Fundamental Limits of Cognitive Networks: Tutorial and Tour

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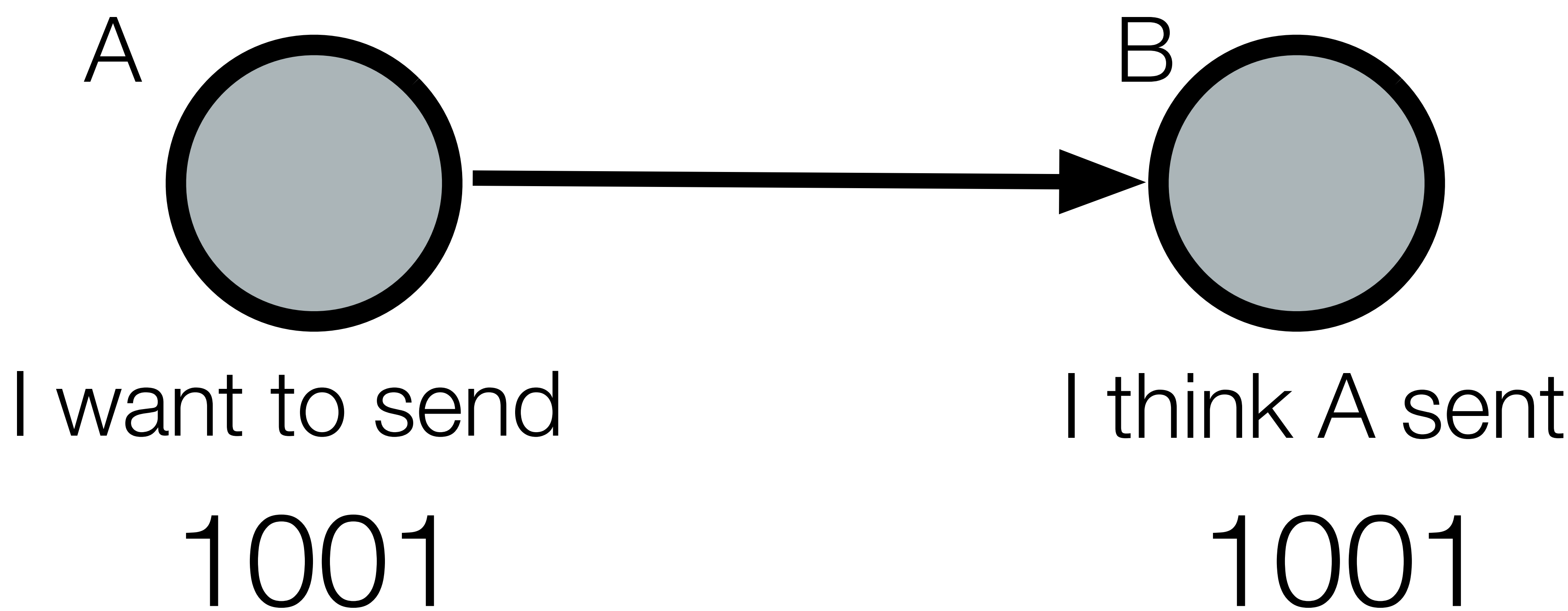
Motorola
7/10/2009

What is communication?

"The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point."

-C.E. Shannon, 1948



Big Open Questions in the 40’s

- is there a general **methodology** for designing communication systems?

- can we communicate reliably in **noise**?

- how **fast** can we communicate?

Claude E. Shannon

A Mathematical Theory of Communication. **Bell System Technical Journal, *27, 379–423 & 623–656, 1948.***

- Introduced a new field: information theory

How fast can we communicate?

What is information?

RANDOMNESS

How much can we compress information?

What is communication?

Information theory’s claims to fame

Source coding

zip

- Source = random variable
- Ultimate data compression limit is the source’s entropy H

jpg

mp3

Channel coding

Turbo-codes

- Channel = conditional distributions
- Ultimate transmission rate is the channel capacity C

GGGOOOOOOODDD

FADING CHANNEL

Reliable communication possible ↔ H<C

Where are we now?

Forney, G.D. and Costello, D.J., ``Channel Coding: The Road to Channel Capacity,'' Proceedings of the IEEE, Volume 95, Issue 6, pp.1150-1177, June 2007.

Claude Shannon — *Born on the planet Earth (Sol III) in the year 1916 A.D. Generally regarded as the father of the Information Age, he formulated the notion of channel capacity in 1948 A.D. Within several decades, mathematicians and engineers had devised practical ways to communicate reliably at data rates within 1% of the Shannon limit . . .*

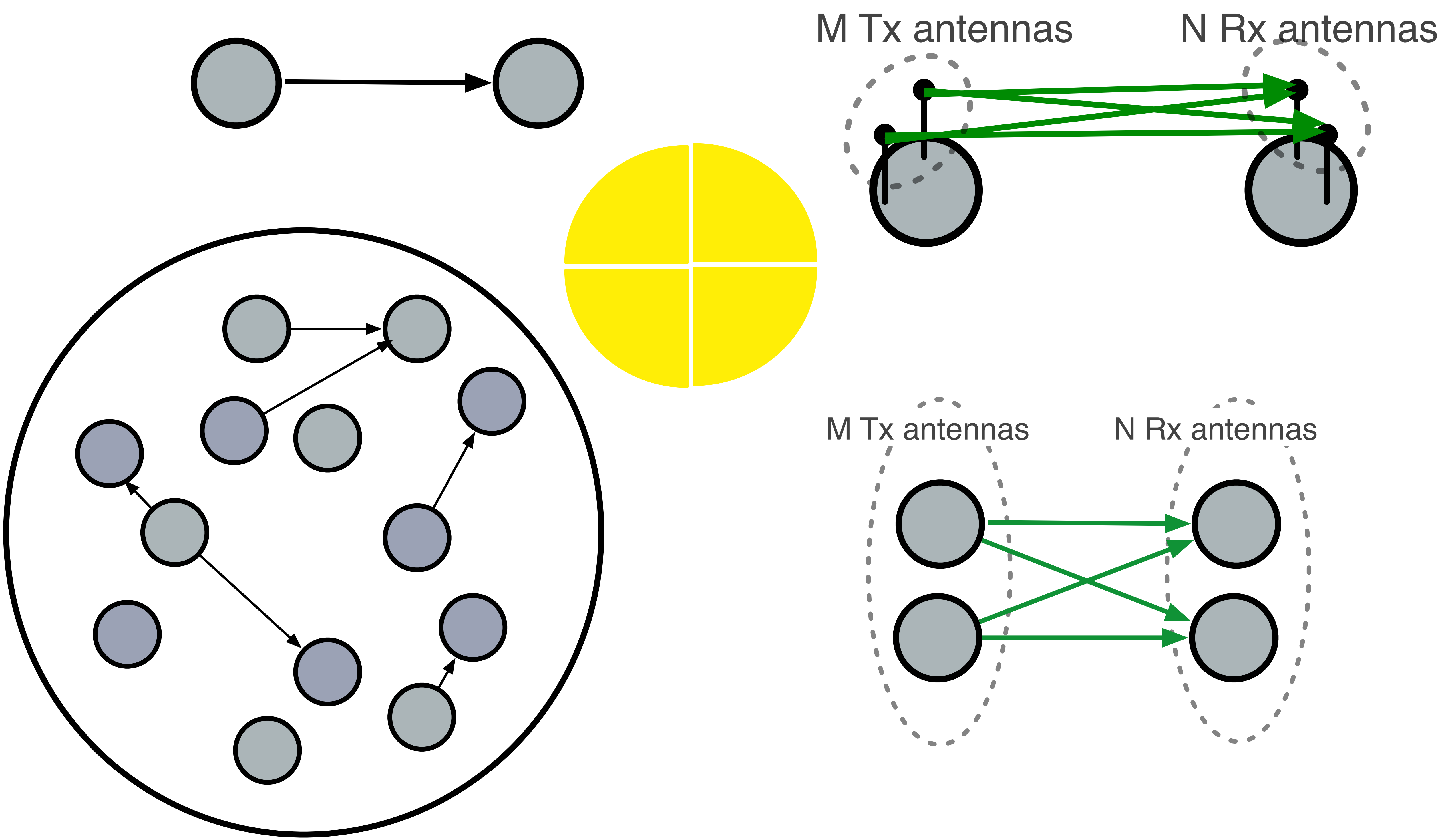
Encyclopedia Galactica, 166th ed.

- algebraic codes
- convolutional codes
- iterative codes (LDPC, turbo)

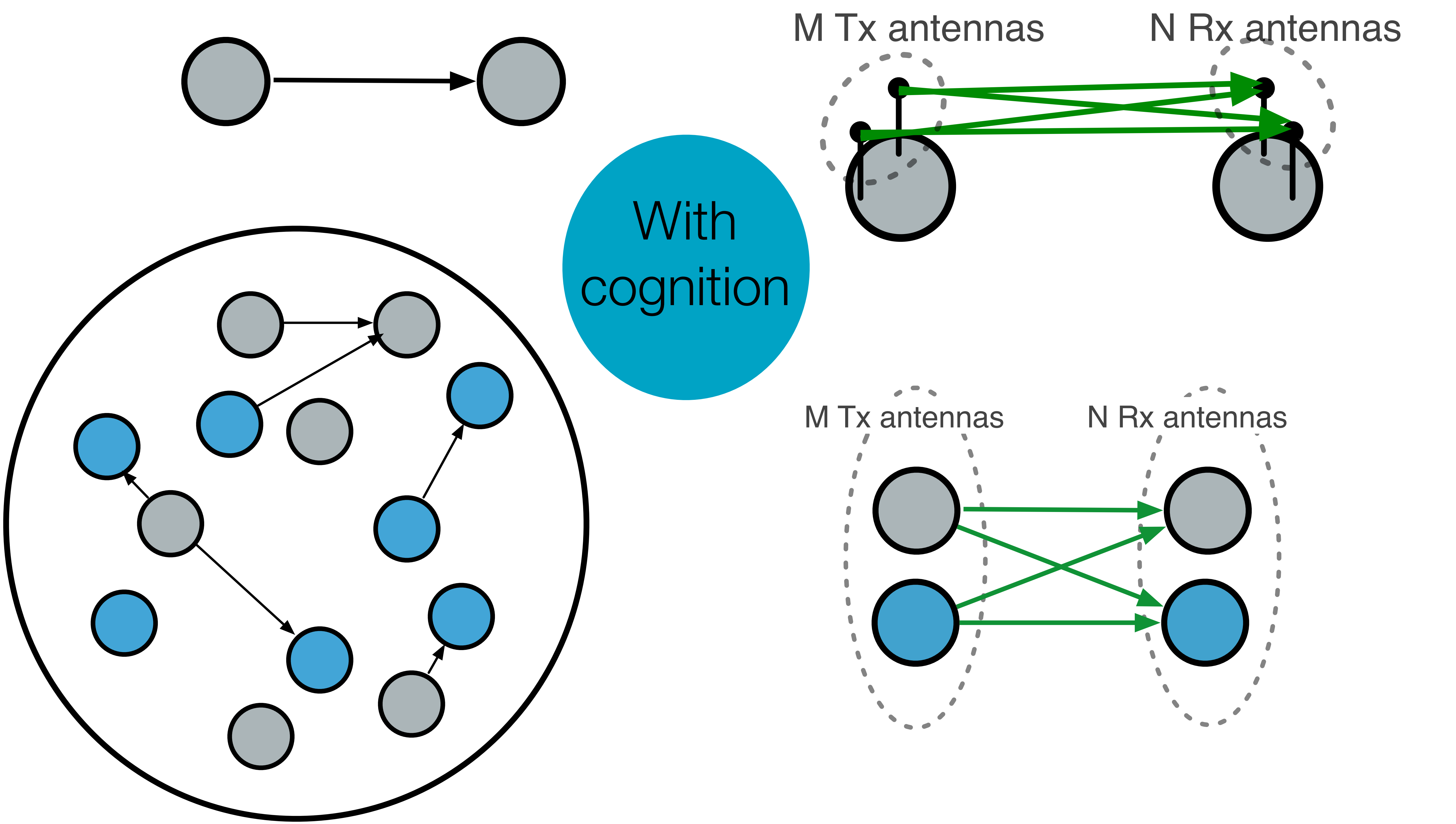
-Robert J. McEliece, Allerton 2000

So now what?

Efficient, reliable communications



Efficient, reliable communications

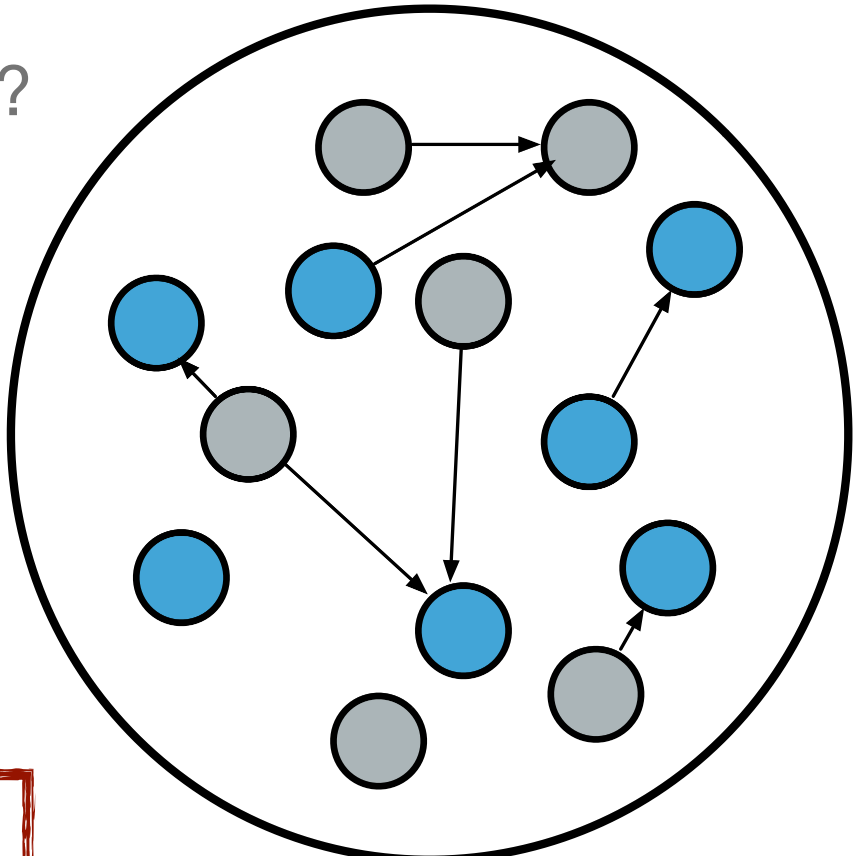


Wireless networks with cognition

- We DON'T know the network capacity
- Ad-hoc solutions, are we going in the right direction?

multi-hop *network coding* *interference*

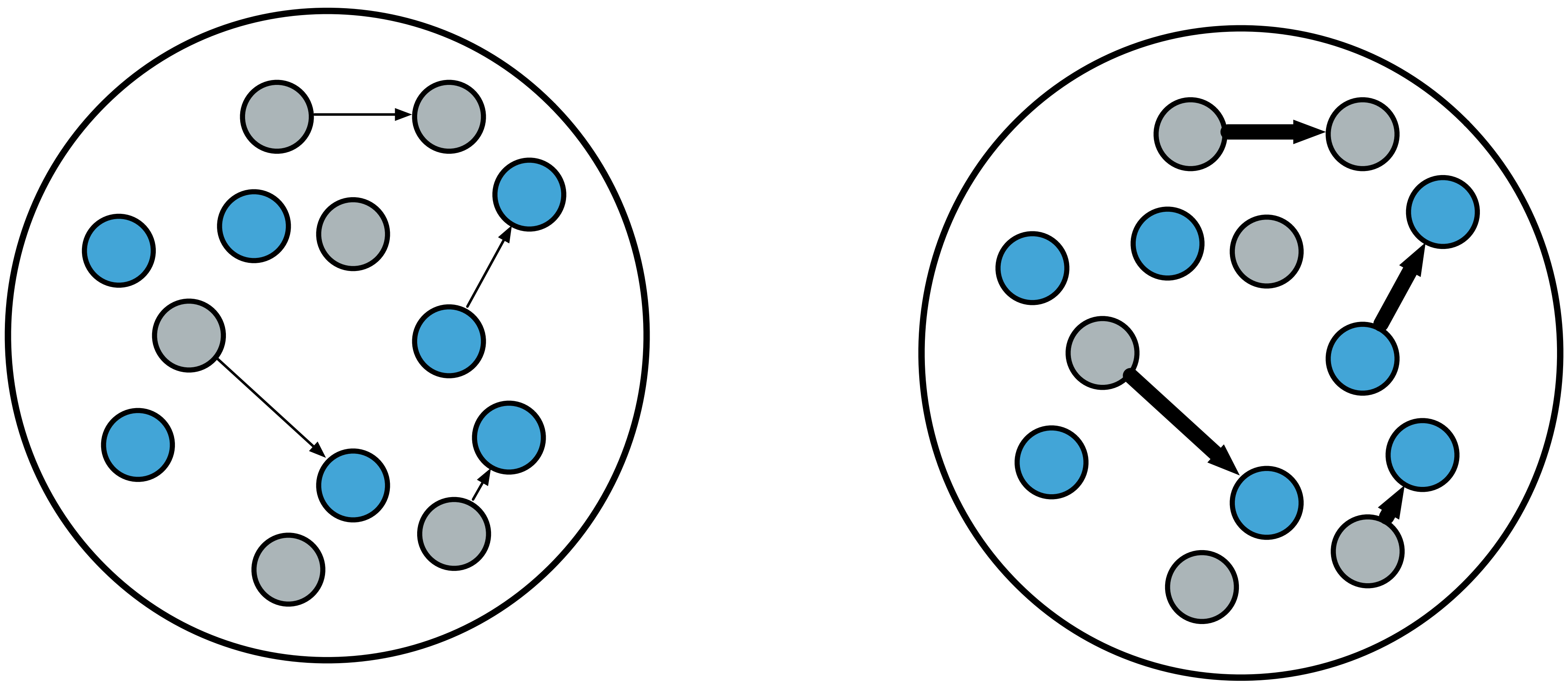
orthogonalize *relaying*



Network/multi-user information theory is the next frontier

Traditional approaches

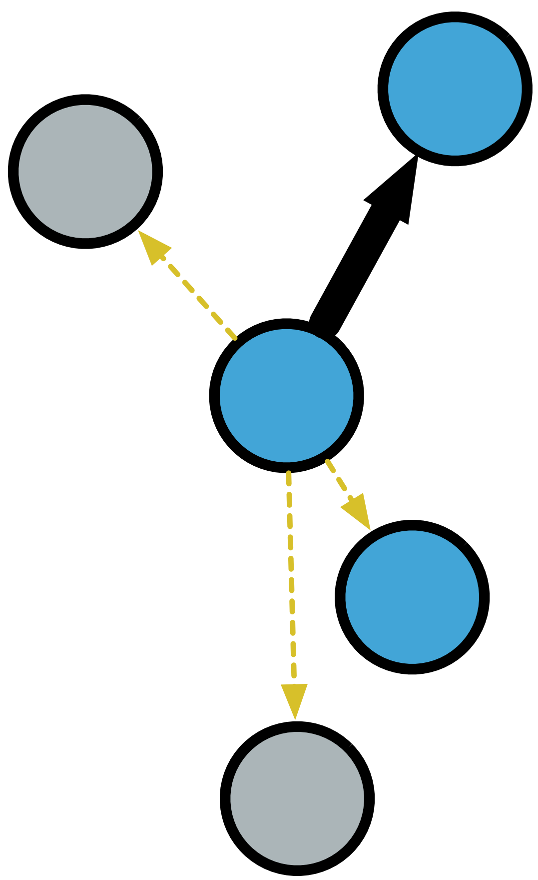
- Network (cellular, WiFi) = bunch of point-to-point links



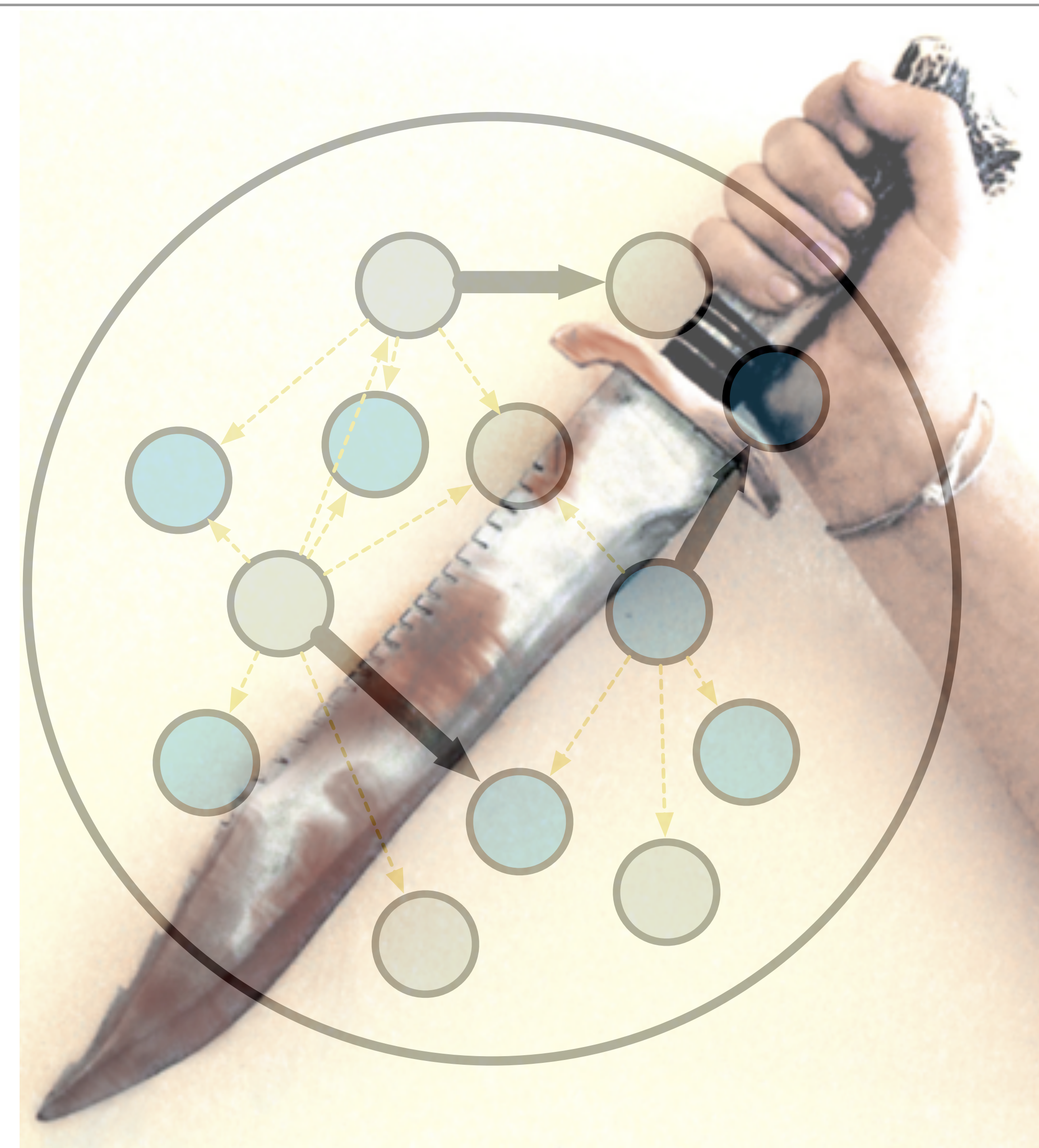
- Increase capacity?

Interference: traditional view

- Broadcast nature of wireless channels

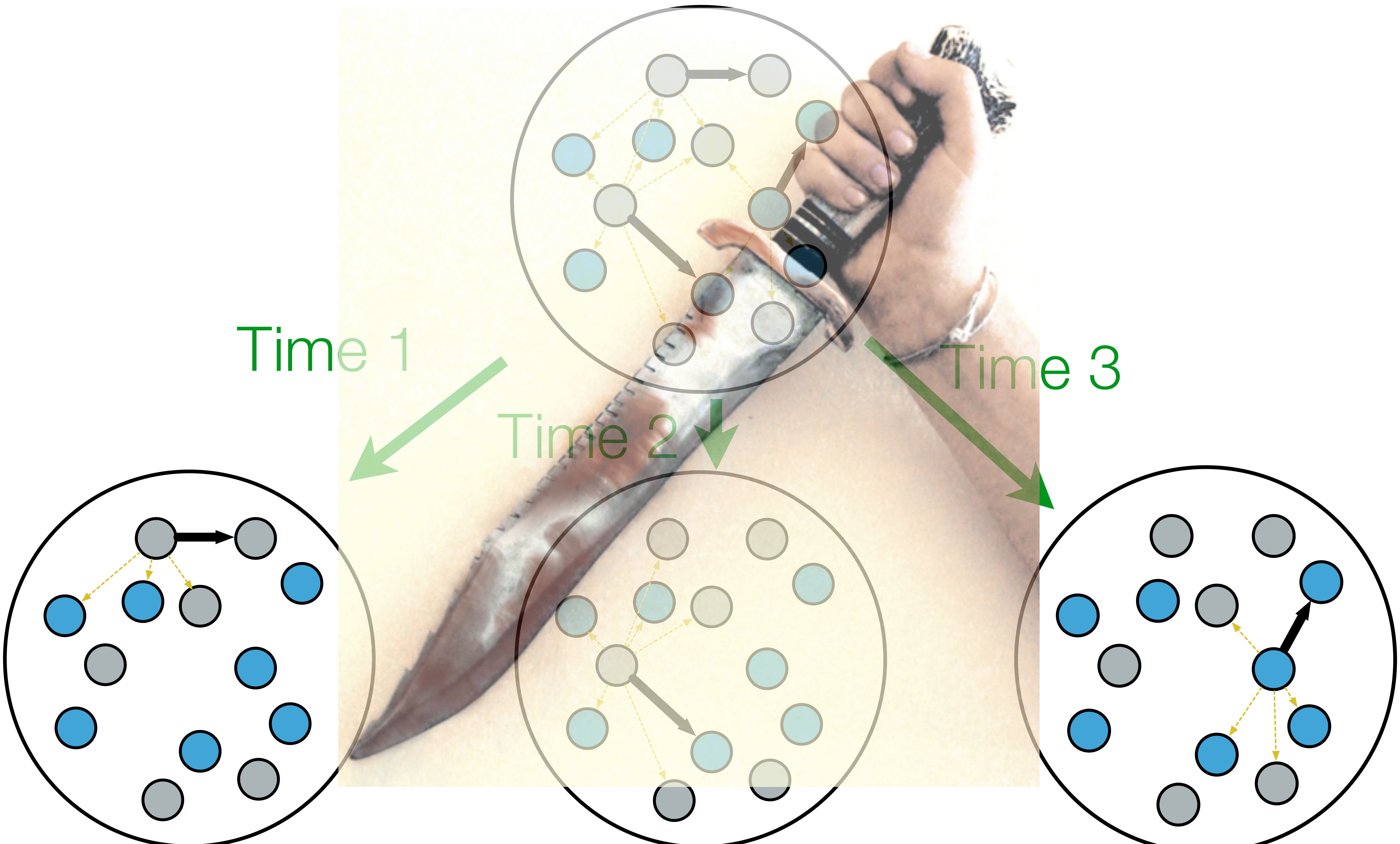


- Transmissions overheard by neighbors



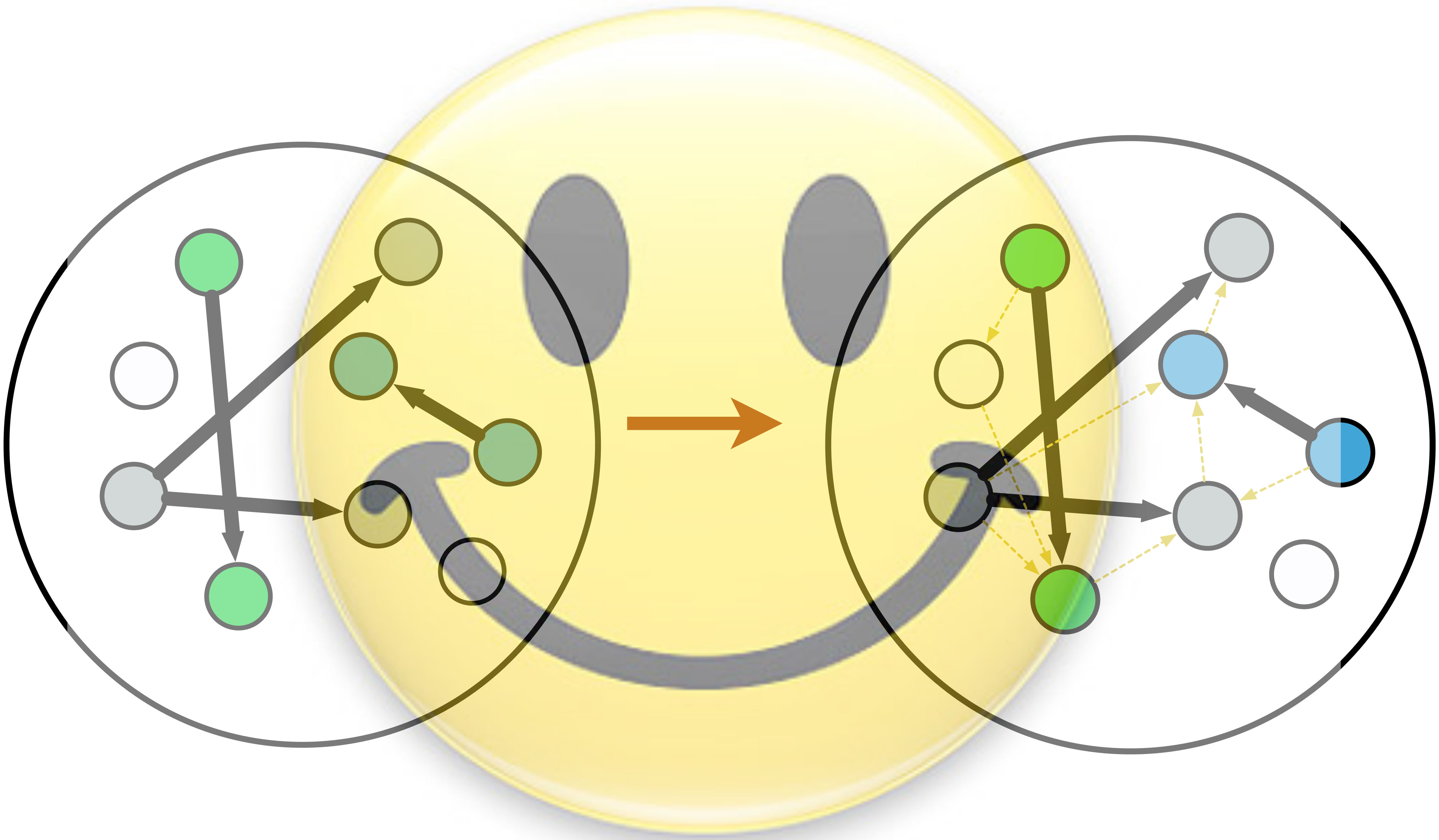
Extremely harmful to current network designs

