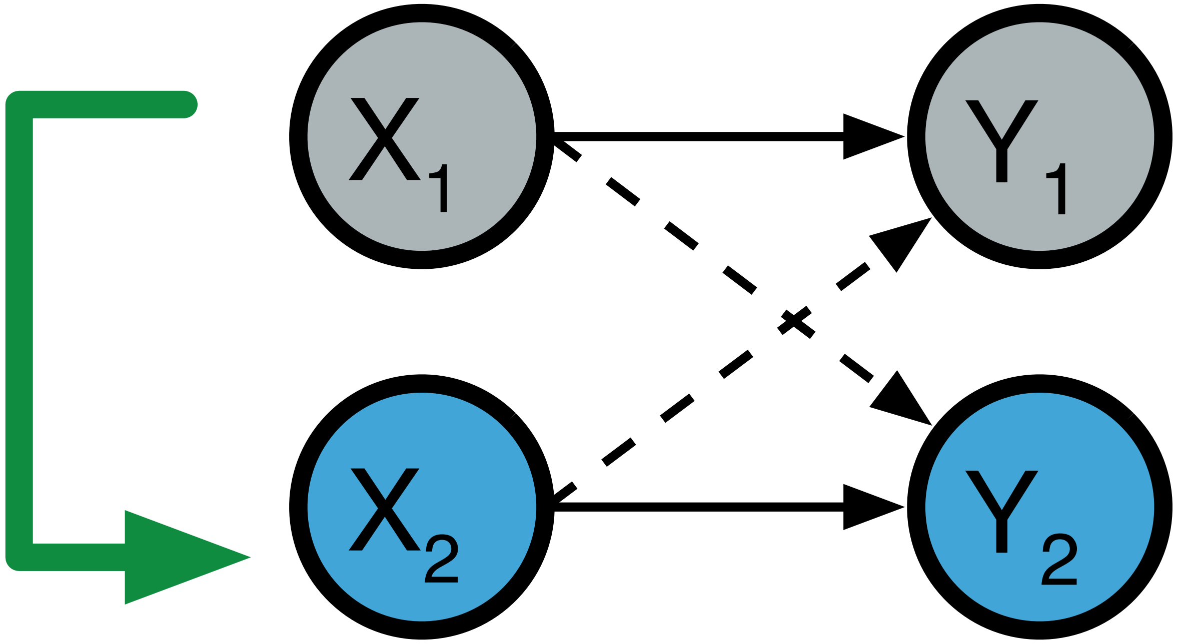
Intuition



A priori message knowledge

aid
transmission
SELFLESS

Intuition



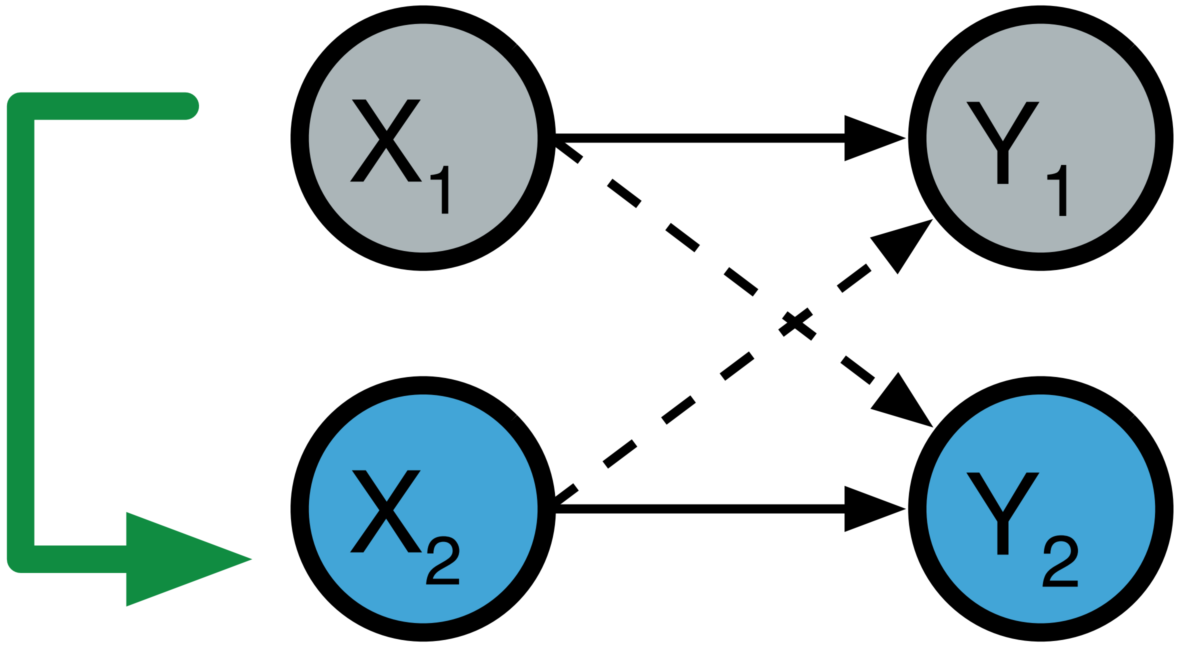
A priori message knowledge

aid
transmission
SELFLESS

mitigate
interference
SELFISH

Intuition

P_1'



A priori message knowledge

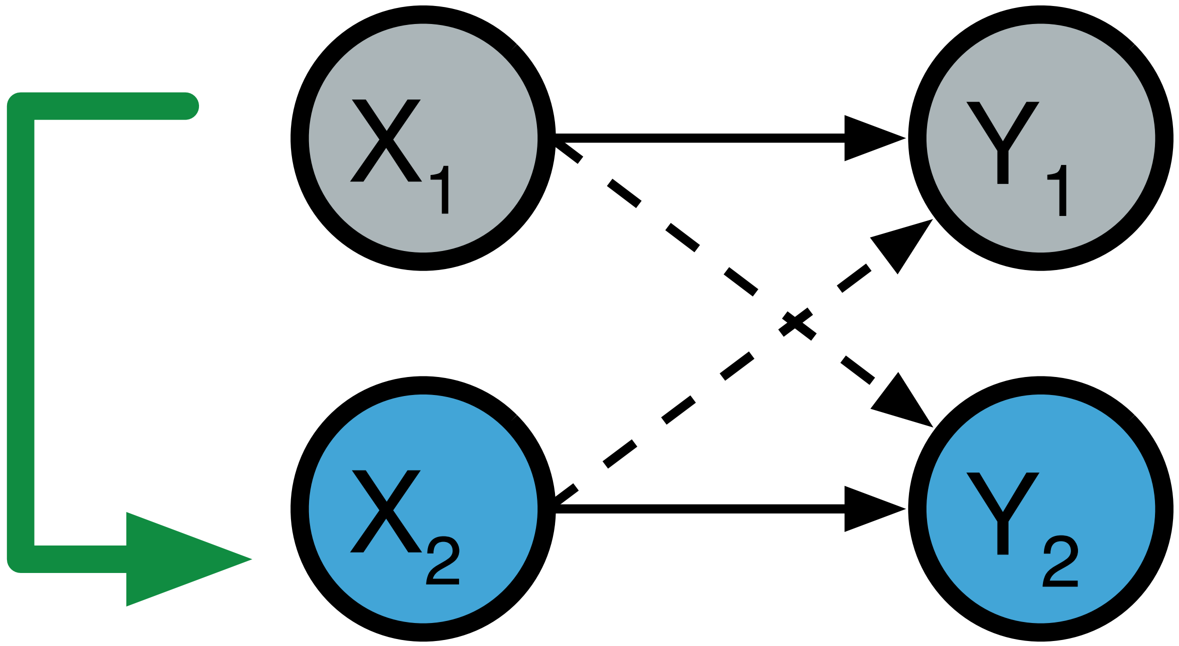
aid
transmission
SELFLESS

mitigate
interference
SELFISH

Intuition

$$P_1'$$

$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$



A priori message knowledge

aid
transmission
SELFLESS

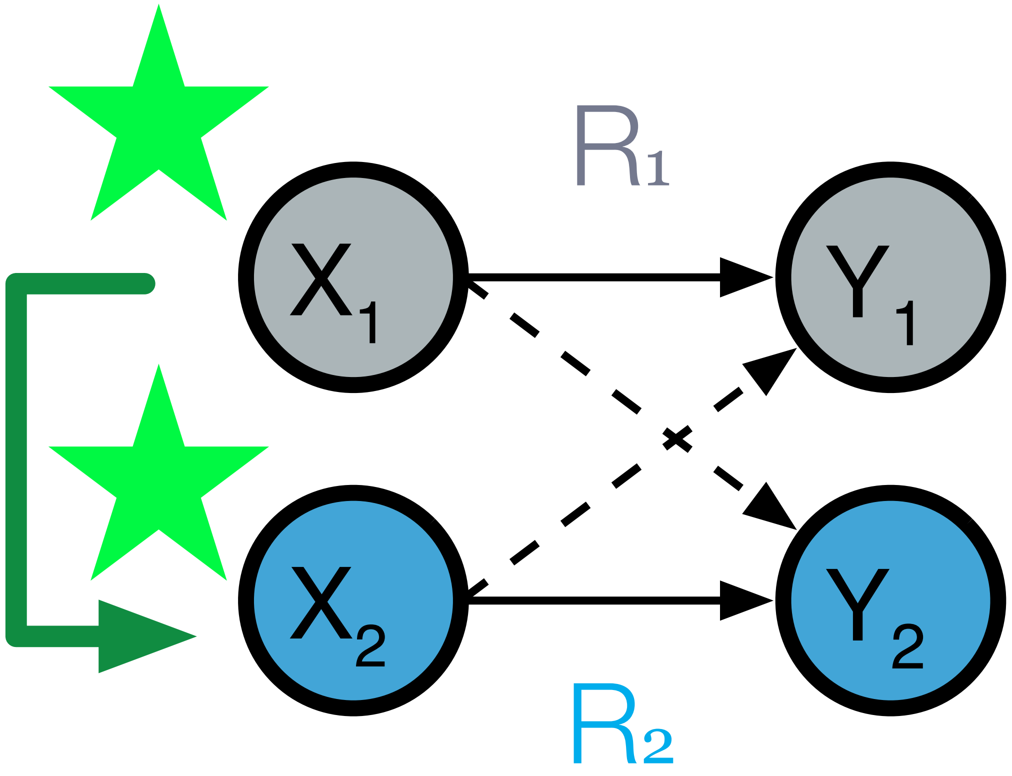
mitigate
interference
SELFISH

Message 1

Message 1: encoded by a codeword which is generated jointly Gaussian according to $\mathcal{N}(0, B_1)$

$$B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}$$

message 1



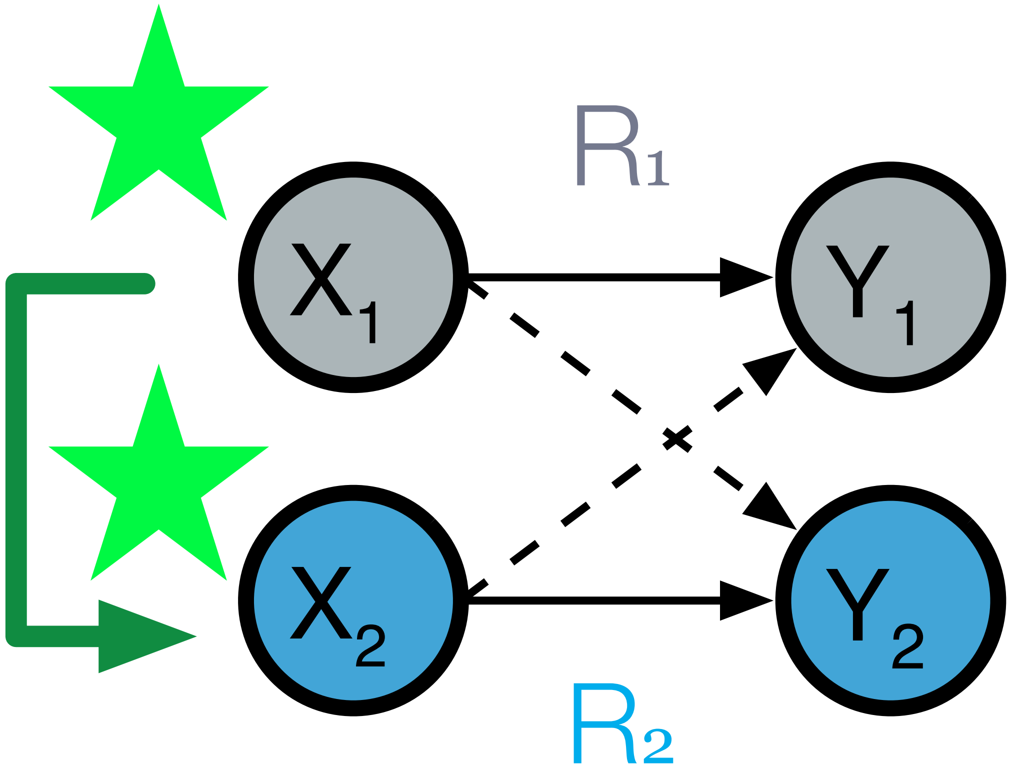
Message 1

Message 1: encoded by a codeword which is generated jointly Gaussian according to $\mathcal{N}(0,B_1)$

$$B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}$$

message 1

$$\begin{bmatrix} E[|X_1|^2] & E[X_1X_2] \\ E[X_1X_2] & E[|X_2|^2] \end{bmatrix}$$

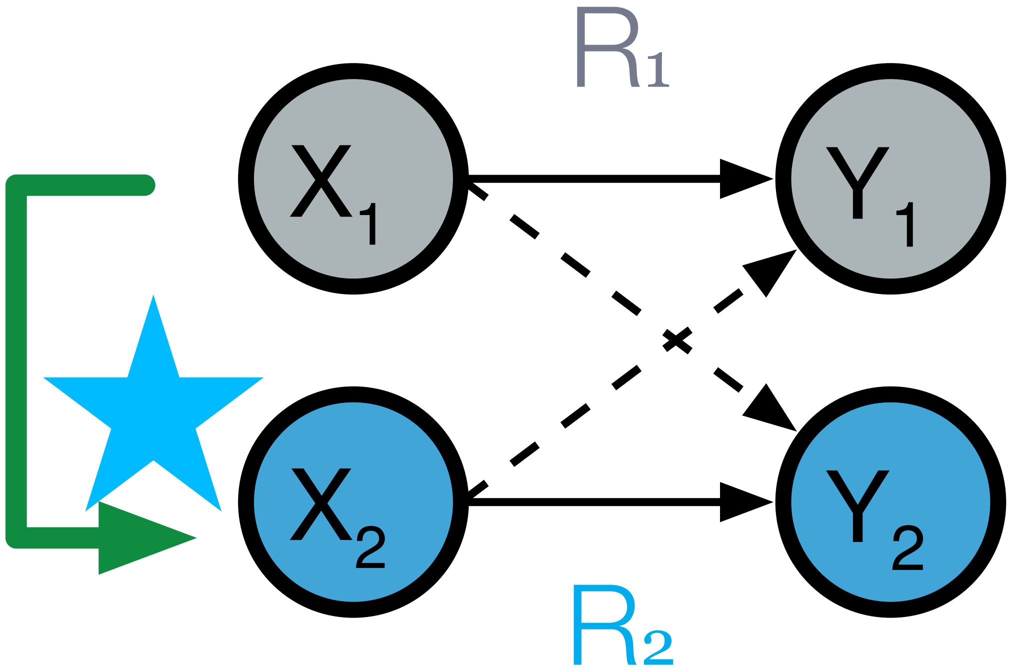


Message 2 ★

Message 2: encoded by a codeword which is generated as jointly Gaussian according to $\mathcal{N}(0,B_2)$

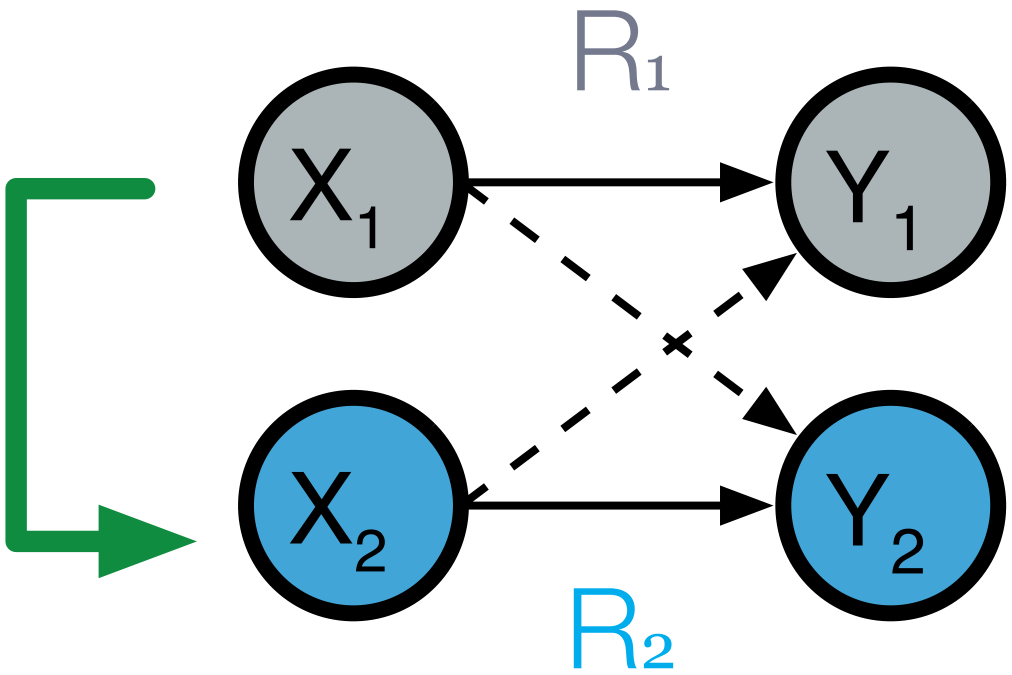
$$\boxed{B_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_2'' \end{bmatrix}}$$

message 2



Send the superposition

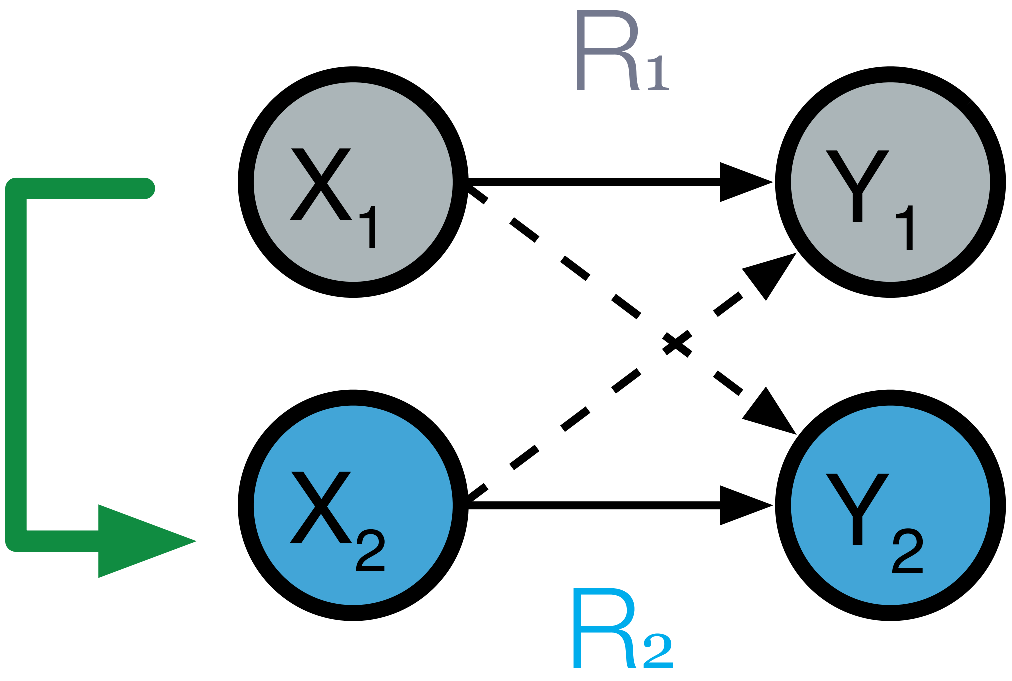
$\boxed{B_1 + B_2}$ Overall transmit covariance matrix



Send the superposition

$$B_1 + B_2 \preceq \begin{bmatrix} \boxed{P_1} & z \\ z & \boxed{P_2} \end{bmatrix}$$

Per antenna power constraints

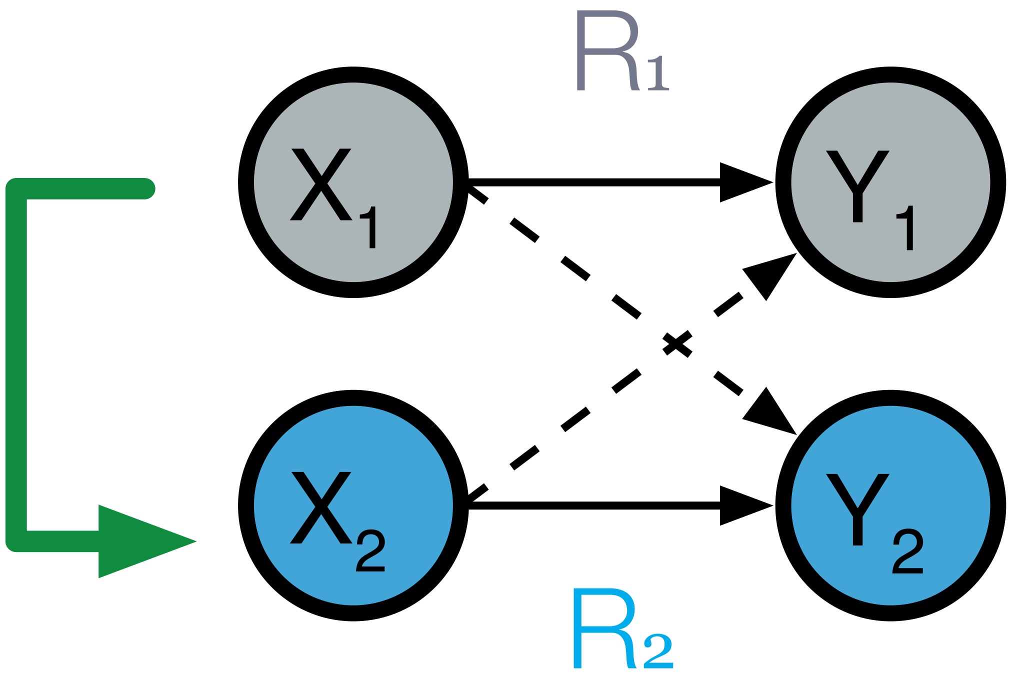


Send the superposition

$$B_1 + B_2 \preceq \begin{bmatrix} P_1 & \boxed{z} \\ \boxed{z} & P_2 \end{bmatrix}, \quad \boxed{z^2 \le P_1 P_2}$$

Correlation between two antennas

Ensures Tx covariance matrix is positive semi-definite

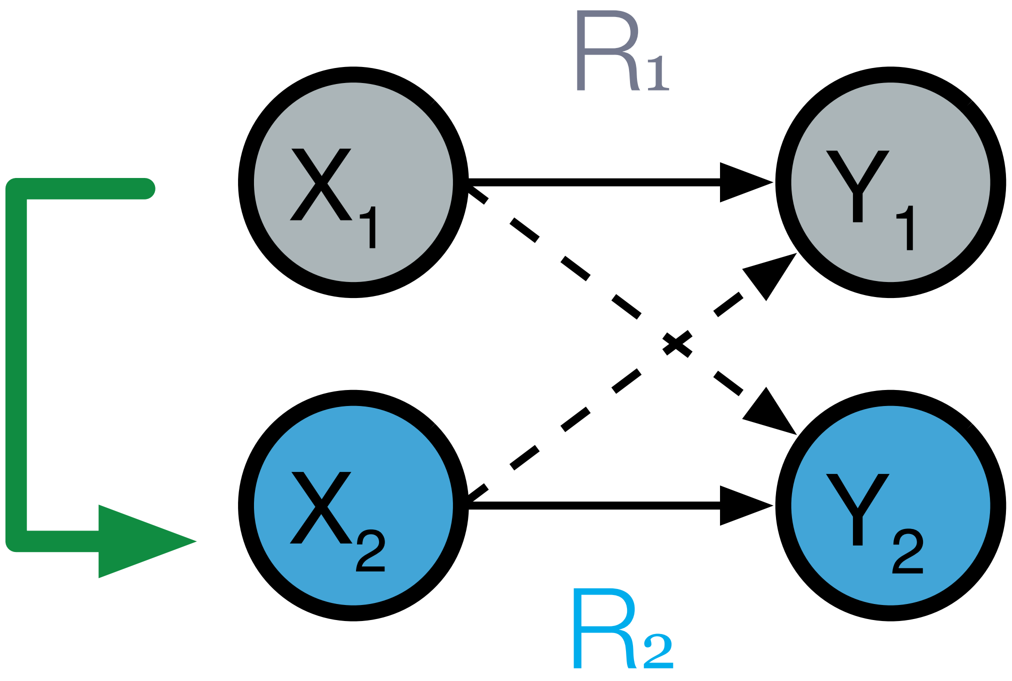


Send the superposition

$$B_1 + B_2 \preceq \begin{bmatrix} P_1 & \boxed{z} \\ \boxed{z} & P_2 \end{bmatrix}, \quad \boxed{z^2 \leq P_1 P_2}$$

Correlation between two antennas

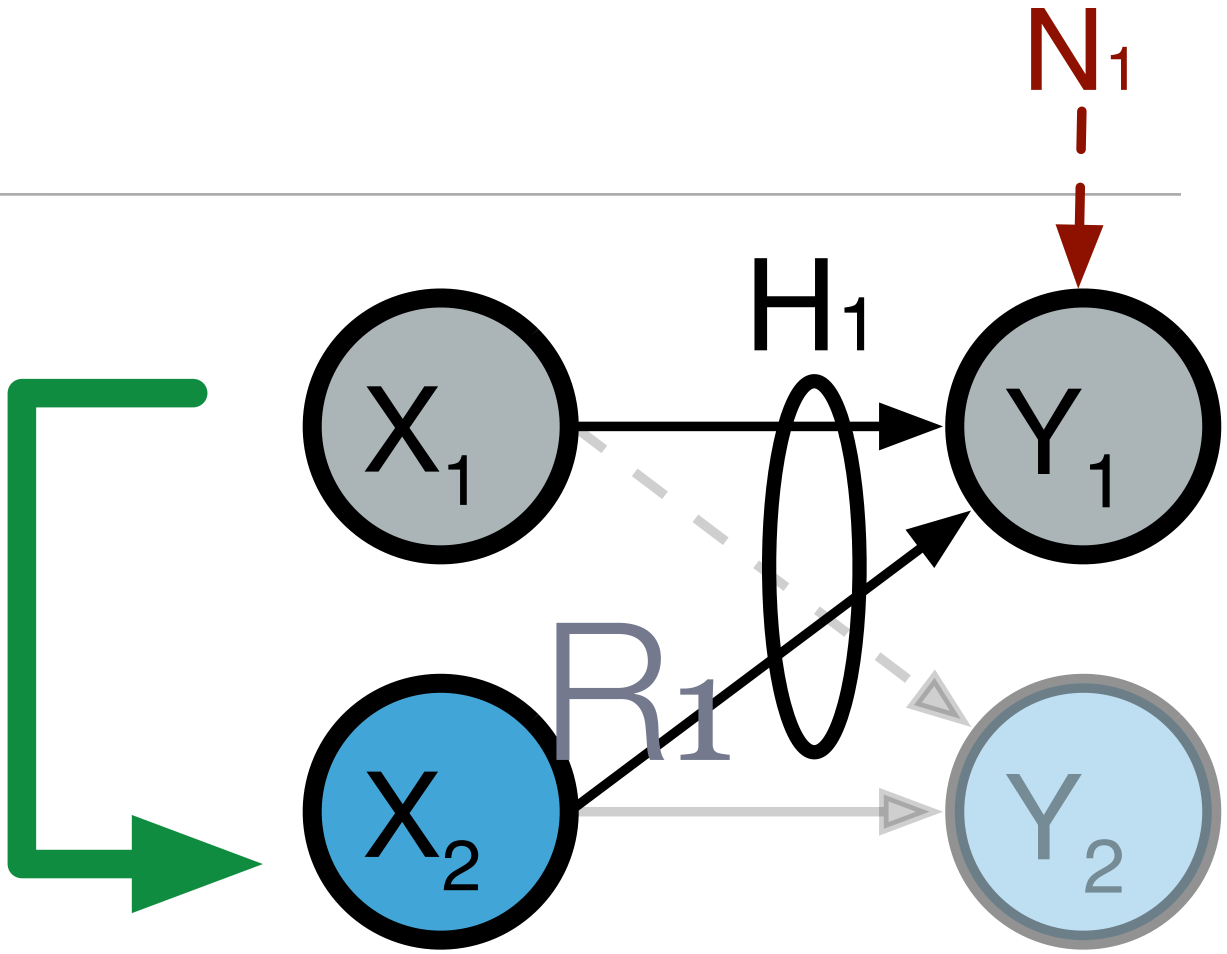
Ensures Tx covariance matrix is positive semi-definite



What rates R_1, R_2 are achievable?

R_1: Rate of message 1

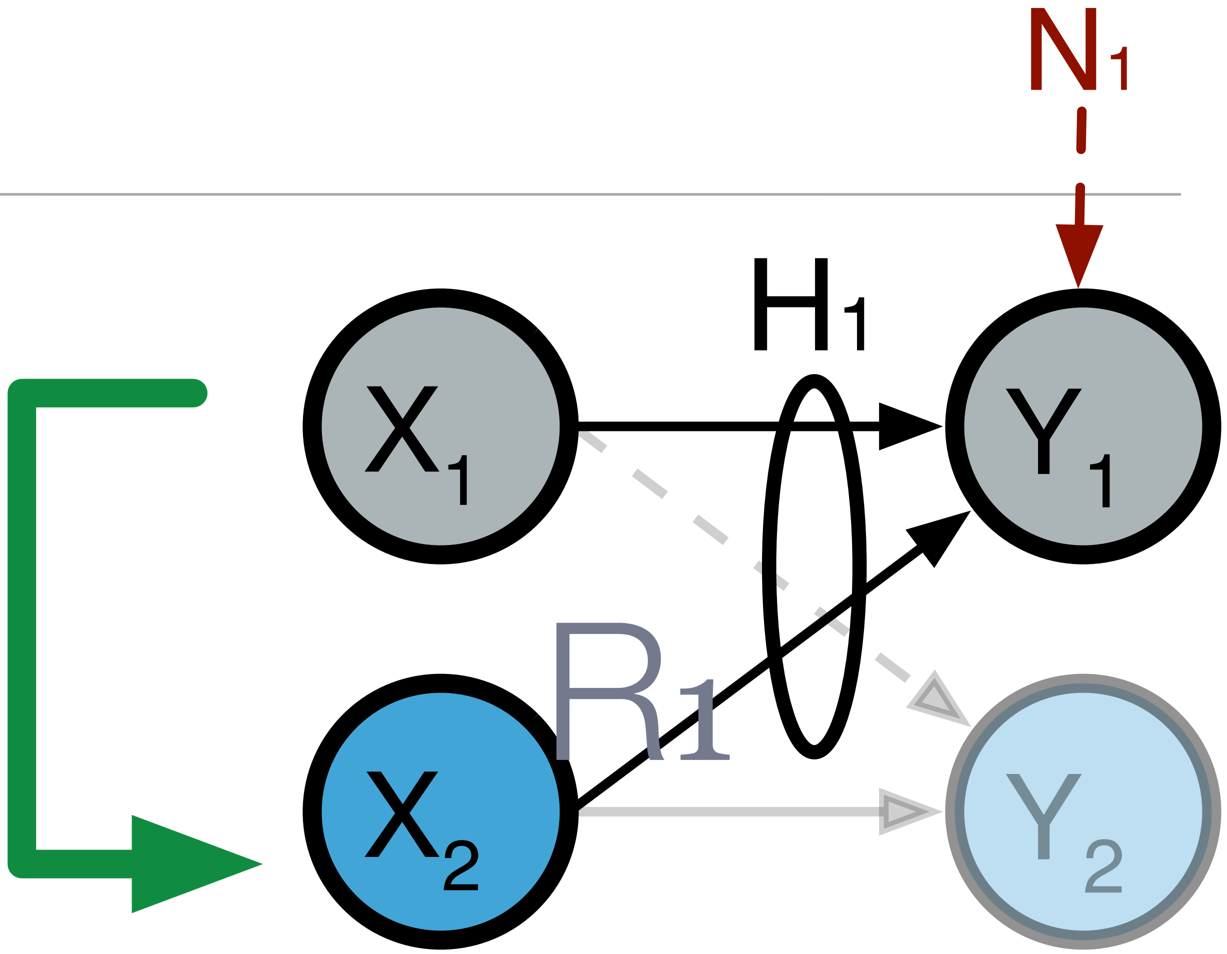
$$B = \boxed{B_1} + B_2$$



$$Y_1 = H_1 X + N_1$$

R₁: Rate of message 1

$$B = \boxed{B_1} + B_2$$

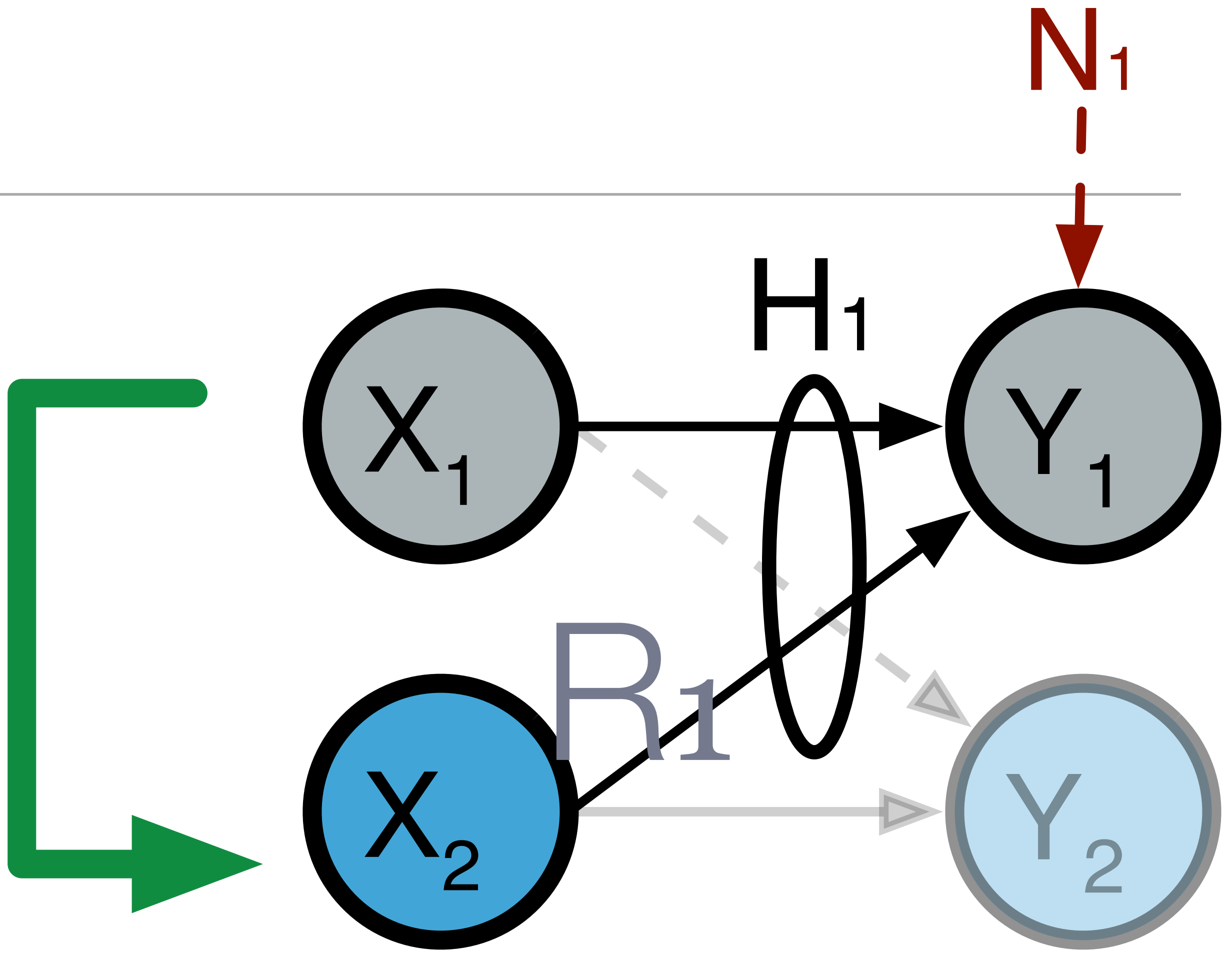


Signal power at Y₁

$$R_1 \leq \frac{1}{2}\log_2\left(\frac{\boxed{H_1(B_1+B_2)H_1^\dagger + P_{N_1}}}{\boxed{H_1(B_2)H_1^\dagger + P_{N_1}}}\right)$$

