Information theory + cognitive radio?

More relevant nowadays

Source coding

<table>
<thead>
<tr>
<th>Source</th>
<th>ASCII</th>
<th>Coded source</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>110 0001</td>
<td>00</td>
</tr>
<tr>
<td>b</td>
<td>110 0010</td>
<td>101</td>
</tr>
<tr>
<td>c</td>
<td>110 0011</td>
<td>011</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>z</td>
<td>111 1010</td>
<td>10011</td>
</tr>
</tbody>
</table>

Communication theoretic perspective

Q: What is the ultimate data compression?
A: The entropy, H

Q: What is the ultimate transmission rate of communication?
A: The capacity, C

Last time

Information theory

Source coding

Q: What is the ultimate data compression?
A: The entropy, H

Channel coding

Q: What is the ultimate transmission rate of communication?
A: The capacity, C

This time

Information theory

Source coding

Q: What is the ultimate data compression?
A: The entropy, H

Channel coding

Q: What is the ultimate transmission rate of communication?
A: The capacity, C
What is communication?

A communicates with B if the physical acts of A induce a desired physical state in B.

I want to send 1001

I think A sent 1001

Communication channel

Medium over which communication takes place:

Communication channel

Joint knowledge: model a noisy channel as a set of conditional probability distributions

Send X=0 or X=1

Receives Y=0 or Y=1

Binary symmetric channel

Conditional distributions

p(y=0|x=0) = p(y=1|x=1)=1-f
p(y=1|x=0) = p(y=0|x=1)=f
Idea: channel coding

Source, sender

Encoder

0       1-f
 0
 1-f

Decoder

1

Receiver

Noisy channel

Even bigger picture

Source, sender

Compressor

Encoder

0       1-f
 0
 1-f

Decoder

Channel coding

Receiver

Source coding

Decompressor

Noisy channel

Example: channel coding

REDUNDANT GLASS

\[ \begin{array}{c}
0 \rightarrow 0 \\
1 \rightarrow 1-f \\
1 \rightarrow 1 \\
\end{array} \]

Rate?

\[ R = \frac{\text{# source bits}}{\text{# coded bits}} \]

Can communicate "reliably" even in noisy channels!

Probability of decoding error < \( \varepsilon \) as the number of "channel uses" \( \rightarrow \infty \)

Channel uses

1 channel use

0 \rightarrow 1

2 channel uses

01 \rightarrow 11

3 channel uses

001 \rightarrow 011

n channel uses

101.....001 \rightarrow 101.....001

Intuition?
Capacity

Achievable rate $\leq$ Capacity $\leq$ Outer bound

highest rate $R$, in (bits/channel use) at which information can be sent ``reliably''

Discrete channel capacity

Mutual information

\[ I(X;Y) = \sum_{x,y} p(x,y) \log \left( \frac{p(x,y)}{p(x)p(y)} \right) \]

[Recall $p(x,y) = p(x)p(y)$ if $X$ and $Y$ are independent]

More examples

Noiseless channel

Noisy channel with non-overlapping outputs

Capacity? 1 bit/channel use
More examples

**Binary symmetric channel**

\[
p(x=0) = q \quad p(x=0) = 1-q
\]

0 at \(f=0,1\) and 1 at \(f=1/2\)

Capacity? \(C = 1-H(f)\) bits/channel use

\[
p(y=0|x=0) = p(y=1|x=1)=1-f
\]

\[
p(y=1|x=0) = p(y=0|x=1)=f
\]

More examples

**Noisy typewriter**

\[
C = \log(9)
\]

More examples

**Binary erasure channel**

\[
\begin{align*}
0 & \quad 1-f \quad o \\
0 & \quad f \quad e \\
1 & \quad f \quad 1
\end{align*}
\]

Capacity? \(C = 1-f\) bits/channel use

More examples

**Discrete channel capacity**

\[
\text{Capacity} \quad C = \max_{p(x)} I(X; Y) \quad \text{bits/channel use}
\]

\[
I(X; Y) = \sum_{x,y} p(x, y) \log \frac{p(x, y)}{p(x)p(y)}
\]

\[
\text{Channel: } p(y|x)
\]

Michael Mitzenmacher
Continuous alphabet channel capacity

Channel: $p(y|x)$

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

"mutual information" between $X$ and $Y$

$X$ and $Y$ are not bits, but real numbers

Example

Continuous alphabet channel

$N \sim N(0,P_N)$

$Y = X + N$

Another example

Wireless channel with fading

Fading example

Multiple scaled delayed copies of $X$ are received at $Y$

Fading model

Wireless channel with fading

The wireless signal undergoes:
- large scale variations: path loss
- mid scale variations: slow fading (shadowing)
- small scale variations: fast fading (multipath)
The channel: \( p(y|x) \)

**Continuous alphabet channel**

\[ Y = hX + N \]

**Gaussian noise channel capacity**

\[ C = \max_{p(x)} I(X;Y) \]

\[ = \frac{1}{2} \log_2 \left( \frac{h^2 P + P_N}{P_N} \right) \]

\[ = \frac{1}{2} \log_2 (1 + SNR) \]

Cognitive radios to reduce, reuse and recycle unused spectrum.

Information theory + cognitive radio?

Unused spectrum?
Spectrum?

\[ f = \frac{c}{\lambda} \]
Spectrum?

\[ f = \frac{c}{\lambda} \]

Used for radio communication!

Primary spectrum licensing

Avoid interference

Unlicensed bands

- Industrial, Scientific and Medical band (ISM band)
- 915 MHz, 2450 MHz, and 5800 MHz

Enormous success, filling up fast!
Neither FCC nor NTIA routinely quantify actual spectrum usage by users under their jurisdiction. However, during the Summer of 2002 the FCC’s Enforcement Bureau took limited measurements of spectrum use in certain urban areas which allow a partial view of actual spectrum use. This effort was limited in duration and only used one site in each city studied, and hence generally underestimate actual spectrum use to some degree. However, the Working Group believes that the general observations made here are likely to have broad applicability and should be verified in a broader measurement program, possibly in conjunction with noise measurements.

