Information theory

+

cognitive radio?

A	.-	N	-.	0	-----
B	-...	O	---	1	.----
C	-.-.	P	.--.	2	..---
D	-..	Q	--.-	3	...--
E	.	R	.-.	4	....-
F	..-.	S	...	5	.....
G	--.	T	-	6	-....
H	....	U	..-	7	--...
I	..	V	...-	8	---..
J	.---	W	.--	9	----.
K	-.-	X	-..-	Fullstop	.-.-.-
L	.-..	Y	-.--	Comma	--..--
M	--	Z	--..	Query	..--..

What is this?

What's the point?

Is it efficient?

Is it used nowadays?

More relevant nowadays

Binary	Oct	Dec	Hex	Glyph
010 0000	040	32	20	SP
010 0001	041	33	21	!
010 0010	042	34	22	"
010 0011	043	35	23	#
010 0100	044	36	24	$
010 0101	045	37	25	%
010 0110	046	38	26	&
010 0111	047	39	27	'
010 1000	050	40	28	(
010 1001	051	41	29	)
010 1010	052	42	2A	*
010 1011	053	43	2B	+
010 1100	054	44	2C	,
010 1101	055	45	2D	-

Binary	Oct	Dec	Hex	Glyph
100 0000	100	64	40	@
100 0001	101	65	41	A
100 0010	102	66	42	B
100 0011	103	67	43	C
100 0100	104	68	44	D
100 0101	105	69	45	E
100 0110	106	70	46	F
100 0111	107	71	47	G
100 1000	110	72	48	H
100 1001	111	73	49	I
100 1010	112	74	4A	J
100 1011	113	75	4B	K
100 1100	114	76	4C	L
100 1101	115	77	4D	M

What is this?

What's the point?

Is it efficient?

Source coding

Source	*ASCII*	*Coded source*
a	110 0001	00
b	110 0010	101
c	110 0011	011
⋮	⋮	⋮
z	111 1010	10011

Communication theoretic perspective

Information theory

Last time

Source coding

Q: What is the ultimate data compression?

A: The entropy, H

Channel coding

Q: What is the ultimate transmission rate of communication?

A: The capacity, C

Communication theoretic perspective

Information theory

Source coding

Q: What is the ultimate data compression?

A: The entropy, H

This time

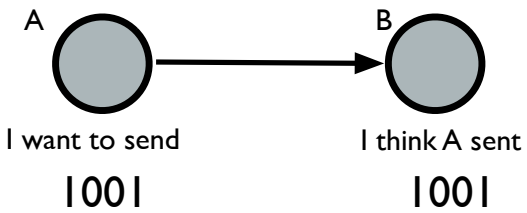
Channel coding

Q: What is the ultimate transmission rate of communication?

A: The capacity, C

What is communication?

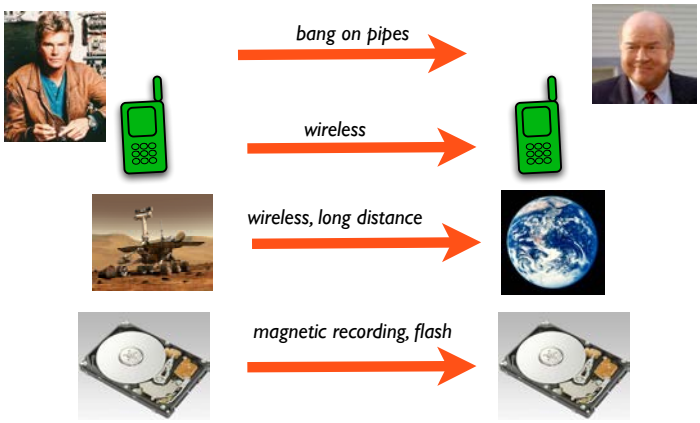
A communicates with B if the
physical acts of A induce a desired physical state in B



Communication channel

Medium over which communication takes place:

Communication channel



Noise

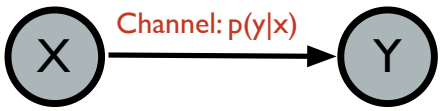
Communication is fundamentally limited by noise

Will errors always be unavoidable?

NO. Can communicate "reliably" even in noisy channels!

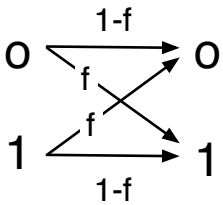
Can communicate "reliably" even in noisy channels!

Key abstraction: model a noisy channel as a
set of conditional probability distributions



Send X=0 or X=1 Receives Y=0 or Y=1

Binary symmetric channel

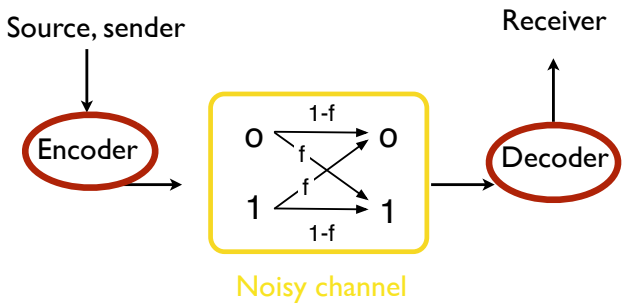


Conditional distributions

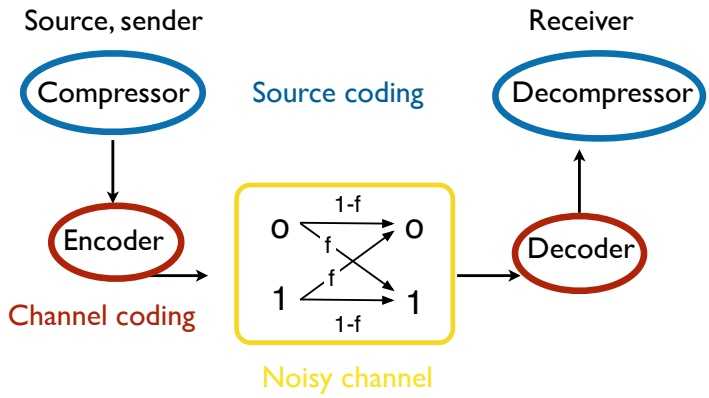
p(y=0|x=0) = p(y=1|x=1)=1-f

p(y=1|x=0) = p(y=0|x=1)=f

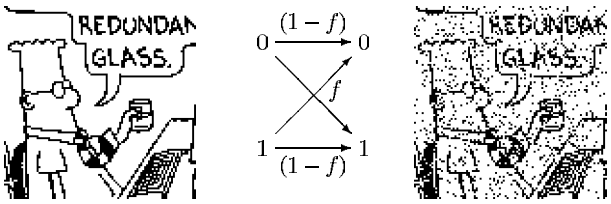
Idea: channel coding



Even bigger picture

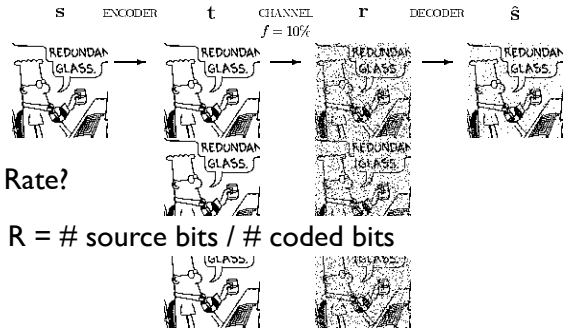


Example: channel coding



With permission from David J.C. Mackay

Example: channel coding



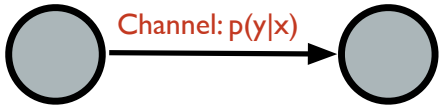
Rate?

R = # source bits / # coded bits

With permission from David J.C. Mackay

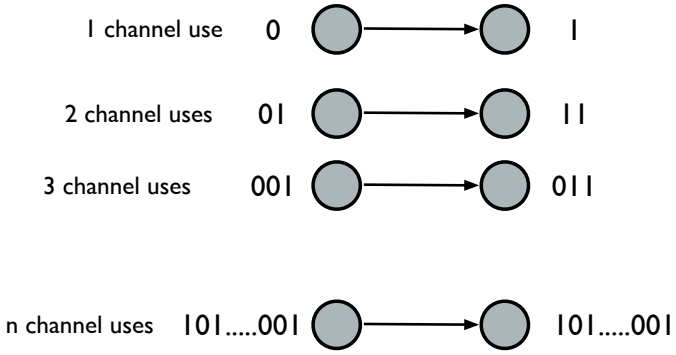
Can communicate "reliably" even in noisy channels!

Probability of decoding error < ε
as the number of "channel uses" → ∞



Intuition?

Channel uses



Capacity

Capacity
II

highest rate R, in (bits/channel use) at which information can be sent ``reliably''

Capacity

"Achievable rate" ≤ Capacity ≤ "Outer bound"
II

any rate at which information can be sent ``reliably''

Discrete channel capacity

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

"mutual information" between X and Y

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

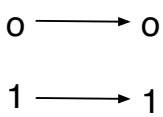
Mutual information

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

[Recall p(x,y) = p(x)p(y) if X and Y are independent]

More examples

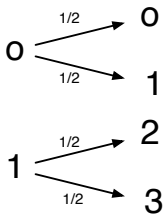
Noiseless channel



Capacity? 1 bit/channel use

More examples

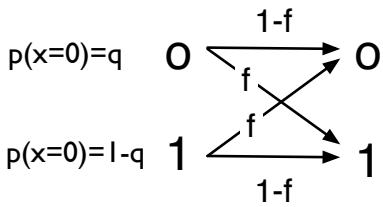
Noisy channel with non-overlapping outputs



Capacity? 1 bit/channel use

More examples

Binary symmetric channel



Conditional distributions

p(y=0|x=0) = p(y=1|x=1)=1-f

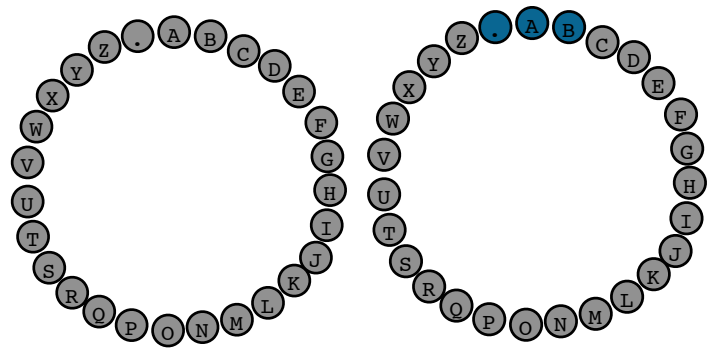
p(y=1|x=0) = p(y=0|x=1)=f

0 at f=0,1 and 1 at f=1/2

Capacity? C = 1-H(f) bits/channel use

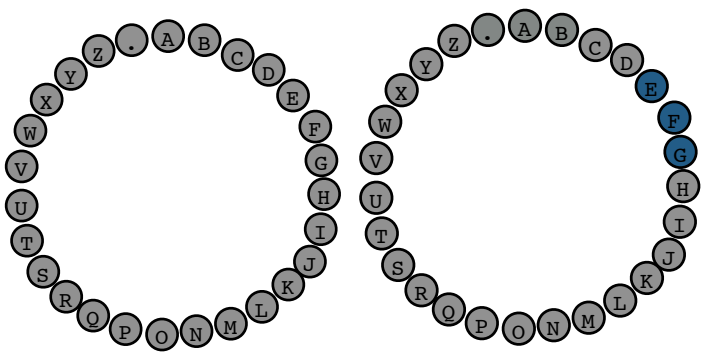
More examples

Noisy typewriter



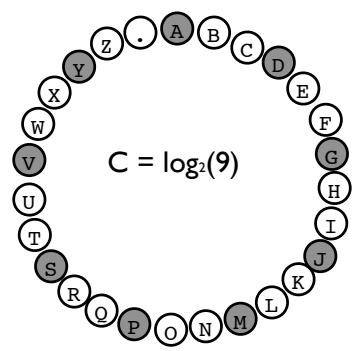
More examples

Noisy typewriter



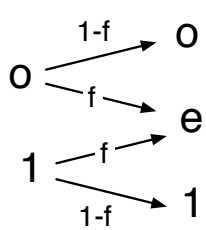
More examples

Noisy typewriter



More examples

Binary erasure channel



Capacity? C = 1-f bits/channel use

Discrete channel capacity

Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

"mutual information" between X and Y

$$I(X;Y) = \sum_{x,y} p(x,y) \log \left(\frac{p(x,y)}{p(x)p(y)} \right)$$

Michael Mitzenmacher

Continuous alphabet channel capacity

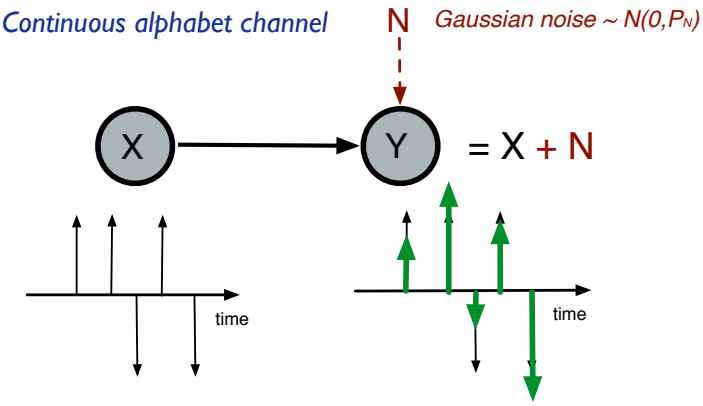
Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

"mutual information" between X and Y

X and Y are not bits, but real numbers

Example

Continuous alphabet channel

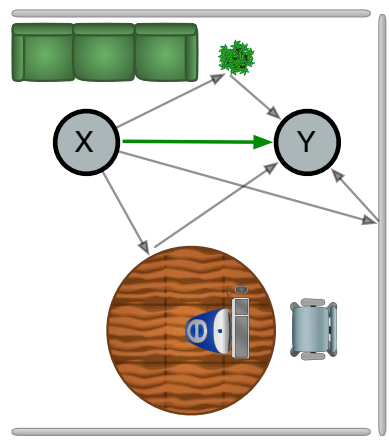


Another example

Wireless channel with fading

Fading example

Multiple scaled delayed copies of X are received at Y

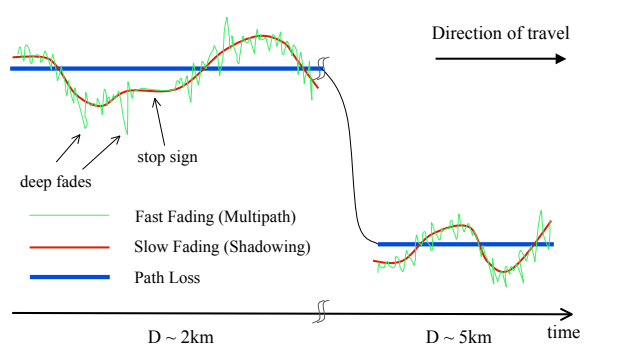


Fading model

The wireless signal undergoes:

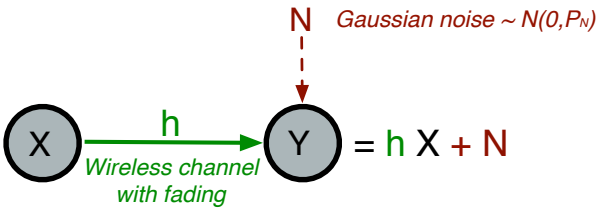
- large scale variations: *path loss*
- mid scale variations: *slow fading (shadowing)*
- small scale variations: *fast fading (multipath)*

Fading example

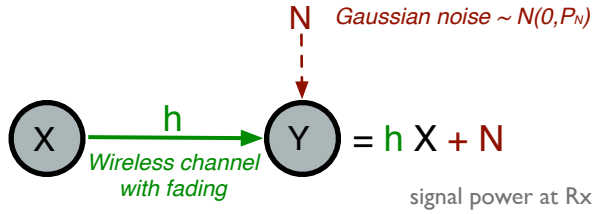


The channel: p(y|x)

Continuous alphabet channel



Gaussian noise channel capacity



signal power at Rx

noise power at Rx

Capacity

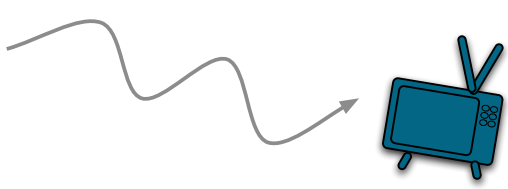
$$C = \max_{p(x):E[|X|^2]\le P} I(X;Y)$$
$$= \frac{1}{2}\log_2\left(\frac{|h|^2P + P_N}{P_N}\right)$$
$$= \frac{1}{2}\log_2(1+\mathrm{SNR})$$

(bits/channel use)

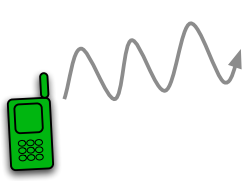
[redacted]

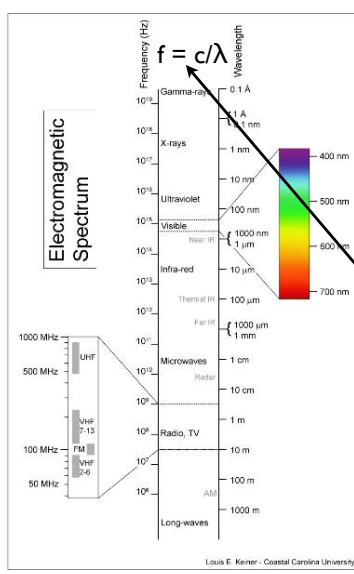
Cognitive radios to
reduce, reuse and recycle
unused spectrum.

Information theory
+
cognitive radio?

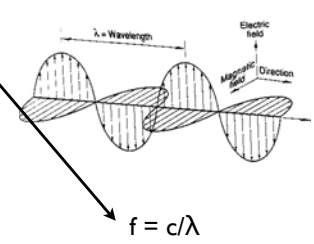


Unused spectrum?

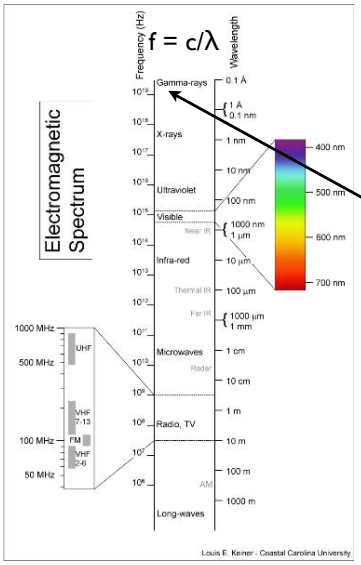




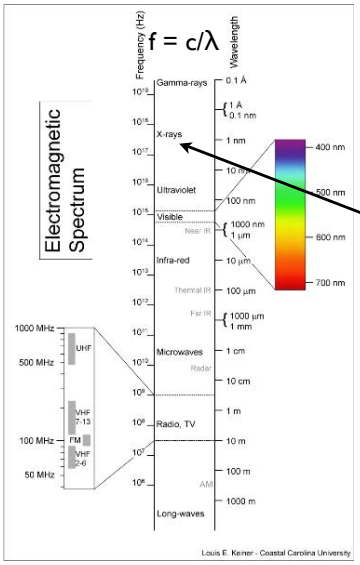
Spectrum?



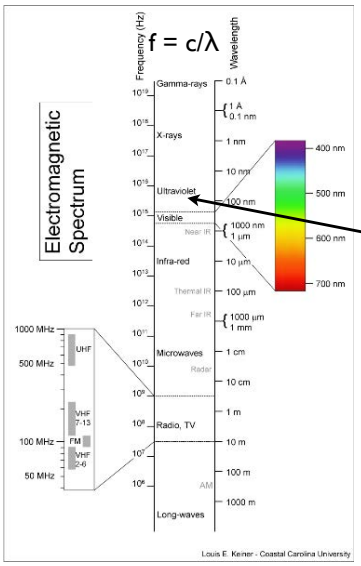
Spectrum?



