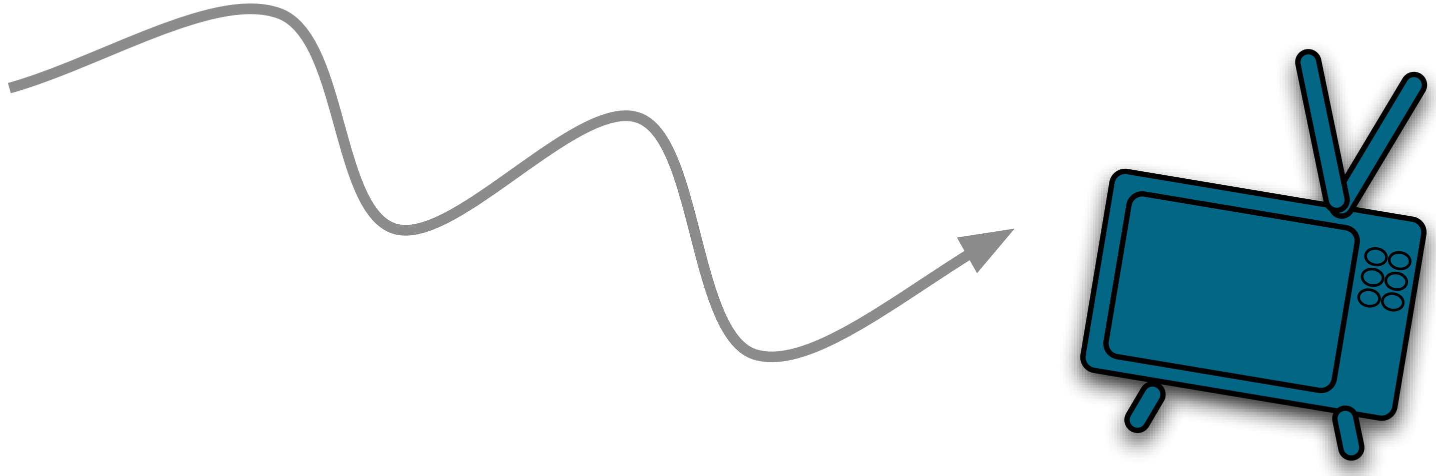
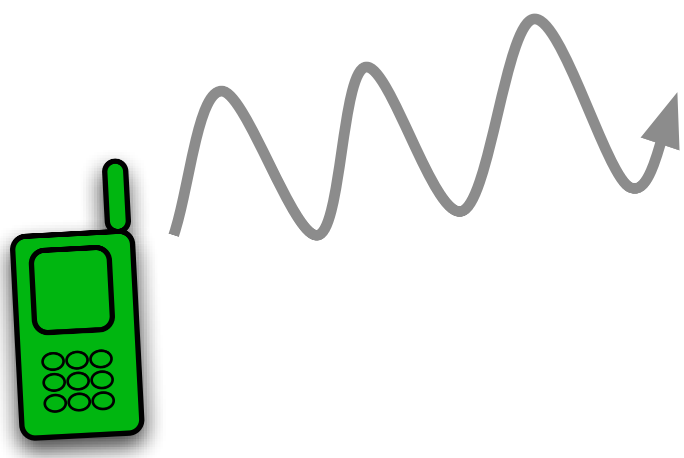
Cognitive radios to reduce, reuse and recycle unused spectrum.

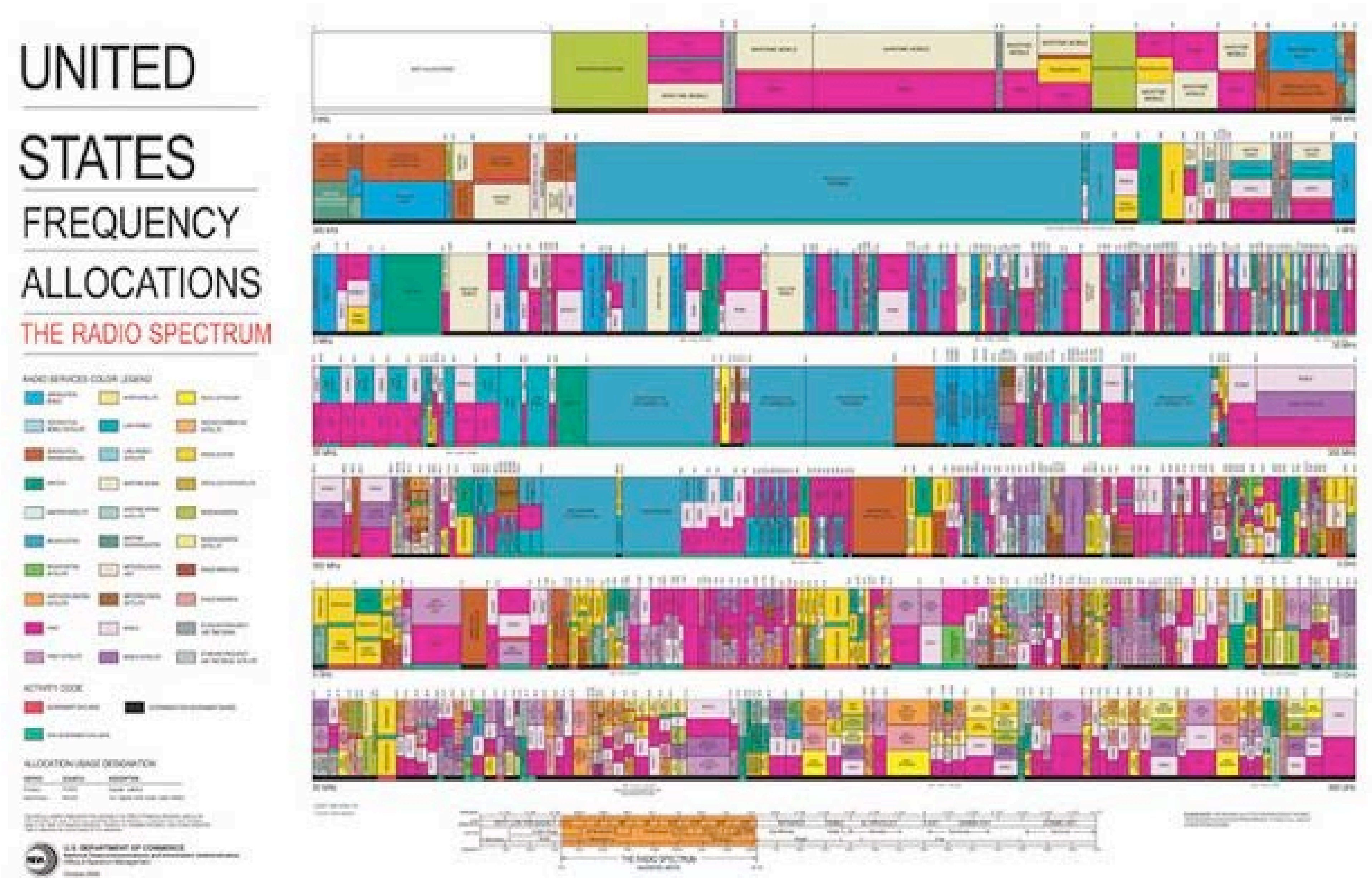
Natasha Devroye
Harvard University



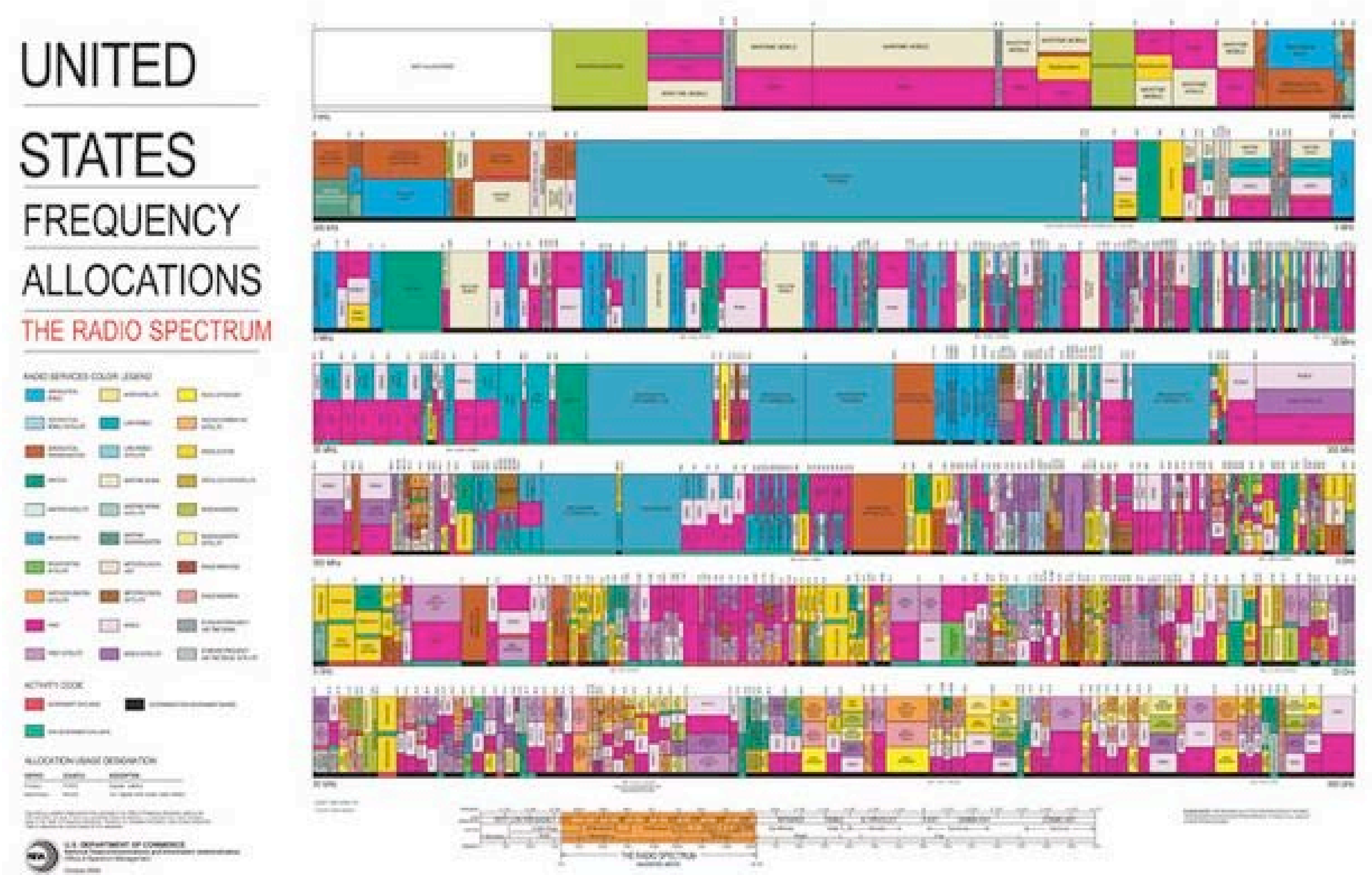
Unused spectrum?