Figure 1 shows the general nature of spectrum occupancy in an approximately 700 megahertz block of spectrum below 1 GHz in Atlanta, New Orleans, and San Diego. This data was taken by FCC’s Enforcement Bureau at its offices in each city in June 2002. The addresses of the measurement locations are: Atlanta - 3575 Koger Blvd, Duluth GA; New Orleans - 2424 Edenborn Avenue, Metairie LA; and San Diego - 4542 Ruffner Street, San Diego CA.

Types of secondary licensing

- long term (sublet)
- opportunistic
- interruptible (emergency bands)

Spectrum licensing: future

Primary users/primary license holders

Secondary users ↔ Cognitive radios
Cognitive Radio

Use information theory to determine fundamental limits of cognitive networks

Wireless channels: my notation

Wireless channel

Encoder
Transmitter, Tx

Decoder
Receiver, Rx

Wireless channel: $p(y|x)$

X: transmitted signal

Y: received signal

Wireless channel: $p(y|x)$
Wireless channels: capacity

Capacity
= highest rate, in (bits/channel use) at which information can be sent with arbitrarily low probability of error

\[ C = \max_{p(x)} I(X; Y) \]

Discrete channel capacity

\[ I(X; Y) = \sum_{x,y} p(x, y) \log \left( \frac{p(x, y)}{p(x)p(y)} \right) \]

Gaussian noise channel capacity

\[ C = \frac{1}{2} \log \left( 1 + \frac{|h|^2 P}{P_N} \right) \]

Capacity (region)

Achievable region

Capacity region

Outer bound

Achievable rate ≤ Capacity ≤ Outer bound

\[ \text{any rate at which information can be sent with arbitrarily low probability of error} \]
Efficient, reliable communications

1. Fundamental limits
2. Limit approaching codes/protocols
3. Practical demonstration
Secondary spectrum usage

What can the cognitive link do?

1. White spaces

2. Just transmit

Interfere with each other!
2. Just transmit

\[ R_1 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\text{Power of signal 1}}{\text{Interference from signal 2 + Noise}} \right) \]

\[ R_2 \leq \frac{1}{2} \log_2 \left( 1 + \frac{\text{Power of signal 2}}{\text{Interference from signal 1 + Noise}} \right) \]

3. Opportunistic “cognitive” decoding

4. Cognitive transmission

Interference!

Interference can be reduced
Our proposal: Simultaneous Cognitive Transmission

Assumption: Tx 2 knows message encoded by X1 a-priori

Cognitive Tx may obtain primary’s message in a fraction of the time

Our proposal: Simultaneous Cognitive Transmission

Cognitive Tx may overhear primary’s message

Primary transmission
Re-transmission

“Competitive”
Interference channel

Our proposal: “Cooperative”
Broadcast channel

“Cognitive”
Cognitive channel

What rates \( (R_1, R_2) \) are achievable?
Gaussian noise channel capacity

Power limited to \( P \)

\[
Y = hX + N
\]

Gaussian noise \( \sim N(0, P_N) \)

Capacity

\[
C = \max_{p(x)} \frac{1}{2} \log_2 \left( \frac{|h|^2 P + P_N}{P_N} \right)
\]

\[
= \frac{1}{2} \log_2 (1 + \text{SNR})
\]

(bits/channel use)

Intuition

A priori message knowledge

aid transmission \text{SELFLESS}

mitigate interference \text{SELFISH}

What rates are achievable?

Primary only

\[
P_2 = P_2' + P_2''
\]

Primary aided by cognitive transmitter

\[
P_2 = P_2' + 0
\]

aid transmission \text{SELFLESS}
Cognitive transmitter mitigates interference

\[ P_2 = \begin{bmatrix} 0 \\ P_2' \end{bmatrix} \]

Since Tx 2 knows message 1, it can mitigate interference!

Dirty paper coding

Dirty-paper coding

Dirty-paper coding

Dirty-paper coding

[Costa, 1983]

[Gelfand, Pinsker, 1980]
Dirty-paper coding

Example of dirty-paper coding

Interference

Power limited

Send 2 bits:

- 11
- 10
- 01
- 00

Example of dirty-paper coding

How to send 01?

Interference

Power limited

Do NOT have enough power to subtract off the interference!

Example of dirty-paper coding

How to send 11?

Interference

Power limited

NO power penalty!

NOT subtracting off interference!
\[ P_2 = P_2' + P_2'' \]

**The Multiplexing Gain of MIMO X-Channels with Partial Transmit Side-Information**

Natasha Devroye, Masoud Sharif

**This talk**

Side-information increases the achievable rates

What about the *multiplexing gains (degrees of freedom)*?

**Degrees of freedom (DOF)**

Capacity of a single input, single output Gaussian noise channel:

\[ C = \frac{1}{2} \log_2 (1 + \text{SNR}) \]

Scales like \( \log(\text{SNR}) \) as \( \text{SNR} \to \infty \)

**Degrees of freedom**

- Multiple-input multiple-output (MIMO) channels may have many information streams
Degrees of freedom

- Multiple-input multiple-output (MIMO) channels may have many information streams
- As SNR → ∞ interference, rather than noise becomes the limiting factor

Degrees of freedom (DOF) measures the number of point-to-point Gaussian channels contained in a MIMO channel as SNR → ∞

MIMO channel

DOF = \lim_{\text{SNR} \to \infty} \frac{\text{Sum capacity(SNR)}}{\log(\text{SNR})}

point-to-point channel

Degrees of freedom

\[ \text{DOF} = \min(M,N) = 2 \]

This talk

How does asymmetric side-information affect the degrees of freedom?