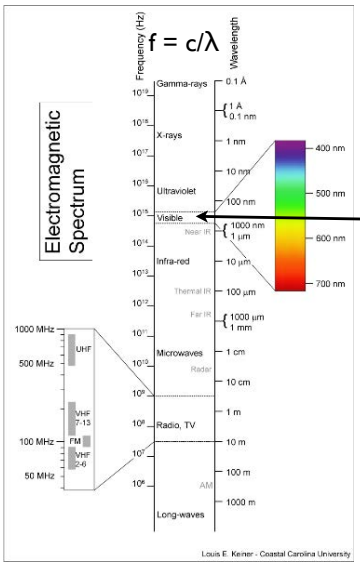
Spectrum?



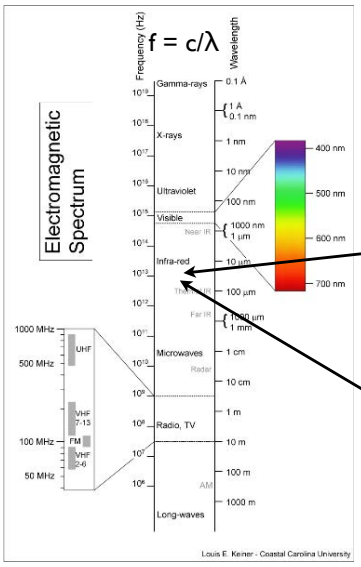
Spectrum?



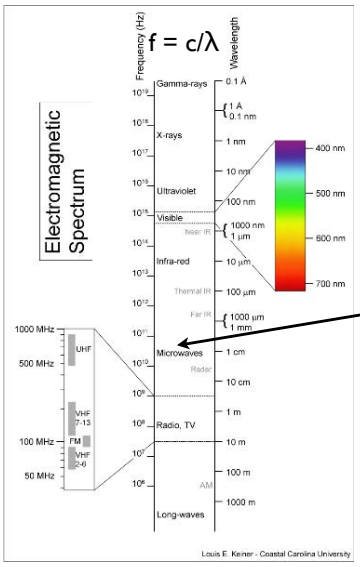
Spectrum?

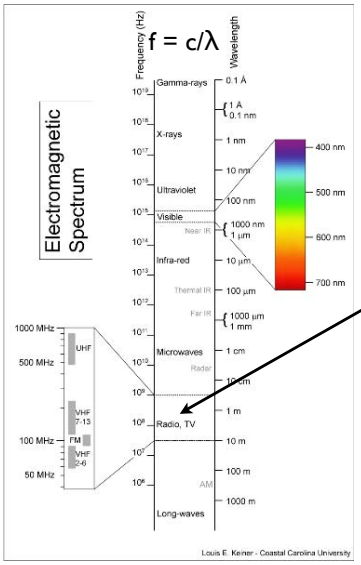


Spectrum?

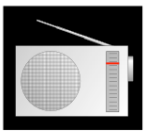


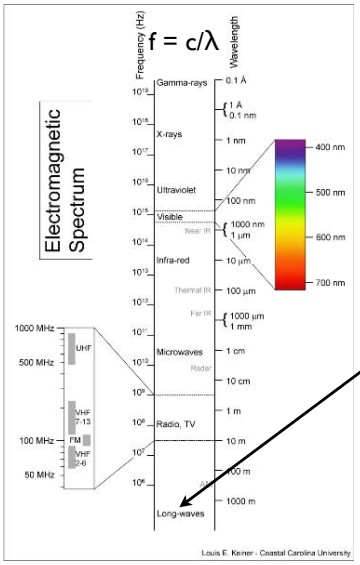
Spectrum?



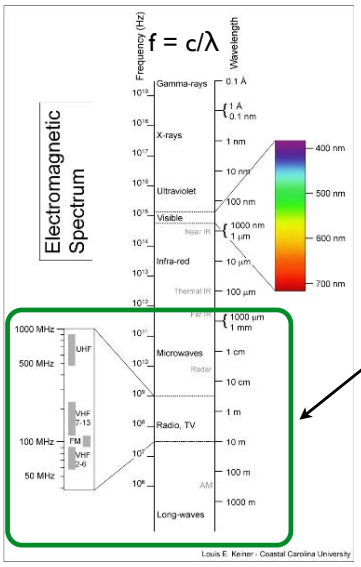


Spectrum?





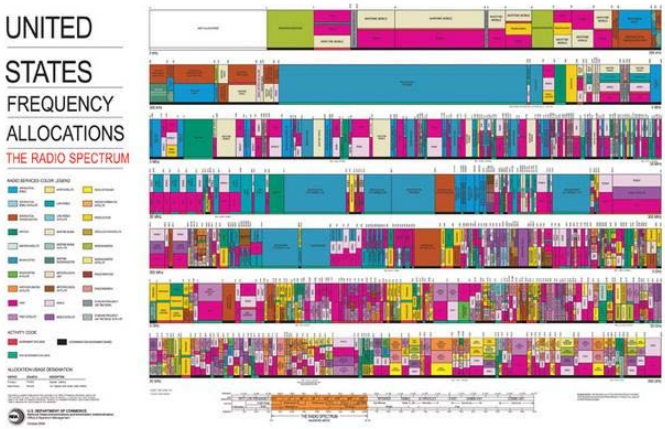
Spectrum?



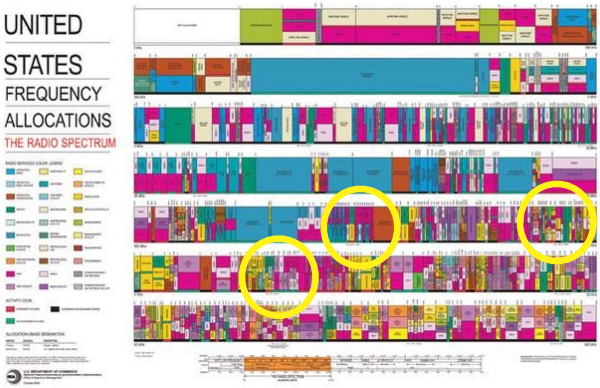
Spectrum?

Used for radio communication!

Unused spectrum?



Primary spectrum licensing

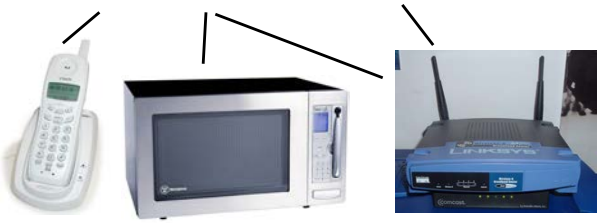


Avoid interference

$$$$$$$$$$$$

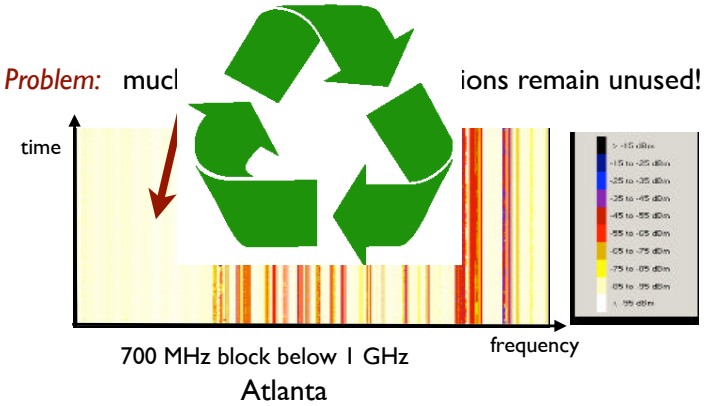
Unlicensed bands

- Industrial, Scientific and Medical band (ISM band)
- 915 MHz, 2450 MHz, and 5800 MHz



Enormous success, filling up fast!

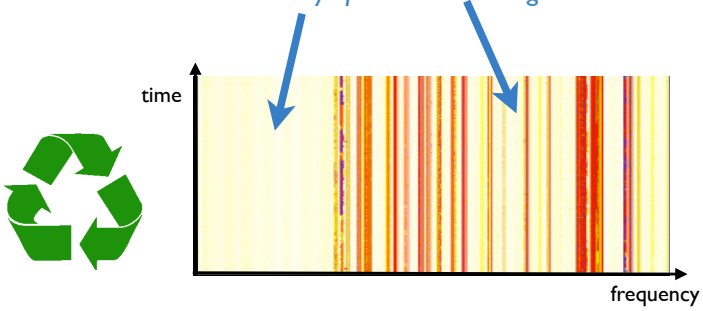
Licensed bands



Spectrum licensing

NOW *Primary spectrum licensing*

FUTURE *Secondary spectrum licensing*

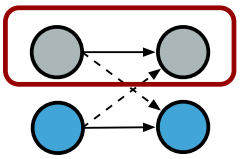


Types of secondary licensing

- long term *(sublet)*
- opportunistic
- interruptible *(emergency bands)*

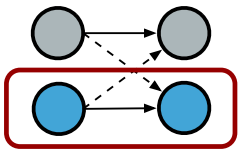
Spectrum licensing: future

Primary users/ primary license holders



Spectrum licensing: future

Primary users/ primary license holders



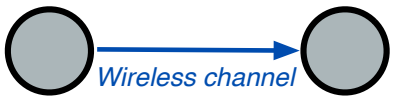
Secondary users ⟷ Cognitive radios

Cognitive Radio

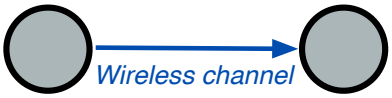
Use information theory to determine fundamental limits of cognitive networks

Wireless channels: my notation

Wireless channels: my notation



Wireless channels: my notation



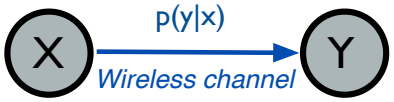
Encoder

Transmitter, Tx

Decoder

Receiver, Rx

Wireless channels: my notation



Encoder

Transmitter, Tx

X: transmitted signal

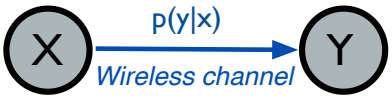
Decoder

Receiver, Rx

Y: received signal

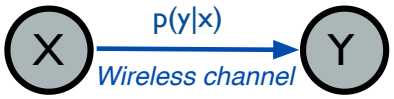
Wireless channel: $p(y|x)$

Wireless channels: capacity



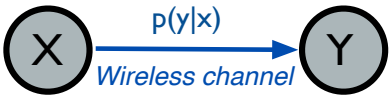
Capacity
II
highest rate, in (bits/channel use) at which information can be sent with arbitrarily low probability of error

Wireless channels: capacity



"Achievable rate" ≤ Capacity ≤ "Outer bound"
II
any rate at which information can be sent with arbitrarily low probability of error

Discrete channel capacity

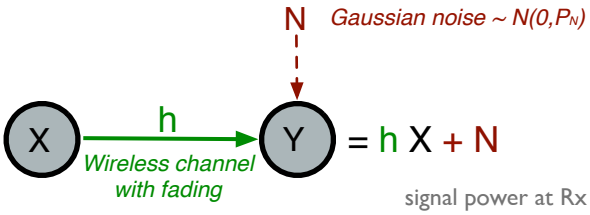


Capacity $C = \max_{p(x)} I(X;Y)$ bits/channel use

"mutual information" between X and Y

$$I(X;Y) = \sum_{x,y} p(x,y) \log\left(\frac{p(x,y)}{p(x)p(y)}\right)$$

Gaussian noise channel capacity

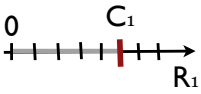


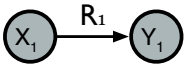
signal power at Rx

noise power at Rx

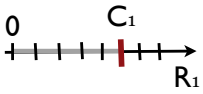
Capacity
$$C = \max_{p(x):E[|X|^2]\le P} I(X;Y)$$
$$= \frac{1}{2}\log_2\left(\frac{|h|^2P + P_N}{P_N}\right)$$
$$= \frac{1}{2}\log_2(1+\mathrm{SNR})$$
(bits/channel use)

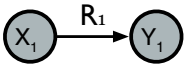
Capacity

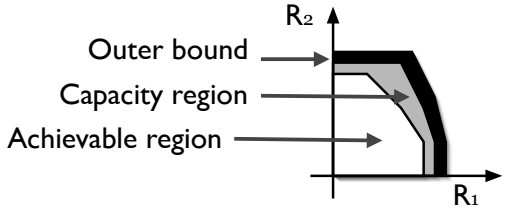


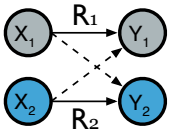


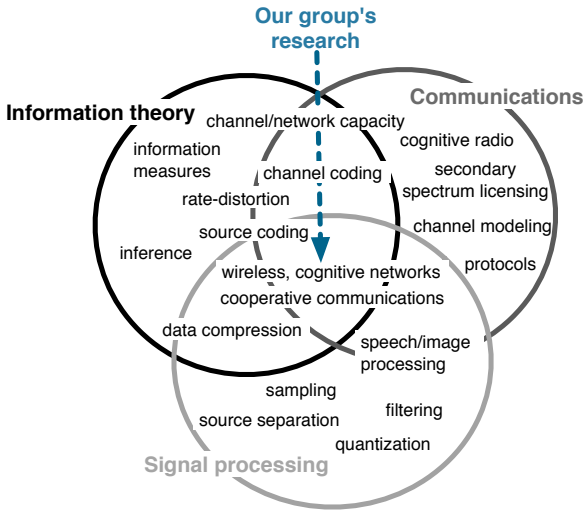
Capacity region



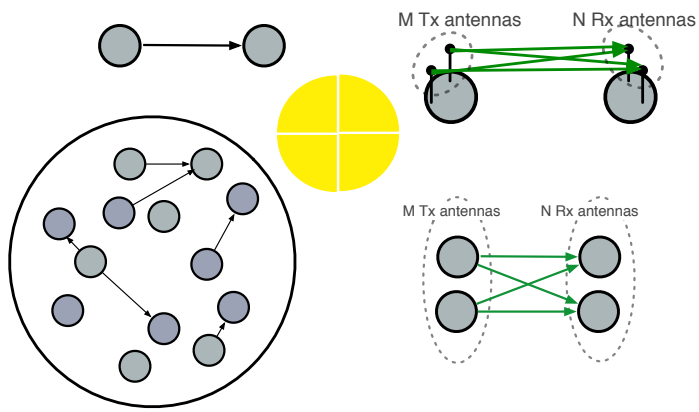




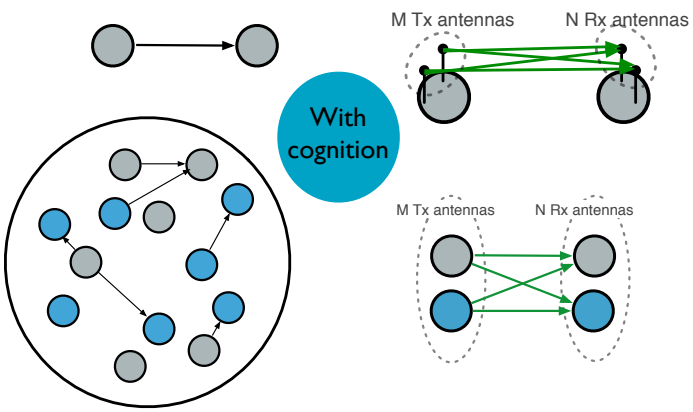




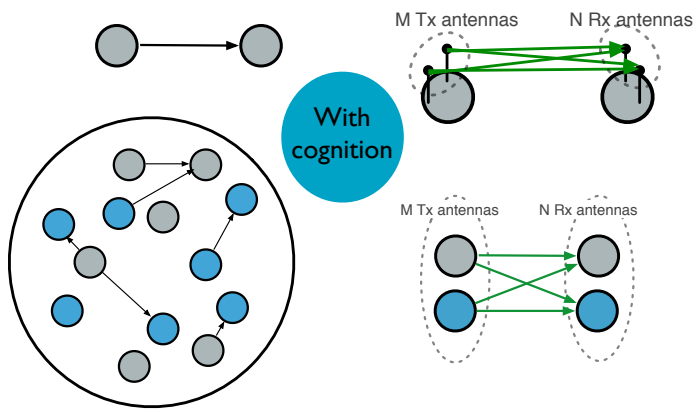
Efficient, reliable communications



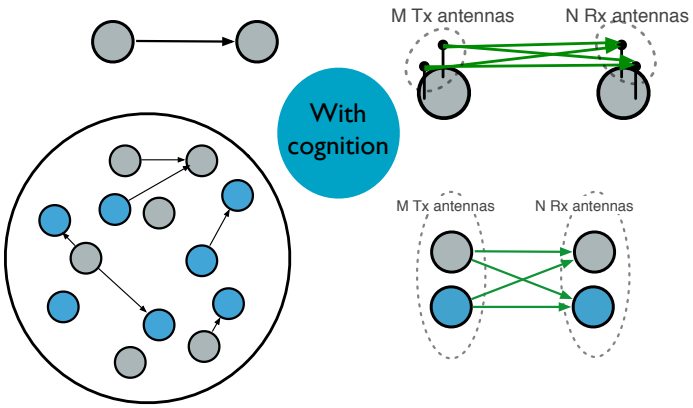
Efficient, reliable communications



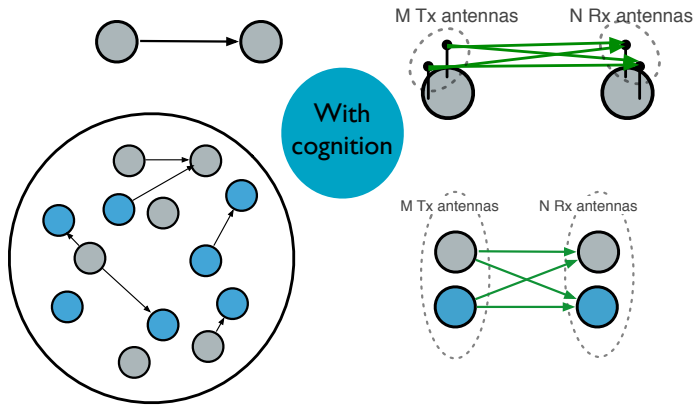
1. Fundamental limits



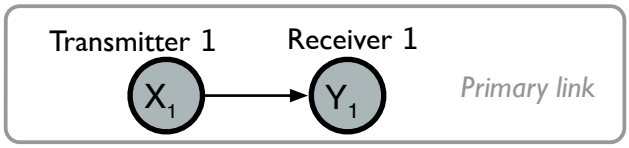
1. Fundamental limits
2. Limit approaching codes/protocols



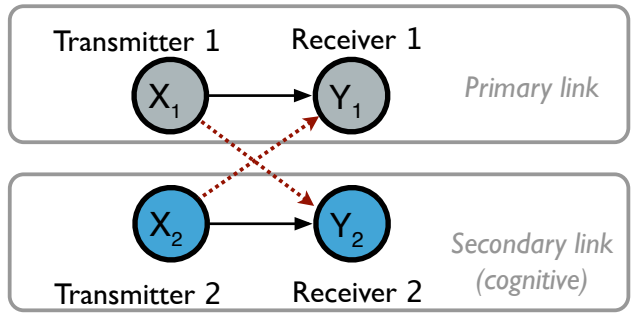
1. Fundamental limits
2. Limit approaching codes/protocols
3. Practical demonstration



Secondary spectrum usage

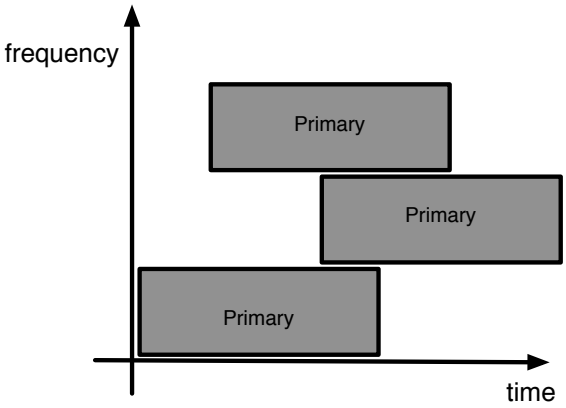


Secondary spectrum usage

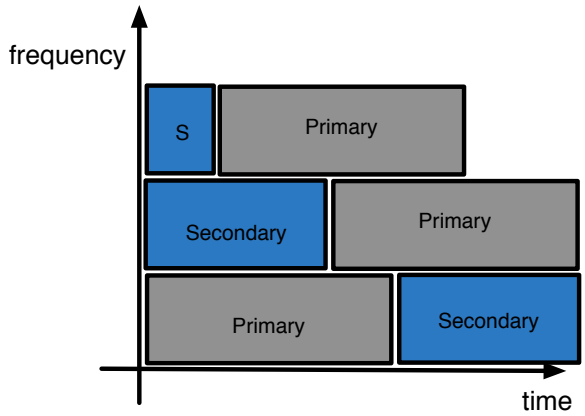


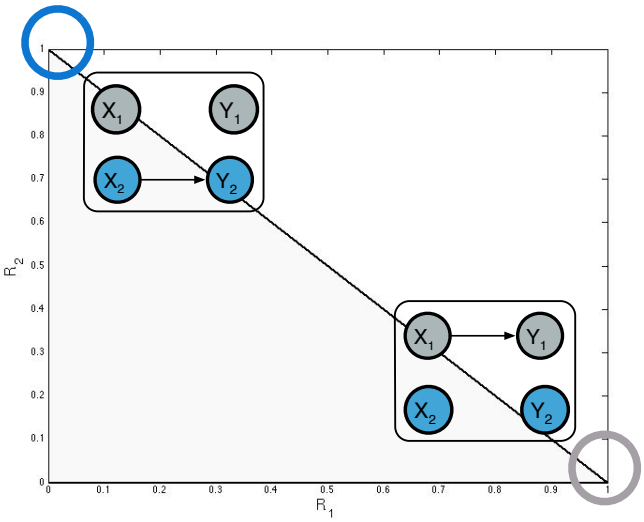
What can the cognitive link do?