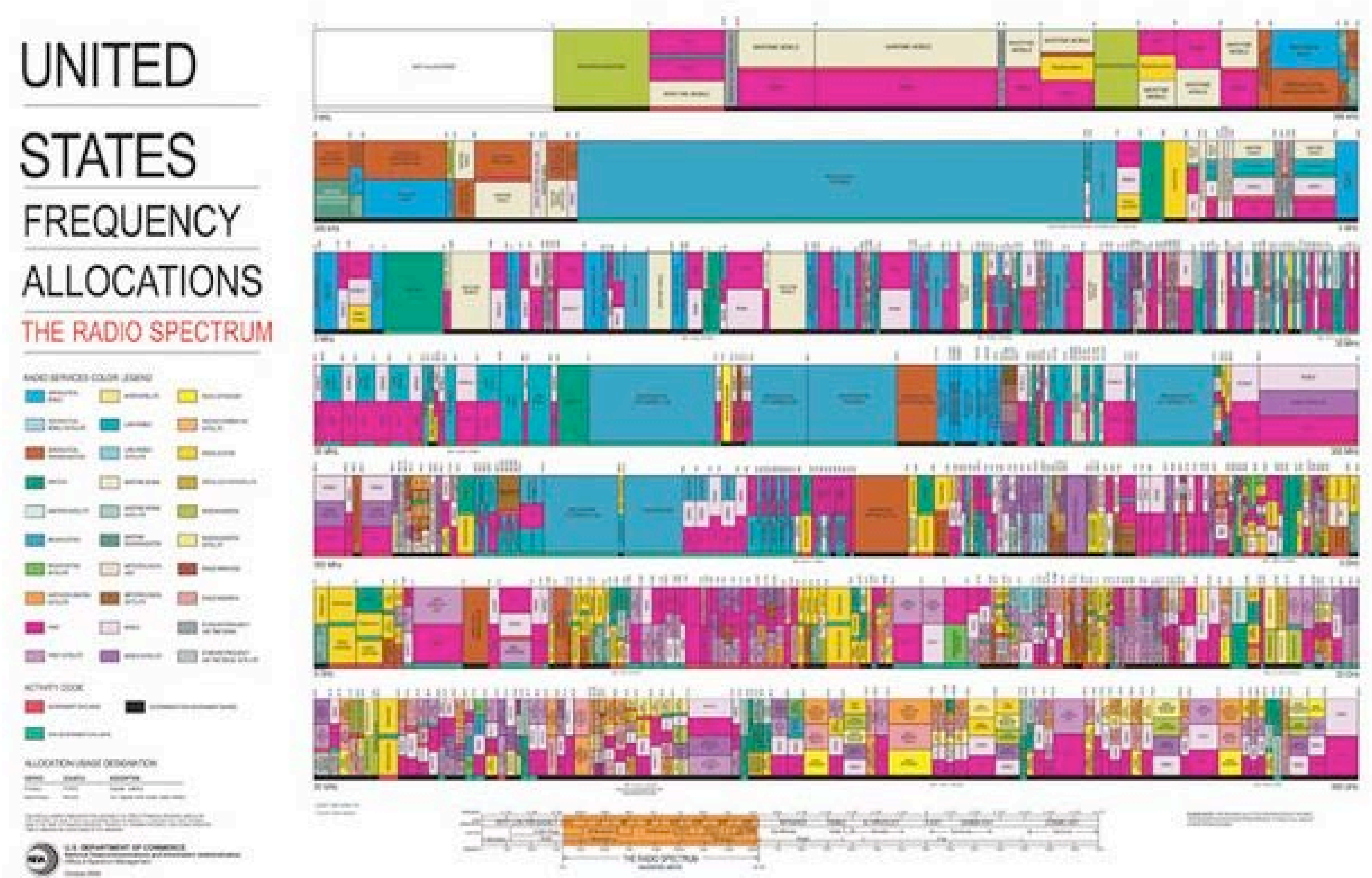
Unused spectrum?



Primary spectrum licensing

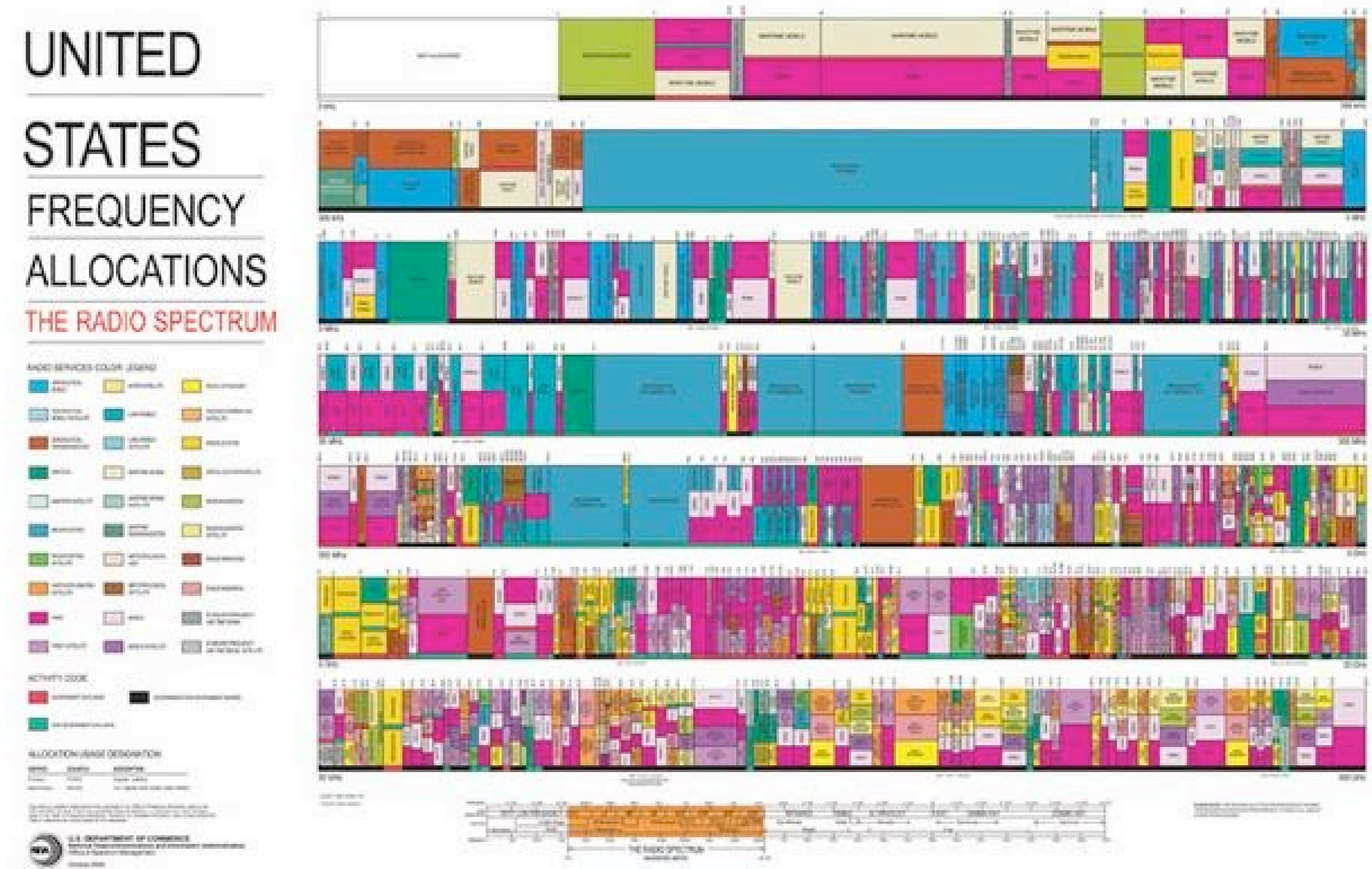


Primary spectrum licensing



Avoid interference

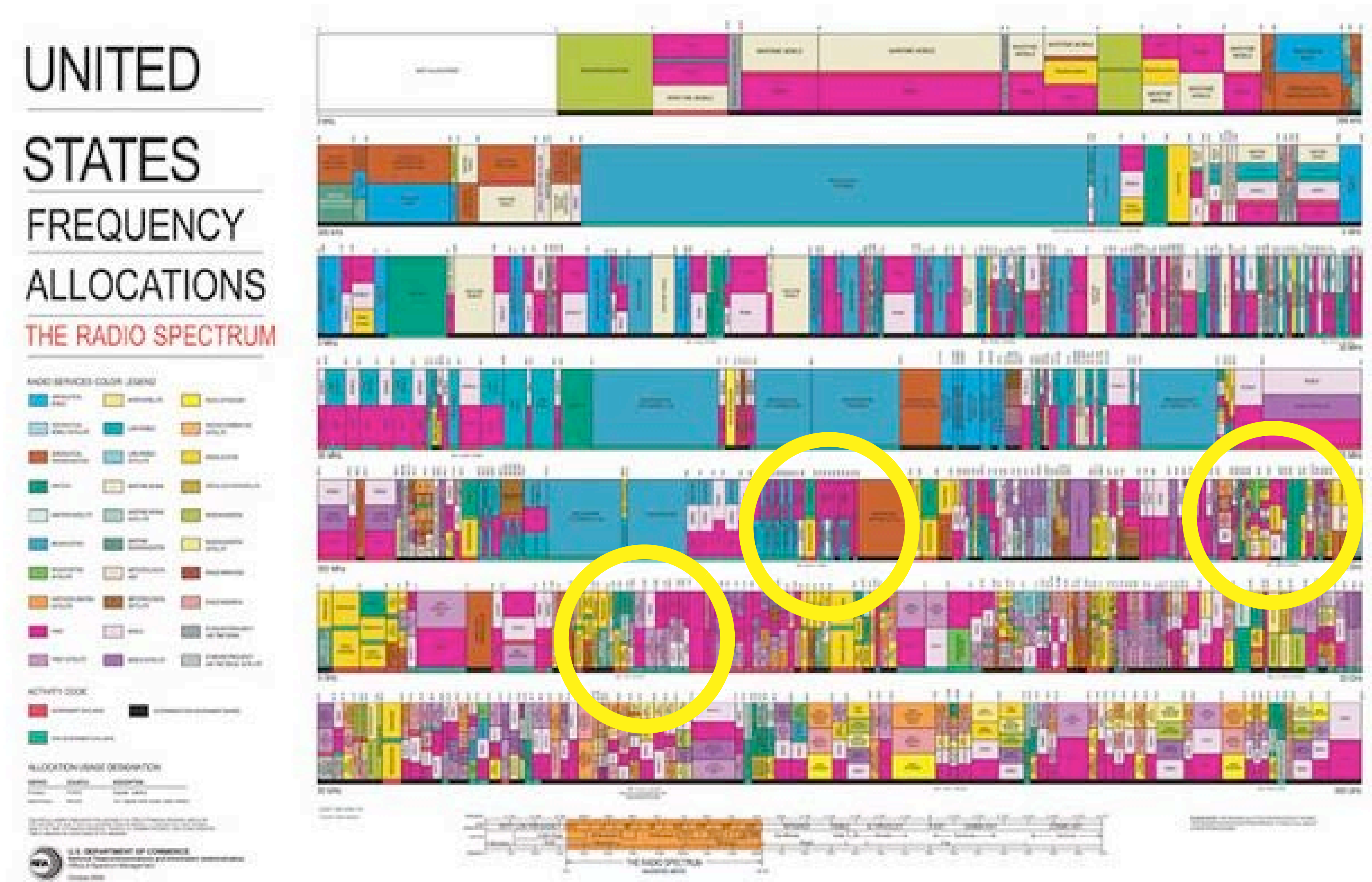
Primary spectrum licensing



Avoid interference

$$$$$$$$$$$$

Primary spectrum licensing



Avoid interference

$$$$$$$$$$$$

Unlicensed bands

- Industrial, Scientific and Medical band (ISM band)
- 915 MHz, 2450 MHz, and 5800 MHz

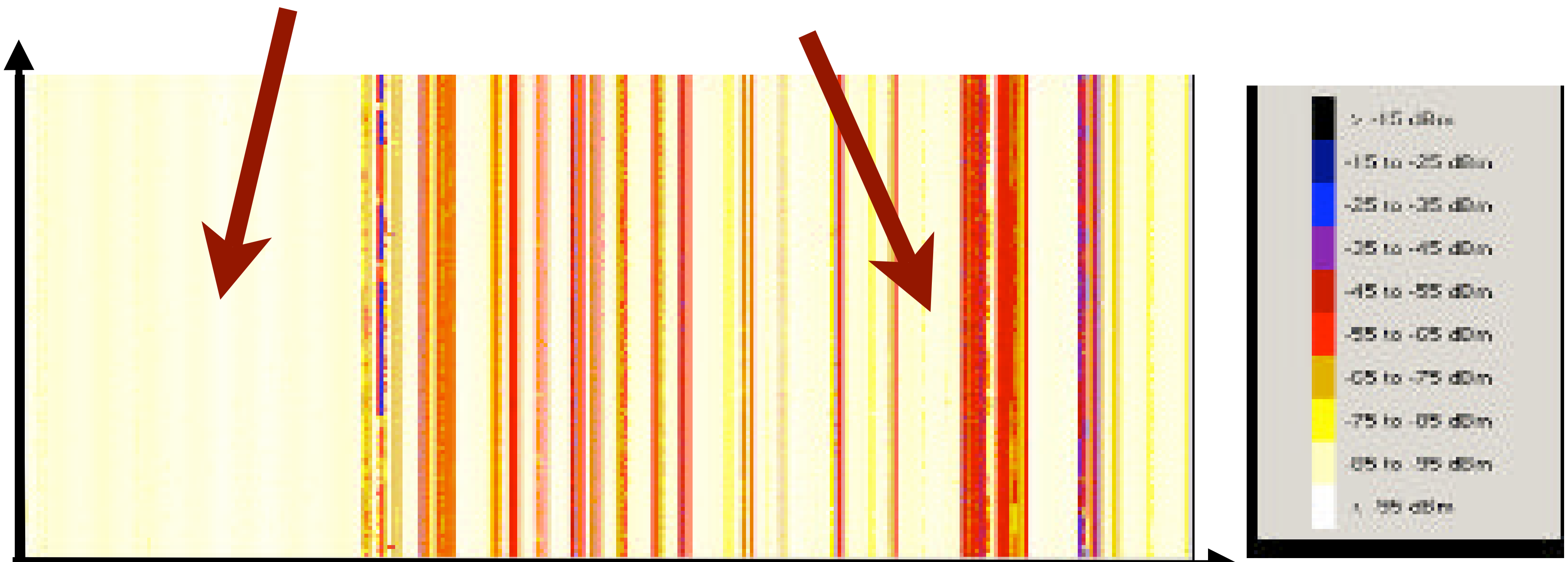
Unlicensed bands

- Industrial, Scientific and Medical band (ISM band)
- 915 MHz, 2450 MHz, and 5800 MHz


Enormous success, filling up fast!