Side-information: Gaussian noise channels

\[ C = \max_{p(m_2,x_2|X_1)} I(M_2;Y_2) - I(M_2;X_1) \]

[Gelfand, Pinsker 1980]

\[ C = \max_{p(x_2,\gamma|x_1)} I(M_2;Y_2) - I(M_2;X_1) \]

[Gelfand, Pinsker 1980]

[Cover's "Writing on Dirty Paper" 1983]

- Assume \( M_2 = X_2 + \gamma X_1 \)
- Optimize \( \gamma \) to obtain \( \gamma = \frac{P_2}{P_2 + Q_2} \)
- Capacity is that of interference-free channel! \( C = \frac{1}{2} \log_2 \left( 1 + \frac{P_2}{Q_2} \right) \)
Degrees of freedom in the cognitive channel

The cognitive channel

Degrees of freedom in cognitive channels

The X channel

• Capacity region is known in Gaussian noise in the weak interference regime ($a_{11} < 1$)
• Sum-rate capacity is known for $a_{11} > 1$

2 Tx antenna MIMO BC

SNR 10, $a_{11} = a_{22} = 0.55$

DOF = 1

Interference channel

Cognitive channel

DOF = 1

2 Tx antenna broadcast channel

DOF = 2

Interference channel

Cognitive channel

Message 11

Message 12

Message 21

Message 22

The X channel

Degrees of freedom in the cognitive X channel
Degrees of freedom in cognitive X channels

\[ I \leq \text{DOF} \leq \frac{4}{3} \]

[Cognitive X Channel]

\[ \text{DOF} \quad \text{DOF} = 2 \]

{Maddah-Ali, Motahari, Khandani 2006}  
{Jafar, Shamai 2007}

Several possibilities for the side-information S at the cognitive Tx 2:

1. Tx 2 knows message 11
2. Tx 2 knows message 11 and message 12
3. Tx 2 knows codeword which encodes message 11

This talk

[Costa’s “Writing on Dirty Paper” 1980]

\[ C = \max_{p(x_1, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1) \]

This talk

- Assume \( M_1 \) to be \( M_2 = X_2 + \gamma X_1 \) (Gaussian), and one dirty-paper coding parameter \( \gamma \)
- Optimize \( \gamma \)
- Evaluate capacity

Achievable rate region for a cognitive X channel

\[ (R_{11}, R_{12}, R_{21}, R_{22}) \text{ such that:} \]

\[ R_{11} \leq I(M_{11}; Y_1 | M_{21}) - I(M_{11}; M_{12}) \]
\[ R_{21} \leq I(M_{21}; Y_1 | M_{11}) \]
\[ R_{11} + R_{21} \leq I(M_{11}, M_{21}; Y_1) + I(M_{11}; M_{21}) - I(M_{11}; M_{12}) \]
\[ R_{12} \leq I(M_{12}; Y_2 | M_{22}) \]
\[ R_{22} \leq I(M_{22}; Y_2 | M_{12}) - I(M_{22}; M_{11}, M_{21}) \]
\[ R_{12} + R_{22} \leq I(M_{12}, M_{22}; Y_2) + I(M_{12}; M_{22}) - I(M_{22}; M_{11}, M_{21}) \]

\{ MAC+Gelfand-Pinsker to Rx 1 \}
\{ MAC+Gelfand-Pinsker to Rx 2 \}

Assumed input variables

\[ M_{11} = U_{11} + \gamma U_{12} \]
\[ M_{12} = U_{12} \]
\[ M_{21} = U_{21} \]
\[ M_{22} = U_{22} + \gamma_2 (U_{21} + \alpha_{12} U_{11}) \]

\[ U_{11} \sim N(0, P_{11}) \]
\[ U_{12} \sim N(0, P_{12}) \]
\[ U_{21} \sim N(0, P_{21}) \]
\[ U_{22} \sim N(0, P_{22}) \]

\[ P_1 = P_{11} + P_{12} \]
\[ \beta P_2 = P_{21} + P_{22} \]

\[ \gamma_2 = \frac{P_{22}}{2} \]

\[ a_{12} = \sqrt{\frac{P_2}{P_{21}}} \]

\[ X_1 = U_{11} + U_{12} \]
\[ X_2 = U_{21} + U_{22} + \sqrt{\frac{2 P_{22}}{P_2}} U_{11} \]
\[ Y_1 = \left( 1 + a_{12} \sqrt{\frac{P_2}{P_{21}}} \right) U_{11} + U_{12} + \alpha_{12} (U_{21} + U_{22}) + N_1 \]
\[ Y_2 = a_{12} + \sqrt{\frac{P_{22}}{P_2}} U_{11} + a_{12} U_{12} + (U_{21} + U_{22}) + N_2. \]
**Results**

DOF of this cognitive X channel is 2

- Power allocation that achieves this is
  
  \[ P_{11} = P_{12} = P_{21} = P \rightarrow \infty \]
  
  \[ P_{22} = \text{constant} \]

**Conclusion**

- They prove that for \( a < 1 \) (weak interference) the capacity region when both nodes are power limited to \( P \) is the set of all rate pairs \((R_1, R_2)\) such that, for all \( 0 \leq \alpha \leq 1 \),

\[
R_1 \leq \log \left( 1 + \frac{(\sqrt{P} + a\sqrt{\alpha P})^2}{1 + a^2(1 - \alpha)P} \right)
\]

\[
R_2 \leq \log_2 (1 + (1 - \alpha)P)
\]

**Extra**

- For any \( 0 \leq \alpha \leq 1 \) the cognitive radio spends \( \alpha P \) of its power amplifying the primary message, and \((1-\alpha)P\) of its power dirty-paper coding its own message.

\[
R_1 \leq \log \left( 1 + \frac{(\sqrt{P} + a\sqrt{\alpha P})^2}{1 + a^2(1 - \alpha)P} \right)
\]

\[
R_2 \leq \log_2 \left( 1 + (1 - \alpha)P \right)
\]

Interference at primary receiver due to cognitive transmission.
DOF=1

- Moreover, they find the maximum rate the cognitive user may transmit its own messages, conditioned on the primary user's interference caused by the cognitive interference and the cognitive regions for cases the cognitive user is not the case: in the interference channel with asymmetric channels, the capacity (5) is

\[ \gamma_1 = \frac{P_{11}(1 + a_{21}\theta)}{P_{11}(1 + a_{21}\theta)^2 + a_{22}^2 P_{22} + N_1}, \quad \gamma_2 = \frac{P_{22}}{P_{22} + N_2}. \]

\[ \theta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}. \]

Selected DPC parameters

- \( Y_1 \) selected so as to maximize the sum-rate to Rx 1
- \( Y_2 \) selected so as to minimize the denominator of the sum-rate to Rx 2

\[ \delta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}. \]