1. White spaces

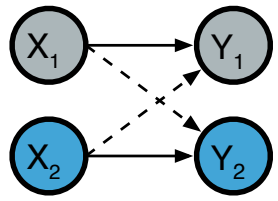


1. White spaces





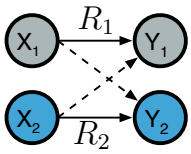
2. Just transmit



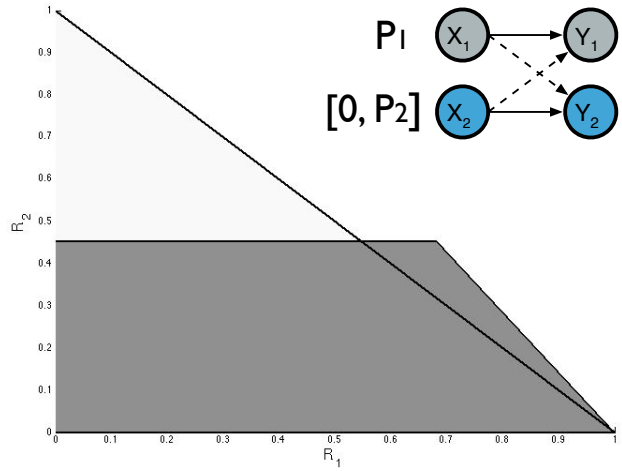
Interfere with each other!

2. Just transmit

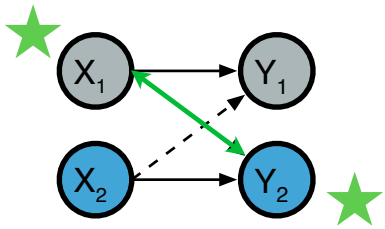
$$R_1 \leq \frac{1}{2}\log_2\left(1+\frac{\text{Power of signal 1}}{\text{Interference from signal 2 + Noise}}\right)$$

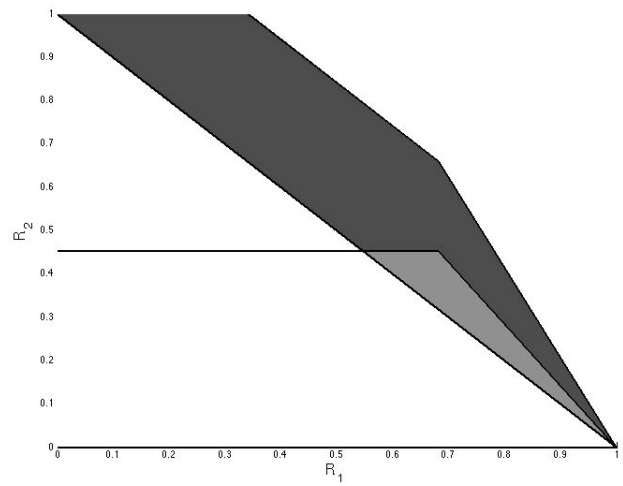


$$R_2 \leq \frac{1}{2}\log_2\left(1+\frac{\text{Power of signal 2}}{\text{Interference from signal 1 + Noise}}\right)$$

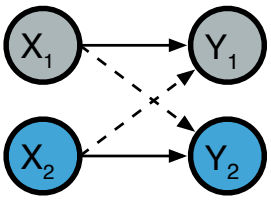


3. Opportunistic "cognitive" decoding



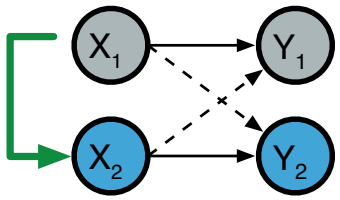


4. Cognitive transmission



Interference!

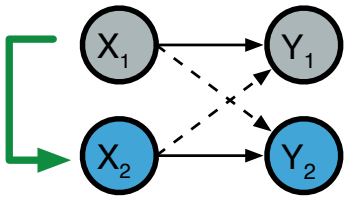
4. Cognitive transmission



Interference can be reduced

Our proposal:

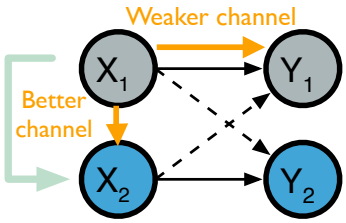
Simultaneous Cognitive Transmission



Assumption: Tx 2 knows message encoded by X_1 a-priori

Our proposal:

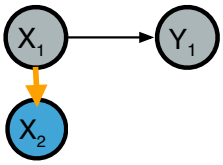
Simultaneous Cognitive Transmission



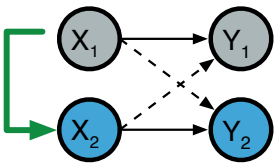
Cognitive Tx may obtain primary's message in a fraction of the time

Our proposal:

Simultaneous Cognitive Transmission

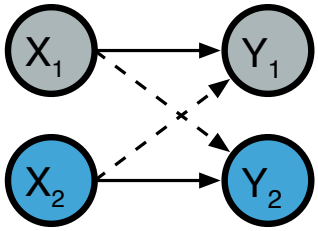


Primary transmission



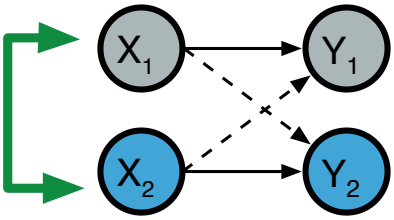
Re-transmission

Cognitive Tx may overhear primary's message



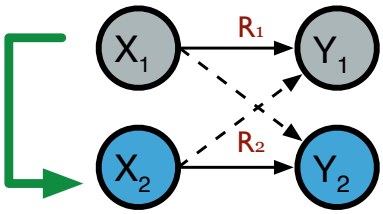
"Competitive"

Interference channel



"Cooperative"

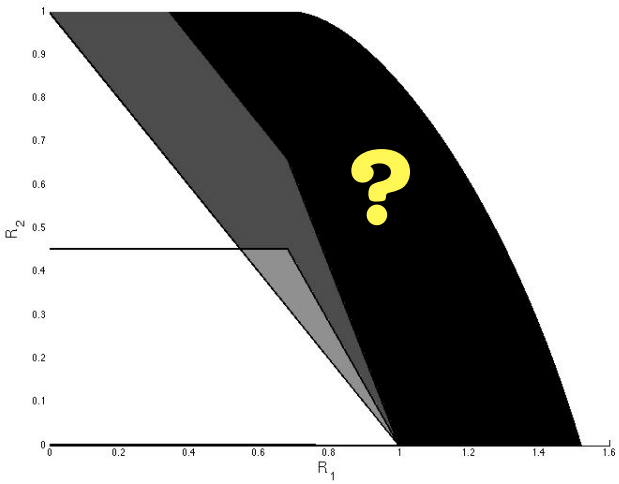
Broadcast channel



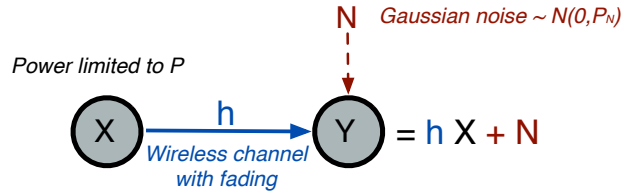
"Cognitive"

Cognitive channel

What rates (R_1, R_2) are achievable?



Gaussian noise channel capacity

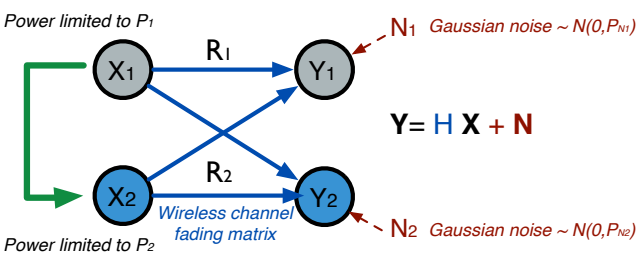


Capacity

$$C = \max_{p(x):E[|X|^2]\le P} I(X;Y)$$
$$= \frac{1}{2}\log_2\left(\frac{|h|^2P + P_N}{P_N}\right)$$
$$= \frac{1}{2}\log_2(1+\mathrm{SNR})$$

(bits/channel use)

Gaussian cognitive channel

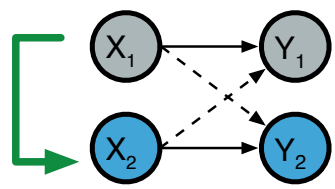


What rates are achievable?

Intuition

$$P_1'$$

$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$

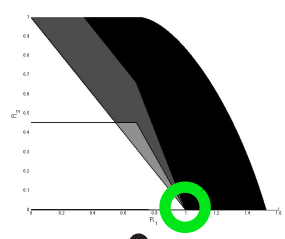


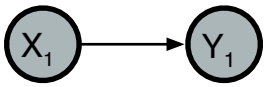
A priori message knowledge

aid transmission *SELFLESS*

mitigate interference *SELFISH*

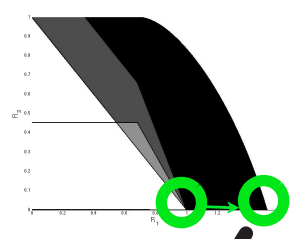
Primary only



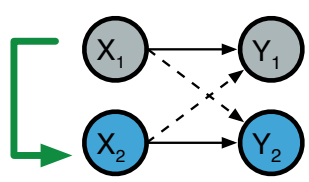


aid transmission *SELFLESS*

Primary aided by cognitive transmitter

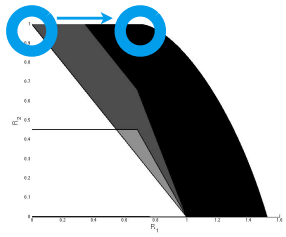


$$P_2 = \boxed{P_2'} + \boxed{0}$$

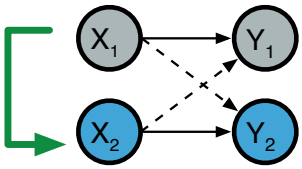


aid transmission *SELFLESS*

Cognitive transmitter mitigates interference



$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$

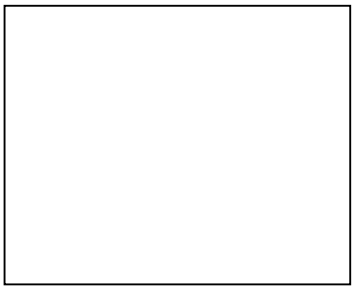


mitigate interference
SELFISH

Since Tx 2 knows message 1, it can mitigate interference!

Dirty paper coding

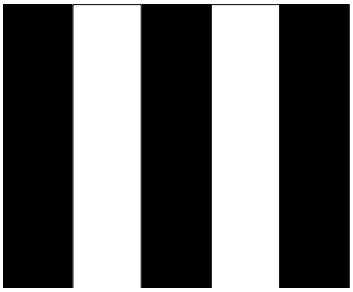
Dirty-paper coding



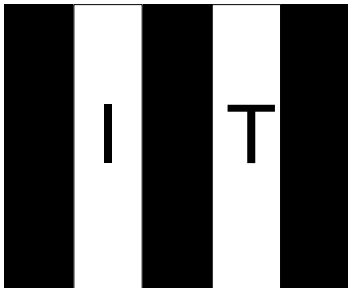
[Costa, 1983]
[Gel'fand, Pinsker, 1980]

Dirty-paper coding

Dirty-paper coding



Dirty-paper coding

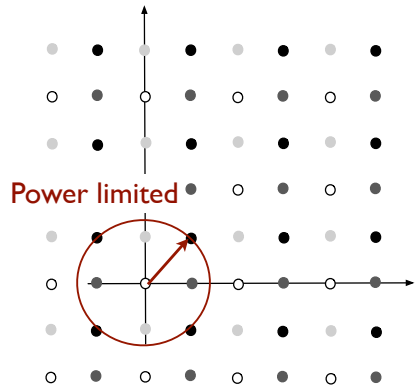


write in black ink?

Dirty-paper coding

adjust your ink ✓

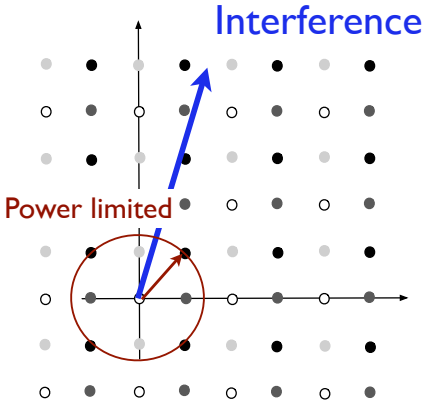
Example of dirty-paper coding



Send 2 bits:

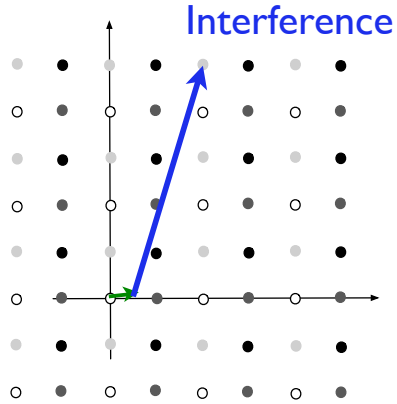
- ● 11
- ● 10
- ● 01
- ○ 00

Example of dirty-paper coding



Do NOT have enough power to subtract off the interference!

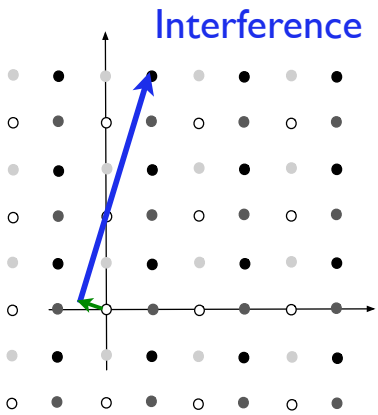
Example of dirty-paper coding



How to send 01?

- ● 11
- ● 10
- ● 01
- ○ 00

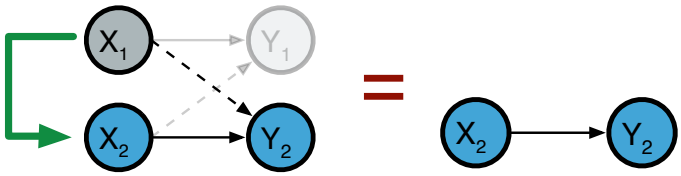
Example of dirty-paper coding



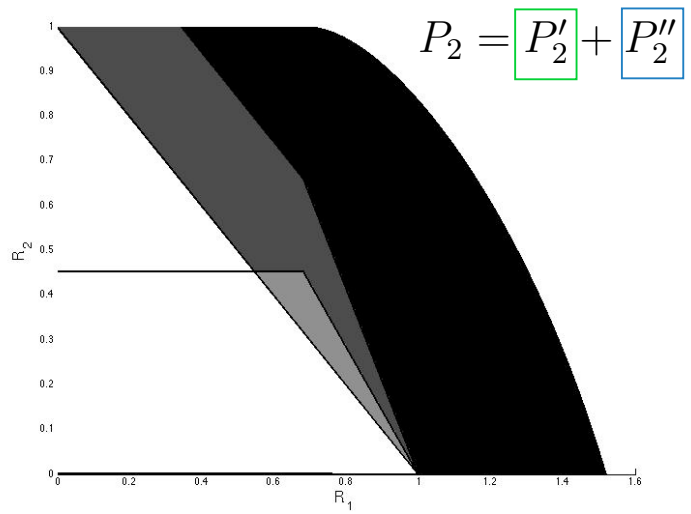
How to send 11?

- ● 11
- ● 10
- ● 01
- ○ 00

Dirty-paper coding



NO power penalty!
NOT subtracting off interference!