Licensed bands

Problem: much of the time large portions remain unused!



700 MHz block below 1 GHz

Atlanta

Licensed bands

Problem: mucl ions remain unused!

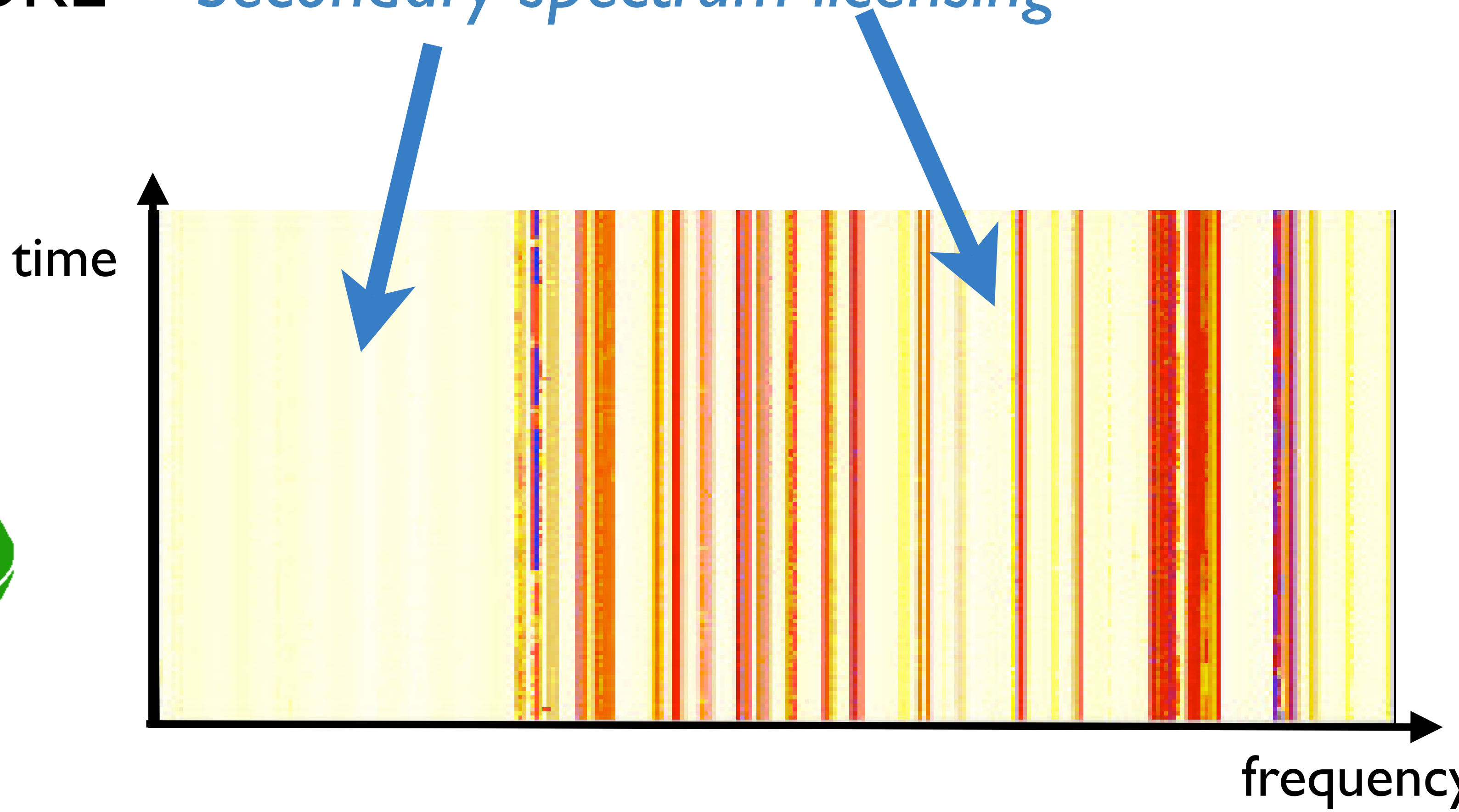
700 MHz block below 1 GHz

Atlanta

Spectrum licensing

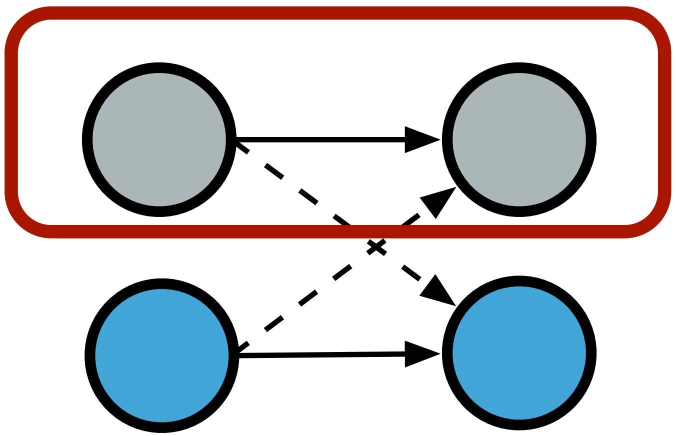
NOW *Primary spectrum licensing*

FUTURE *Secondary spectrum licensing*



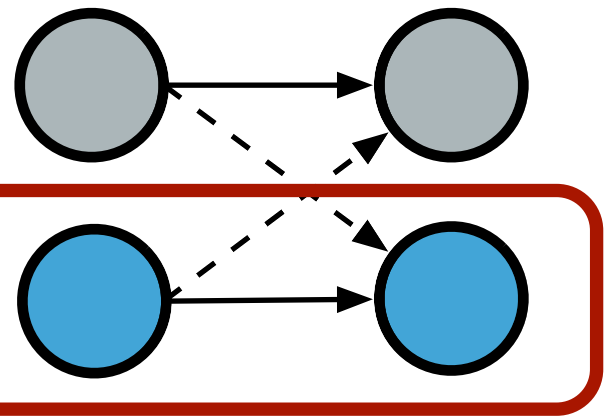
Spectrum licensing: future

Primary users/ primary license holders



Spectrum licensing: future

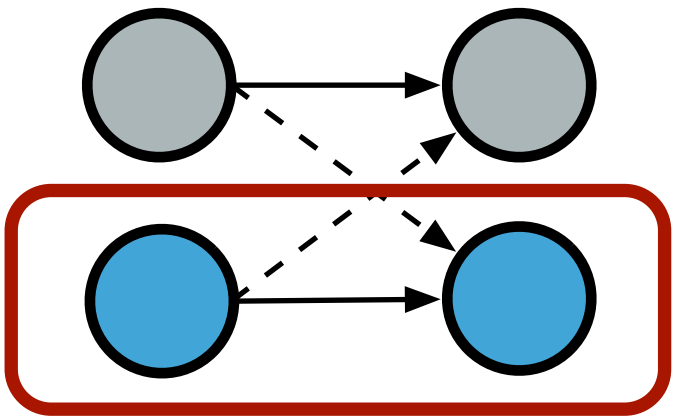
Primary users/ primary license holders



Secondary users

Spectrum licensing: future

Primary users/ primary license holders



Secondary users ⟷ Cognitive radios

Cognitive Radio

Use information theory to determine fundamental limits of cognitive networks

Wireless channels: my notation

Wireless channels: my notation

Wireless channels: my notation

Encoder

Transmitter, Tx

Decoder

Receiver, Rx

Wireless channels: my notation

Encoder

Transmitter, Tx

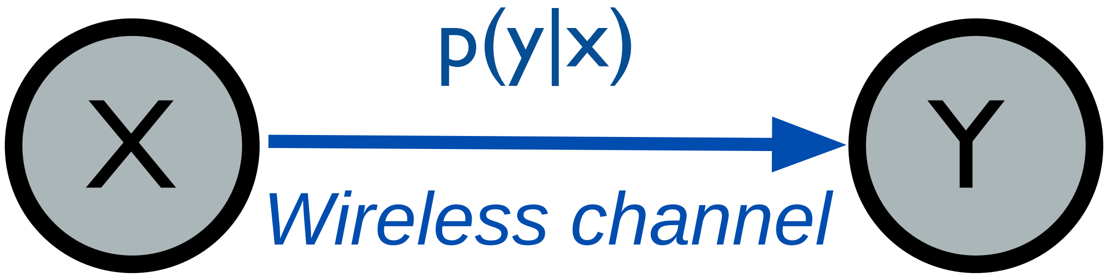
X: transmitted signal

Decoder

Receiver, Rx

Y: received signal

Wireless channels: my notation



Encoder

Decoder

Transmitter, Tx

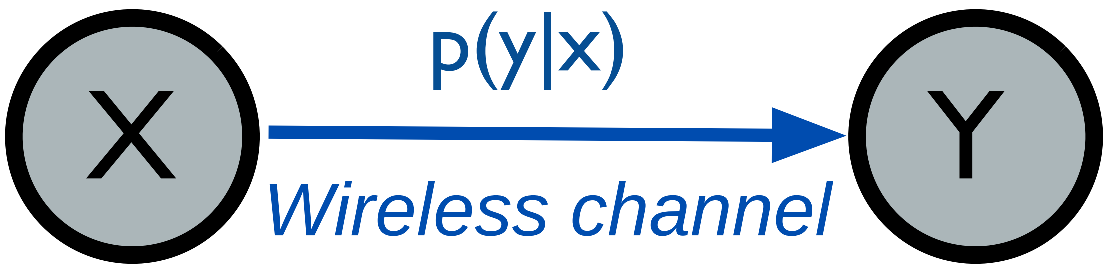
Receiver, Rx

X: transmitted signal

Y: received signal

Wireless channel: p(y|x)

Wireless channels: capacity

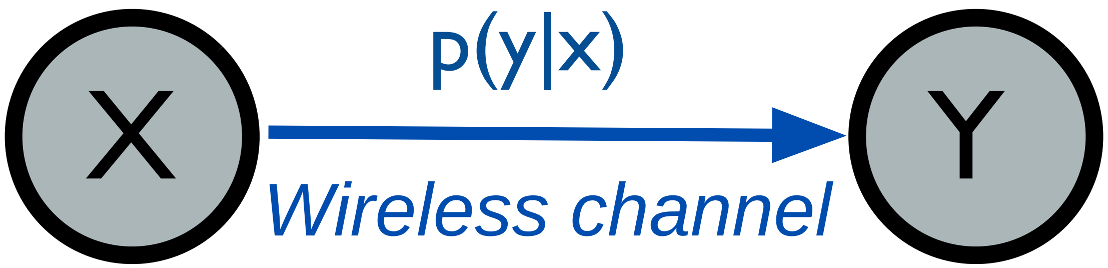


Capacity

=

highest rate, in (bits/channel use) at which information can be sent with arbitrarily low probability of error