Sum-rates

\[ r_{11} + r_{21} \leq \frac{1}{2} \log \left( \frac{(P_{11}(1 + a_{21}\theta^2) + P_{22} + N_1)(P_{11}(1 + a_{21}\theta^2) + P_{22} + N_1) + P_{11}(1 + a_{21}\theta^2) + P_{22} + N_1}{(21K^2 + a_{21}\theta^2) + P_{22} + N_1} \right) \quad (5) \]

\[ r_{11} + r_{21} \leq \frac{1}{2} \log \left( \frac{(P_{11}(1 + a_{21}\theta^2) + P_{22} + N_1)(P_{11}(1 + a_{21}\theta^2) + P_{22} + N_1) + P_{11}(1 + a_{21}\theta^2) + P_{22} + N_1}{(21K^2 + a_{21}\theta^2) + P_{22} + N_1} \right) \quad (5) \]

Side-information: discrete memoryless channels

\[ C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1) \]

\[ \text{Point-to-point} \quad \downarrow \quad \text{Penalty for using non-causal side info} \]

Marton’s Region

- Used in [Caire, Shamai 2003]

\[ \bigcup_{0 \leq R_1 \leq I(U_1; Y_1), 0 \leq R_2 \leq I(U_2; Y_2), R_1 + R_2 \leq I(U_1 U_2), I(U_2 Y_2) - I(U_1 U_2)} \]

Formal Theorem 1

Theorem 1: Let \( Z \triangleq (Y_1, Y_2, X_1, X_2, M_{11}, M_{12}, M_{21}, M_{22}) \), and let \( P \) be the set of distributions on \( Z \) that can be decomposed into the form

\[ p(m_{11} | m_{12}) p(m_{22} | m_{21}, m_{22}) \]

\[ p(x_1 | m_{11}, m_{12}) p(x_2 | x_1, m_{11}, m_{22}) \]

\[ p(y_1 | x_1, x_2) p(y_2 | x_2). \]

where we additionally require \( p(m_{12}, m_{22}) = p(m_{12})p(m_{22}) \). For any \( Z \in \mathcal{P} \), let \( S(Z) \) be the set of all tuples \( (R_{11}, R_{12}, R_{21}, R_{22}) \) of non-negative real numbers such that:

\[ R_{11} \leq I(M_{11}; Y_1) - I(M_{11}; M_{12}) \]

\[ R_{12} \leq I(M_{12}; Y_1) - I(M_{11}; M_{12}) \]

\[ R_{21} \leq I(M_{21}; Y_2) - I(M_{21}; M_{22}) \]

\[ R_{22} \leq I(M_{22}; Y_2) - I(M_{21}; M_{22}) \]

Any element in the closure of \( \bigcup_{Z \in \mathcal{P}} S(Z) \) is achievable.
Degrees of freedom in classical channels

DOF = min(M_T, N_R)

Degrees of freedom in classical channels

Broadcast channel

Multiple access channel

Interference channel

min(M_T, N_R0 + N_R2)

min(M_T1 + M_T2, N_R)

min(M_T1 + M_T2, N_R0 + N_R2, max(M_T1, N_R2), max(M_T2, N_R1))

Wireless networks

Cognitive networks

Scaling laws

• [Gupta+Kumar 2000]: Non-cooperative ad hoc networks
  • per-node throughput \( \sim O(1/\sqrt{n}) \)
  • Degradation is due to multi-hop and interference between nodes

• [Ozgur, Leveque, Tse 2007]: Cooperative ad hoc networks
  • nodes may cooperate as in a MIMO system
  • per-node throughput \( \sim O(1) \) (constant)

What about cognitive networks?

[Vu, Devroye, Tarokh 2007]
Exact calculation of interference as an inequality and establish an explicit dependence of the values of $E[I_0]_{\alpha=4}$ with integer $k$.

\[ E[I_0]_{\alpha=4} = \lambda \pi P \left[ -\frac{R^2}{(R^2 - R_0^2)^2} + \frac{(R_0 + \epsilon_p)^2}{\epsilon_p^1(2R_0 + \epsilon_p)^2} \right] \]

Single hop

Primary nodes

Secondary nodes

What we guarantee

Primary nodes don’t suffer too much

$\Pr[\text{primary user’s rate } \leq C_0] \leq \beta$

What we prove

Sum-throughput per cognitive user scale as $O(J)$ as $n \to \infty$

while guaranteeing $\Pr[\text{primary user’s rate } \leq C_0] \leq \beta$

\[ \{Vu, Devroye, Tarokh 2007\} \]

Lower bound on interference
Primary exclusive radius $R_0$ for $\alpha = 4$

\[ E[I_0](dB) \]

Lower bound on interference 2

Outer bound on interference

How to pick parameters

What $R_0$ and $\epsilon$ will guarantee $\Pr[\text{primary user's rate } \leq C_0] \leq \beta$ ?

Tradeoffs!

Model 1
How to pick parameters

![Graphs showing relationship between parameters](image1)

Ad hoc cognitive networks

![Network diagram](image2)

What we guarantee

Primary nodes act as if cognitive network does not exist
Primary nodes achieve same scaling law as if cognitive network does not exist

What we prove

\[ T_F(n) = \Theta \left( \sqrt{\frac{1}{n \log n}} \right) , \quad T_s(m) = \Theta \left( \sqrt{\frac{1}{m \log m}} \right) \]

[Jeon, Devroye, Yu, Chung, Tarokh 2008]

Improving Cellular Downlink Capacity

practical application of asymmetric cooperation
Motivation

- Cellular providers are introducing relays to
  - extend cell coverage
  - boost transmission rates
  - improve spectral efficiency

All at lower costs than building new full-fledged base stations

Downlink cellular system

Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages
Problem setup: phase 1

Phase 1 may result in asymmetry

Phase 2

Equivalence to cognitive radio channel

4 message knowledge cases

Asymmetric message knowledge

Symmetric message knowledge
Equivalence to cognitive radio channel

\[
\begin{array}{c}
\text{Asymmetric cases} \\
\text{linear precoding} \\
\text{dirty-paper coding}
\end{array}
\]

\begin{align*}
B & = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\
B & = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \\
B & = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}
\end{align*}