Interference + noise power

R_1: Rate of message 1



$$B = \boxed{B_1} + B_2$$

Signal power at Y_1

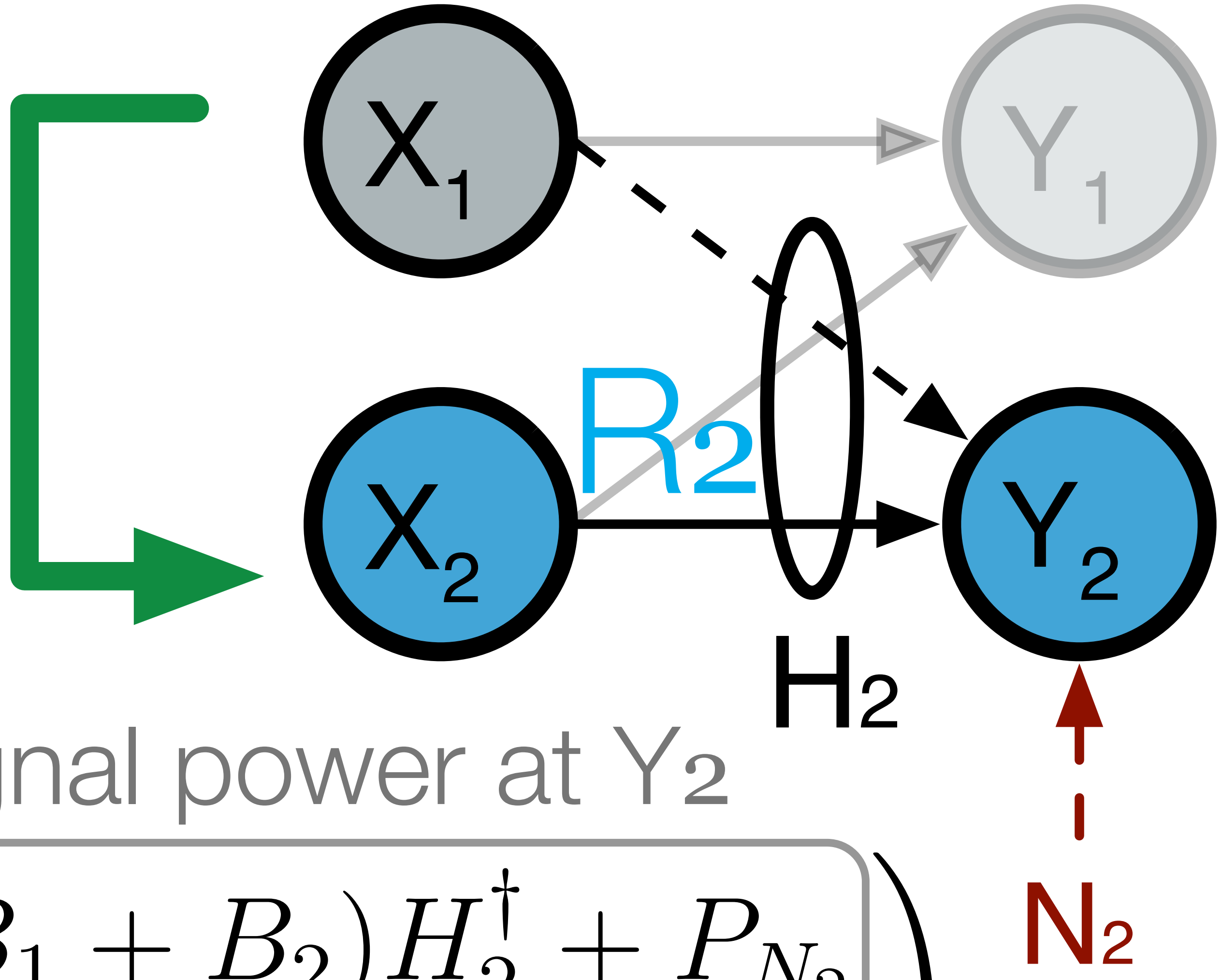
$$R_1 \le \frac{1}{2}\log_2\left(\frac{\boxed{H_1(B_1+B_2)H_1^\dagger + P_{N_1}}}{\boxed{H_1(B_2)H_1^\dagger + P_{N_1}}}\right)$$

Interference + noise power

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_2'' \end{bmatrix}$$

R_2: Rate of message 2

$$B = B_1 + \boxed{B_2}$$



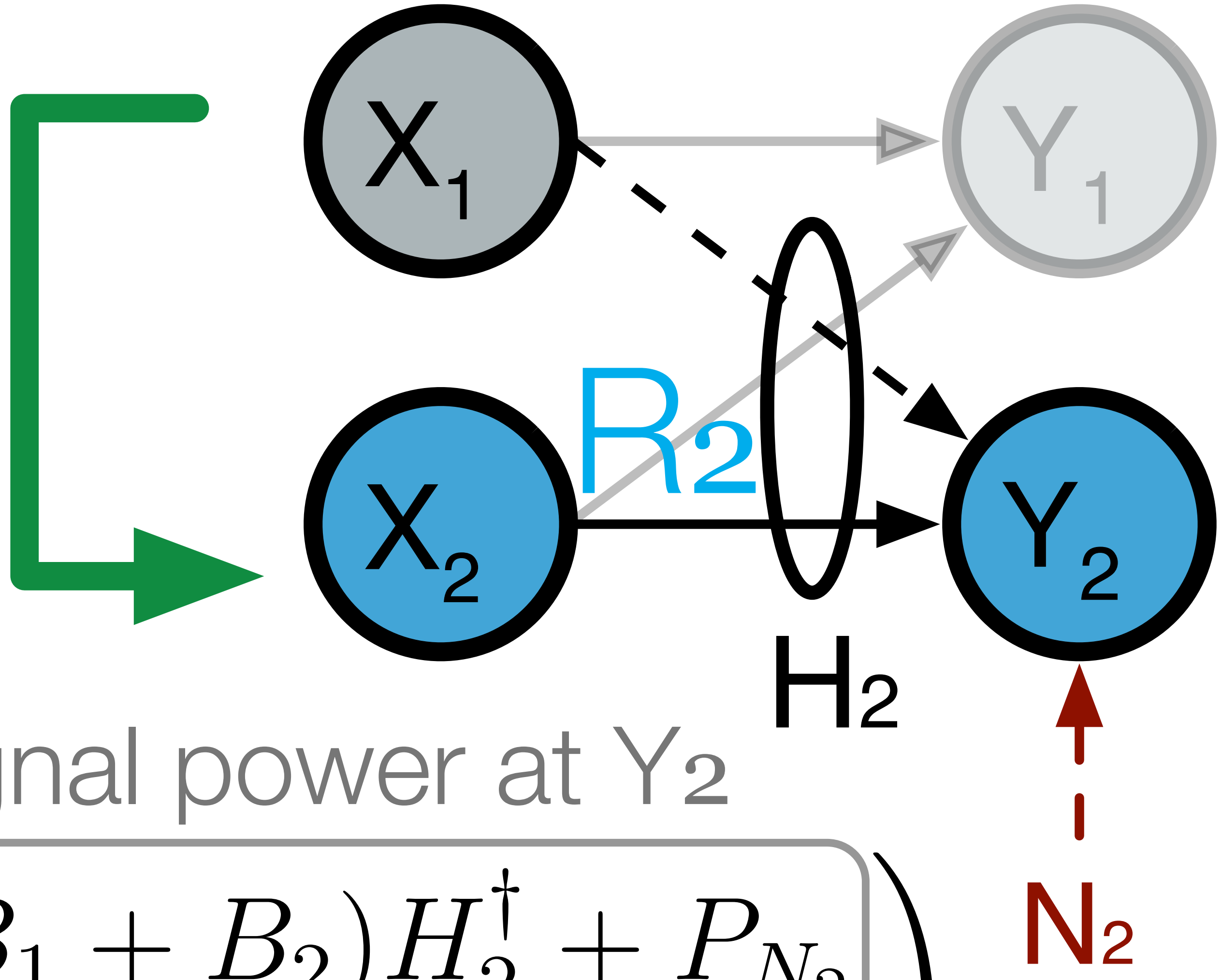
Signal power at Y_2

$$R_2 \le \frac{1}{2}\log_2\left(\frac{\boxed{H_2(B_1+B_2)H_2^\dagger + P_{N_2}}}{\boxed{H_2(B_1)H_2^\dagger + P_{N_2}}}\right)$$

Interference + noise power

R$_2$: Rate of message 2

$$B = B_1 + \boxed{B_2}$$



Signal power at Y$_2$

$$R_2 \le \frac{1}{2}\log_2\left(\frac{\boxed{H_2(B_1+B_2)H_2^\dagger + P_{N_2}}}{\boxed{H_2(B_1)H_2^\dagger + P_{N_2}}}\right)$$

$$B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}$$

Interference + noise power

Since Tx 2 knows message $\mathbf{1}$,
it can mitigate interference!

Since Tx 2 knows message **1**,
it can mitigate interference!

Dirty paper coding

Dirty-paper coding

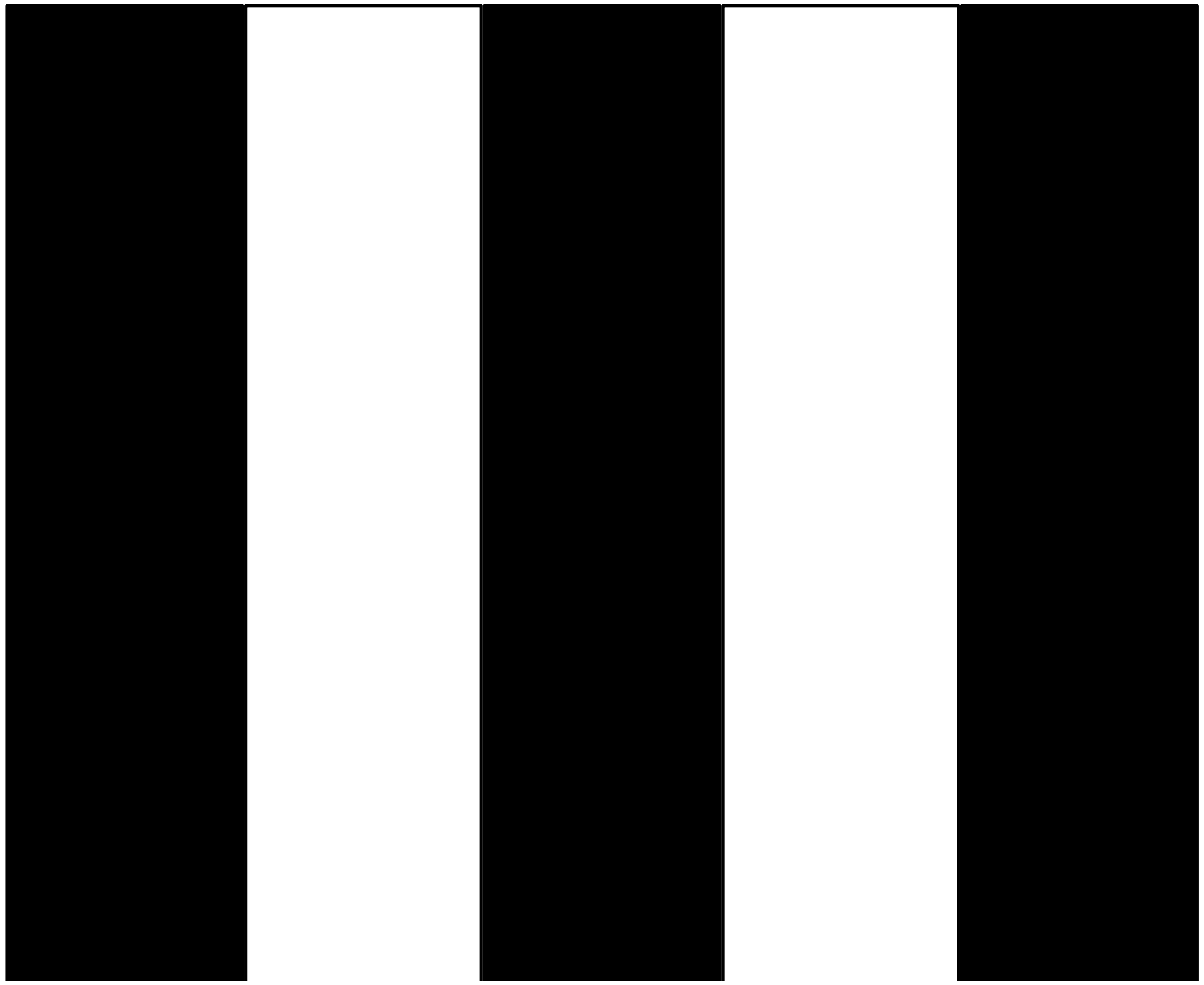
[Gel’fand, Pinsker, 1980]

[Costa, 1983]

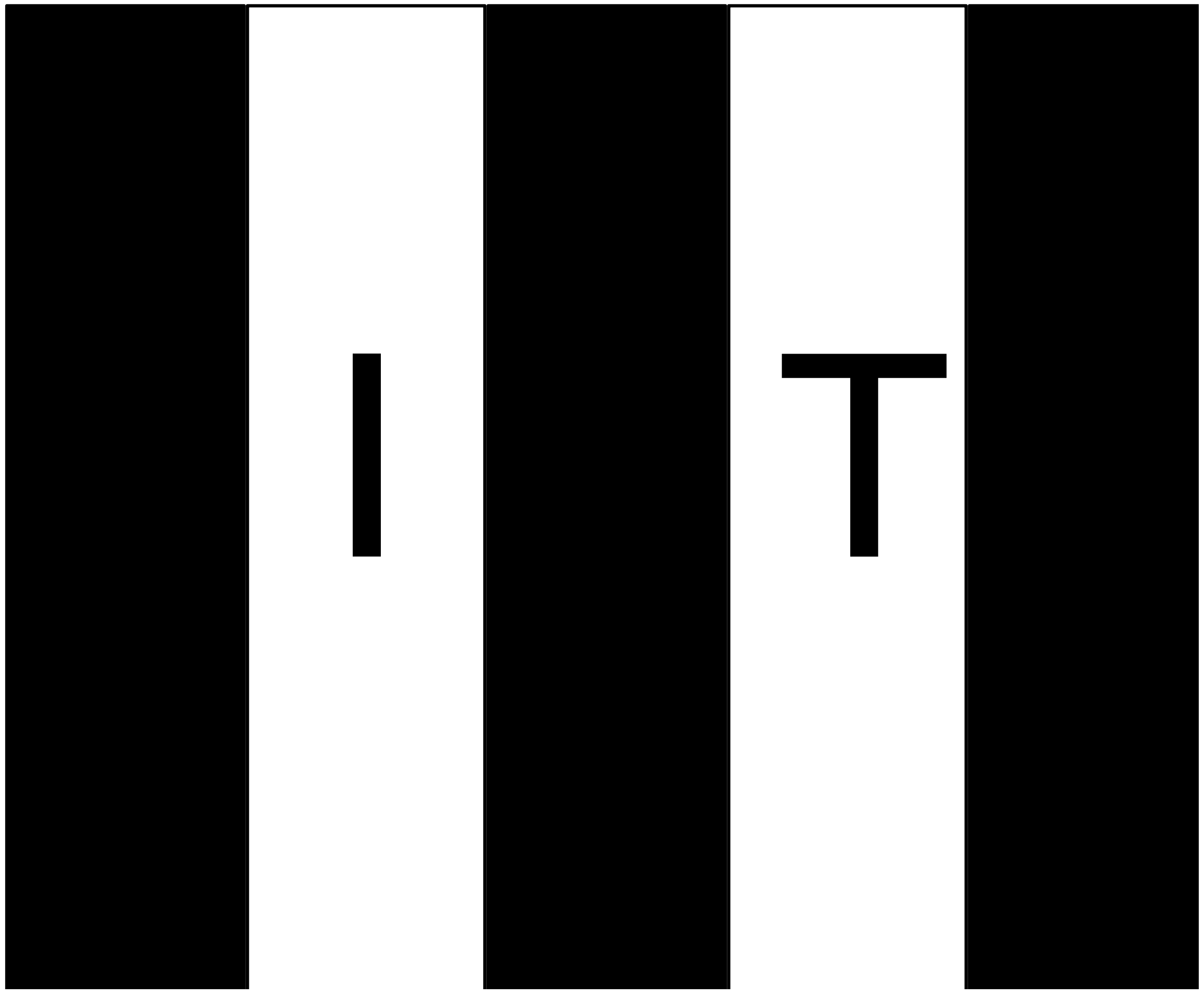
Dirty-paper coding

DIRTY

Dirty-paper coding



Dirty-paper coding

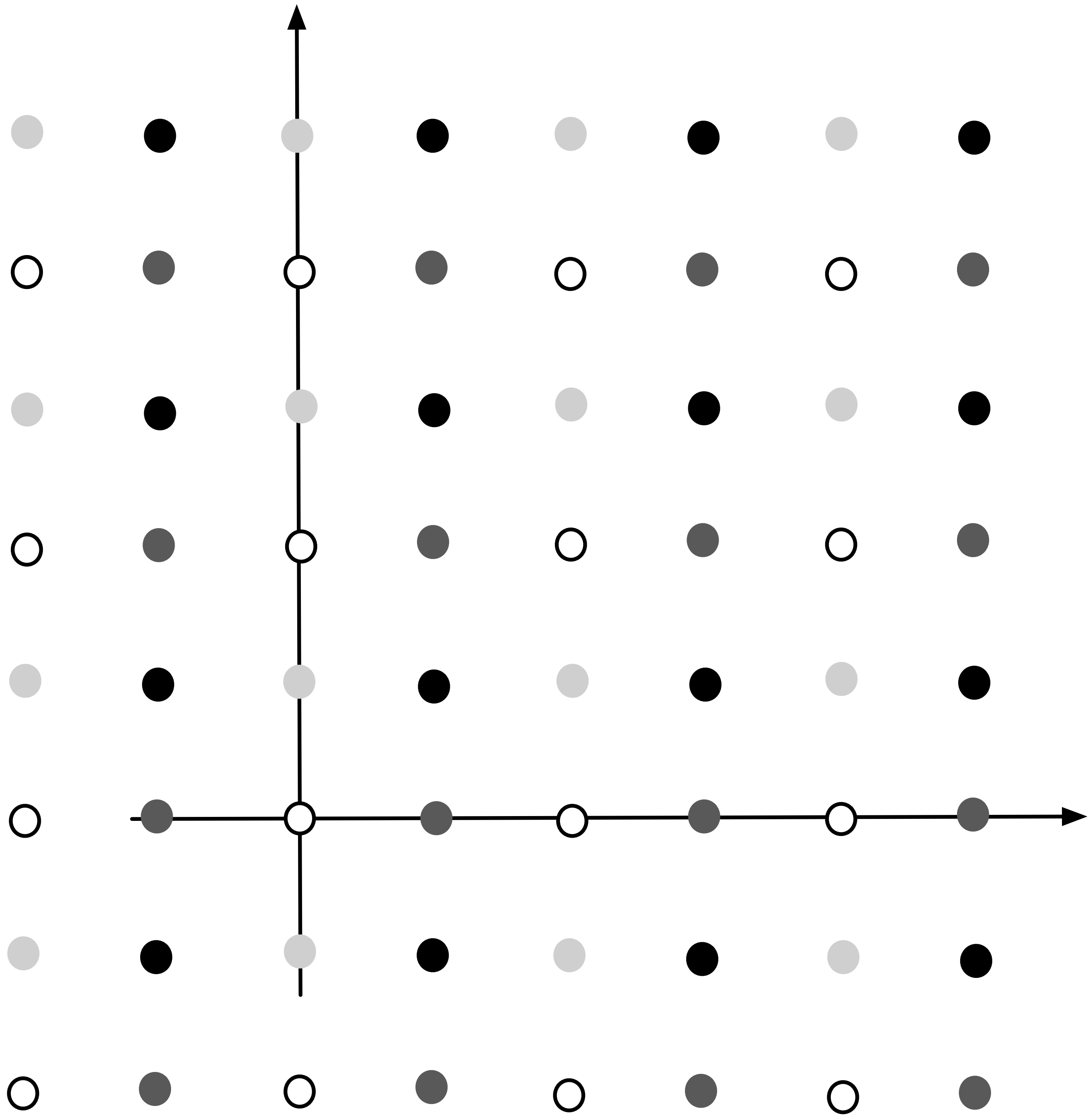


write in black ink?

Dirty-paper coding

adjust your ink ✓

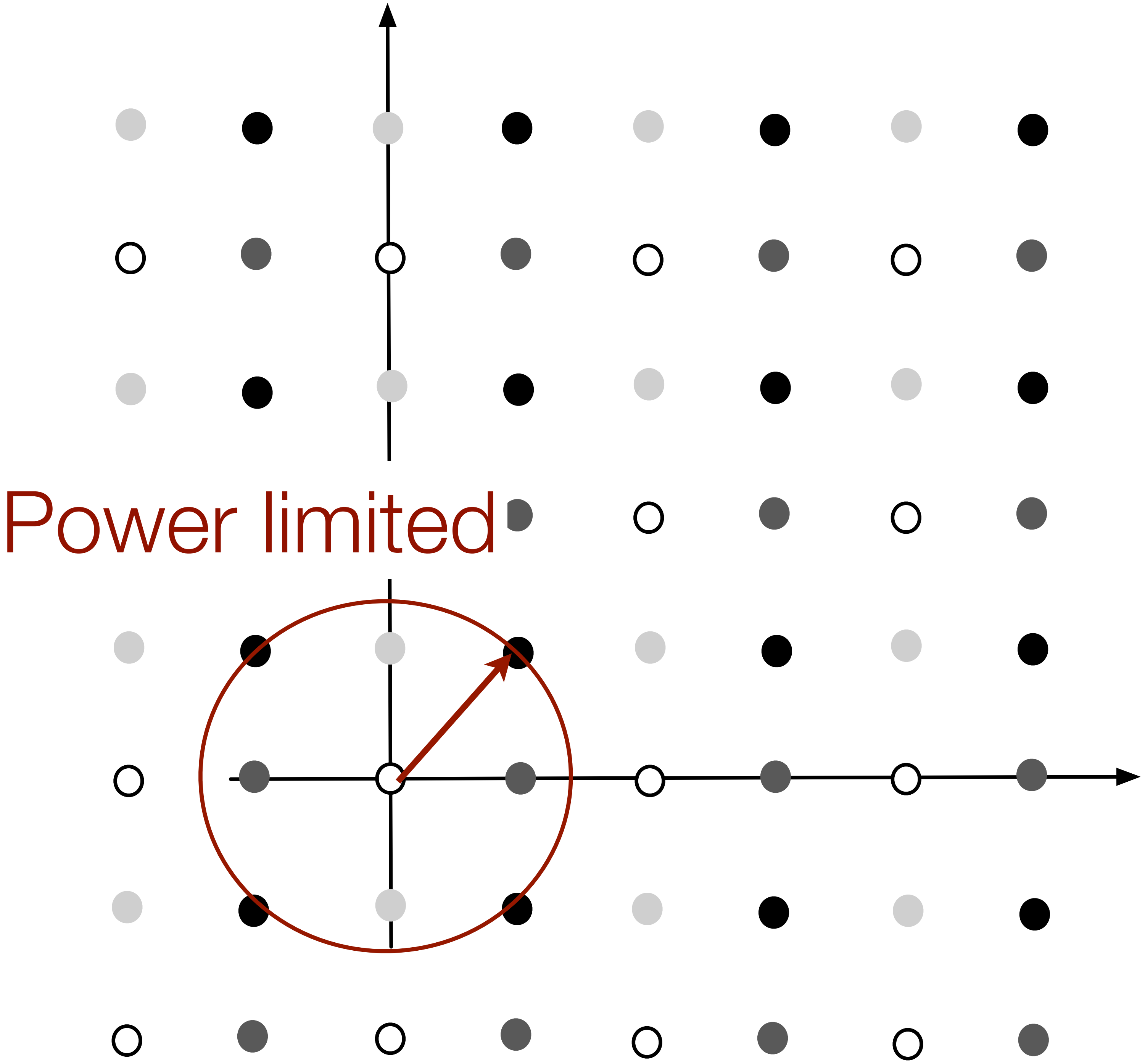
Example of dirty-paper coding



Send 2 bits:

- ● 11
- ● 10
- ● 01
- ○ 00

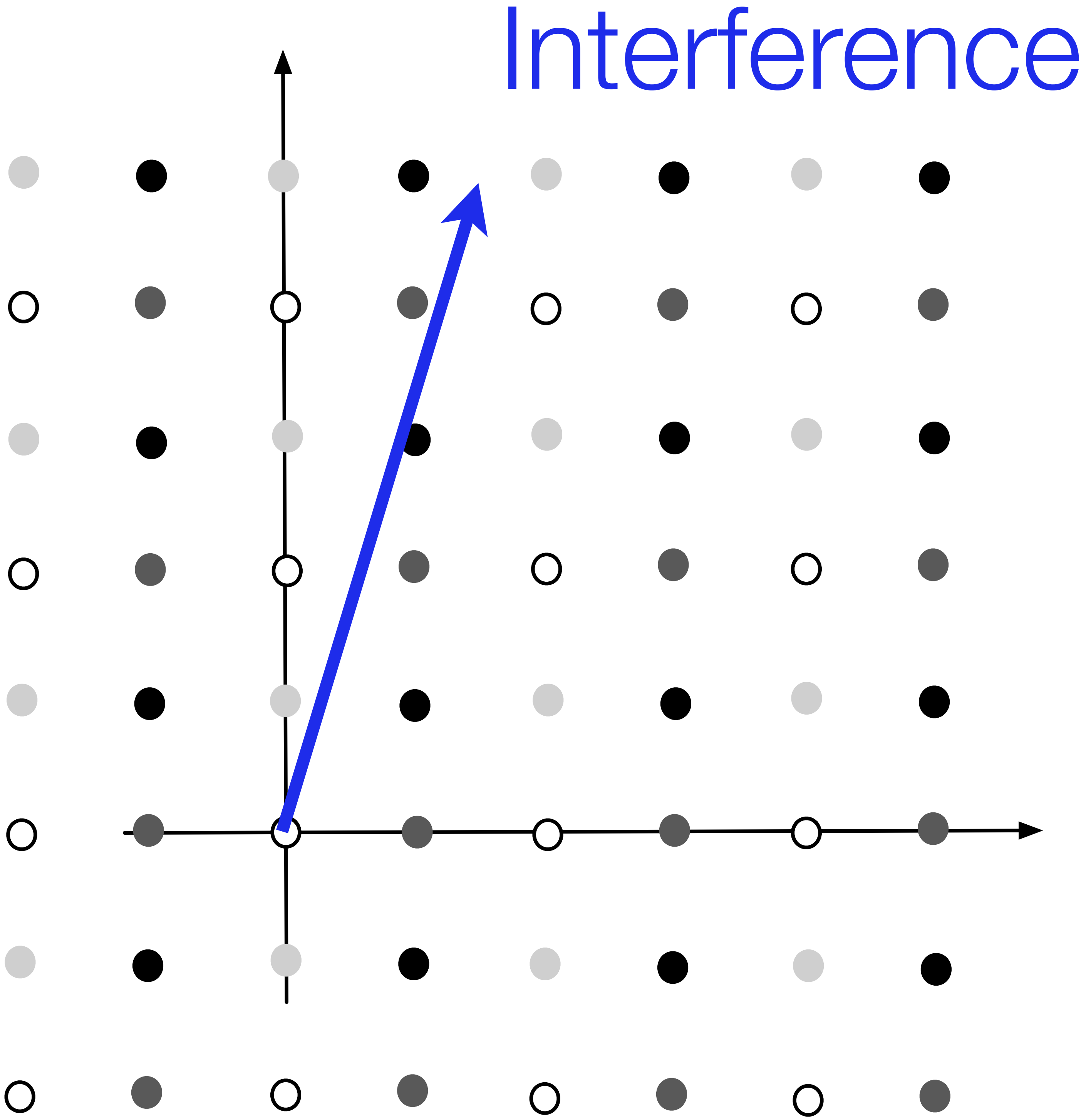
Example of dirty-paper coding



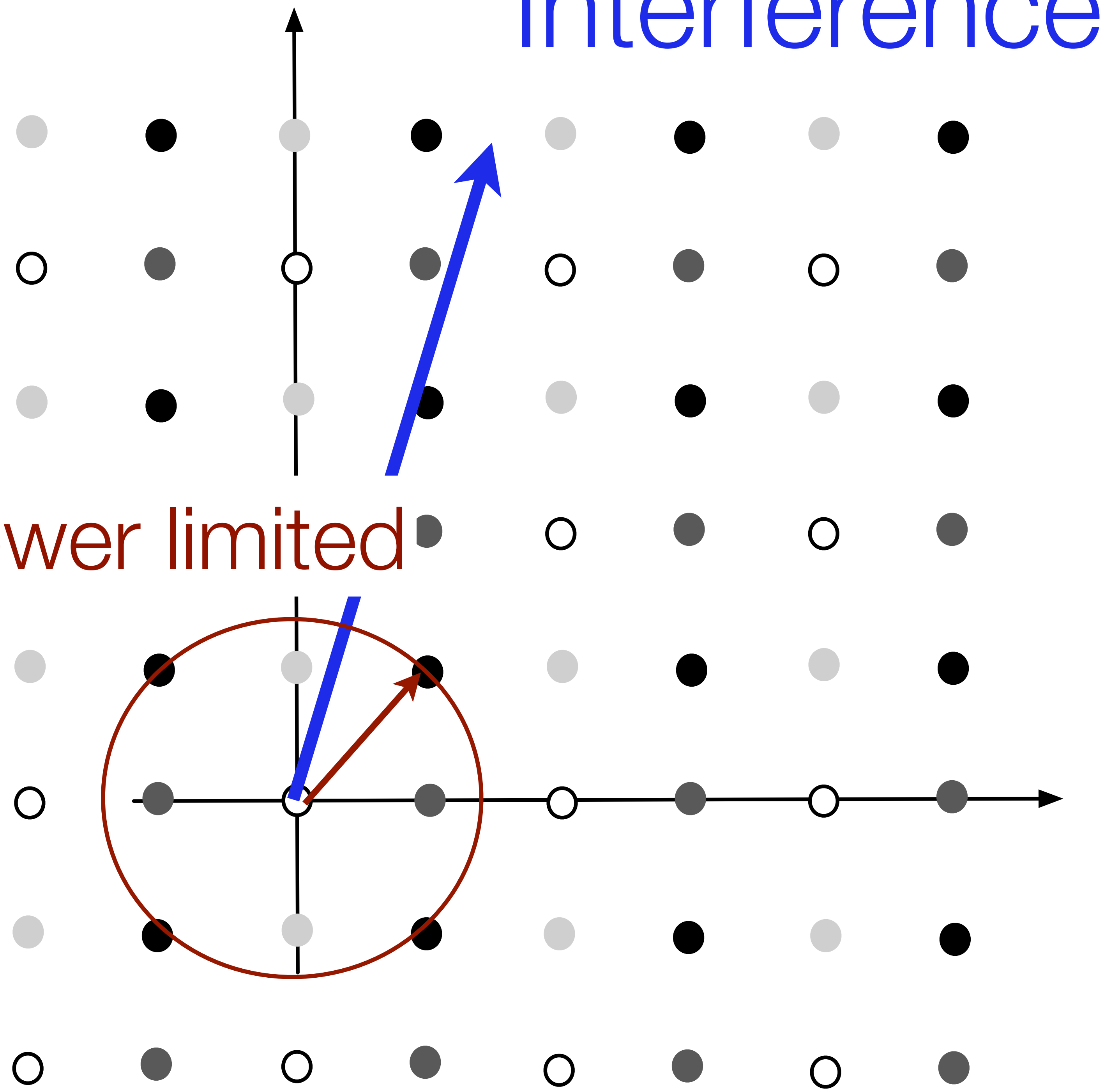
Send 2 bits:

● 11
◉ 10
◌ 01
○ 00

Example of dirty-paper coding

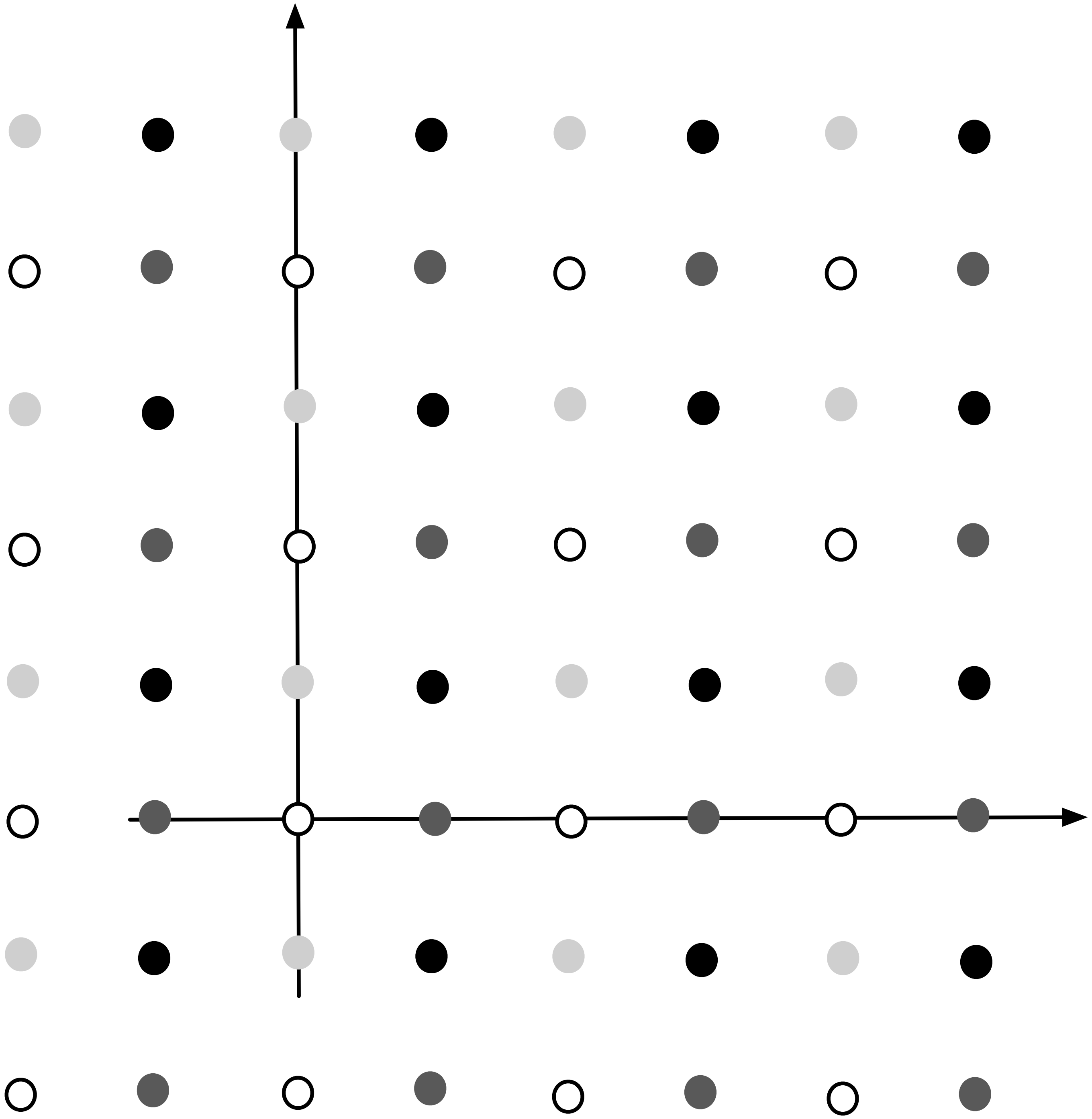


Example of dirty-paper coding



Do NOT have enough power to subtract off the interference!