Wireless channels: capacity



“Achievable rate” ≤ Capacity ≤ “Outer bound”

||

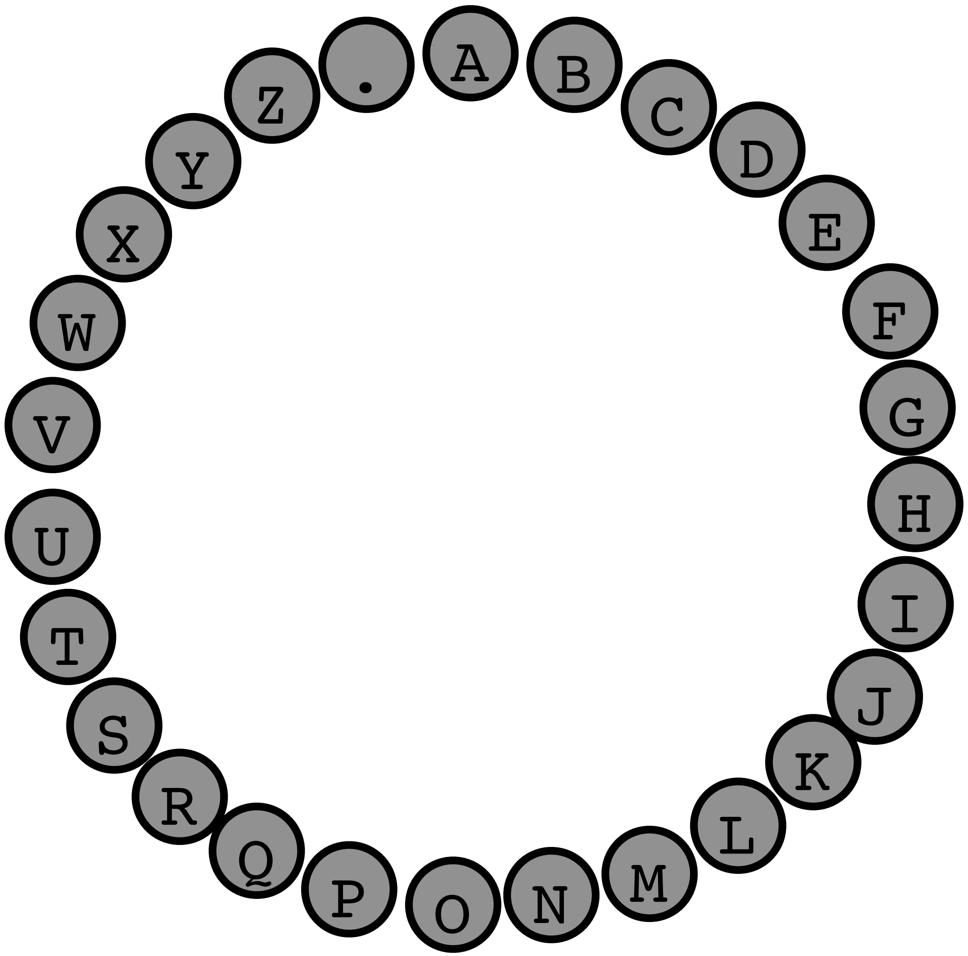
any rate at which information can be sent with arbitrarily low probability of error

The channel: $p(y|x)$

Discrete memoryless channel

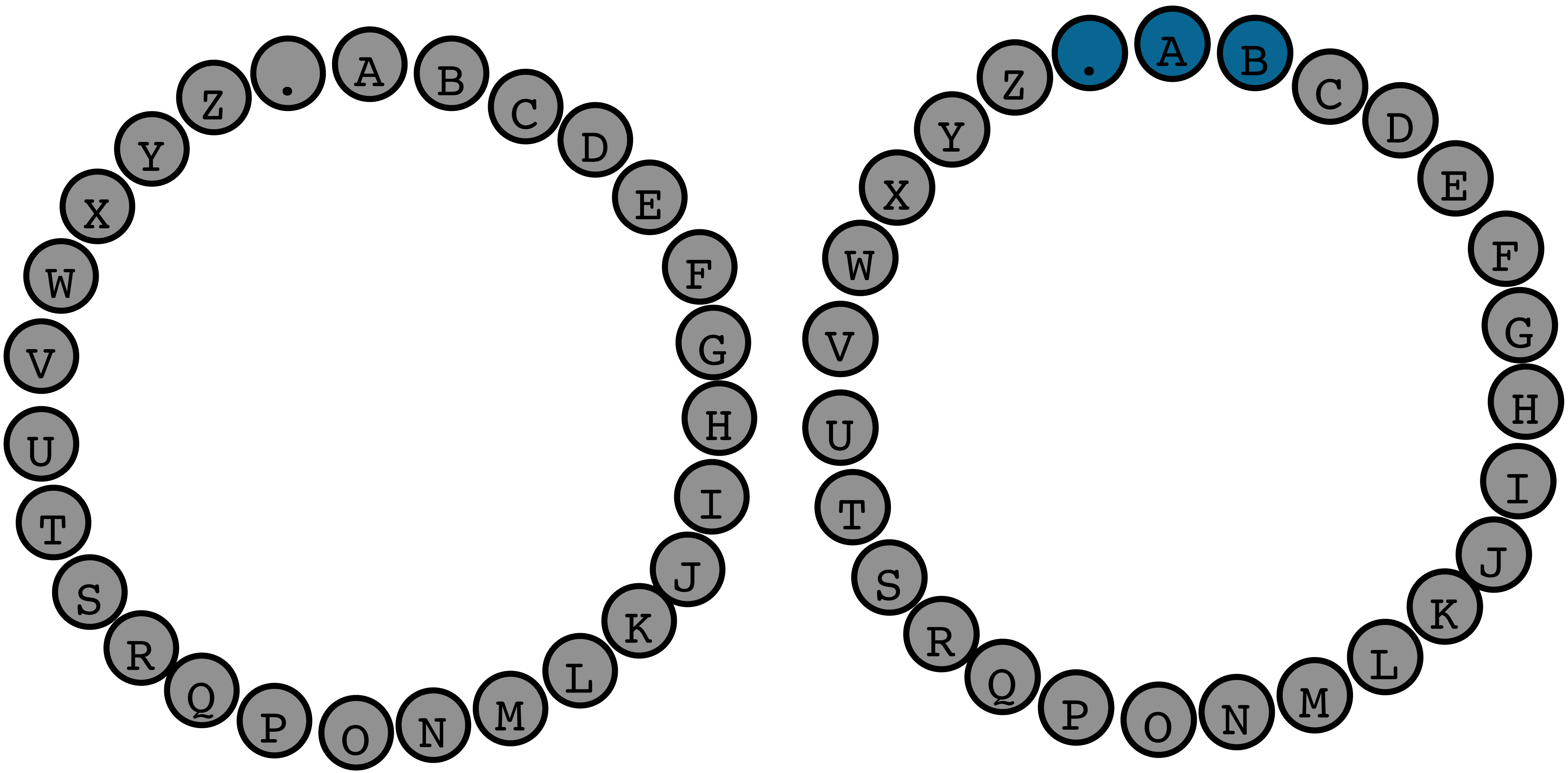
The channel: $p(y|x)$

Discrete memoryless channel



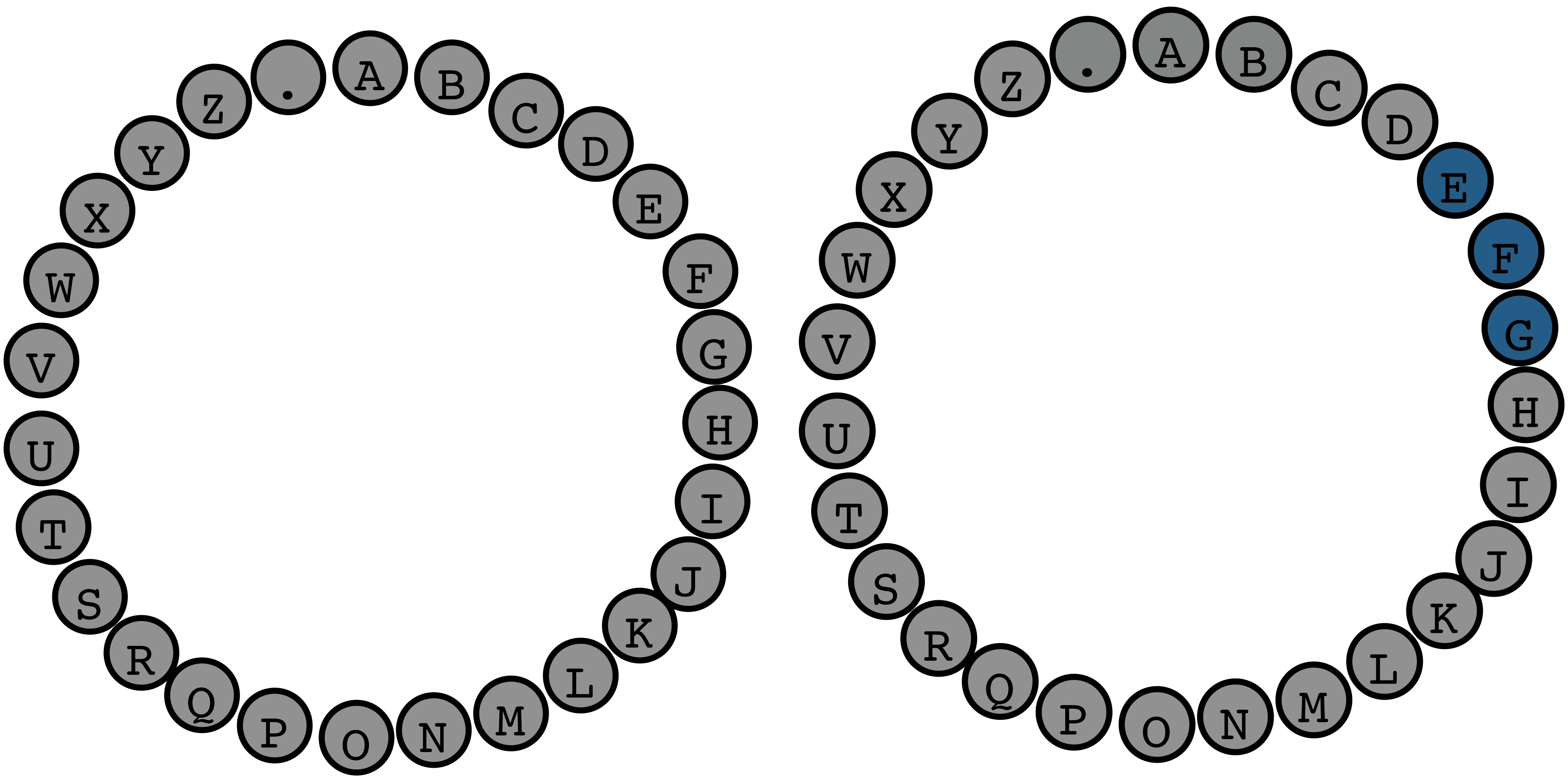
The channel: p(y|x)

Discrete memoryless channel



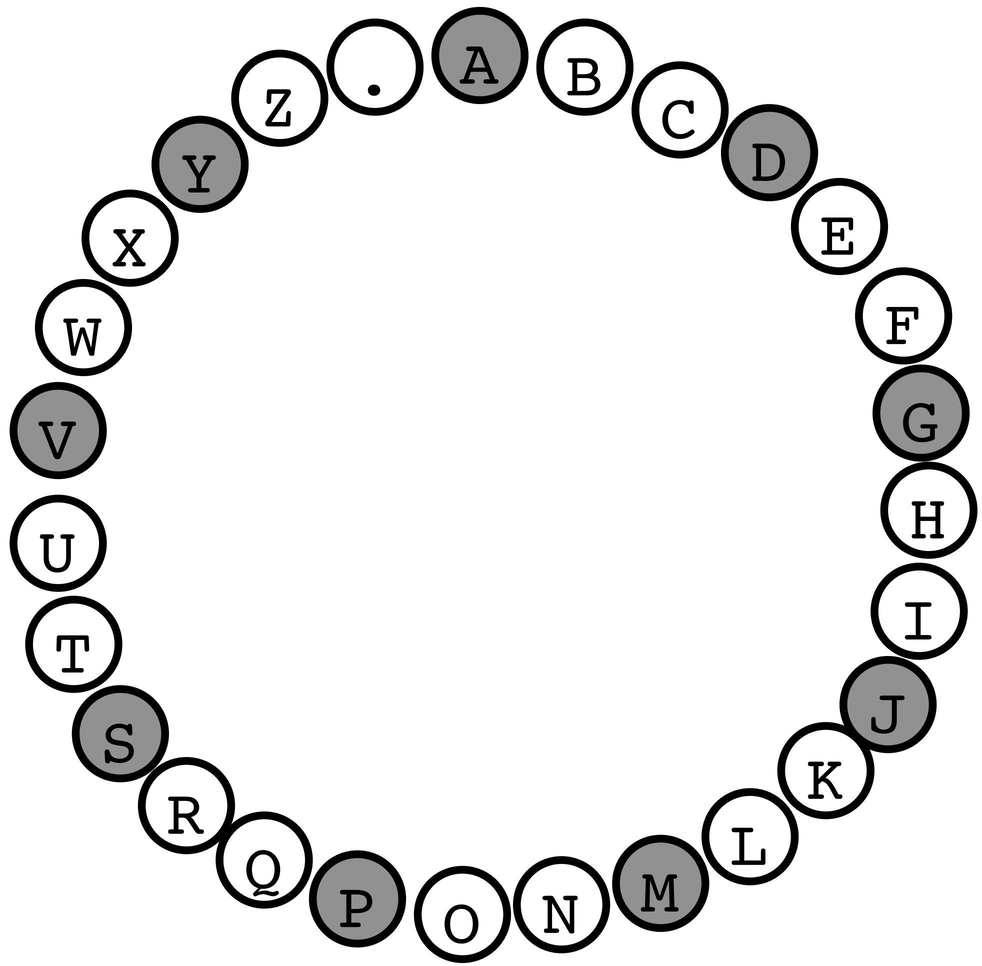
The channel: p(y|x)