Linear precoding

Assume that MS 1 broadcasts both messages, and then the SNR at MS 2, are then given

\[
Y = HX + N = HBU + N
\]

Linear precoding

Constraints on the precoding matrices imposed by phase 1

\[
\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}
\]

Linear precoding

Optimization

Need to select:
Phase 1: \( t_1^{(1)}, t_2^{(1)}, R_1^{(1)}, R_2^{(1)}, n_1, n_2 \)
Phase 2: linear precoding \( B \) matrix

We will do so according to two optimization criteria

- Power constraints: \( |b_{11}|^2 + |b_{12}|^2 + |b_{21}|^2 + |b_{22}|^2 \leq P_R \)
- Message knowledge constraints: \( B \) has zeros
Max throughput criterion

Maximize the throughput over 2 phases

\[
\max \frac{n_1 + n_2}{R_1^{(1)} + R_2^{(1)} + \max(n_1/R_1^{(2)}, n_2/R_2^{(2)})}
\]

s.t. \( n_1, n_2 \geq 0 \)

\( R_1^{(1)}, R_2^{(1)} \) are phase 1 rates

\( R_1^{(2)}, R_2^{(2)} \) are phase 2 rates

\( n_1, n_2 \) are number of bits sent to each mobile, variables to be optimized over

Extreme fairness criterion

Maximize the throughput when forced to send one unit of information to each mobile

\[
\max \frac{n_1 + n_2}{R_1^{(1)} + R_2^{(1)} + \max(n_1/R_1^{(2)}, n_2/R_2^{(2)})}
\]

s.t. \( n_1, n_2 \geq 0 \)

\( R_1^{(1)}, R_2^{(1)} \) are phase 1 rates

\( R_1^{(2)}, R_2^{(2)} \) are phase 2 rates

Optimization reduction

We reduce the non-linear optimization problem from one over 8 variables to one over 2 variables.

Max throughput optimization:

\[
\max \left\{ \frac{x}{R_1^{(1)} + R_2^{(1)}} + \max\left( \frac{x}{\log_2 \left( 1 + \frac{x}{R_1^{(2)}} \right)}, \frac{x}{\log_2 \left( 1 + \frac{x}{R_2^{(2)}} \right)} \right) \right\}
\]

s.t. \( x \geq 0, \ |h_1|^2 \leq P_B, \ \gamma_1 |x|^2 + 2y_1 |x| \cos(\phi_1 + \theta) + y_1 |\theta|^2 \leq P_B |h_1|^2 - 1 \)

Simplified max throughput optimization:

\[
\max \left\{ \frac{x}{R_1^{(1)} + R_2^{(1)}} + \max\left( \frac{x}{\log_2 \left( 1 + \frac{x}{R_1^{(2)}} \right)}, \frac{x}{\log_2 \left( 1 + \frac{x}{R_2^{(2)}} \right)} \right) \right\}
\]

s.t. \( x \in [0, x^*, \infty), \ |h_1|^2 \leq P_B, \ t \in [0, \pi], \ \theta \in (-\phi_1, \pi - \phi_1) \)

Optimization summary

Step 1: Single message

Step 2: Dual message

Step 3: Select best

Max throughput

Simulation Setup

Rayleigh fading, pathloss

Simulation Results

Random MS placement

Fixed MS placement

Random MS

Asymmetric cases are throughput optimal often!
Optimization

Nodes a and b want to exchange messages over a shared half-duplex memoryless channel with the help of a relay.

Protocols in time

Protocols in relaying

- Decode and Forward (D&F)

- Compress and Forward (C&F)

- Mixed Forward: One way uses D&F and the other used C&F

Results

Find inner and outer bounds of capacity region

- Inner bound (achievable region)

- Outer bound

- Capacity C lies in
In constructing an achievable rate region we will assume all transmitted
channel gains to the primary Rx, with two independent cognitive Tx's
 wishing to communicate with a single cognitive Rx, as shown in Figure 1.

Motivation

- Rather than Tx-side cognition, can the Rx's behave in a cognitive fashion?
- Assume the cognitive Rx knows the primary’s codebook
- Assume the cognitive Tx knows at what power it may transmit so as not to harm the primary Rx
- Assume primary system does not change

Scenario 1: MAC

Scenario 2: Interference

Scenario 3: Broadcast
Key idea

- The channel to the cognitive Rx is a multiple access channel: opportunistically decode the primary signal if its rate is below the capacity from the primary to secondary.
- Obtain the rate region described by the MAC but with the MAC rate set to the primary rate. Project down 1 dimension.

Nice technical result

- Can achieve all points on the boundary without time sharing with the primary
- Achieved by:
  - MAC: single message split at cognitive Tx
  - Int: cannot in general achieve
  - BC: cannot in general achieve

Gains for opportunistic MAC

![Achievable rate regions for opportunistic MAC channel with equal, fixed powers](image1.png)

(a) Rate regions for opportunistic MAC channel with equal, fixed powers $P_1 = P_2$.

![Achievable rate regions for opportunistic MAC channel](image2.png)

(b) Rate regions for opportunistic MAC channel with $(P_1, P_2) \in P_{MAC}$.

Gains for opportunistic interference

![Achievable rate regions for opportunistic interference channel](image3.png)

Rate regions for opportunistic interference channel with equal, fixed powers $P_1 = P_2$.

![Achievable rate regions for opportunistic interference channel](image4.png)

Rate regions for opportunistic interference channel with $(P_1, P_2) \in P_{INT}$.

The question

What is the capacity of the cognitive radio channel?

![Diagram](image5.png)
Achievable region: graphical

\[ Y_1 = X_1 + a_{21}X_2 + N_1, \quad Y_2 = a_{12}X_1 + X_2 + N_2, \]

\[ a_{21} = 0.8, \quad a_{12} = 0.2 \]

Different cross-over parameters

\[ a_{21} = 0.2, \quad a_{12} = 0.8 \]

\[ a_{21} = 0.55, \quad a_{12} = 0.55 \]

Protocol 1

Causal message knowledge?

What if this link is not free?