Information theory

Source coding (compression)

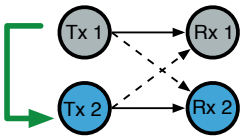
Channel coding (add redundancy)

Application: cognitive radio channels

The Multiplexing Gain of MIMO X-Channels with Partial Transmit Side-Information

Natasha Devroye, Masoud Sharif

This talk



Side-information increases the achievable rates

What about the *multiplexing gains (degrees of freedom)?*

Degrees of freedom (DOF)

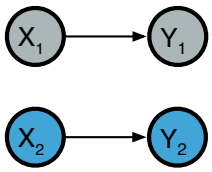
Capacity of a single input, single output Gaussian noise channel:

$$C = \frac{1}{2}\log_2(1 + \mathrm{SNR})$$

Scales like log(SNR) as SNR $\to \infty$

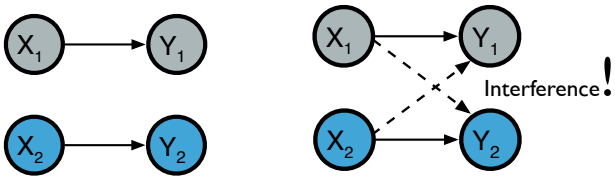
Degrees of freedom

- Multiple-input multiple-output (MIMO) channels may have many information streams



Degrees of freedom

- Multiple-input multiple-output (MIMO) channels may have many information streams
- As SNR $\to \infty$ interference, rather than noise becomes the limiting factor



Degrees of freedom

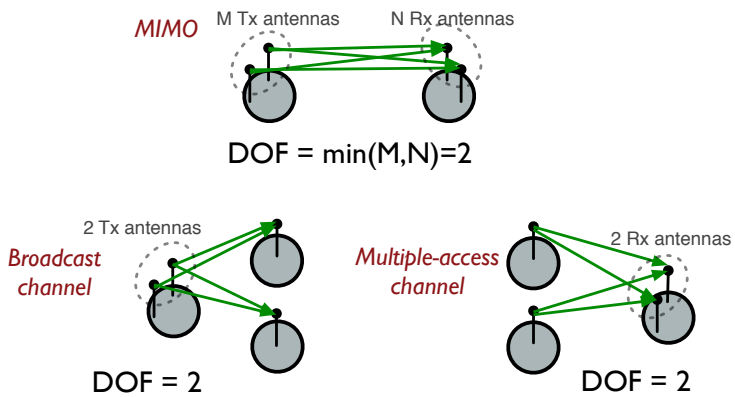
Degrees of freedom (DOF) measures the number of point-to-point Gaussian channels contained in a MIMO channel as SNR $\to \infty$

MIMO channel

$$\mathrm{DOF} = \lim_{\mathrm{SNR}\to\infty} \frac{\text{Sum capacity}(\mathrm{SNR})}{\log(\mathrm{SNR})}$$

point-to-point channel

Degrees of freedom



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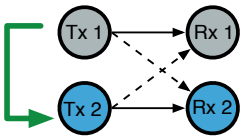
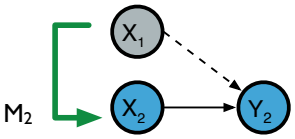


How does *asymmetric* side-information affect the *degrees of freedom?*

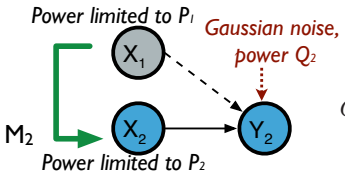
Side-information:
Gaussian noise channels



$$C = \max_{p(m_2,x_2|x_1)} I(M_2;Y_2) - I(M_2;X_1)$$

[Gel'fand, Pinsker 1980]

Side-information:
Gaussian noise channels



$$C = \max_{p(m_2,x_2|x_1)} I(M_2;Y_2) - I(M_2;X_1)$$

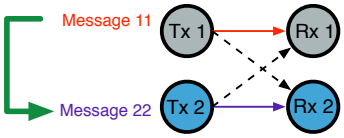
[Gel'fand, Pinsker 1980]

[Costa's "Writing on Dirty Paper" 1983]

- Assume $M_2 = X_2 + \gamma X_1$
- Optimize Y to obtain $\gamma = \dfrac{P_2}{P_2 + Q_2}$
- Capacity is that of interference-free channel! $C = \frac{1}{2}\log_2\left(1 + \frac{P_2}{Q_2}\right)$

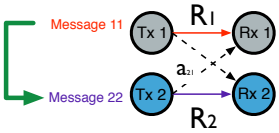
Degrees of freedom in the cognitive channel

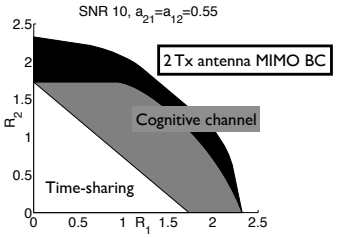
The cognitive channel



Interference channel with degraded message sets
Interference channel with partially-cognitive transmitter
Interference channel with unidirectional cooperation

The cognitive channel

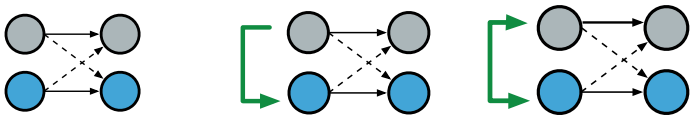




- Capacity region is known in Gaussian noise in the weak interference regime ($a_{21}<1$)
- Sum-rate capacity is known for $a_{21}>1$

[Jovicic, Viswanath 2007]

Degrees of freedom in *cognitive* channels



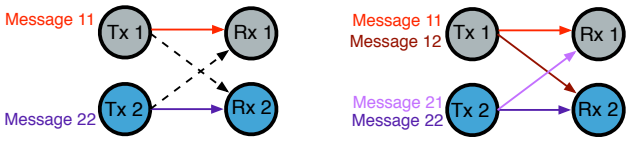
DOF = 1 — Interference channel

DOF = 1 — Cognitive channel

DOF = 2 — 2 Tx antenna broadcast channel

Degrees of freedom in the cognitive X channel

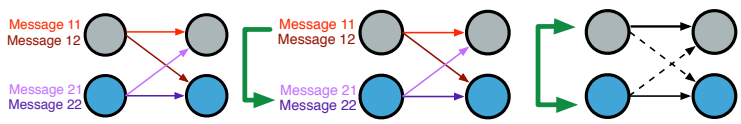
The X channel



Interference channel
2 messages

X channel
4 messages

Degrees of freedom in *cognitive* X channels

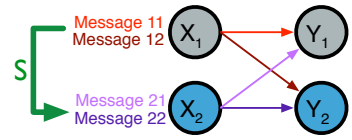


$1 \leq \text{DOF} \leq 4/3$ DOF ? DOF = 2

[Maddah-Ali, Motahari, Khandani 2006]
[Jafar, Shamai 2007]

Cognitive X channel

Several possibilities for the side-information S at the cognitive Tx 2:



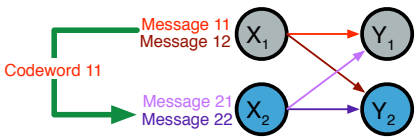
1. Tx 2 knows message 11
2. Tx 2 knows message 11 and message 12

[Jafar, Shamai 2007]

This talk

3. Tx 2 knows codeword which encodes message 11

Cognitive X Channel



Tx 1
- Encode message 12 as codeword M_{12}
- Encode message 11 as codeword M_{11} by dirty-paper coding against M_{12}

Tx 2
- Encode message 21 as codeword M_{21}
- Encode message 22 as codeword M_{22} by dirty-paper coding against M_{11} and M_{21}

Achievable rate region for a cognitive X channel

$(R_{11}, R_{12}, R_{21}, R_{22})$ such that:

MAC+Gel'fand-Pinsker to Rx 1

$$\left.\begin{aligned} R_{11} &\leq I(M_{11}; Y_1|M_{21}) - I(M_{11}; M_{12}) \\ R_{21} &\leq I(M_{21}; Y_1|M_{11}) \\ R_{11} + R_{21} &\leq I(M_{11}, M_{21}; Y_1) + I(M_{11}; M_{21}) - I(M_{11}; M_{12}) \end{aligned}\right\}$$

MAC+Gel'fand-Pinsker to Rx 2

$$\left.\begin{aligned} R_{12} &\leq I(M_{12}; Y_2|M_{22}) \\ R_{22} &\leq I(M_{22}; Y_2|M_{12}) - I(M_{22}; M_{11}, M_{21}) \\ R_{12} + R_{22} &\leq I(M_{12}, M_{22}; Y_2) + I(M_{12}; M_{22}) - I(M_{22}; M_{11}, M_{21}) \end{aligned}\right\}$$

Approach

[Costa's "Writing on Dirty Paper" 1980]

$$C = \max_{p(m_2, x_2|x_1)} I(M_2; Y_2) - I(M_2; X_1)$$

- Assume M_2 to be $M_2 = X_2 + \gamma X_1$ (Gaussian), and one dirty-paper coding parameter Υ
- Optimize Υ
- Evaluate capacity

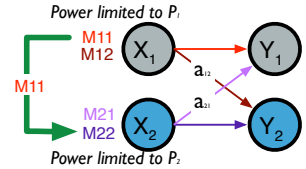
[This talk]

$$\begin{aligned} R_{11} &\leq I(M_{11}; Y_1|M_{21}) - I(M_{11}; M_{12}) \\ R_{21} &\leq I(M_{21}; Y_1|M_{11}) \\ R_{11} + R_{21} &\leq I(M_{11}, M_{21}; Y_1) + I(M_{11}; M_{21}) - I(M_{11}; M_{12}) \end{aligned}$$

$$\begin{aligned} R_{12} &\leq I(M_{12}; Y_2|M_{22}) \\ R_{22} &\leq I(M_{22}; Y_2|M_{12}) - I(M_{22}; M_{11}, M_{21}) \\ R_{12} + R_{22} &\leq I(M_{12}, M_{22}; Y_2) + I(M_{12}; M_{22}) - I(M_{22}; M_{11}, M_{21}) \end{aligned}$$

- Assume M_{11}, M_{12}, M_{21}, M_{22} Gaussian and 2 dirty paper coding parameters Υ_1, Υ_2
- Optimize Υ_1, Υ_2
- Evaluate achievable sum-rate

Assumed input variables



$$\begin{aligned} M_{11} &= U_{11} + \gamma_1 U_{12} \\ M_{12} &= U_{12}, \\ M_{21} &= U_{21}, \\ M_{22} &= U_{22} + \gamma_2(U_{21} + a_{12}U_{11}) \end{aligned} \qquad \begin{aligned} U_{11} &\sim \mathcal{N}(0, P_{11}) \\ U_{12} &\sim \mathcal{N}(0, P_{12}) \\ U_{21} &\sim \mathcal{N}(0, P_{21}) \\ U_{22} &\sim \mathcal{N}(0, P_{22}) \end{aligned} \qquad \begin{aligned} P_1 &= P_{11} + P_{12} \\ \beta P_2 &= P_{21} + P_{22} \end{aligned}$$

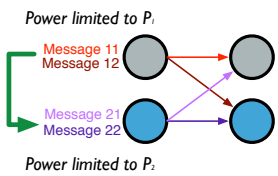
$$\begin{aligned} X_1 &= U_{11} + U_{12} \\ X_2 &= U_{21} + U_{22} + \sqrt{\tfrac{(1-\beta)P_2}{P_{11}}} U_{11} \\ Y_1 &= \left(1 + a_{21}\sqrt{\tfrac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + U_{12} + a_{21}(U_{21} + U_{22}) + N_1 \\ Y_2 &= \left(a_{12} + \sqrt{\tfrac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + a_{12}U_{12} + (U_{21} + U_{22}) + N_2. \end{aligned}$$

Results

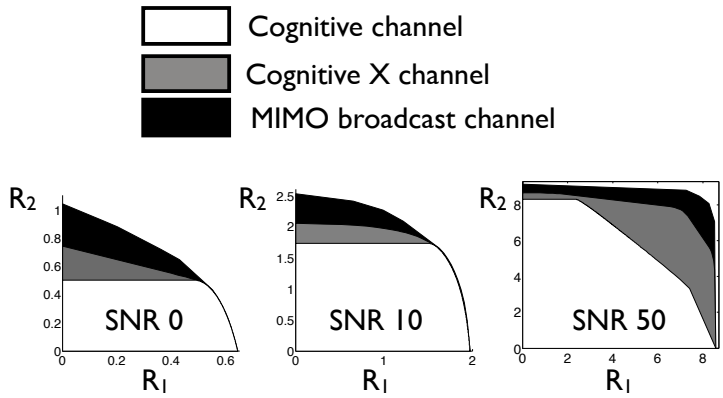
DOF of this cognitive X channel is **2**

- Power allocation that achieves this is

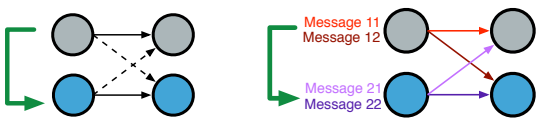
$$P_{11} = P_{12} = P_{21} = P \to \infty$$
$$P_{22} = \text{constant}$$



Rate regions



Conclusion



DOF = 1

Cognitive channel

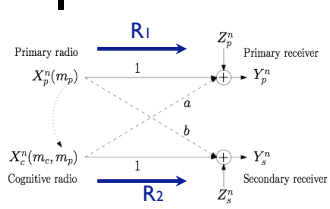
DOF = 2

Cognitive X channel
Side-information is one of Tx 1's codewords

Extra

DOF=1

- They prove that for a<1 *(weak interference)* the **capacity region** when both nodes are power limited to P is the set of all rate pairs (R_1, R_2) such that, for all $0 \le \alpha \le 1$,



$$R_1 \le \log\left(1 + \frac{(\sqrt{P} + a\sqrt{\alpha P})^2}{1 + a^2(1-\alpha)P}\right)$$

$$R_2 \le \log_2\left(1 + (1-\alpha)P\right)$$

DOF=1

– For any $0 \le \alpha \le 1$ the cognitive radio spends αP of its power amplifying the primary message, and $(1-\alpha)P$ of its power dirty-paper coding its own message

$$R_1 \le \log\left(1 + \frac{(\sqrt{P} + a\sqrt{\alpha P})^2}{1 + \boxed{a^2(1-\alpha)P}}\right)$$

$$R_2 \le \log_2\left(1 + (1-\alpha)P\right)$$

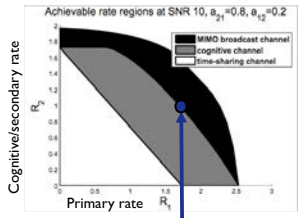
Interference at primary receiver due to cognitive transmission

DOF=1

- Moreover, they find the maximum rate the cognitive user may transmit at such that the primary user suffers no loss in rate (compared to cognitive-free transmission)

- This determines the optimal α* as:

$$\alpha^* = \left(\frac{\sqrt{P}\left(\sqrt{1+a^2P(1+P)}-1\right)}{a\sqrt{P}(1+P)} \right)^{\frac{1}{2}}$$



Optimal α* yields this point

Selected DPC parameters

- γ_1 selected so as to maximize the sum-rate to Rx 1
- γ_2 selected so as to minimize the denominator of the sum-rate to Rx 2

$$\gamma_1 = \frac{P_{11}(1+a_{21}\theta)}{P_{11}(1+a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1}, \quad \gamma_2 = \frac{P_{22}}{P_{22} + N_2}$$

$$\theta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}.$$

Sum-rates

$$R_{11} + R_{21} \le \frac{1}{2}\log_2\left(\frac{(P_{11})(P_{11}(1+a_{21}\theta)^2 + P_{12} + \alpha_{21}^2(P_{21}+P_{22}) + N_1)}{\gamma_1^2 P_{12}(P_{11}(1+a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1) - 2\gamma_1 P_{11}P_{12}(1+a_{21}\theta) + P_{11}(P_{12} + \alpha_{21}^2 P_{22} + N_1)}\right) \quad (5)$$

$$R_{12} + R_{22} \le \frac{1}{2}\log_2\left(\frac{(P_{11}(a_{12}+\theta)^2 + \alpha_{12}^2 P_{12} + P_{21} + P_{22} + N_2)(\gamma_1^2 P_{12}(P_{22} + \gamma_2^2(a_{12}+\theta)^2 P_{11}) + P_{11}P_{22})}{(P_{11} + \gamma_1^2 P_{12})(\gamma_2^2((P_{22}+N_2)(P_{21}+\alpha_{12}^2) + P_{11}P_{21}\theta^2) + \gamma_2(-2P_{22}(P_{11}(a_{12}+\theta)^2 + P_{21})) + P_{22}(P_{11}(a_{12}+\theta)^2 + P_{21} + N_2))}\right) \quad (6)$$

$$R_{11} + R_{21} \le \frac{1}{2}\log_2\left(\frac{(P_{11}(1+a_{21}\theta)^2 + P_{12} + \alpha_{21}^2(P_{21}+P_{22}) + N_1)(P_{11}(1+a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1)}{(\alpha_{21}^2 P_{22} + N_1)(P_{11}(1+a_{21}\theta)^2 + P_{12} + \alpha_{21}^2 P_{22} + N_1)}\right) \quad (7)$$

after substituting γ_i

$$\theta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}.$$

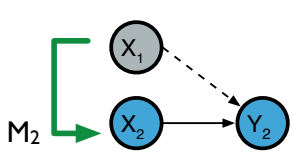
Marton's Region

- Used in *[Caire, Shamai 2003]*

$$\mathrm{co} \bigcup_{P_{U_1,U_2,X,Y_1,Y_2} \in \mathcal{P}} \left\{ \begin{array}{l} 0 \le R_1 \le I(U_1;Y_1) \\ 0 \le R_2 \le I(U_2;Y_2) \\ R_1 + R_2 \le I(U_1;Y_1) \\ \quad +I(U_2;Y_2) - I(U_1,U_2) \end{array} \right\}$$

Side-information:

discrete memoryless channels



$$C = \max_{p(m_2,x_2|x_1)} \underbrace{I(M_2;Y_2)}_{\text{Point-to-point}} - \underbrace{I(M_2;X_1)}_{\downarrow}$$

Penalty for using non-causal side info

[Gel'fand, Pinsker 1980]

Formal Theorem 1

Theorem 1: Let $Z \triangleq (Y_1, Y_2, X_1, X_2, M_{11}, M_{12}, M_{21}, M_{22})$, and let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form

$$\begin{array}{c} p(m_{11}|m_{12})p(m_{12})p(m_{21})p(m_{22}|m_{11},m_{21}) \\ p(x_1|m_{11},m_{12})p(x_2|m_{11},m_{21},m_{22}) \\ p(y_1|x_1,x_2)p(y_2|x_1,x_2), \end{array} \quad (3)$$

where we additionally require $p(m_{12}, m_{22}) = p(m_{12})p(m_{22})$. For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all tuples $(R_{11}, R_{12}, R_{21}, R_{22})$ of non-negative real numbers such that:

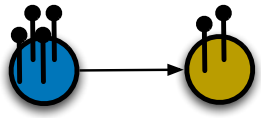
$$\begin{array}{rl} R_{11} & \le I(M_{11};Y_1|M_{21}) - I(M_{11};M_{12}) \\ R_{21} & \le I(M_{21};Y_1|M_{11}) \\ R_{11}+R_{21} & \le I(M_{11},M_{21};Y_1) - I(M_{11};M_{12}) \end{array}$$

$$\begin{array}{rl} R_{12} & \le I(M_{12};Y_2|M_{22}) \\ R_{22} & \le I(M_{22};Y_2|M_{12}) - I(M_{22};M_{11},M_{21}) \\ R_{12}+R_{22} & \le I(M_{12},M_{22};Y_2) - I(M_{22};M_{11},M_{21}) \end{array}$$

Any element in the closure of $\cup_{Z\in\mathcal{P}} S(Z)$ is achievable.

Degrees of freedom in *classical* channels

M_T transmit antennas

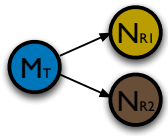


N_R receive antennas

$$\text{DOF} = \min(M_T, N_R)$$

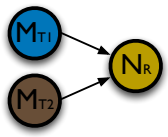
Degrees of freedom in *classical* channels

Broadcast channel



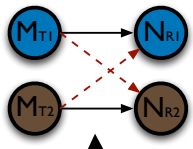
$\min(M_T, N_{R1} + N_{R2})$

Multiple access channel



$\min(M_{T1} + M_{T2}, N_R)$

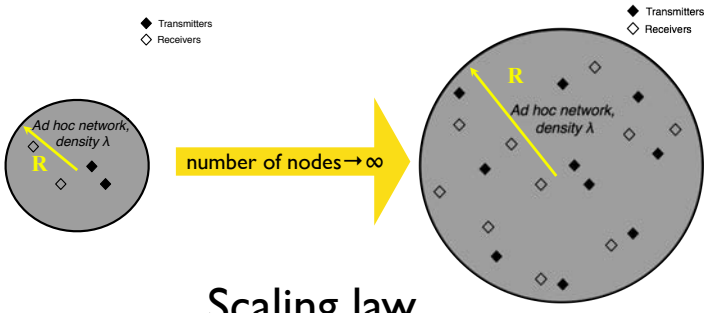
Interference channel



$\min(M_{T1} + M_{T2}, N_{R1} + N_{R2}, \max(M_{T1}, N_{R2}), \max(M_{T2}, N_{R1}))$

Cognitive networks

Wireless networks



Scaling law

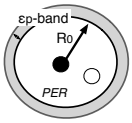
Throughput(number of nodes)

Scaling laws

- **[Gupta+Kumar 2000]:** Non-cooperative ad hoc networks
  - per-node throughput ~ $O(1/\sqrt{n})$
  - Degradation is due to multi-hop and **interference** between nodes
- **[Ozgur, Leveque, Tse 2007]:** Cooperative ad hoc networks
  - nodes may cooperate as in a MIMO system
  - per-node throughput ~ O(1) *(constant)*

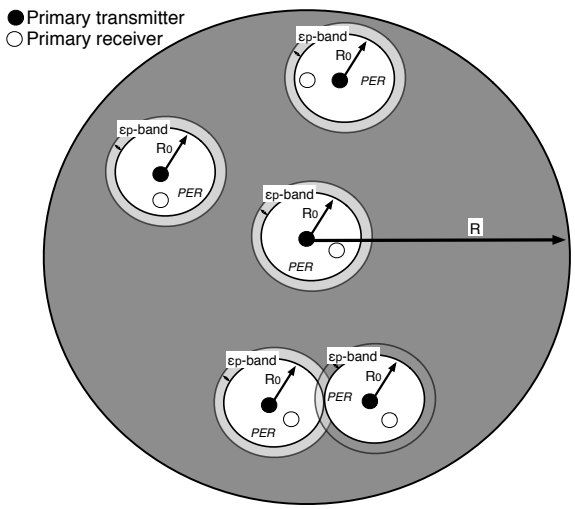
What about cognitive networks?