Discrete memoryless channel



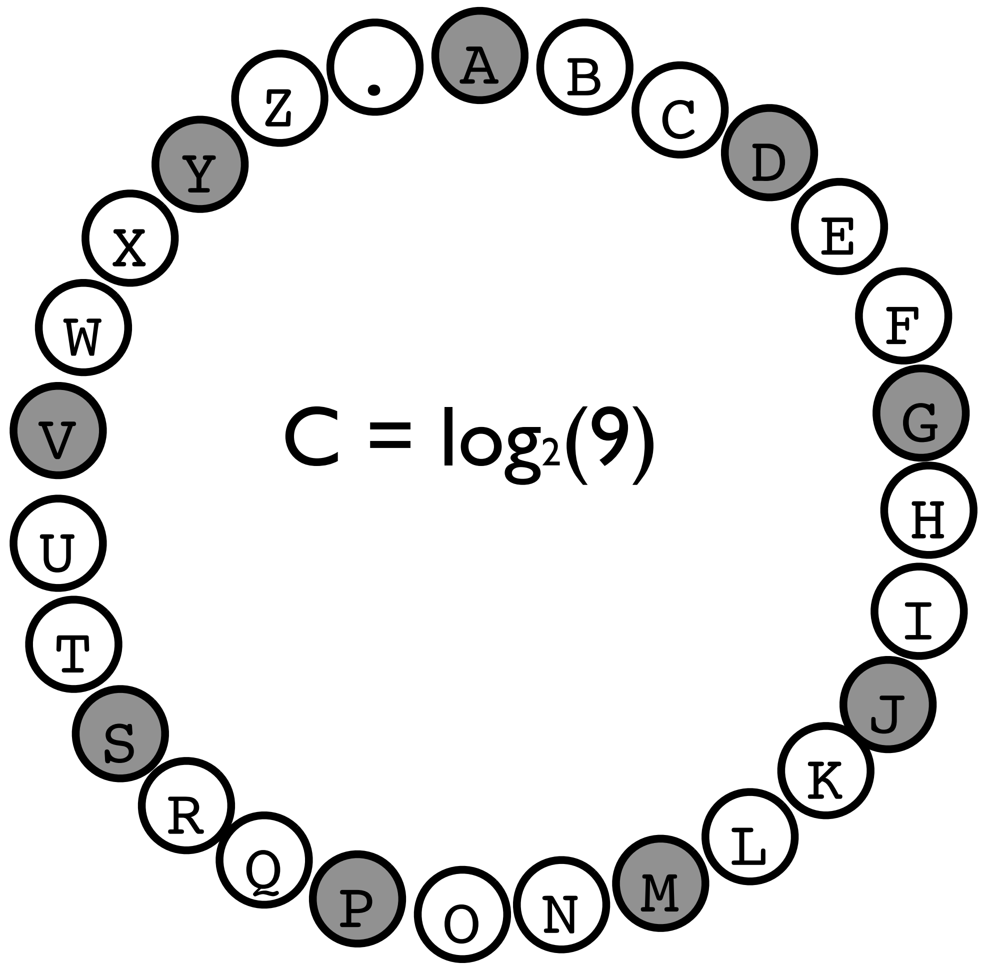
The channel: p(y|x)

Discrete memoryless channel

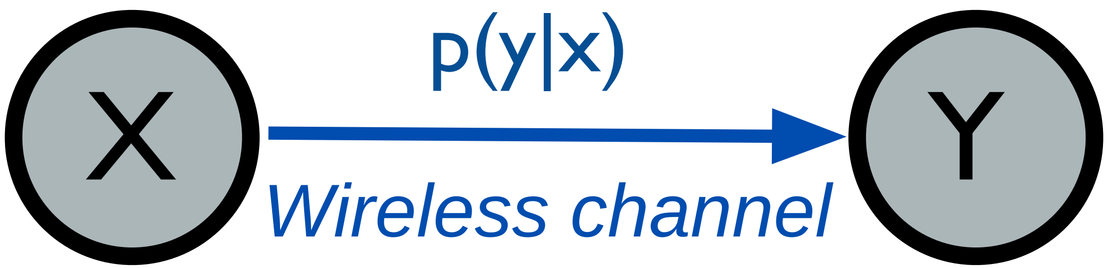


The channel: $p(y|x)$

Discrete memoryless channel



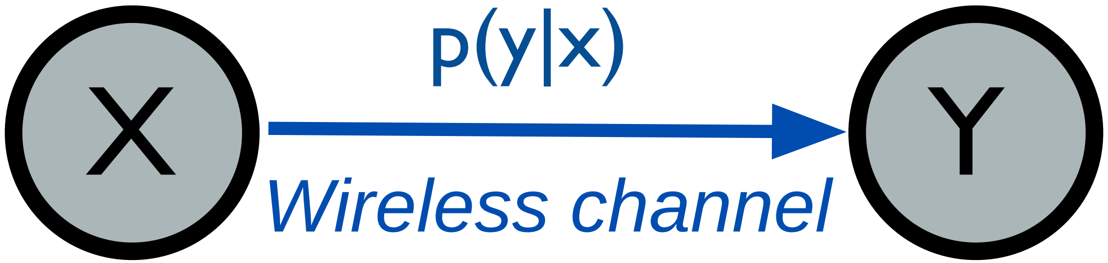
Discrete channel capacity



Capacity $$C = \max_{p(x)} I(X;Y)$$ bits/channel use

"mutual information" between X and Y

Discrete channel capacity



Capacity $$C = \max_{p(x)} I(X;Y)$$ bits/channel use

"mutual information" between X and Y

$$I(X;Y) = \sum_{x,y} p(x,y) \log\left(\frac{p(x,y)}{p(x)p(y)}\right)$$

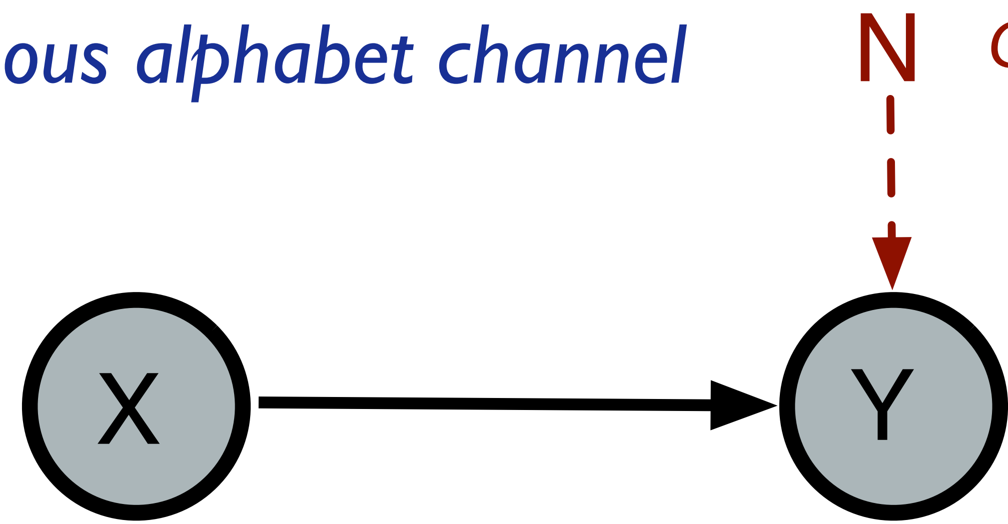
The channel: $p(y|x)$

Continuous alphabet channel

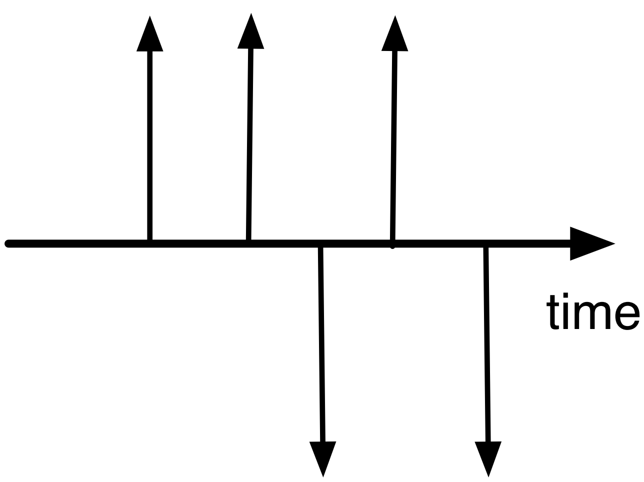
The channel: p(y|x)