Phase 1: broadcast channel

Phase 2: cognitive radio channel

Protocol 2

Phase 1: multiple access channel

Phase 2: cognitive radio channel

Protocol 3

Only one phase: interference channel.

\[ X_2 \] does not know \[ X_1 \] and does not dirty paper code against it.
**Protocol 4**

**Phase 1:**
- broadcast channel

**Phase 2:**
- $X_2$ aids $X_1$ in sending its message. $X_2$ does not send any information of its own.

**Causal achievable regions**

<table>
<thead>
<tr>
<th>Blue: $G=1$</th>
<th>Yellow: $G=10$</th>
<th>Cyan: Genie-Aided</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1=P_2=6$</td>
<td>$P_1=6, P_2=1.5$</td>
<td></td>
</tr>
</tbody>
</table>

**Unused spectrum**

**Formal definitions**

A rate pair $(R_1, R_2)$ is achievable if, for any $\epsilon > 0$, there exists an $(2^{[nR_1]}, 2^{[nR_2]}, n, P_{e})$ code such that $P_{e} \leq \epsilon$.

An $(2^{[nR_1]}, 2^{[nR_2]}, n, P_{e})$ code consists of encoding functions that map messages $W_1 \in \{1, 2, \ldots, 2^{[nR_1]}\}$ and $W_2 \in \{1, 2, \ldots, 2^{[nR_2]}\}$ and decoding functions that recover these messages such that the average error probability is less than $P_{e}$.

Achievable rate region: set of achievable rate pairs $(R_1, R_2)$.

Capacity region: the closure of the set of all achievable rate pairs $(R_1, R_2)$. 

---

**Efficiency Working Group**

Spectrum Policy Task Force

Report of the Spectrum Efficiency Working Group

November 15, 2002

Why Cooperate to Communicate?

Channel capacity (bits/channel use):

\[ C = \begin{cases} \frac{1}{2} \log_2(1 + |h|^2 P/P_n) & \text{Instantaneous} \\ \mathbb{E}_h \left[ \frac{1}{2} \log_2(1 + |h|^2 P/P_n) \right] & \text{Ergodic} \end{cases} \]

Motivation:

Power limited to \( P \)

\[ h X + N \]

Gaussian noise \( \sim N(0,1) \)

\[ X \]

Wireless channel with fading

\[ Y = h X + N \]

Why Cooperate to Communicate?

2 Tx antenna, 2 Rx antenna Multiple Input Multiple Output (MIMO) fading channel with Gaussian noise

\[ H X + N \]

Gaussian noise

Power limited to \( P \)

\[ X \]

Wireless channel fading matrix

\[ Y = H X + N \]

Channel capacity: \( C = \begin{cases} \max_{Q \sim \mathcal{Q} = F \text{ rank}(Q) = 1} \frac{1}{2} \log_2 \left| H Q H^T \right| & \text{Instantaneous} \\ \max_{Q \sim \mathcal{Q} = F \text{ rank}(Q) = 1} \mathbb{E}_H \left[ \frac{1}{2} \log_2 \left| H Q H^T \right| \right] & \text{Ergodic} \end{cases} \]

Which of 4 schemes is optimal

Extra

Asymmetric cooperation downlink
Gains of relay cooperation over non-cooperative schemes

Dirty-paper coding

Baselines for comparison

- Depends on criterion
- Round robin with relay
- Best 2 hop overall

Dirty-paper coding

Optimal encoding scheme for a broadcast channel

Power constraint: \( \text{trace}(B_1 + B_2) \leq P_T \)
For each of the 4 cases, find the parameters that maximize throughput or extreme fairness criteria.

Optimization

Cognitive multiple access networks
Natasha Devroye, Patrick Mitran, Vahid Tarokh

A priori message knowledge aid transmission mitigate interference OR BOTH

Disjoint clusters interfere (inter-cluster)
Nodes within a cluster interfere (intra-cluster)

Competitive: MAC
Cognitive: Van der Meulen
Cooperative: 2x1 vector

Competitive: generalized interference
Cognitive: THIS TALK
Cooperative: almost, but not quite broadcast!
• Motivation and definition
• Relation to previous work
  • Theorem intuition
  • Achievable region in Gaussian case

Traditionally

The catch: interference

If $X_{11}$ is close to $X_{11}, X_{12}$ it can obtain their messages

Genie assumption: $X_{11}$ knows $X_{11}, X_{12}$ a priori

$X_{11}$ knows the messages of $X_{11}$ and $X_{12}$ a priori.

Asymmetric problem.

During simultaneous transmission, what rates $R_{11}, R_{12}, R_{13}$ are achievable?

Private variables $M_{11a}, M_{12a}, M_{121a}$
(=intended for one receiver only)

Public variables $M_{11b}, M_{12b}, M_{121b}$
(=intended for all receivers)

$\bigcap_{T \in T_1} \left( \sum_{i \in T} L_i \right) \leq I(Y_i; M_T | M_T')$
$T_1 = \{11a, 11b, 12a, 12b, 13b\}$

$L_t$ is the rate of $M_t$
\[
\bigcap_{T \subset T_2} \left( \sum_{i \in T} L_{ik} \right) \leq I(Y_2; M_2 | M_1)
\]

\[
T_2 = \{ 11, \beta, 12\beta, 21\alpha, 21\beta \}
\]

Secondary transmitter has prior knowledge of the primary's message and can:

1. Aid the primary transmitters
2. Use knowledge of interference to mitigate it

\[
C = \max_{p(m_1, m_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)
\]

\[
R_{1ik} = L_{1ik}
\]

\[
R_{2jk} \leq L_{2jk} - I(V_{2jk}; V_1)
\]
Achievable rate region:

\[
R_{ik} = L_{ik},
\]

\[
R_{2i} \leq L_{2i} - I(V_{2j}; V_1)
\]

\[
\bigcap_{T \subseteq \mathcal{T}, V_{2j} \in T} \left( \sum_{T \subseteq \mathcal{T}} L_{ij} \right) \leq I(Y_1; V_2|X_2)
\]

\[
\bigcap_{T \subseteq \mathcal{T}, V_{2j} \in T} \left( \sum_{T \subseteq \mathcal{T}} L_{ij} \right) \leq I(Y_2; V_2|X_2)
\]