Primary transmitter
Primary receiver

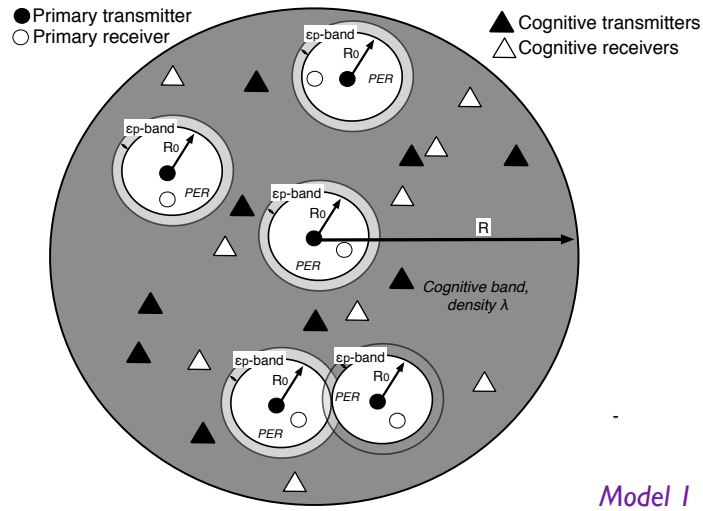


[Vu, Devroye, Tarokh 2007]

Model 1



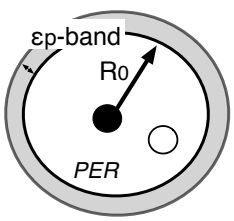
Model 1



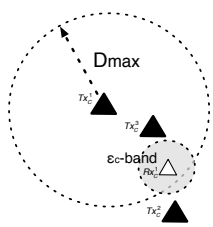
Model 1

Single hop

Primary nodes



Secondary nodes



Model 1

What we guarantee

Primary nodes don't suffer too much

$$\Pr\left[\text{primary user's rate} \leq C_0\right] \leq \beta$$

What we prove

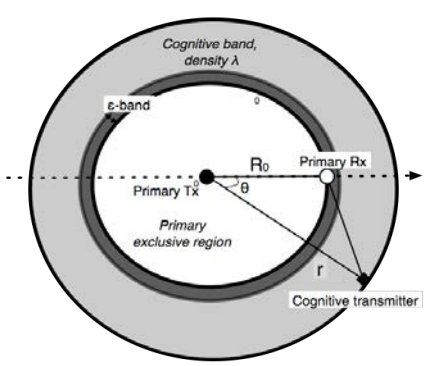
Sum-throughput per cognitive user scale as

$O(1)$ as $n \to \infty$

while guaranteeing $\Pr\left[\text{primary user's rate} \leq C_0\right] \leq \beta$

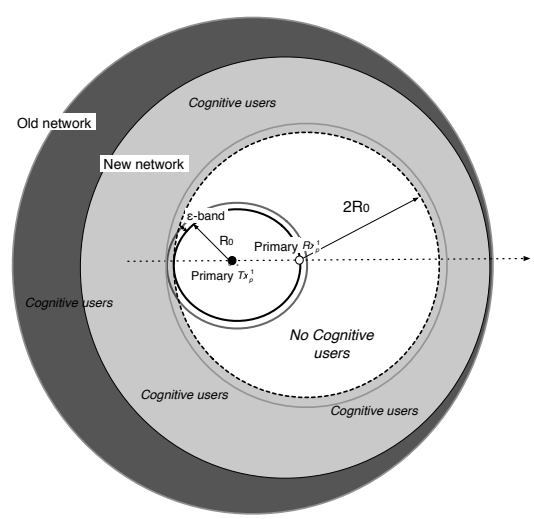
[Vu, Devroye, Tarokh 2007]

Model 1

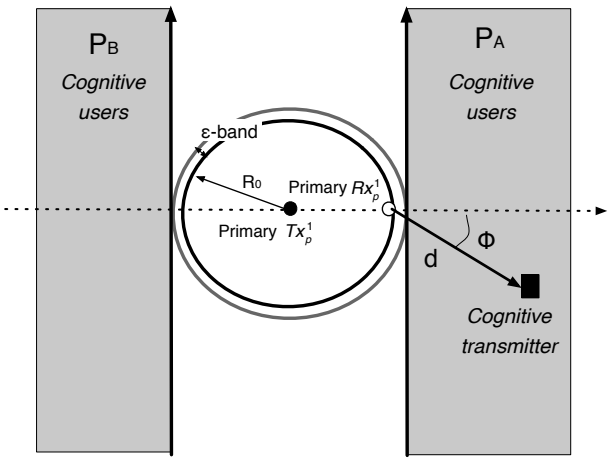


$$E[I_0]_{\alpha=4} = \lambda \pi P \left[-\frac{R^2}{(R^2 - R_0^2)^2} + \frac{(R_0 + \epsilon_p)^2}{\epsilon_p^2 (2R_0 + \epsilon_p)^2} \right]$$

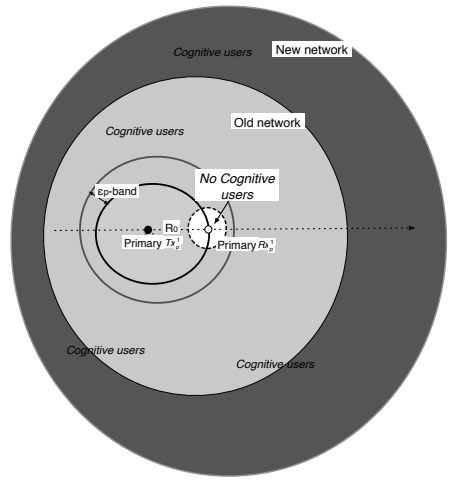
Exact calculation of interference



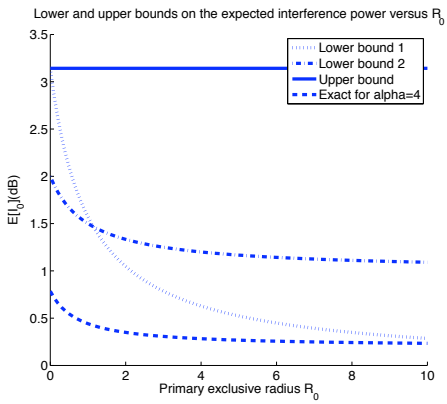
Lower bound on interference I



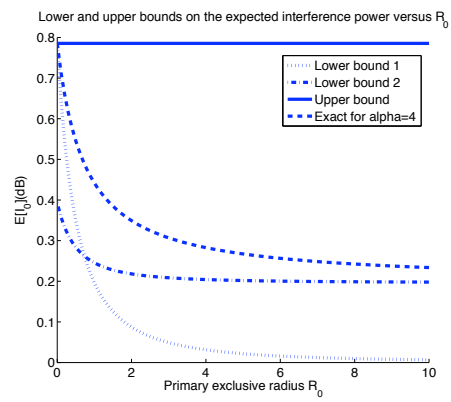
Lower bound on interference 2



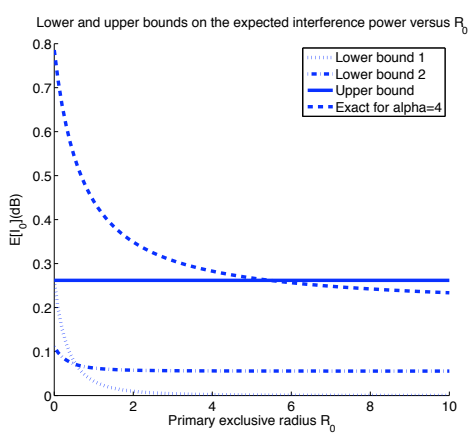
Outer bound on interference



α =3

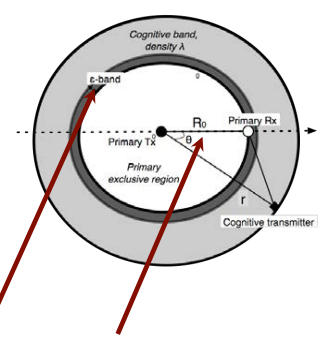


α =4



α =5

How to pick parameters

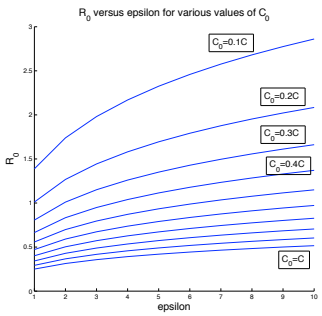


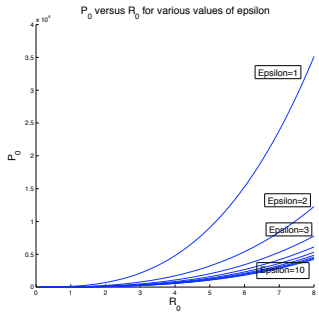
What R0 and ε will guarantee $\Pr\left[\text{primary user's rate} \leq C_0\right] \leq \beta$?

Tradeoffs!

Model 1

How to pick parameters



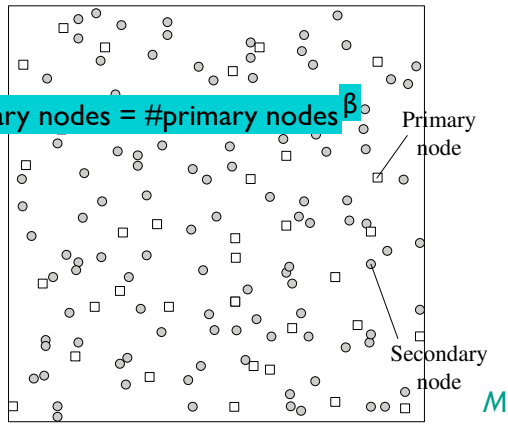


Model 1

Model 2

Ad hoc cognitive networks

#secondary nodes = #primary nodes β



Model 2

What we guarantee

Primary nodes act as if cognitive network does not exist

Primary nodes achieve same scaling law as if cognitive network does not exist

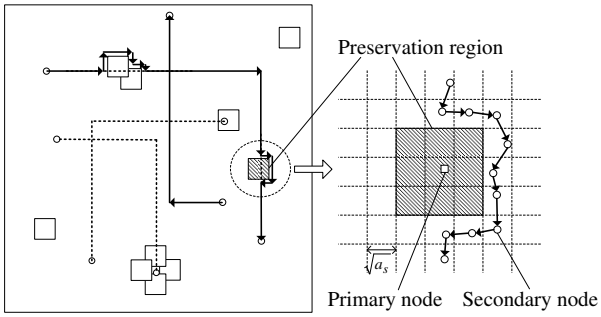
What we prove

$$T_p(n) = \Theta\left(\sqrt{\frac{1}{n \log n}}\right); \quad T_s(m) = \Theta\left(\sqrt{\frac{1}{m \log m}}\right)$$

*[Jeon, Devroye, Vu, Chung, Tarokh 2008]**

Model 2

How



Improving Cellular Downlink Capacity

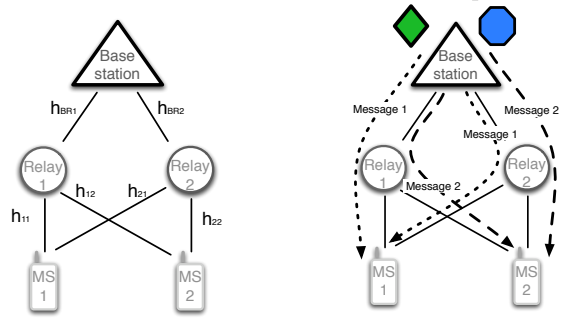
practical application of asymmetric cooperation

Motivation

- Cellular providers are introducing *relays* to
  - extend cell coverage
  - boost transmission rates
  - improve spectral efficiency

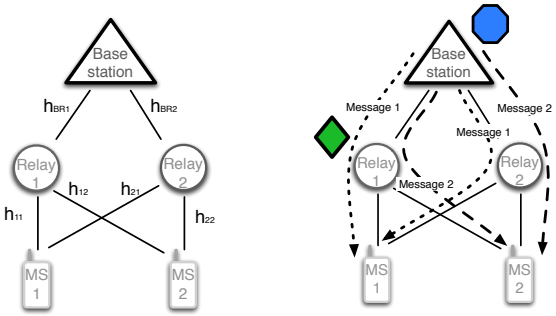
All at lower costs than building new
full-fledged base stations

Downlink cellular system



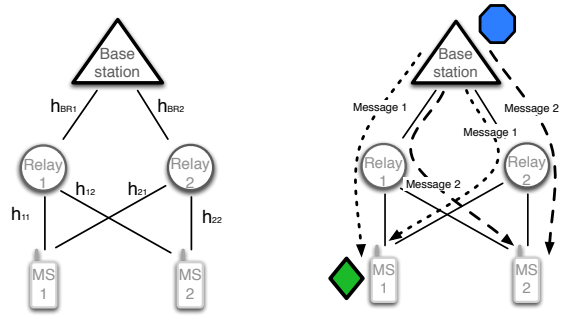
Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink cellular system



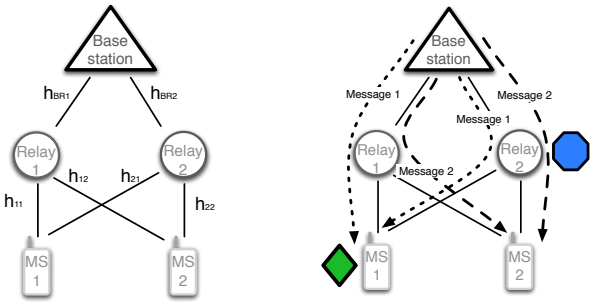
Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink cellular system



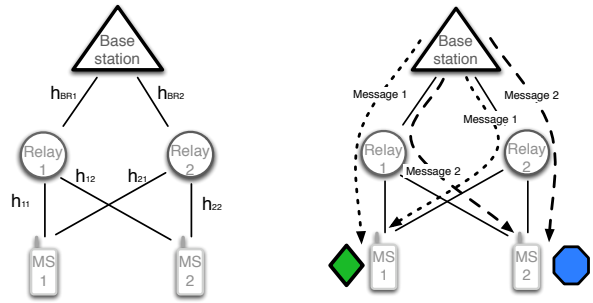
Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink cellular system



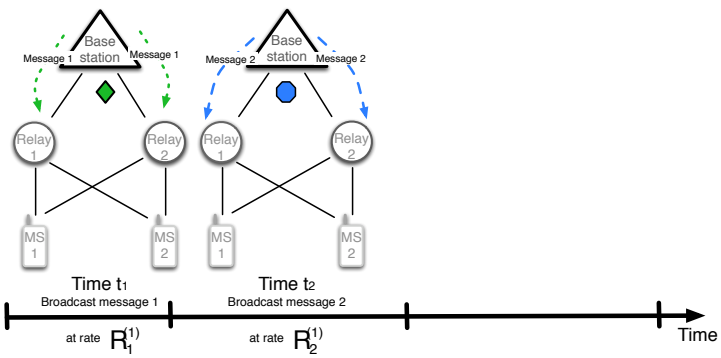
Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink cellular system

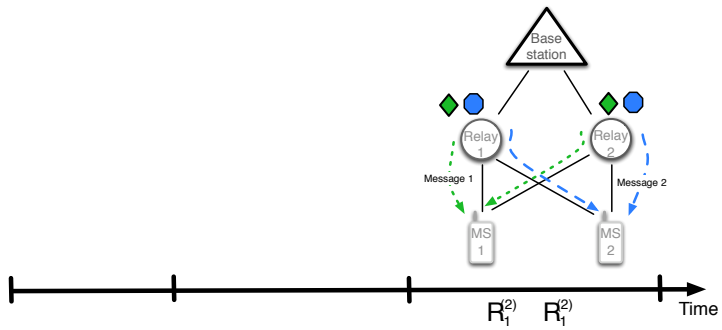


Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

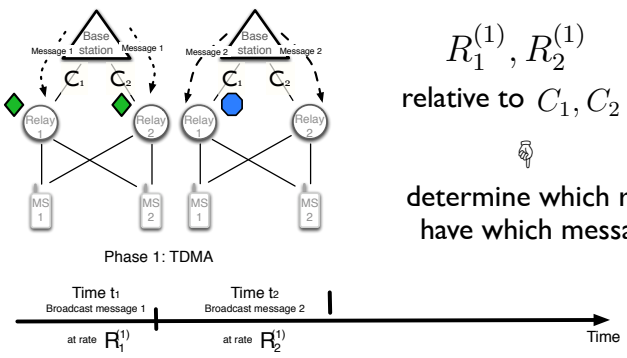
Problem setup: phase 1



Problem setup: phase 2



Phase 1 may result in asymmetry

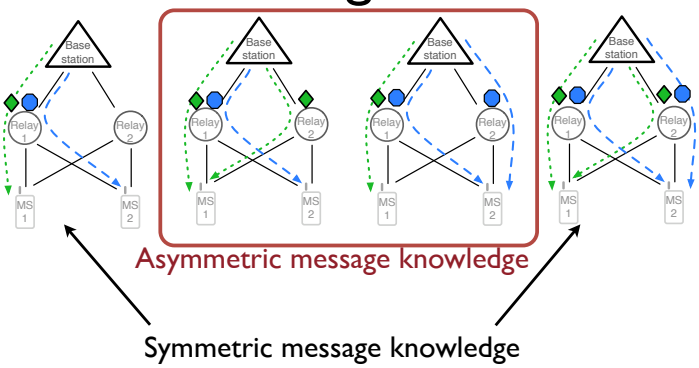


$R_1^{(1)}, R_2^{(1)}$

relative to C_1, C_2

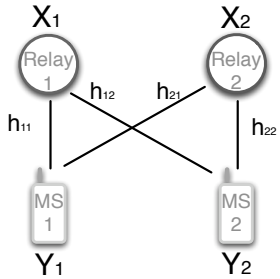
determine which relays have which messages!

4 message knowledge cases

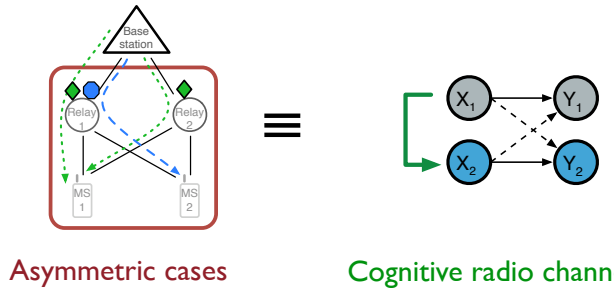


Phase 2

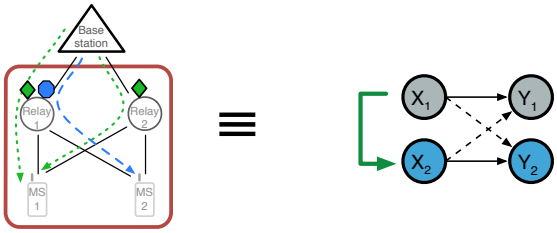
$$\underbrace{\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}}_{\text{Output}} = \underbrace{\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix}}_{\text{Channel given, known}} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{\text{to be determined, depends on what messages the relays have!}} + \underbrace{\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}}_{\text{Noise}}$$



Equivalence to cognitive radio channel



Equivalence to cognitive radio channel



Asymmetric cases

☞ linear precoding

Cognitive radio channel

☞ dirty-paper coding

Linear precoding

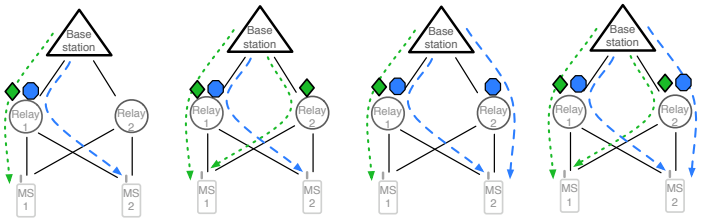
Transmitted signal at the 2 relays, resp.