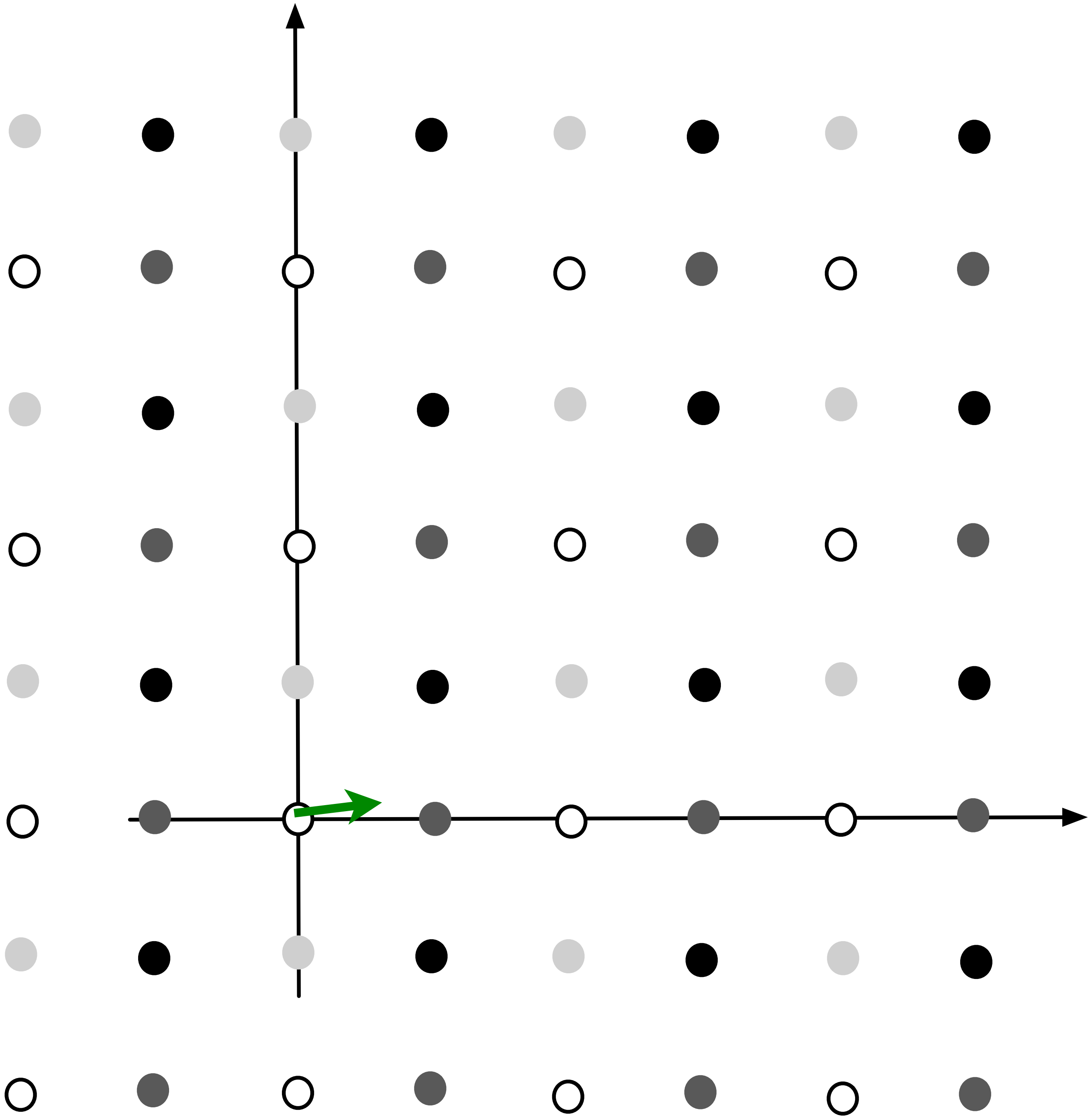
Example of dirty-paper coding



How to send 01?

● 11
◍ 10
◌ 01
○ 00

Example of dirty-paper coding



How to send 01?

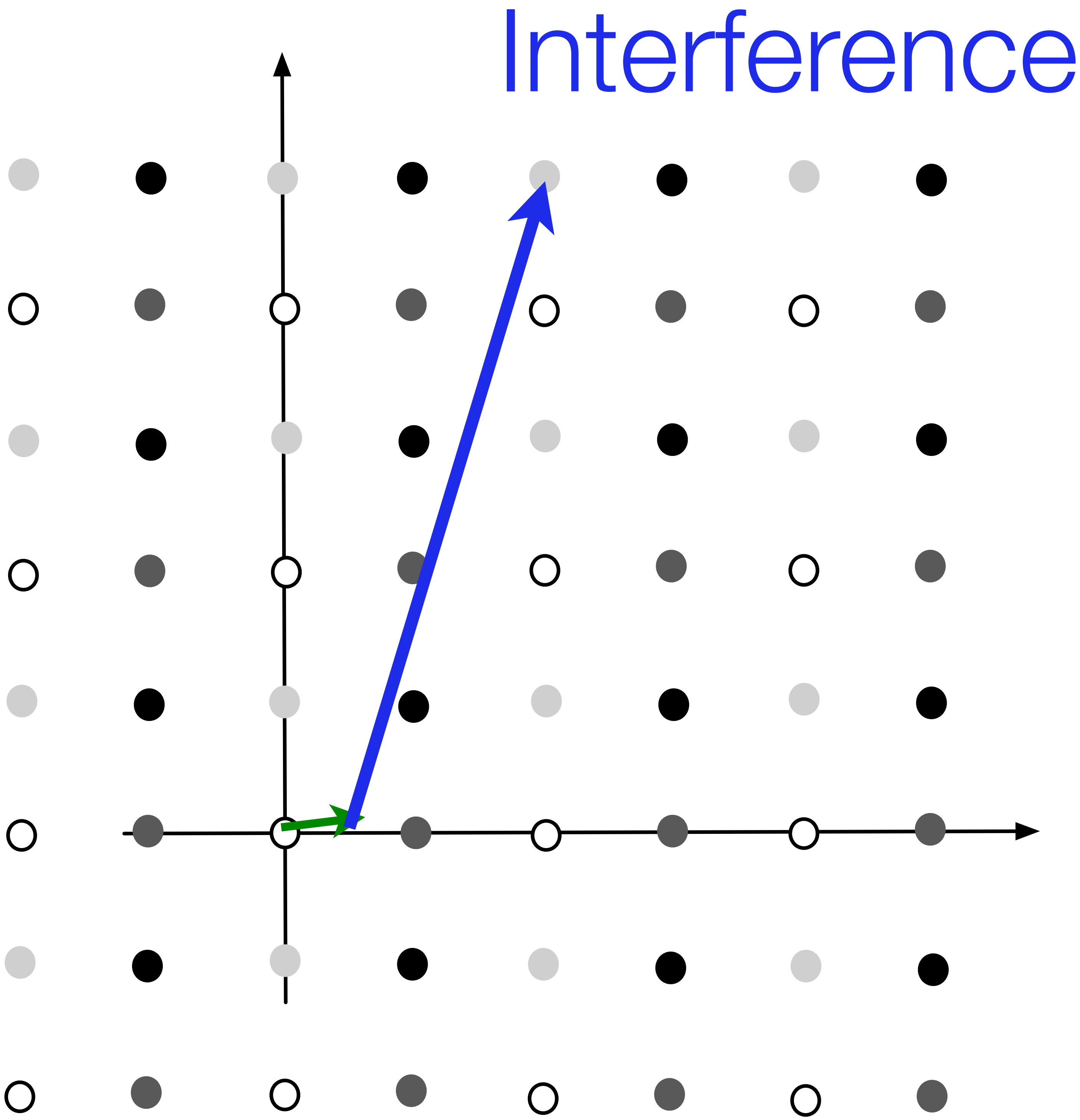
● 11

▩ 10

░ 01

○ 00

Example of dirty-paper coding



How to send 01?

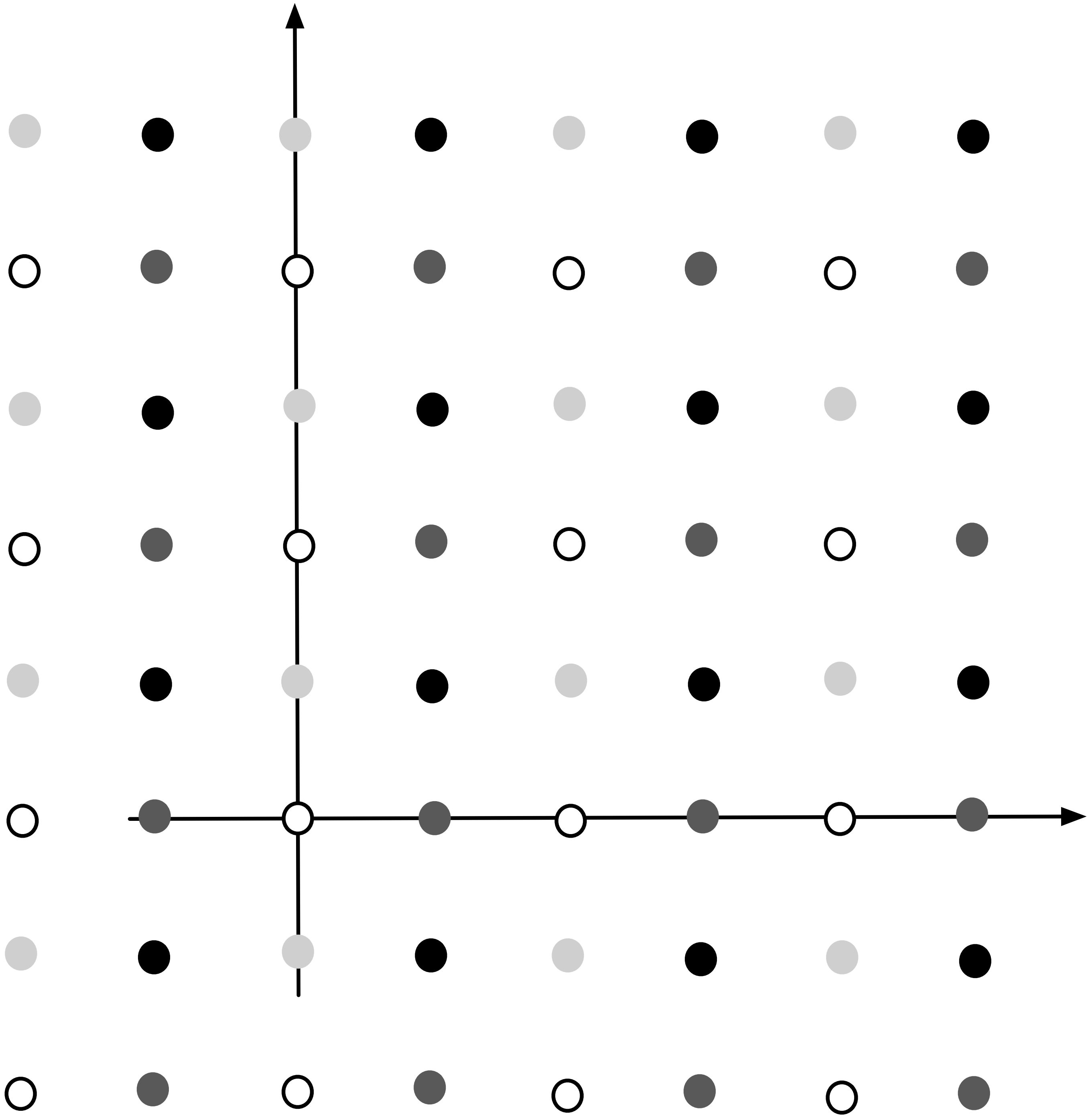
● 11

● 10

● 01

○ 00

Example of dirty-paper coding



How to send 11?

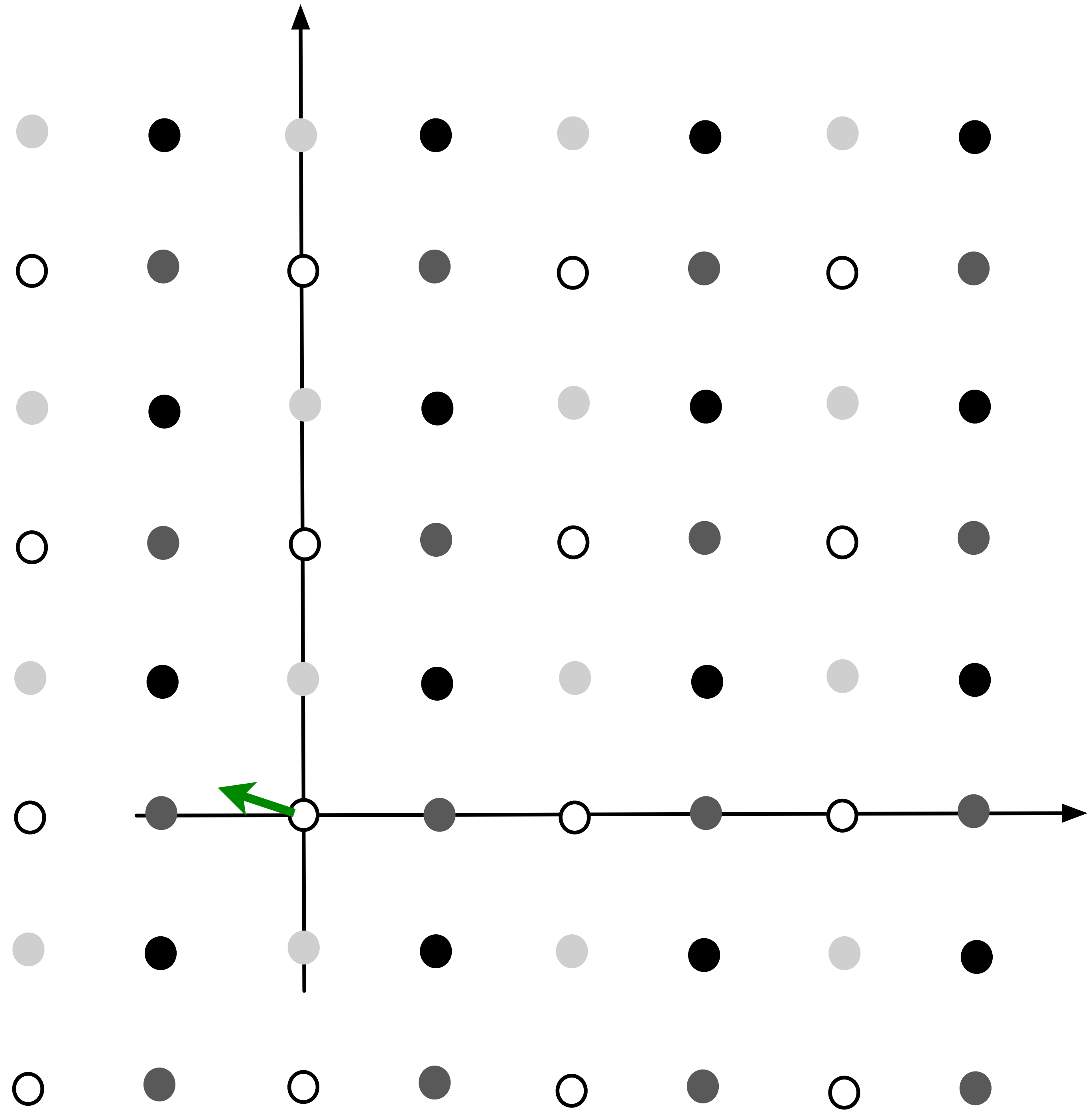
● 11

● 10

● 01

○ 00

Example of dirty-paper coding



How to send 11?

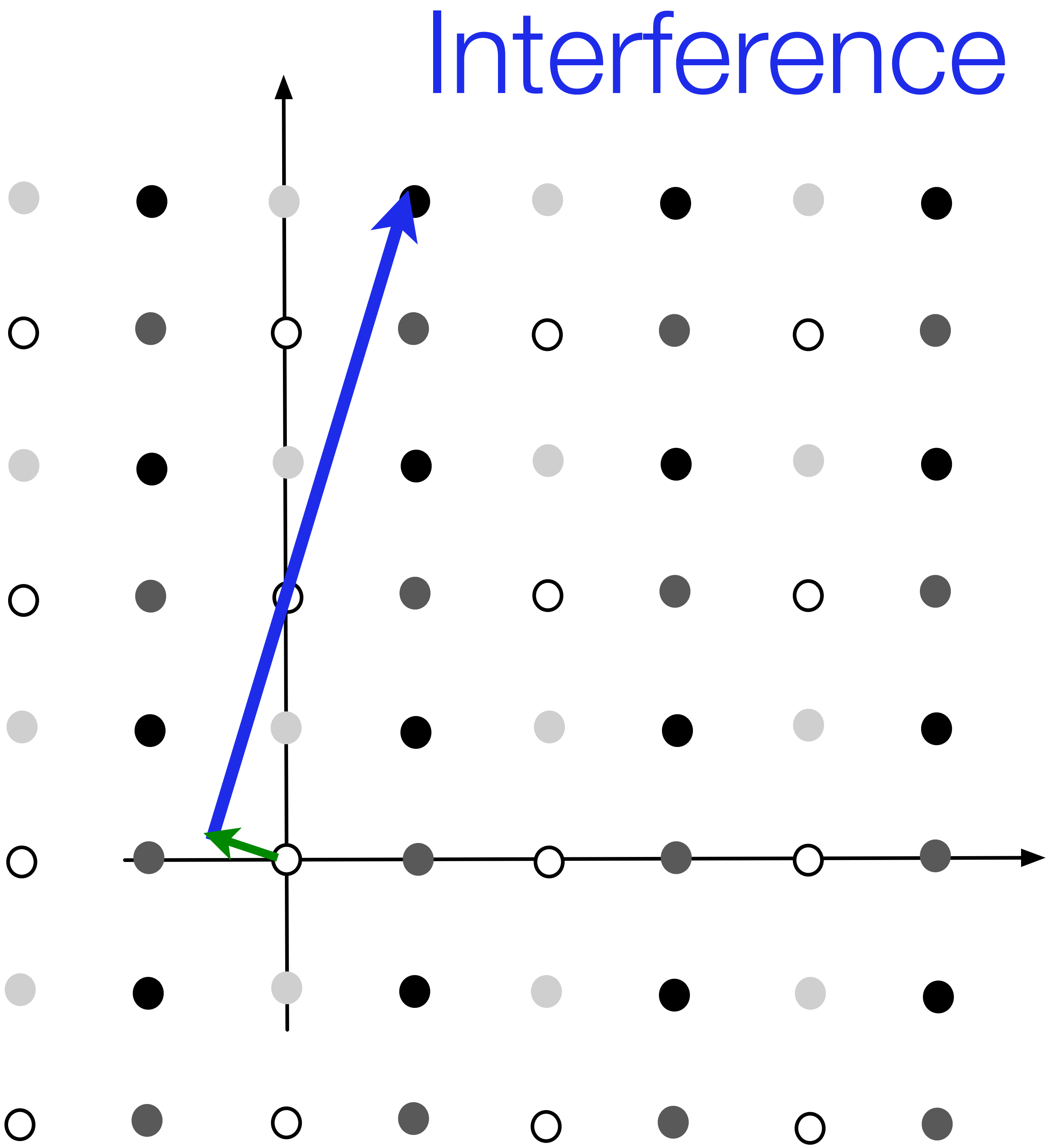
● 11

◉ 10

◌ 01

○ 00

Example of dirty-paper coding



How to send 11?

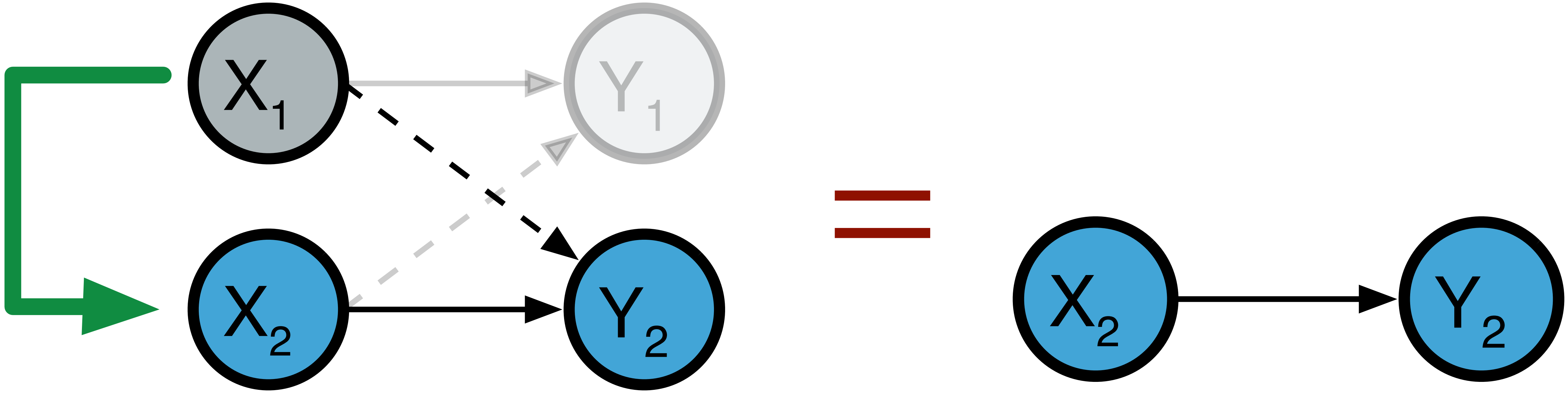
● 11

● 10

● 01

○ 00

Dirty-paper coding



NO power penalty!

NOT subtracting off interference!

Rate of message 2:
WITHOUT and **WITH** dirty-paper coding

WITHOUT

Signal power at Y_2

$$R_2 \le \frac{1}{2}\log_2\left(\frac{\boxed{H_2(B_1 + B_2)H_2^\dagger + P_{N_2}}}{\boxed{H_2(B_1)H_2^\dagger + P_{N_2}}}\right)$$

Interference + noise power

Rate of message 2:
WITHOUT and **WITH** dirty-paper coding

WITHOUT

$$R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_1 + B_2)H_2^\dagger + P_{N_2}}{H_2(B_1)H_2^\dagger + P_{N_2}}\right)$$

WITH

$$R_2 \leq \frac{1}{2}\log_2\left(\frac{\overset{\text{Signal at } Y_2}{\boxed{H_2(B_2)H_2^\dagger + P_{N_2}}}}{\underset{\text{No interference + noise}}{\boxed{P_{N_2}}}}\right)$$

Gaussian **cognitive** channel

$$\text{Cognitive region} = \text{Convex hull of}$$

$$\left\{ \begin{array}{c} (R_1, R_2) : \\ \\ R_1 \leq \frac{1}{2} \log_2 \left(\frac{H_1(B_1+B_2)H_1^\dagger + Q_1}{H_1(B_2)H_1^\dagger + Q_1} \right) = R_1(\pi_{12}) \\ R_2 \leq \frac{1}{2} \log_2 \left(\frac{H_2(B_2)H_2^\dagger + Q_2}{Q_2} \right) = R_2(\pi_{12}) \\ \\ B_1, B_2 \succeq 0, \ \boxed{B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_2'' \end{bmatrix}}, \ B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \ z^2 \leq P_1 P_2 \end{array} \right\}$$

Matrices with zeros

Gaussian **broadcast** channel, multi-antenna

Permutation 1

$$\left\{ \begin{array}{c} (R_1, R_2) : \\ \boxed{\begin{array}{c} R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \end{array}} \bigcup \begin{array}{c} R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger+Q_1}{Q_1}\right) = R_1(\pi_{21}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^\dagger+Q_2}{H_2(B_1)H_2^\dagger+Q_2}\right) = R_2(\pi_{21}) \end{array} \\ B_1, B_2 \succeq 0, \quad B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}, \; B_2 = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}, \; B_1 + B_2 \preceq S \end{array} \right\}$$

Gaussian **broadcast** channel, multi-antenna

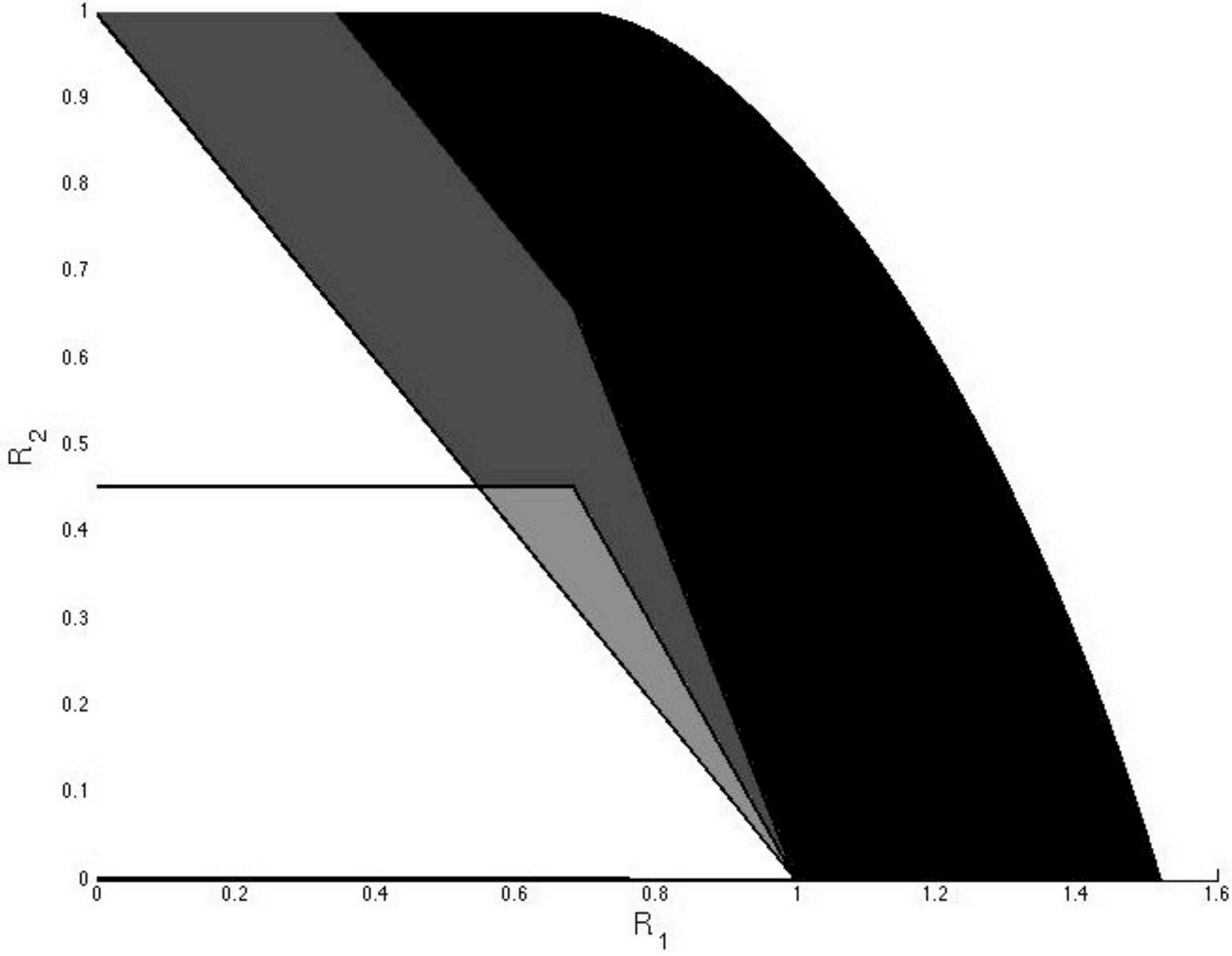
Permutation 2

$$\left\{ \begin{array}{c} (R_1, R_2) : \\[2ex] \begin{array}{c} R_1 \le \frac{1}{2} \log_2 \left(\frac{H_1(B_1+B_2)H_1^\dagger + Q_1}{H_1(B_2)H_1^\dagger + Q_1} \right) = R_1(\pi_{12}) \\ R_2 \le \frac{1}{2} \log_2 \left(\frac{H_2(B_2)H_2^\dagger + Q_2}{Q_2} \right) = R_2(\pi_{12}) \end{array} \cup \boxed{\begin{array}{c} R_1 \le \frac{1}{2} \log_2 \left(\frac{H_1(B_1)H_1^\dagger + Q_1}{Q_1} \right) = R_1(\pi_{21}) \\ R_2 \le \frac{1}{2} \log_2 \left(\frac{H_2(B_1+B_2)H_2^\dagger + Q_2}{H_2(B_1)H_2^\dagger + Q_2} \right) = R_2(\pi_{21}) \end{array}} \\[3ex] B_1, B_2 \succeq 0, \quad B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}, \; B_2 = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}, \quad B_1 + B_2 \preceq S \end{array} \right\}$$

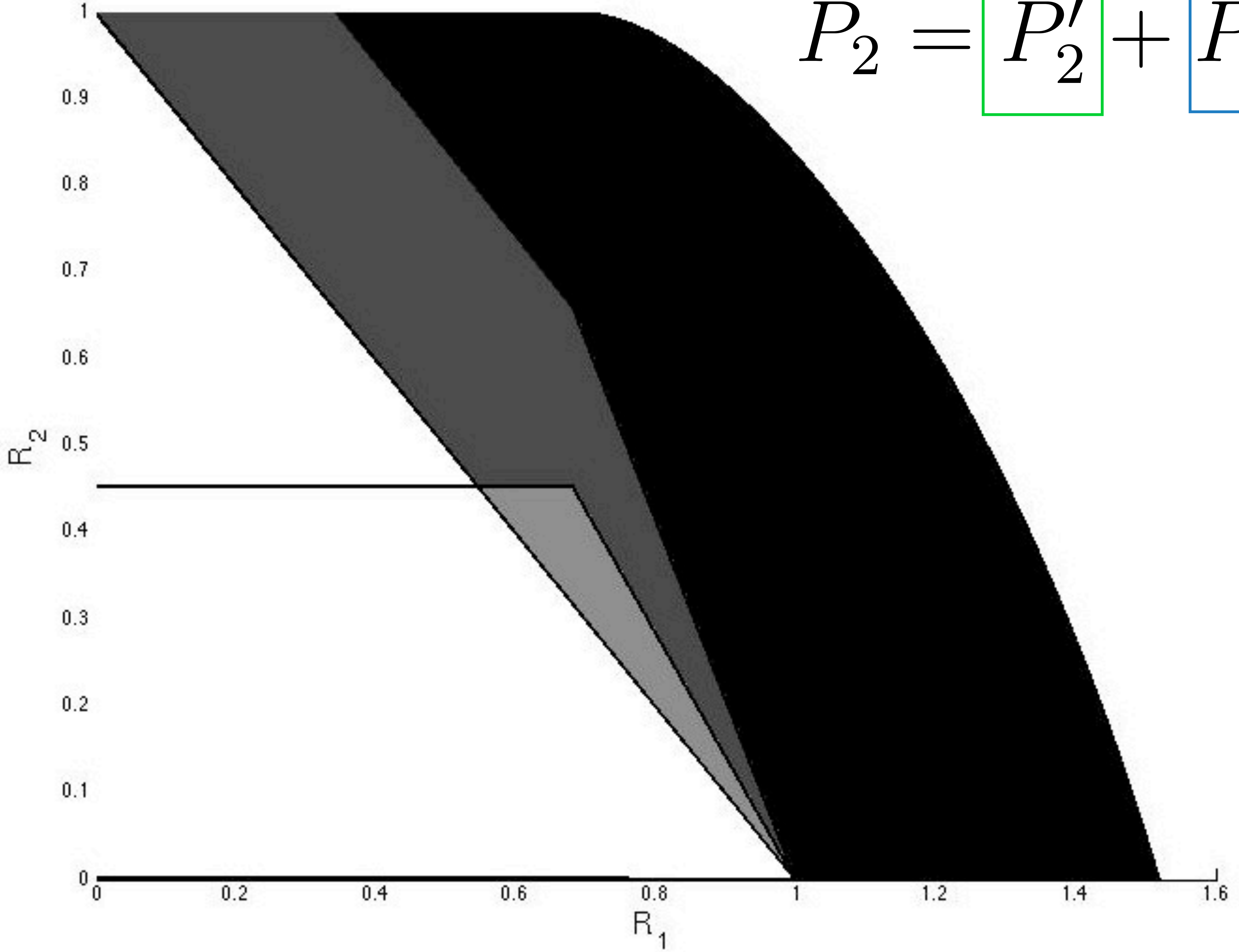
Gaussian **broadcast** channel, multi-antenna

$$\left\{ \begin{array}{c} (R_1, R_2) : \\ \\ \begin{array}{c} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \end{array} \quad \cup \quad \begin{array}{c} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger+Q_1}{Q_1}\right) = R_1(\pi_{21}) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^\dagger+Q_2}{H_2(B_1)H_2^\dagger+Q_2}\right) = R_2(\pi_{21}) \end{array} \\ \boxed{B_1, B_2 \succeq 0, \quad B_1 = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix}, \; B_2 = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix}, \quad B_1 + B_2 \preceq S} \end{array} \right\}$$

No zeros!