\{\text{Gelfand-Pinsker coding}\}

\{\text{Overlapping MACs}\}

C. Terminology and definitions

- Competitive MAC channel
- Cognitive MAC channel
- Outer bound 3x2 MIMO + inequalities

A priori message knowledge

- aid transmission
- mitigate interference

OR BOTH

![Diagram of achievable rate region](Image)

![Diagram of competitive MAC channel](Image)

![Diagram of cognitive MAC channel](Image)

![Diagram of outer bound 3x2 MIMO + inequalities](Image)

To send bin number \(i\), given interference \(X_1\), look in bin \(i\) for \(M_2\) such that \((M_2, X_1)\) are jointly typical.

\[
C = \max_{p(M_2, X_2 | X_1)} I(M_2; Y_2) - I(M_2; X_1)
\]
Background: 1D-GPS

Rate Guarantees

\[
\begin{align*}
\Phi_1 &= 0.2 \\
\Phi_2 &= 0.3 \\
\Phi_3 &= 0.4 \\
\Phi_4 &= 0.1
\end{align*}
\]

The goal of Packet-by-Packet GPS (PGPS) server is to mimic the result of GPS

2D-GPS: Overview

Rate Guarantees

\[
\Phi = \begin{bmatrix}
0.2 & 0.1 & 0.3 & 0.4 \\
0.3 & 0.5 & 0.1 & 0.4 \\
0.2 & 0.4 & 0.2 & 0.2
\end{bmatrix}
\]

Routing Matrix

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

Substochastic matrix

\[
R_y = \Phi \Phi R
\]

2D PGPS Scheduler
**Proportional Fairness**

\[ \min \sum_{i,j} |R_{ij} - k\phi_{ij}| \quad \text{subjected to} \]

- \( R_{ij} \geq \phi_{ij} \)
- \( R_{ij} = 0 \) if \( \phi_{ij} = 0 \)
- \( \sum_j R_{ij} \leq 1 \forall q \)
- \( \sum_i R_{ij} \leq 1 \forall p \)
- \( k \geq 1 \)

**Birkhoff Decomposition**

- Any NxN doubly stochastic matrix can be represented by a convex sum of at most \((N-1)^2+1\) permutation matrices.

\[ A = \sum_i \lambda_i P^i, \quad \sum_i \lambda_i = 1 \]

**Lemma**

- Any non-augmentable matrix \( R \) from \( \Phi \) can be represented by a convex sum of non-absorbable zero-enforced permutation matrices of \( \Phi \).

\[ A = \sum_i \lambda_i P^i, \quad \sum_i \lambda_i = 1 \]

**Transform problem**

Lemma 2: Let \( P = \{P^1, \ldots, P^s\} \) be the set of all non-augmentable zero-enforced permutation matrices of \( \Phi \), and let \( p^1_{ij} \) be the entries of matrix \( P^i \) for \( n \in \{1, \ldots, s\} \). Let \( \lambda = (\lambda_1, \ldots, \lambda_s) \) be a vector of dimension \( s = |P| \). Suppose \( \lambda \) and \( k \) are the solution to the optimization problem:

\[ \min \sum_{i,j} \sum_{n=1}^s \lambda_n \phi_{ij} - k \phi_{ij}, \quad \sum_i \lambda_i = 1 \]

subjected to

- \( \sum_{n=1}^s \lambda_n \geq \phi_{ij} \forall i, j \)
- \( \sum_n \lambda_n = 1 \)
- \( k \geq 0 \)

and the service rate matrix \( R = \sum_{n=1}^s \lambda_n P^n \) satisfies the non-augmentability constraint. Then, we say \( R \) is the proportional fair solution to \( \Phi \), and \( k \) is the proportional increase of \( \Phi \).
Example 2: Using $\Phi$ from Example 1. We first write the service rate matrix $R$ as a convex combination of non-absorbable zero-enforced permutation matrices. That is,

$$R = \lambda_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \lambda_4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}. $$

The non-augmentability constraint is $\lambda_2 \lambda_3 = 0$. Solve the two linear optimization problems, one with $\lambda_2 = 0$ and one with $\lambda_3 = 0$, the minimum cost result from $\lambda_2 = 0$. The resulting $R = \begin{bmatrix} 0.4 & 0.85 & 0.15 \\ 0.6 & 0 & 0.4 \end{bmatrix}$.

\[ \Phi = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & \frac{1}{2} \\ 2 & 0 & \frac{1}{2} \end{bmatrix} \]

Max-min augmentation

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 \\
0 & \frac{1}{2} & 0 \\
\frac{1}{2} & 0 & \frac{1}{2} 
\end{bmatrix}
\]

PF augmentation

\[
\begin{bmatrix}
0.4 & 0 & 0 \\
0 & 0.85 & 0.15 \\
0.6 & 0 & 0.4 
\end{bmatrix}
\]

- Results on the capacities of channels with side information known at the transmitter assume full channel knowledge

- In realistic cognitive radio channels, we should not assume full channel knowledge. Results for cognitive radio channels use results from channels with known side-info at the transmitter

Example

\[ \text{Compound Dirty Paper Channel} \]

Patrick Mitran, Natasha Devroye, Vahid Tarokh
Harvard University

- Channel with side information at the transmitter
- Fading channels, where the channel state is unknown to the transmitter, but known to the receiver
- What are bounds on the capacity?

Theorem 2: Let $p_{X,Y|S}$ be a compound channel continuously parametrized in $\beta \in \mathcal{C}$ (C compact) where the input and output alphabets are standard. Then, the capacity of the compound channel with side-information at the transmitter, $C$, is bounded by $C_L \leq C \leq C_U$ where

\[ C_L = \sup_{P_{X,Y,S,W}} \left[ I(U;Y|W) - I(U;S|W) \right] \]  

\[ C_U = \sup_{P_{X,Y,S,W}} \left[ I(U;Y|W) - I(U;S|W) \right] \]

and the supremum are over all distributions on standard alphabets $U$, all distributions on finite alphabet random variables $W$ and all quantizers $q_0$ for $S$. \hfill \square
Returning to the cognitive radio scenario, we consider the problem of encoding a message $V$ with knowledge of a Gaussian interfering signal $S$ of power $Q$. The encoder output $X$ is also power constrained to $P = Q$ and the signal received at the decoder is $Y = \beta_1X + \beta_2S + Z$ where $Z$ is independent Gaussian noise and the compound parameter is $\beta := (\beta_1, \beta_2)$.