Unit power messages

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \underbrace{\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}}_{\text{Precoding matrix B}} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

- Power constraints: $|b_{11}|^2 + |b_{12}|^2 + |b_{21}|^2 + |b_{22}|^2 \leq P_R$
- Message knowledge constraints: B has zeros

Linear precoding



$$B = \begin{bmatrix} b_{11} & b_{12} \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Constraints on the precoding matrices imposed by phase 1

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \overbrace{\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}}^{\text{B}} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Linear precoding

$$Y = HX + N$$
$$= HBU + N$$

Linear precoding

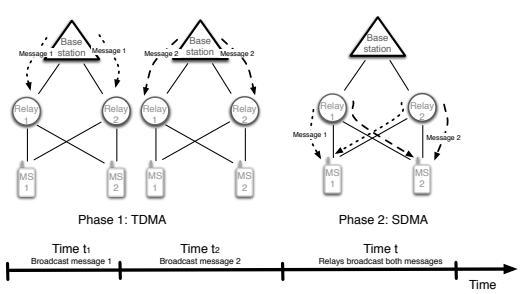
$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$
$$= \begin{bmatrix} (h_{11}b_{11} + h_{21}b_{21})U_1 + (h_{11}b_{12} + h_{21}b_{22})U_2 \\ (h_{12}b_{11} + h_{22}b_{21})U_1 + (h_{12}b_{12} + h_{22}b_{22})U_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}.$$

Receive SINR at Y_1 $\quad \gamma_1 = \dfrac{(h_{11}b_{11} + h_{21}b_{21})^2}{(h_{11}b_{12} + h_{21}b_{22})^2 + N_1}$

Receive SINR at Y_2 $\quad \gamma_2 = \dfrac{(h_{12}b_{12} + h_{22}b_{22})^2}{(h_{12}b_{11} + h_{22}b_{21})^2 + N_2}$

$$R_1^{(2)} = \frac{1}{2}\log_2(1+\gamma_1), \qquad R_2^{(2)} = \frac{1}{2}\log_2(1+\gamma_2)$$

Optimization



Need to select:

Phase 1: $t_1^{(1)},\ t_2^{(1)},\ R_1^{(1)},\ R_2^{(1)},\ n_1,\ n_2$

Phase 2: linear precoding B matrix

☞ We will do so according to two optimization criteria

Max throughput criterion

☞ Maximize the throughput over 2 phases

$$\max \quad \frac{n_1+n_2}{\frac{n_1}{R_1^{(1)}}+\frac{n_2}{R_2^{(1)}}+\max(\frac{n_1}{R_1^{(2)}},\frac{n_2}{R_2^{(2)}})}$$

$$\text{s.t.} \quad n_1, n_2 \geq 0$$

$$R_1^{(1)}, R_2^{(1)} \text{ are phase 1 rates}$$

$$R_1^{(2)}, R_2^{(2)} \text{ are phase 2 rates}$$

n_1, n_2 are number of bits sent to each mobile, variables to be optimized over

Extreme fairness criterion

☞ Maximize the throughput when forced to send one unit of information to each mobile

$$\max \quad \frac{n_1+n_2}{\frac{n_1}{R_1^{(1)}}+\frac{n_2}{R_2^{(1)}}+\max(\frac{n_1}{R_1^{(2)}},\frac{n_2}{R_2^{(2)}})}$$

$$\text{s.t.} \quad n_1, n_2 \geq 0$$

$$R_1^{(1)}, R_2^{(1)} \text{ are phase 1 rates}$$

$$R_1^{(2)}, R_2^{(2)} \text{ are phase 2 rates}$$

$n_1 = n_2$

Optimization reduction

We reduce the non-linear optimization problem from one over 8 variables to one over 2 variables.

Max throughput optimization:

$$\max_{x, b_{11}, \alpha, \beta, \theta} \quad \frac{x+1}{x/R_1^{(1)}+1/R_2^{(1)}+\max\left(x/\log_2\left(1+\frac{|h_{11}|^2}{|\alpha|^2+N_1/|b_{11}|^2}\right),\ 1/\log_2\left(1+\frac{|\beta|^2}{|h_{12}|^2+N_2/|b_{11}|^2}\right)\right)}$$

$$\text{s.t.} \quad x \geq 0 \quad |b_{11}|^2 \leq P_R$$

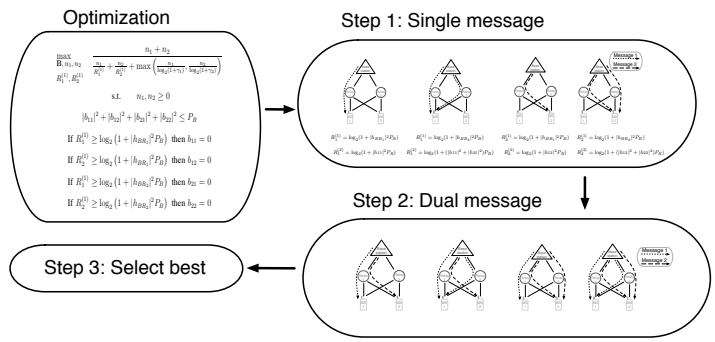
$$g_1|\alpha|^2 + 2|g_{12}||\alpha||\beta|\cos(\theta_G+\theta) + g_2|\beta|^2 \leq P_R/|b_{11}|^2 - 1$$

Simplified max throughput optimization:

$$\max_{t, |b_{11}|} \quad \frac{x+1}{x/R_1^{(1)}+1/R_2^{(1)}+\max\left(x/\log_2\left(1+\frac{|h_{11}|^2}{|\alpha(t,b_{11},\theta^*)|^2+N_1/|b_{11}|^2}\right),\ 1/\log_2\left(1+\frac{|\beta(t,b_{11},\theta^*)|^2}{|h_{12}|^2+N_2/|b_{11}|^2}\right)\right)}$$

$$\text{s.t.} \quad x \in \{0, x^*, \infty\}, \quad |b_{11}|^2 \leq P_R, \quad t \in [0,\pi], \quad \theta^* \in \{-\theta_G, \pi-\theta_G\}$$

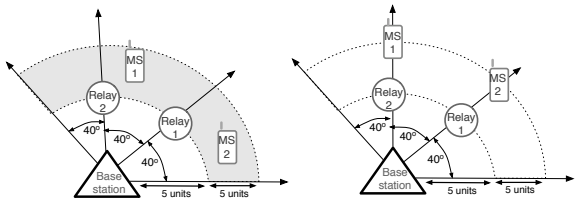
Optimization summary



Max throughput

Simulation Setup

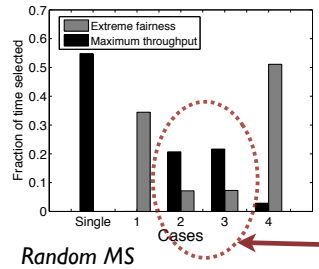
Rayleigh fading, pathloss



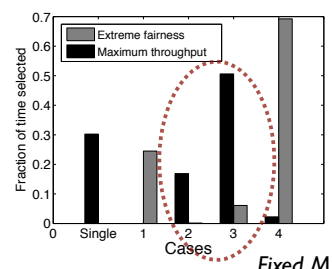
Random MS placement

Fixed MS placement

Simulation Results



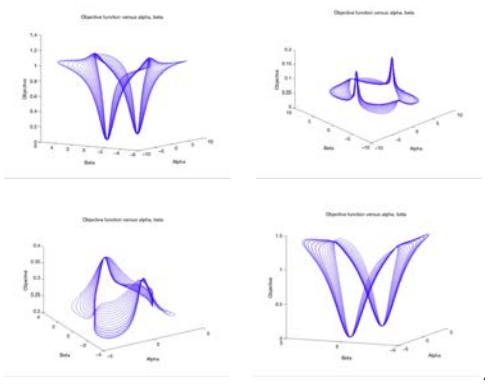
Random MS



Fixed MS

Asymmetric cases are throughput optimal often!

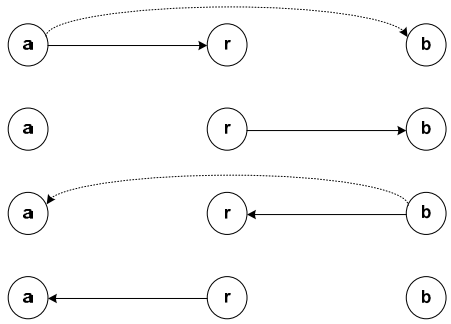
Optimization



Bi-directional relaying

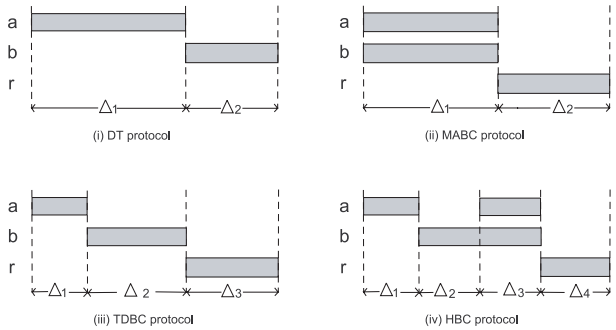
Sang Joon Kim, Patrick Mitran,
Natasha Devroye, Vahid Tarokh

Nodes a and b want to exchange messages over a *shared half-duplex memoryless* channel with the help of a relay



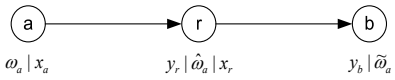
[Naïve four phase bi-directional cooperation]

Protocols in time

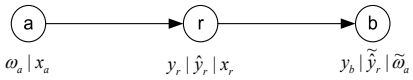


Protocols in relaying

- Decode and Forward (D&F)



- Compress and Forward (C&F)



- Mixed Forward : One way uses D&F and the other used C&F

Results

Find inner and outer bounds of capacity region

- Inner bound (achievable region)

$$R \le C_{in} \Rightarrow \lim_{n\to\infty} P_e = 0$$

- Outer bound

$$R \le C_{out} \Leftarrow \lim_{n\to\infty} P_e = 0$$

- Capacity C lies in

$$C_{in} \le C \le C_{out}$$

Plots

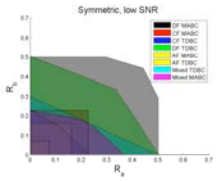


Fig. 3. Comparison of bi-directional regions with $h_{ar} = h_{br} = 1$, $h_{ab} = 0.2$, $P_a = P_b = P_r = 1$ and $N_a = N_b = N_r = 1$.

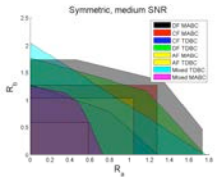


Fig. 4. Comparison of bi-directional regions with $h_{ar} = h_{br} = 1$, $h_{ab} = 0.2$, $P_a = P_b = P_r = 10$ and $N_a = N_b = N_r = 1$.

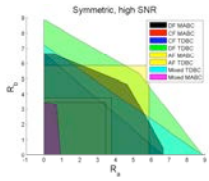


Fig. 5. Comparison of bi-directional regions with $h_{ar} = h_{br} = 1$, $h_{ab} = 0.2$, $P_a = P_b = P_r = 10^5$ and $N_a = N_b = N_r = 1$.

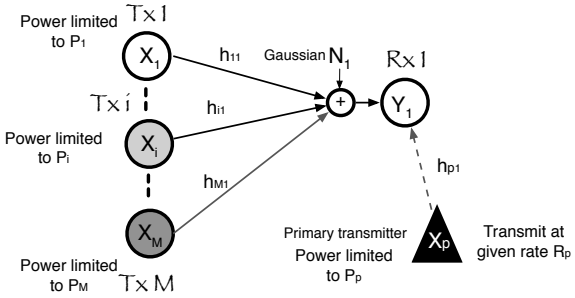
Opportunistic interference cancellation

Petar Popovski, Natasha Devroye

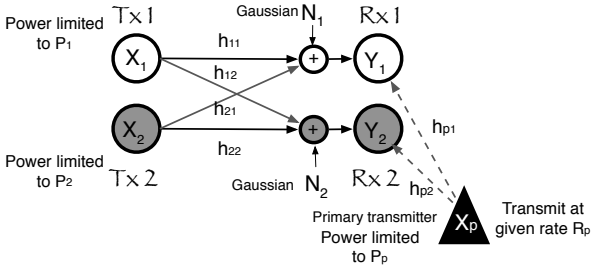
Motivation

- Rather than Tx-side cognition, can the Rxs behave in a cognitive fashion?
- Assume the cognitive Rx knows the primary's codebook
- Assume the cognitive Tx knows at what power it may transmit so as not to harm the primary Rx
- Assume primary system does not change

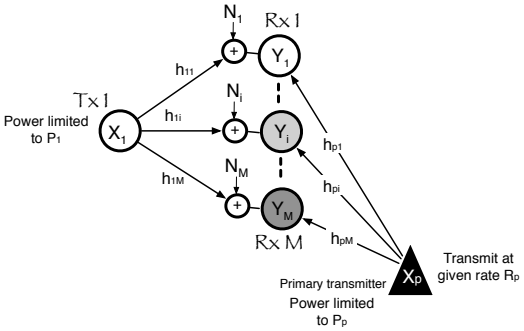
Scenario 1: MAC



Scenario 2: Interference



Scenario 3: Broadcast



Key idea

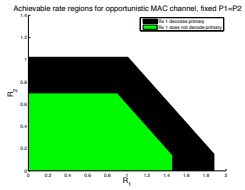
- The channel to the cognitive Rx is a multiple access channel: opportunistically decode the primary signal if its rate is below the capacity from the primary to secondary.
- Obtain the rate region described by the MAC but with the MAC rate set to the primary rate. Project down 1 dimension.

Nice technical result

- Can achieve all points on the boundary without time sharing with the primary
- Achieved by:
  - MAC: single message split at cognitive Tx
  - Int: cannot in general achieve
  - BC: cannot in general achieve

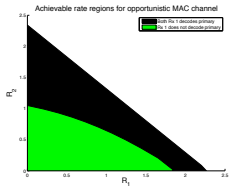
Gains for opportunistic MAC

11



(a)

Rate regions for opportunistic MAC channel with equal, fixed powers $P_1 = P_2$.

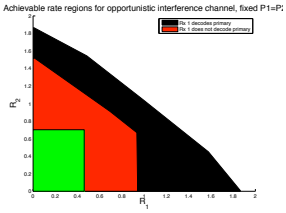


(b)

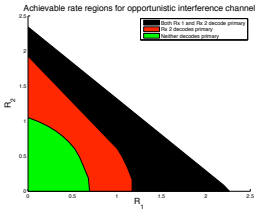
Rate regions for opportunistic MAC channel with $(P_1, P_2) \in \mathcal{P}_{MAC}$.

Fig. 5. Opportunistic MAC channel regions.

Gains for opportunistic interference



Rate regions for opportunistic interference channel with equal, fixed powers $P_1 = P_2$.



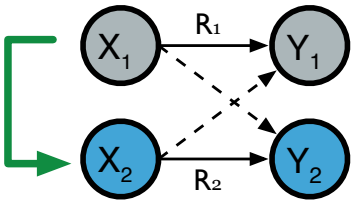
Rate regions for opportunistic interference channel with $(P_1, P_2) \in \mathcal{P}_{INT}$.

Fig. 6. Opportunistic interference channel regions.

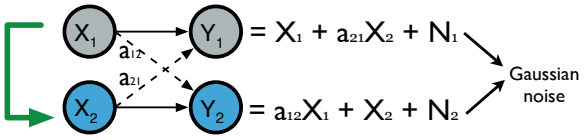
Causal cognitive radio channel

The question

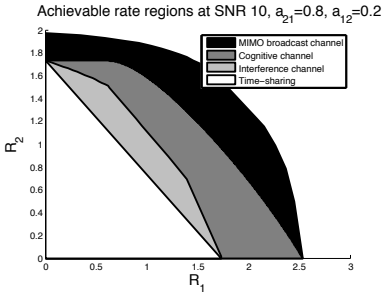
What is the capacity of the cognitive radio channel?



Achievable region: graphical

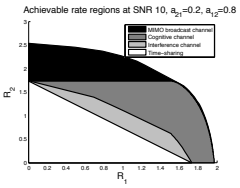


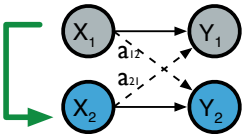
$a_{21}=0.8$
$a_{12}=0.2$



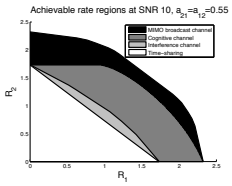
Different cross-over parameters

$a_{21}=0.2$
$a_{12}=0.8$



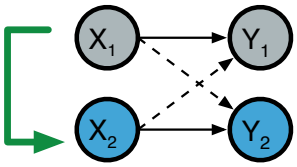


$a_{21}=0.55$
$a_{12}=0.55$



Causal message knowledge?

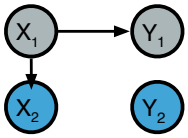
What if this link is not free?



Protocol 1

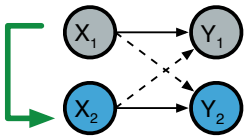
Phase 1:

broadcast channel



Phase 2:

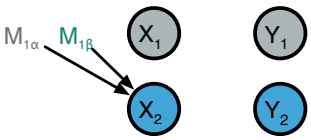
cognitive radio channel



Protocol 2

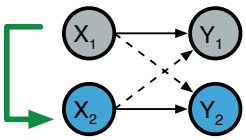
Phase 1:

multiple access channel



Phase 2:

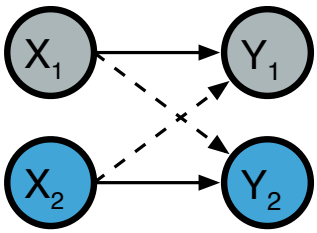
cognitive radio channel



Protocol 3

Only one phase: interference channel.

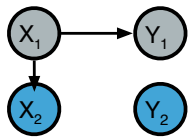
X_2 does not know X_1 and does not dirty paper code against it.



Protocol 4

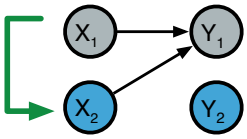
Phase 1:

broadcast channel



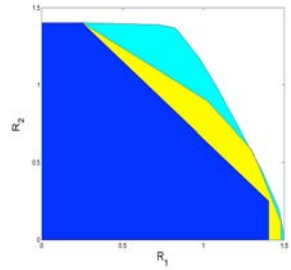
Phase 2:

X_2 aids X_1 in sending it message. X_2 does not send any information of its own.