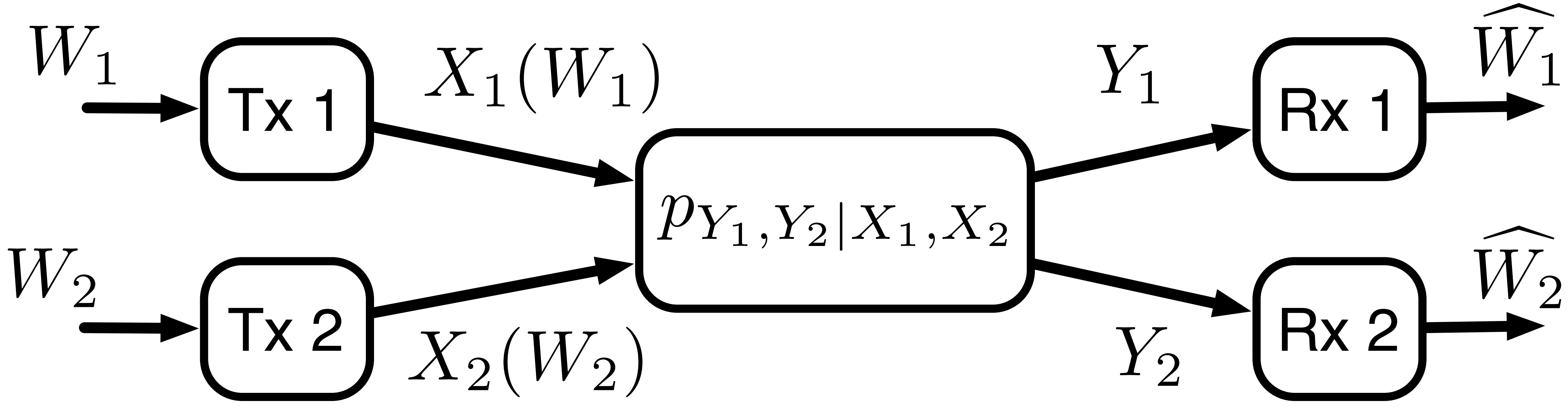

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1

$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$

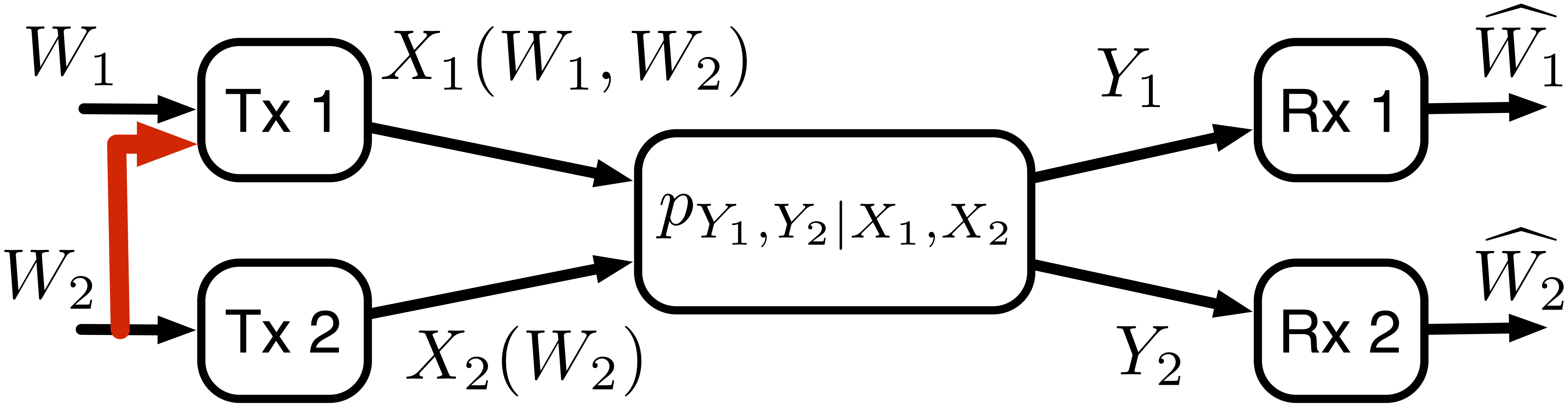


Information theoretic abstraction

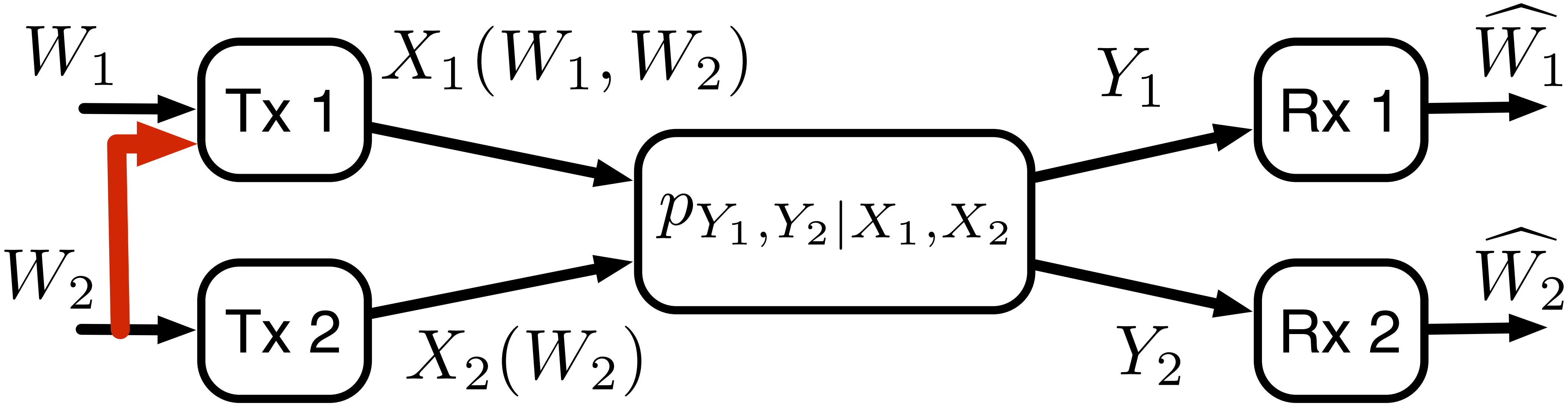
Interference channel



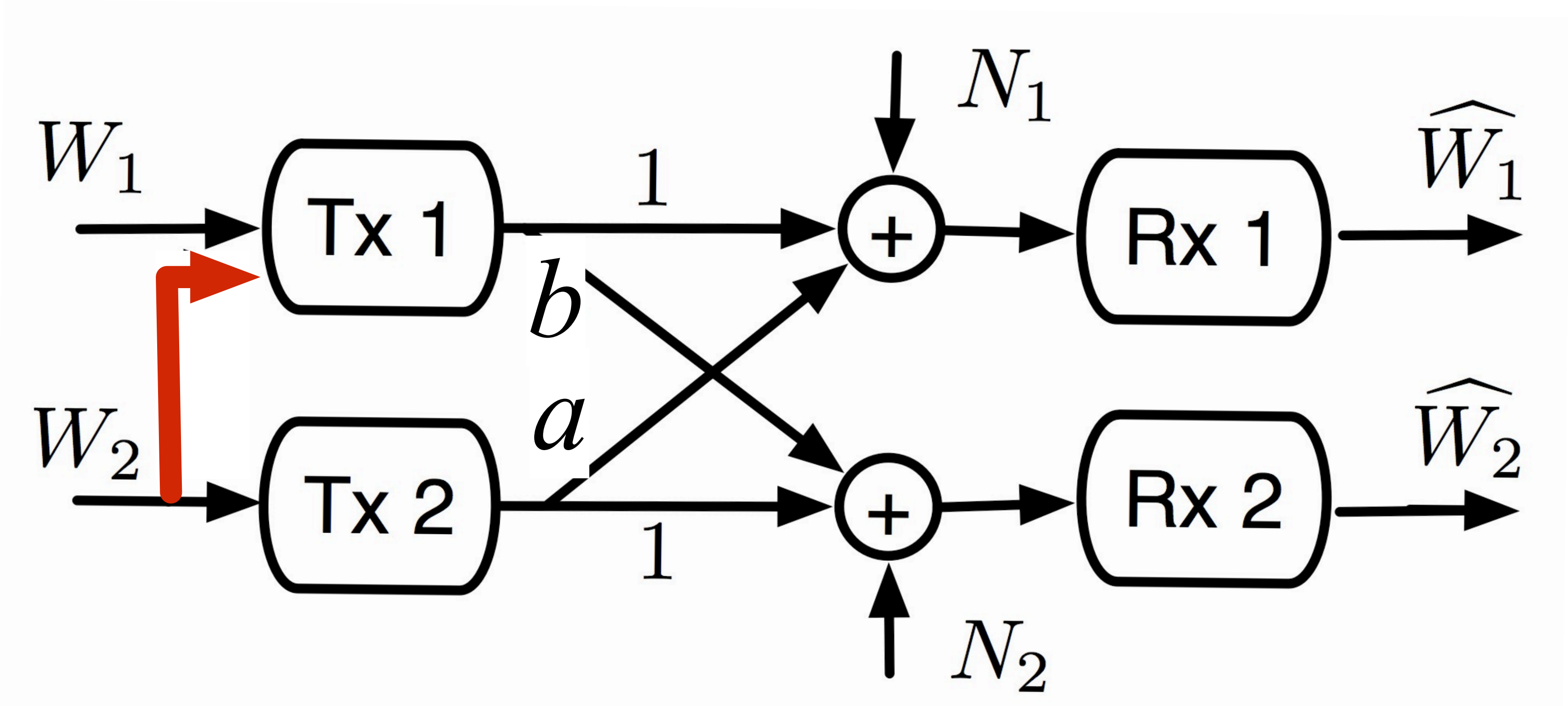
DM Cognitive interference channel



DM Cognitive interference channel



Gaussian Cognitive interference channel



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Large unified region

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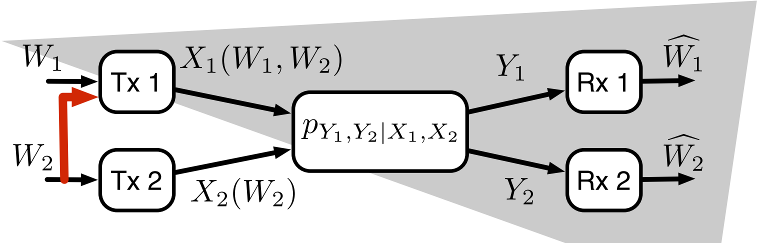
Broadcast channel is contained



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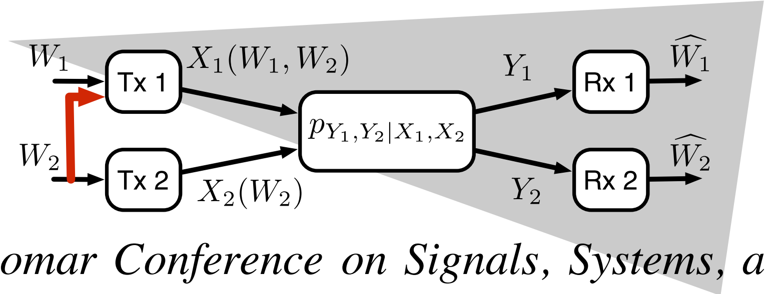

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Interference channel with cognitive relay

J. Jiang, I. Maric, A. Goldsmith and S. Cui, “Achievable Rate Regions for Broadcast Channels with Cognitive Radios,” *IEEE Information Theory Workshop (ITW)*, Taormina, Italy, Oct. 2009.

Causal cognitive interference channel

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Cognitive interference channels with secrecy

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Cognitive Z interference channel

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Degrees of Freedom of Cognitive Channels

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Wyner-type cognitive networks

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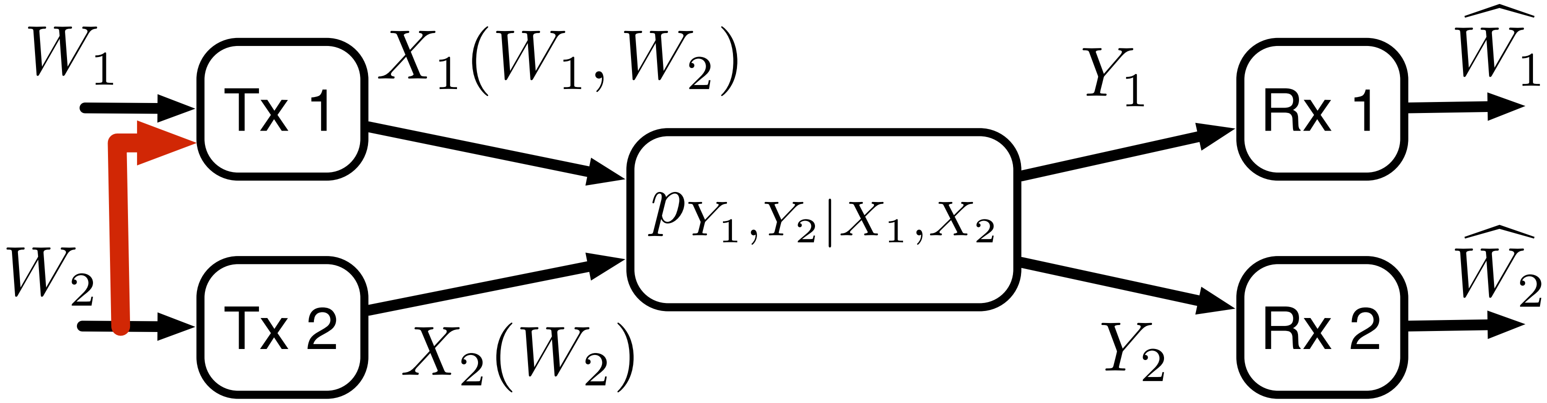
Cognition

↕

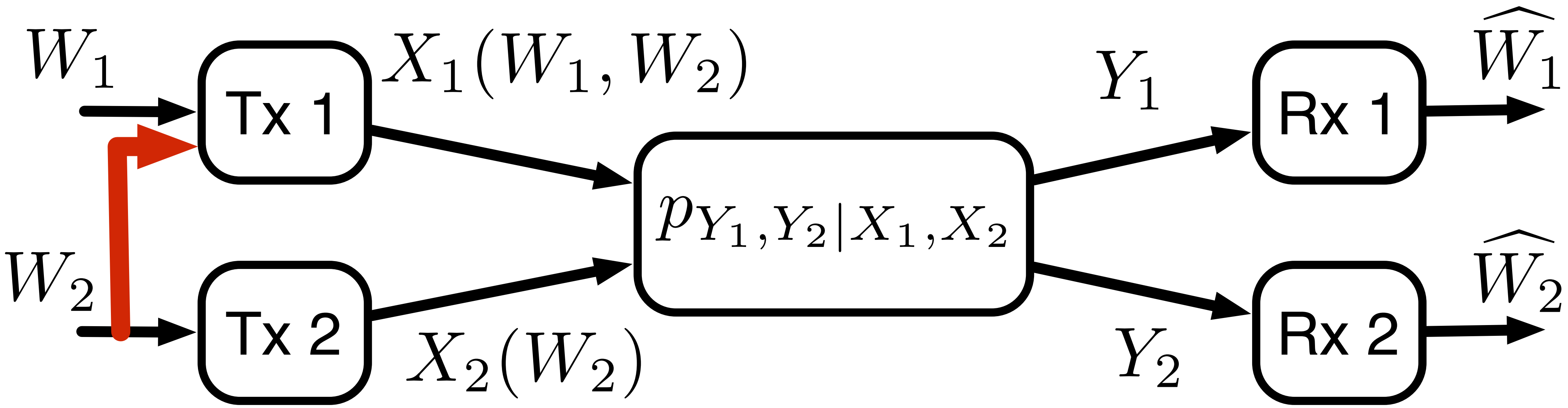
Non-causal side information at Tx/Rxs

Contributions



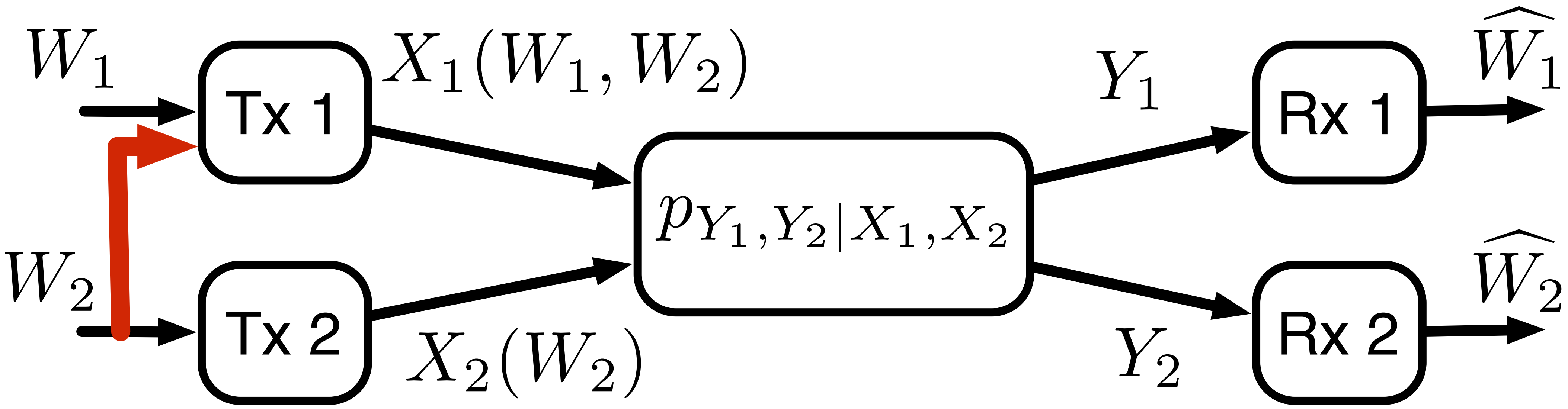
- new inner bound *(largest region)*
- new outer bound *(not tightest, but computable)*
- capacity for deterministic channels *(also semi-deterministic)*
- 1.8 bit gap result for Gaussian channels *(preliminary simulations show smaller gap)*

Achievable scheme (inner bound)



- rate-splitting
- Gel'fand-Pinkser binning
- superposition coding

Achievable scheme (inner bound)



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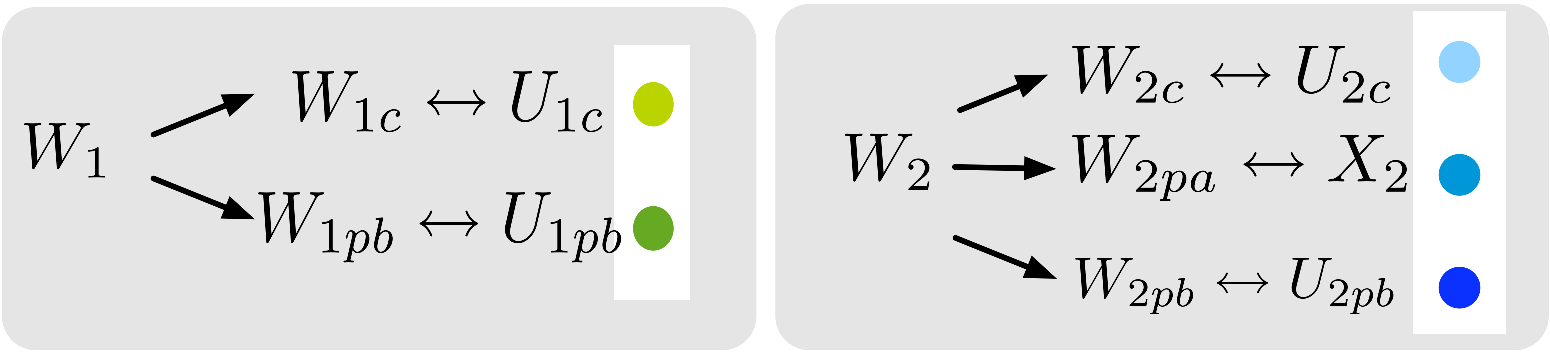
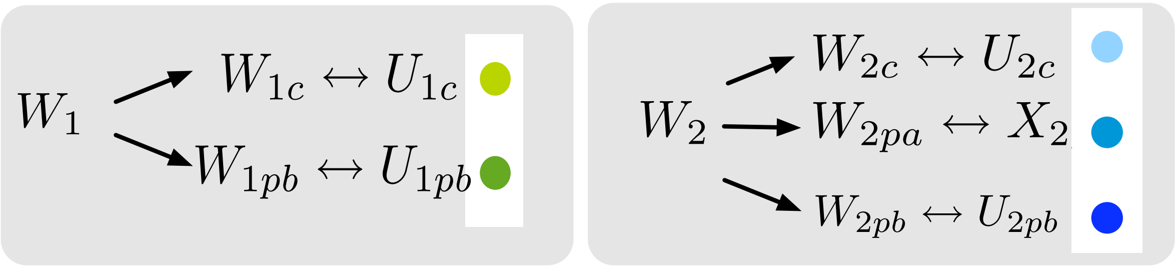


c = common, p = private,
a = alone, b = broadcast



$W_1 \to W_{1c} \leftrightarrow U_{1c}$ ●

$W_1 \to W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \to W_{2c} \leftrightarrow U_{2c}$ ●

$W_2 \to W_{2pa} \leftrightarrow X_2$ ●

$W_2 \to W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,

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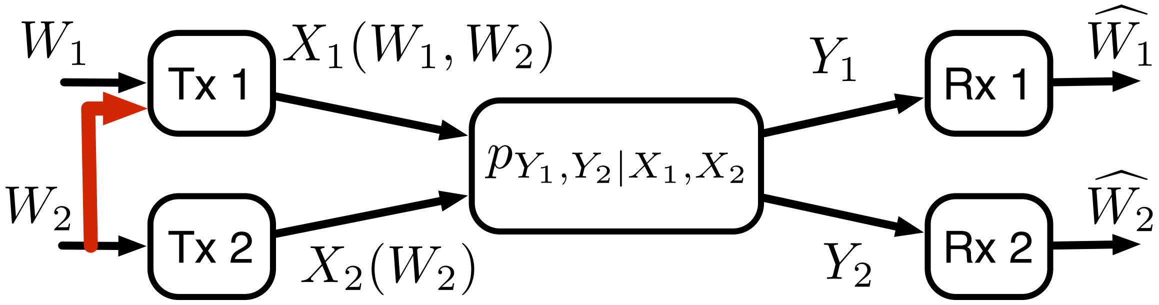

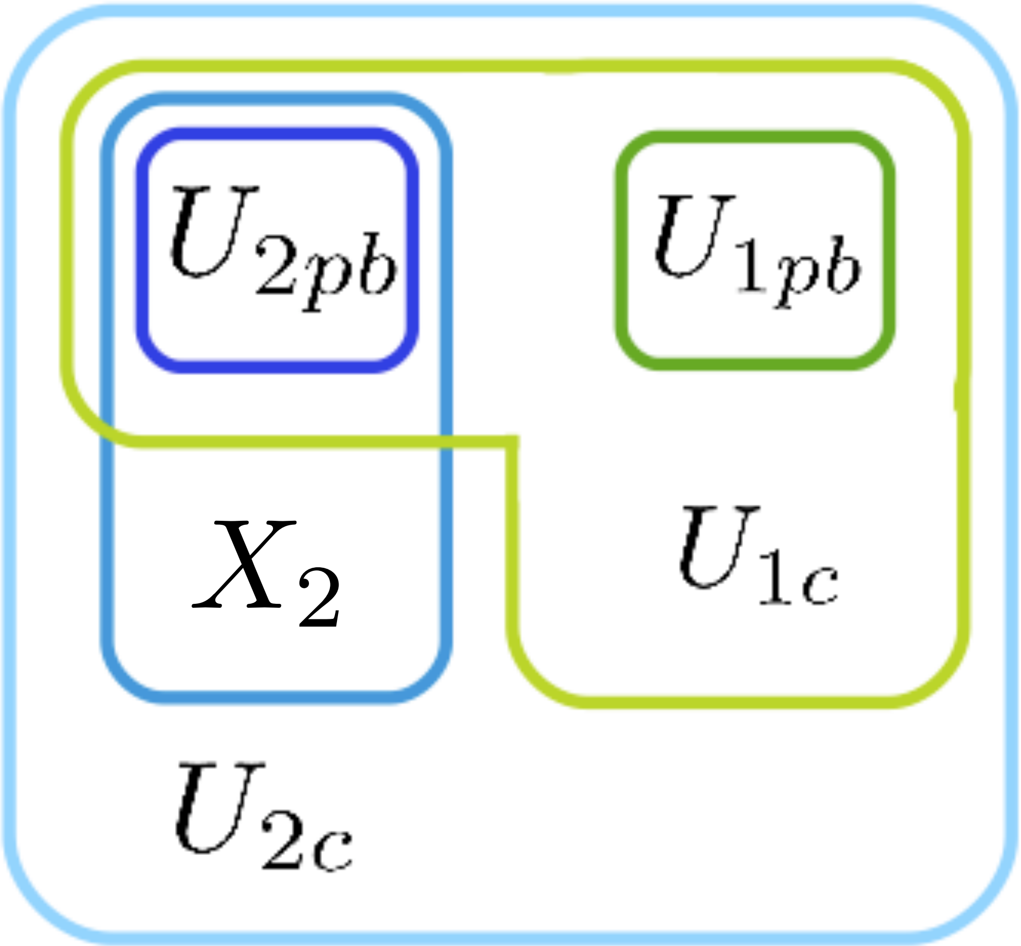

Superposition

$W_1 \to W_{1c} \leftrightarrow U_{1c}$ ●

$W_1 \to W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \to W_{2c} \leftrightarrow U_{2c}$ ●

$W_2 \to W_{2pa} \leftrightarrow X_2$ ●

$W_2 \to W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,

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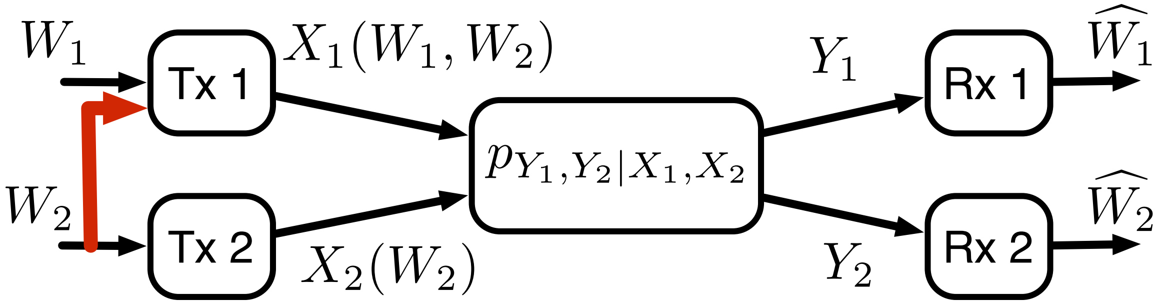
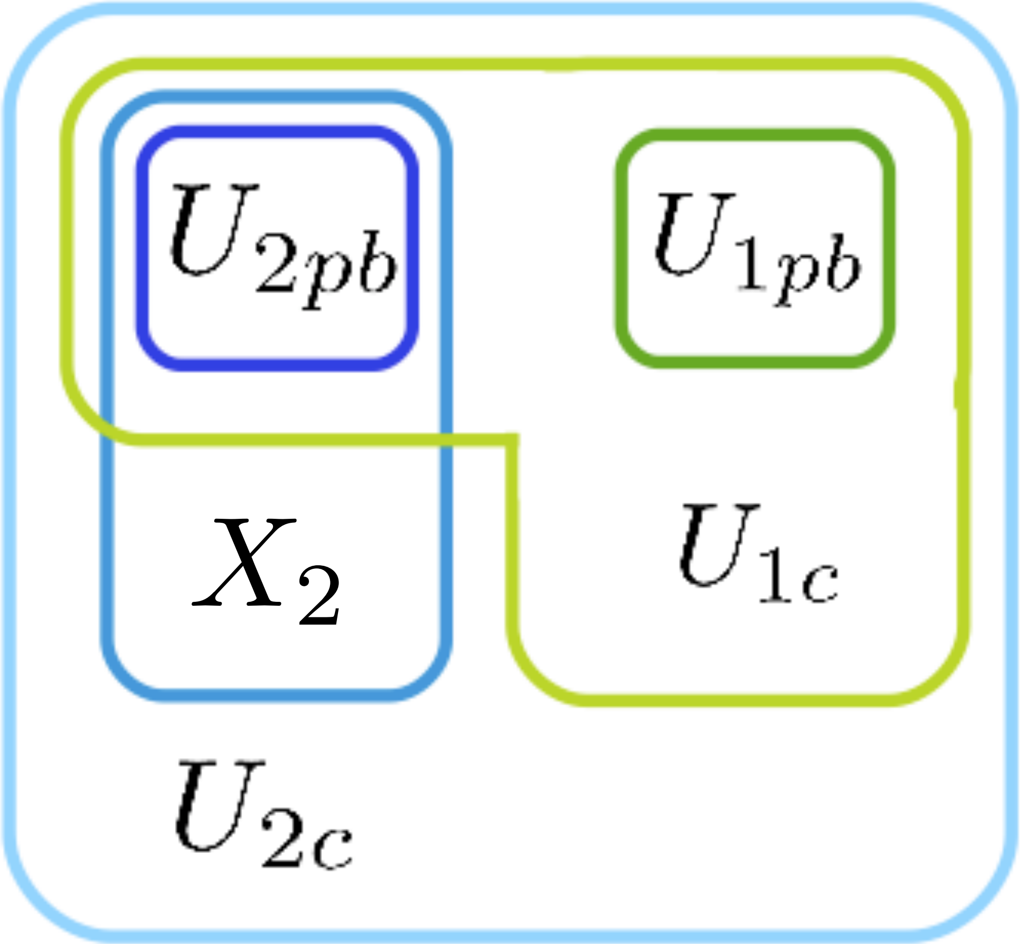
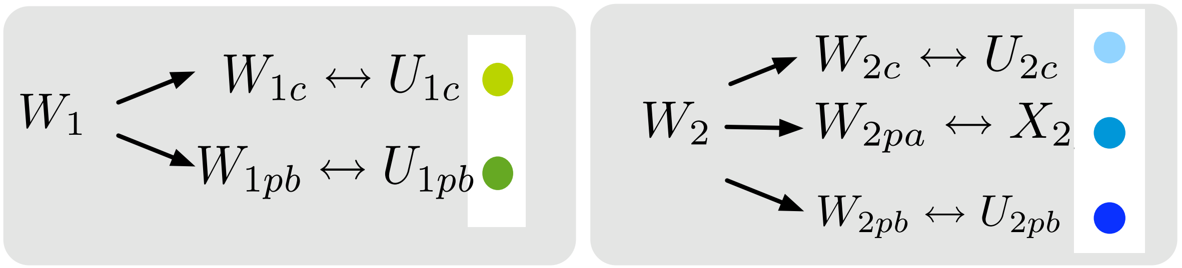


$p_{u_{2c}}$



Superposition



$W_1 \rightarrow W_{1c} \leftrightarrow U_{1c}$ ●

$W_1 \rightarrow W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \rightarrow W_{2c} \leftrightarrow U_{2c}$ ●

$W_2 \rightarrow W_{2pa} \leftrightarrow X_2$ ●

$W_2 \rightarrow W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,
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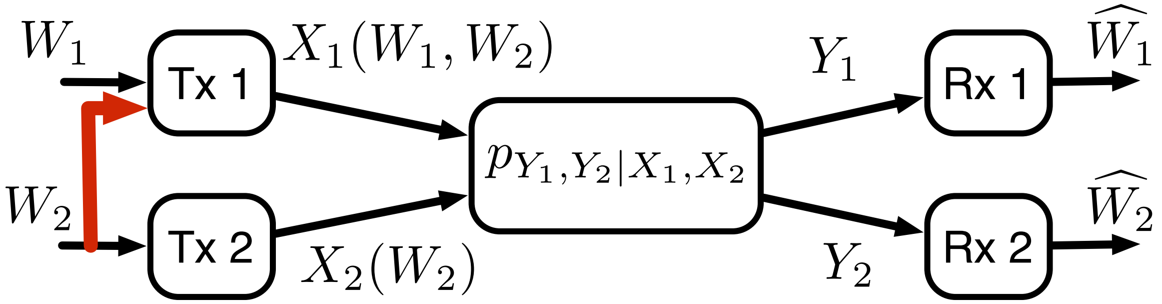
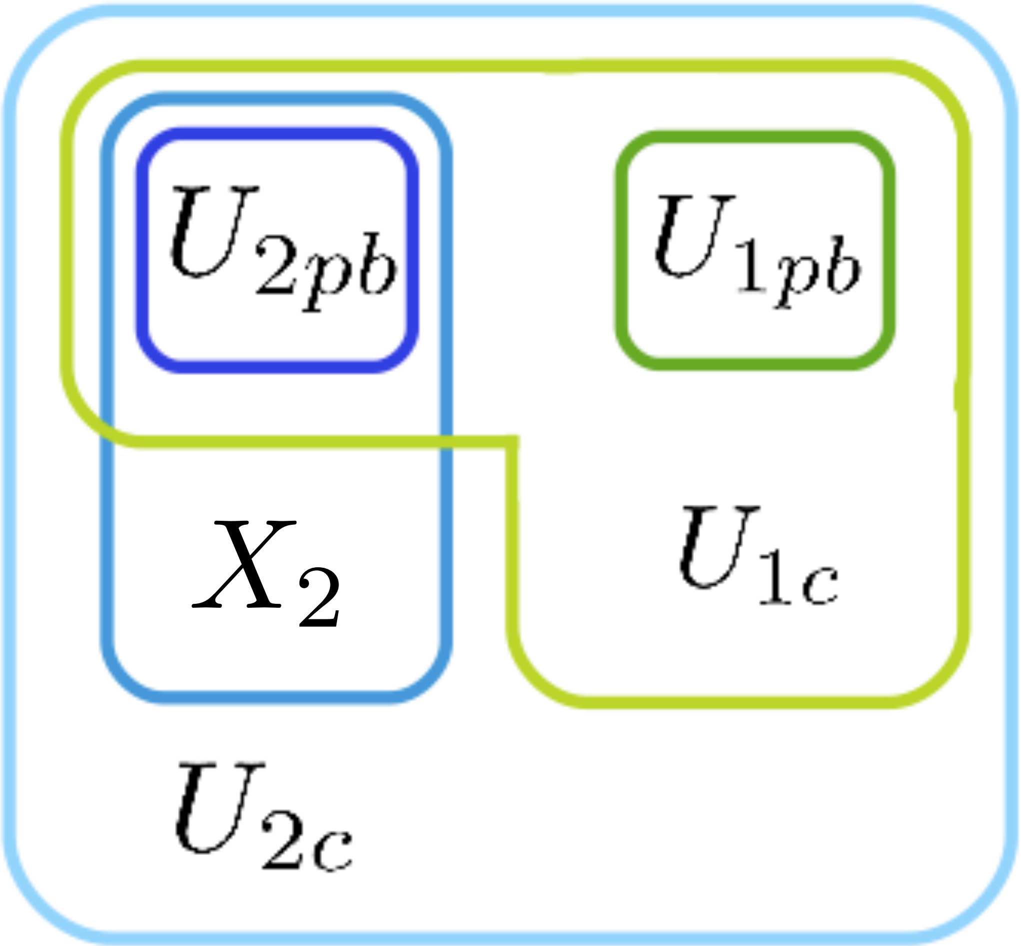
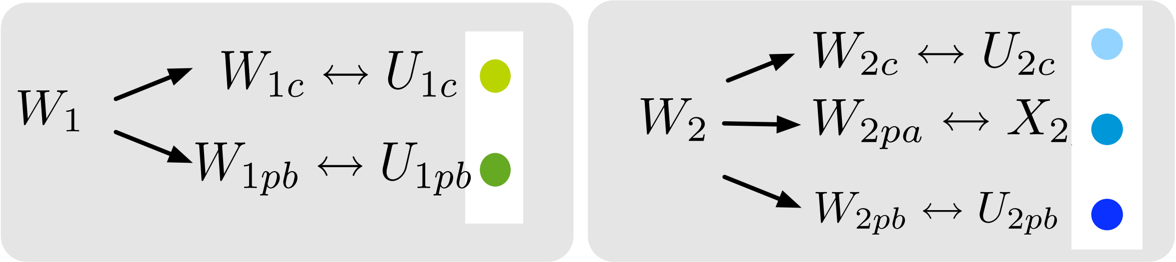




$p_{x_2|u_{2c}}$

$p_{u_{2c}}$

Superposition



$W_1 \to W_{1c} \leftrightarrow U_{1c}$ ●

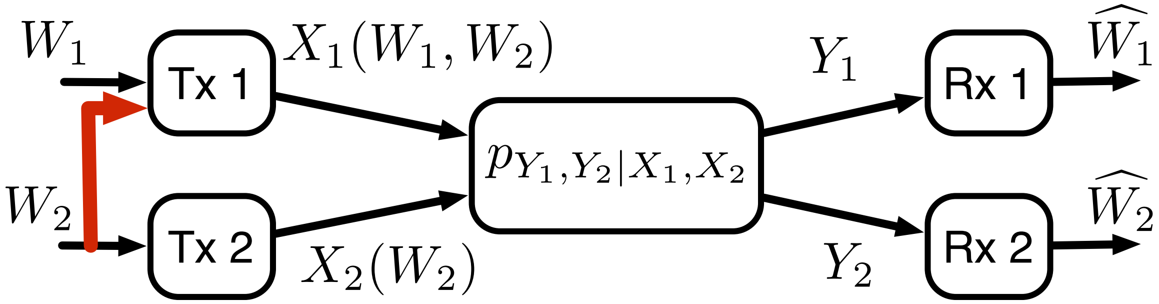
$W_1 \to W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \to W_{2c} \leftrightarrow U_{2c}$ ●

$W_2 \to W_{2pa} \leftrightarrow X_2$ ●

$W_2 \to W_{2pb} \leftrightarrow U_{2pb}$ ●

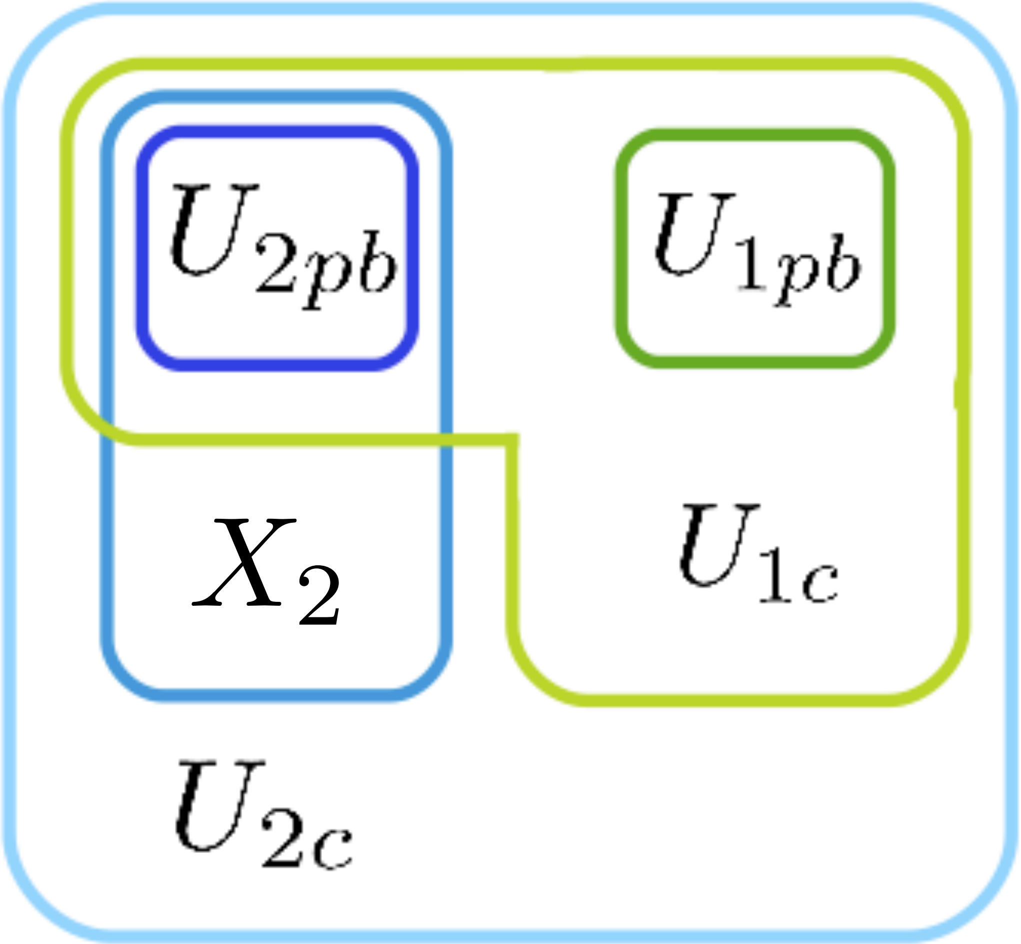
c = common, p = private,
a = alone, b = broadcast