Interference: one solution - orthogonalize



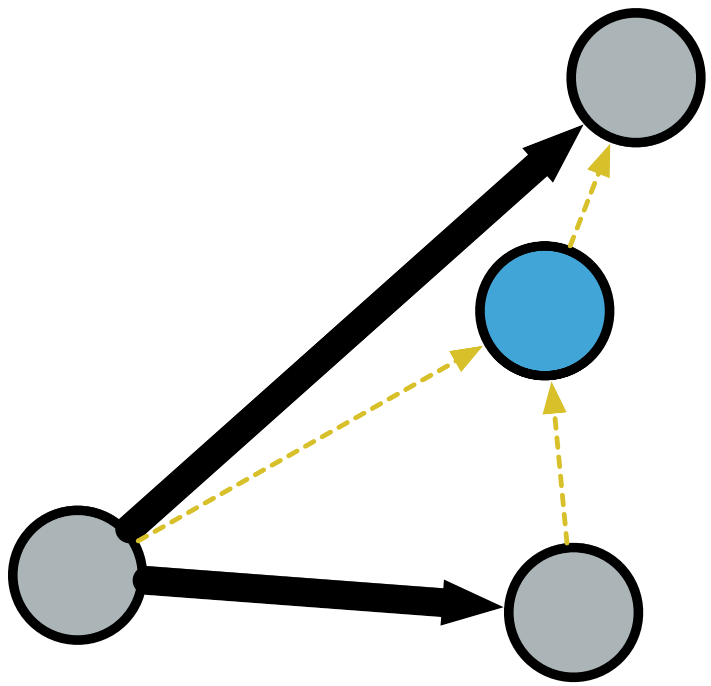
Interference: let's use it to our advantage

- Nodes which are not the source/destination of a message may help

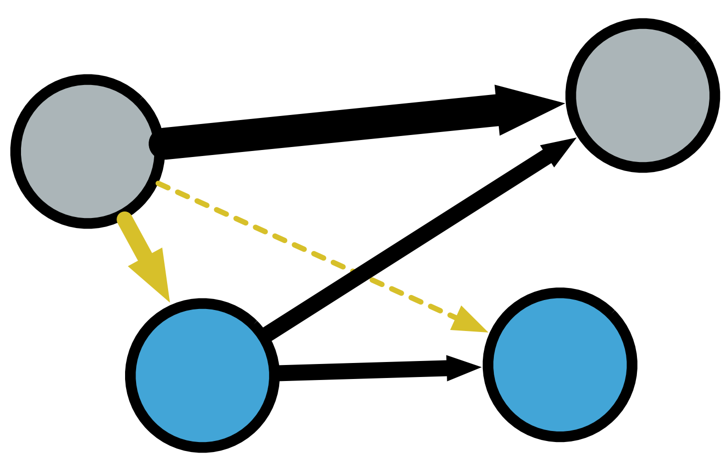


Cooperation and cognition

- Nodes may use overheard information to cooperate

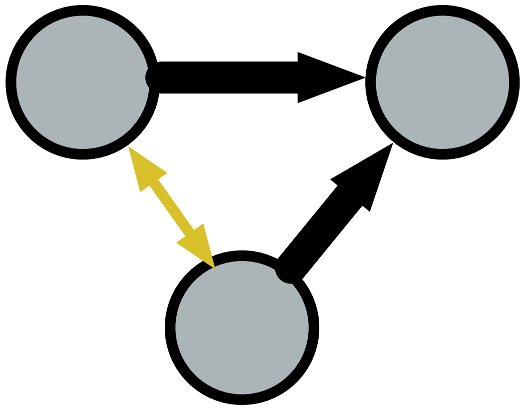


- Some nodes may be endowed with extra ``cognitive” capabilities



Intuition: why cooperate?

- Improved rates (power, signal alignment)



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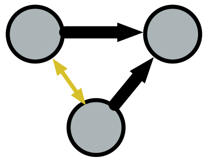
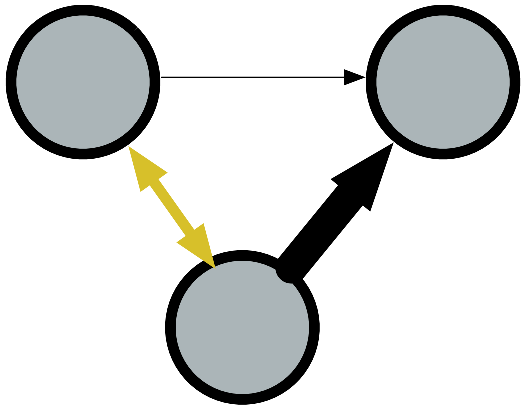


- Improved robustness (combat fading)



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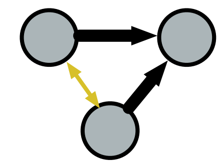

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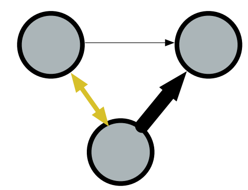
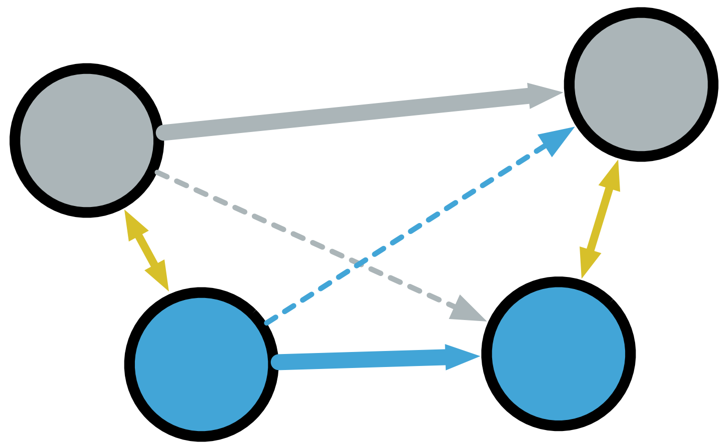


- Information multiplexing



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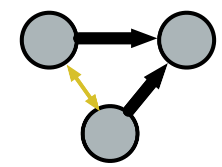

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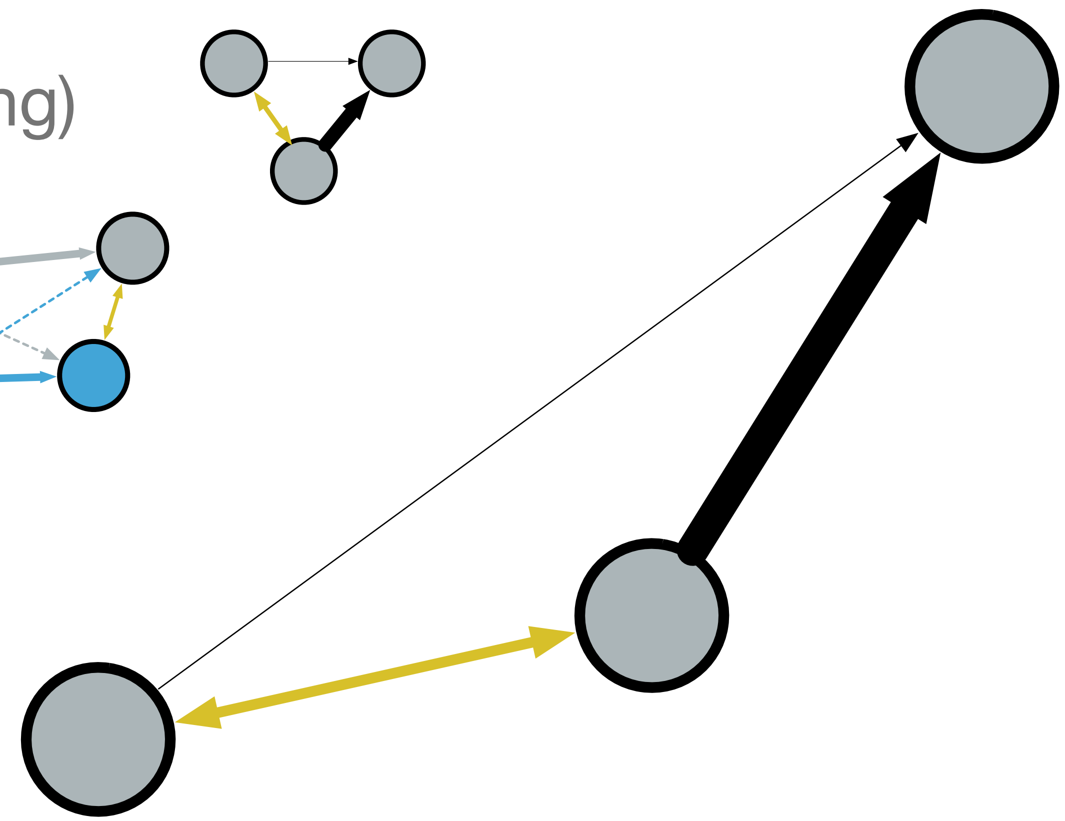

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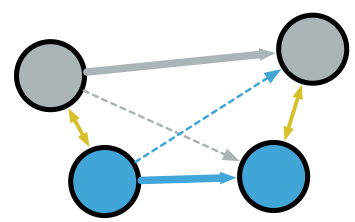

- Range extension

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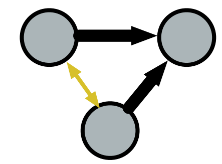

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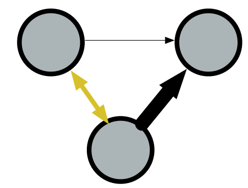

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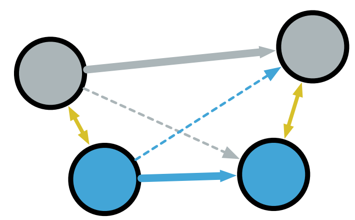

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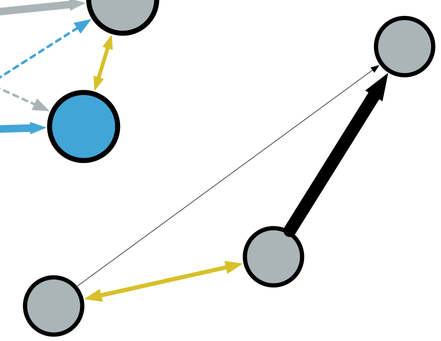
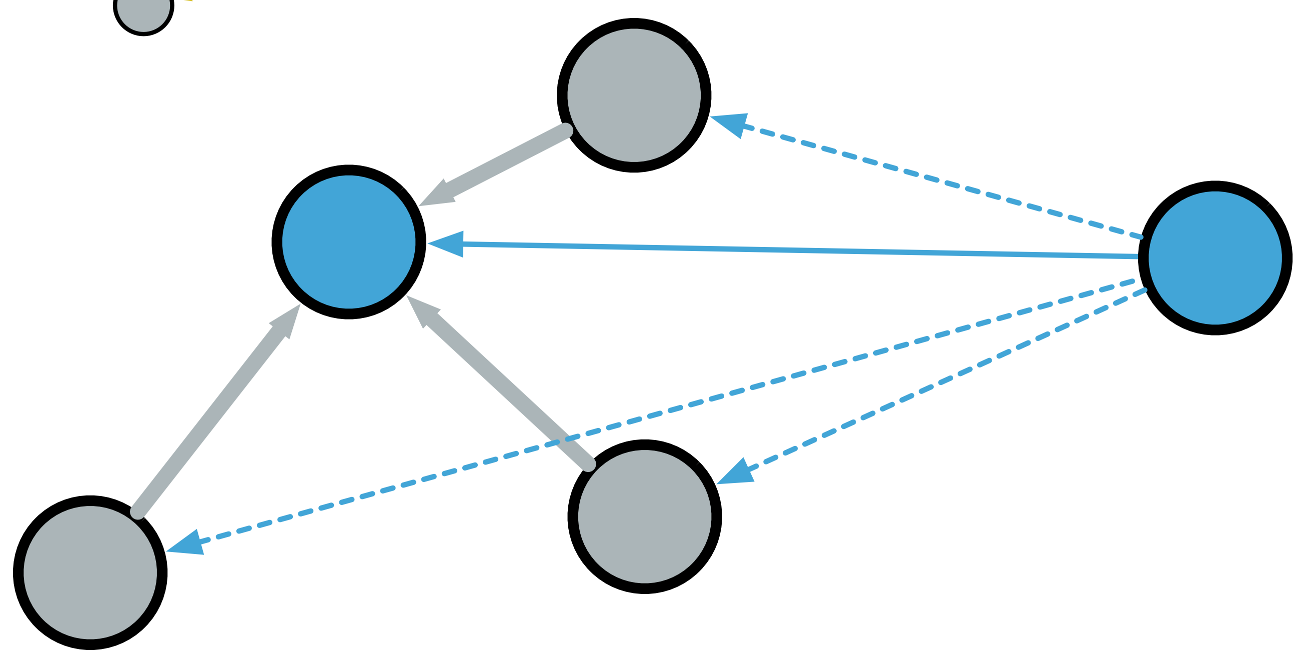


- Cooperative detection



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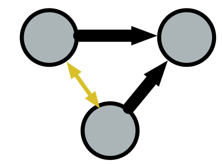

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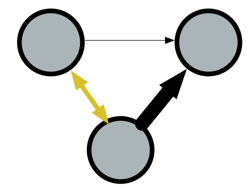

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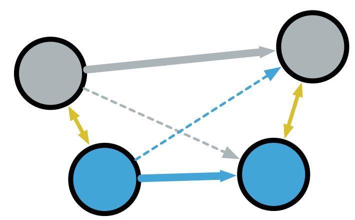

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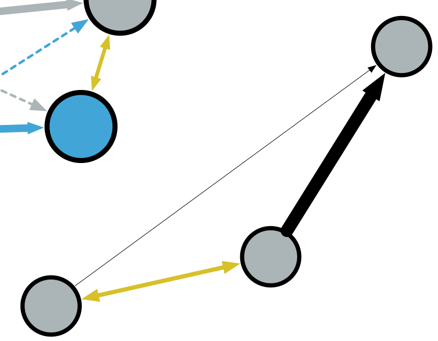

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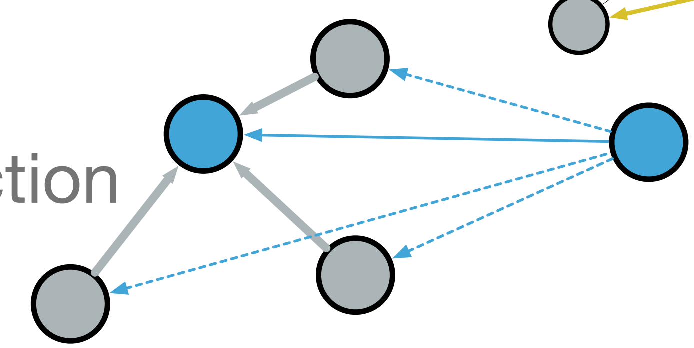
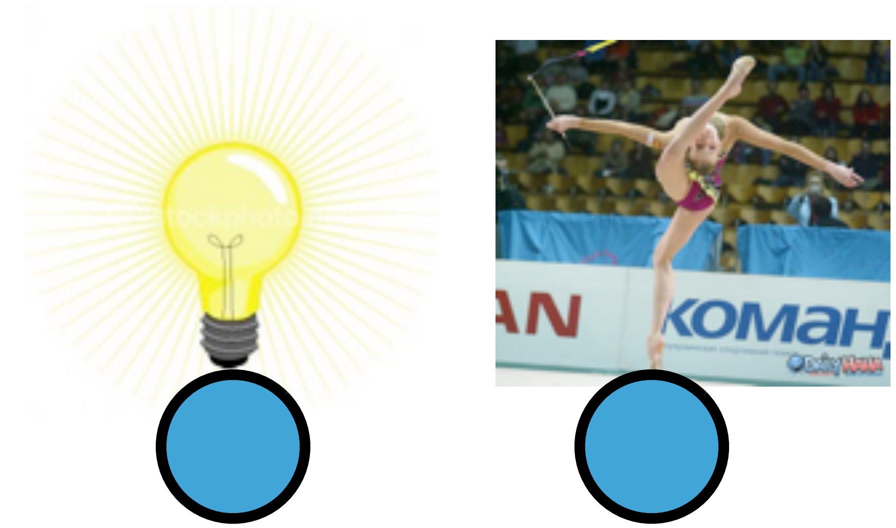


- Understand how networks work!

Intuition: why cognition?

- Exploit recent developments in SDR



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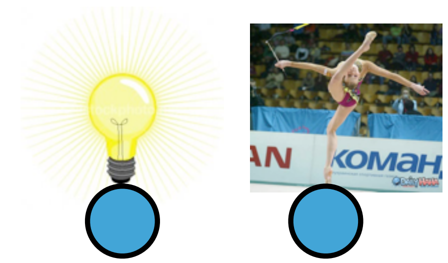
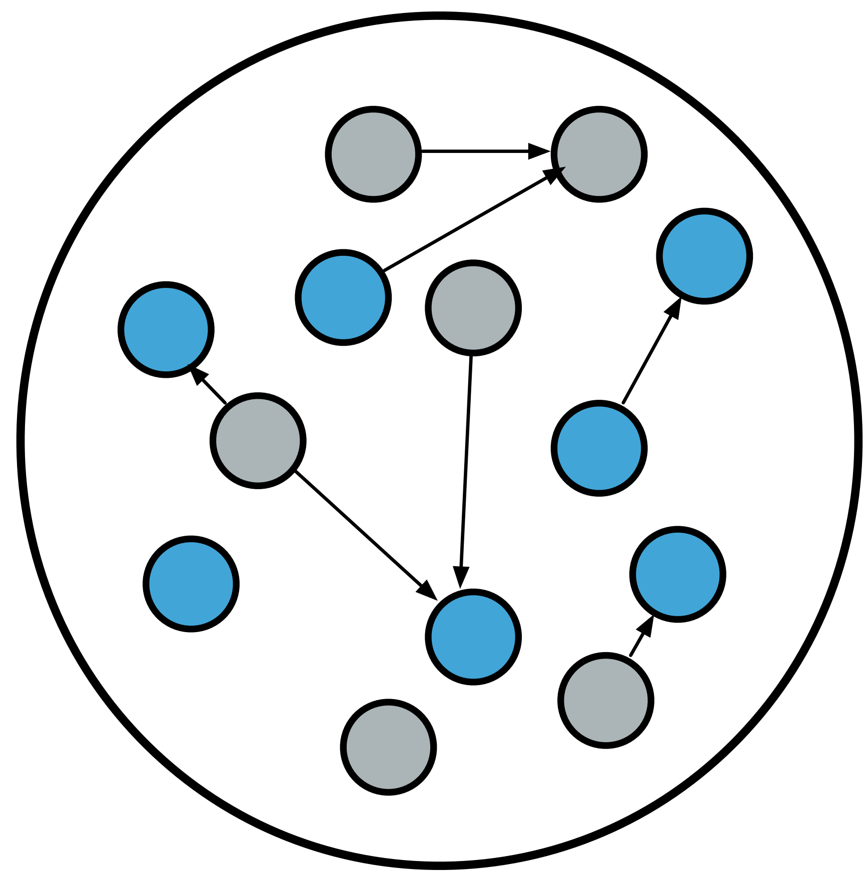


- Heterogeneous networks



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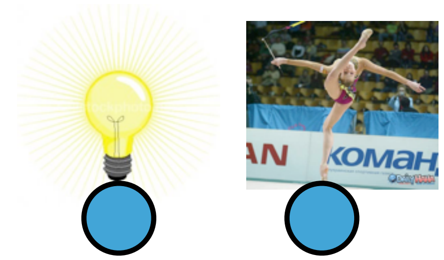

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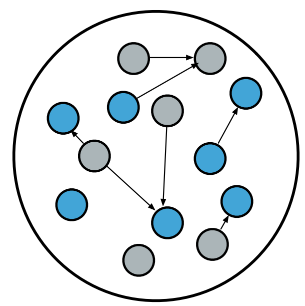
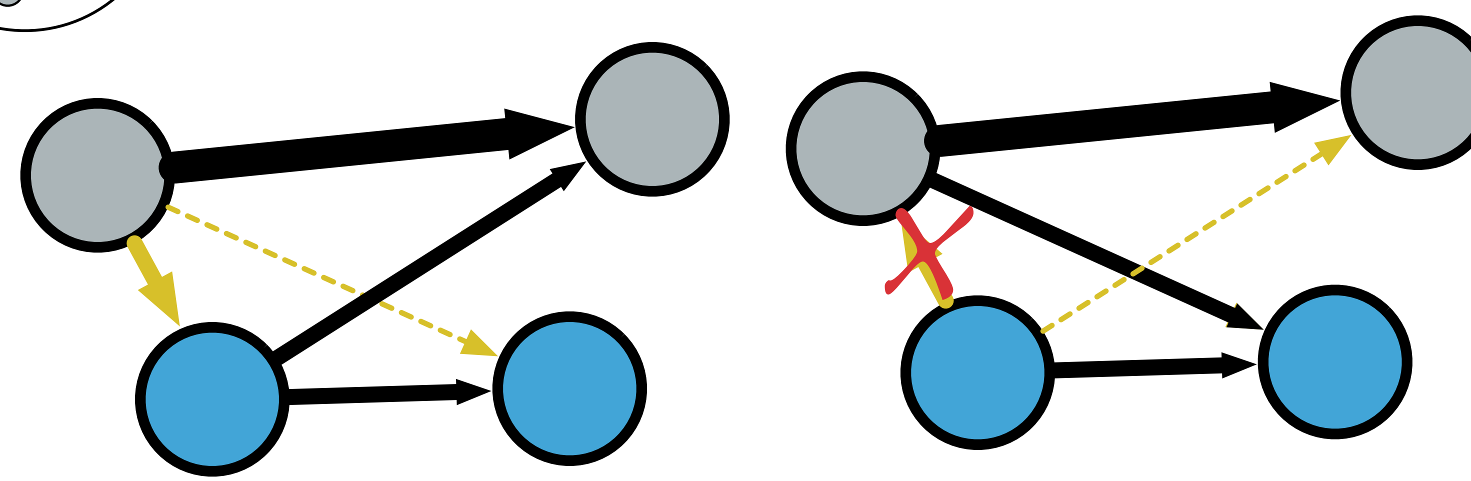


- Asymmetric cooperation



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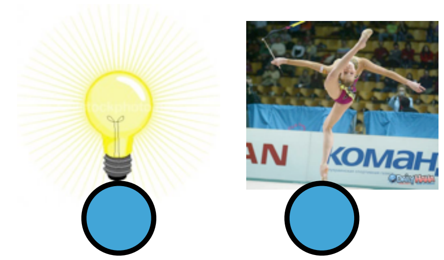

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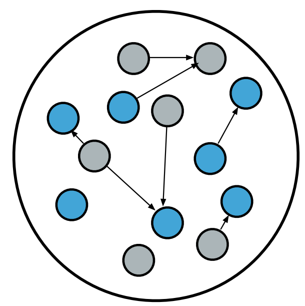

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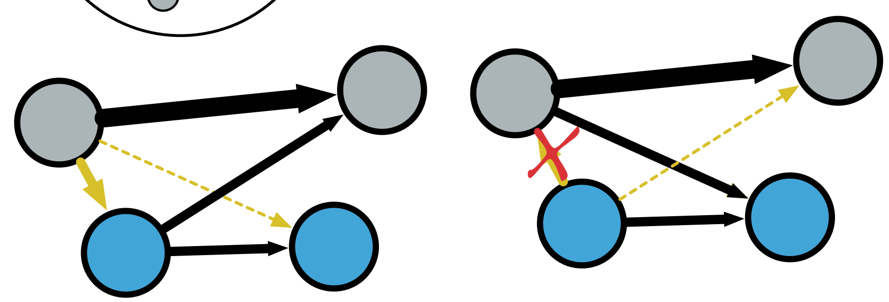


- Spectral efficiency and/or robustness

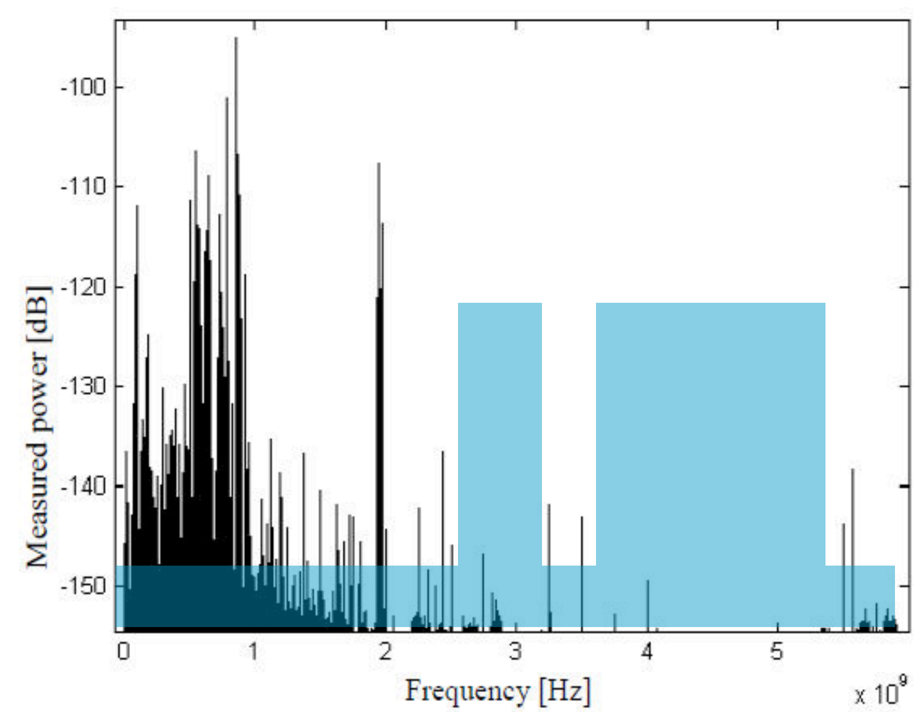
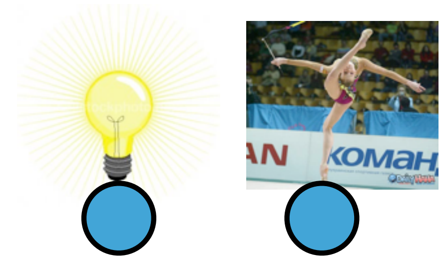


Figure 1. Spectrum utilization measurement at BWRC

Intuition: why cognition?