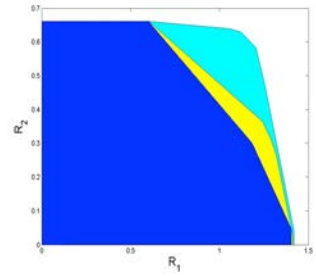


Causal achievable regions

Blue: G=1 Yellow: G=10 Cyan: Genie-Aided



$P_1=P_2=6$



$P_1=6, P_2=1.5$

Extra

Cognitive radio channel

Unused spectrum

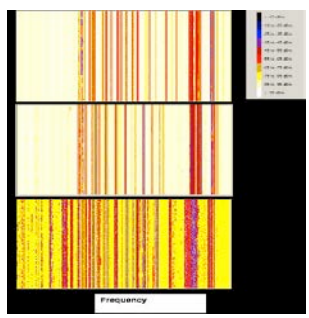


Figure 1: Occupancy of approximately 700 megahertz of spectrum below 1 GHz

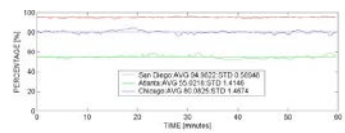


Figure 3: Use of a 7 megahertz band below 1 GHz (percentage of a 30-second window by 7 megahertz block that was idle)

Federal Communications Commission
Spectrum Policy Task Force
Report of the Spectrum
Efficiency Working Group
November 15, 2002
http://www.fcc.gov/sptf/files/SEWGFinalReport_1.pdf

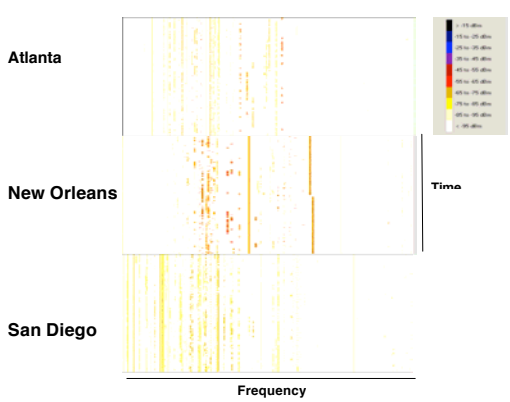


Figure 2: Occupancy of a 7.5 megahertz UHF Land Mobile band

Formal definitions

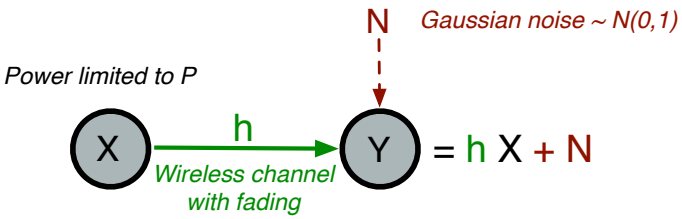
A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there exists an $(2^{\lceil nR_1 \rceil}, 2^{\lceil nR_2 \rceil}, n, P_e)$ code such that $P_e \leq \epsilon$.

An $(2^{\lceil nR_1 \rceil}, 2^{\lceil nR_2 \rceil}, n, P_e)$ code consists of encoding functions that map messages $W_1 \in \{1, 2, \cdots 2^{\lceil nR_1 \rceil}$ and $W_2 \in \{1, 2, \cdots 2^{\lceil nR_2 \rceil}$ and decoding functions that recover these messages such that the average error probability is less than P_e.

Achievable rate region: set of achievable rate pairs (R_1, R_2).

Capacity region: the closure of the set of all achievable rate pairs (R_1, R_2).

Why Cooperate to Communicate?

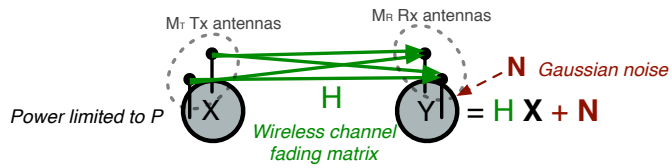


Channel capacity (bits/channel use):

$$C = \begin{cases} \frac{1}{2}\log_2(1+|h|^2 P/P_N) & \text{Instantaneous} \\ E_h\left[\frac{1}{2}\log_2(1+|h|^2 P/P_N)\right] & \text{Ergodic} \end{cases}$$

Why Cooperate to Communicate?

2 Tx antenna, 2 Rx antenna Multiple Input Multiple Output (MIMO) fading channel with Gaussian noise

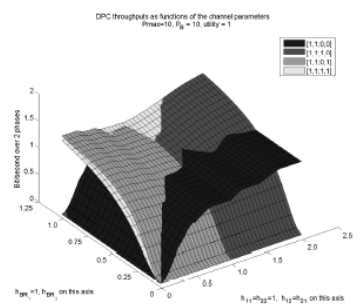


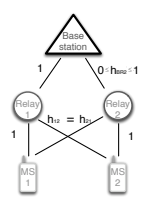
Channel capacity: $C = \begin{cases} \max_{\mathbf{Q}:Tr(\mathbf{Q})=P} \frac{1}{2}\log_2\left|\mathbf{I}_{\mathbf{M_R}} + \mathbf{HQH}^\dagger\right| & \text{Instantaneous} \\ \max_{\mathbf{Q}:Tr(\mathbf{Q})=P} E_{\mathbf{H}}\left[\frac{1}{2}\log_2\left|\mathbf{I}_{\mathbf{M_R}} + \mathbf{HQH}^\dagger\right|\right] & \text{Ergodic} \end{cases}$

Extra

Asymmetric cooperation downlink

Which of 4 schemes is optimal





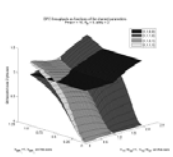


Fig. 15. DPC, Max throughput, $P_{max} = 10, P_H = 5$

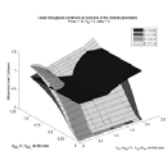


Fig. 16. Linear, Max throughput, $P_{max} = 10, P_H = 5$

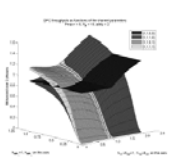


Fig. 17. DPC, Max throughput, $P_{max} = 5, P_H = 10$

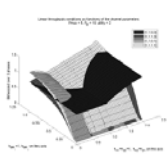


Fig. 18. Linear, Max throughput, $P_{max} = 5, P_H = 10$

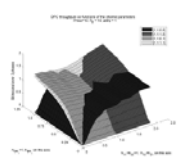


Fig. 7. DPC, Extreme Fairness, $P_{max} = 10, P_H = 10$

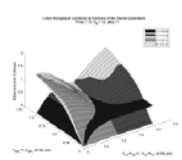


Fig. 8. Linear, Extreme Fairness, $P_{max} = 10, P_H = 10$

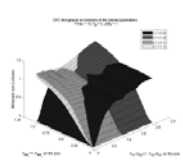


Fig. 9. DPC, Extreme Fairness, $P_{max} = 10, P_H = 5$

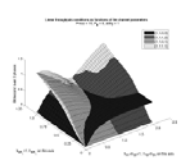
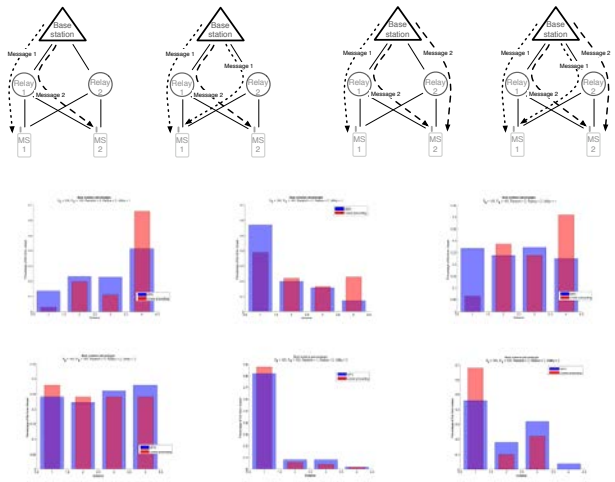
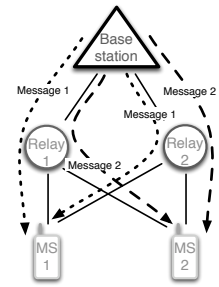


Fig. 10. Linear, Extreme Fairness, $P_{max} = 10, P_H = 5$



Baselines for comparison

- Depends on criterion
- Round robin with relay
- Best 2 hop overall



Gains of relay cooperation over non-cooperative schemes

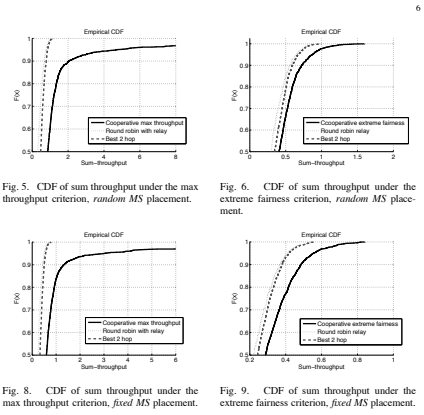


Fig. 5. CDF of sum throughput under the max throughput criterion, *random MS* placement.

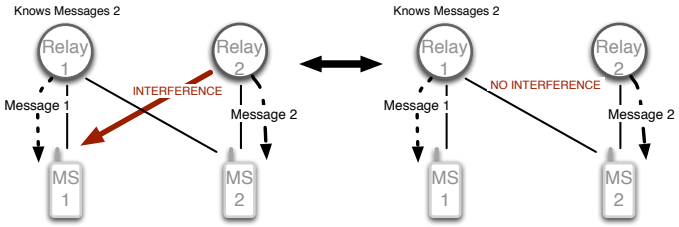
Fig. 6. CDF of sum throughput under the extreme fairness criterion, *random MS* placement.

Fig. 8. CDF of sum throughput under the max throughput criterion, *fixed MS* placement.

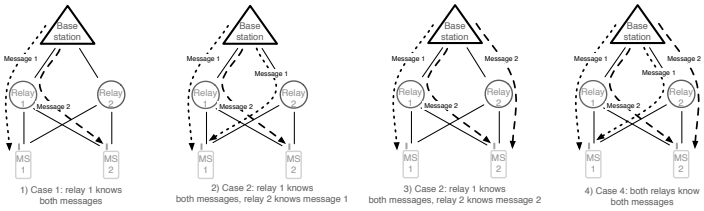
Fig. 9. CDF of sum throughput under the extreme fairness criterion, *fixed MS* placement.

Dirty-paper coding

Optimal encoding scheme for a broadcast channel



Dirty-paper coding

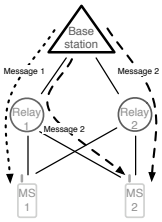


$$B_1 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \quad B_2 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \qquad B_1 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \qquad B_1 = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \quad B_2 = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$$

Power constraint: $trace(B_1 + B_2) \leq P_R$

Dirty-paper coding

- The achievable rates can be written in terms of the transmit covariance matrices B_1 and B_2 as:



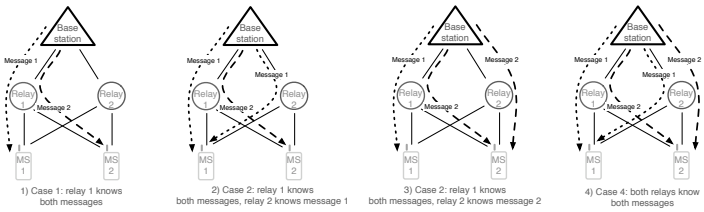
3) Case 3: relay 1 knows both messages, relay 2 knows message 2

$$R_1 \leq \frac{1}{2}\log_2\left(\frac{|H_1(B_1)H_1^T + N_1|}{|N_1|}\right), \quad R_2 \leq \frac{1}{2}\log_2\left(\frac{|H_2(B_1+B_2)H_2^T|}{|H_2B_1H_2^T + N_2|}\right)$$

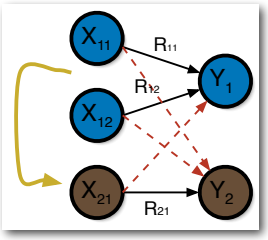
Cognitive multiple access networks

Natasha Devroye, Patrick Mitran, Vahid Tarokh

Optimization



For **each** of the 4 cases, find the parameters that maximize throughput or extreme fairness criteria

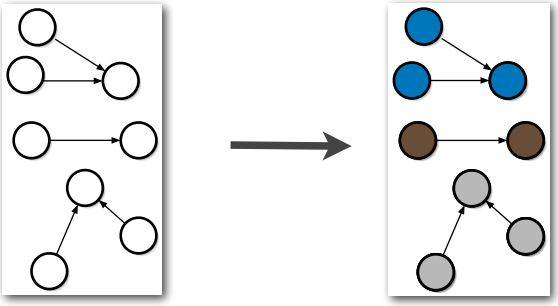


A priori message knowledge

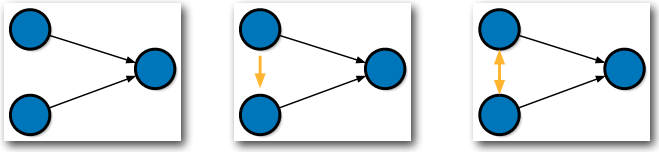
aid transmission

mitigate interference

OR BOTH



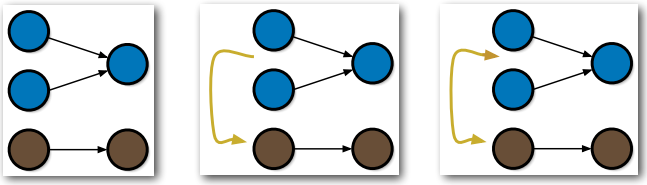
Disjoint clusters interfere (inter-cluster)
Nodes within a cluster interfere (intra-cluster)



Competitive: MAC

Cognitive: Van der Meulen Prelov

Cooperative: 2x1 vector MISO



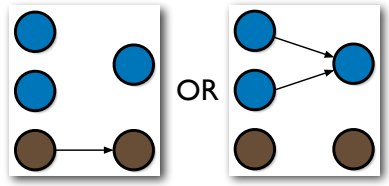
Competitive: generalized interference

Cognitive: THIS TALK

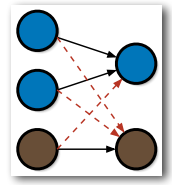
Cooperative: almost, but not quite broadcast!

- Motivation and definition
- Relation to previous work
- Theorem intuition
- Achievable region in Gaussian case

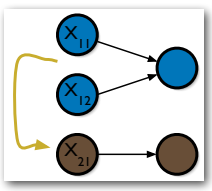
Traditionally



Proposed: simultaneous



The catch: interference

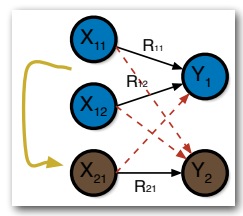


If X_{21} is close to X_{11}, X_{12}
it can obtain their messages

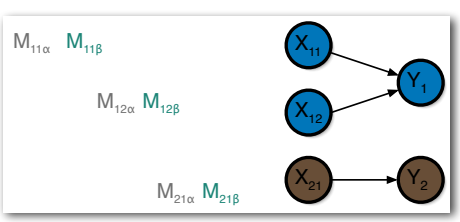
Genie assumption: X_{21} knows X_{11}, X_{12} *a priori*

X_{21} knows the messages
of X_{11} and X_{12} a priori.

Asymmetric problem.

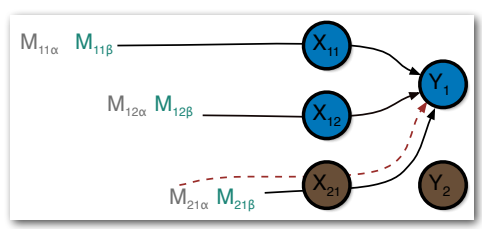


During simultaneous transmission,
what rates R_{11}, R_{12}, R_{21} are achievable?



Private variables $M_{11\alpha}$, $M_{12\alpha}$, $M_{21\alpha}$
(intended for one receiver only)

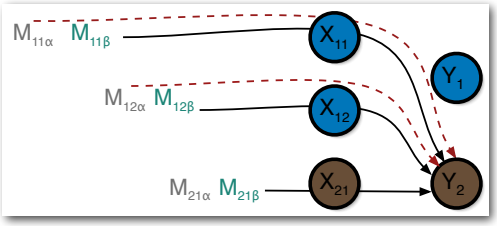
Public variables $M_{11\beta}$, $M_{12\beta}$, $M_{21\beta}$
(intended for all receivers)



$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1; \mathbf{M}_T | \mathbf{M}_{\overline{T}})$$

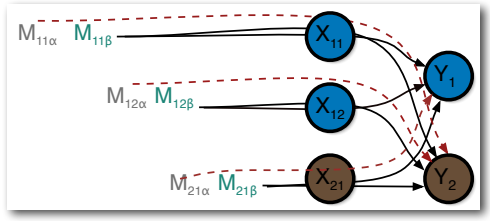
$$T_1 = \{11\alpha, 11\beta, 12\alpha, 12\beta, 21\beta\}$$

L_t is the rate of M_t



$$\bigcap_{T\subset T_2}\left(\sum_{t_2\in T} L_{t_2}\right) \le I(Y_2;\mathbf{M}_T|\mathbf{M}_{\overline{T}})$$

$$T_2 = \{11\beta, 12\beta, 21\alpha, 21\beta\}$$

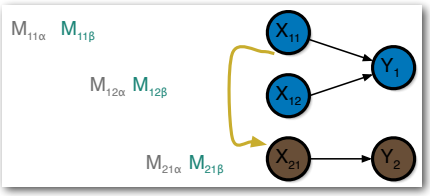


$$\bigcap_{T\subset T_1}\left(\sum_{t_1\in T} L_{t_1}\right) \le I(Y_1;\mathbf{M}_T|\mathbf{M}_{\overline{T}})$$

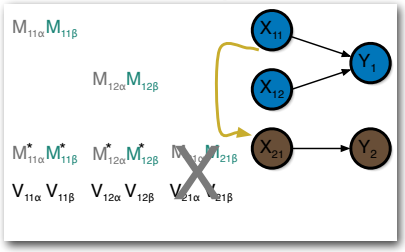
$$\bigcap_{T\subset T_2}\left(\sum_{t_2\in T} L_{t_2}\right) \le I(Y_2;\mathbf{M}_T|\mathbf{M}_{\overline{T}})$$

Secondary transmitter has prior knowledge of the primary's message and can:

1. Aid the primary transmitters
2. Use knowledge of interference to mitigate it

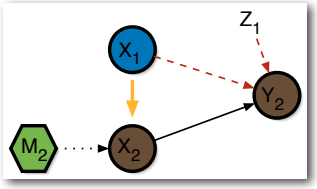


a priori message knowledge allows secondary transmitter to act as relay

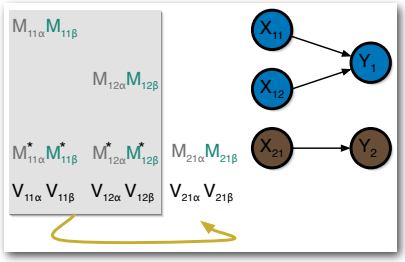


$M^*_{11\alpha}M^*_{11\beta}$ $M^*_{12\alpha}M^*_{12\beta}$ ``aid" random variables

$V_{11\alpha}$ $V_{11\beta}$ $V_{12\alpha}$ $V_{12\beta}$ (M,M*) combined for notational clarity



$$C = \max_{p(m_2,x_2|x_1)} I(M_2;Y_2) - I(M_2;X_1)$$



R_t is the rate of V_t

L_t is the rate of M_t

$$R_{1ik} = L_{1ik}$$

$$R_{2jk} \le L_{2jk} - I(V_{2jk};\mathbf{V_1})$$

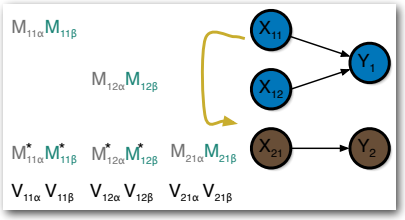
Achievable rate region:

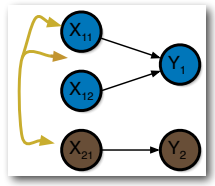
$$R_{1ik} = L_{1ik}$$
$$R_{2jk} \leq L_{2jk} - I(V_{2jk}; \mathbf{V_1})$$

Gel'fand-Pinsker coding

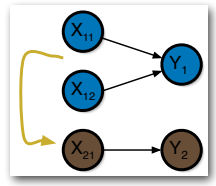
$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1; \mathbf{V}_T | \mathbf{V}_{\overline{T}})$$
$$\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) \leq I(Y_2; \mathbf{V}_T | \mathbf{V}_{\overline{T}})$$

Overlapping MACs



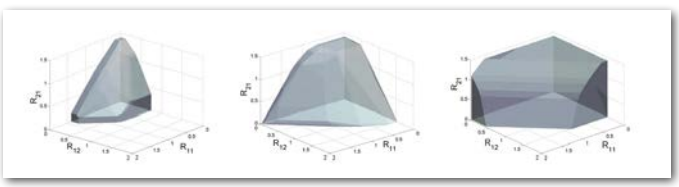


is an outer bound for



along with the bounds

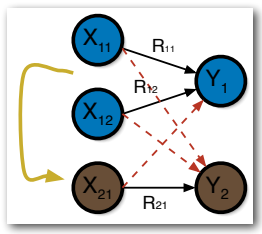
$$R_{11} \leq I(Y_1; X_{11}, X_{21} | X_{12})$$
$$R_{12} \leq I(Y_1; X_{12}, X_{21} | X_{11})$$
$$R_{21} \leq I(Y_2; X_{21} | X_{11}, X_{12})$$