Continuous alphabet channel

Gaussian noise ~ $N(0,P_N)$

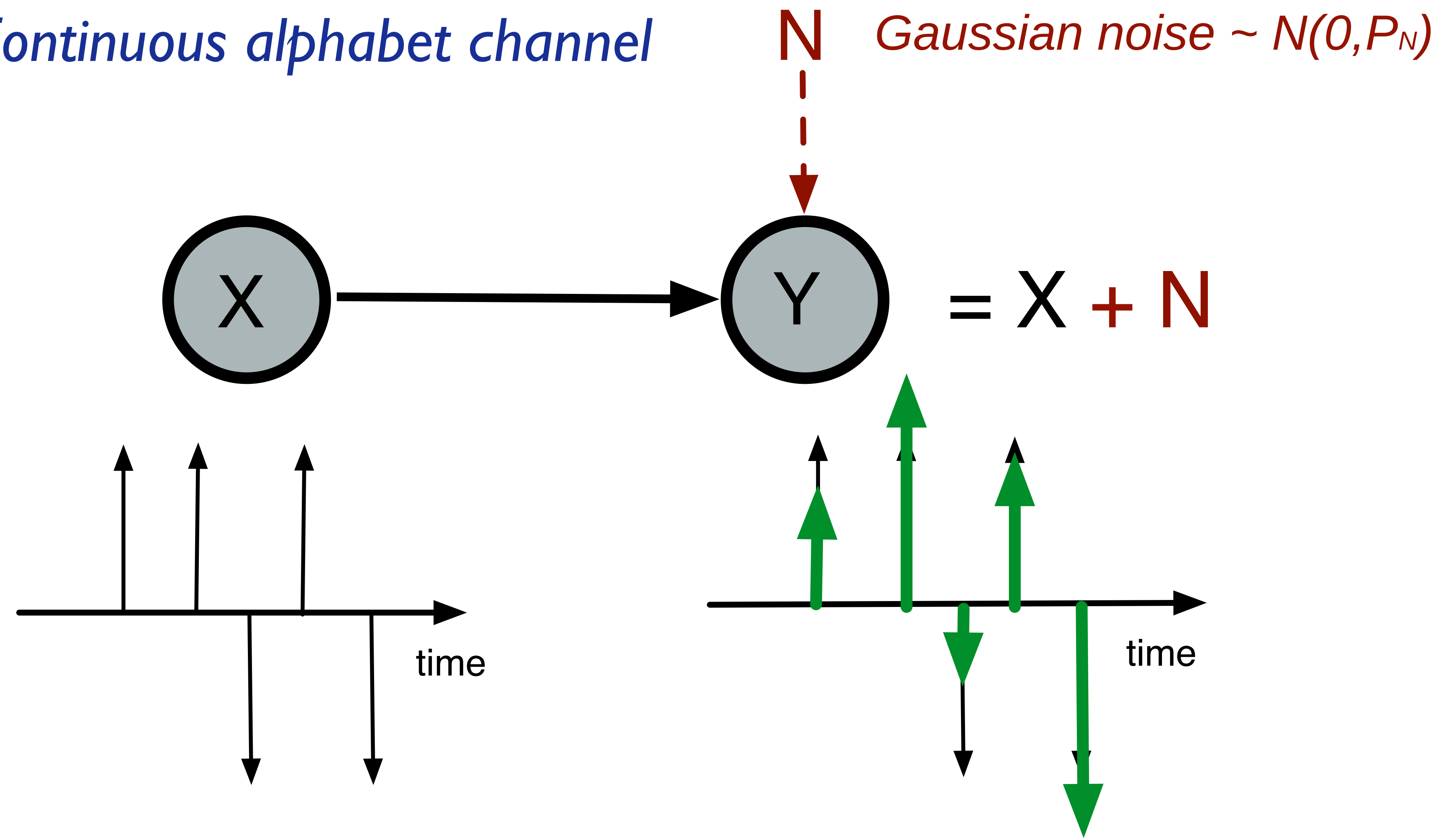


$= X + N$



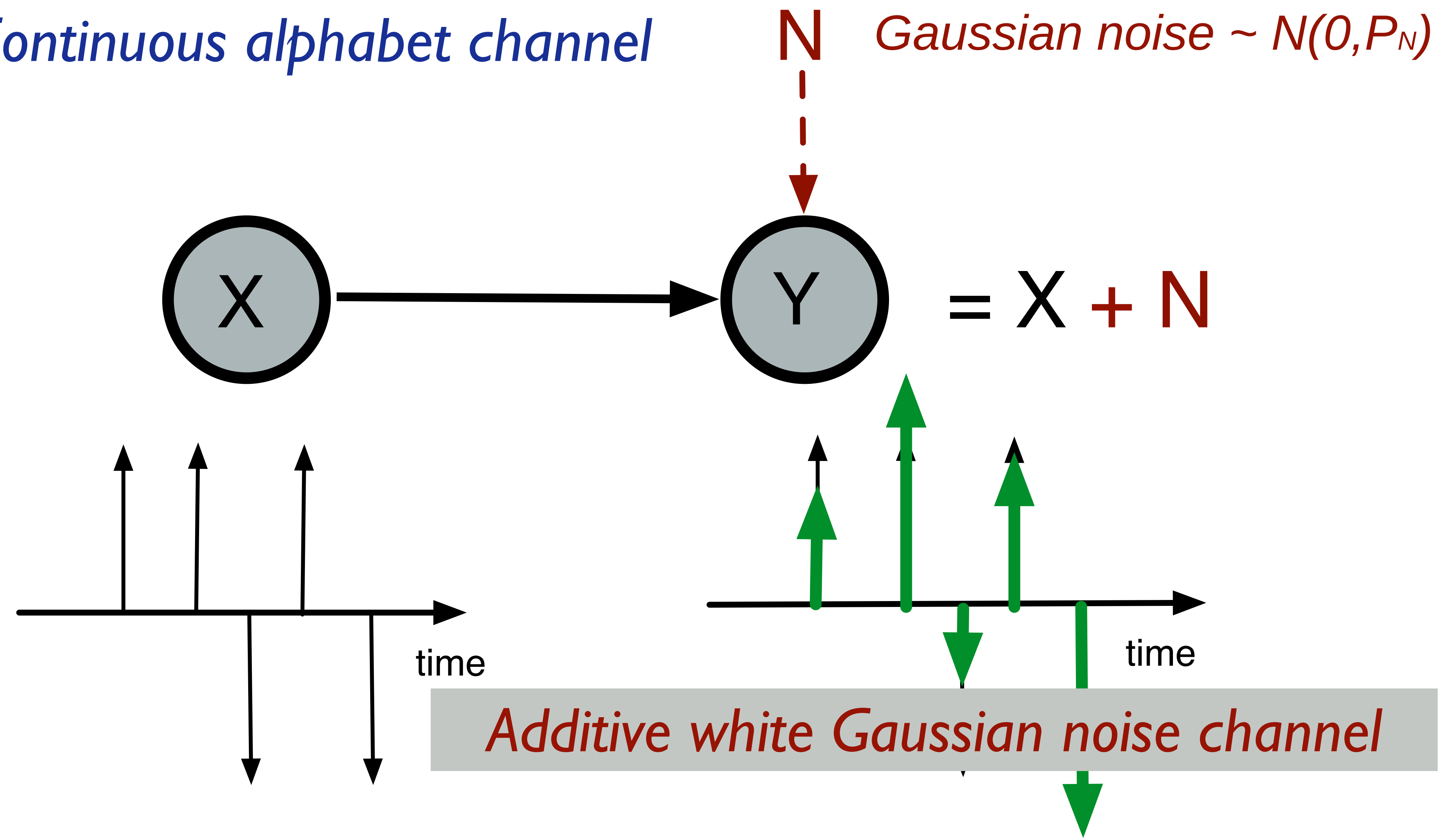
The channel: p(y|x)

Continuous alphabet channel

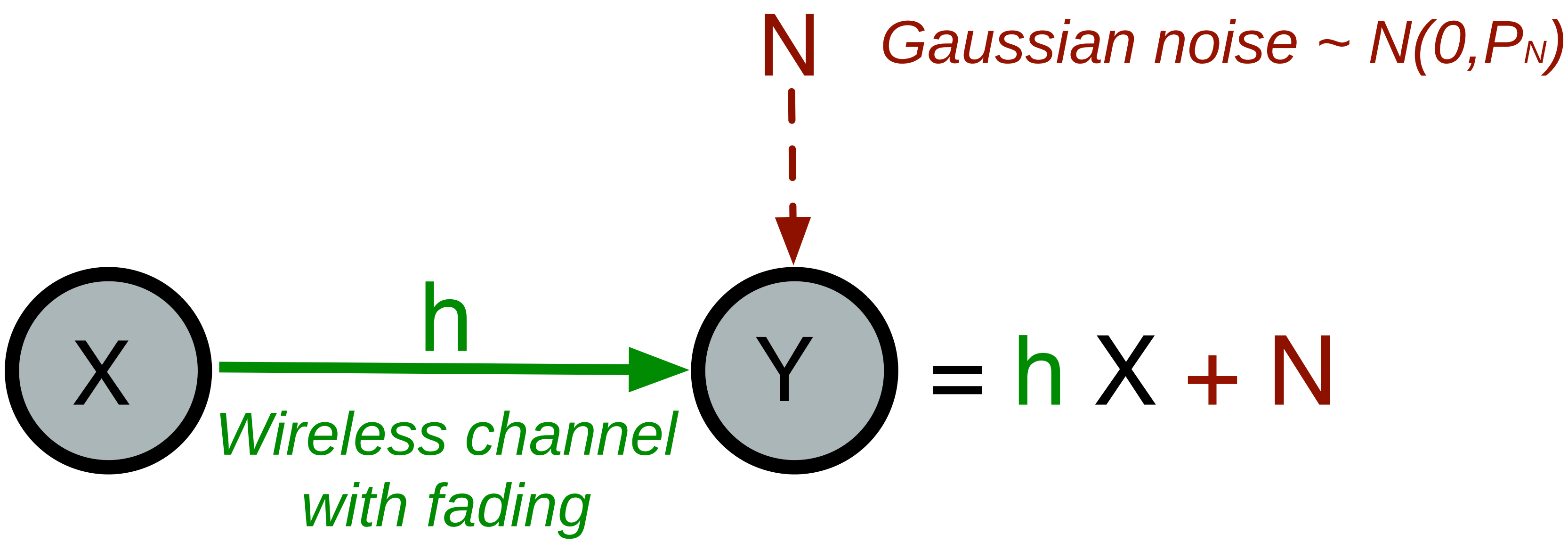


The channel: p(y|x)

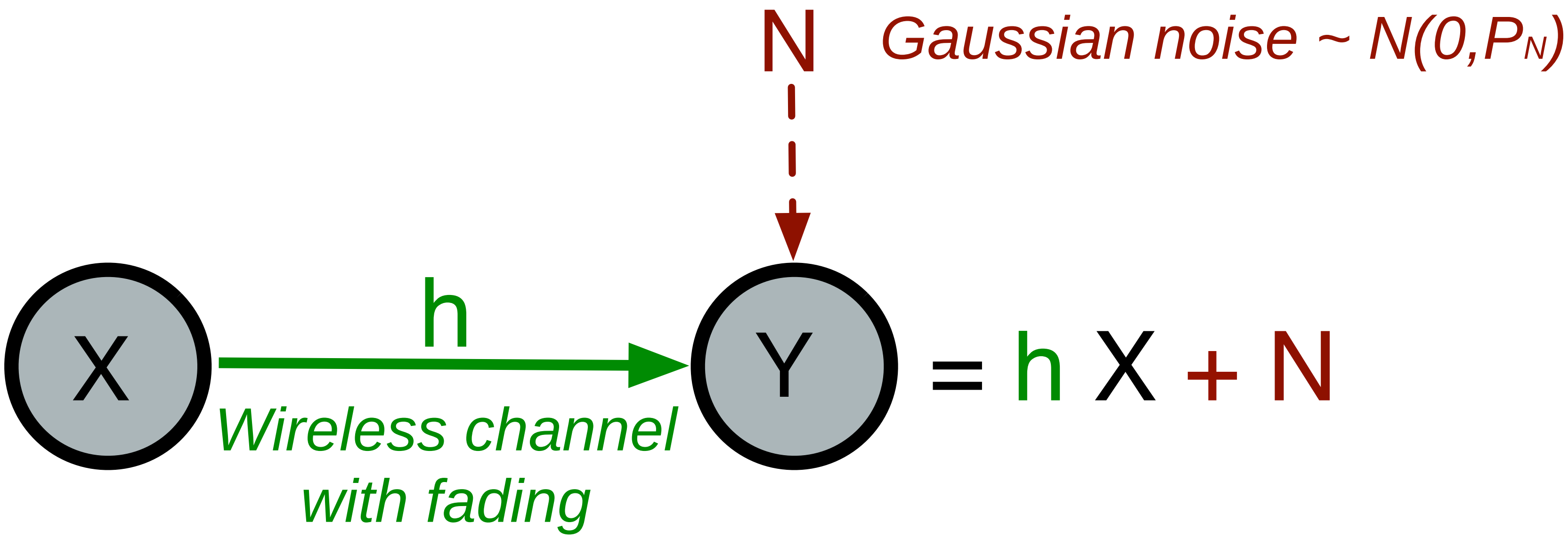
Continuous alphabet channel



Gaussian noise channel capacity



Gaussian noise channel capacity



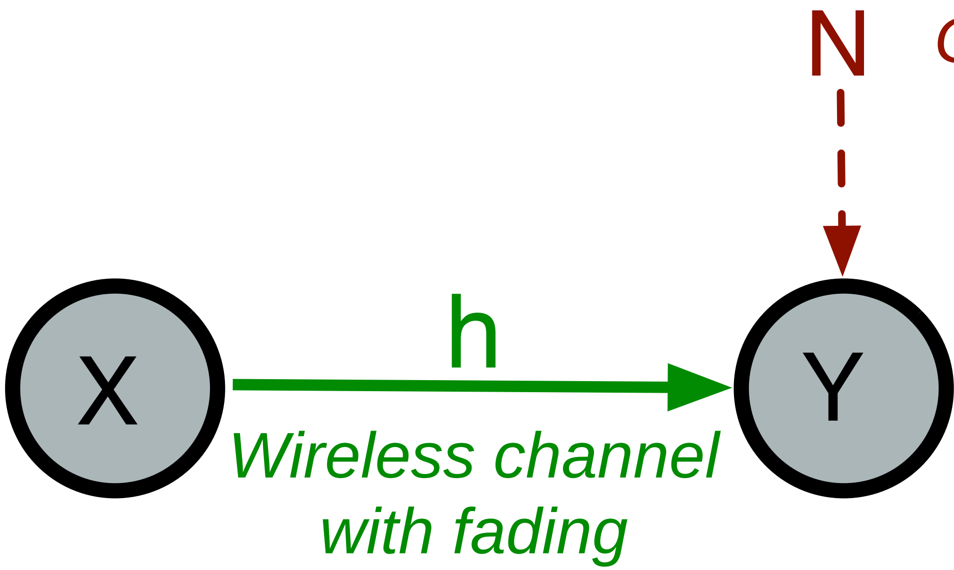
Capacity

$$C = \max_{p(x):E[|X|^2]\leq P} I(X;Y)$$

$$= \frac{1}{2}\log_2\left(\frac{|h|^2 P + P_N}{P_N}\right)$$

$$= \frac{1}{2}\log_2(1+\mathrm{SNR})$$ (bits/channel use)

Gaussian noise channel capacity



$= h\,X + N$

signal power at Rx

noise power at Rx

Capacity

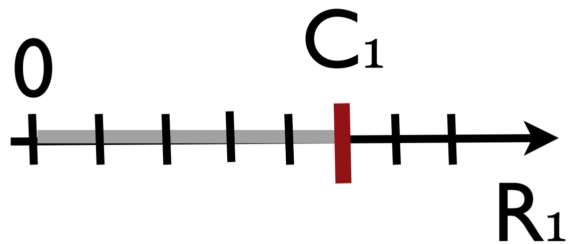
$$C = \max_{p(x):E[|X|^2]\leq P} I(X;Y)$$

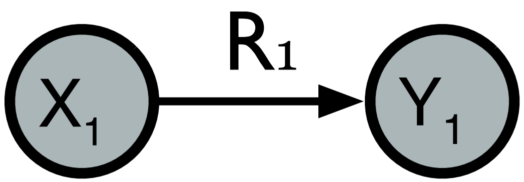
$$= \frac{1}{2}\log_2\left(\frac{\boxed{|h|^2 P + P_N}}{\boxed{P_N}}\right)$$

$$= \frac{1}{2}\log_2(1 + \text{SNR})$$

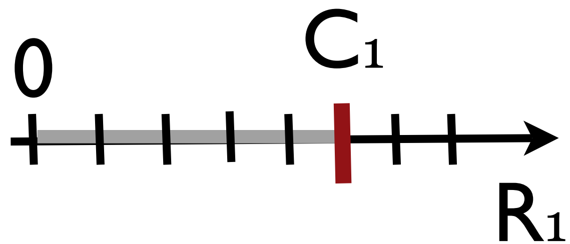
(bits/channel use)