- Exploit recent developments in SDR



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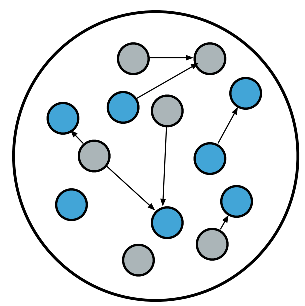

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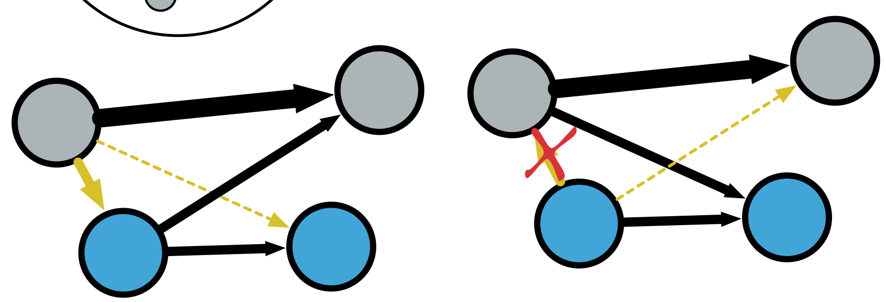

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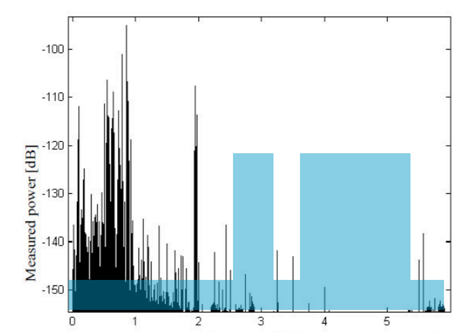
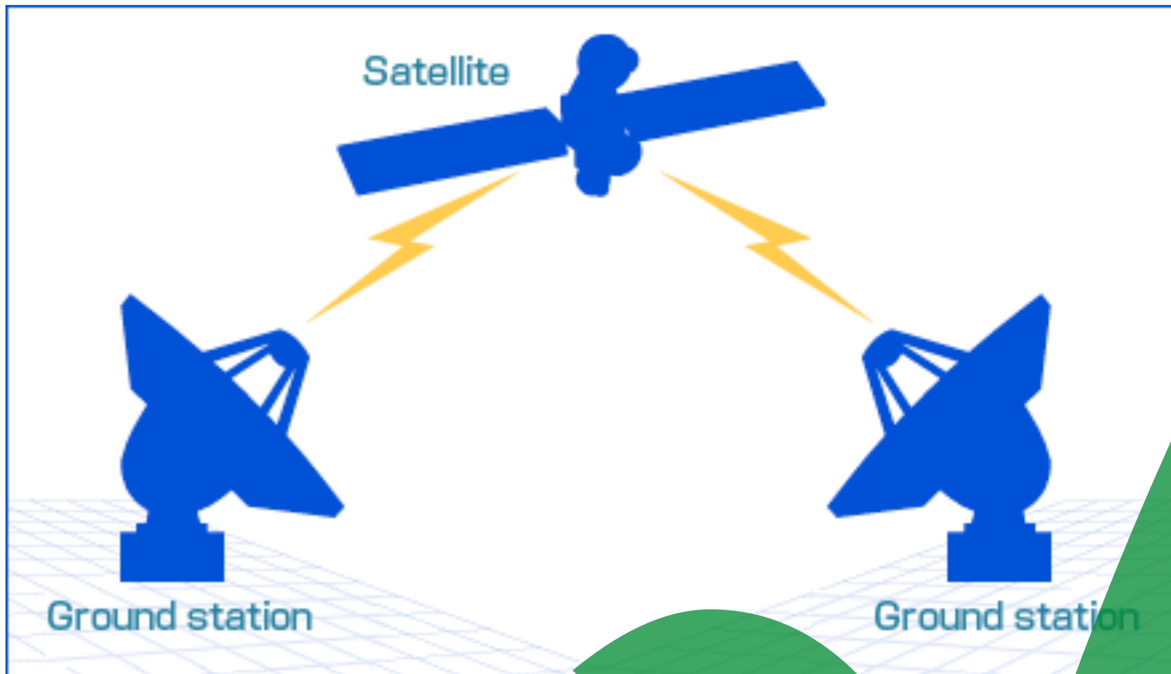


Figure 1. Spectrum utilization measurement at BWRC

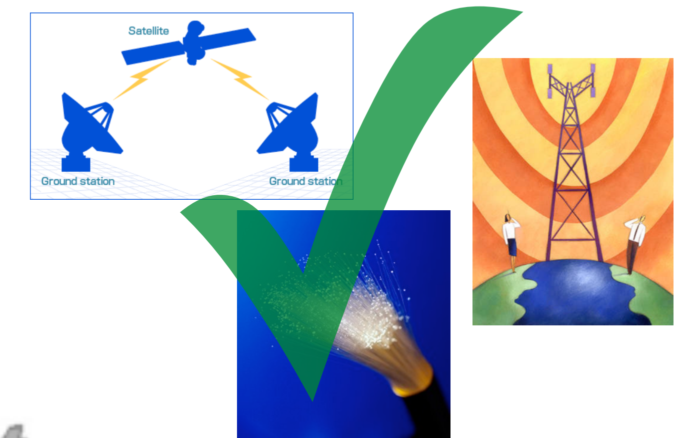
- To understand and explore all the possibilities of cognitive radio!

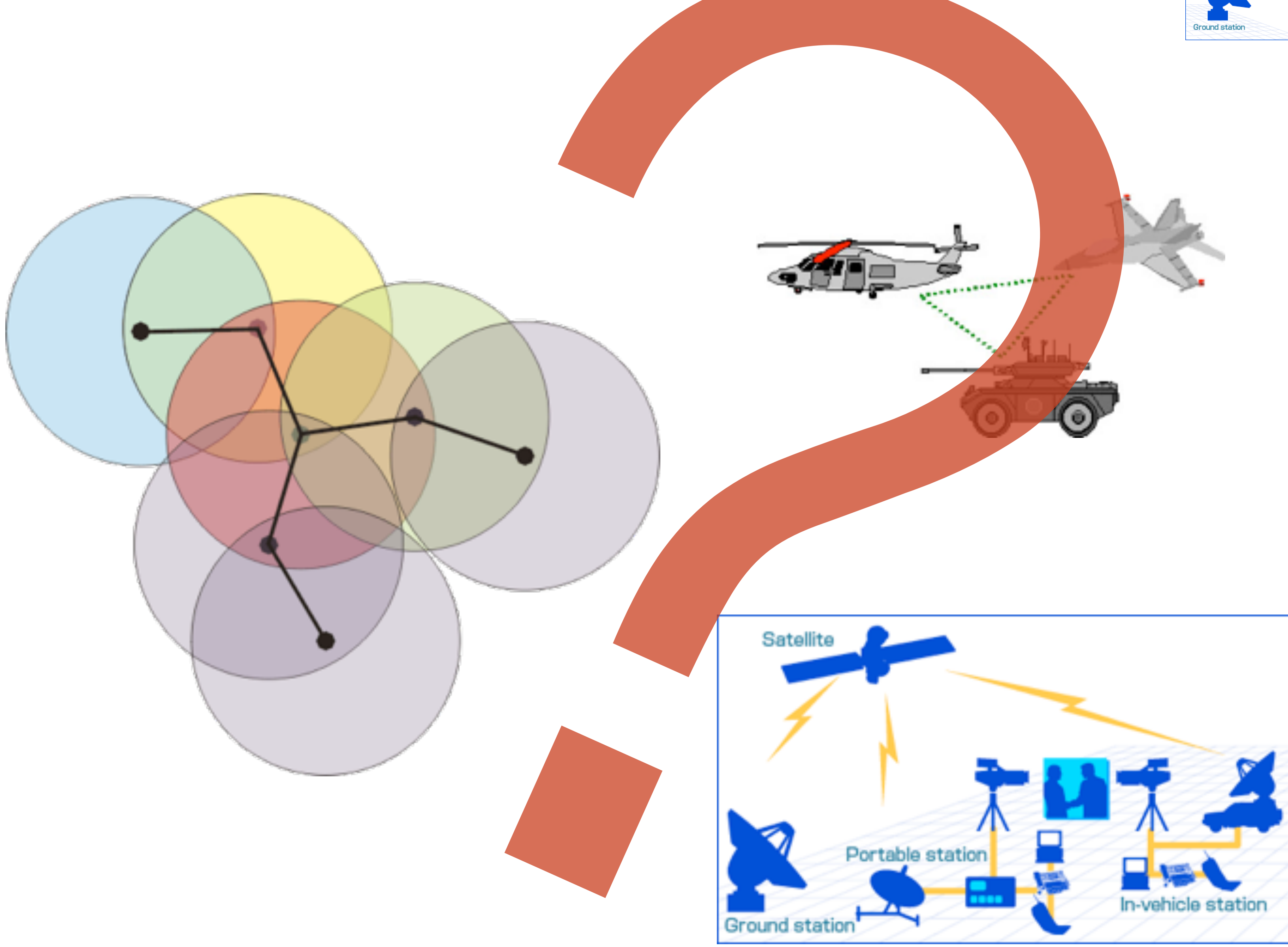
Intuition: why information theory?



Intuition: why information theory?

- Understand some systems well enough to engineer





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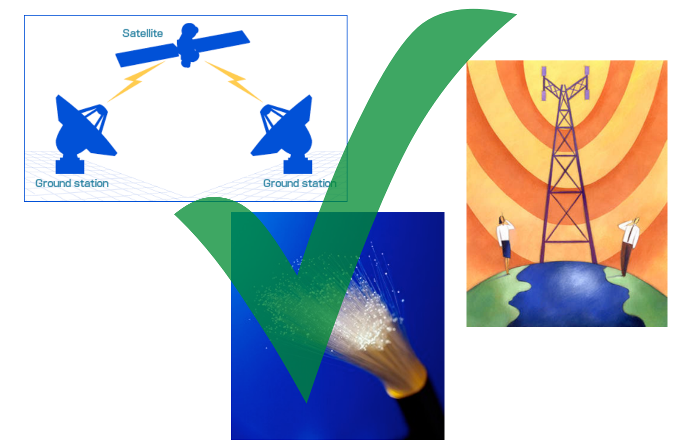
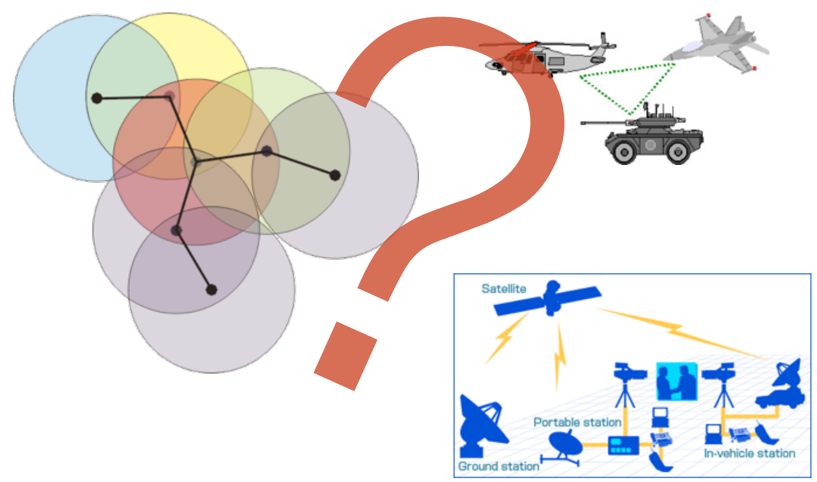
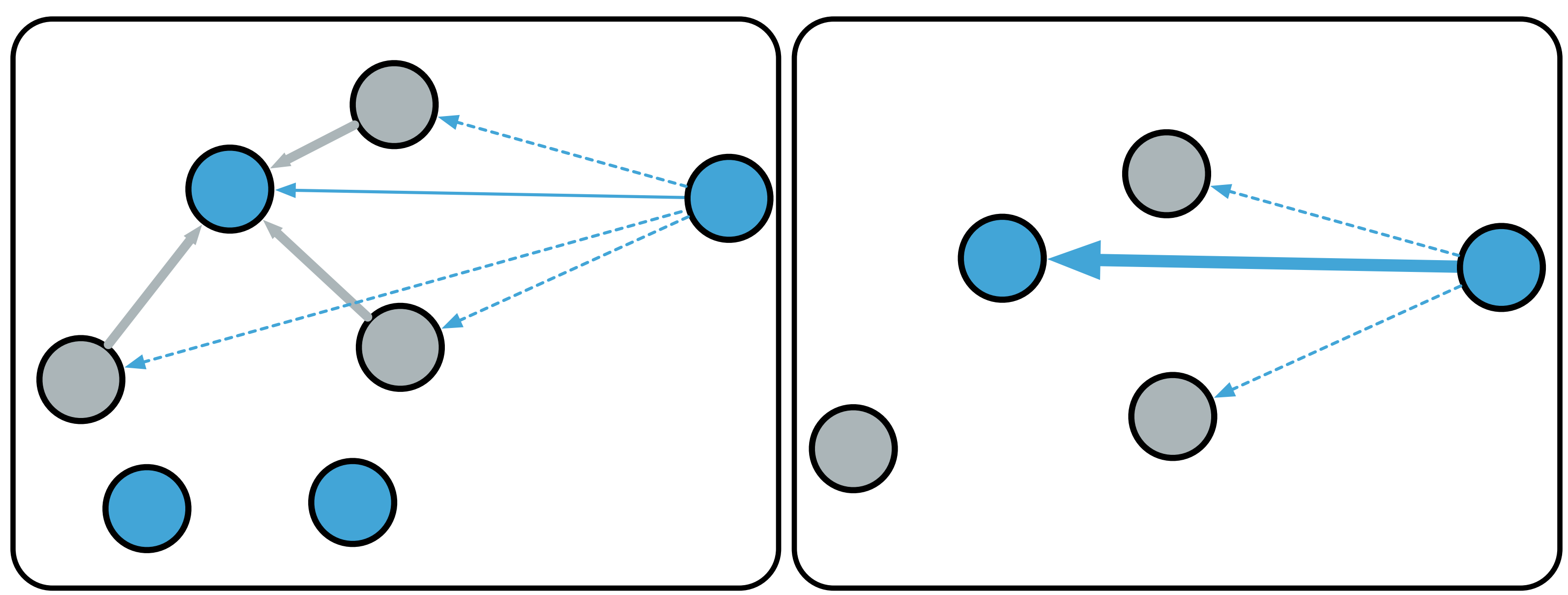


- Need to get a grip



- Which schemes perform well?



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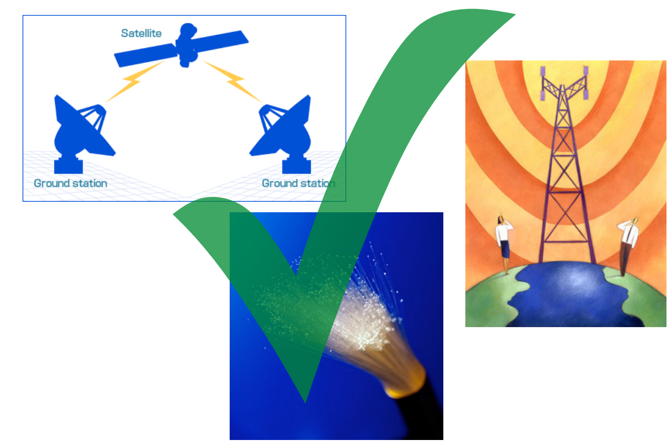

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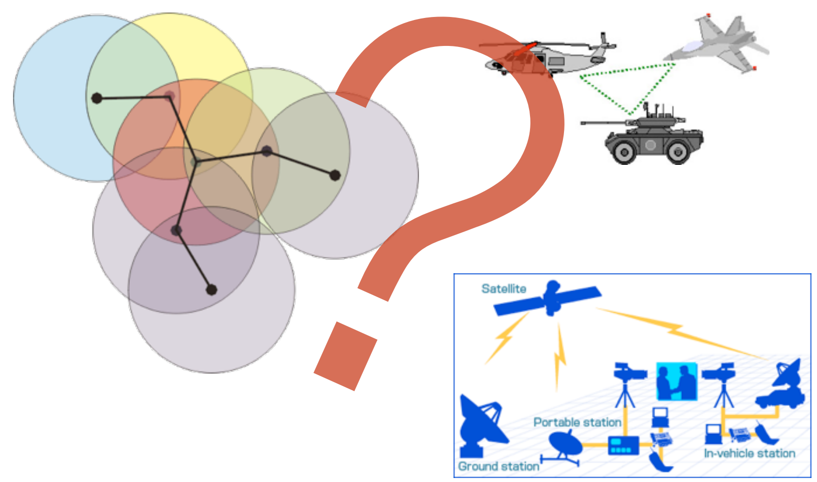

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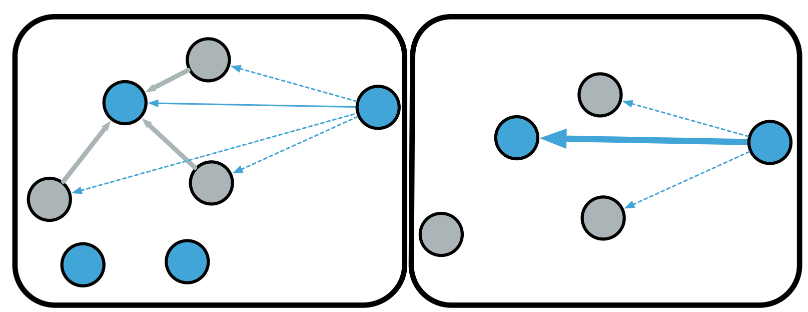
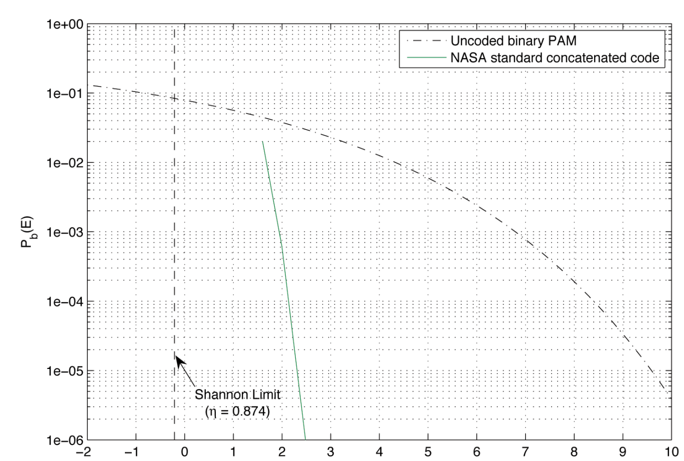


- Benchmark to engineer towards

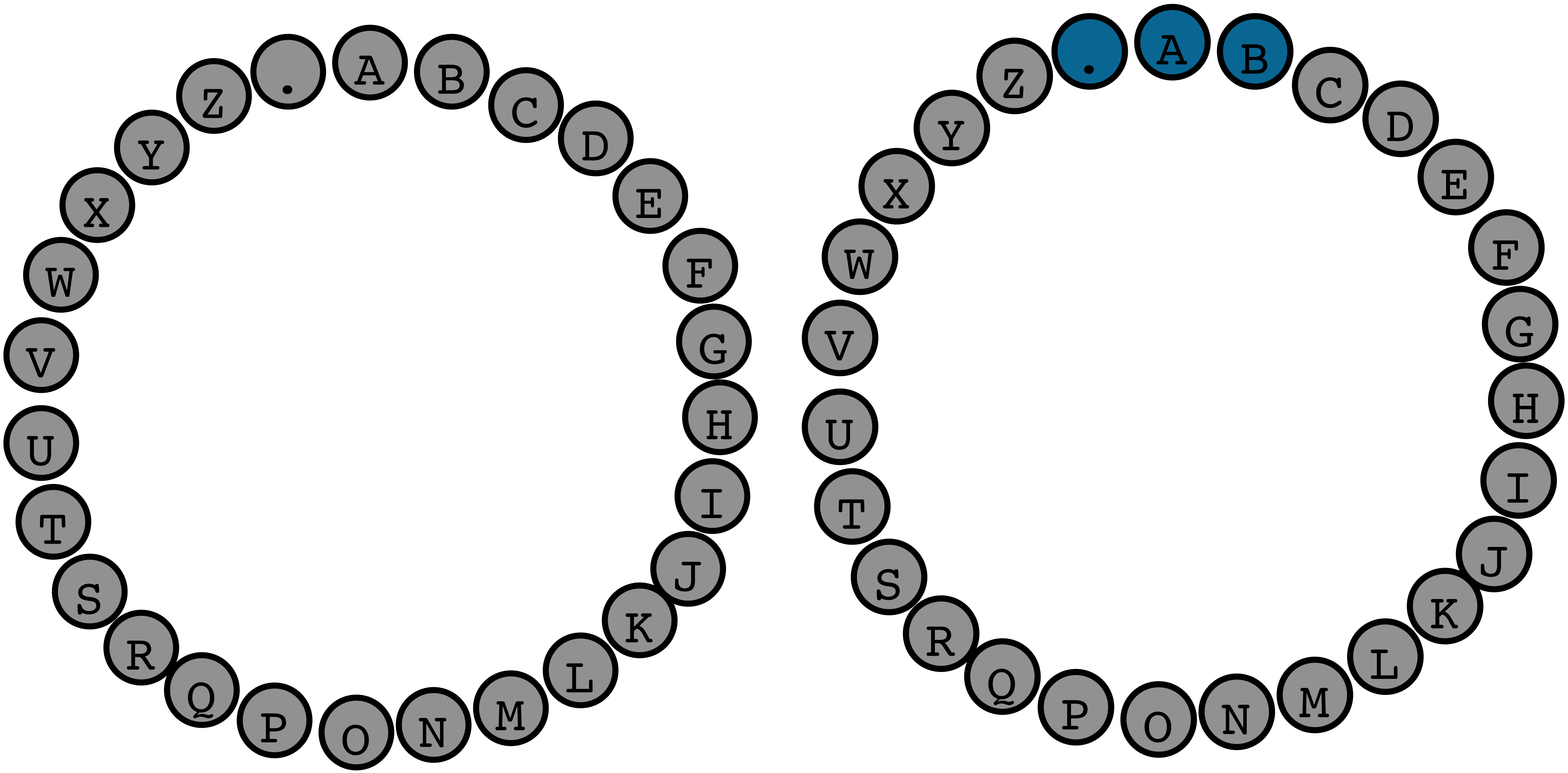


- Fundamental insight!

This talk

- The tutorial: information theory basics
    - channels + channel capacity
    - known multi-user channels
- The tour: Information theoretic limits of cognition in wireless networks
    - small networks
    - large networks
- Other interesting problems?

Channel capacity: a cute example



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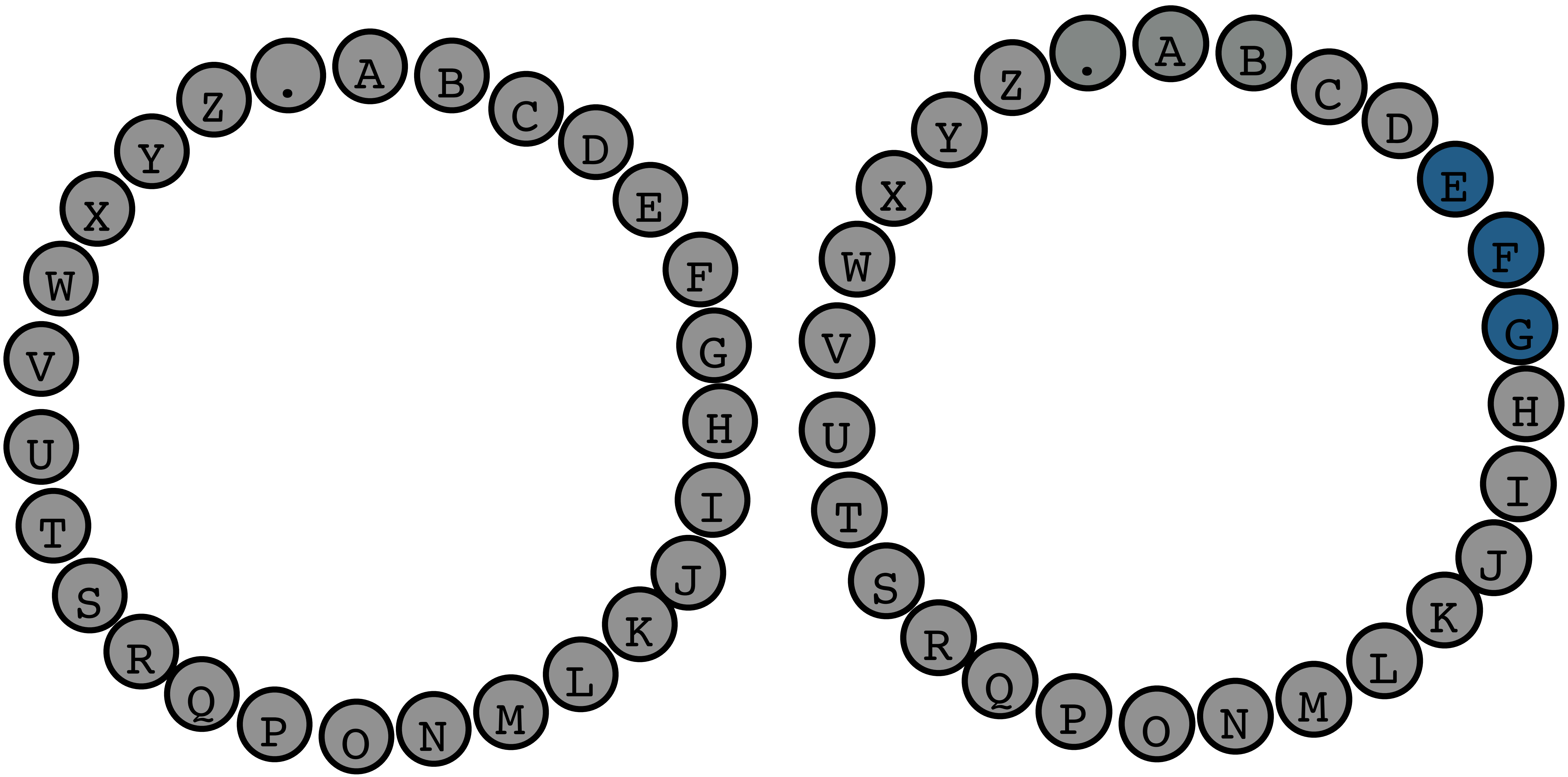

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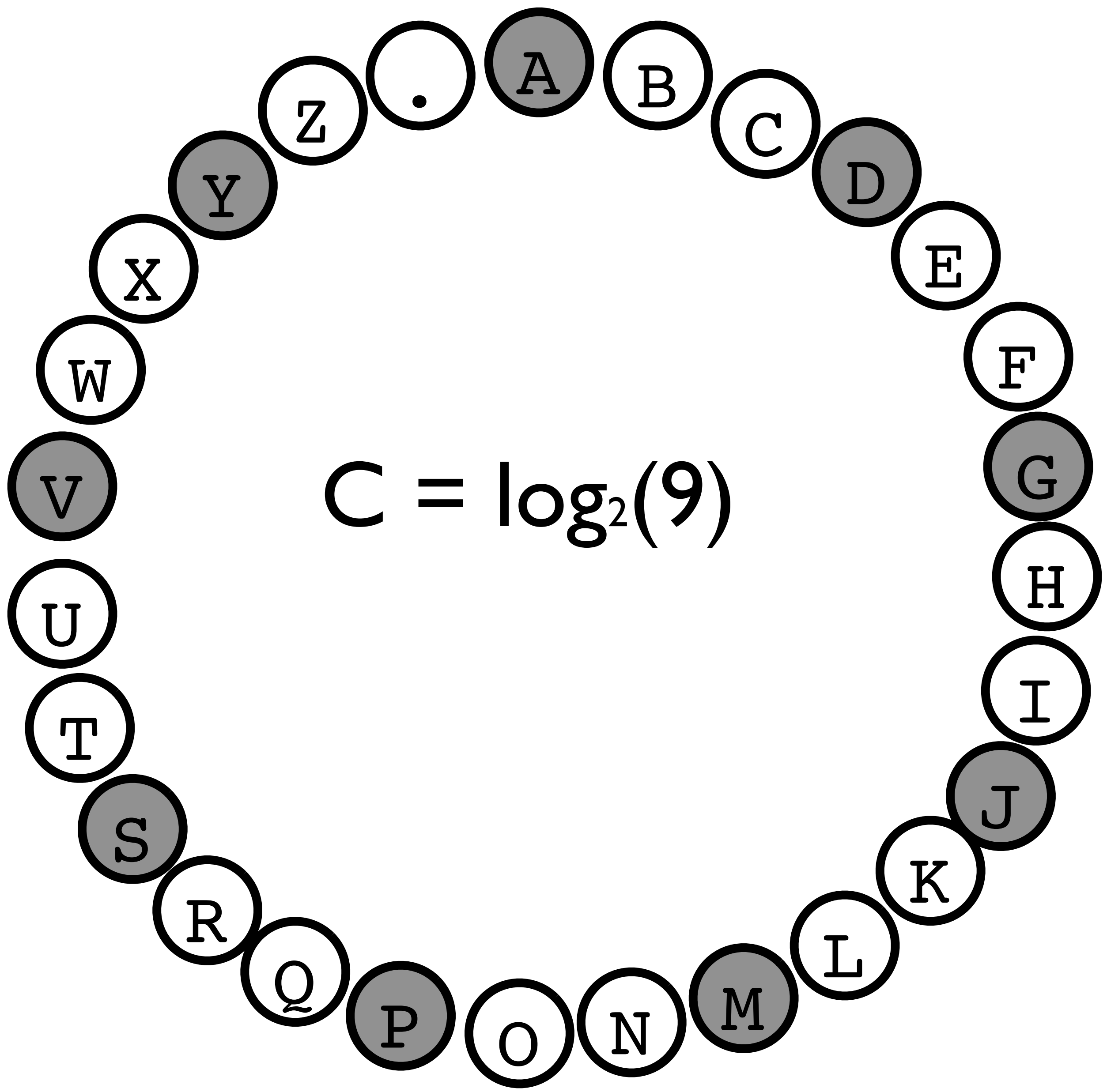
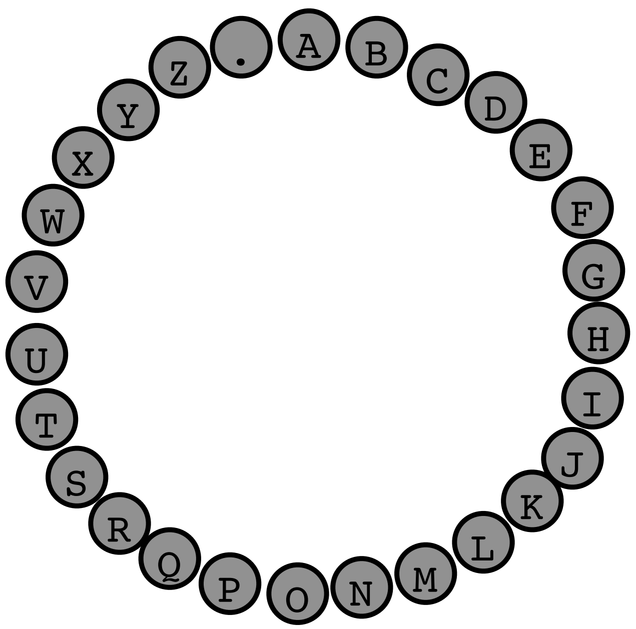
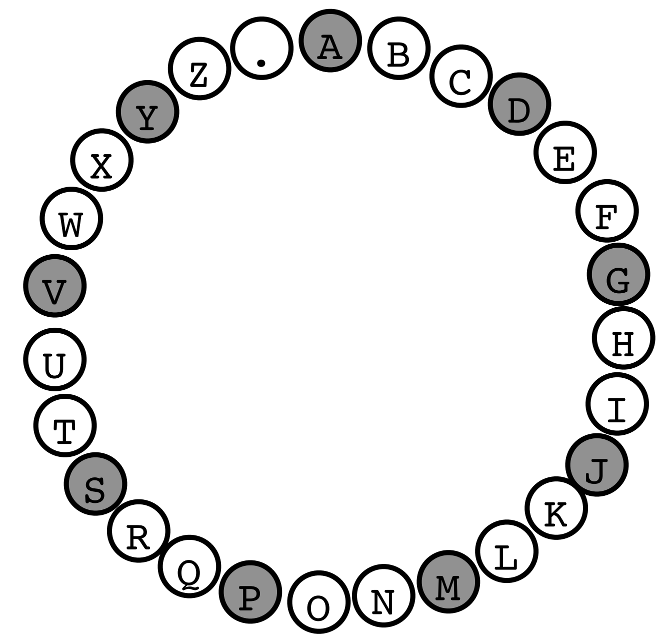