Similar to Costa's scheme, we suggest $U = X + \alpha S$, where $\alpha$ is now chosen as a function of the second order statistics of $\beta_1$ and $\beta_2$. The scheme proposed in Section V selects

$$
\alpha = \frac{\mu_1 P_{SNR}}{(\mu_1^2 + \mu_2^2)P_{SNR} + 1}
$$

(5)

We note the following three facts about this choice for Ricean fading channels where $\beta_1$ and $\beta_2$ have $K$ factors $K_1$ and $K_2$ respectively:

1) If $K_1, K_2 \to \infty$, then the scheme is identical to Costa's with $\alpha = P/(P + N)$ and the interference is perfectly mitigated.

2) If either $K_1 \to 0$ or $K_2 \to 0$, the scheme treats the interferer as noise.

3) The performance does not depend on the phase difference between $\mu_1$ and $\mu_2$ as this choice of $\alpha$ rotates the mean channels so that their phases are aligned.

Fig. 6. Communications over a fading channel with a fading interferer whose signal, but not fading coefficient, is known at the transmitter for $SNR = 10$ dB with $P = Q = 1$.

### SDR Implementation of Collaborative Communications

Oh-Soon Shin, H.T. Kung, Vahid Tarokh
Harvard University

John Chapin's Team
Vanu Inc.

- **Design and implementation of an OFDM-based space-time collaborative system**
- **Our lab completed the simulations in Matlab/C, and Vanu Inc. will do the software radio platform design and implementation.**

- **Carrier frequencies: 902-928 MHz**
- **Bandwidth: < 5 MHz signal**
- **64 subcarriers, spaced at 72kHz**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total bandwidth ($B$)</td>
<td>4.625 MHz</td>
</tr>
<tr>
<td>Total number of subcarriers ($N_s$)</td>
<td>64</td>
</tr>
<tr>
<td>Number data subcarriers ($N_d$)</td>
<td>48</td>
</tr>
<tr>
<td>Number pilot subcarriers ($N_p$)</td>
<td>4</td>
</tr>
<tr>
<td>Number of guard or null subcarriers ($N_g$)</td>
<td>12</td>
</tr>
<tr>
<td>Subcarrier frequency spacing ($\Delta f$)</td>
<td>72.27 kHz</td>
</tr>
<tr>
<td>IFFT/IFFT period ($T_{IFFT}$)</td>
<td>13.828 usec (64 samples)</td>
</tr>
<tr>
<td>Guard interval duration ($T_{guard}$)</td>
<td>2.162 usec (10 samples)*</td>
</tr>
<tr>
<td>OFDM symbol duration ($T_{symbol}$)</td>
<td>16.0 usec ($T_{IFFT} + T_{guard}$)</td>
</tr>
</tbody>
</table>

- Each PHY Frame consists of two subsequent phases
- T-R configuration during each phase

* S: Source, D: Destination, R: Relay
• Listening phase
  – The source broadcasts an information frame
  – The destination and relay try to decode the frame
  – If the destination can decode it correctly, it will ignore the following collaboration frame
• Collaboration phase
  – If the relay succeeded in decoding of the listening frame, the source and relay will transmit space-time coded signal
  – Otherwise, the relay should be silent and only the source transmits as in the listening phase
  – For more reliable decision, the destination can combine the signals from the listening and collaboration phases

Behaves more like a 2x1 system than a 1x1 system

• Develop an equivalent system for cognitive transmission
• Toughest to tackle will be developing codes that layer relaying and dirty-paper coding schemes in the same transmission
• Same synchronization issues
• 2 phase protocols seem practical

To cooperate or to select?

Natasha Devroye, Sumeet Sandhu
Intel Research

• Evaluate gains of cooperation between base stations in an 802.16d wireless network over no-cooperation, as well as selection.
• System level simulations that take shadowing, fading, sectorization, pathloss, realistic channel models, and interference from other base-stations/users into account.
Cooperation performs only slightly better than selection.

Shadowing drastically alters all simulations.

Shadowing diversity exists.

**Goal:** Produce a stable system of queues that is stochastically smaller that any queue length process produced by another stable policy.

- Found necessary conditions on Poisson arrival rates of various length packets that ensure the stability the queue length process.
- Simulated various scheduling algorithms, and saw their throughput.
- Base the decision of who to schedule on the current parameters and queue lengths only.
- If there exists a pair \((i^*, j^*)\) with \(C_{i^*j^*} = 1\), and non-empty queues, send the pair such that:

Criteria 1: \(i^*, j^*\) = arg max\(_{i,j\in\{1,\ldots,K\}}\) \(q_{i*} q_{j*}\)

Criteria 2: \(i^*, j^*\) = arg max\(_{i,j\in\{1,\ldots,K\}}\) \(\frac{q_{i*} q_{j*}}{q_{i*} + q_{j*}}\) \(\frac{t_{i*} + t_{j*}}{\max(t_{i*}, t_{j*})}\)

Criteria 3: \(i^*, j^*\) = arg max\(_{i,j\in\{1,\ldots,K\}}\) \(\frac{q_{i*} q_{j*}}{q_{i*} + q_{j*}}\) \(\frac{t_{i*} + t_{j*}}{\max(t_{i*}, t_{j*})}\)

Criteria 4: \(i^*, j^*\) = arg max\(_{i,j\in\{1,\ldots,K\}}\) \(\frac{q_{i*} q_{j*}}{q_{i*} + q_{j*}}\) \(\frac{t_{i*} + t_{j*}}{\max(t_{i*}, t_{j*})}\)

Learned all background material on image compression techniques, and coded the described algorithm, which forms the basis of JPEG2000 in C++.

- 2-D Wavelet Transform
- Bit-plane encoding
- Adaptive Arithmetic Coding
- On each bit plane, uses Quad-tree, and 4 more coding passes
- Post-compression rate distortion optimization