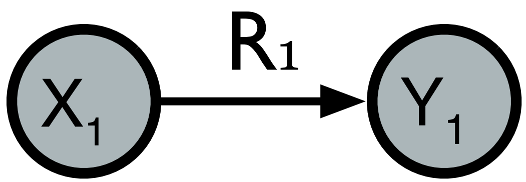
Capacity

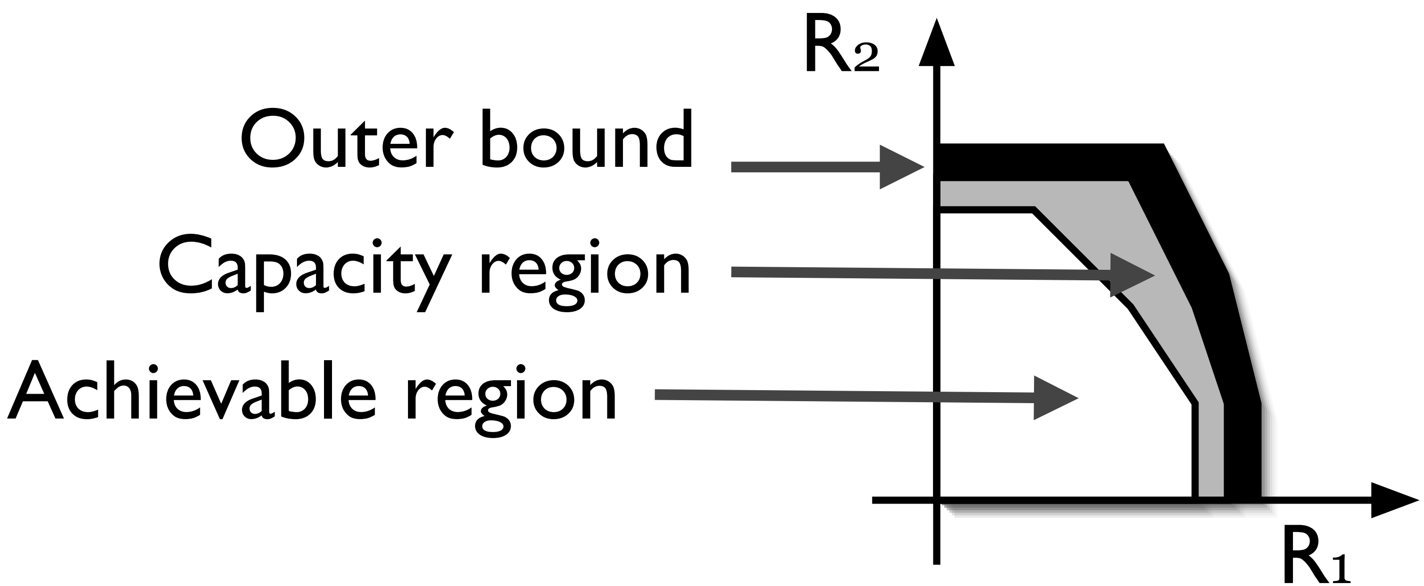


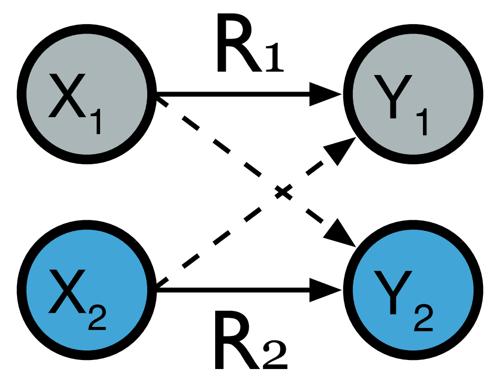


Capacity region

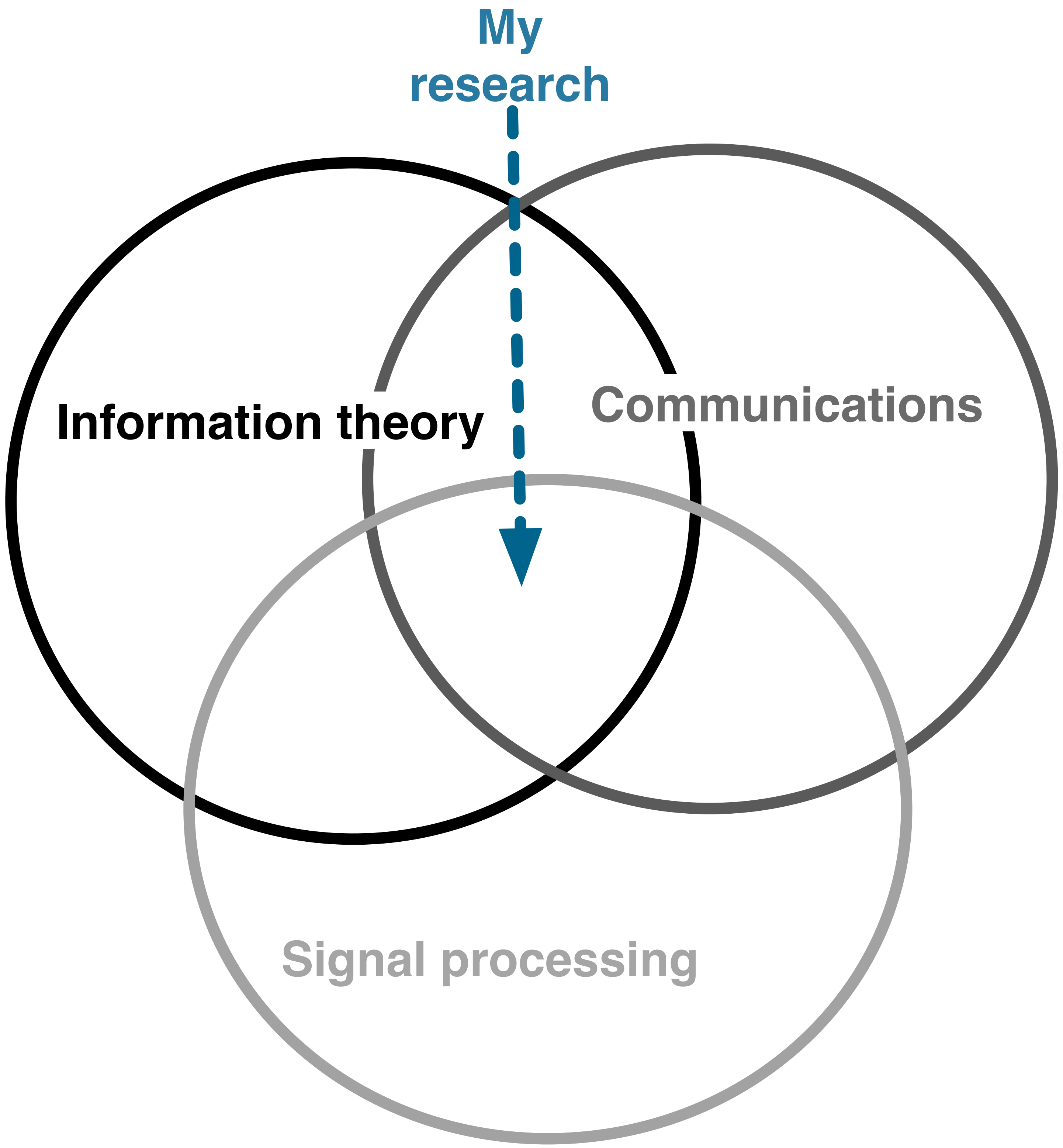








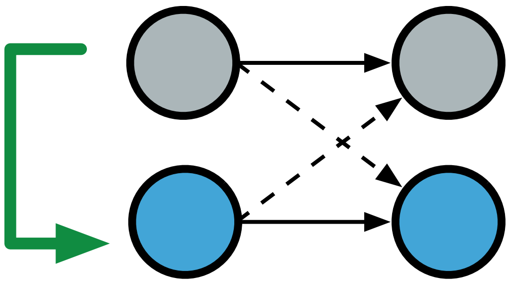
Fundamental limits of communication in networks of primary users and secondary/cognitive users.


My
research
Information theory
Communications
Signal processing

Cognitive radio channels

achievable rate regions

multiplexing gains, MIMO X channel



Cognitive networks

scaling laws of ad-hoc cognitive networks

parameter design to guarantee performance

Cooperative relaying

bi-directional relaying rate regions

asymmetric cooperation in downlink cellular systems

Cognitive radio channels

achievable rate regions

multiplexing gains, MIMO X channel

Cognitive networks

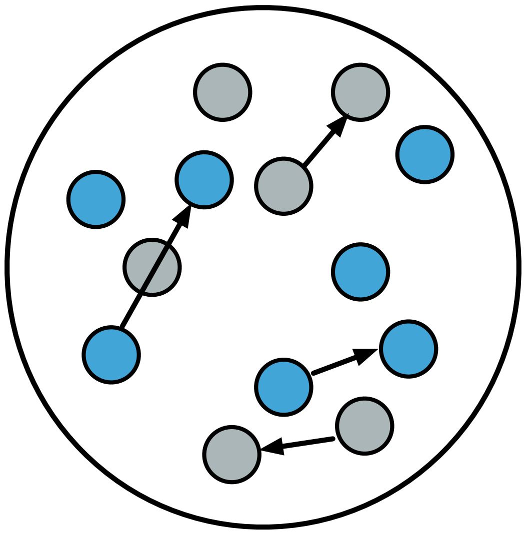
scaling laws of ad-hoc cognitive networks

parameter design to guarantee performance

Cooperative relaying

bi-directional relaying rate regions

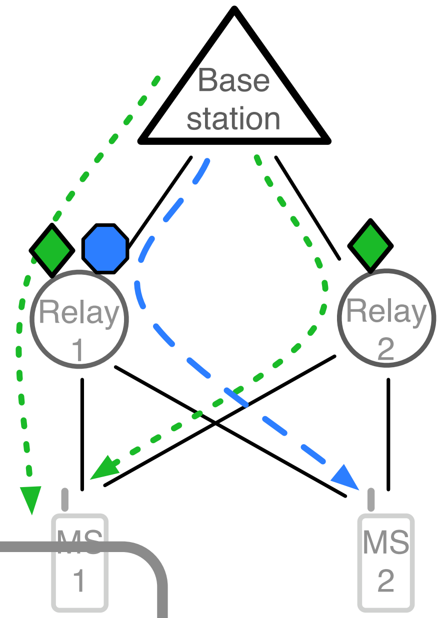
asymmetric cooperation in downlink cellular systems



Cognitive radio channels

achievable rate regions

multiplexing gains, MIMO X channel



Cognitive networks

scaling laws of ad-hoc cognitive networks

parameter design to guarantee performance

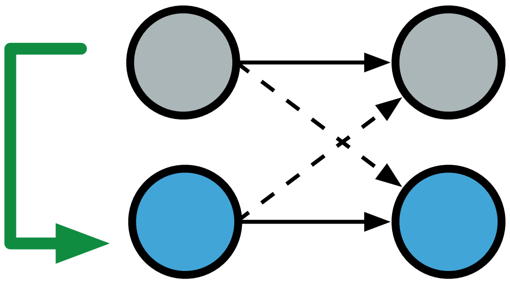
Cooperative relaying

bi-directional relaying rate regions

asymmetric cooperation in downlink cellular systems

Cognitive radio channels

achievable rate regions



multiplexing gains, MIMO X channel

Cognitive networks

scaling laws of ad-hoc cognitive networks

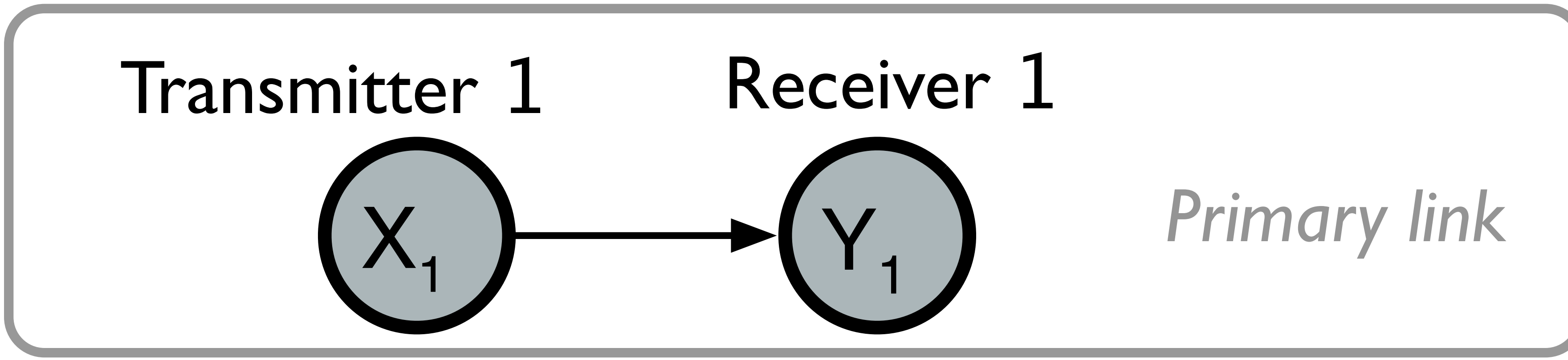
parameter design to guarantee performance