Competitive MAC channel

Cognitive MAC channel

Outer bound 3x2 MIMO + inequalities

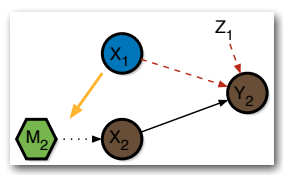


A priori message knowledge

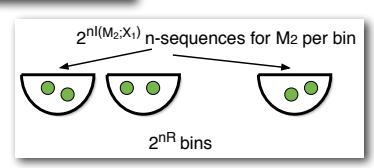
aid transmission

mitigate interference

OR BOTH



To send *bin number i*, given interference X_1 look in bin *i* for M_2 such that (M_2, X_1) are jointly typical.



$$C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)$$

$$\bigcap_{T\subset T_G}\left(\sum_{t\in T} R_t\right) \le I(g(\mathbf{X_1});\mathbf{M}_T|\mathbf{M}_{\overline{T}})$$

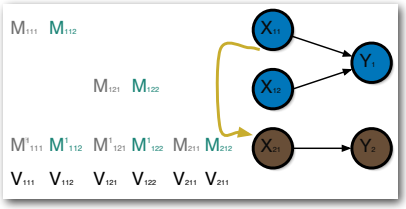
MAC equations for S_2 to get X_1's messages

$$R_{1ik} = L_{1ik}$$
$$R_{2jk} \le L_{2jk} - I(V_{2jk};\mathbf{V_1})$$

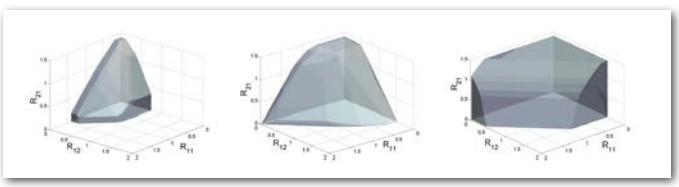
Gel'fand-Pinsker coding

$$\bigcap_{T\subset T_1}\left(\sum_{t_1\in T} L_{t_1}\right) \le I(Y_1,\mathbf{V}_{\overline{T}};\mathbf{V}_T|W)$$
$$\bigcap_{T\subset T_2}\left(\sum_{t_2\in T} L_{t_2}\right) \le I(Y_2,\mathbf{V}_{\overline{T}};\mathbf{V}_T|W),$$

Overlapping MACs



$$[P(m_{11\alpha})P(m_{11\beta})P(x_{11}|m_{11\alpha},m_{11\beta})]$$
$$\times\,[P(m_{12\alpha})P(m_{12\beta})P(x_{12}|m_{12\alpha},m_{12\beta})]$$
$$\times\,\left[P(m^*_{11\alpha}|m_{11\alpha})P(m^*_{11\beta}|m_{11\beta})\right]$$
$$\times\,\left[P(m^*_{12\alpha}|m_{12\alpha})P(m^*_{12\beta}|m_{12\beta})\right]$$
$$\times\,[P(m_{21\alpha}|\mathbf{v_1})P(m_{21\beta}|\mathbf{v_1})]$$
$$\times\,[P(x_{21}|m_{21\alpha},m_{21\beta},\mathbf{m}^*)]\,P(y_1|\mathbf{x_1},\mathbf{x_2})P(y_2|\mathbf{x_1},\mathbf{x_2})$$



Competitive MAC channel	Cognitive MAC channel	Outer bound 3x2 MIMO + inequalities
Volume 0.6536	Volume 1.5064	Volume 2.9127

2-Dimensional Generalized Processor Sharing (2D-GPS)

Raymond Yim

Natasha Devroye

Background: 1D-GPS

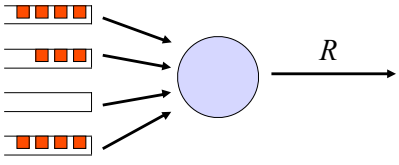
Rate Guarantees

$\phi_1 = 0.2$

$\phi_2 = 0.3$

$\phi_3 = 0.4$

$\phi_4 = 0.1$



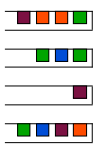
$$R_i = \frac{\phi_i R}{\sum_{j:b_j\neq 0}\phi_j}\ \text{if}\ b_i \neq 0$$

The goal of Packet-by-Packet GPS (PGPS) server is to mimic the result of GPS

2D-GPS: Overview

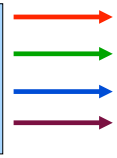
Rate Guarantees

$$\begin{bmatrix} .2 & .1 & .4 & .3 \\ .3 & .5 & .1 & .1 \\ .3 & .2 & .1 & .4 \\ .2 & .2 & .4 & .2 \end{bmatrix}$$



Routing Matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$



2D PGPS Scheduler

$$\phi = \begin{bmatrix} .2 & .1 & 0 & .3 \\ 0 & .5 & .1 & 0 \\ 0 & 0 & 0 & .4 \\ .2 & .2 & .4 & .2 \end{bmatrix}$$

Augmen

Substochastic matrix

$R_{ij} \ge \phi_{ij} R$

$R_{ij} = 0\ \text{if}\ \phi_{ij} = 0$

Fairness in Augmentation

- In 1D-GPS, solutions are both
  - Max-Min Fair
  - Proportionally Fair
- In 2D-GPS, the two fairness metrics will give rise to two different results in general.

Max-Min Fairness

$$\begin{bmatrix} 1/5 & 0 & 0 \\ 0 & 1/2 & 1/10 \\ 2/5 & 0 & 2/5 \end{bmatrix} \xrightarrow{\times \frac{5}{4}} \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 5/8 & 1/8 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 5/6 & 5/6 \\ 1/2 & 0 & 1/2 \end{bmatrix} \xrightarrow{\times \frac{3}{2}} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 5/6 & 5/6 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$\times \frac{4}{3}$$

Proportional Fairness

$$\min \sum_{i,j} \left| R_{ij} - k\phi_{ij} \right| \quad \text{subjected to}$$

$$R_{ij} \geq \phi_{ij}$$

$$R_{ij} = 0 \; if \; \phi_{ij} = 0$$

$$\sum_i R_{iq} \leq 1 \; \forall \; q$$

$$\sum_j R_{pj} \leq 1 \; \forall \; p$$

$$k \geq 1$$

$$\left(\sum_j R_{pj} - 1 \frac{1}{k} \right) \left(\sum_i R_{iq} - 1 \frac{1}{k} \right) = 0 \; if \; \phi_{ij} \neq 0$$

The non-augmentability constraints are non-linear

Birkhoff Decomposition

- Any *N*x*N* doubly stochastic matrix can be represented by a convex sum of at most $(N\text{-}1)^2+1$ permutation matrices.

$$A = \sum_i \lambda_i P^i, \quad \sum_i \lambda_i = 1$$

Lemma

- Any non-augmentable matrix *R* from ϕ can be represented by a convex sum of non-absorbable zero-enforced permutation matrices of ϕ.

$$A = \sum_i \lambda_i P^i, \quad \sum_i \lambda_i = 1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & X & 0 \end{bmatrix}, \begin{bmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & X & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & X \\ X & 0 & 0 \\ 0 & X & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & X \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Transform problem

Lemma 3: Let $P = \{P^1, \cdots, P^z\}$ be the set of all non-absorbable zero-enforced permutation matrices of Φ, and let p^n_{ij} be the entries of matrix P^n for $n \in \{1, \cdots z\}$. Let $\lambda = (\lambda_1, \cdots, \lambda_z)$ be a vector of dimension $z = |P|$. Suppose λ and k are the solution to the optimization problem:

$$\min \sum_{i,j} \left| \sum_{n: p^n_{ij}=1} \lambda_n - k\phi_{ij} \right|, \tag{7}$$

subjected to

$$\sum_{n: p^n_{ij}=1} \lambda_n \geq \phi_{ij} \;\; \forall \;\; i, j, \tag{8}$$

$$\sum_n \lambda_n = 1, \tag{9}$$

$$\lambda_n \geq 0 \;\; \forall \;\; n, \tag{10}$$

$$k \geq 0, \tag{11}$$

and the service rate matrix $R = \sum_{n=1}^{z} \lambda_n P^n$ satisfies the non-augmentability constraint. Then, we say R is the proportional fair solution to Φ, and k is the proportional increase of Φ.

Example 2: Using Φ from Example 1. We first write the service rate matrix R as a convex combination of non-absorbable zero-enforced permutation matrices. That is, $R = \lambda_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \lambda_4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. The non-augmentability constraint is $\lambda_2\lambda_3 = 0$. Solve the two linear optimization problems, one with $\lambda_2 = 0$ and one with $\lambda_3 = 0$, the minimum cost result from $\lambda_2 = 0$. The resulting $R = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.85 & 0.15 \\ 0.6 & 0 & 0.4 \end{bmatrix}$.

Example

$$\Phi = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{10} \\ \frac{2}{5} & 0 & \frac{2}{5} \end{bmatrix}$$

Max-min augmentation

$$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

PF augmentation

$$\begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.85 & 0.15 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

Compound Dirty Paper Channel

Patrick Mitran, Natasha Devroye, Vahid Tarokh
Harvard University

- Results on the capacities of channels with side information known at the transmitter assume *full channel knowledge*
- In realistic cognitive radio channels, we should not assume full channel knowledge. Results for cognitive radio channels use results from channels with known side-info at the transmitter

- Channel with side information at the transmitter
- Fading channels, where the channel state is unknown to the transmitter, but known to the receiver
- What are bounds on the capacity?

Theorem 2: Let $p^{\beta}_{Y|X,S}$ be a compound channel continuously parameterized in $\beta \in \mathcal{C}$ ($\mathcal{C}$ compact) where the input and output alphabets are standard. Then, the capacity of the compound channel with side-information at the transmitter, C, is bounded by $C_L \leq C \leq C_U$ where

$$C_L = \sup_{p_{U|X,q(S),W} \cdot p_{X|q(S),W} \cdot p_W} \left[\inf_{\beta \in \mathcal{C}} \left[I^{\beta}(U;Y|W) - I(U;q(S)|W) \right] \right] \tag{17}$$

$$C_U = \sup_{p^{\beta}_{U|X,S,W} \cdot p_{X|S,W} \cdot p_W} \left[\inf_{\beta \in \mathcal{C}} \left[I^{\beta}(U;Y|W) - I(U;S|W) \right] \right], \tag{18}$$

and the suprema are over all distributions on standard alphabets U, all distributions on finite alphabet random variables W and all quantizers $q(.)$ for S. □

Returning to the cognitive radio scenario, we consider the problem of encoding a message V with knowledge of a Gaussian interfering signal S of power Q. The encoder output X is also power constrained to $P = Q$ and the signal received at the decoder is $Y = \beta_1 X + \beta_2 S + Z$ where Z is independent Gaussian noise and the compound parameter is $\beta := (\beta_1, \beta_2)$.

Similar to Costa's scheme, we suggest $U = X + \alpha S$, where α is now chosen as a function of the second order statistics of β_1 and β_2. The scheme proposed in Section V selects

$$\alpha = \frac{\mu_1^* \mu_2 SNR}{(|\mu_1|^2 + \sigma_1^2) SNR + 1}. \tag{5}$$

We note the following three facts about this choice for Ricean fading channels where β_1 and β_2 have K-factors K_1 and K_2 respectively:

1) If $K_1, K_2 \to \infty$, then the scheme is identical to Costa's with $\alpha = P/(P+N)$ and the interference is perfectly mitigated.
2) If either $K_1 \to 0$ or $K_2 \to 0$, the scheme treats the interferer as noise.
3) The performance does not depend on the phase difference between μ_1 and μ_2 as this choice of α rotates the mean channels so that their phases are aligned.

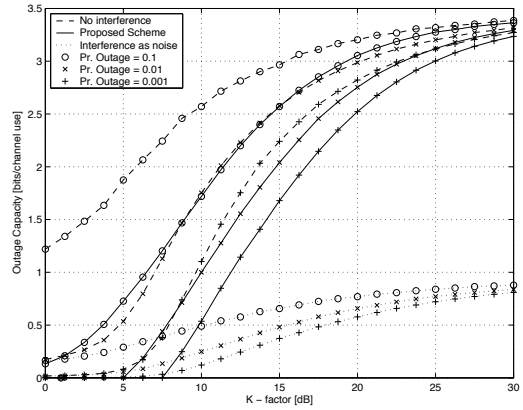


Fig. 6. Communications over a fading channel with a fading interferer whose signal, but not fading coefficient, is known at the transmitter for $SNR = 10$ dB with $P = Q = 1$.

SDR Implementation of Collaborative Communications

Oh-Soon Shin, H.T. Kung, Vahid Tarokh
Harvard University

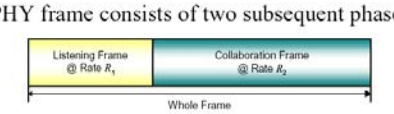
John Chapin's Team
Vanu Inc.

- Design and implementation of an OFDM-based space-time collaborative system
- Our lab completed the simulations in Matlab/C, and Vanu Inc. will do the software radio platform design and implementation.

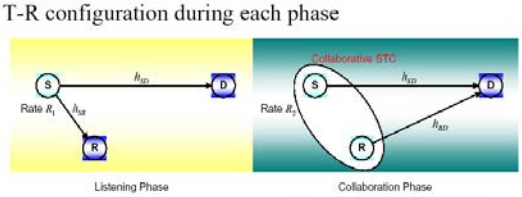
- Carrier frequencies: 902-928 MHz
- Bandwidth: < 5 MHz signal
- 64 subcarriers, spaced at 72kHz

Parameter	Value
Total bandwidth (W)	4.625 MHz
Total number of subcarriers (N_T)	64
Number data subcarriers (N_D)	48
Number pilot subcarriers (N_P)	4
Number of guard or null subcarriers (N_G)	12
Subcarrier frequency spacing (Δ_F)	72.27 kHz
IFFT/FFT period (T_{FFT})	13.838 µsec (64 samples)
Guard interval duration (T_{GI})	2.162 µsec (10 samples)*
OFDM symbol duration (T_{SYM})	16.0 µsec ($T_{FFT} + T_{GI}$)

- Each PHY frame consists of two subsequent phases

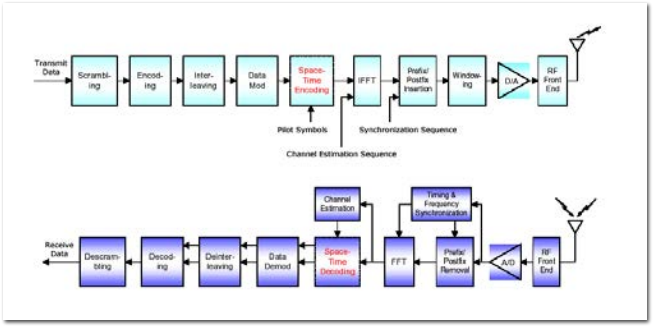


- T-R configuration during each phase

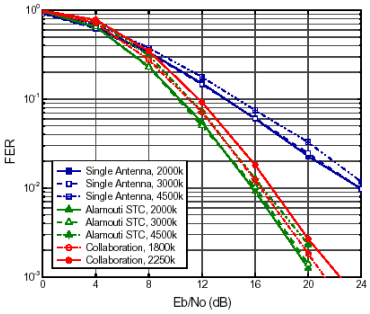


* S: Source, D: Destination, R: Relay

- Listening phase
  - The source broadcasts an information frame
  - The destination and relay try to decode the frame
  - If the destination can decode it correctly, it will ignore the following collaboration frame
- Collaboration phase
  - If the relay succeeded in decoding of the listening frame, the source and relay will transmit space-time coded signal
  - Otherwise, the relay should be silent and only the source transmits as in the listening phase
  - For more reliable decision, the destination can combine the signals from the listening and collaboration phases



Behaves more like a 2x1 system than a 1x1 system



- Develop an equivalent system for cognitive transmission
- Toughest to tackle will be developing codes that layer relaying and dirty-paper coding schemes in the same transmission
- Same synchronization issues
- 2 phase protocols seem practical

To cooperate or to select?

Natasha Devroye, Sumeet Sandhu
Intel Research

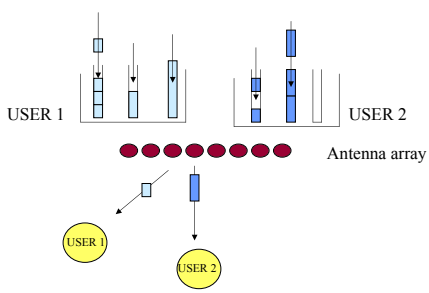
- Evaluate gains of cooperation between base stations in an 802.16d wireless network over no-cooperation, as well as selection.
- System level simulations that take shadowing, fading, sectorization, pathloss, realistic channel models, and interference from other base-stations/users into account.

- Cooperation performs only slightly better than selection.
- Shadowing drastically alters all simulations.
- Shadowing diversity exists.

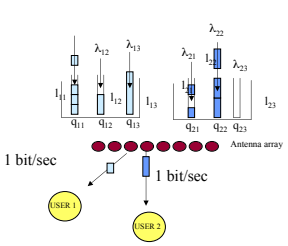
Scheduling for SDMA

Natasha Devroye, Vahid Tarokh
Harvard University

Goal: Produce a stable system of queues that is stochastically smaller that any queue length process produced by another stable policy.



- Found necessary conditions on Poisson arrival rates of various length packets that ensure the stability the queue length process.
- Simulated various scheduling algorithms, and saw their throughput.



The system of queues with arrival rates $\lambda_{11}, \lambda_{12}, \ldots, \lambda_{1K}, \lambda_{21}, \lambda_{22}, \ldots, \lambda_{2K}$ is stable if $\exists \sigma_{ij}$ for $i, j \in \{0, 1, \ldots, K\}$ such that

$$\Updownarrow$$

$$\sigma_{ij} = 0 \text{ if } C_{ij} = 0$$

$$\lambda_{1i} = \sum_{j=0}^{K} \sigma_{ij}$$

$$\lambda_{2j} = \sum_{i=0}^{K} \sigma_{ij}$$

$$\sum_{i=0}^{K} \sum_{j=0}^{K} \sigma_{ij} \max\{l_{1i}, l_{2j}\} < 1$$

- Base the decision of who to schedule on the current parameters and queue lengths only.
- If there exists a pair (i*, j*) with $C_{i^*j^*}=1$, and non-empty queues, send the pair such that:

$$\text{Criteria 1: } (i^*, j^*) = \arg\max_{i,j}(q_{1i}, q_{2j})$$
$$\text{Criteria 2: } (i^*, j^*) = \arg\max_{i,j}(q_{1i} q_{2j}) \left(\frac{l_{1i}+l_{2j}}{\max\{l_{1i}, l_{2j}\}}\right)$$
$$\text{Criteria 3: } (i^*, j^*) = \arg\max_{i,j}\left(\frac{q_{1i} q_{2j}}{q_{1i}+q_{2j}}\right) \left(\frac{l_{1i}+l_{2j}}{\max\{l_{1i}, l_{2j}\}}\right)$$
$$\text{Criteria 4: } (i^*, j^*) = \arg\max_{i,j}\left(\frac{q_{1i} q_{2j}}{q_{1i}+q_{2j}}\right) \left(\frac{l_{1i}+l_{2j}}{\max\{l_{1i}, l_{2j}\}}\right)^2$$

EBCOT Image Compression

Natasha Devroye, Fabrice Labeau
McGill University

- Year long project which consisted of reading D. Taubman's ``High performance scalable image compression with EBCOT'' *IEEE Trans. on Image Processing*, vol.9, 2000.
- Learned all background material on image compression techniques, and coded the described algorithm, which forms the basis of JPEG2000 in C++.

- 2-D Wavelet Transform
- Bit-plane encoding
- Adaptive Arithmetic Coding
- On each bit plane, uses Quad-tree, and 4 more coding passes
- Post-compression rate distortion optimization