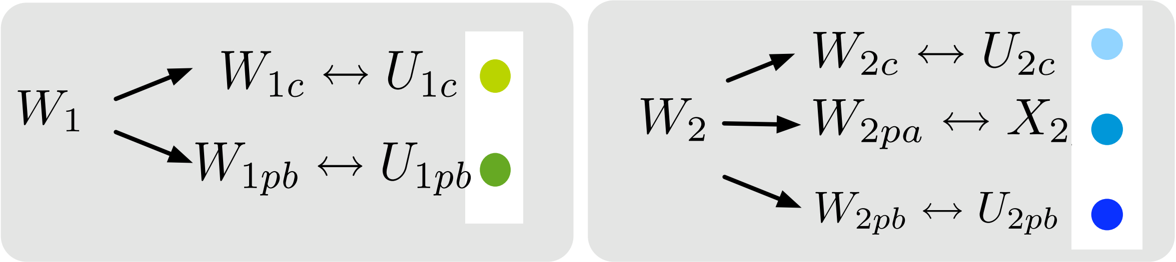
$$p_{u_{2pb}|u_{1c},u_{2c},x_2}$$

$$p_{x_2|u_{2c}}$$

$$p_{u_{2c}}$$



Superposition



W_1

$W_{1c} \leftrightarrow U_{1c}$ ●

$W_{1pb} \leftrightarrow U_{1pb}$ ●

W_2

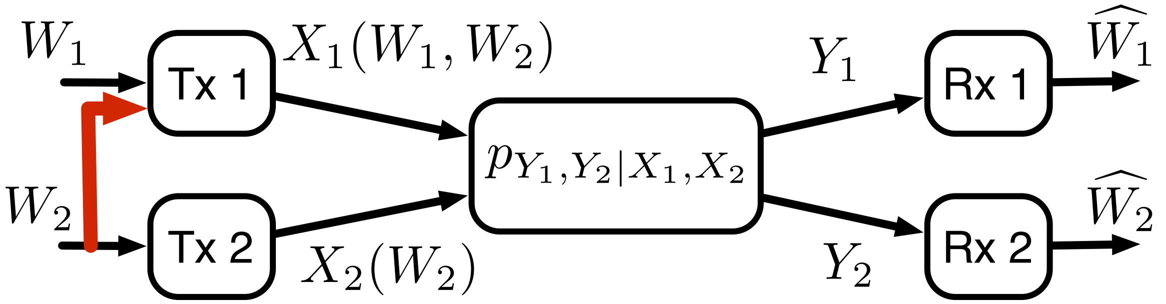
$W_{2c} \leftrightarrow U_{2c}$ ●

$W_{2pa} \leftrightarrow X_2$ ●

$W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,

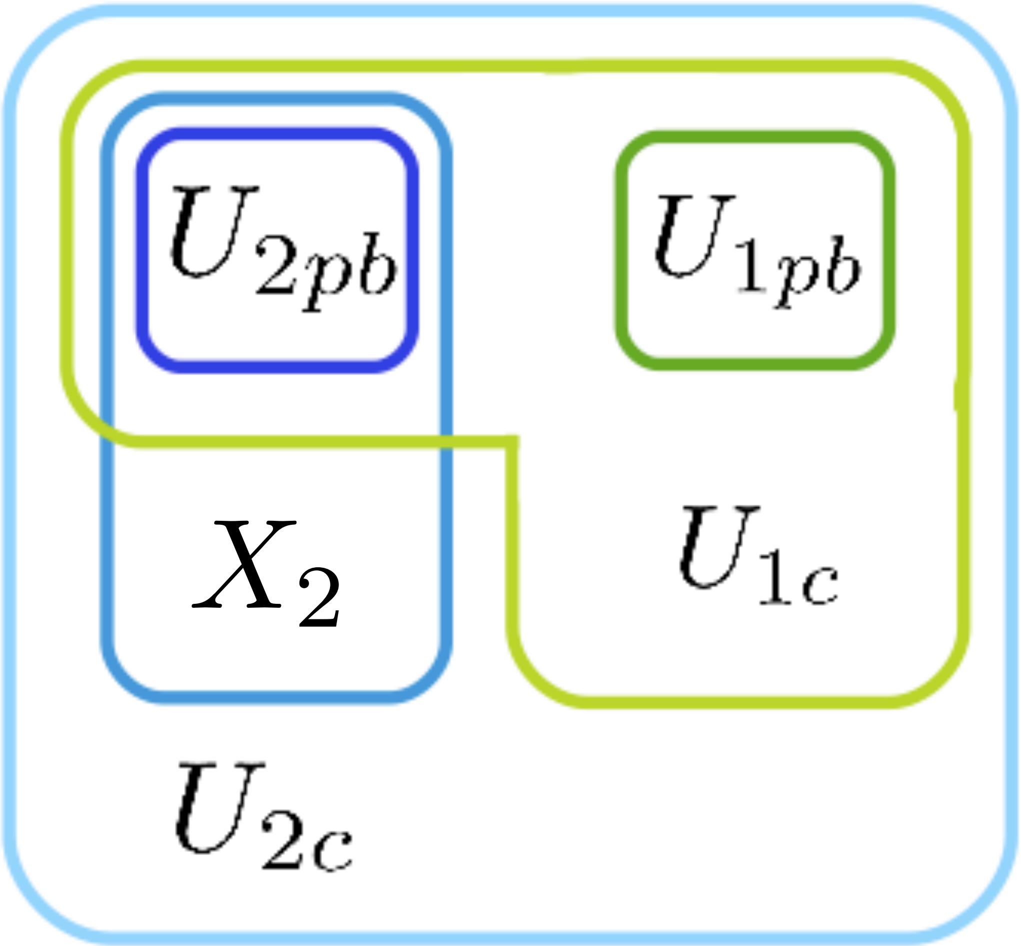
a = alone, b = broadcast



$p_{u_{2pb}|u_{1c},u_{2c},x_2}$

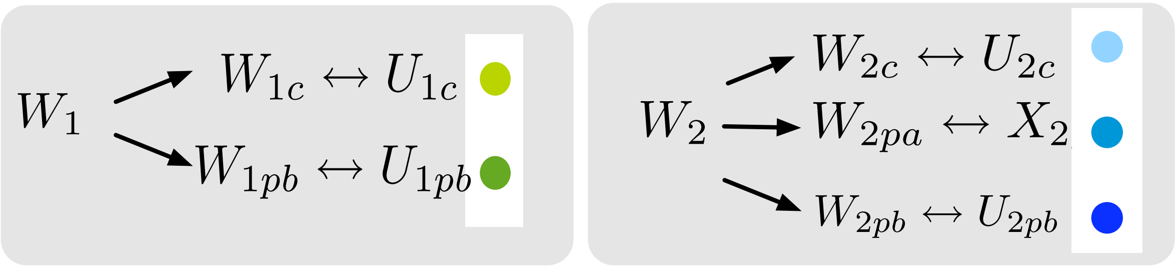
$p_{x_2|u_{2c}}$

$p_{u_{2c}}$



$p_{u_{1c}|u_{2c}}$

Superposition



$W_1 \rightarrow W_{1c} \leftrightarrow U_{1c}$ ●

$W_1 \rightarrow W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \rightarrow W_{2c} \leftrightarrow U_{2c}$ ●

$W_2 \rightarrow W_{2pa} \leftrightarrow X_2$ ●

$W_2 \rightarrow W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,
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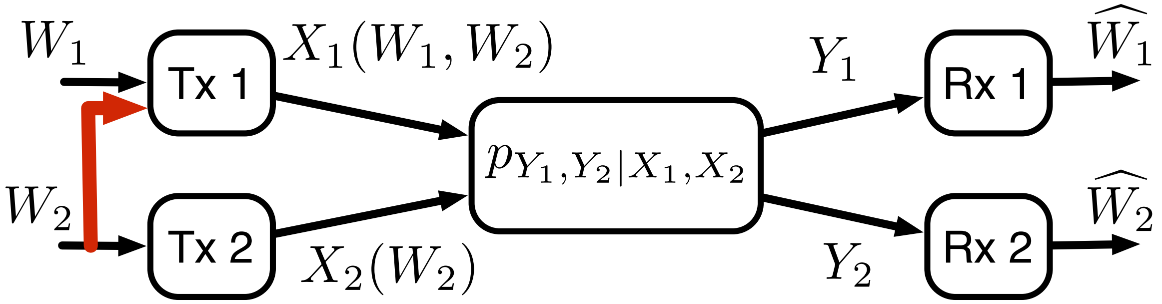
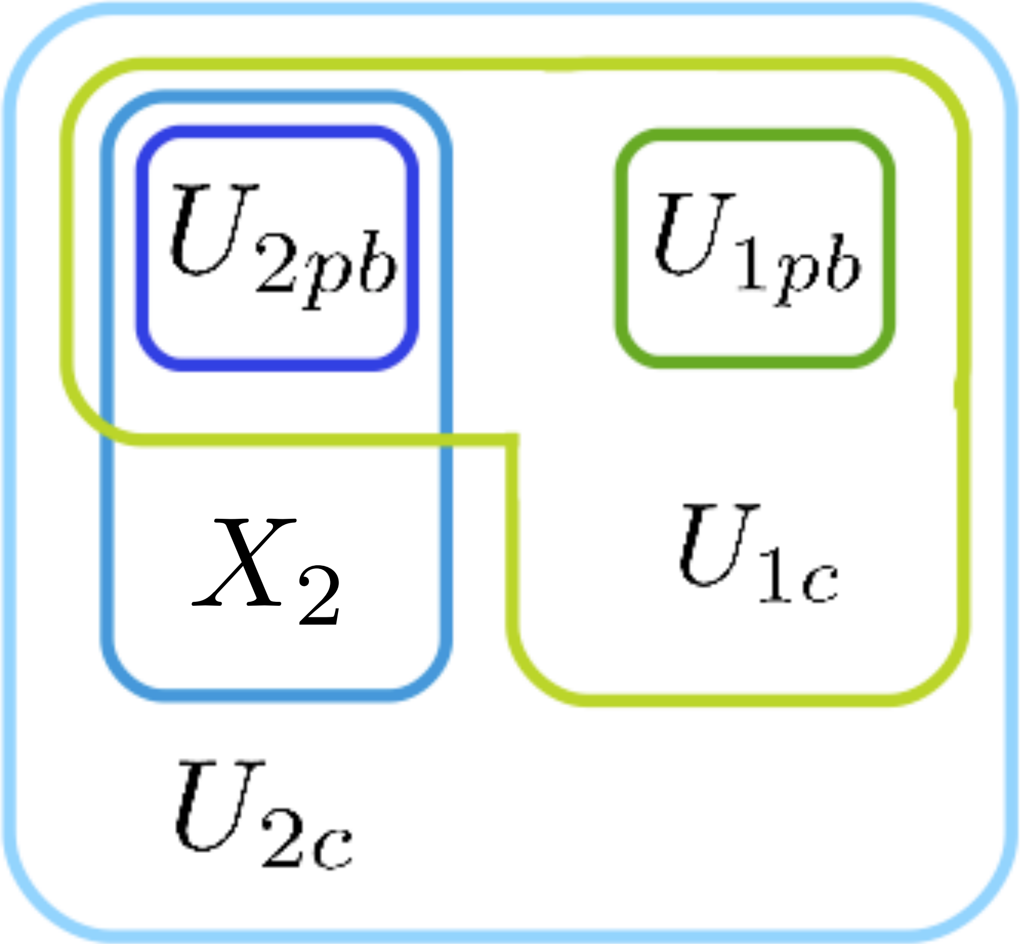




$$p_{u_{2pb}|u_{1c},u_{2c},x_2}$$

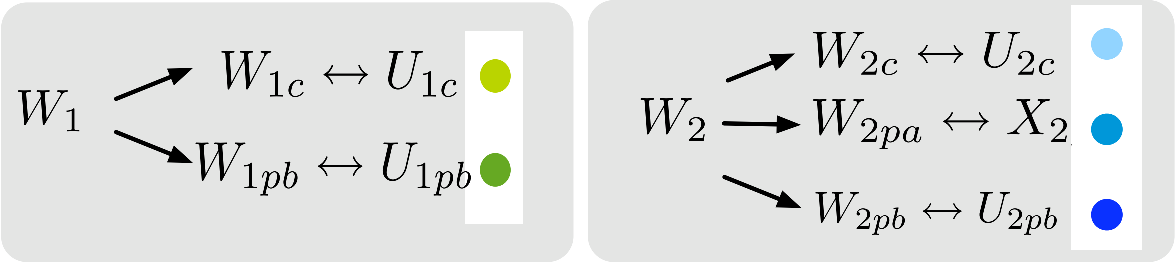
$$p_{u_{1pb}|u_{1c},u_{2c}}$$

$$p_{x_2|u_{2c}}$$

$$p_{u_{1c}|u_{2c}}$$

$$p_{u_{2c}}$$

Superposition



$W_1 \to W_{1c} \leftrightarrow U_{1c}$ ●

$W_1 \to W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \to W_{2c} \leftrightarrow U_{2c}$ ●

$W_2 \to W_{2pa} \leftrightarrow X_2$ ●

$W_2 \to W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,

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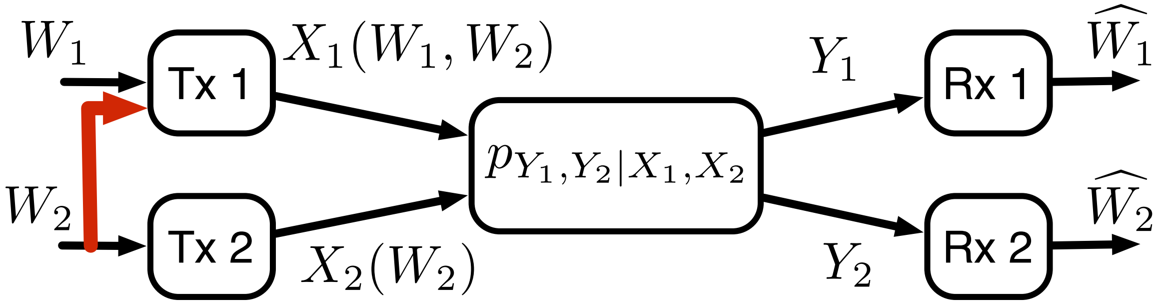
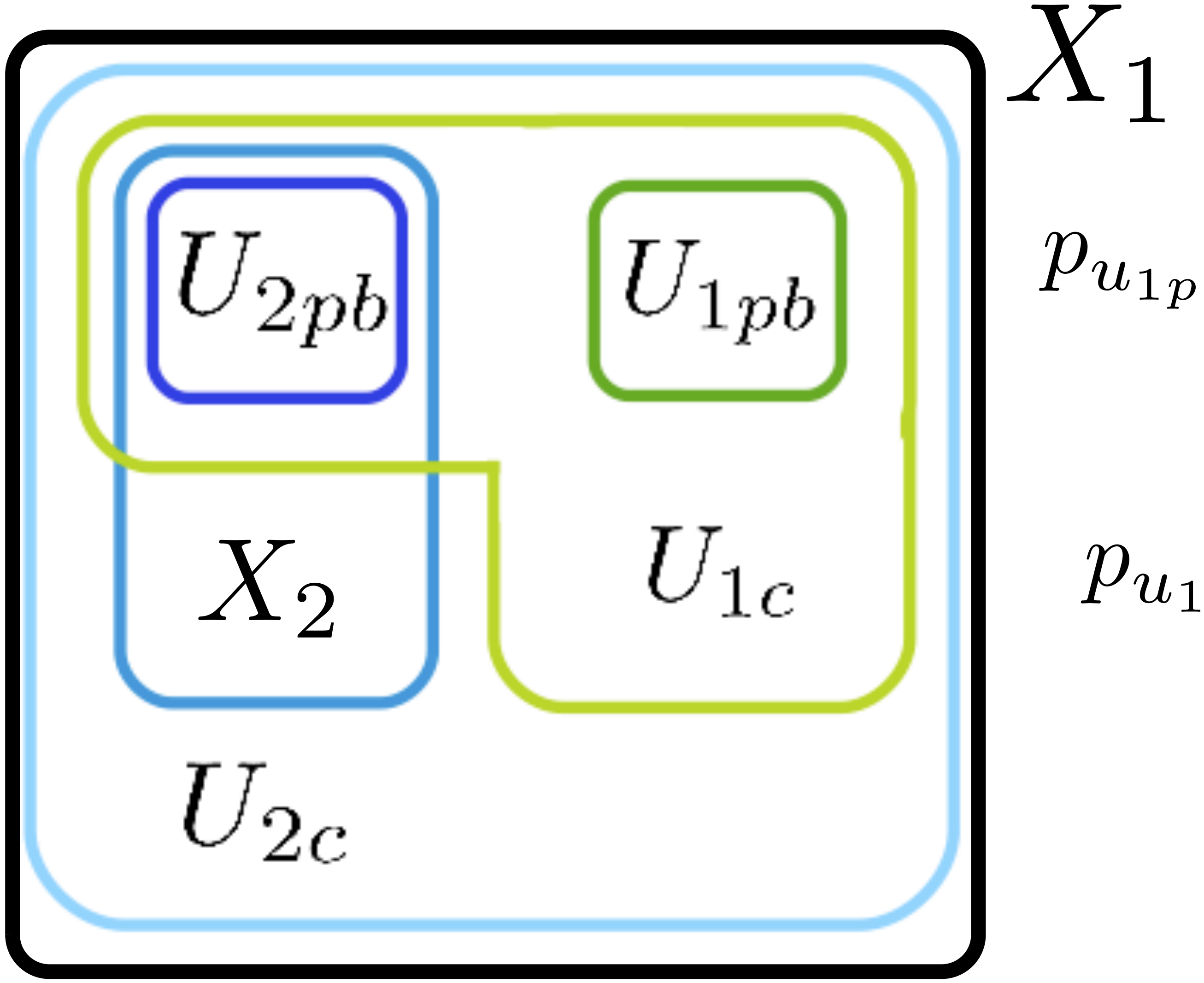


$$p_{x_1|u_{2pb},u_{1pb},u_{1c},u_{2c},x_2}$$

$$p_{u_{2pb}|u_{1c},u_{2c},x_2}$$

$$p_{x_2|u_{2c}}$$

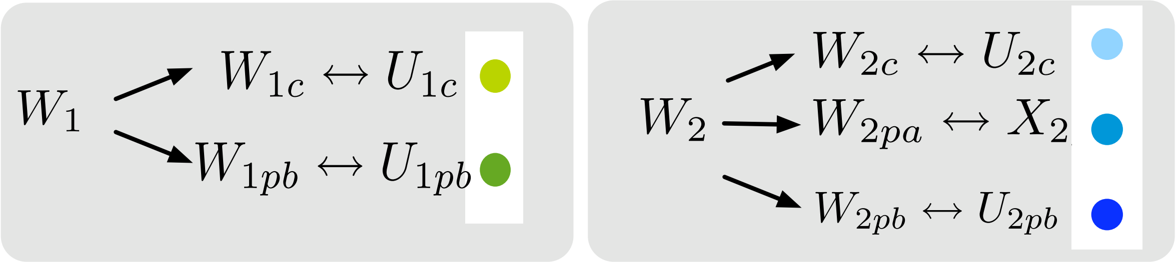
$$p_{u_{2c}}$$



$$p_{u_{1pb}|u_{1c},u_{2c}}$$

$$p_{u_{1c}|u_{2c}}$$

Superposition



c = common, p = private,
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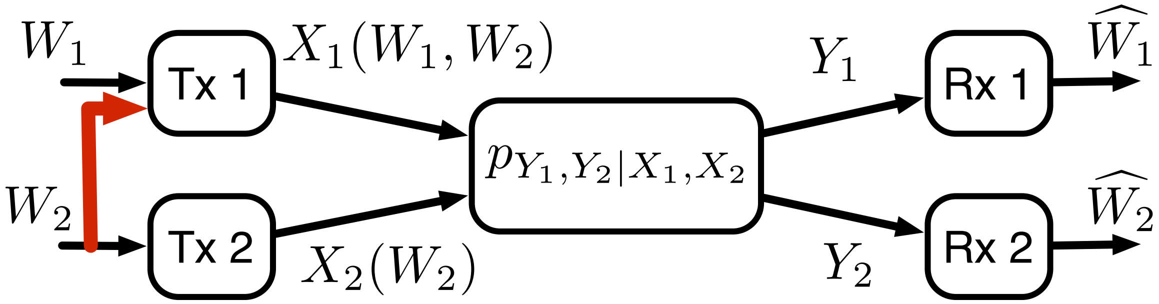
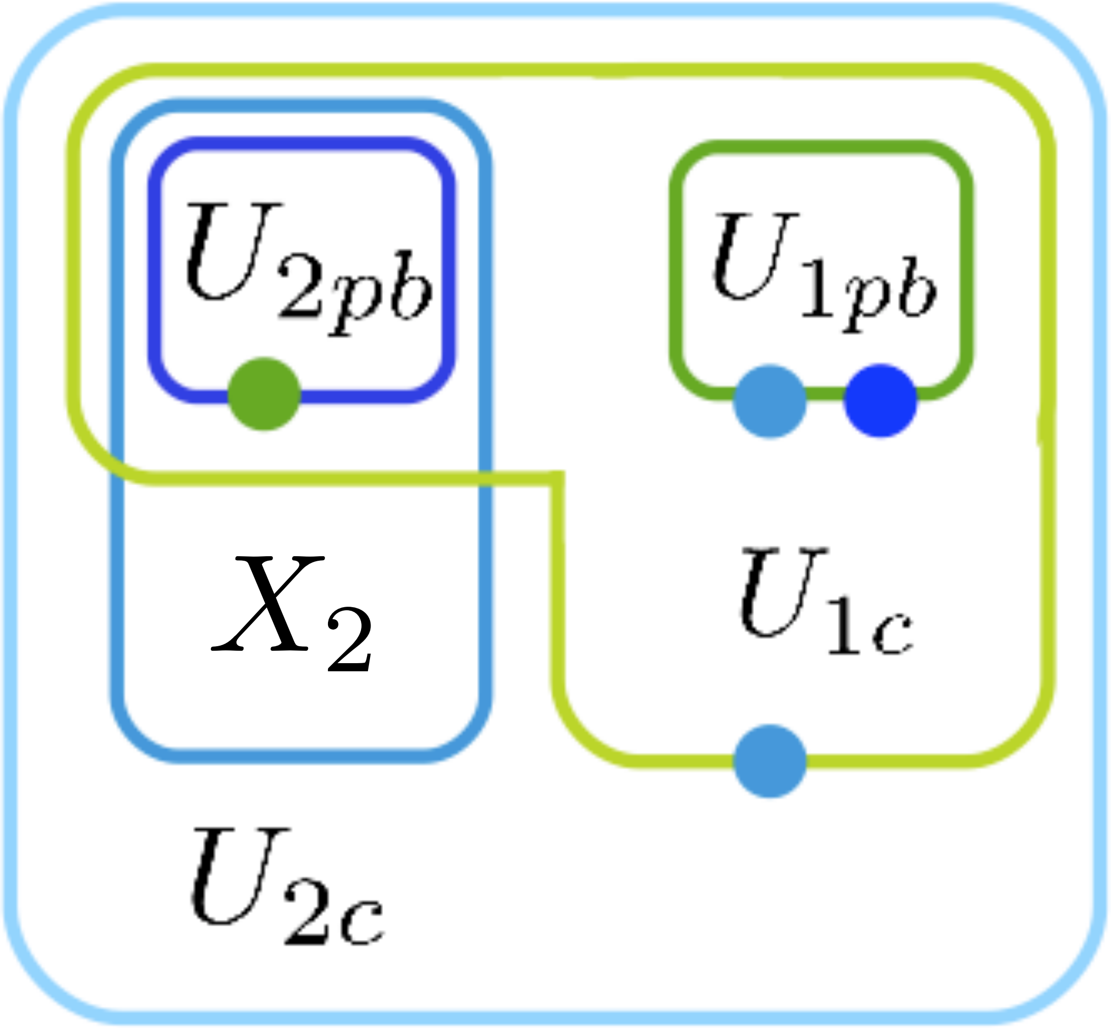
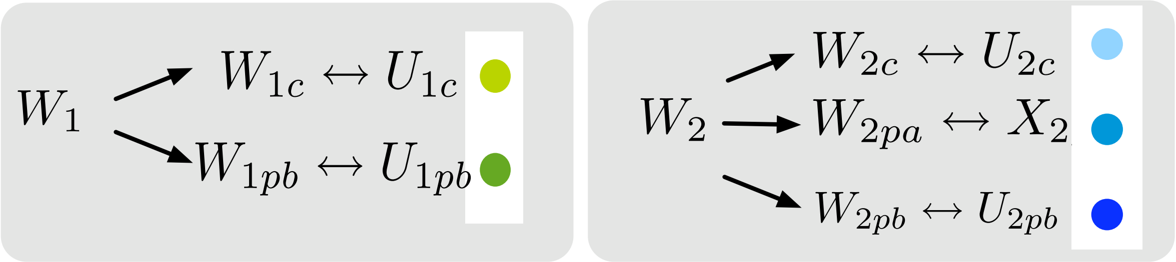


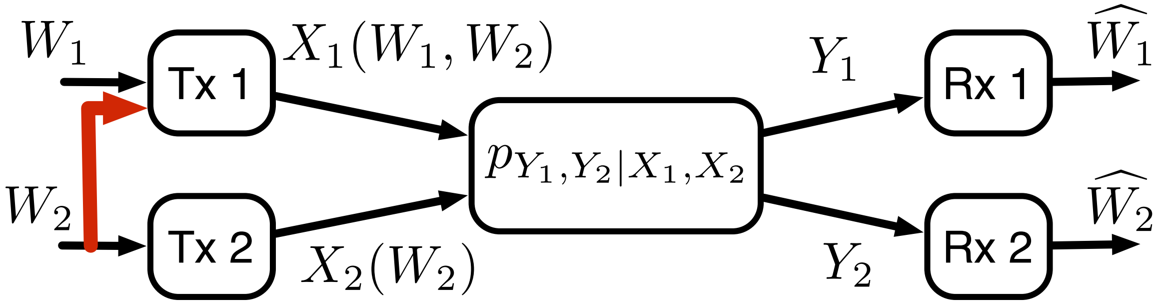


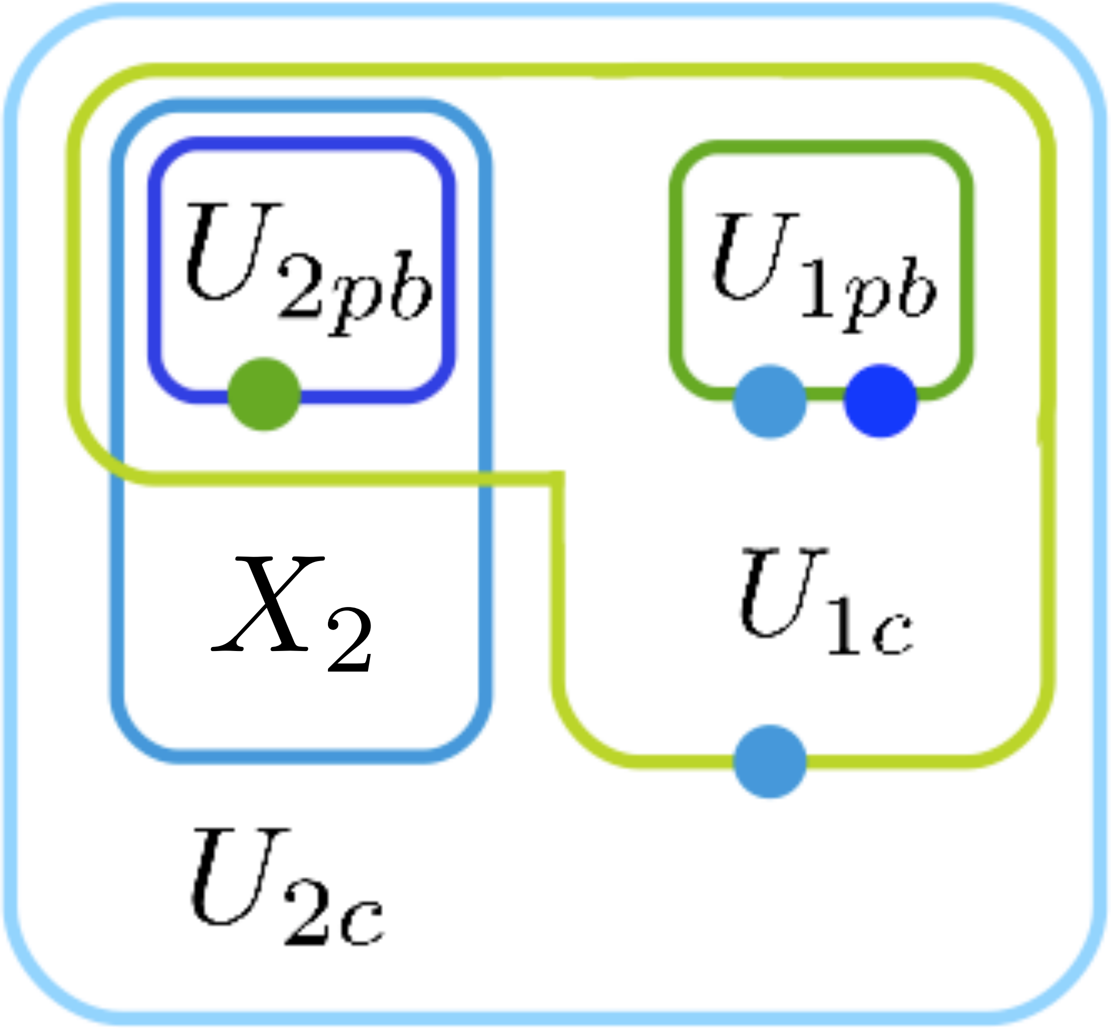
Binning



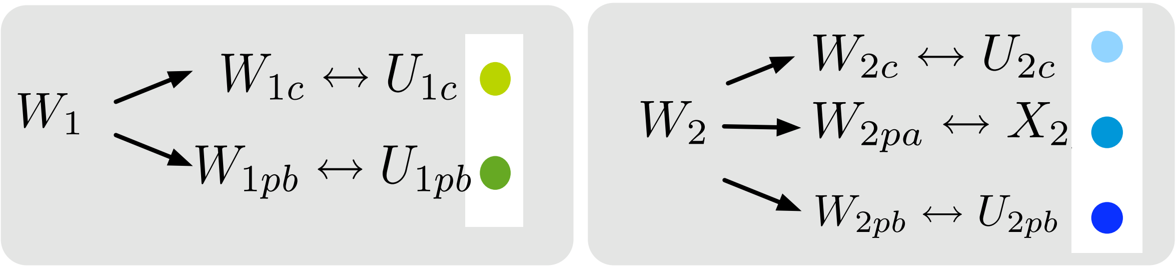
c = common, p = private,

a = alone, b = broadcast





Binning ~ **Dirty paper coding!**



$W_1 \to W_{1c} \leftrightarrow U_{1c}$ ●
$W_1 \to W_{1pb} \leftrightarrow U_{1pb}$ ●

$W_2 \to W_{2c} \leftrightarrow U_{2c}$ ●
$W_2 \to W_{2pa} \leftrightarrow X_2$ ●
$W_2 \to W_{2pb} \leftrightarrow U_{2pb}$ ●

c = common, p = private,
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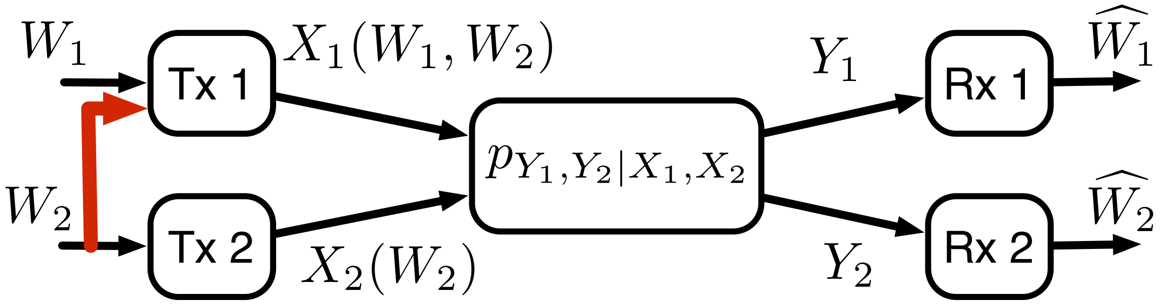
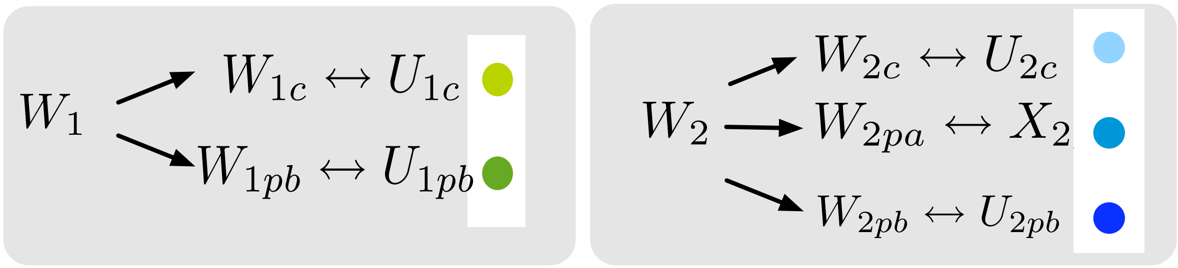


$$\begin{array}{rcl}
R'_{1c} & \geq & I(U_{1c}; X_2|U_{2c}) \\
R'_{1c} + R'_{1pb} & \geq & I(U_{1pb}, U_{1c}; X_2|U_{2c}) \\
R'_{1c} + R'_{1pb} + R'_{2pb} & \geq & I(U_{1pb}, U_{1c}; X_2|U_{2c}) + I(U_{2pb}; U_{1pb}|U_{1c}, U_{2c}, X_2) \\
R_{2c} + R_{2pa} + (R_{1c} + R'_{1c}) + (R_{2pb} + R'_{2pb}) & \leq & I(Y_2; U_{2pb}, U_{1c}, X_2, U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
R_{2pa} + (R_{1c} + R'_{1c}) + (R_{2pb} + R'_{2pb}) & \leq & I(Y_2; U_{2pb}, U_{1c}, X_2|U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
R_{2pa} + (R_{2pb} + R'_{2pb}) & \leq & I(Y_2; U_{2pb}, X_2|U_{1c}, U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
(R_{1c} + R'_{1c}) + (R_{2pb} + R'_{2pb}) & \leq & I(Y_2; U_{2pb}, U_{1c}|X_2, U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
(R_{2pb} + R'_{2pb}) & \leq & I(Y_2; U_{2pb}|U_{1c}, X_2, U_{2c}) \\
R_{2c} + (R_{1c} + R'_{1c}) + (R_{1pb} + R'_{1pb}) & \leq & I(Y_1; U_{1pb}, U_{1c}, U_{2c}), \\
(R_{1c} + R'_{1c}) + (R_{1pb} + R'_{1pb}) & \leq & I(Y_1; U_{1pb}, U_{1c}|U_{2c}), \\
(R_{1pb} + R'_{1pb}) & \leq & I(Y_1; U_{1pb}|U_{1c}, U_{2c}),
\end{array}$$