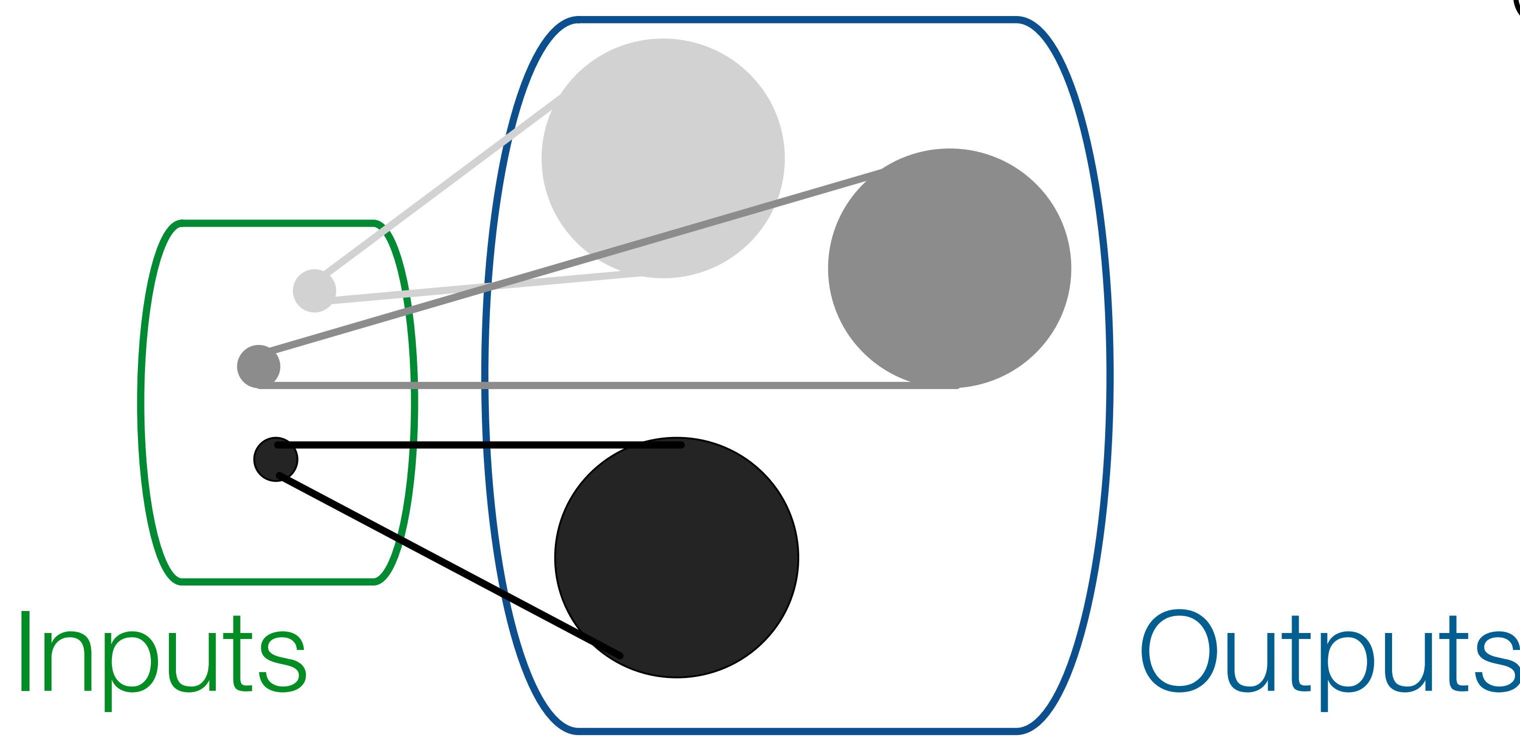
Capacity in general

- Main idea was to reduce the rate (from a 27-letter input per channel use to a 9-letter input per channel use) so as to produce



Non-overlapping outputs!





Mathematical description of capacity

- Information channel capacity:

$$C = \max_{p(x)} I(X;Y)$$

- Operational channel capacity:

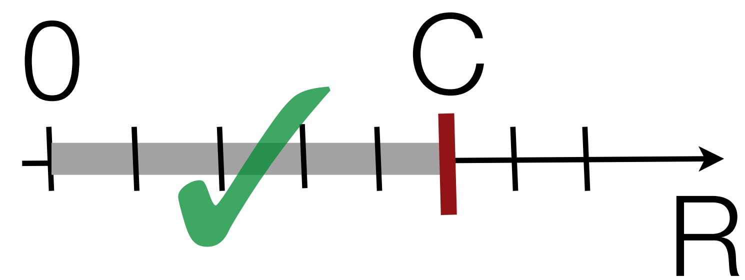
```
Highest rate (bits/channel use) that can
communicate at reliably
```

- Channel coding theorem says: information capacity = operational capacity

Mathematical description of capacity

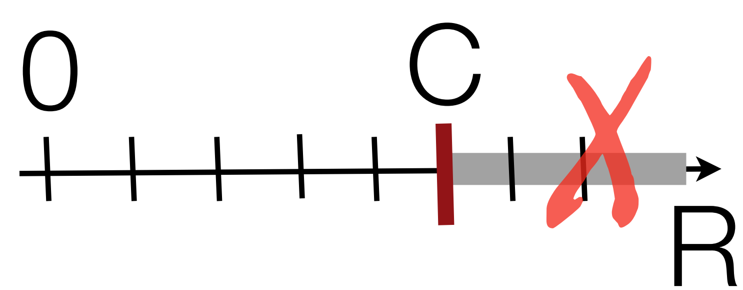
- Can achieve reliable communication for all transmission rates R:

$$R < C$$



- BUT, probability of decoding error always bounded away from zero if

$$R > C$$

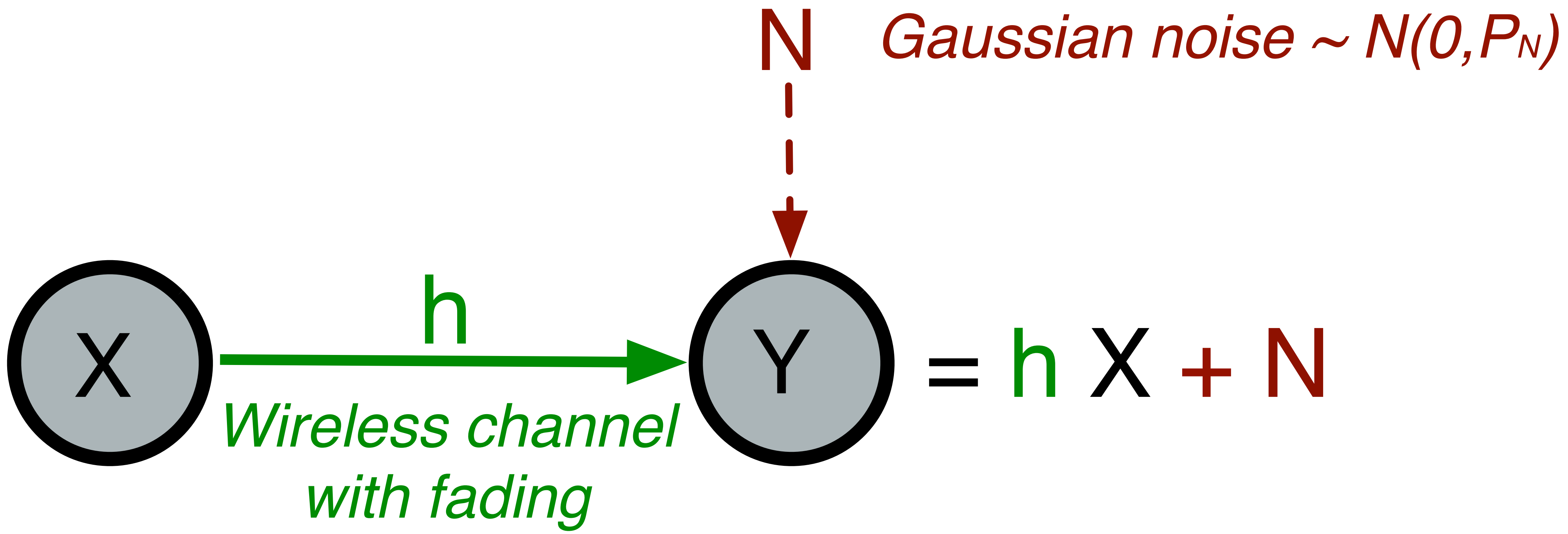


Channel capacity

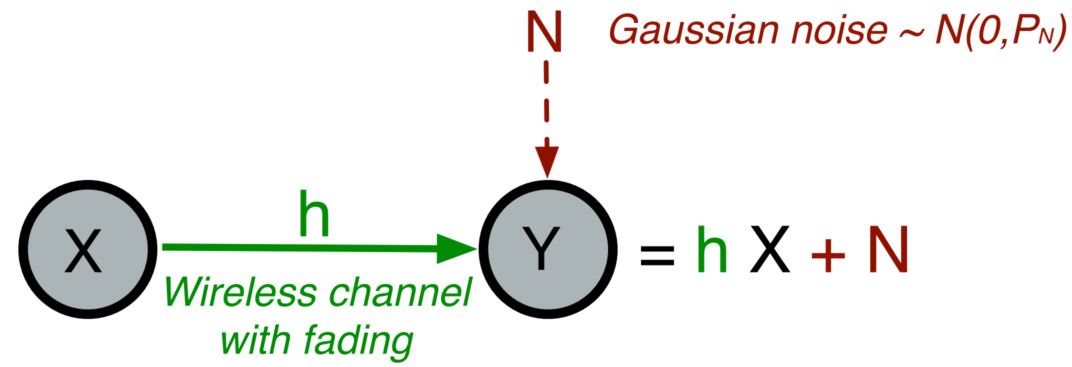
Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

AWGN channel capacity



AWGN channel capacity



$$C = \frac{1}{2}\log\left(\frac{|h|^2 P + P_N}{P_N}\right)$$

$$= \frac{1}{2}\log(1 + SNR) \quad \text{(bits/channel use)}$$

What about bits/second and bandwidth of the channel?

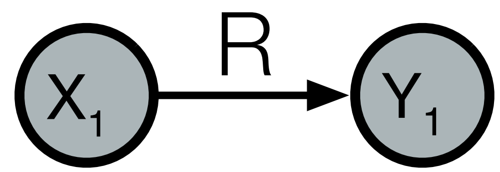
$$C = W\log_2\left(1 + \frac{P}{WN_0}\right) \quad \text{(bits/second)}$$

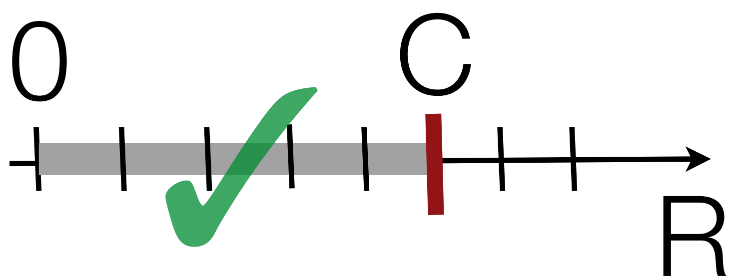
[Bandwidth W, h=1, spectral density $N_0/2$]

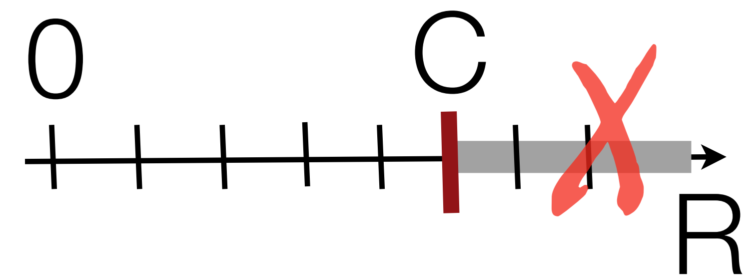
Moving on to multi-user channels

Capacity and capacity regions

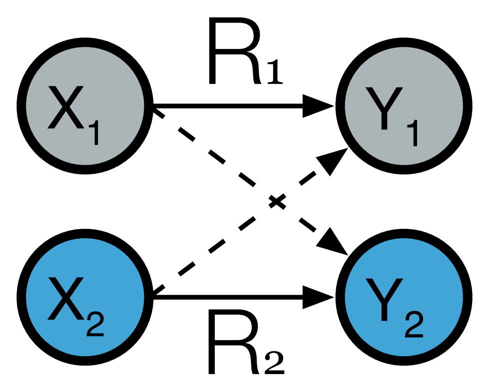
- Point to point **capacity**

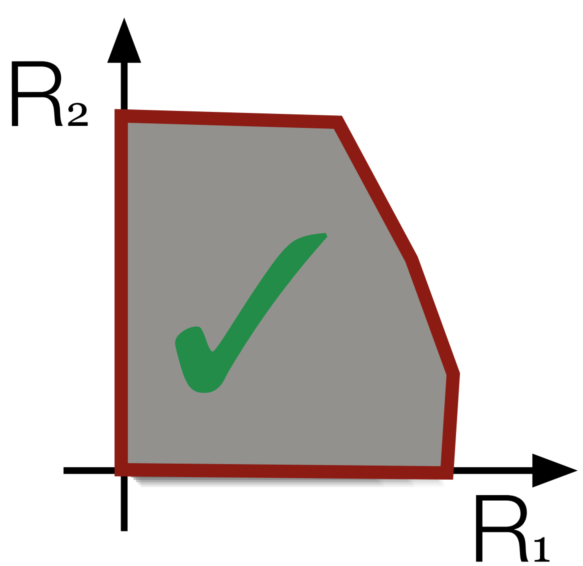


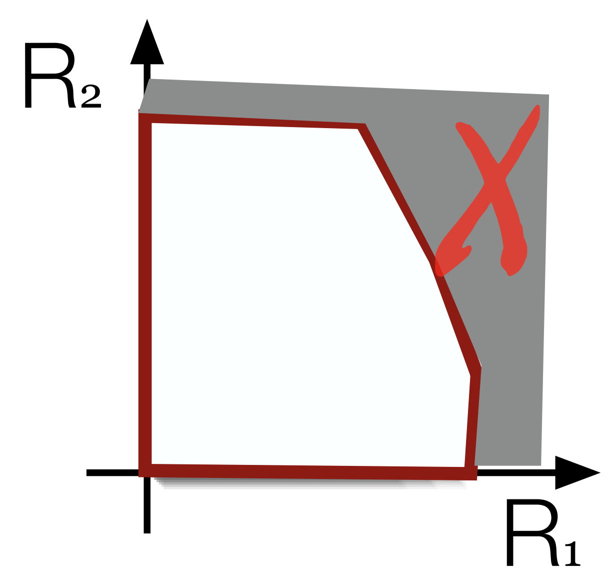




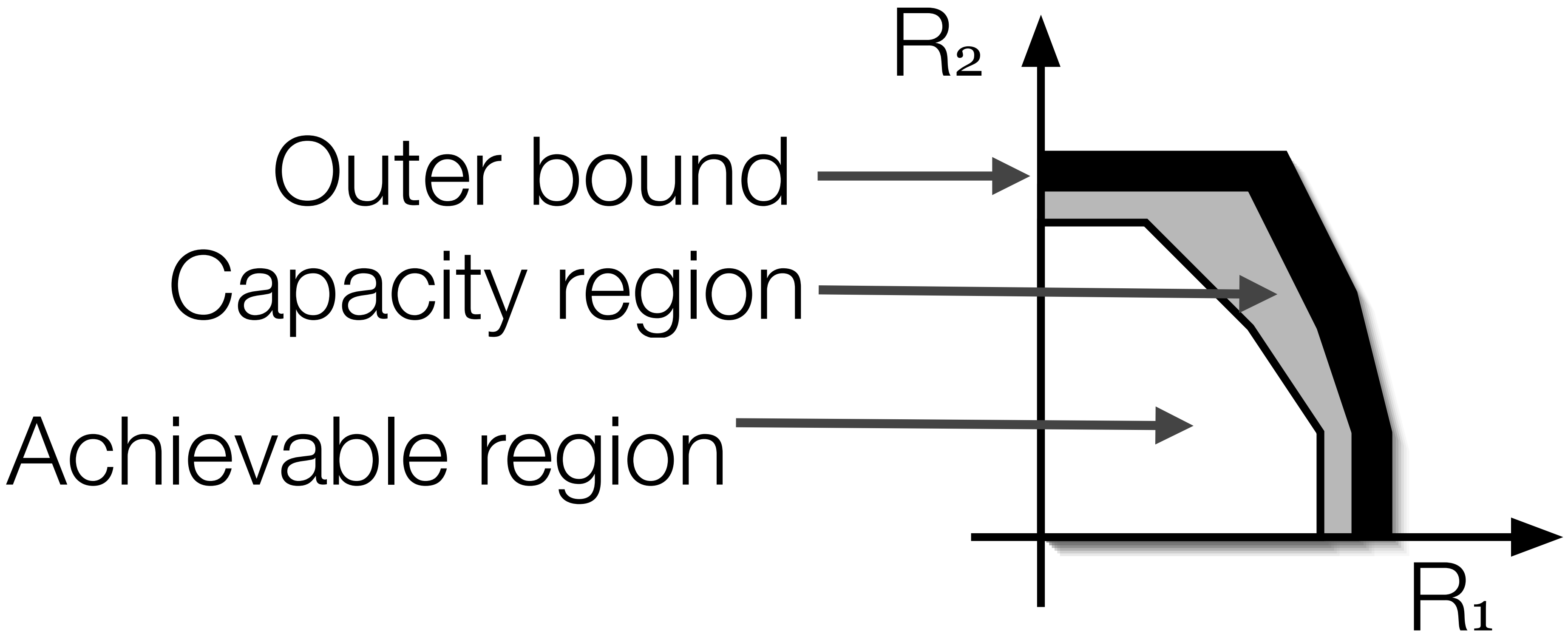
- Multi-user **capacity region**

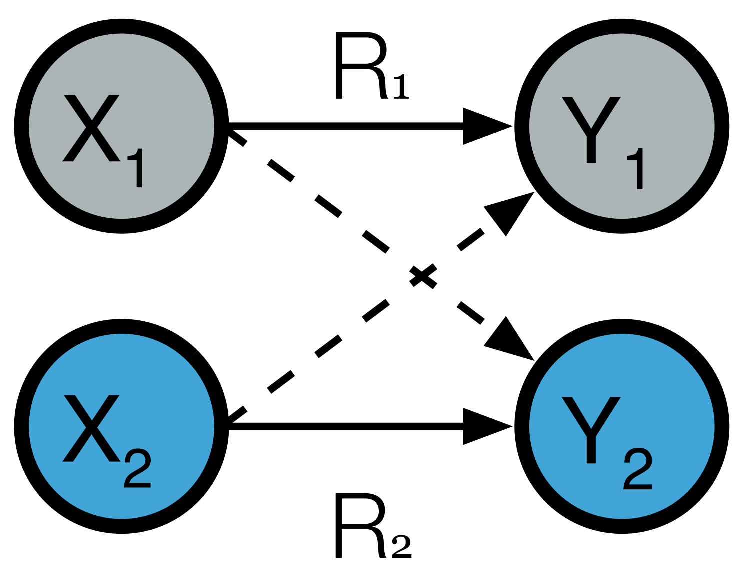




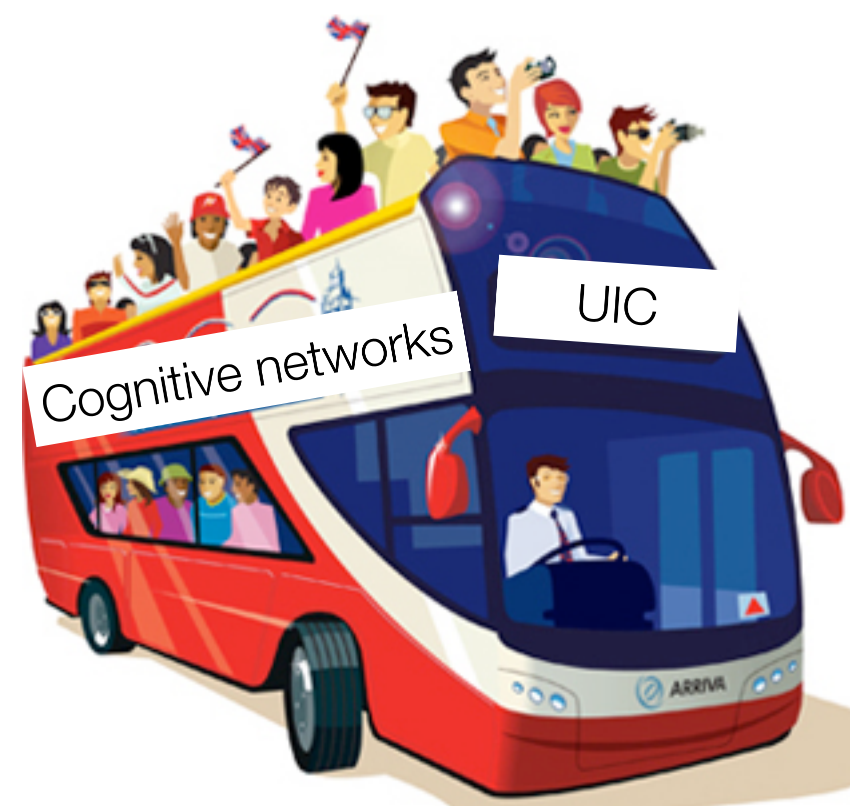


Capacity regions



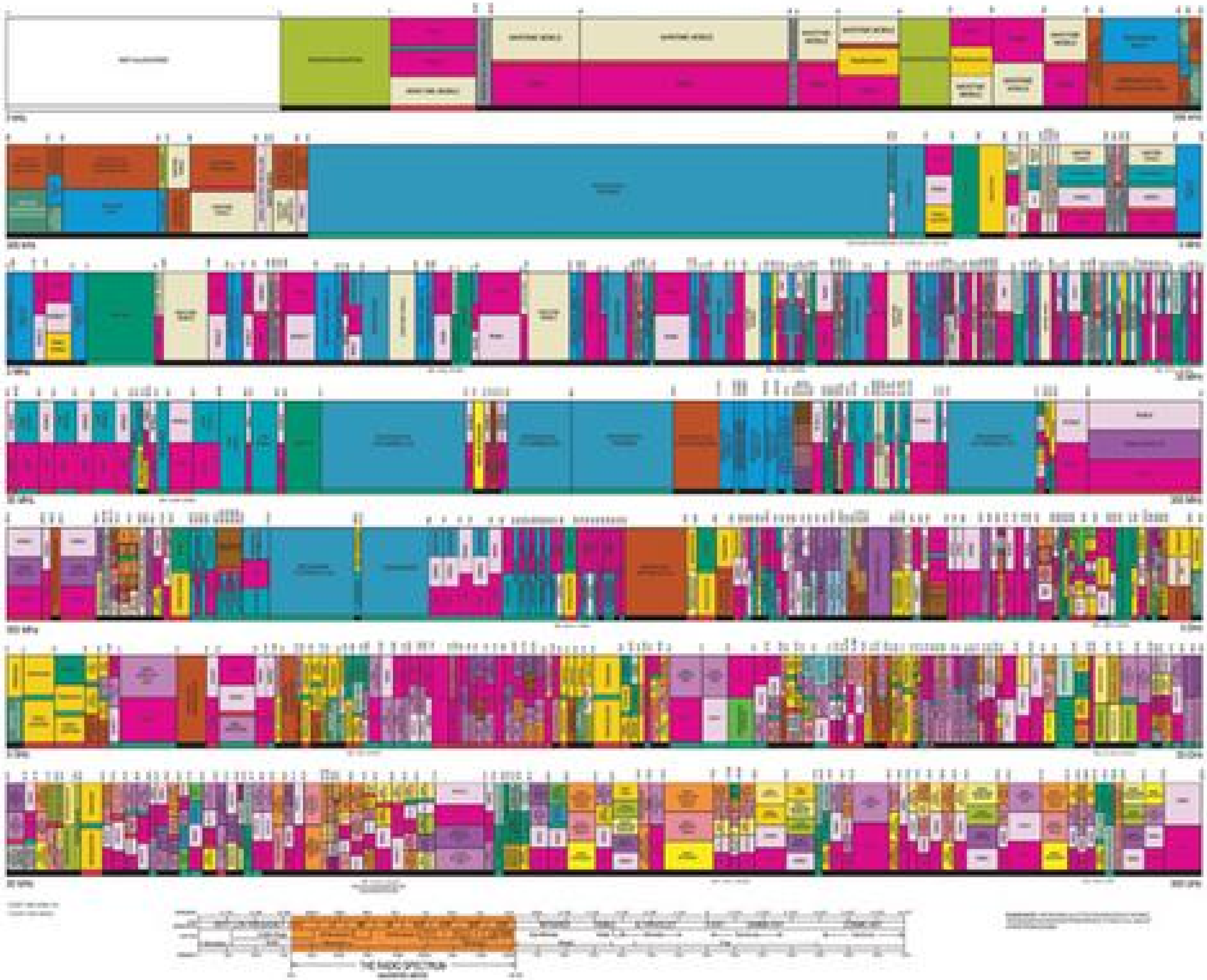


Information theoretic limits of cognition in wireless networks



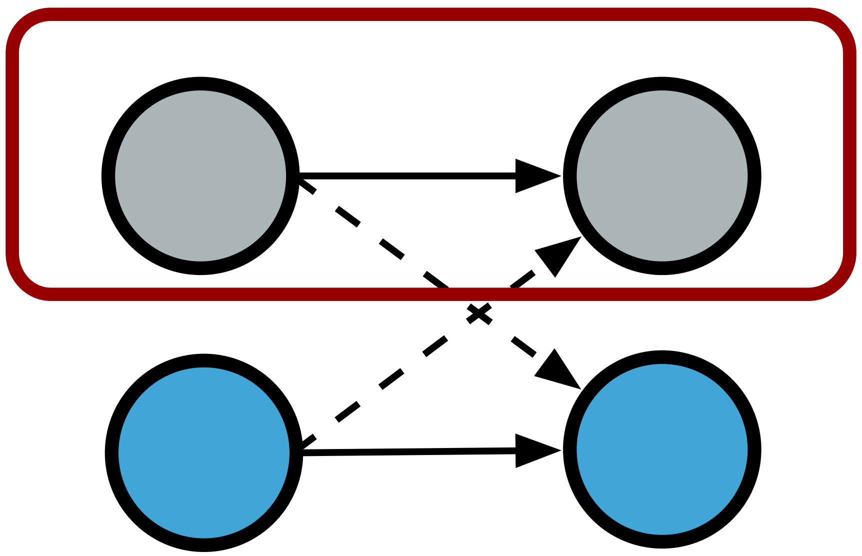
UNITED STATES FREQUENCY ALLOCATIONS

THE RADIO SPECTRUM



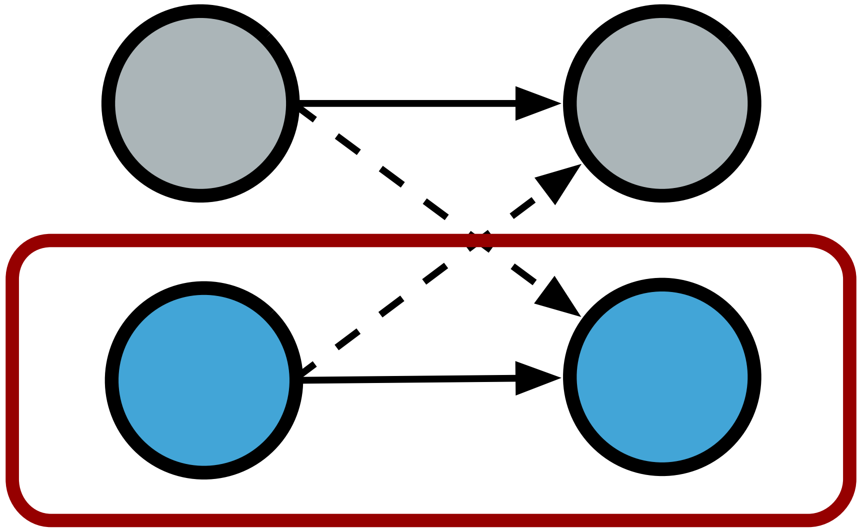
Spectrum licensing: future

Primary users/ primary license holders



Spectrum licensing: future

Primary users/ primary license holders

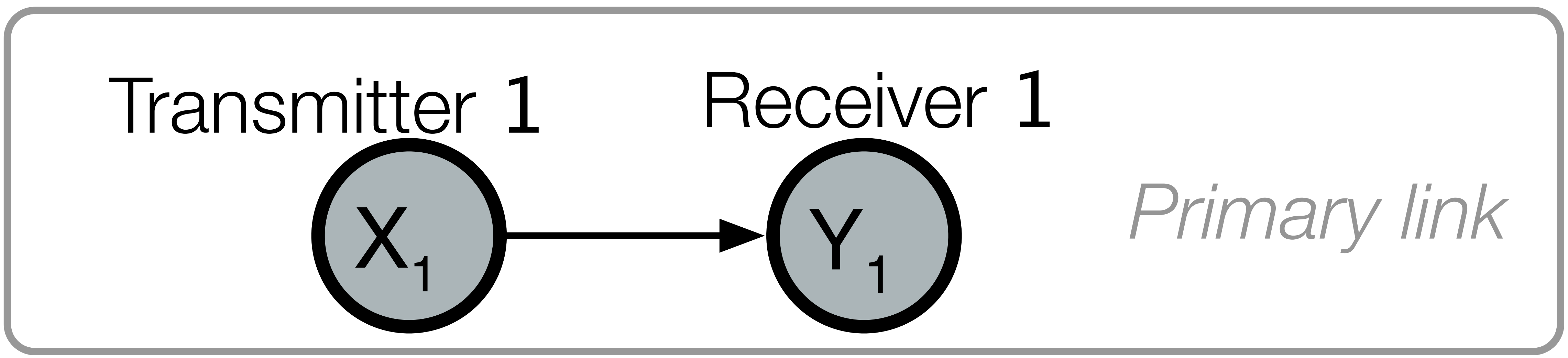


Secondary users ⟷ Cognitive radios

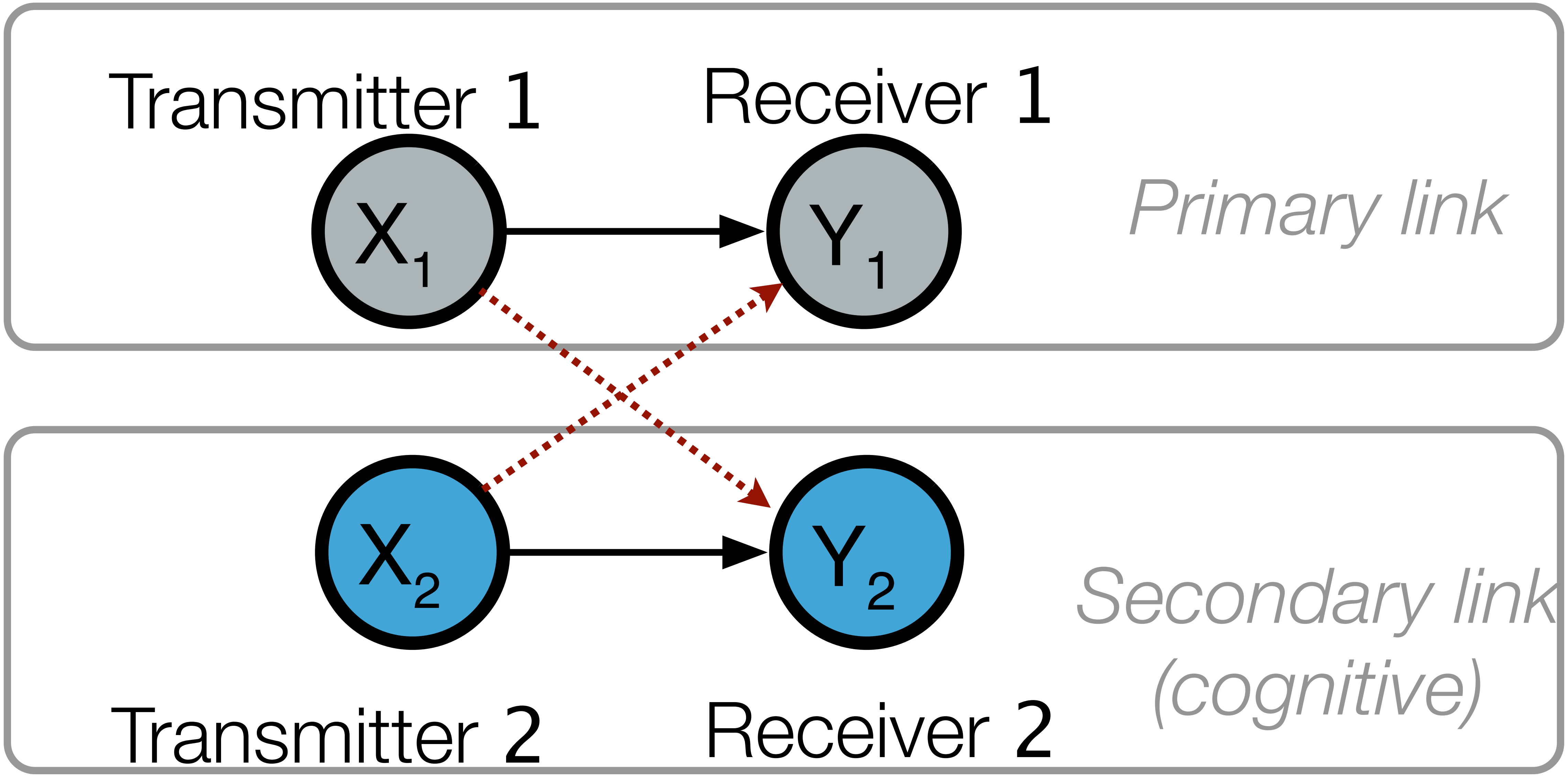
Cognitive Radio

Secondary spectrum usage

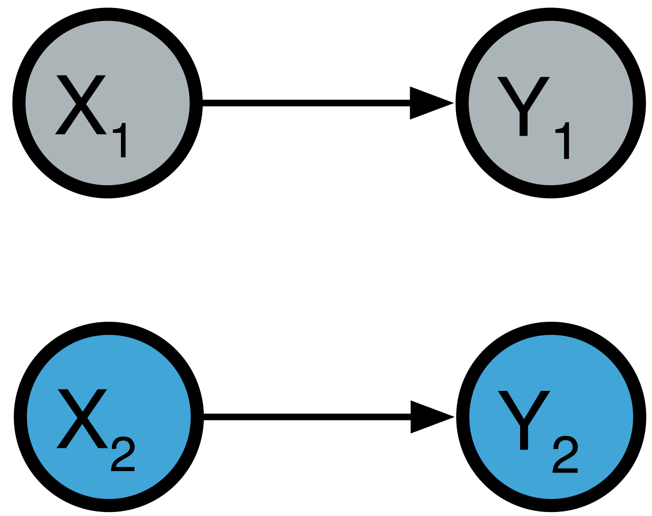


Secondary spectrum usage



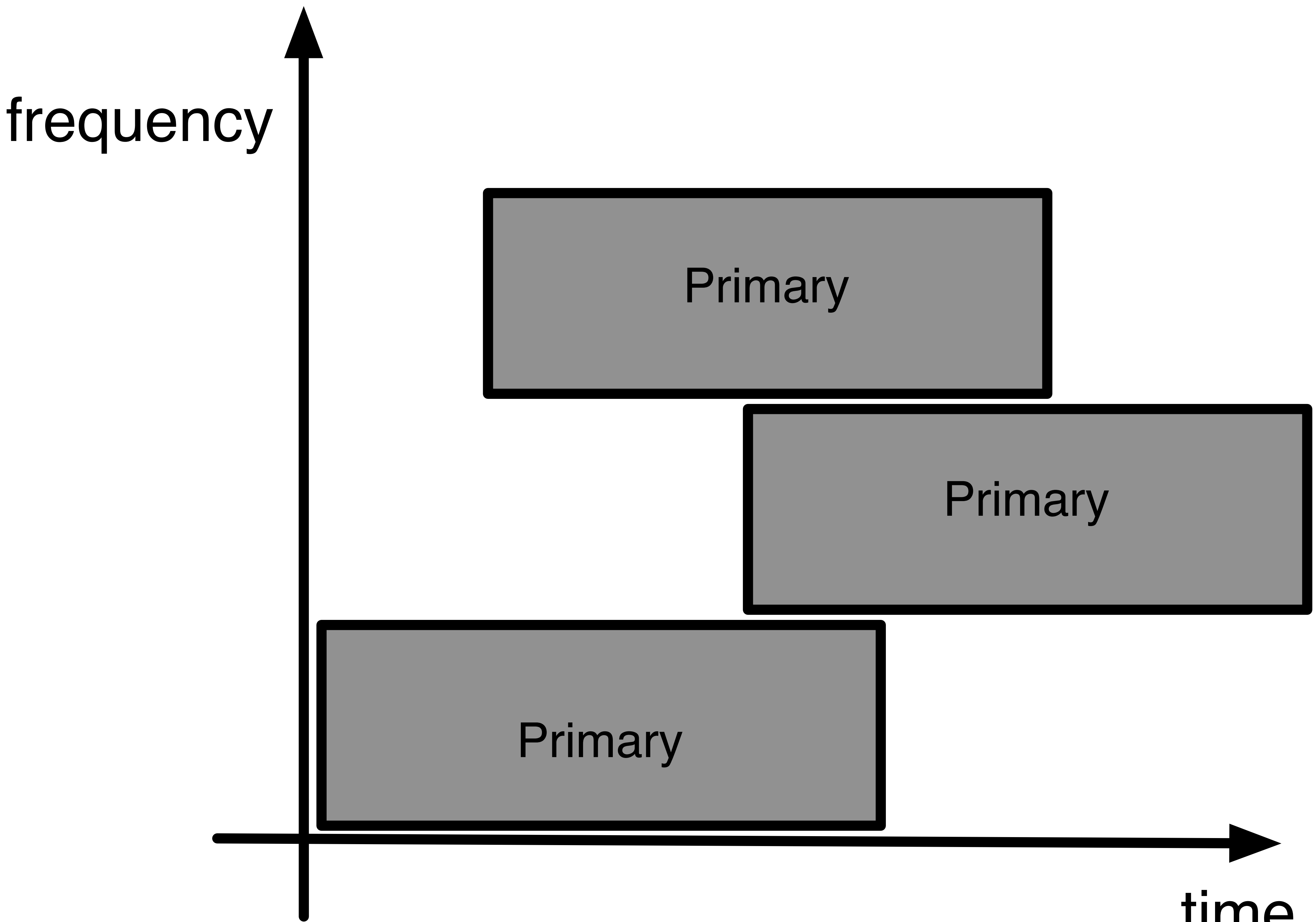
What can the cognitive link do?

Cognition

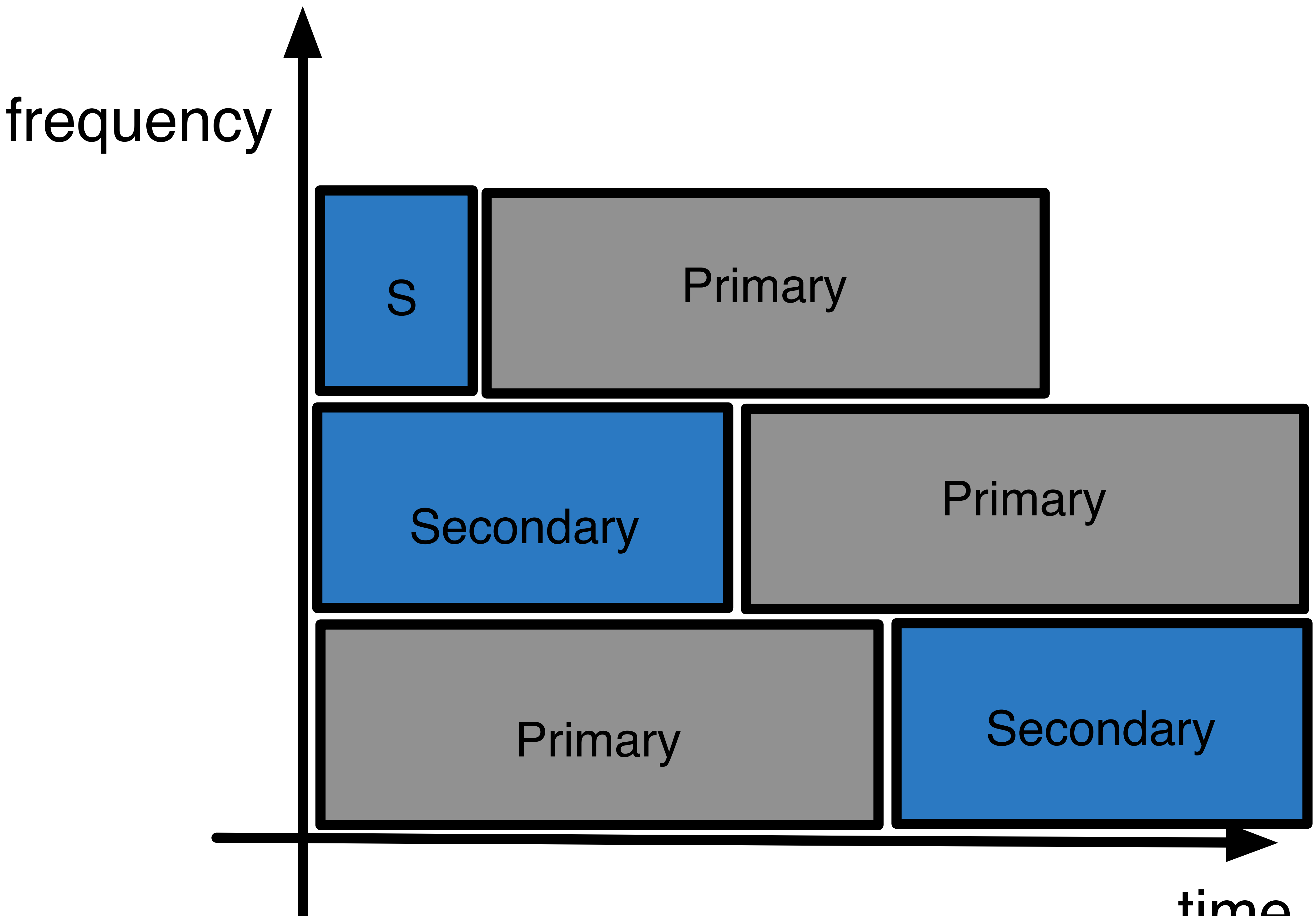


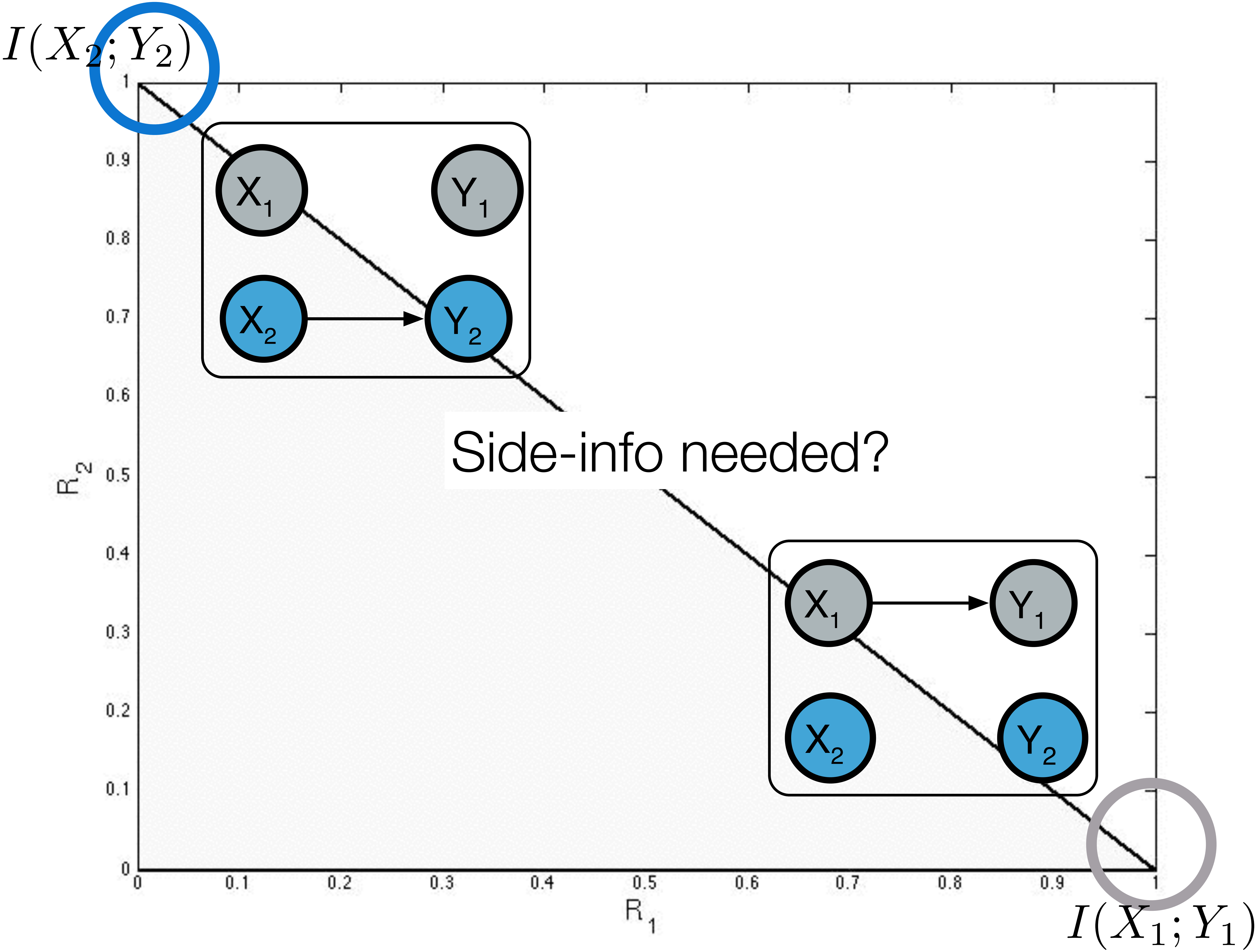
- Assumptions on primary/secondary models will dictate behavior + performance
- Cognition boils down to **side-information** and how to use it
- Use information theory to tell us which techniques are most promising

1. White spaces



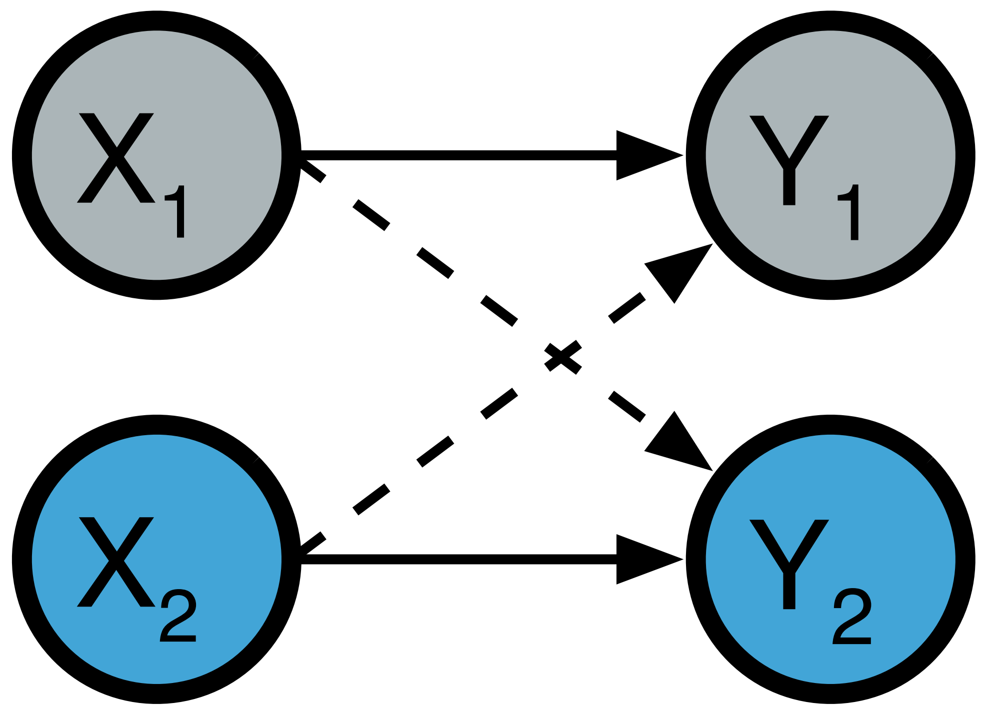
1. White spaces





$I(X_2; Y_2)$
R_2
R_1
X_1
Y_1
X_2
Y_2
Side-info needed?
X_1
Y_1
X_2
Y_2
$I(X_1; Y_1)$

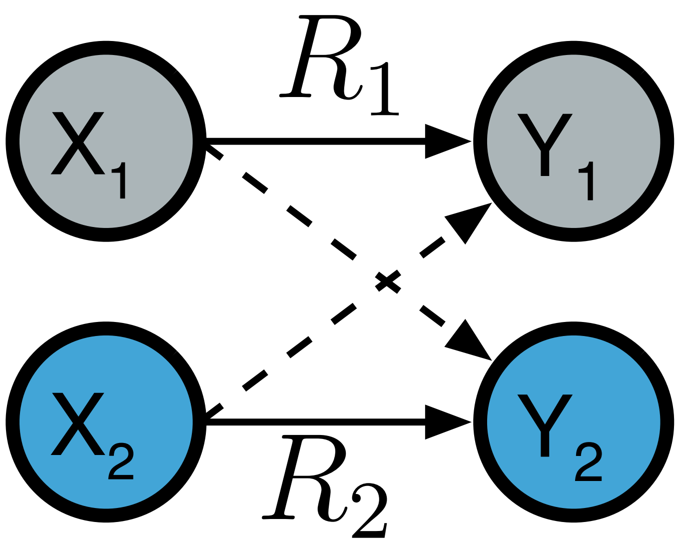
2. Just transmit



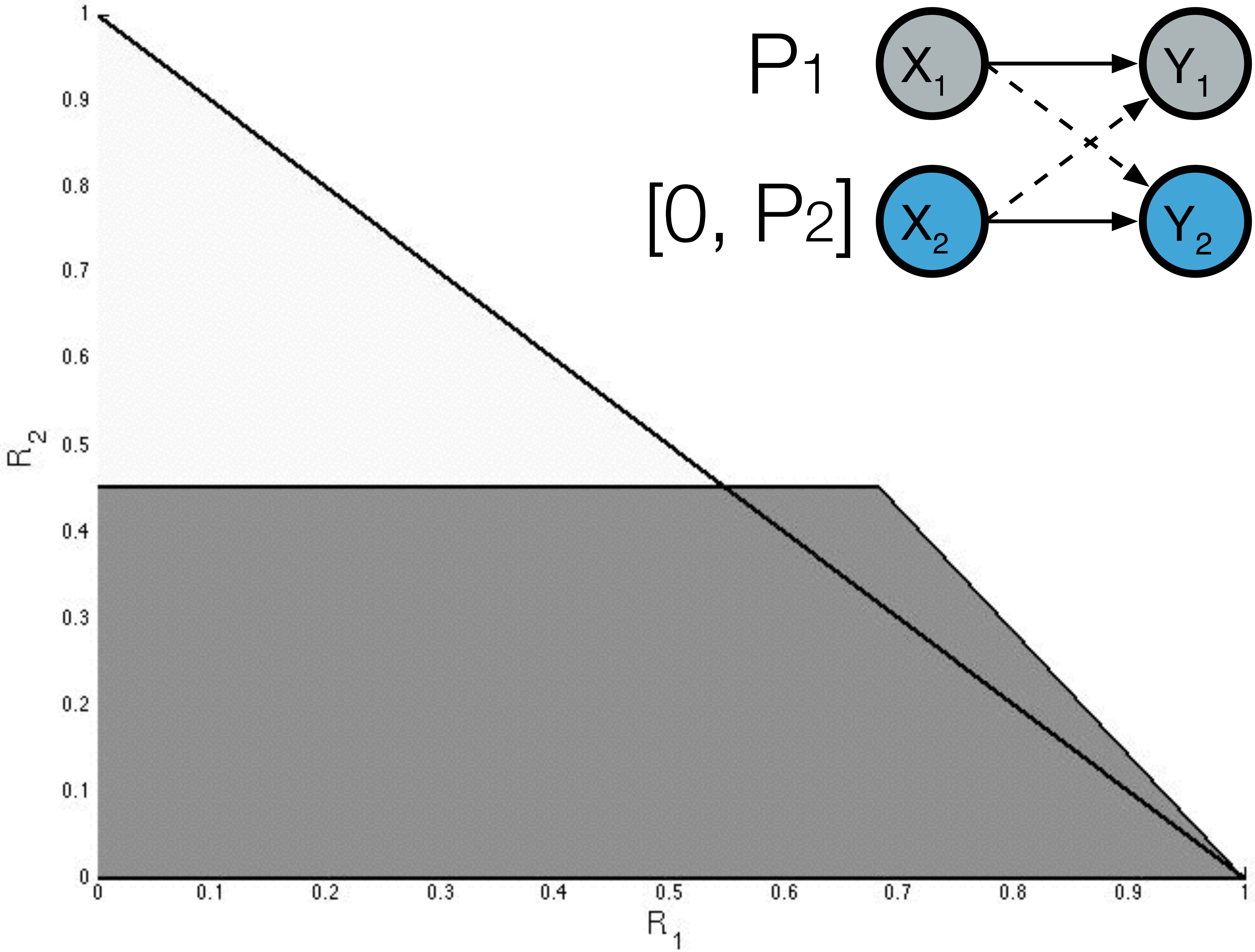
Interfere with each other!