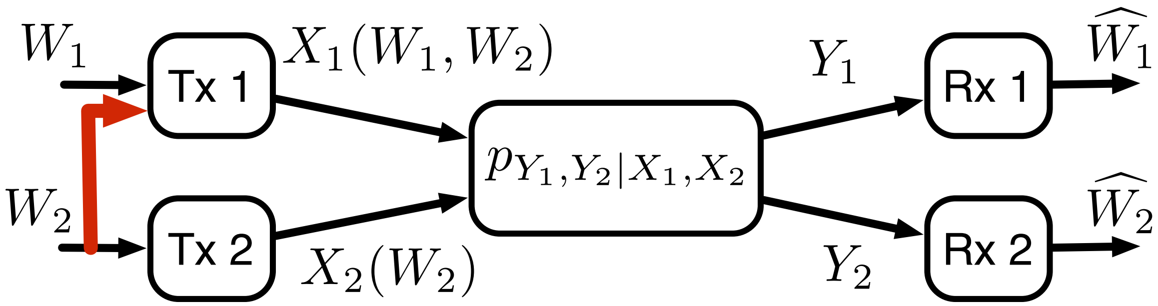
for some input distribution

$$p_{Y_1,Y_2,X_1,X_2,U_{1c},U_{2c},U_{1pb},U_{2pb}} = p_{U_{1c},U_{2c},U_{1pb},U_{2pb}} p_{X_1,X_2|U_{1c},U_{2c},U_{1pb},U_{2pb}} p_{Y_1,Y_2|X_1,X_2}.$$



c = common, p = private,

a = alone, b = broadcast

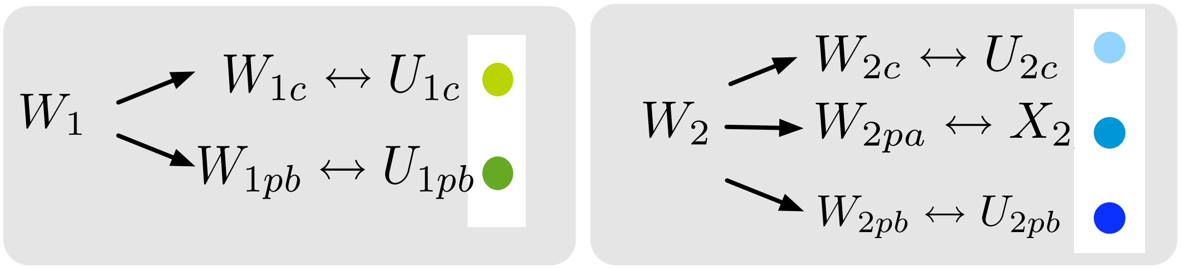


Analytically shown to be largest known region

$$\begin{aligned}
R'_{1c} &\geq I(U_{1c};X_2|U_{2c})\\
R'_{1c}+R'_{1pb} &\geq I(U_{1pb},U_{1c};X_2|U_{2c})\\
R'_{1c}+R'_{1pb}+R'_{2pb} &\geq I(U_{1pb},U_{1c};X_2|U_{2c})+I(U_{2pb};U_{1pb}|U_{1c},U_{2c},X_2)\\
R_{2c}+R_{2pa}+(R_{1c}+R'_{1c})+(R_{2pb}+R'_{2pb}) &\leq I(Y_2;U_{2pb},U_{1c},X_2,U_{2c})+I(U_{1c};X_2|U_{2c})\\
R_{2pa}+(R_{1c}+R'_{1c})+(R_{2pb}+R'_{2pb}) &\leq I(Y_2;U_{2pb},U_{1c},X_2|U_{2c})+I(U_{1c};X_2|U_{2c})\\
R_{2pa}+(R_{2pb}+R'_{2pb}) &\leq I(Y_2;U_{2pb},X_2|U_{1c},U_{2c})+I(U_{1c};X_2|U_{2c})\\
(R_{1c}+R'_{1c})+(R_{2pb}+R'_{2pb}) &\leq I(Y_2;U_{2pb},U_{1c}|X_2,U_{2c})+I(U_{1c};X_2|U_{2c})\\
(R_{2pb}+R'_{2pb}) &\leq I(Y_2;U_{2pb}|U_{1c},X_2,U_{2c})\\
R_{2c}+(R_{1c}+R'_{1c})+(R_{1pb}+R'_{1pb}) &\leq I(Y_1;U_{1pb},U_{1c},U_{2c}),\\
(R_{1c}+R'_{1c})+(R_{1pb}+R'_{1pb}) &\leq I(Y_1;U_{1pb},U_{1c}|U_{2c}),\\
(R_{1pb}+R'_{1pb}) &\leq I(Y_1;U_{1pb}|U_{1c},U_{2c}),
\end{aligned}$$

for some input distribution

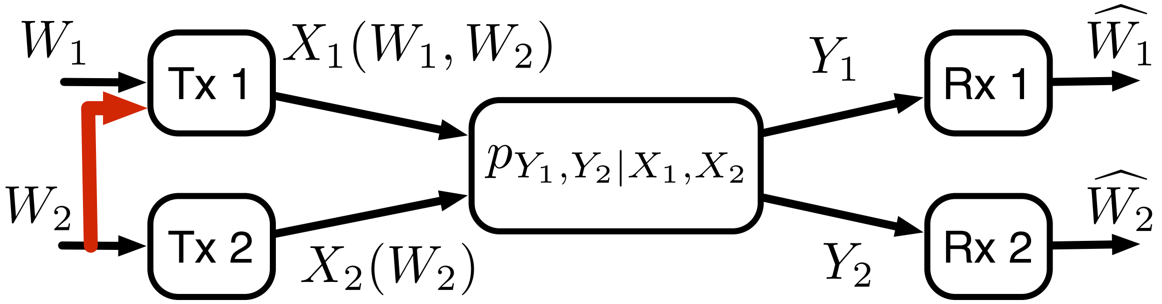
$$p_{Y_1,Y_2,X_1,X_2,U_{1c},U_{2c},U_{1pb},U_{2pb}} = p_{U_{1c},U_{2c},U_{1pb},U_{2pb}}\,p_{X_1,X_2|U_{1c},U_{2c},U_{1pb},U_{2pb}}\,p_{Y_1,Y_2|X_1,X_2}.$$



$$W_1 \to W_{1c} \leftrightarrow U_{1c} \quad ●$$
$$W_1 \to W_{1pb} \leftrightarrow U_{1pb} \quad ●$$
$$W_2 \to W_{2c} \leftrightarrow U_{2c} \quad ●$$
$$W_2 \to W_{2pa} \leftrightarrow X_2 \quad ●$$
$$W_2 \to W_{2pb} \leftrightarrow U_{2pb} \quad ●$$

c = common, p = private,

a = alone, b = broadcast



Analytically shown to be largest known region

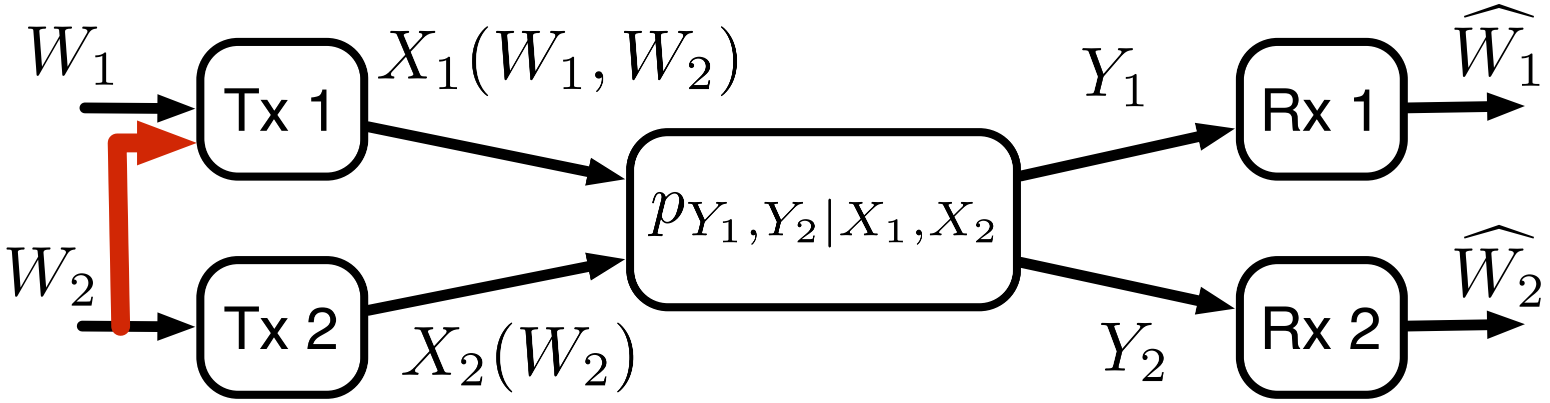
$$\begin{aligned}
{R'}_{1c} &\geq I(U_{1c}; X_2|U_{2c}) \\
{R'}_{1c} + {R'}_{1pb} &\geq I(U_{1pb}, U_{1c}; X_2|U_{2c}) \\
{R'}_{1c} + {R'}_{1pb} + {R'}_{2pb} &\geq I(U_{1pb}, U_{1c}; X_2|U_{2c}) + I(U_{2pb}; U_{1pb}|U_{1c}, U_{2c}, X_2) \\
R_{2c} + R_{2pa} + (R_{1c} + {R'}_{1c}) + (R_{2pb} + {R'}_{2pb}) &\leq I(Y_2; U_{2pb}, U_{1c}, X_2, U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
R_{2pa} + (R_{1c} + {R'}_{1c}) + (R_{2pb} + {R'}_{2pb}) &\leq I(Y_2; U_{2pb}, U_{1c}, X_2|U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
R_{2pa} + (R_{2pb} + {R'}_{2pb}) &\leq I(Y_2; U_{2pb}, X_2|U_{1c}, U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
(R_{1c} + {R'}_{1c}) + (R_{2pb} + {R'}_{2pb}) &\leq I(Y_2; U_{2pb}, U_{1c}|X_2, U_{2c}) + I(U_{1c}; X_2|U_{2c}) \\
(R_{2pb} + {R'}_{2pb}) &\leq I(Y_2; U_{2pb}|U_{1c}, X_2, U_{2c}) \\
R_{2c} + (R_{1c} + {R'}_{1c}) + (R_{1pb} + {R'}_{1pb}) &\leq I(Y_1; U_{1pb}, U_{1c}, U_{2c}), \\
(R_{1c} + {R'}_{1c}) + (R_{1pb} + {R'}_{1pb}) &\leq I(Y_1; U_{1pb}, U_{1c}|U_{2c}), \\
(R_{1pb} + {R'}_{1pb}) &\leq I(Y_1; U_{1pb}|U_{1c}, U_{2c}),
\end{aligned}$$

for some input distribution

$$p_{Y_1,Y_2,X_1,X_2,U_{1c},U_{2c},U_{1pb},U_{2pb}} = p_{U_{1c},U_{2c},U_{1pb},U_{2pb}} p_{X_1,X_2|U_{1c},U_{2c},U_{1pb},U_{2pb}} p_{Y_1,Y_2|X_1,X_2}.$$

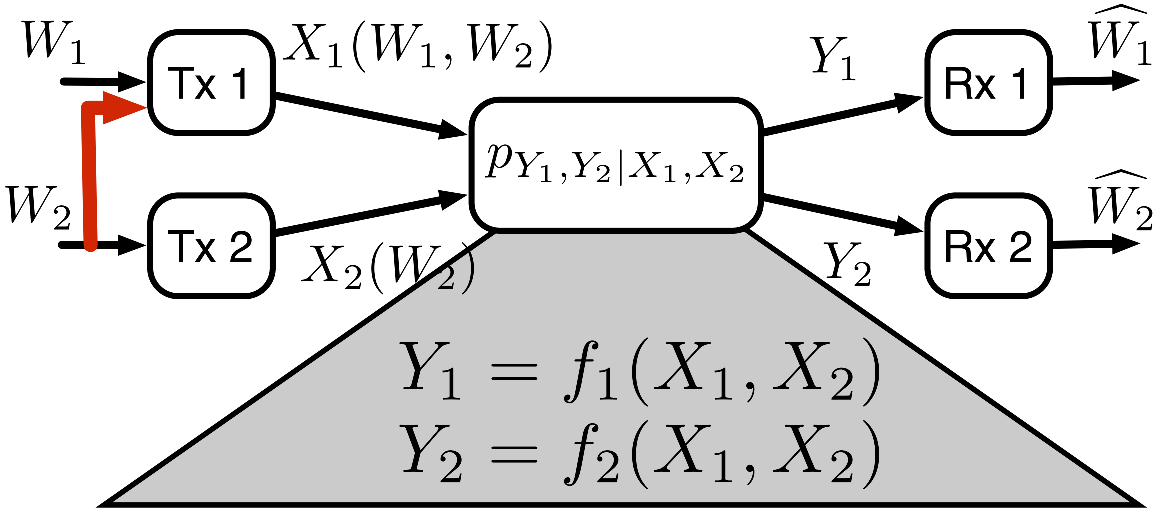
Reduces to capacity whenever it is known

Contributions



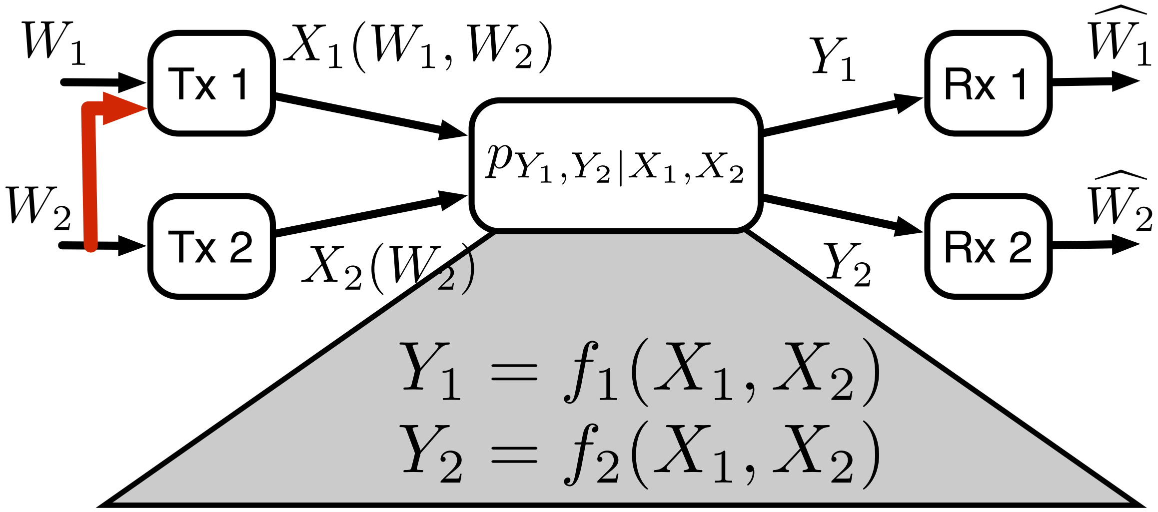
- new inner bound *(largest region)*
- new outer bound *(not tightest, but computable)*
- capacity for deterministic channels *(also semi-deterministic)*
- 1.8 bit gap result for Gaussian channels *(preliminary simulations show smaller gap)*

Capacity region for deterministic channels



$$Y_1 = f_1(X_1, X_2)$$
$$Y_2 = f_2(X_1, X_2)$$

Capacity region for deterministic channels

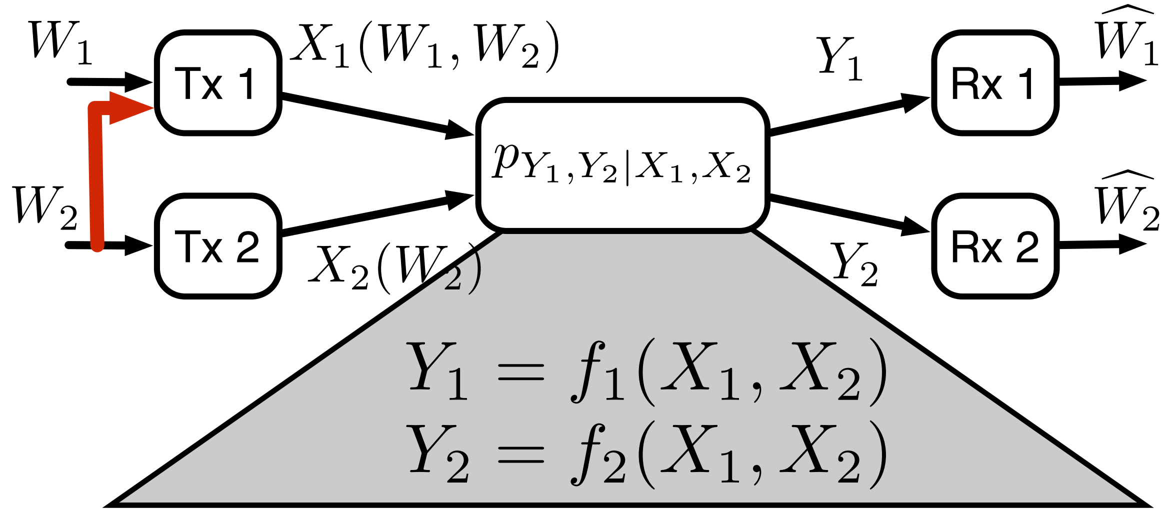


Capacity region is

$$R_1 \leq H(Y_1|X_2)$$
$$R_2 \leq H(Y_2)$$
$$R_1 + R_2 \leq H(Y_2) + H(Y_1|X_2, Y_2)$$

for the deterministic *(semi-deterministic, linear high SNR deterministic)* channel

Capacity region for deterministic channels



Capacity region is

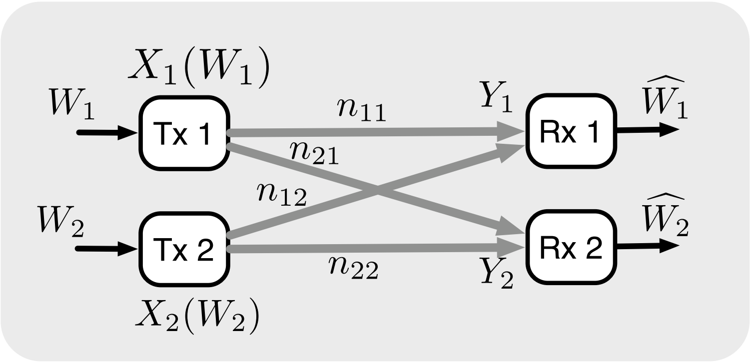
$$
\begin{aligned}
R_1 &\leq H(Y_1|X_2)\\
R_2 &\leq H(Y_2)\\
R_1 + R_2 &\leq H(Y_2) + H(Y_1|X_2, Y_2)
\end{aligned}
$$

for the deterministic *(semi-deterministic, linear high SNR deterministic)* channel

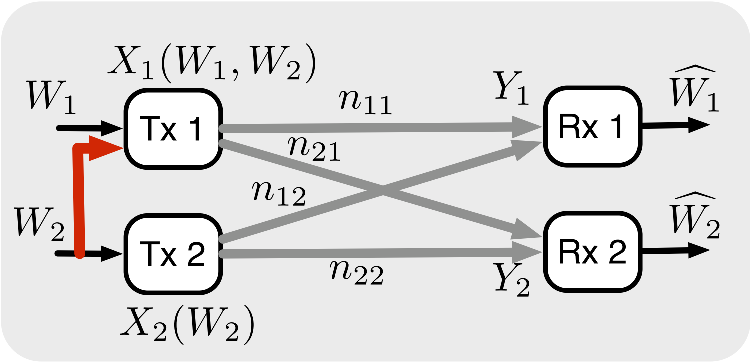
We have capacity!

High-SNR linear deterministic cognitive interference channel

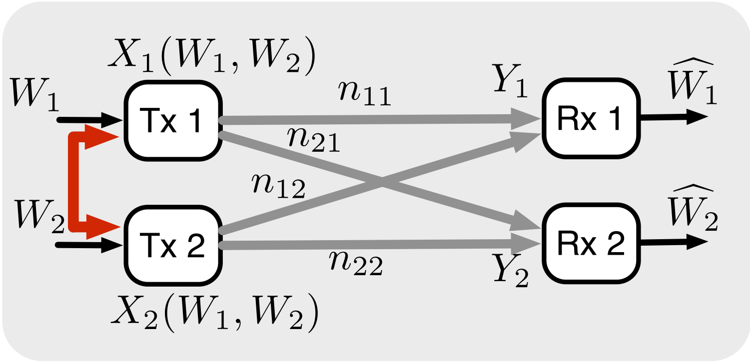
S. Rini, D. Tuninetti, and N. Devroye, “The capacity region of deterministic cognitive radio channels,” *Proc. IEEE ITW Taormina, Italy*, vol. Oct., 2009.



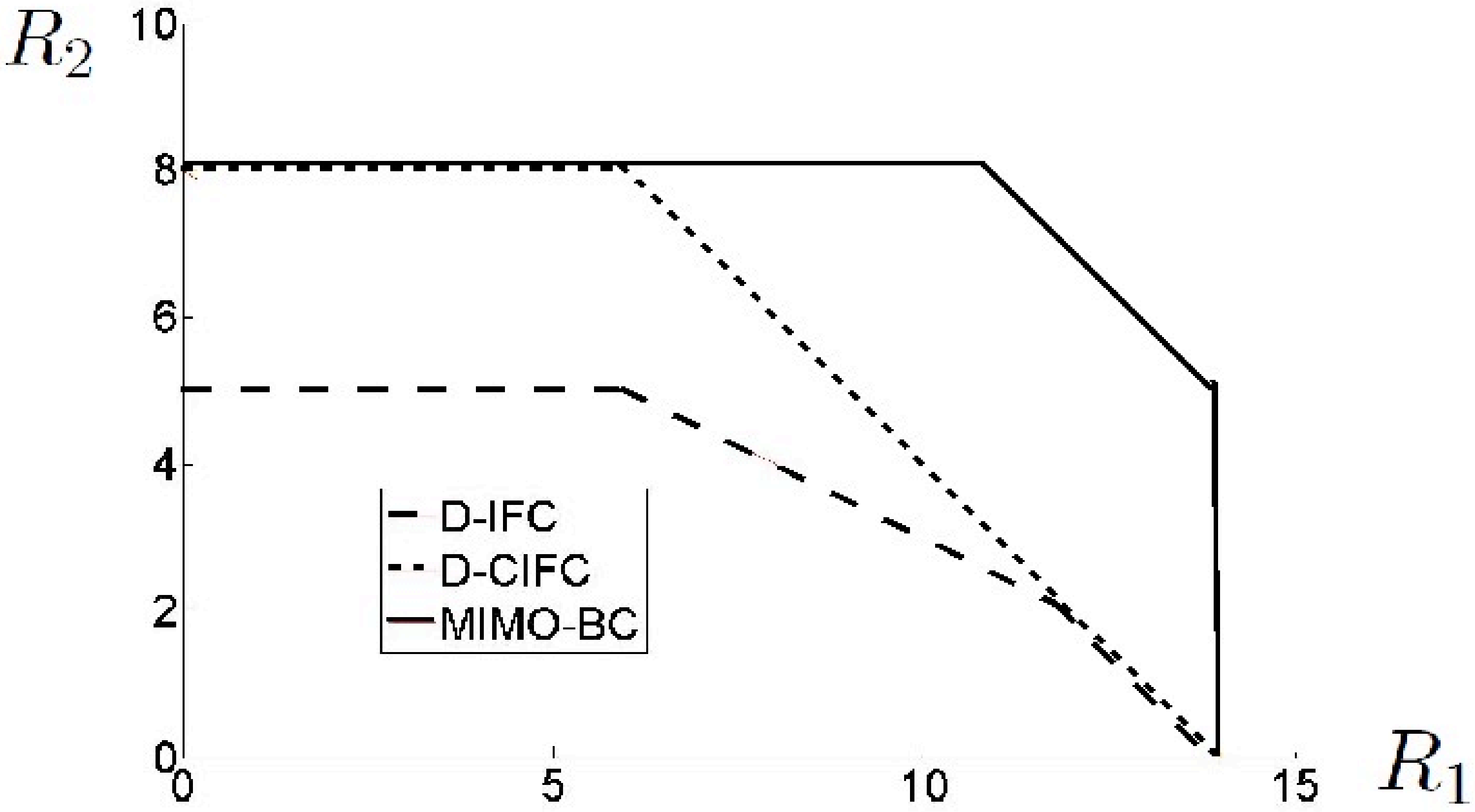
D-IFC



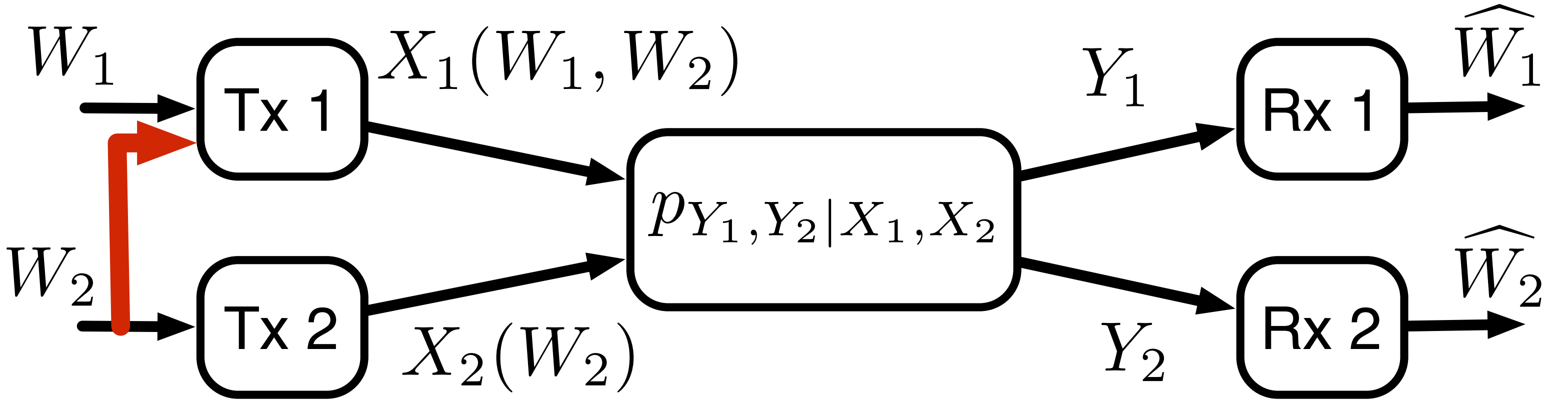
D-CIFC



D-MIMO-BC

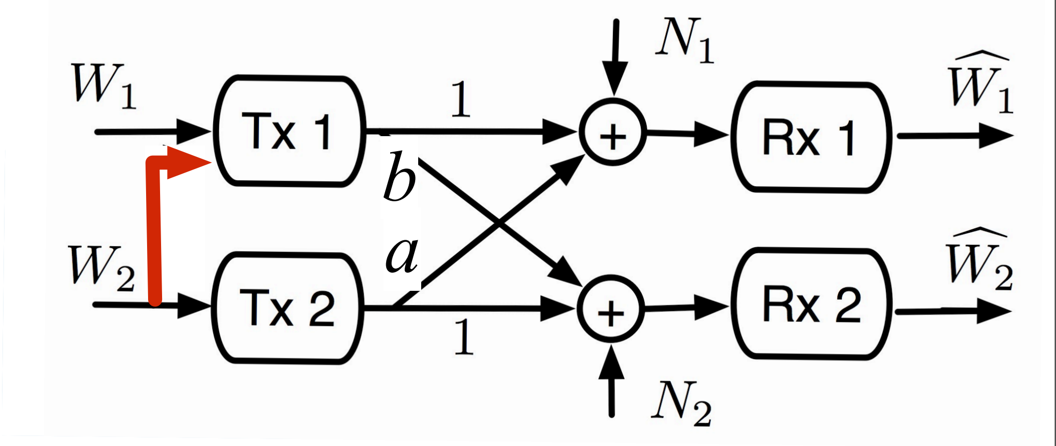


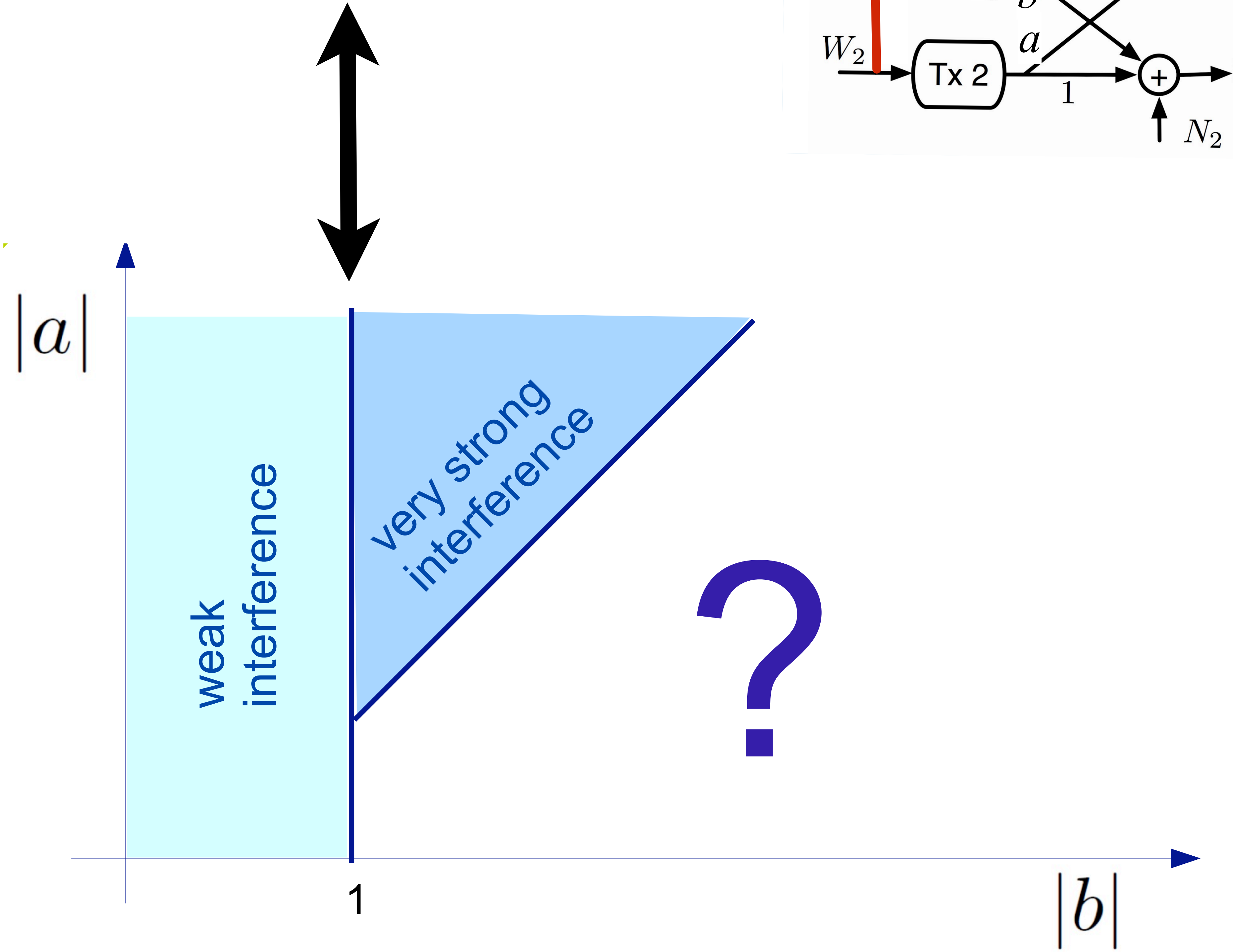
Contributions



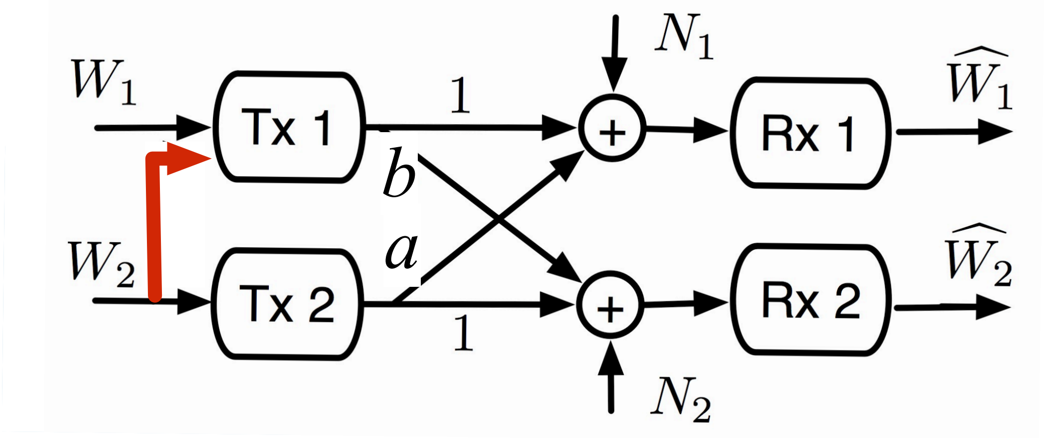
- new inner bound *(largest region)*
- new outer bound *(not tightest, but computable)*
- capacity for deterministic channels *(also semi-deterministic)*
- 1.8 bit gap result for Gaussian channels *(recently reduced to 1 bit gap)*

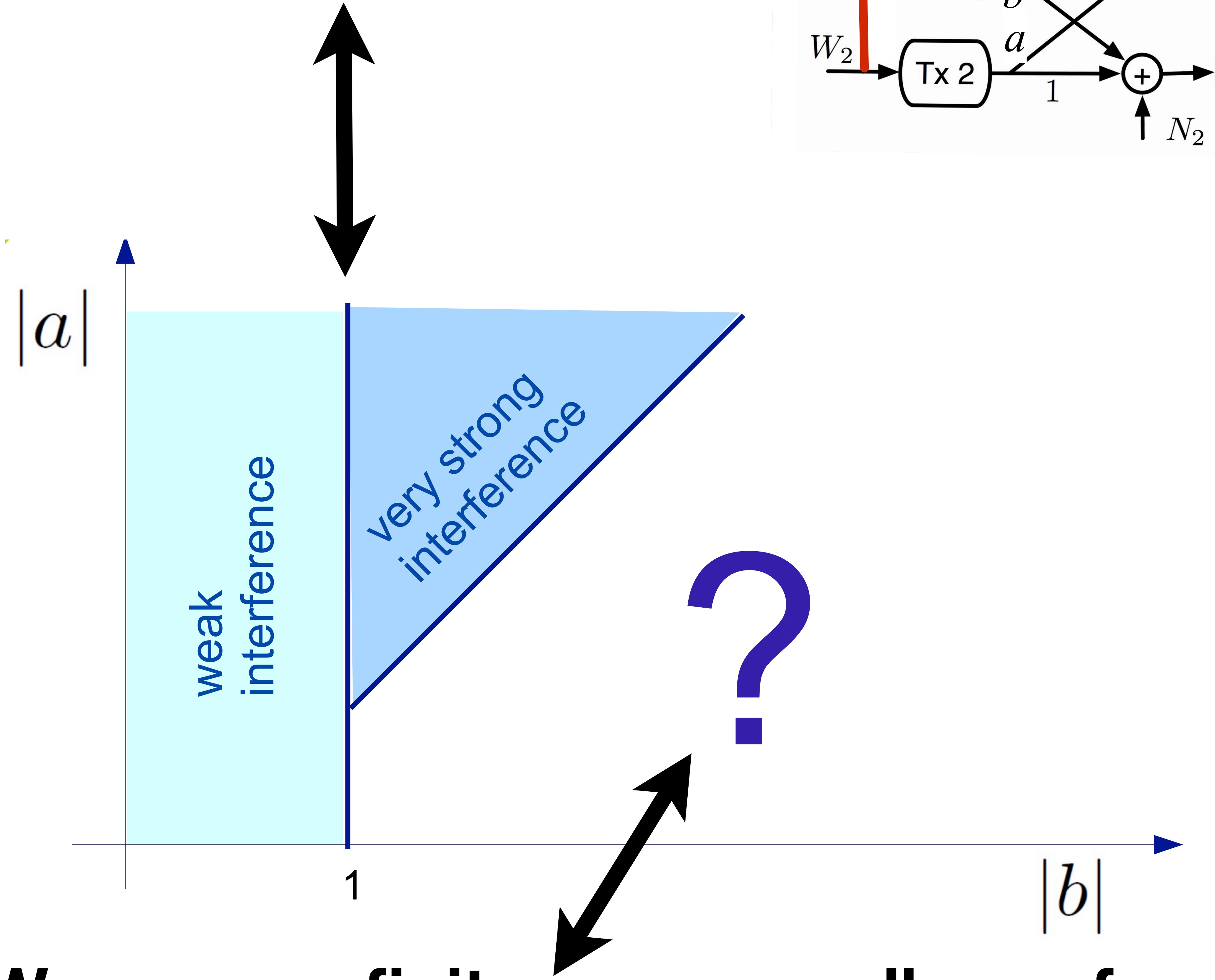
Known Gaussian results





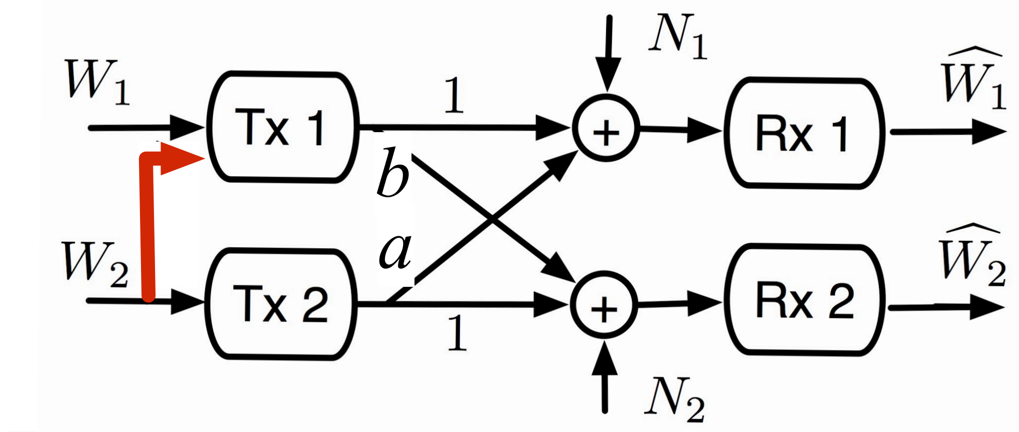
Known Gaussian results

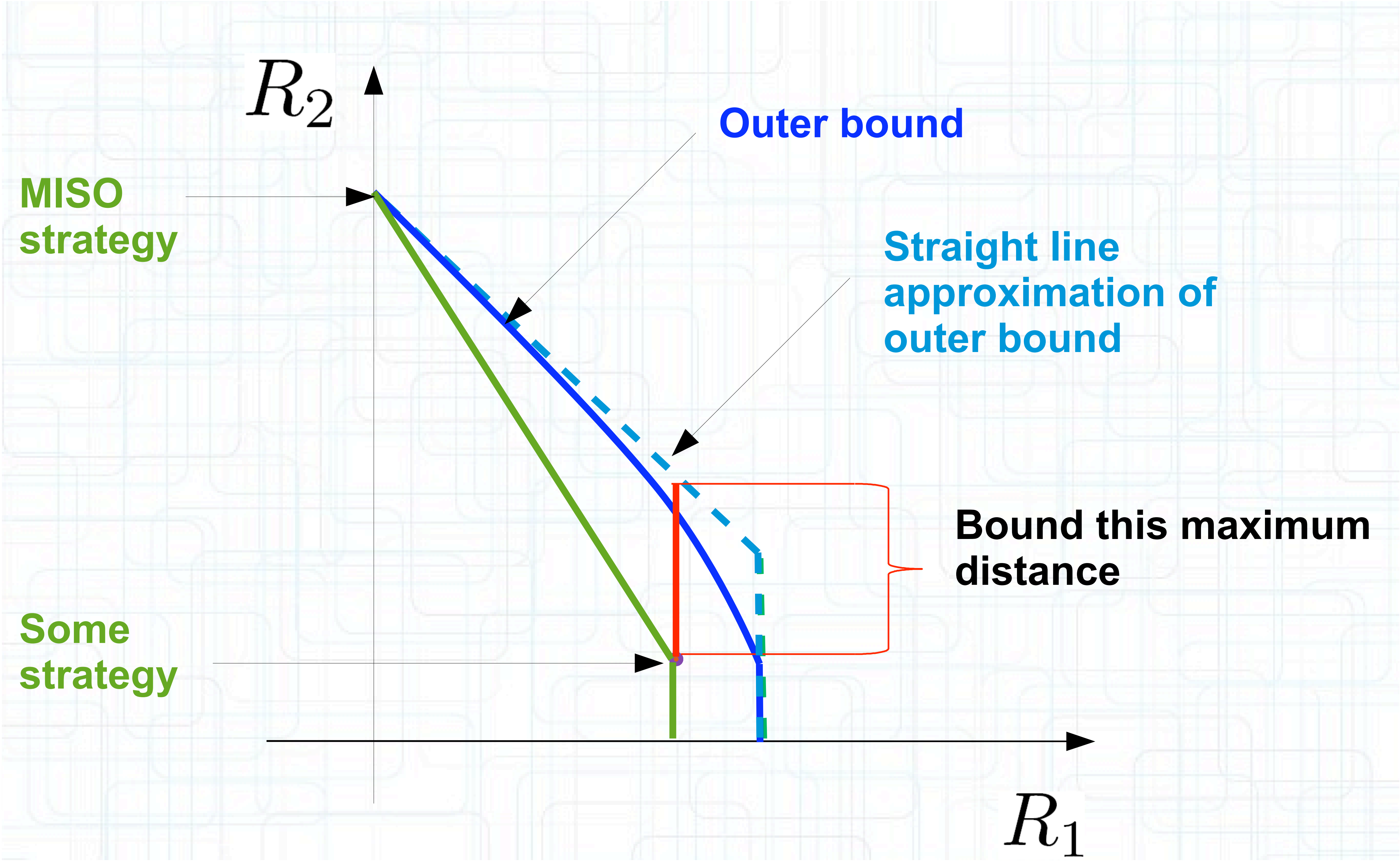




We prove a finite gap regardless of channel parameters!

Proving a finite gap





Outer bound

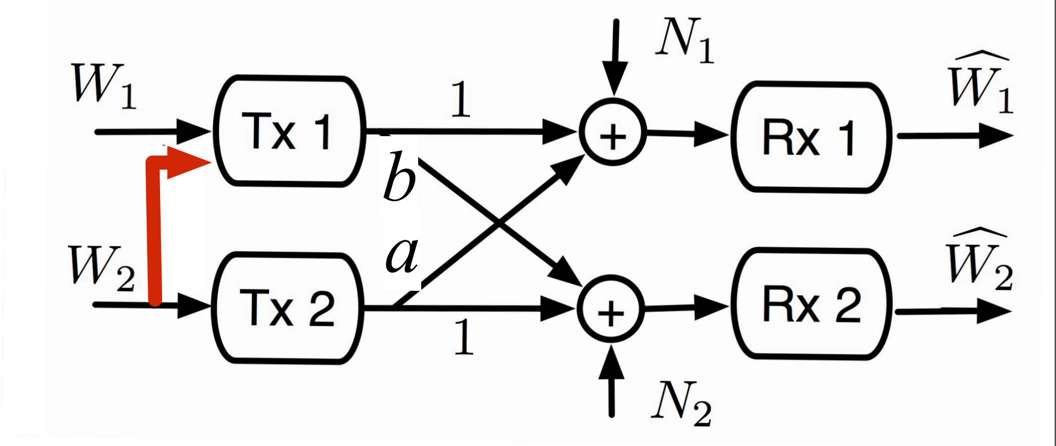
MISO strategy

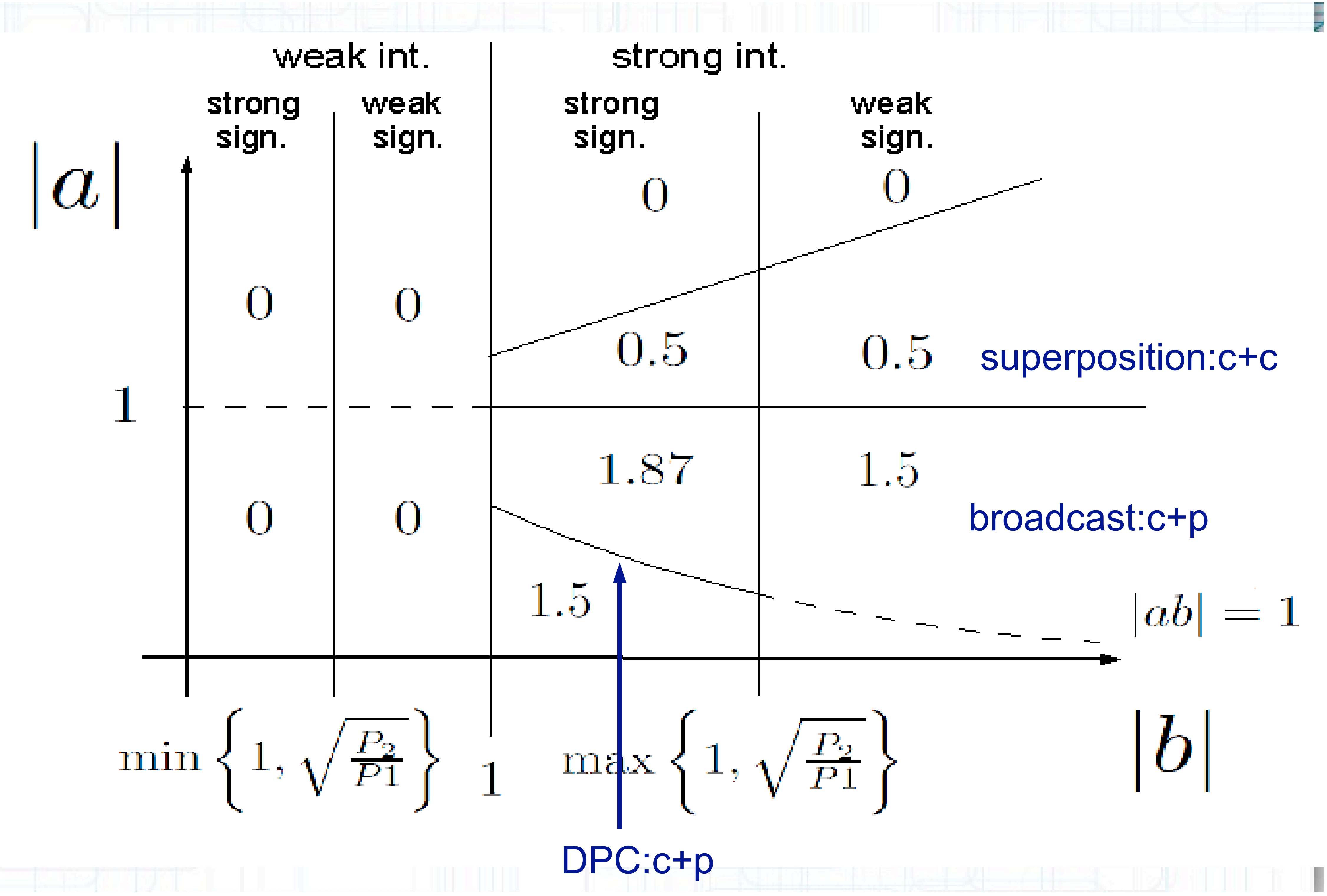
Straight line approximation of outer bound

Bound this maximum distance

Some strategy

Proving a finite gap



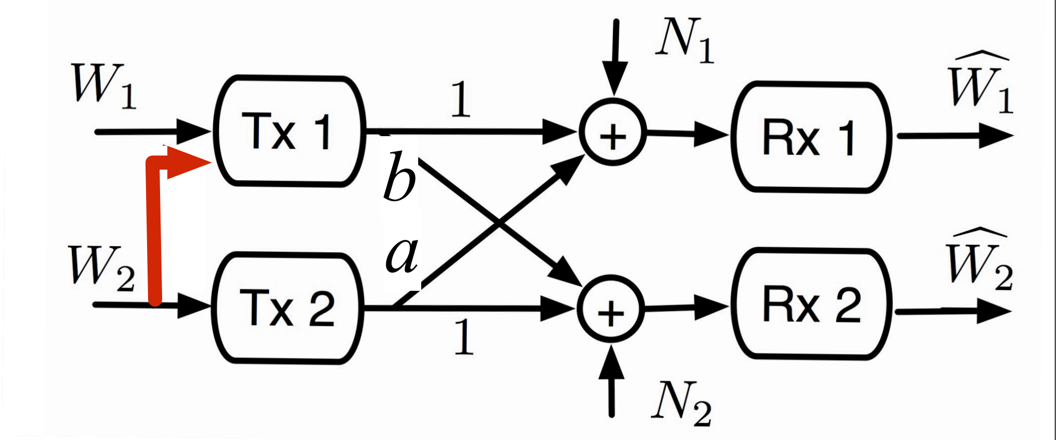


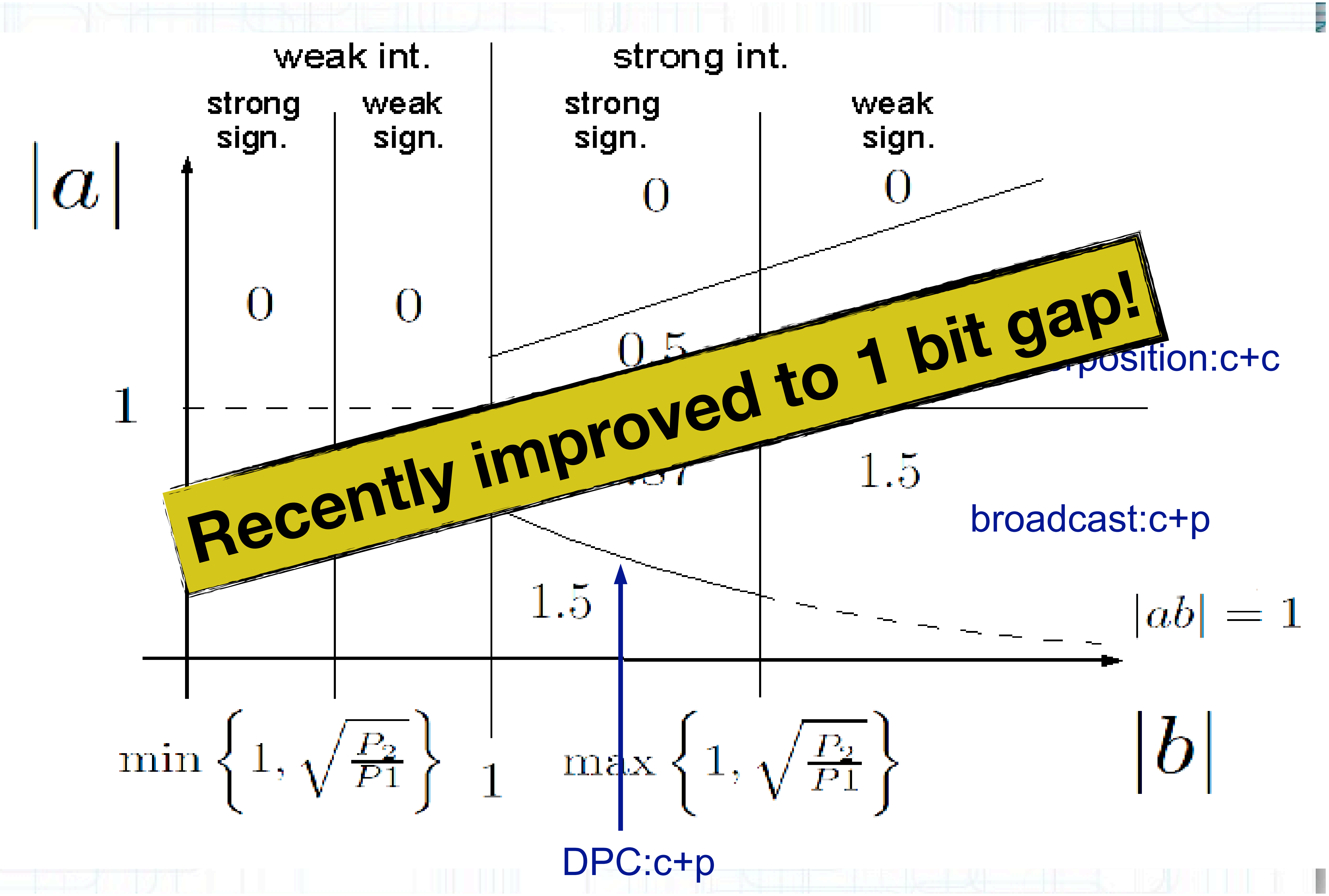
superposition:c+c

broadcast:c+p

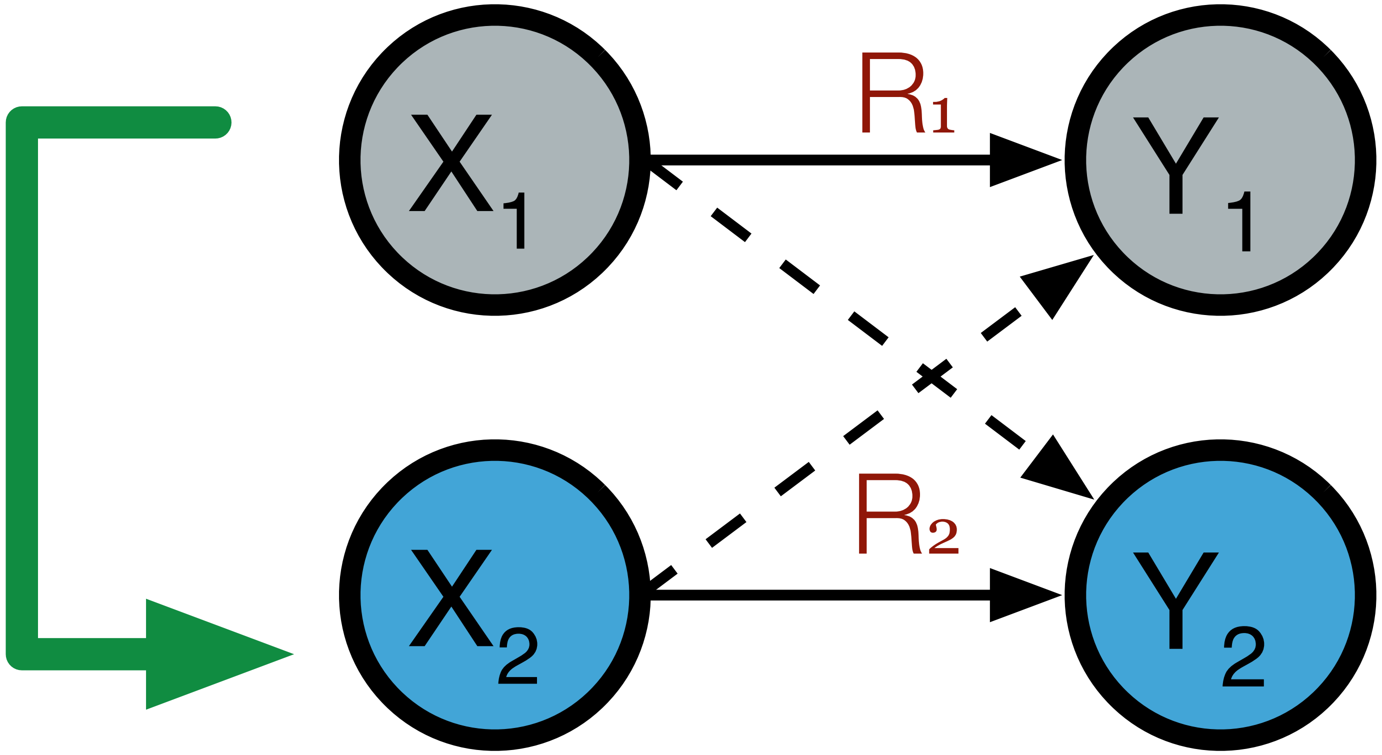
DPC:c+p

Proving a finite gap





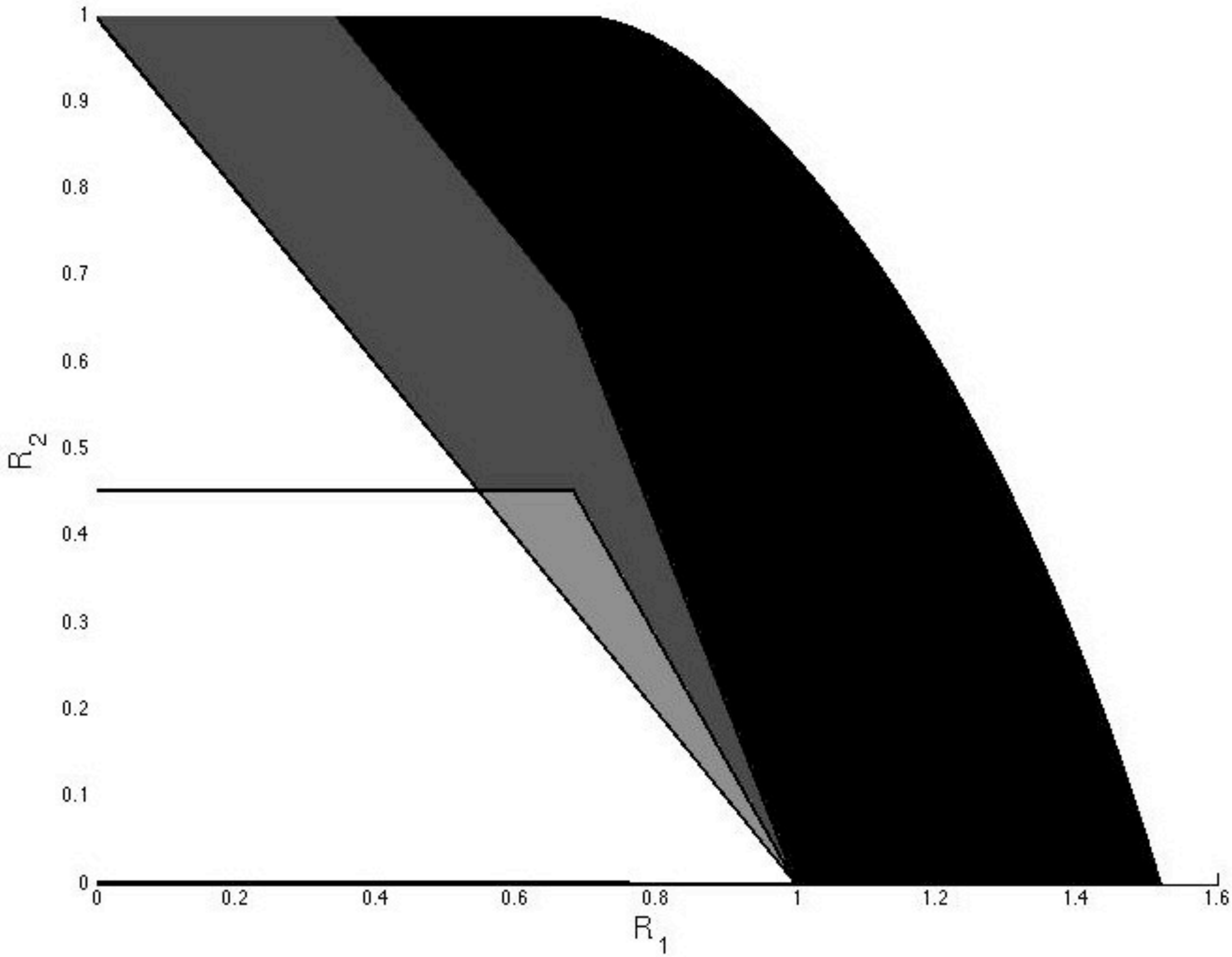
Recently improved to 1 bit gap!



“Cognitive”

Cognitive channel

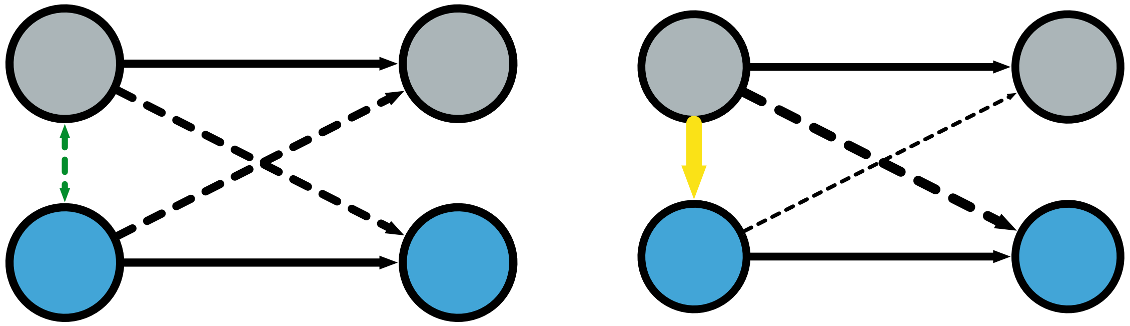
What rates (R_1, R_2) are achievable?



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1

Extensions of “cognition” in multi-user IT

- causal versus non-causal cognition



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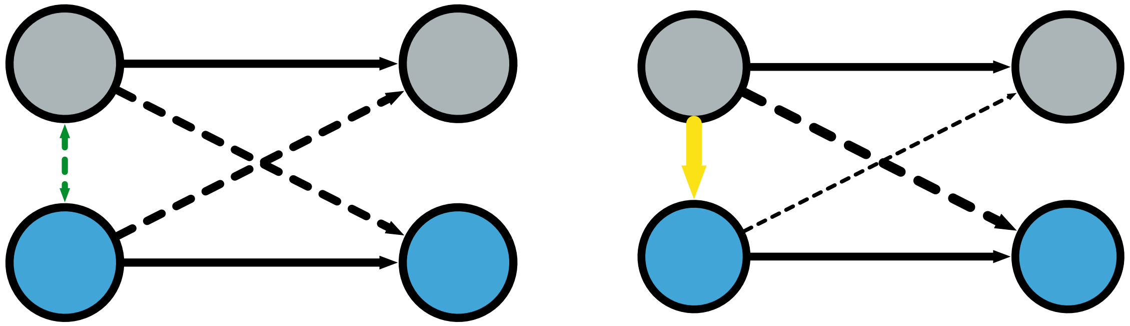
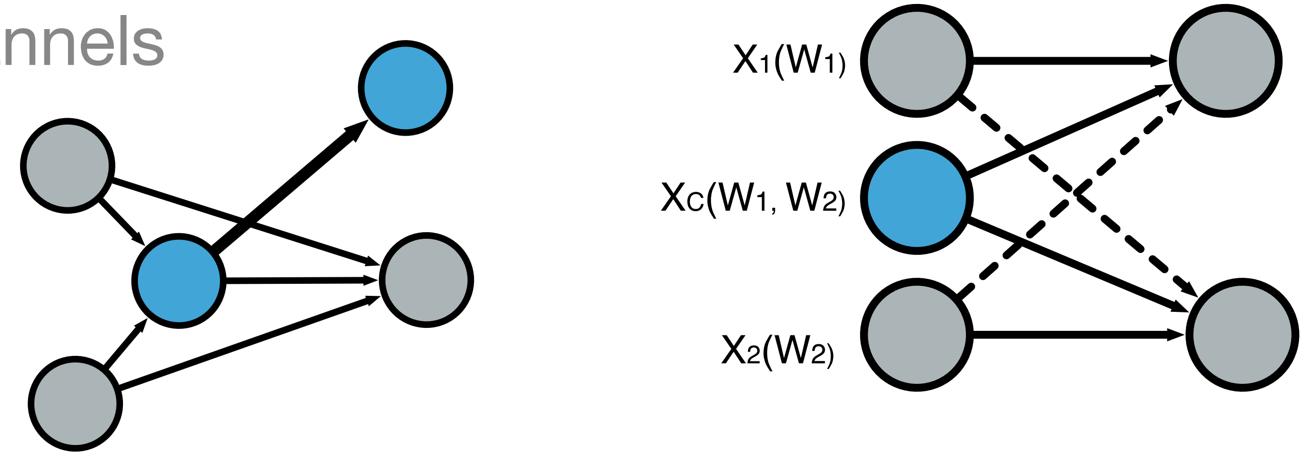


- cognitive relay: interference, relay channels



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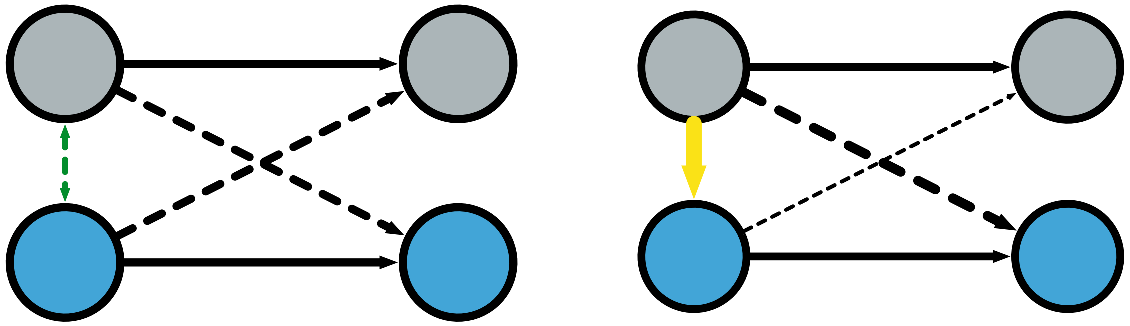

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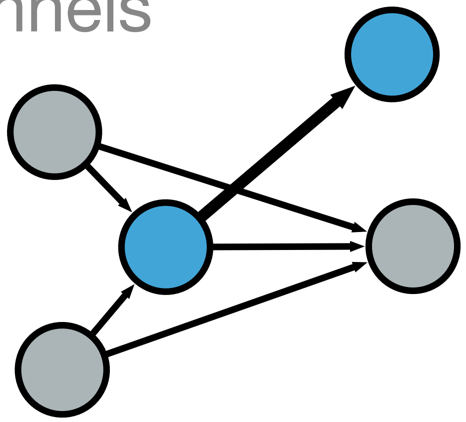
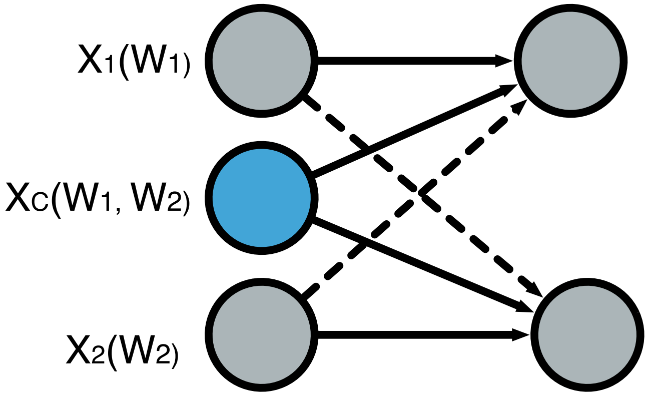
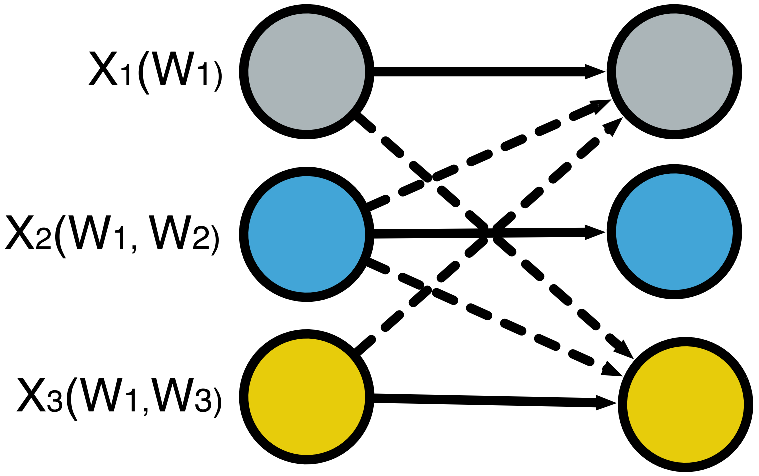
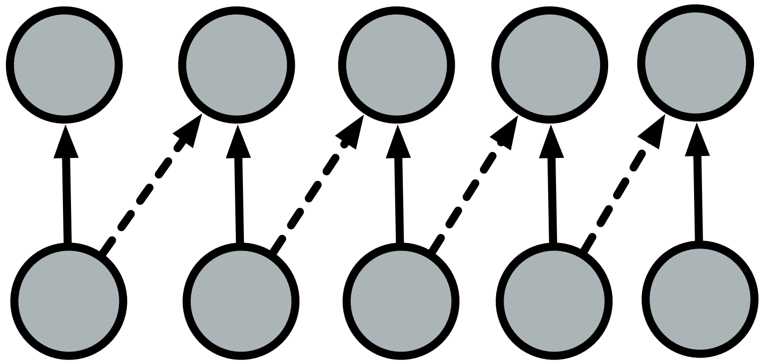
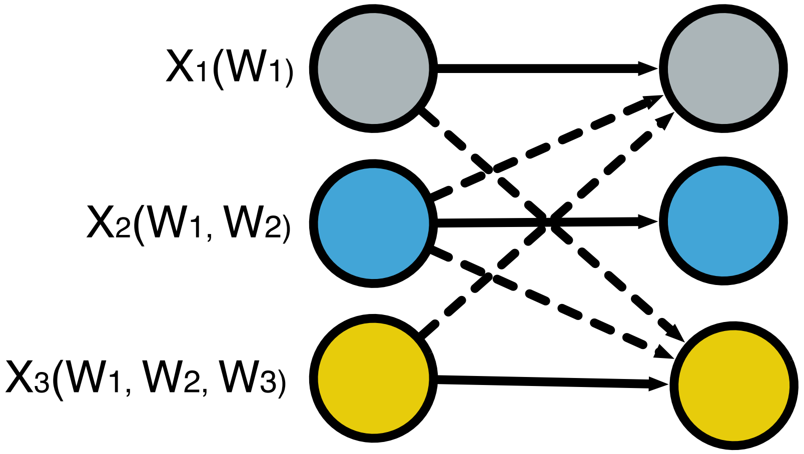




- more cognitive users, more scenarios....



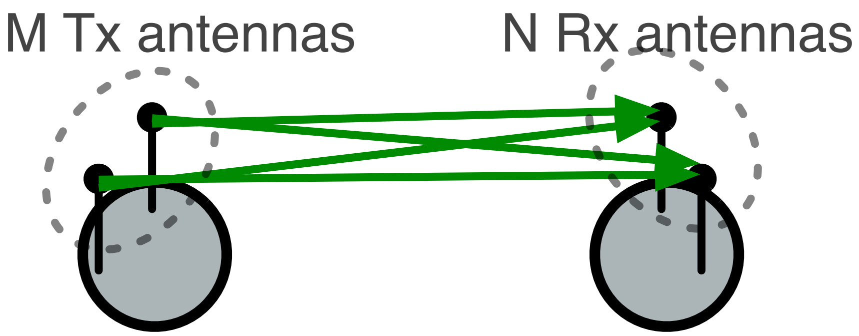




Degrees of freedom: classical

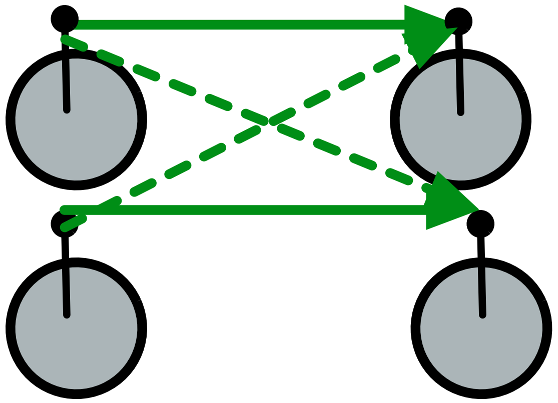
DOF = # “clean” channels in a multi-stream network

MIMO



DOF = min(M,N)=2

Interference channel

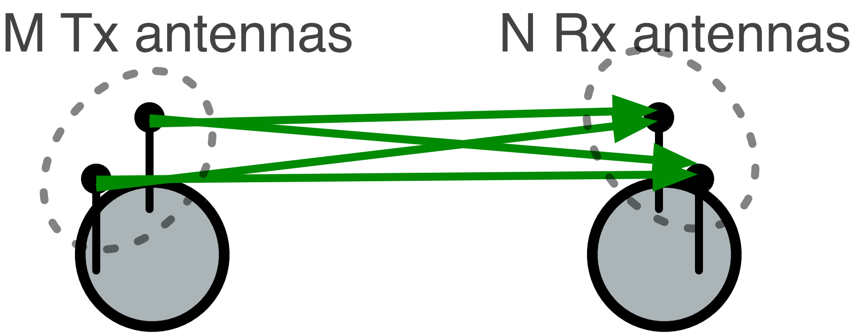


DOF = 1

Degrees of freedom: classical

DOF = # "clean" channels in a multi-stream network

MIMO



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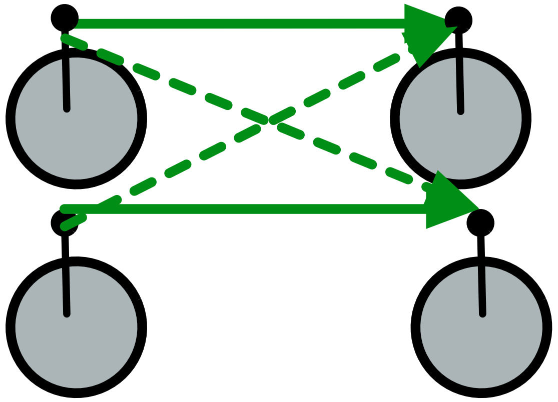
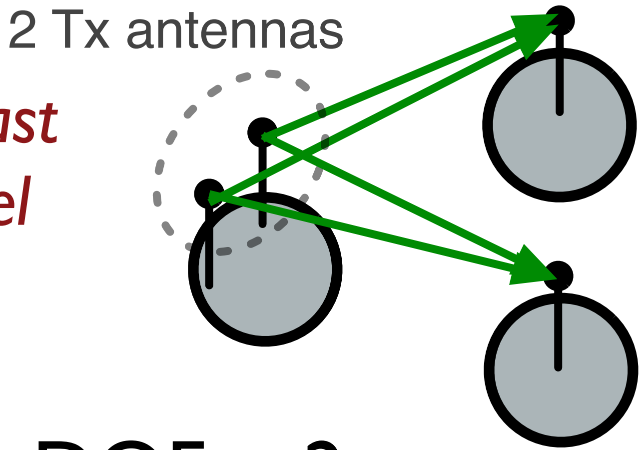
Interference channel



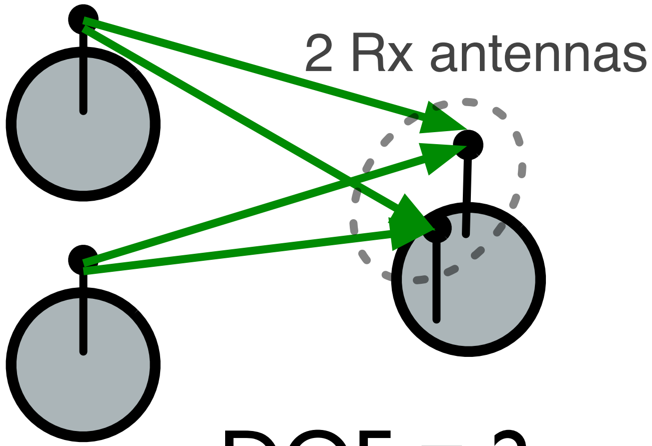
DOF = 1

Broadcast channel



DOF = 2

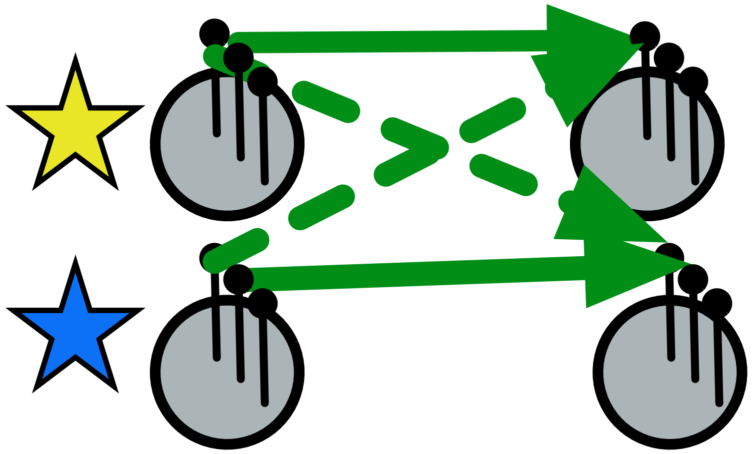
Multiple-access channel



DOF = 2

Degrees of freedom: cognitive, M antennas

MIMO interference channel

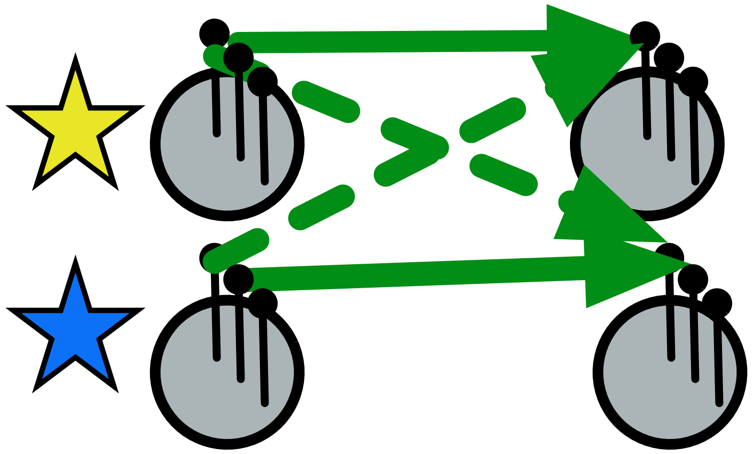


DOF = M

Syed A. Jafar, Shlomo Shamai, Degrees of Freedom Region for the MIMO X Channel , IEEE Transactions on Information Theory, Vol. 54, No. 1, Jan. 2008, Pages: 151-170.