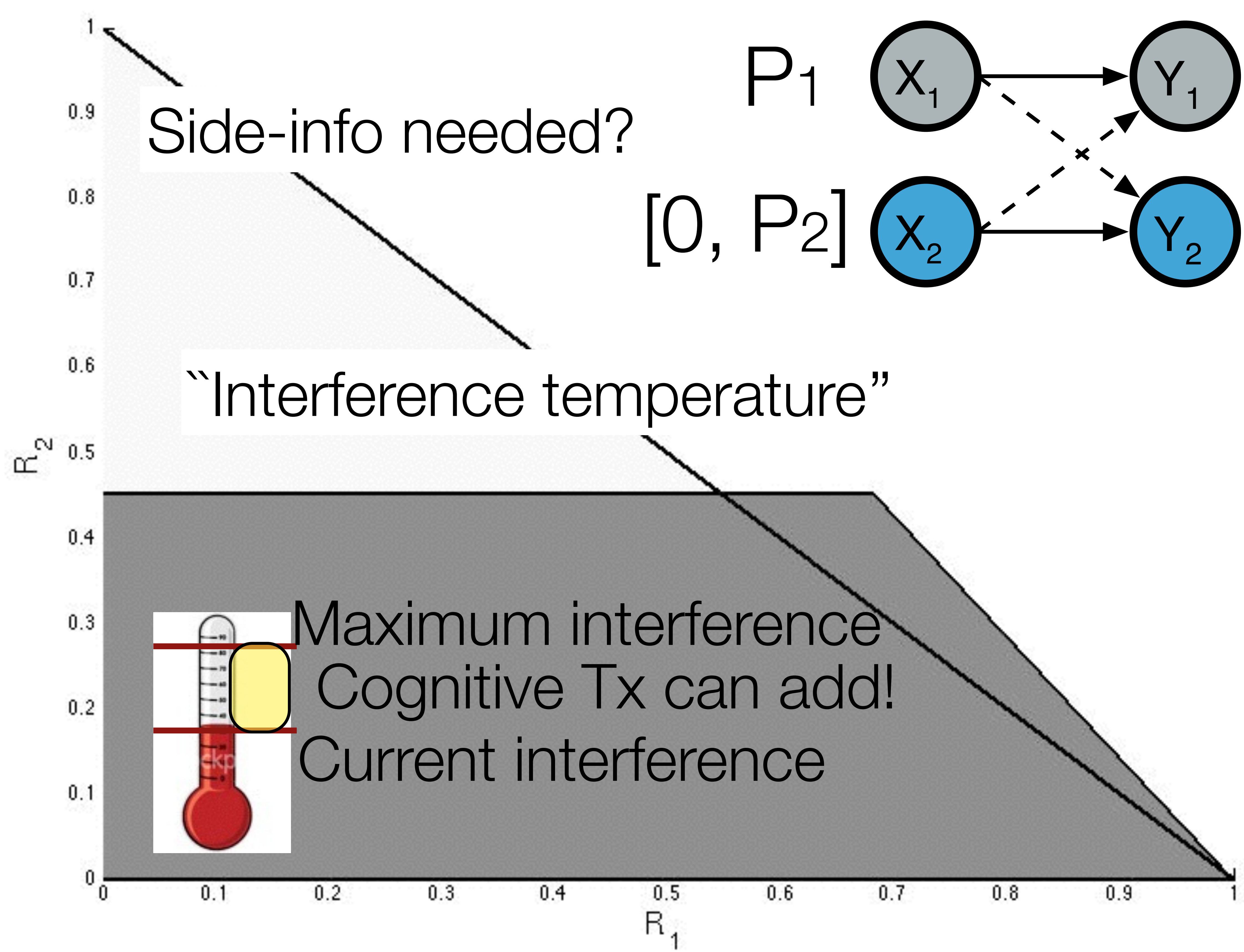
2. Just transmit

$$R_1 \leq \frac{1}{2}\log_2\left(1 + \frac{\text{Power of signal 1}}{\text{Interference from signal 2} + \text{ Noise}}\right)$$

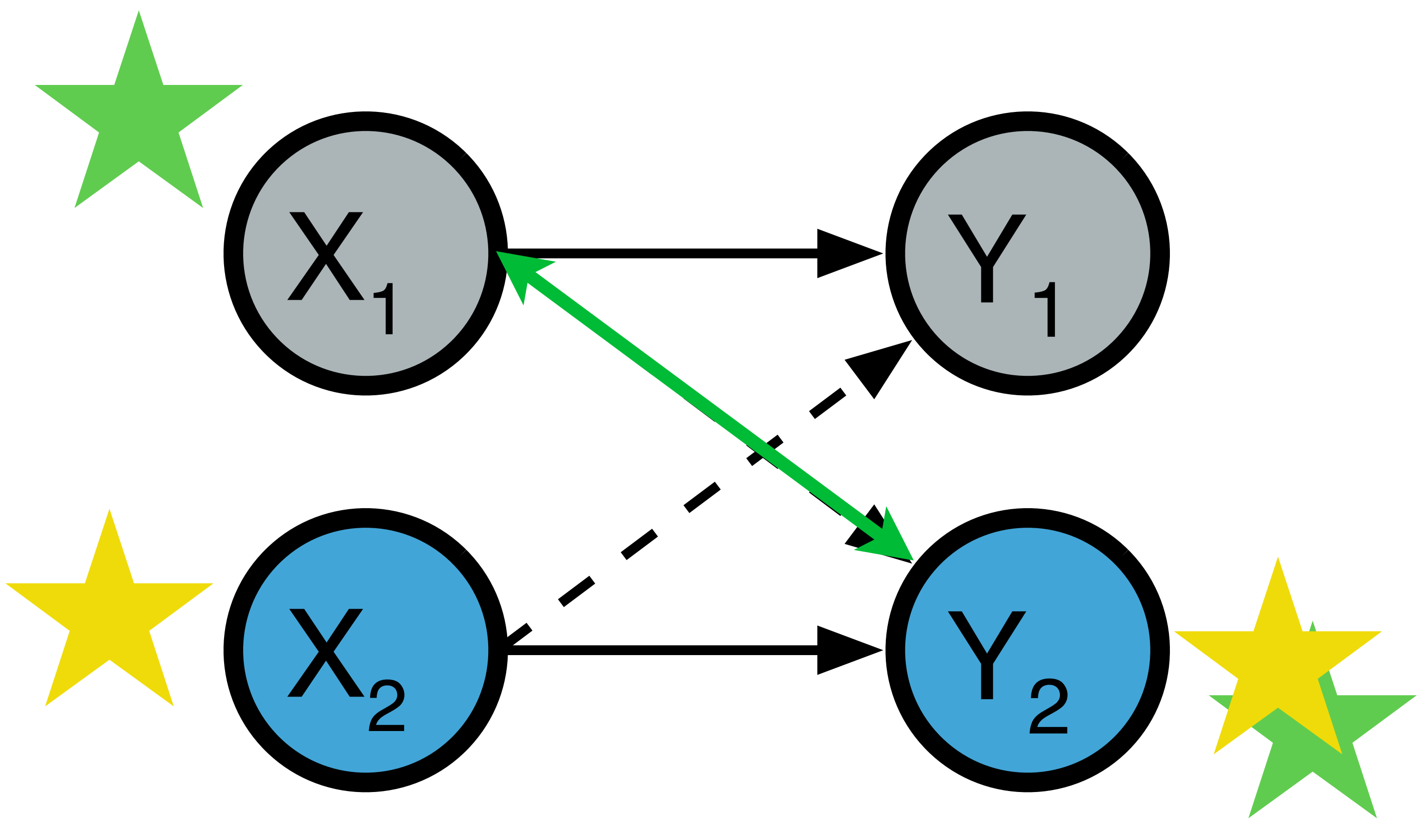


$$R_2 \leq \frac{1}{2}\log_2\left(1 + \frac{\text{Power of signal 2}}{\text{Interference from signal 1} + \text{ Noise}}\right)$$

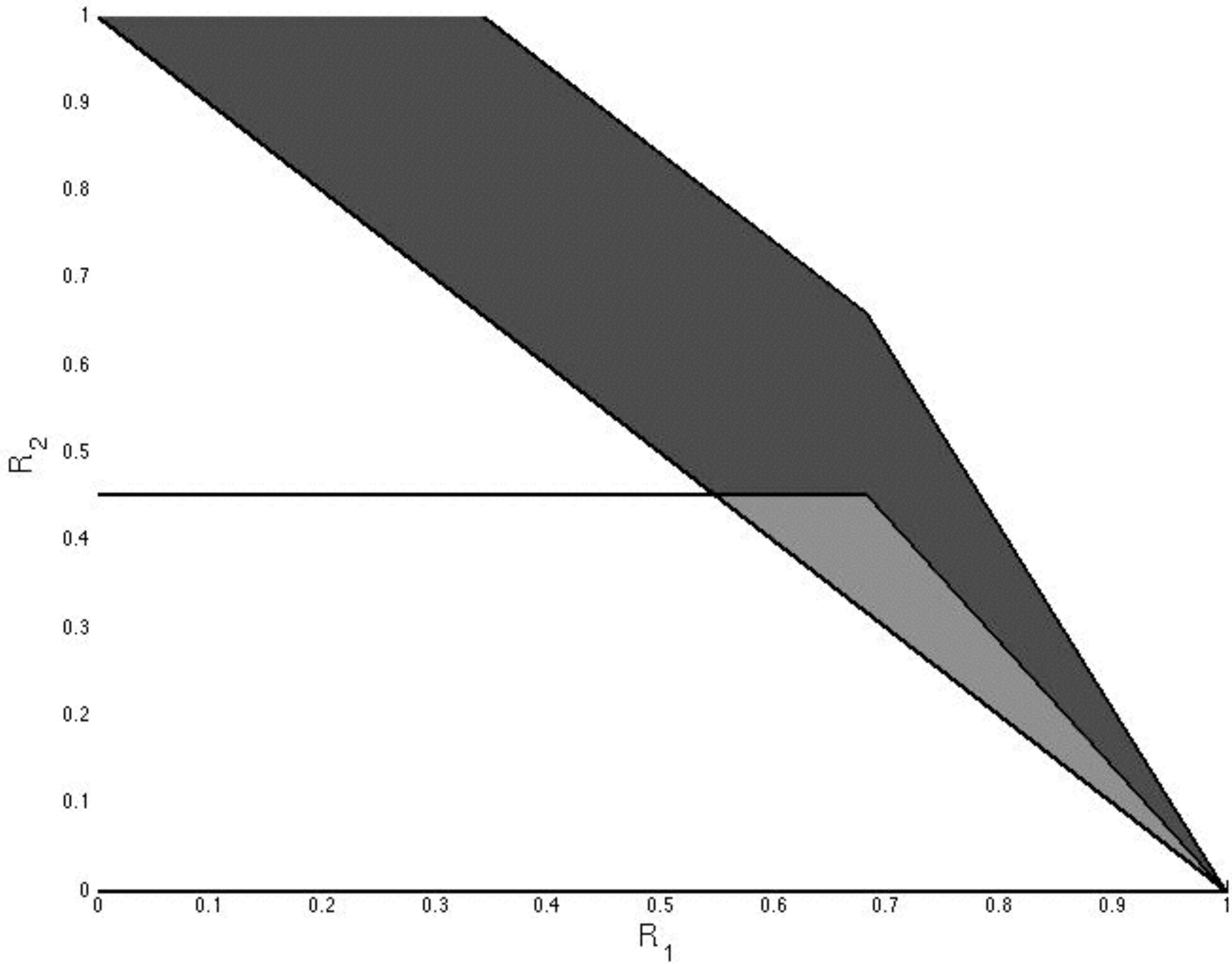

P1
[0, P2]
X1
X2
Y1
Y2
R2
R1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Side-info needed?
P1
[0, P2]
X1
X2
Y1
Y2
"Interference temperature"
Maximum interference
Cognitive Tx can add!
Current interference
R2
R1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

3. Opportunistic “cognitive” decoding

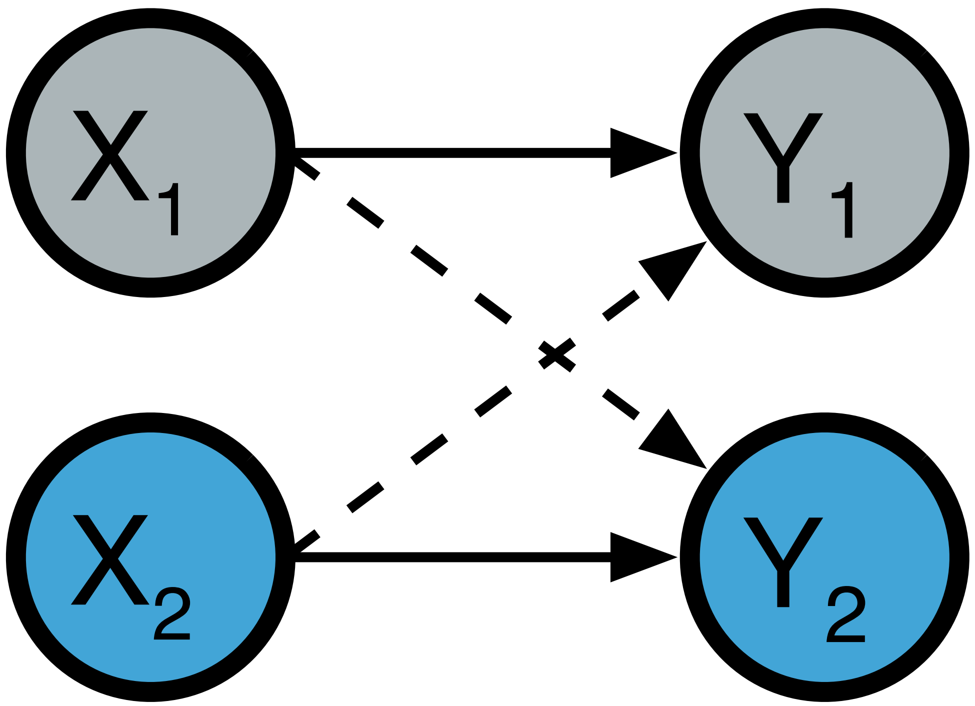


Side-info needed?



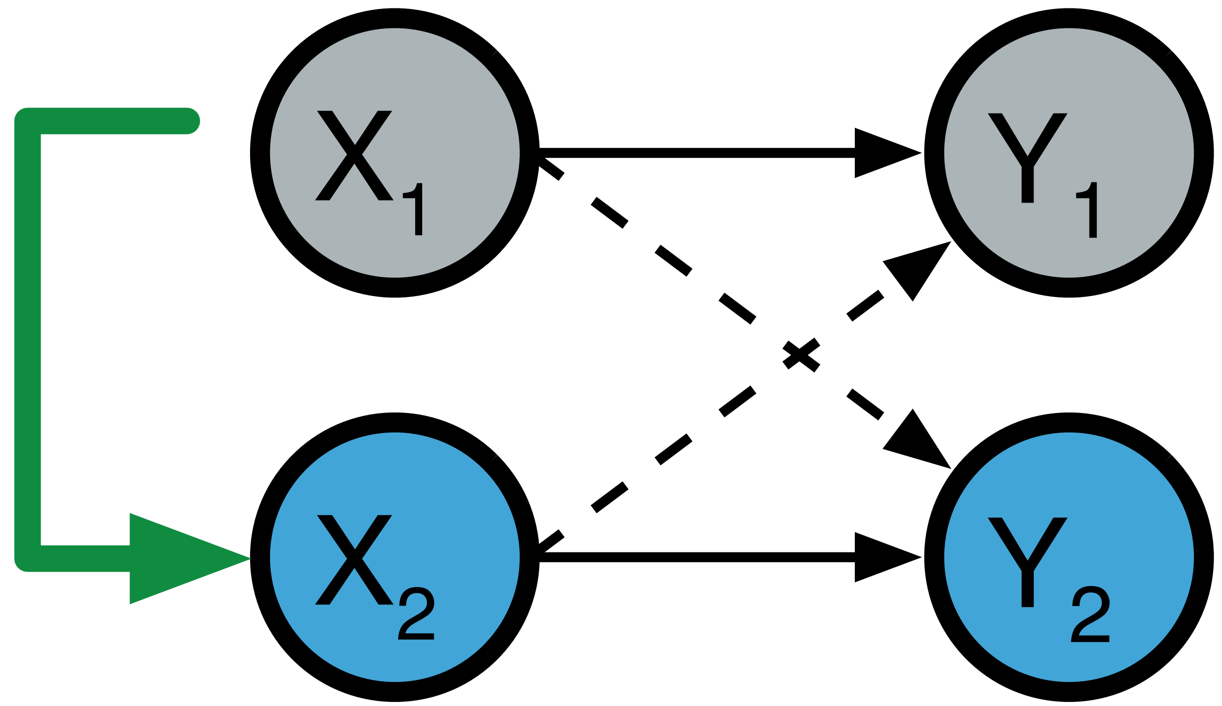
R_2
R_1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

4. Cognitive transmission



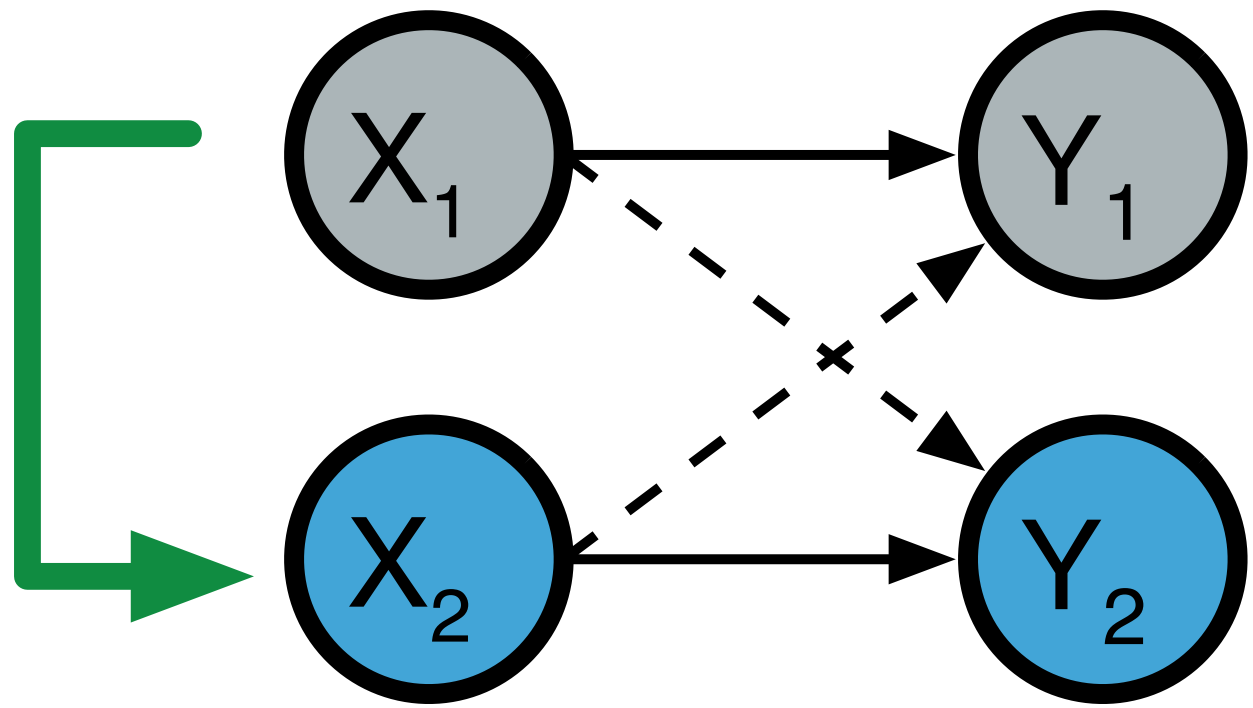
Interference!

4. Cognitive transmission



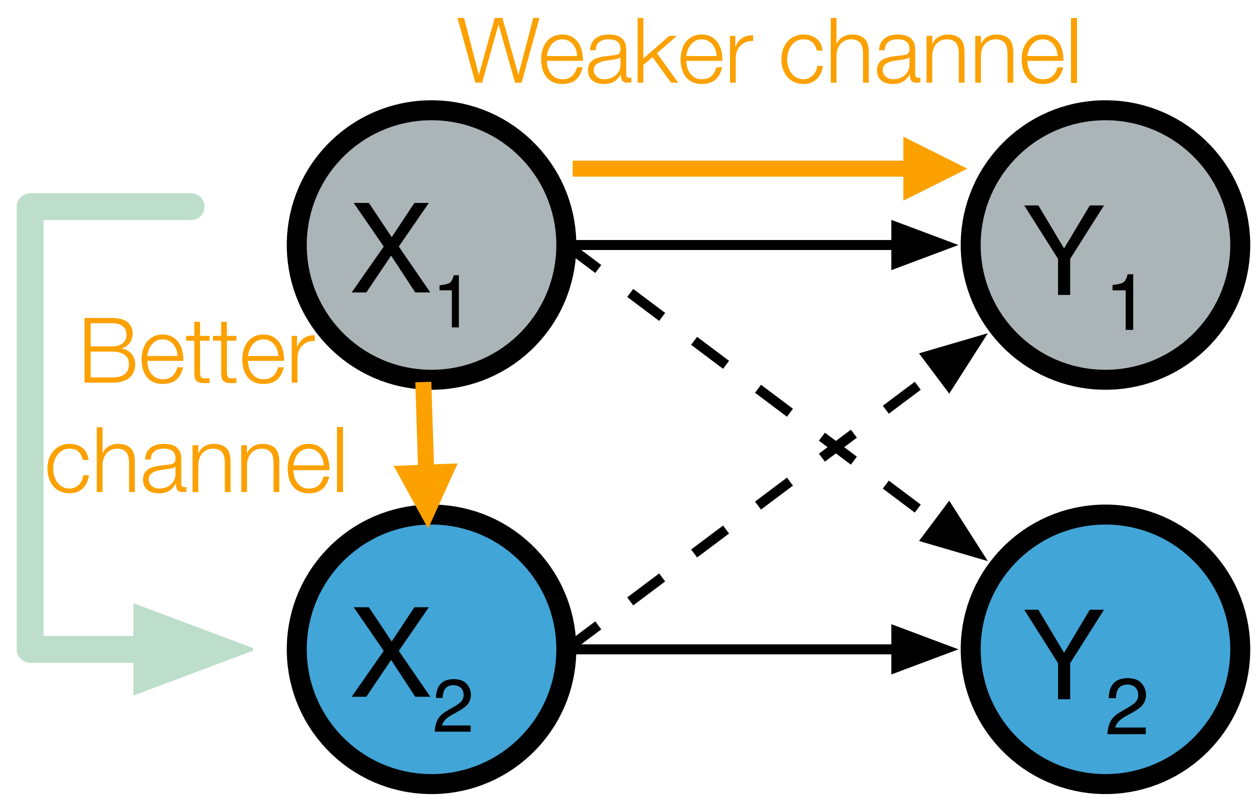
Interference can be reduced

Simultaneous Cognitive Transmission



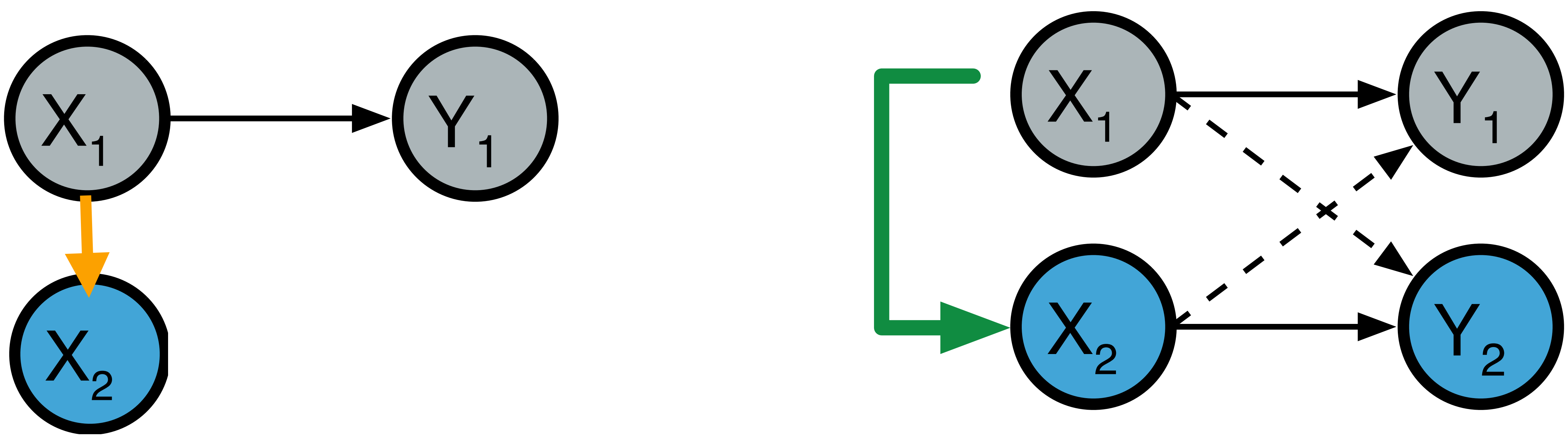
Assumption: Tx 2 knows message encoded by X_1 a-priori

Simultaneous Cognitive Transmission



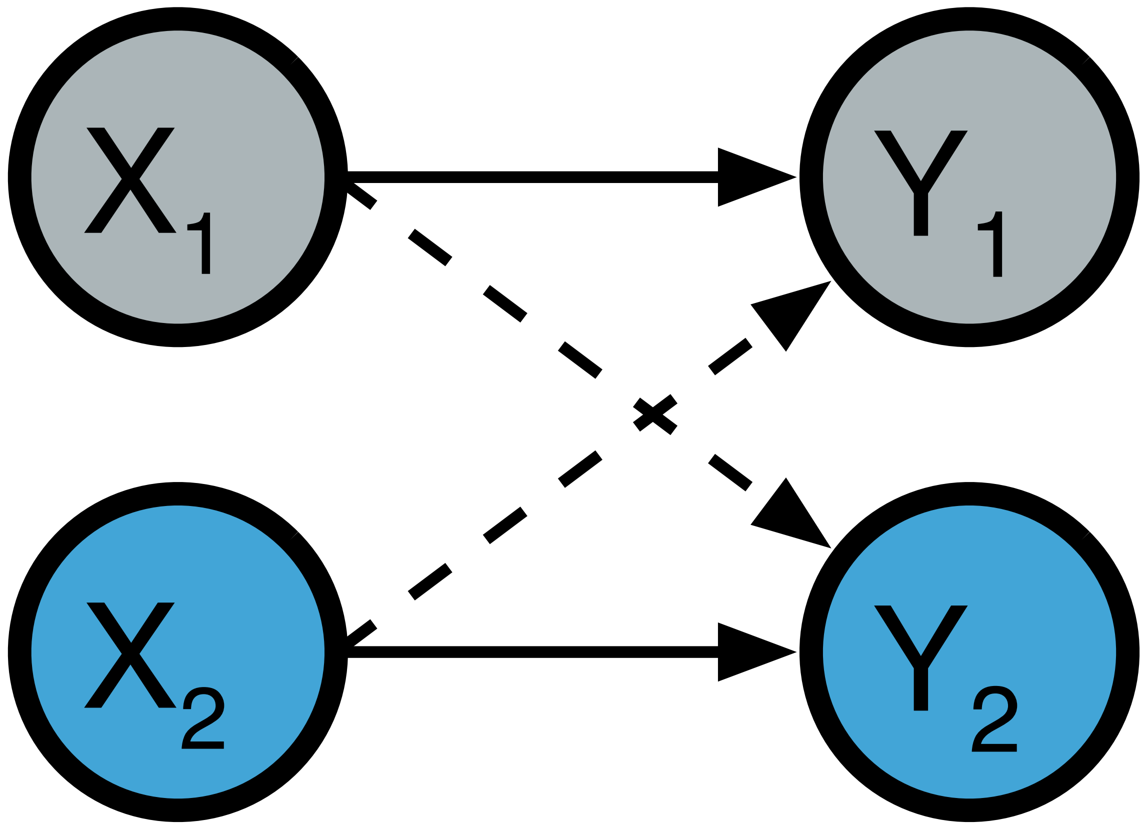
Cognitive Tx may obtain primary's message in a fraction of the time

Simultaneous Cognitive Transmission



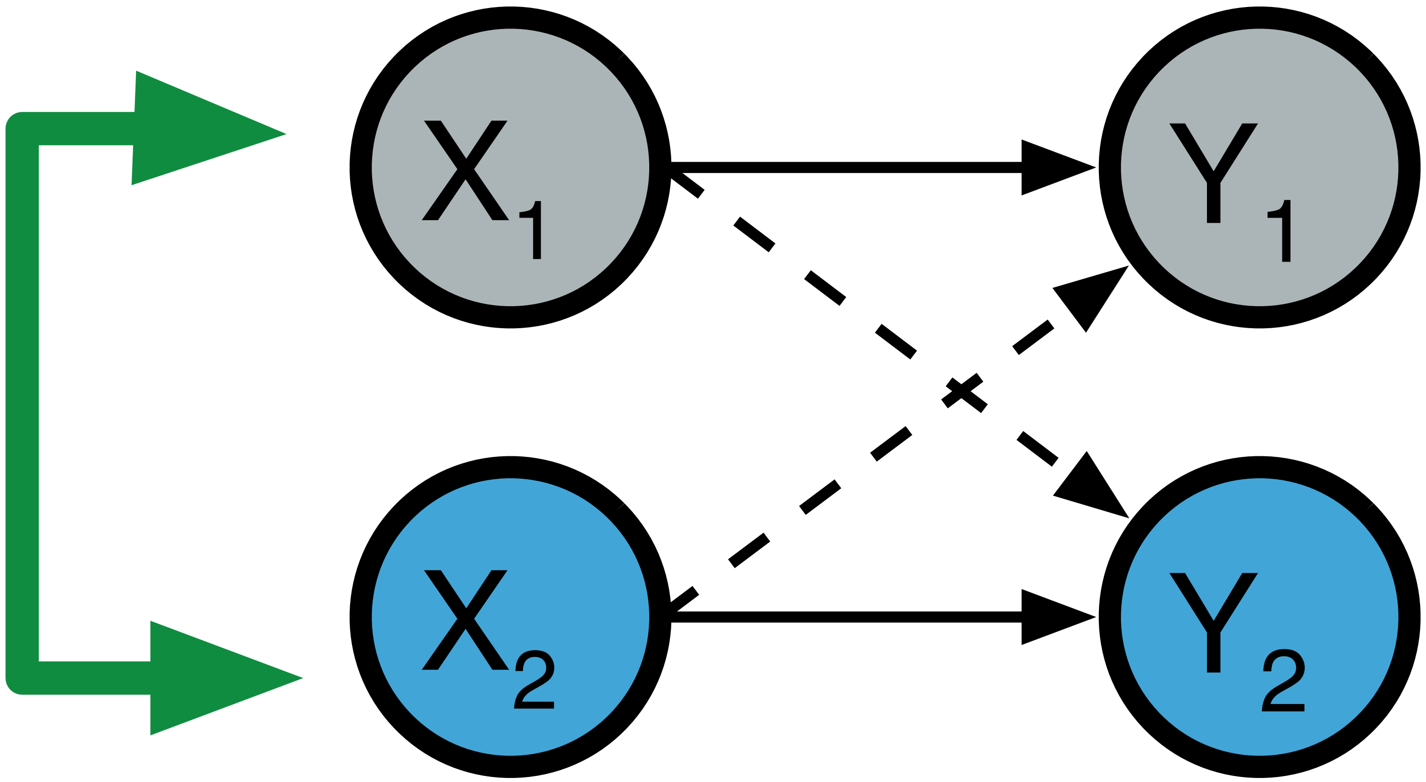
Primary transmission *Re-transmission*

Cognitive Tx may overhear primary's message



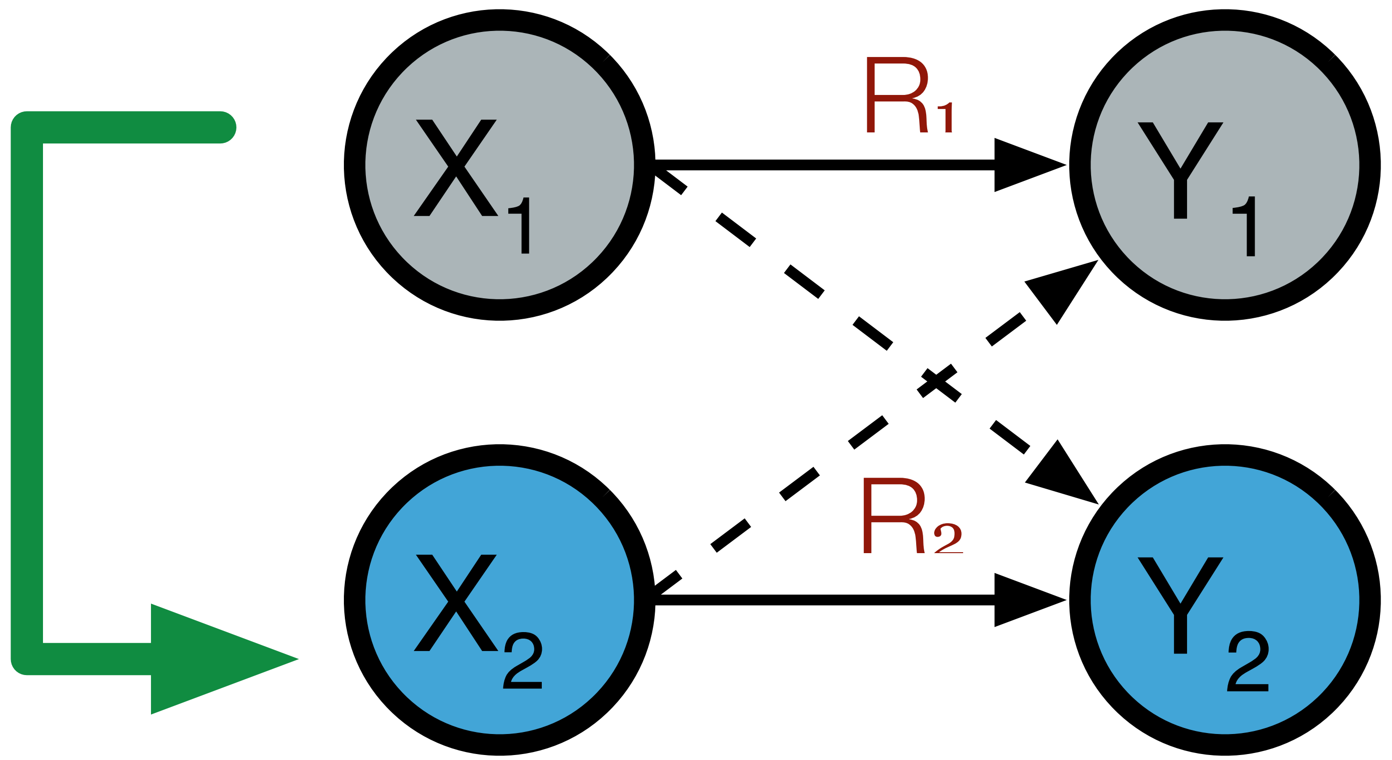
“Competitive”

Interference channel



“Cooperative”

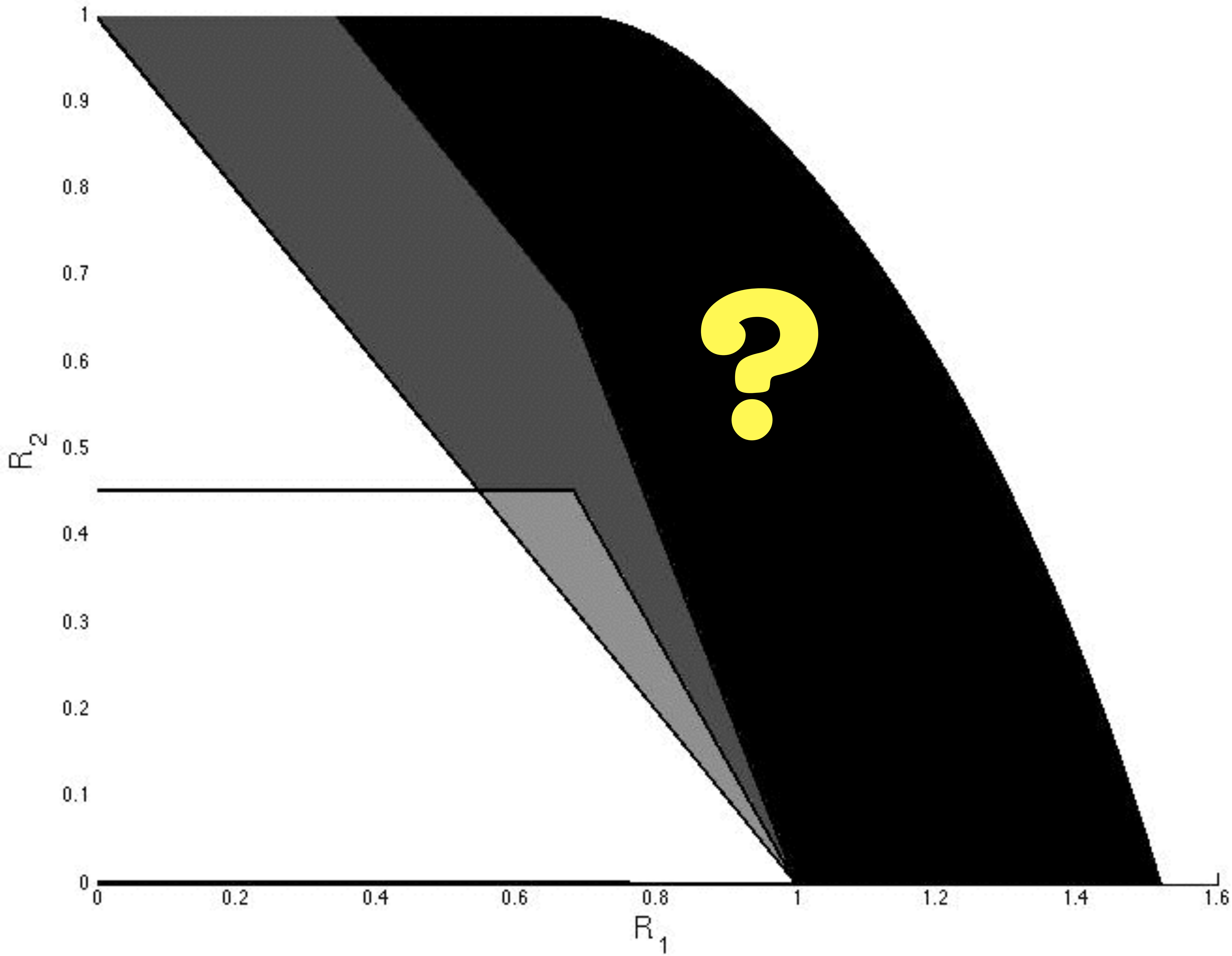
2 Tx antenna
Broadcast channel



“Cognitive”

Cognitive channel

What rates (R_1, R_2) are achievable?

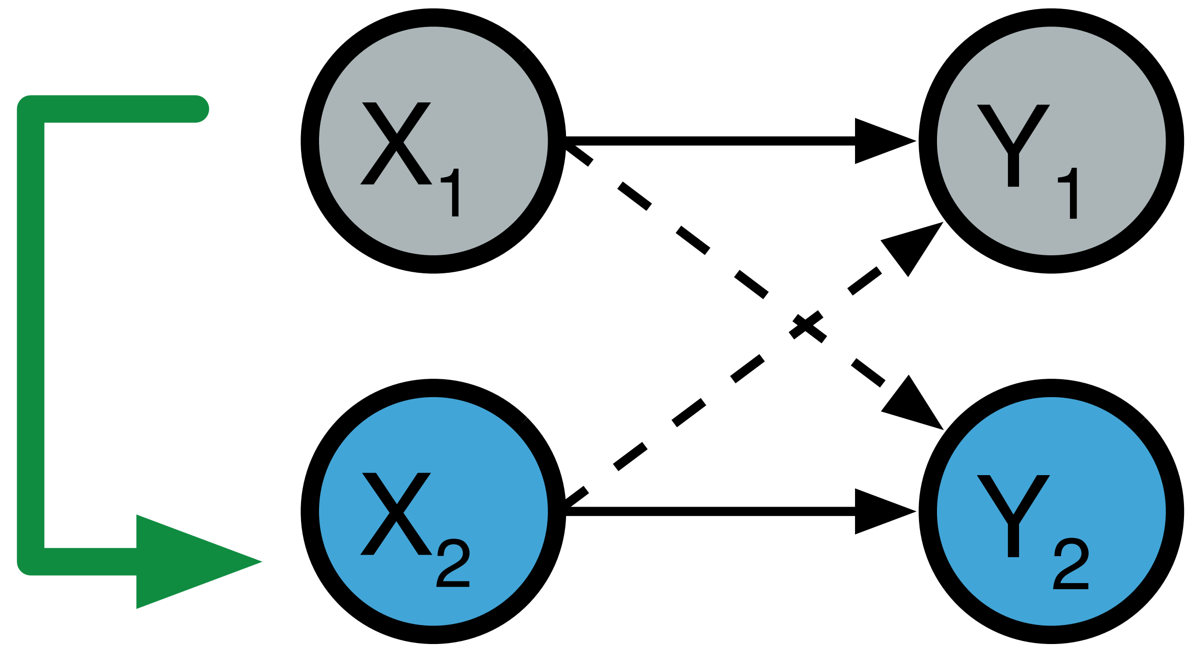


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1
?

Intuition

$$P_1'$$

$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$

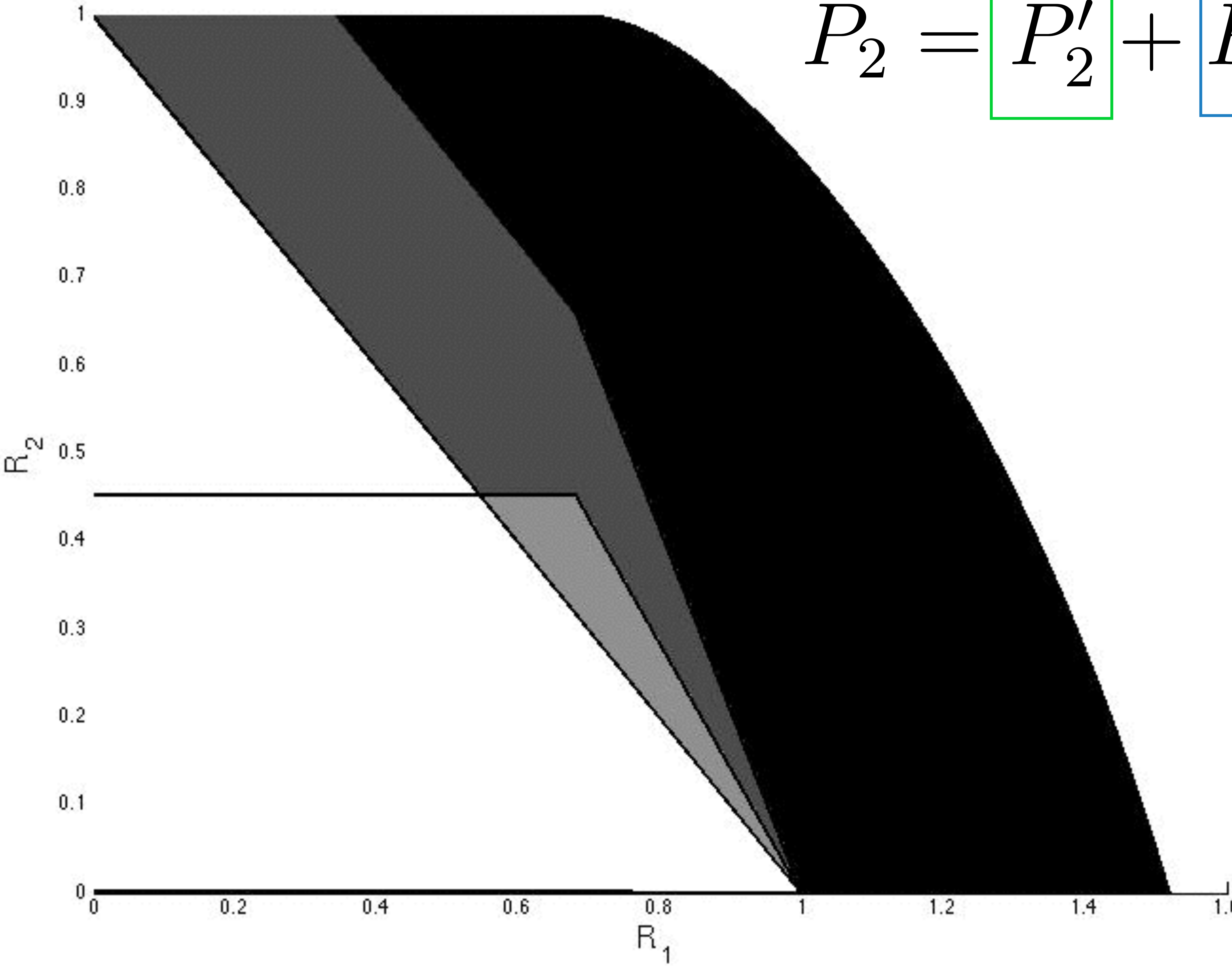


A priori message knowledge

aid transmission
SELFLESS

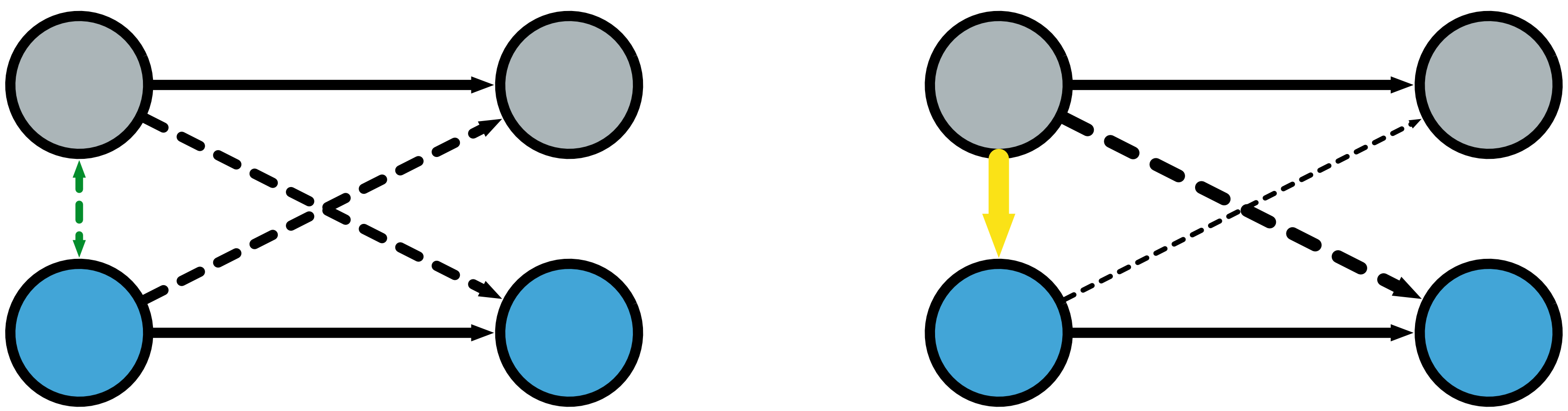
mitigate interference
SELFISH

$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$



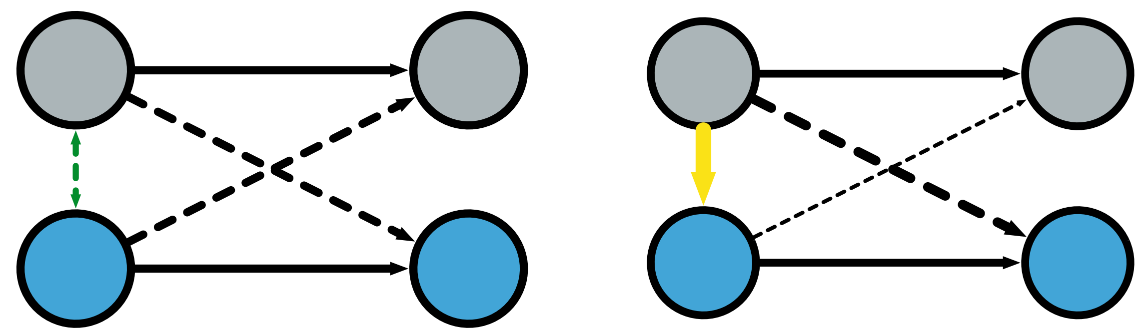
Extensions of "cognition" in multi-user IT

- causal versus non-causal cognition

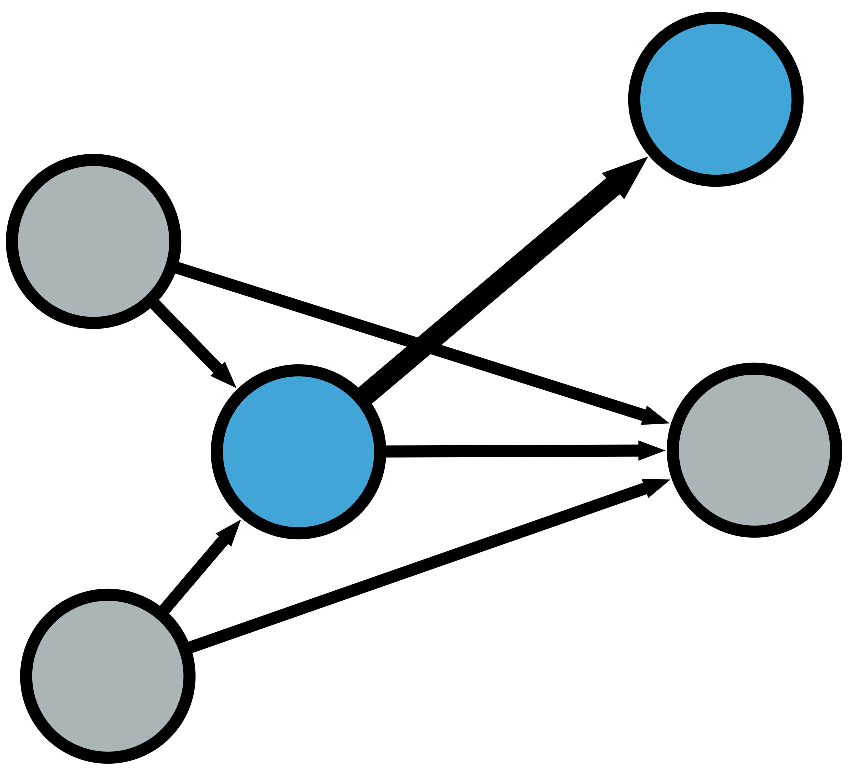


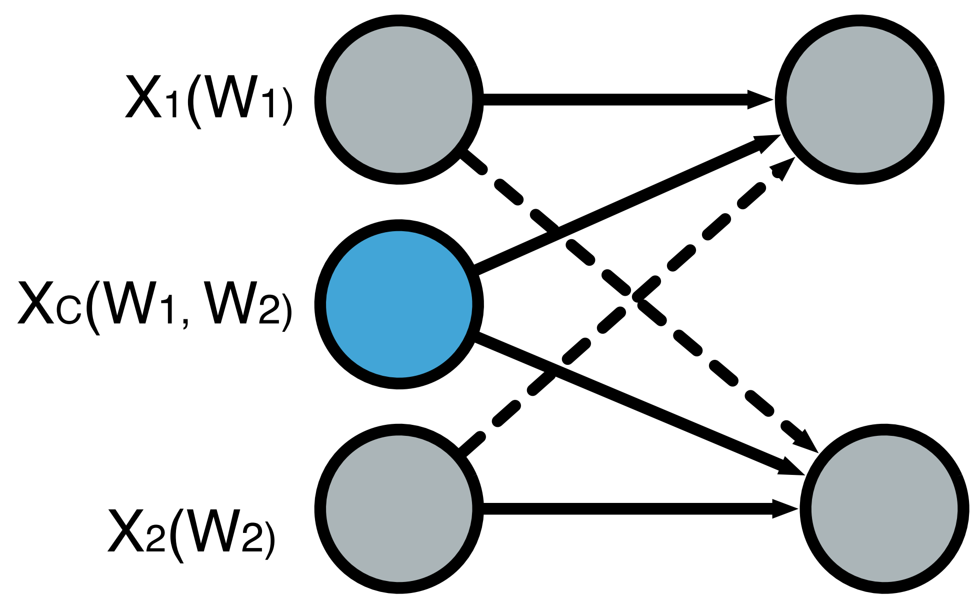
Extensions of “cognition” in multi-user IT

- causal versus non-causal cognition



- cognitive relay: interference, relay channels





Extensions of “cognition” in multi-user IT

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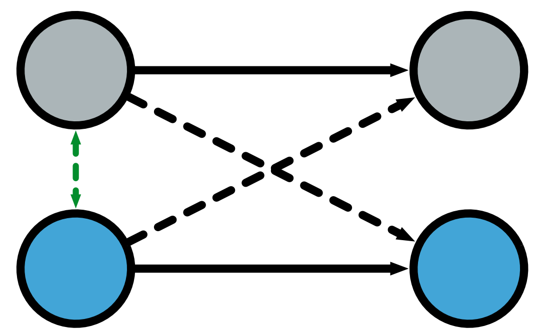
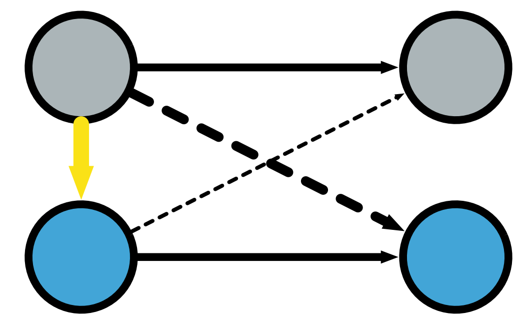
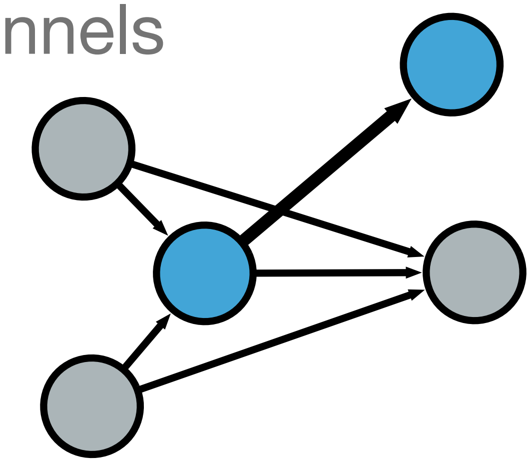
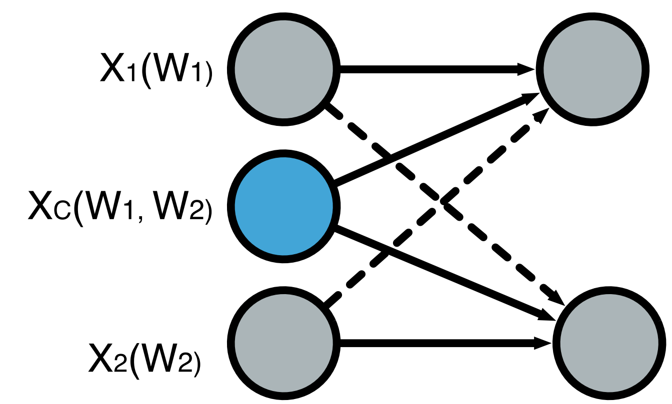


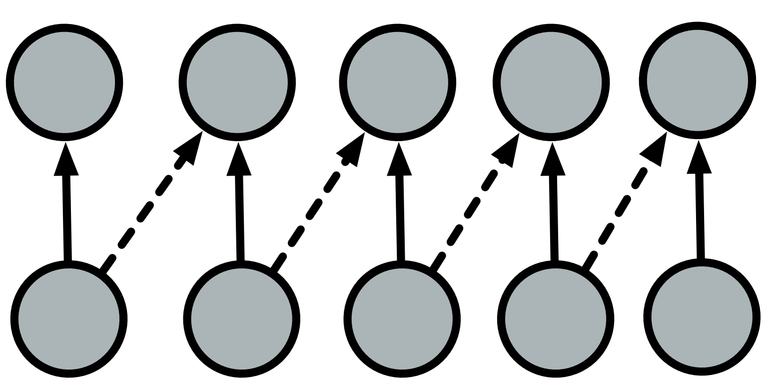


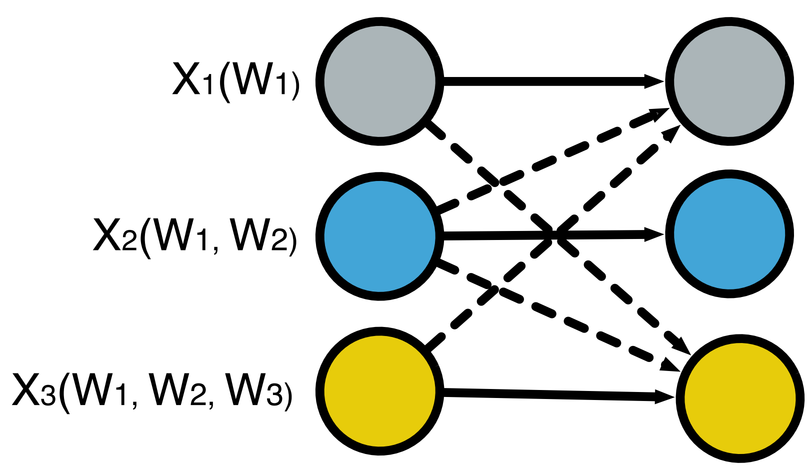
- cognitive relay: interference, relay channels

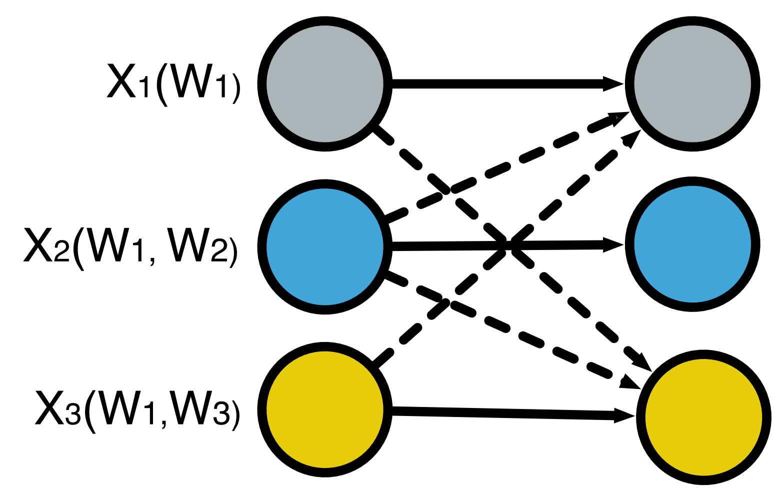




- more cognitive users, more scenarios....