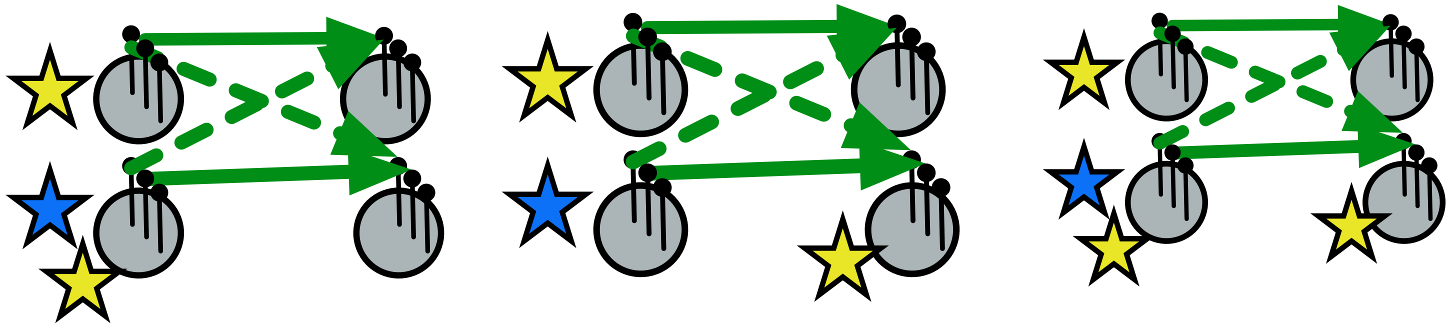
Degrees of freedom: cognitive, M antennas

MIMO interference channel



DOF = M

MIMO cognitive channel, cases a,b,c



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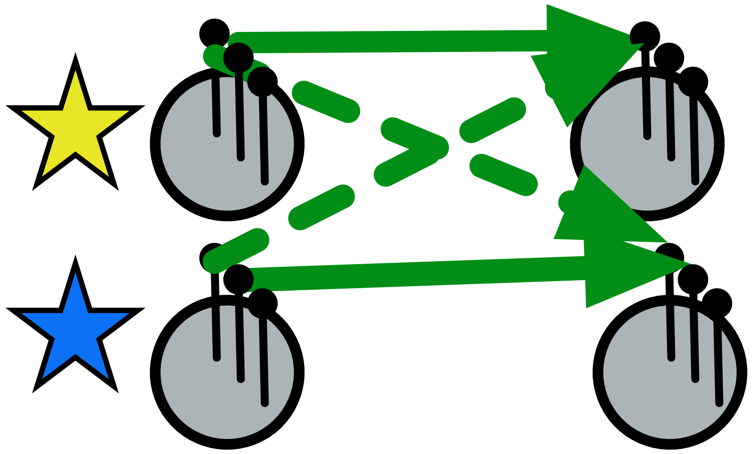

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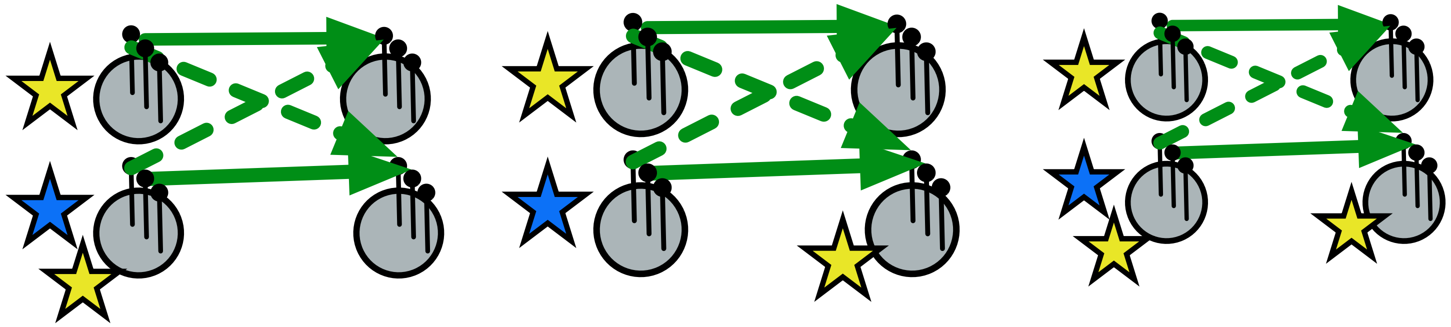
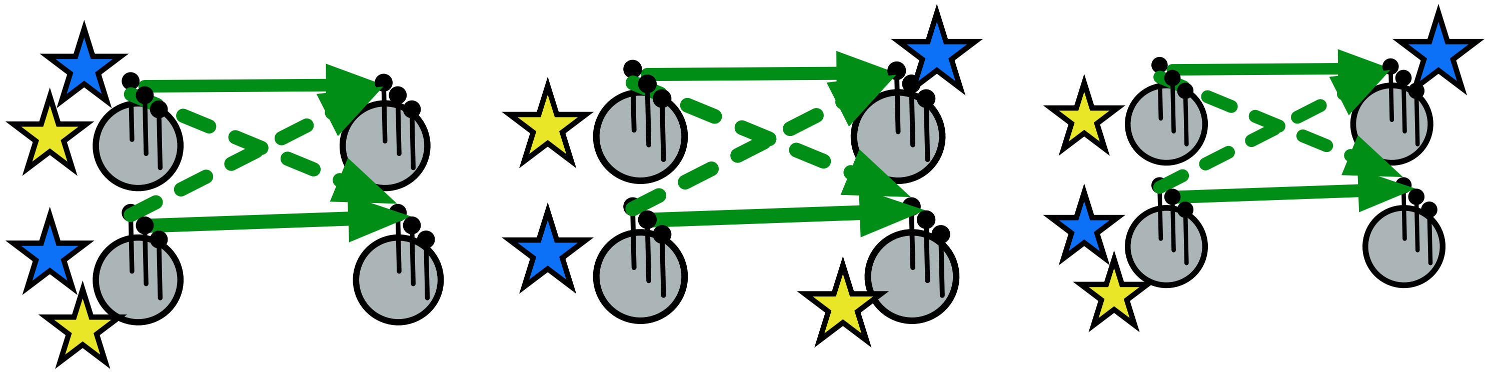


DOF = M

MIMO cognitive channel, cases d,e,f

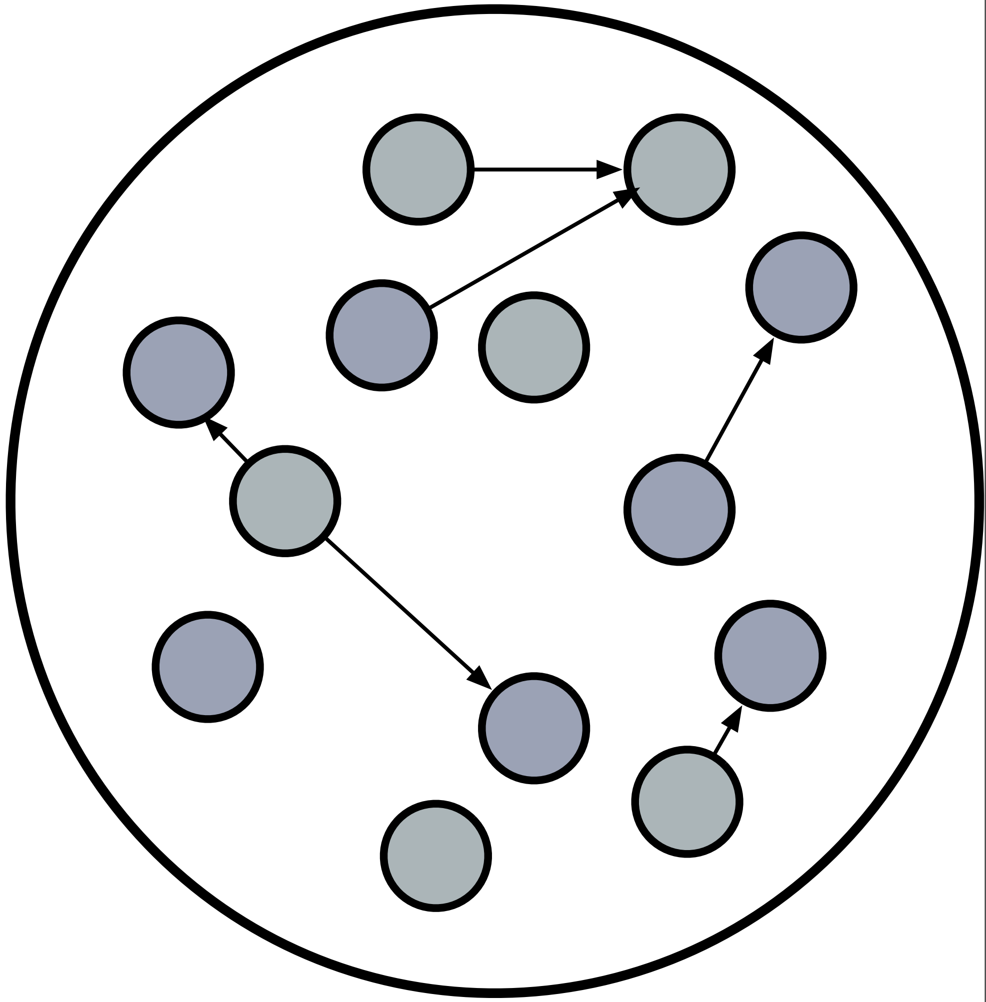


DOF = 2M

Syed A. Jafar, Shlomo Shamai, Degrees of Freedom Region for the MIMO X Channel , IEEE Transactions on Information Theory, Vol. 54, No. 1, Jan. 2008, Pages: 151-170.

Scaling laws

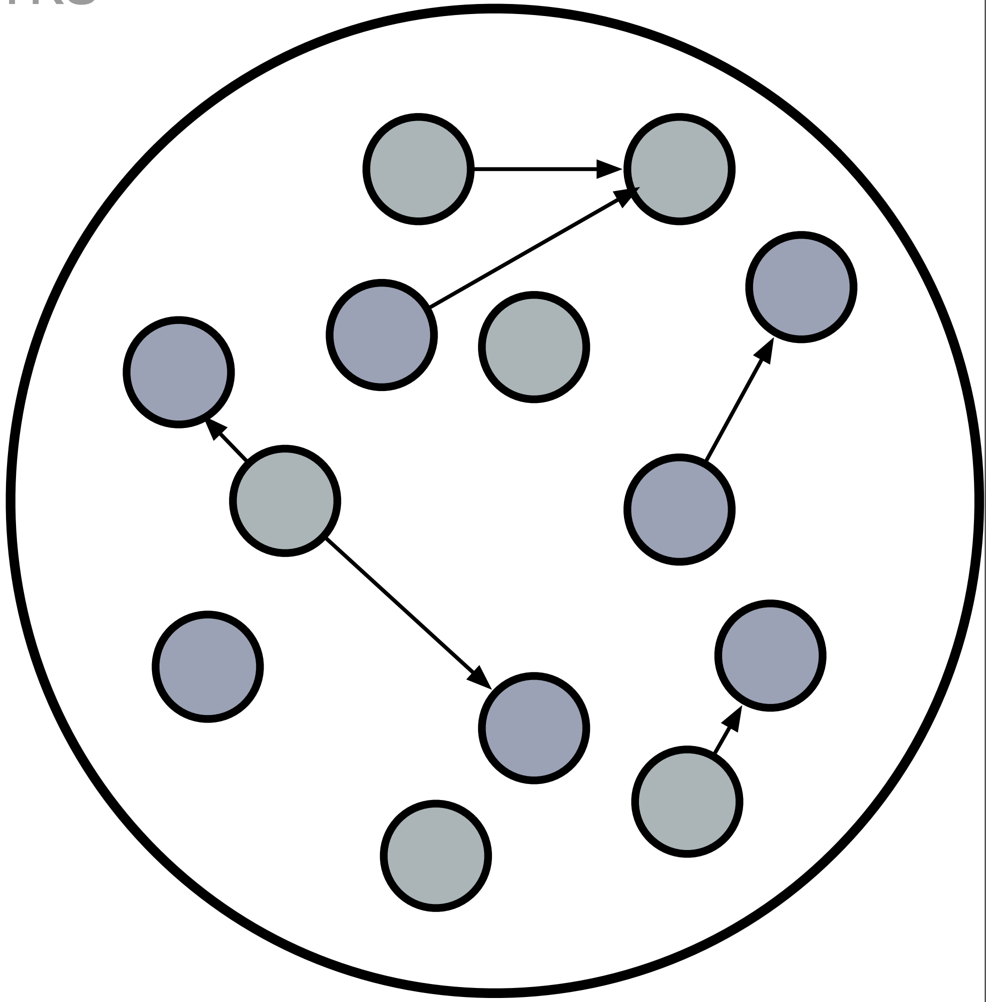
nodes n → ∞



Scaling laws

nodes n → ∞

- [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  - per-node throughput ~ $O(1/\sqrt{n}\log(n))$
  - Degradation is due to multi-hop and interference between nodes



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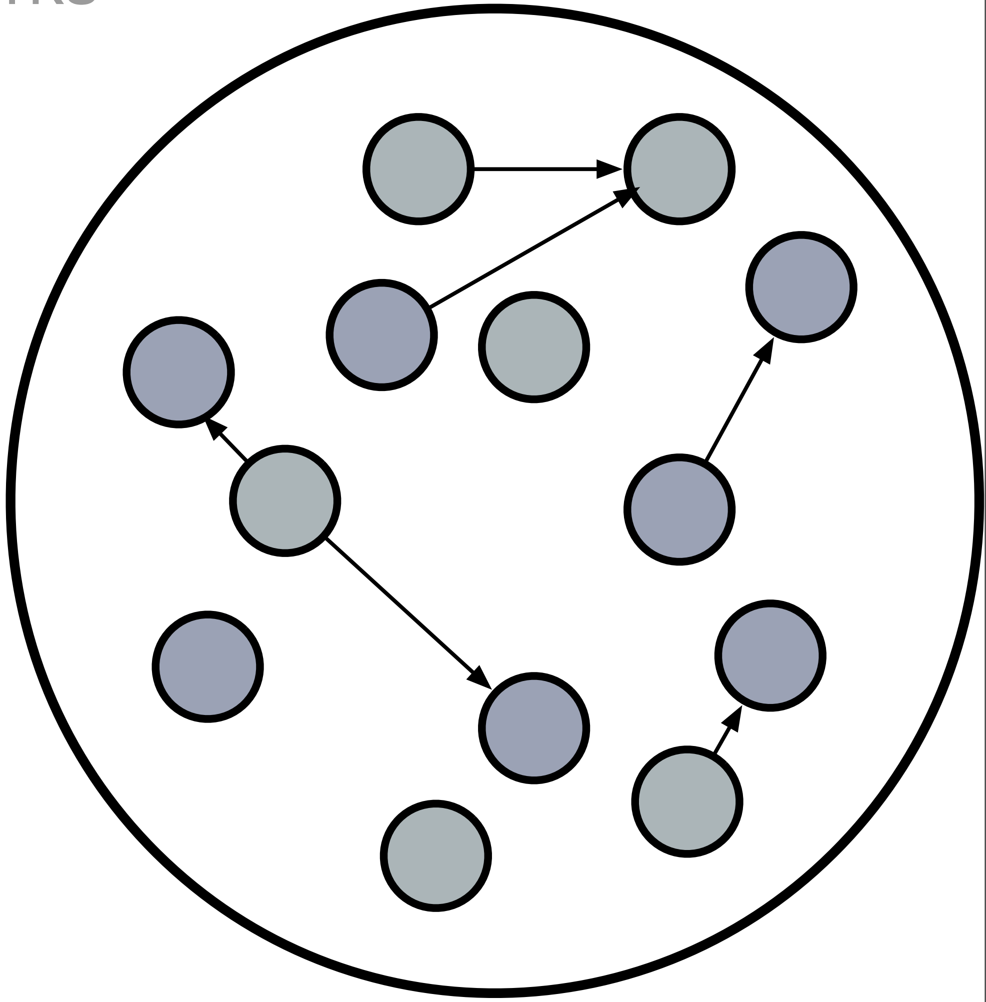

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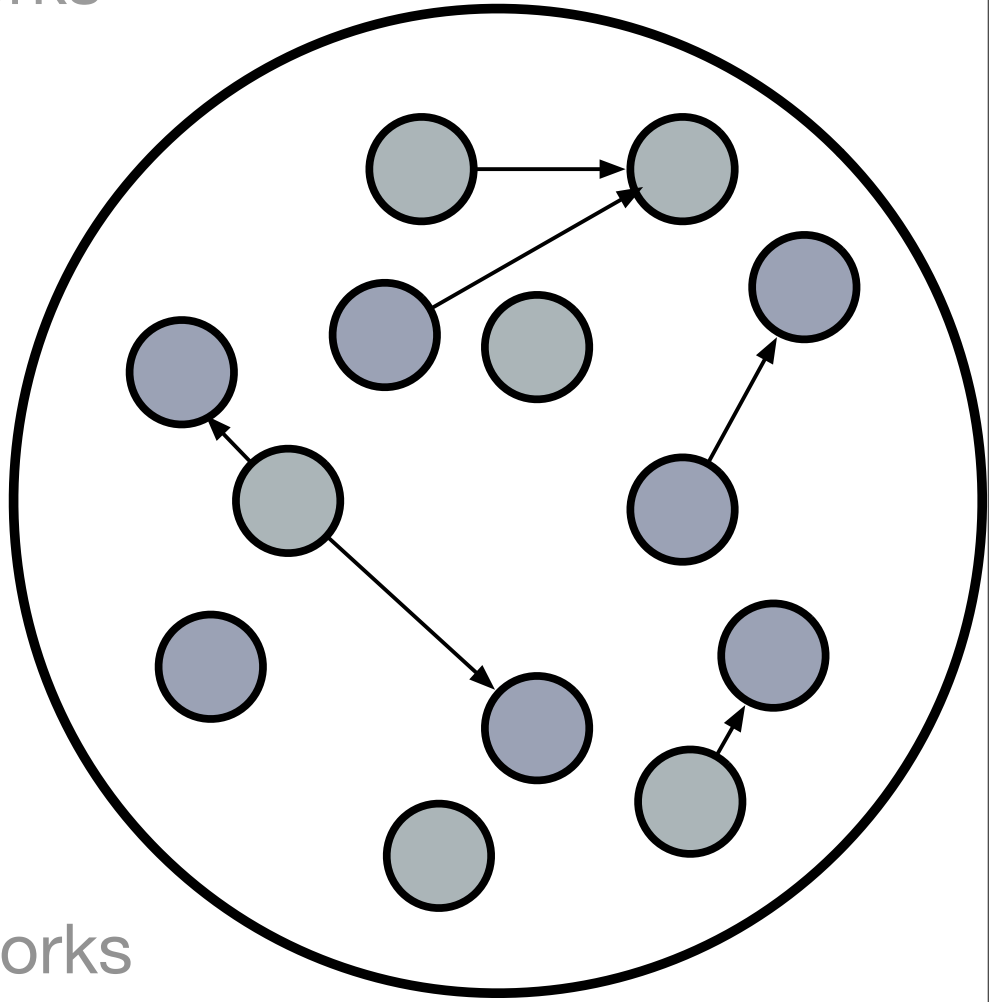

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  - nodes may cooperate as in a MIMO system
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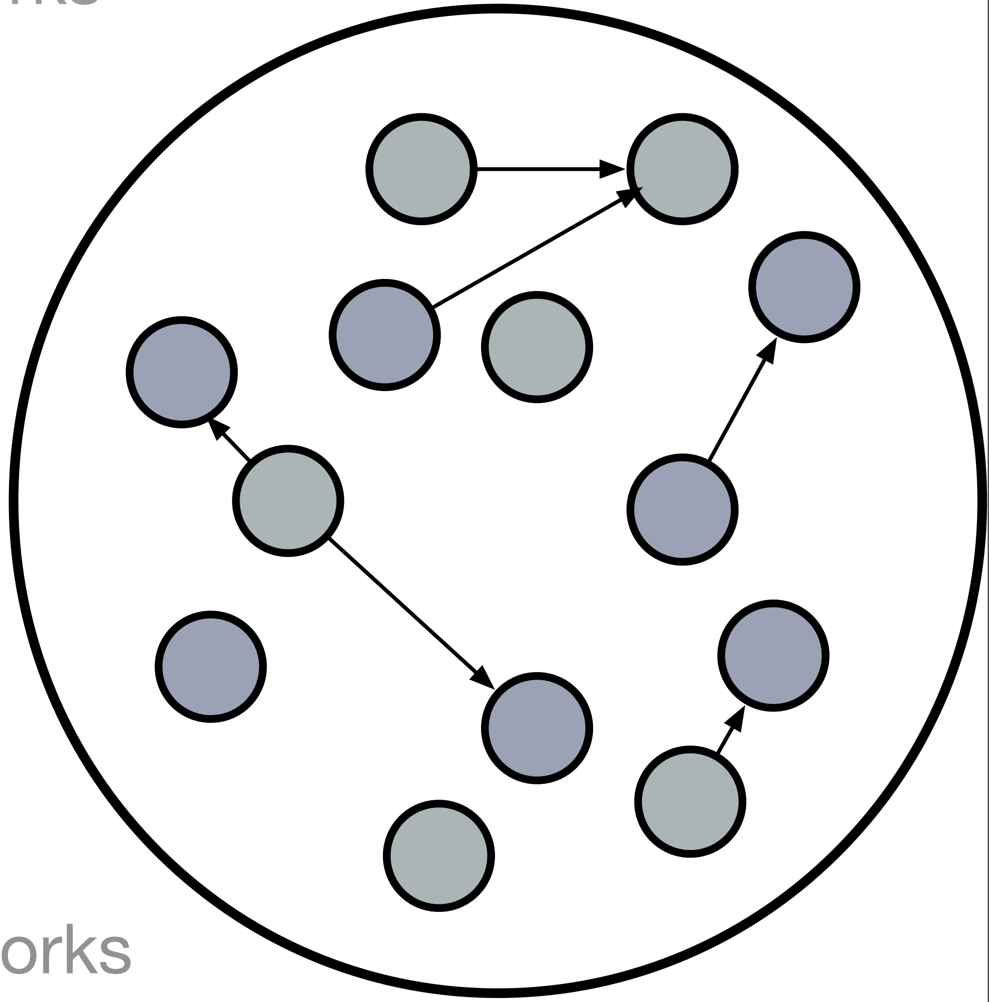

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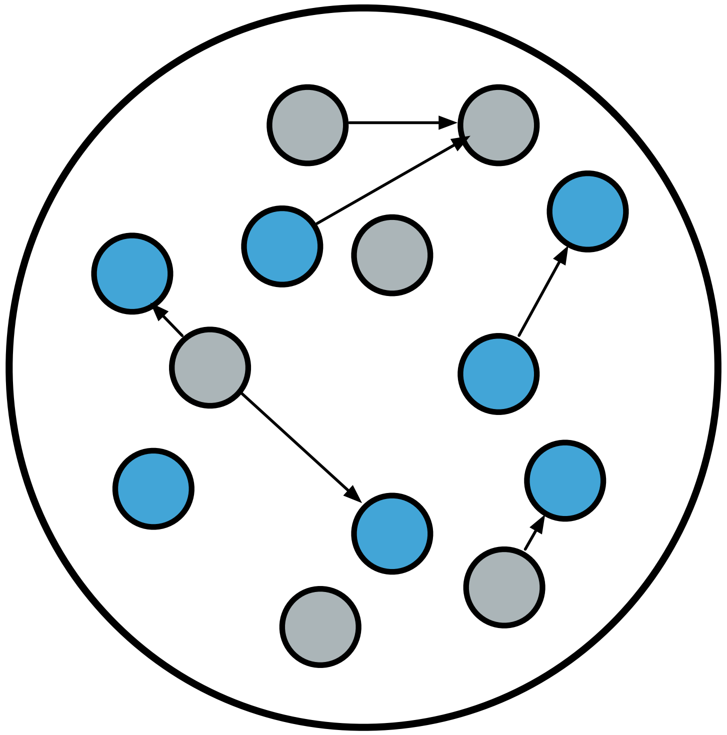
- Many many more...

Scaling laws: with cognition

- What we guarantee:

Primary nodes act as if cognitive network does not exist

Primary nodes achieve same scaling law as if cognitive network does not exist



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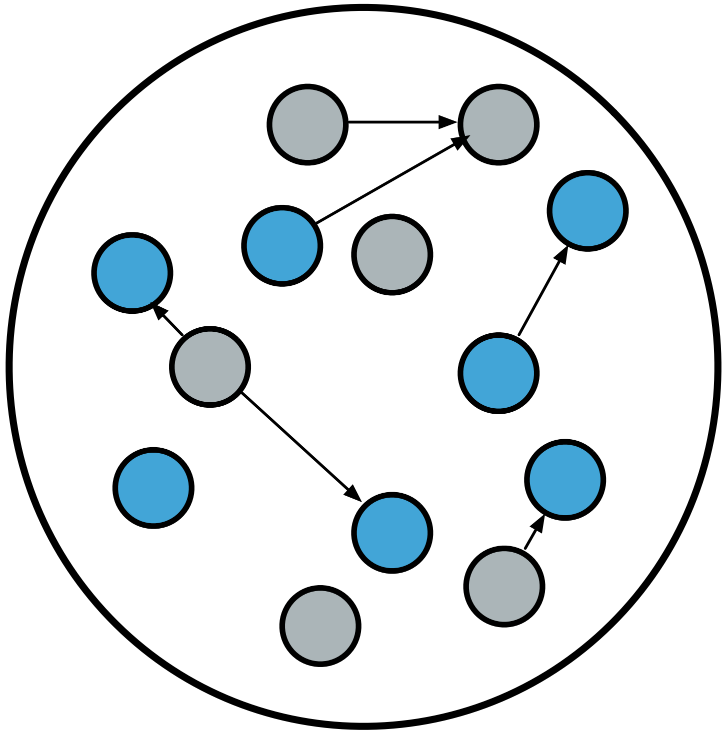
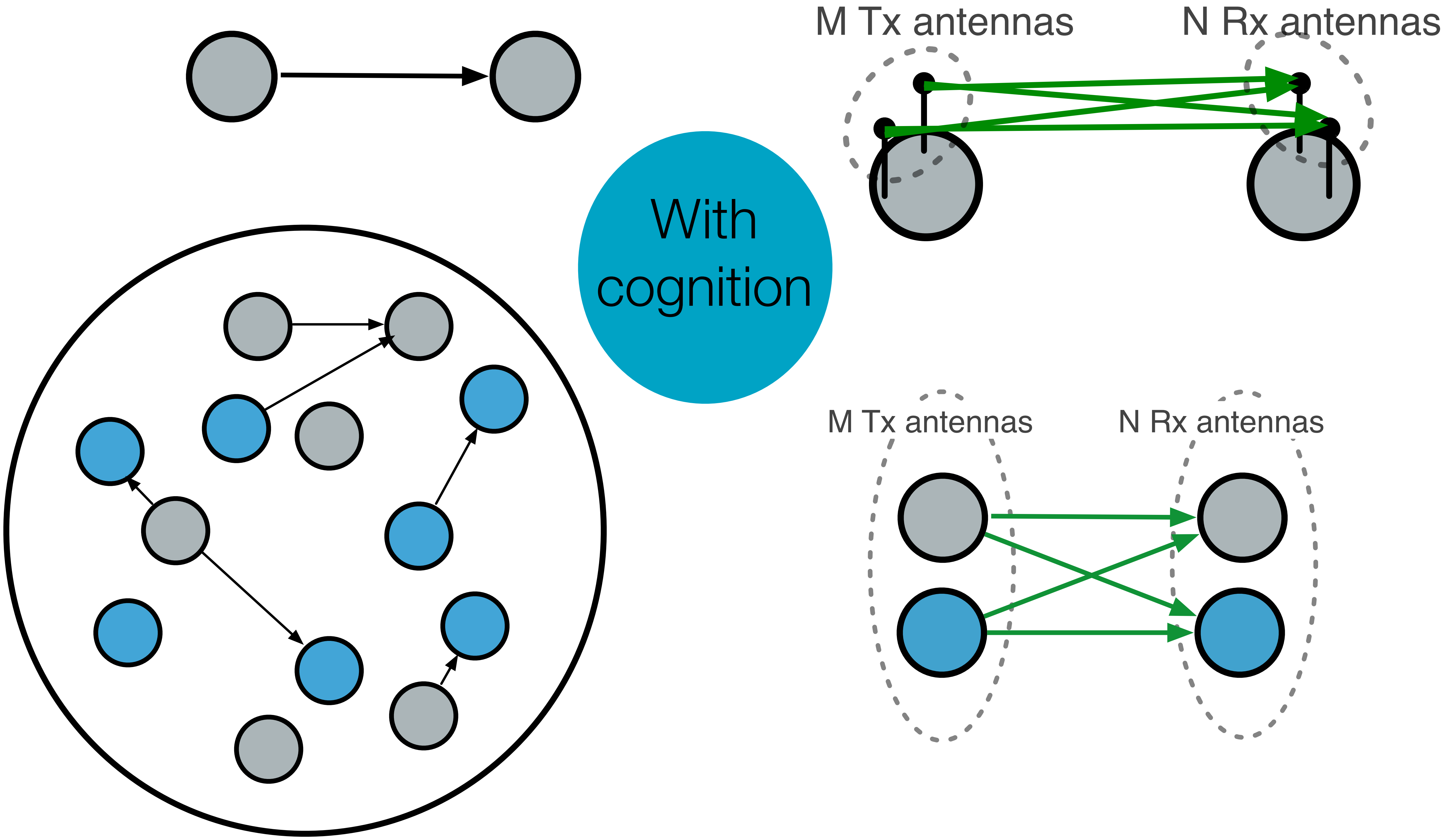


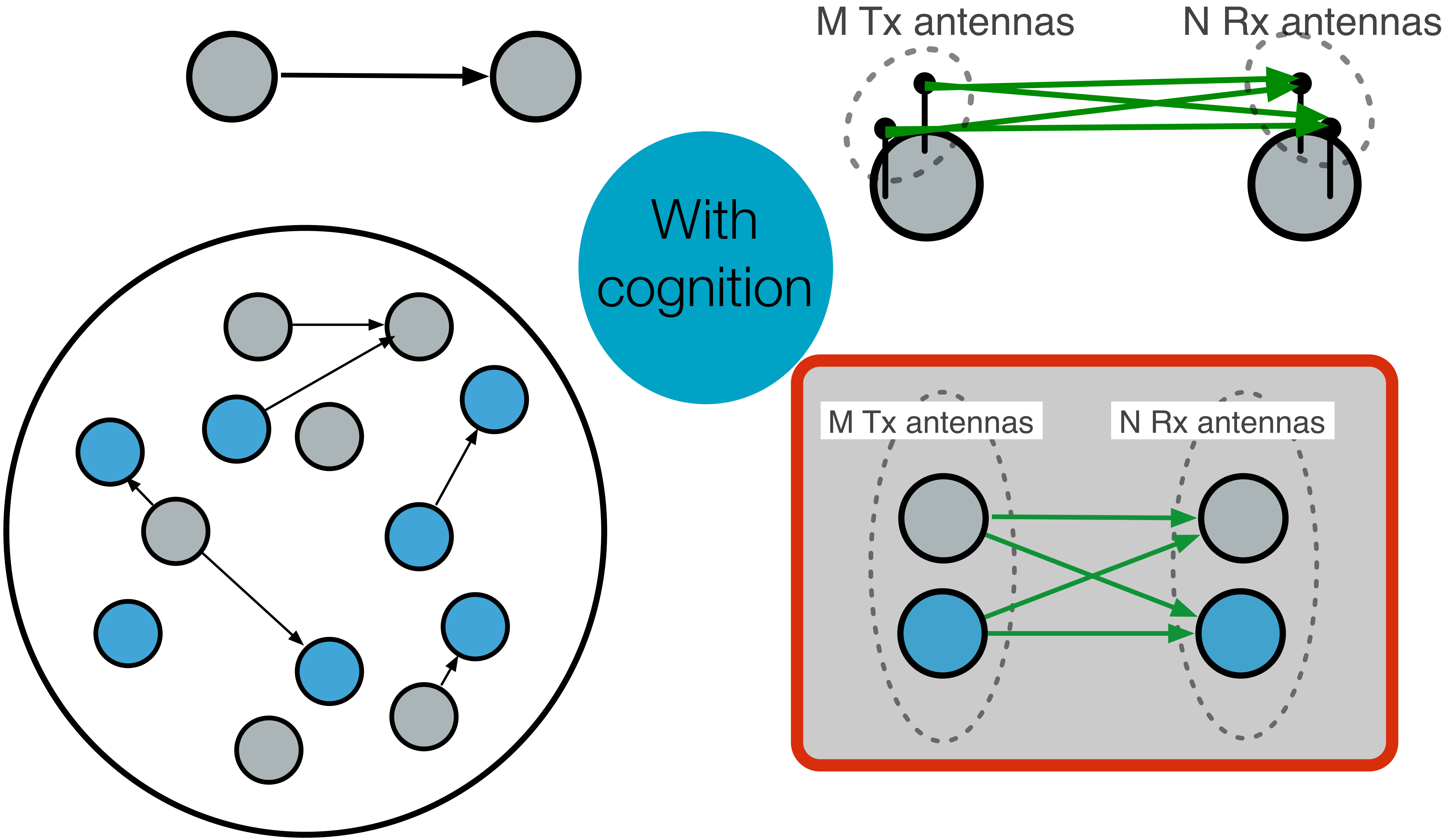
- What we prove:

$$T_p(n) = \Theta\left(\sqrt{\frac{1}{n\log n}}\right), \quad T_s(m) = \Theta\left(\sqrt{\frac{1}{m\log m}}\right)$$

Efficient, reliable communications



Efficient, reliable communications



M Tx antennas

N Rx antennas

With cognition

M Tx antennas

N Rx antennas

Thank you

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5/19/2010