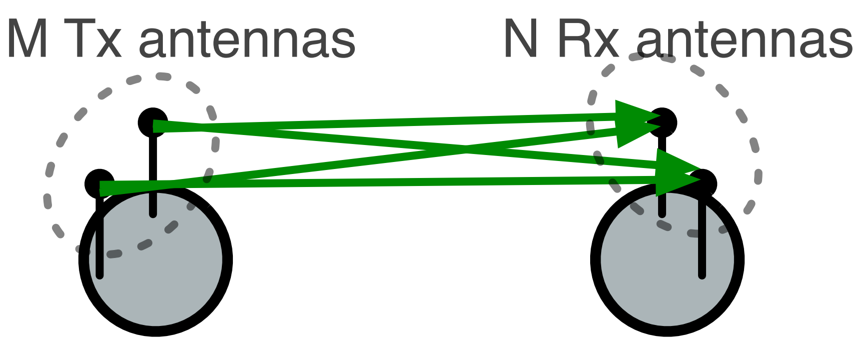


$$\begin{aligned}\eta(\mathrm{INT}) &= \min(M_1, N_1) \\ &+ \min(M_2 - N_1, N_2)^+ \, 1(M_1 > N_1) \\ &+ \min(M_2, N_2 - M_1)^+ \, 1(M_1 < N_1)\end{aligned}$$

Degrees of freedom: classical

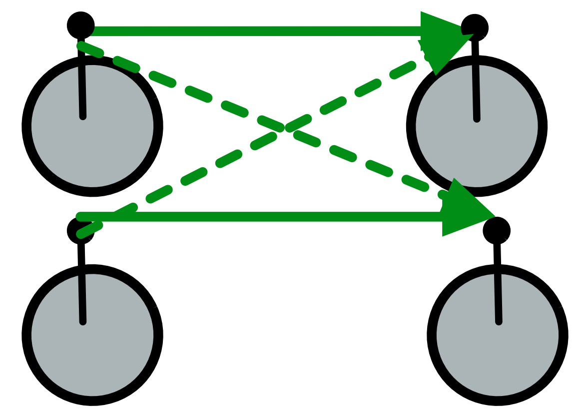
DOF = # "clean" channels in a multi-stream network

MIMO



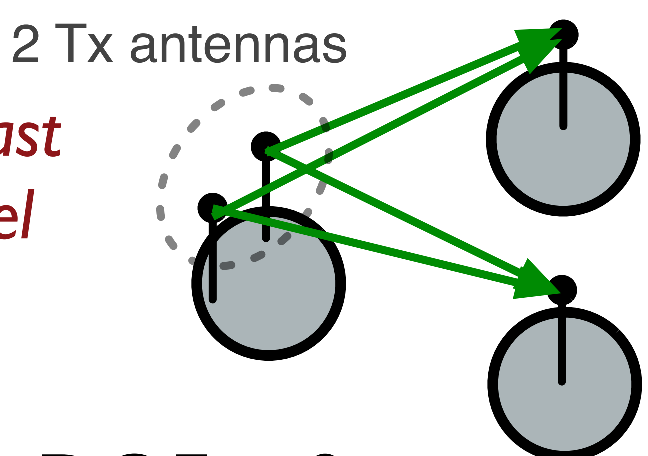
DOF = min(M,N)=2

Interference channel



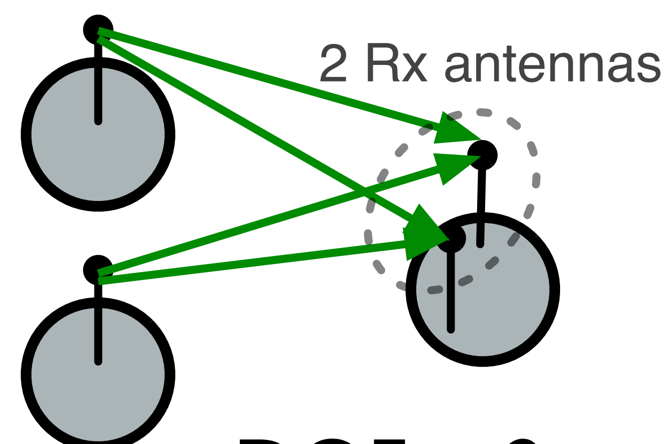
DOF = 1

Broadcast channel



DOF = 2

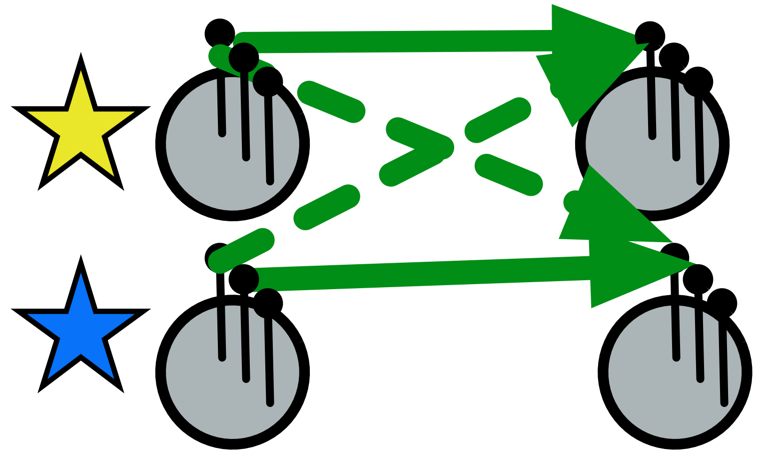
Multiple-access channel



DOF = 2

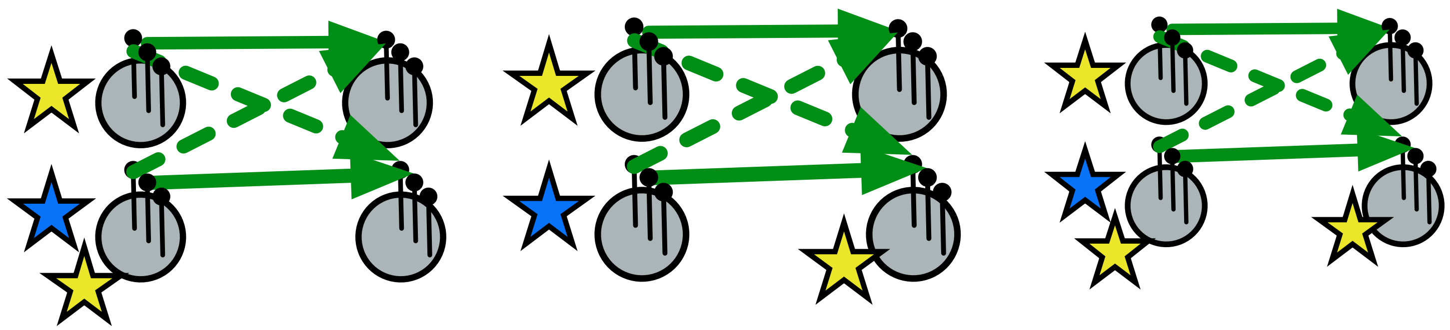
Degrees of freedom: cognitive, M antennas

MIMO interference channel



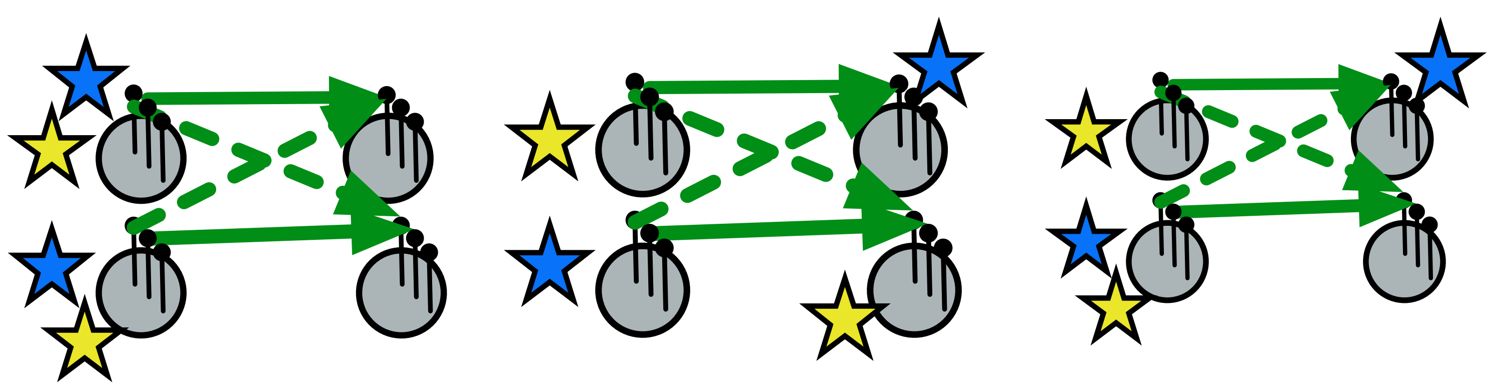
DOF = M

MIMO cognitive channel, cases a,b,c



DOF = M

MIMO cognitive channel, cases d,e,f



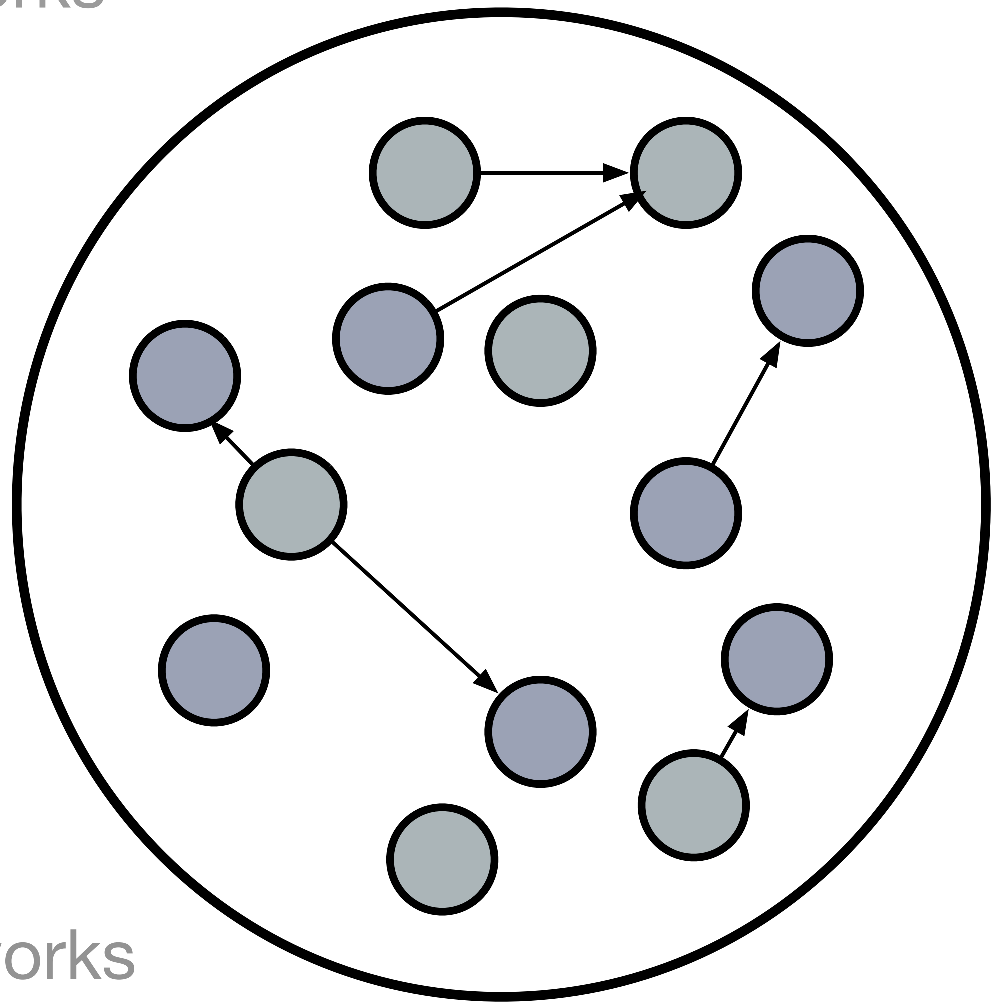
DOF = 2M

Syed A. Jafar, Shlomo Shamai, Degrees of Freedom Region for the MIMO X Channel , IEEE Transactions on Information Theory, Vol. 54, No. 1, Jan. 2008, Pages: 151-170.

Scaling laws # nodes n → ∞

- [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  - per-node throughput ~ $O(1/\sqrt{n} \log(n))$
  - Degradation is due to multi-hop and interference between nodes

- [Franseschetti et al. 2000]: ad hoc networks
  - per-node throughput ~ $O(1/\sqrt{n})$
  - percolation theory



- [Ozgur, Leveque, Tse 2007]: Cooperative ad hoc networks
  - nodes may cooperate as in a MIMO system
  - per-node throughput ~ O(1) (constant)

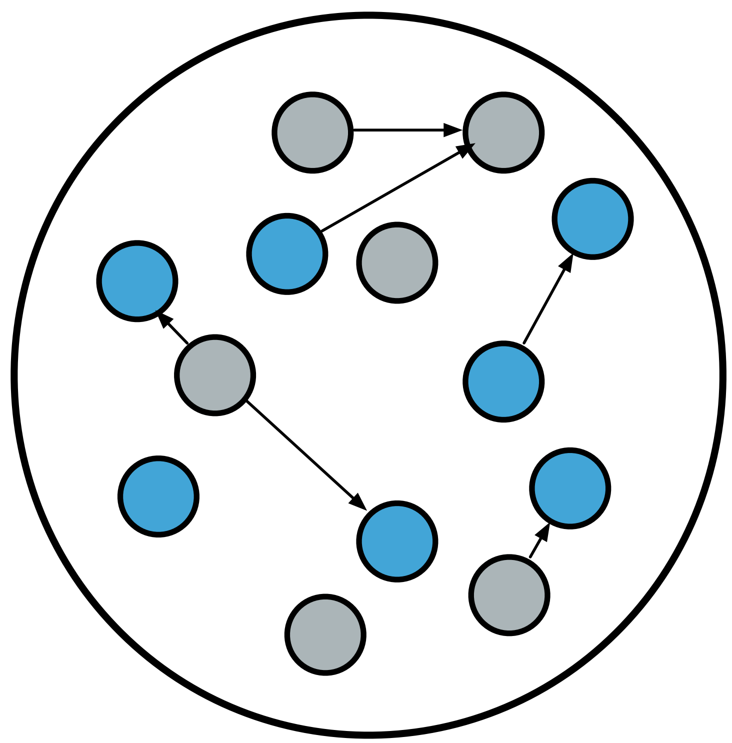
- Many many more...

Scaling laws: with cognition

- What we guarantee:

Primary nodes act as if cognitive network does not exist

Primary nodes achieve same scaling law as if cognitive network does not exist



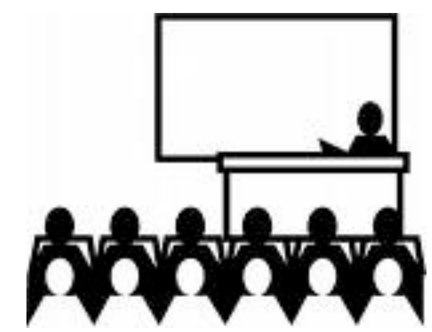
- What we prove:

$$T_p(n) = \Theta\left(\sqrt{\frac{1}{n \log n}}\right), \quad T_s(m) = \Theta\left(\sqrt{\frac{1}{m \log m}}\right)$$

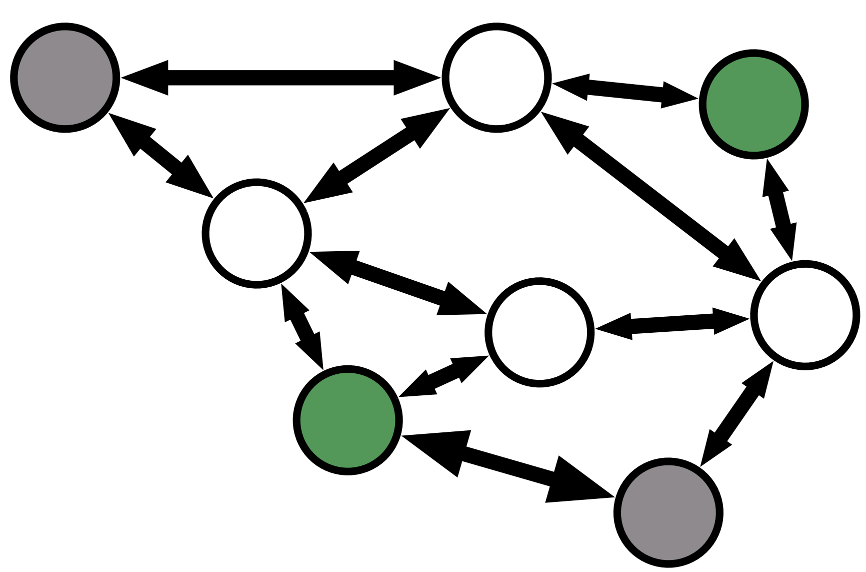
Other interesting problems

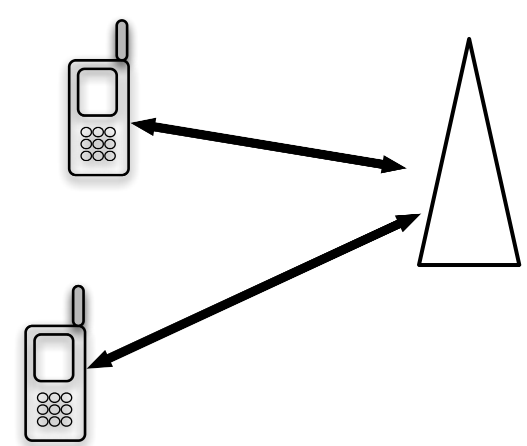
- Two-way wireless networks

- communication inherently a dialogue, not a sequence of monologues



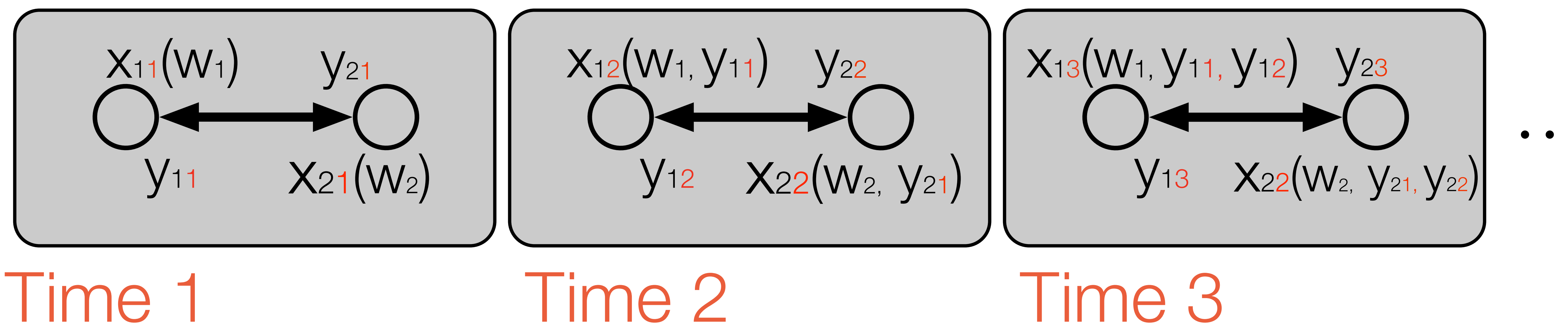
- mostly considered one-way, time to change that!





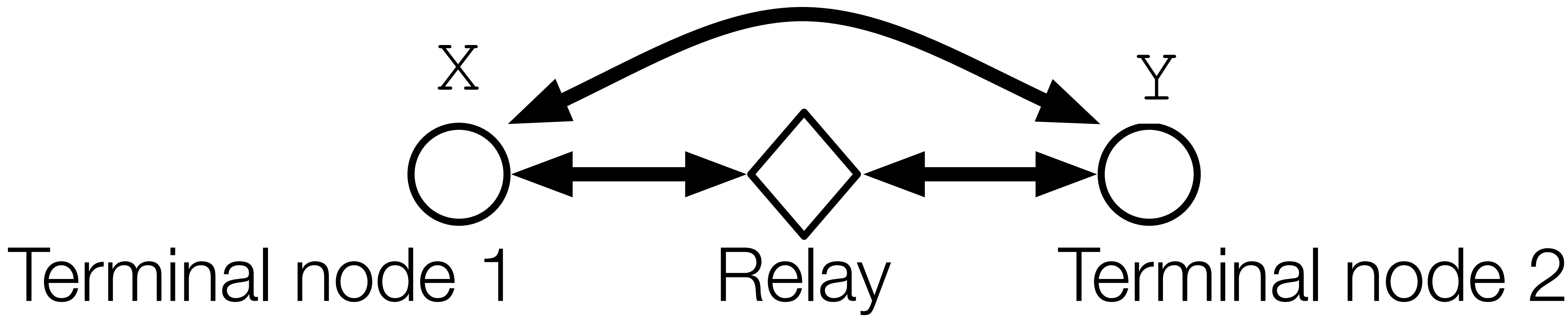
Two-way channel

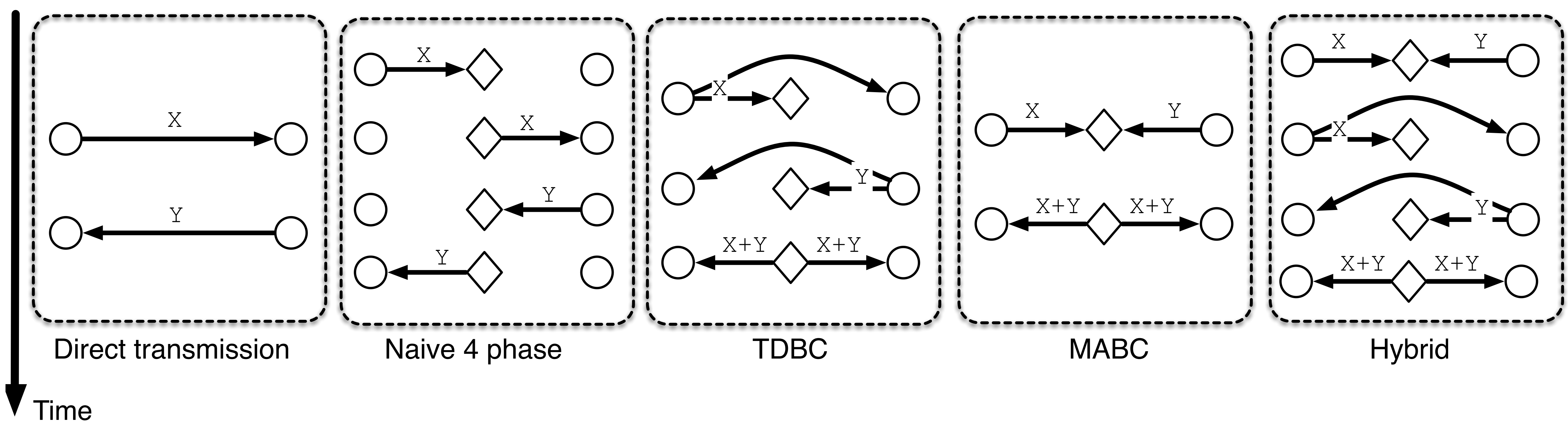
- Simple channel BUT capacity unknown in general!
- Why so hard?



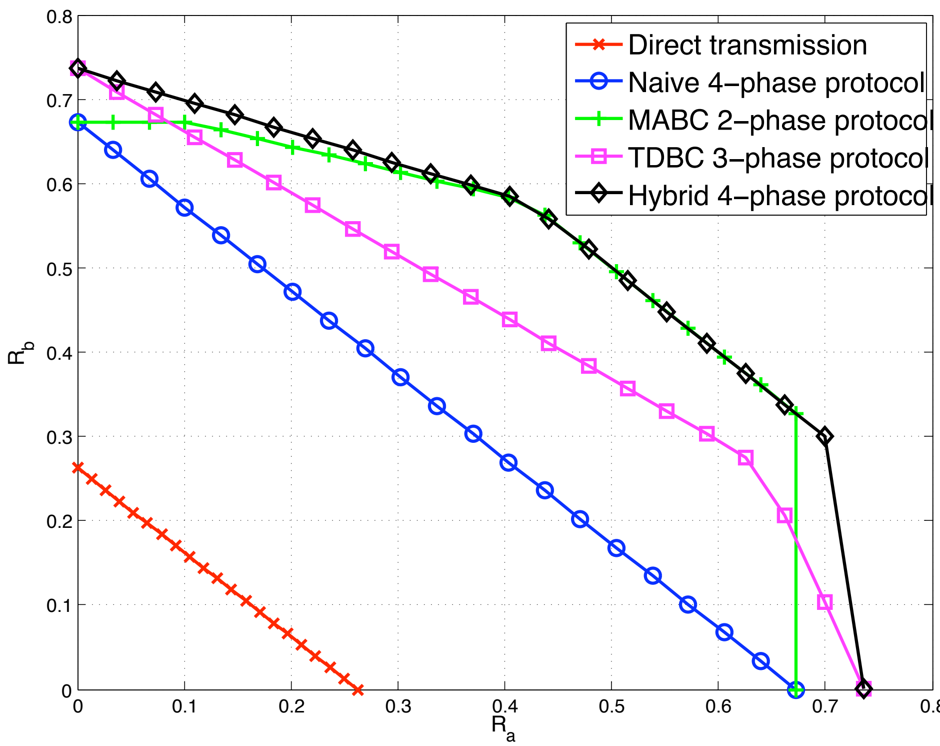
- Known for: restricted, Gaussian and push-to-talk channels

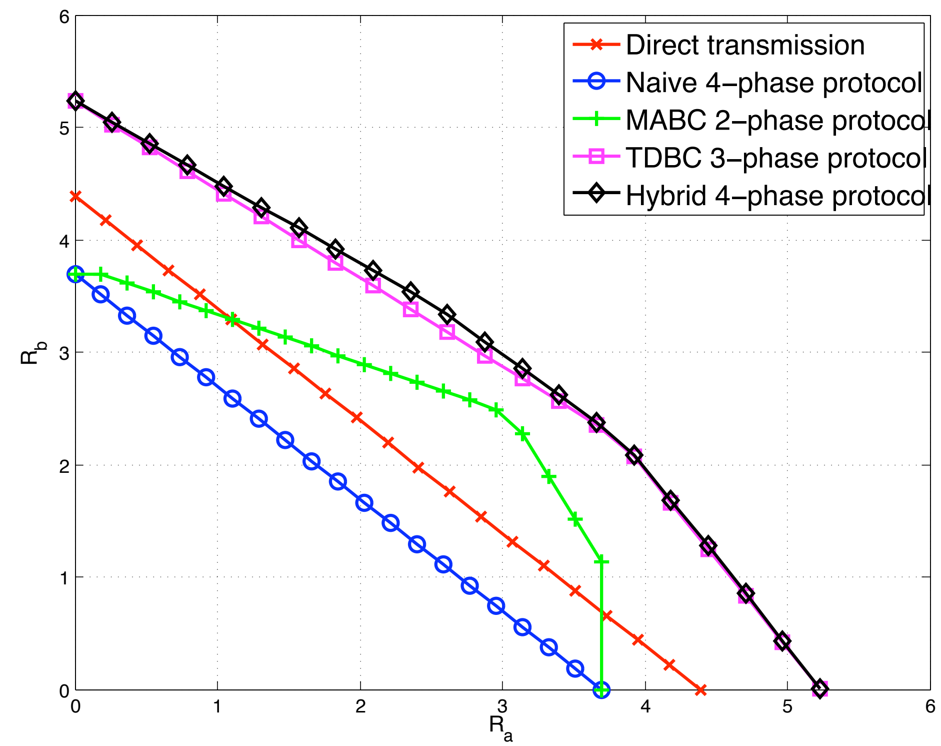
Two-way relay channel

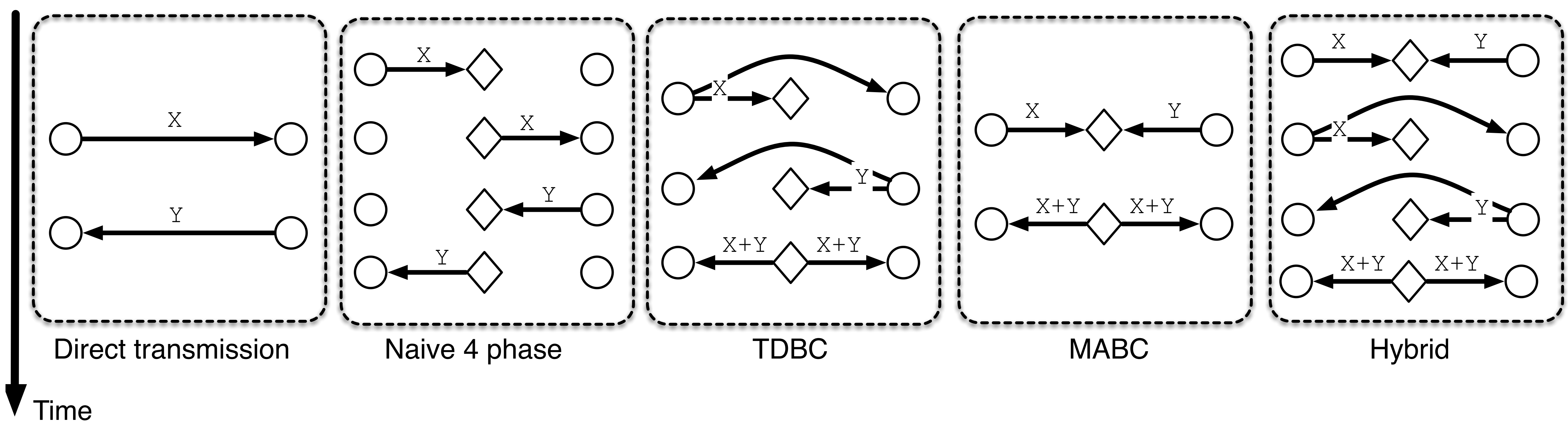




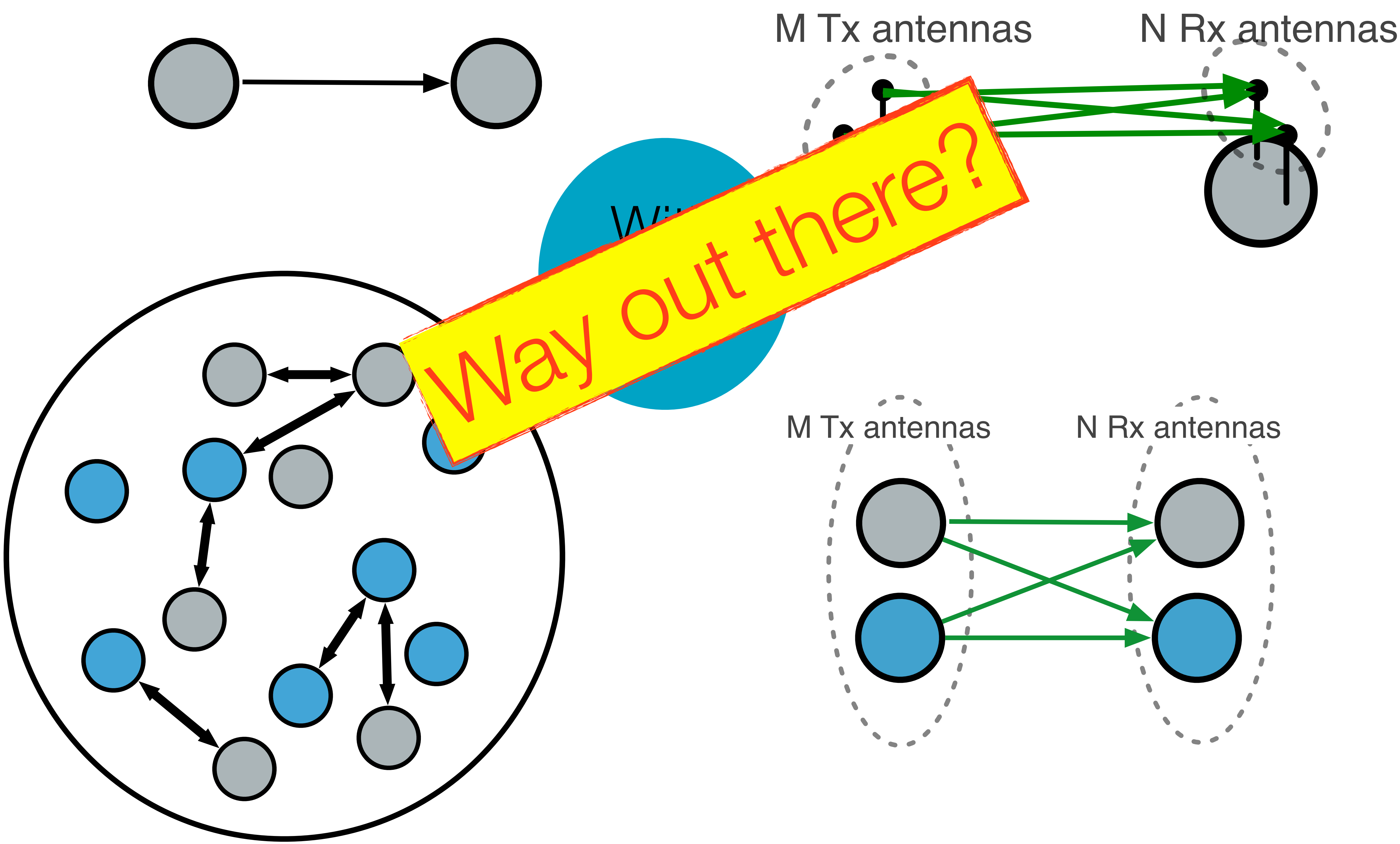
Two-way relay channel







Conclusions



Thank you

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