Cooperative relaying

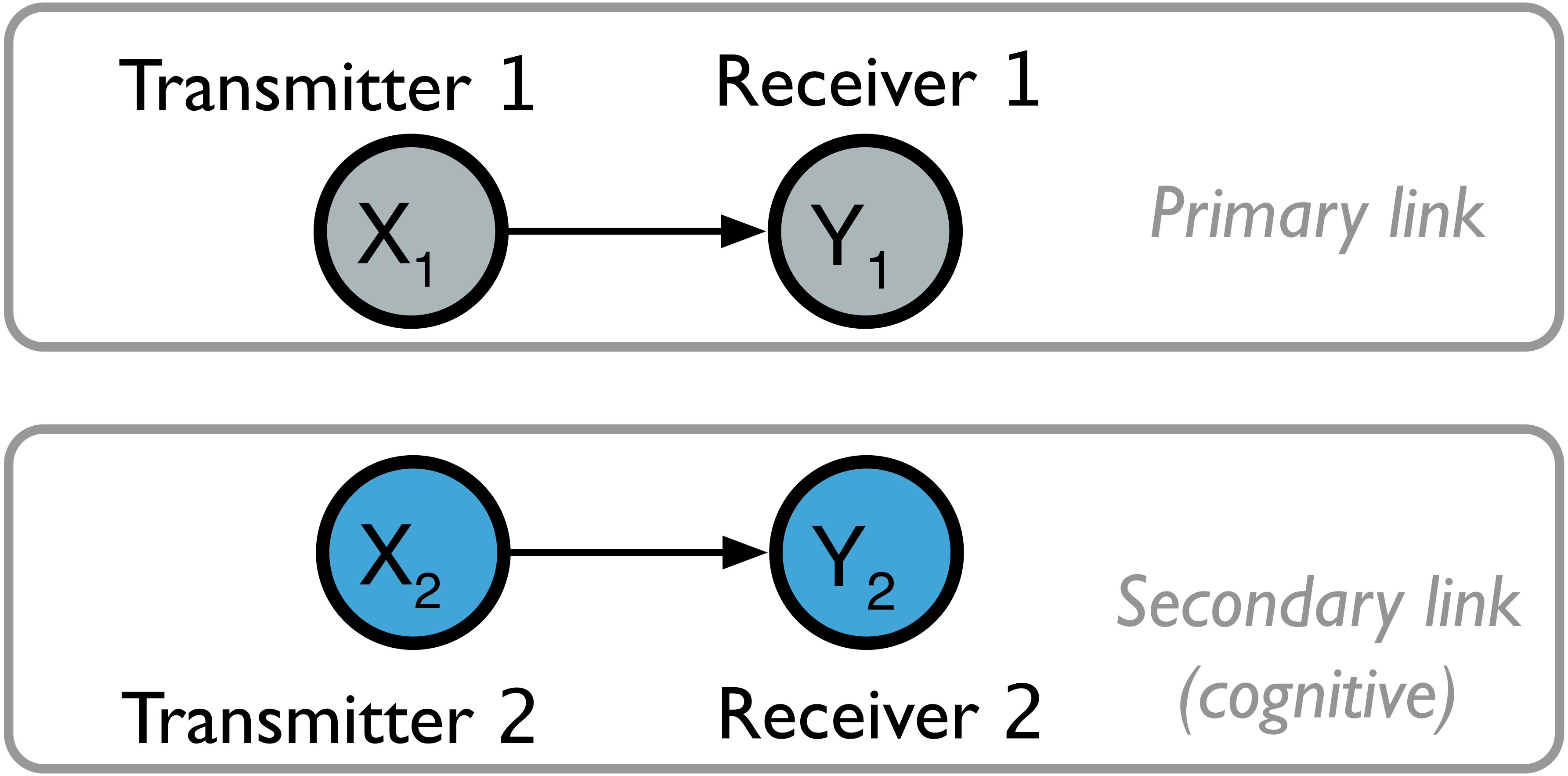
bi-directional relaying rate regions

asymmetric cooperation in downlink cellular systems

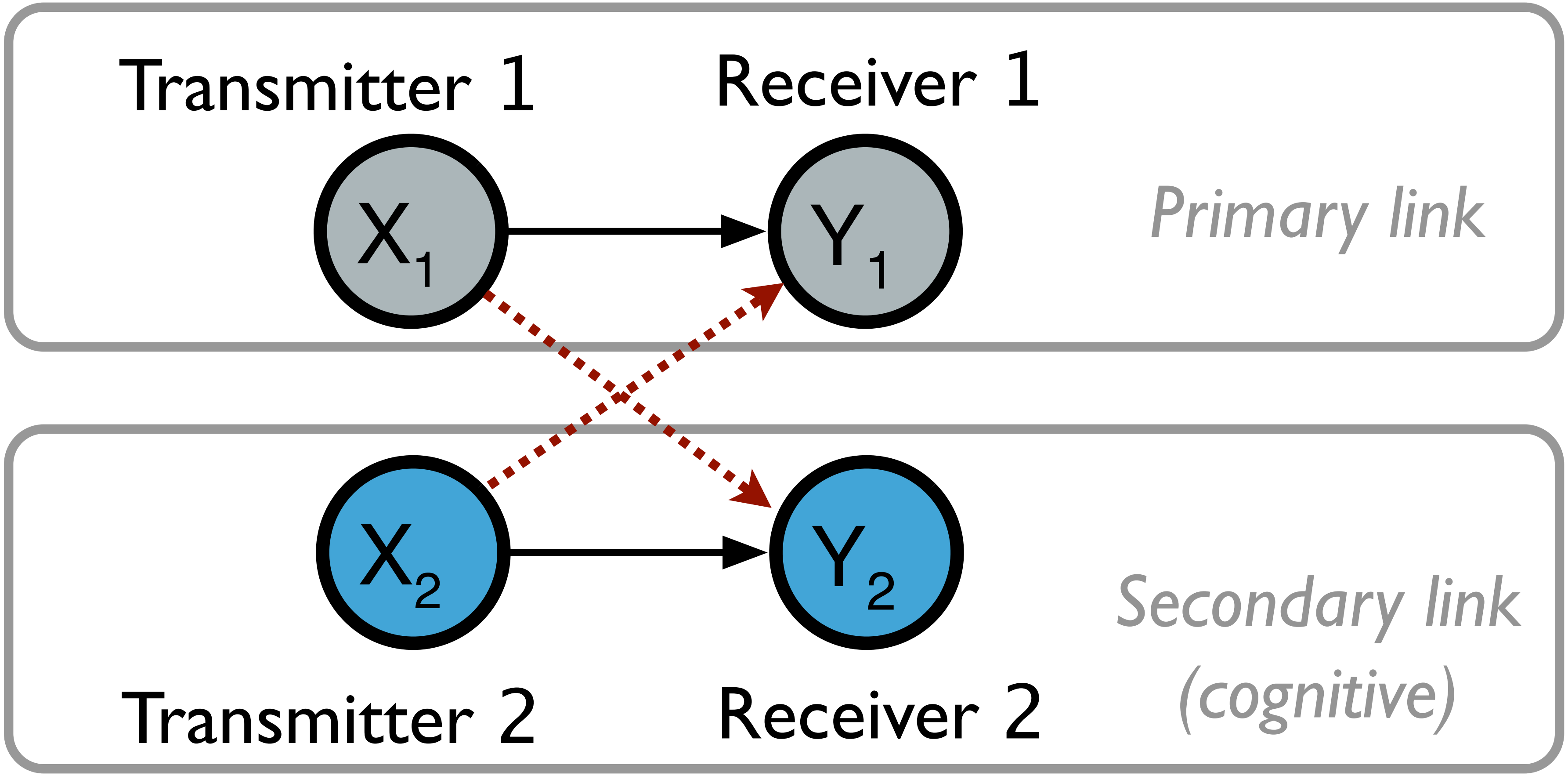
Secondary spectrum usage



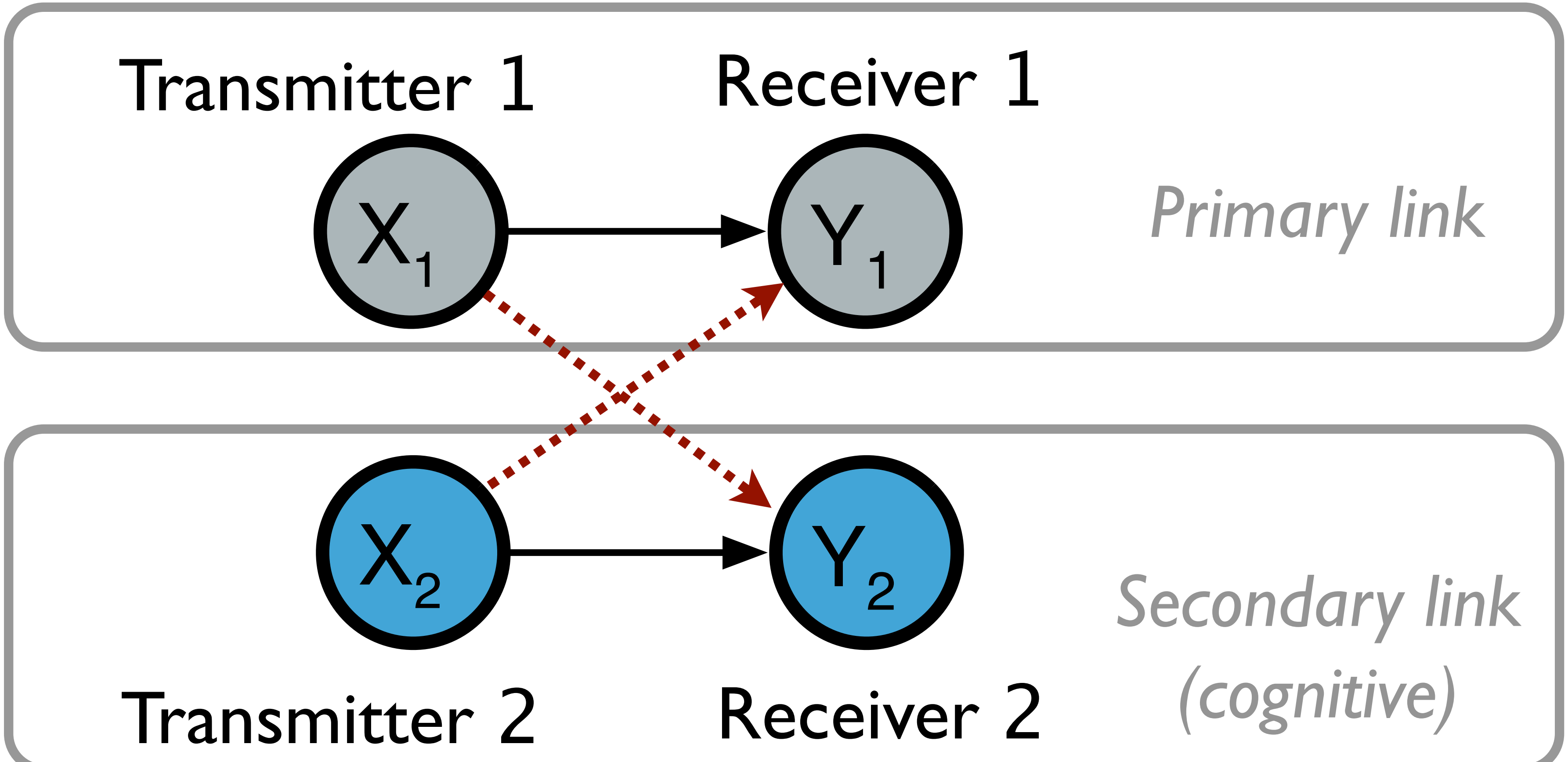
Secondary spectrum usage



Secondary spectrum usage

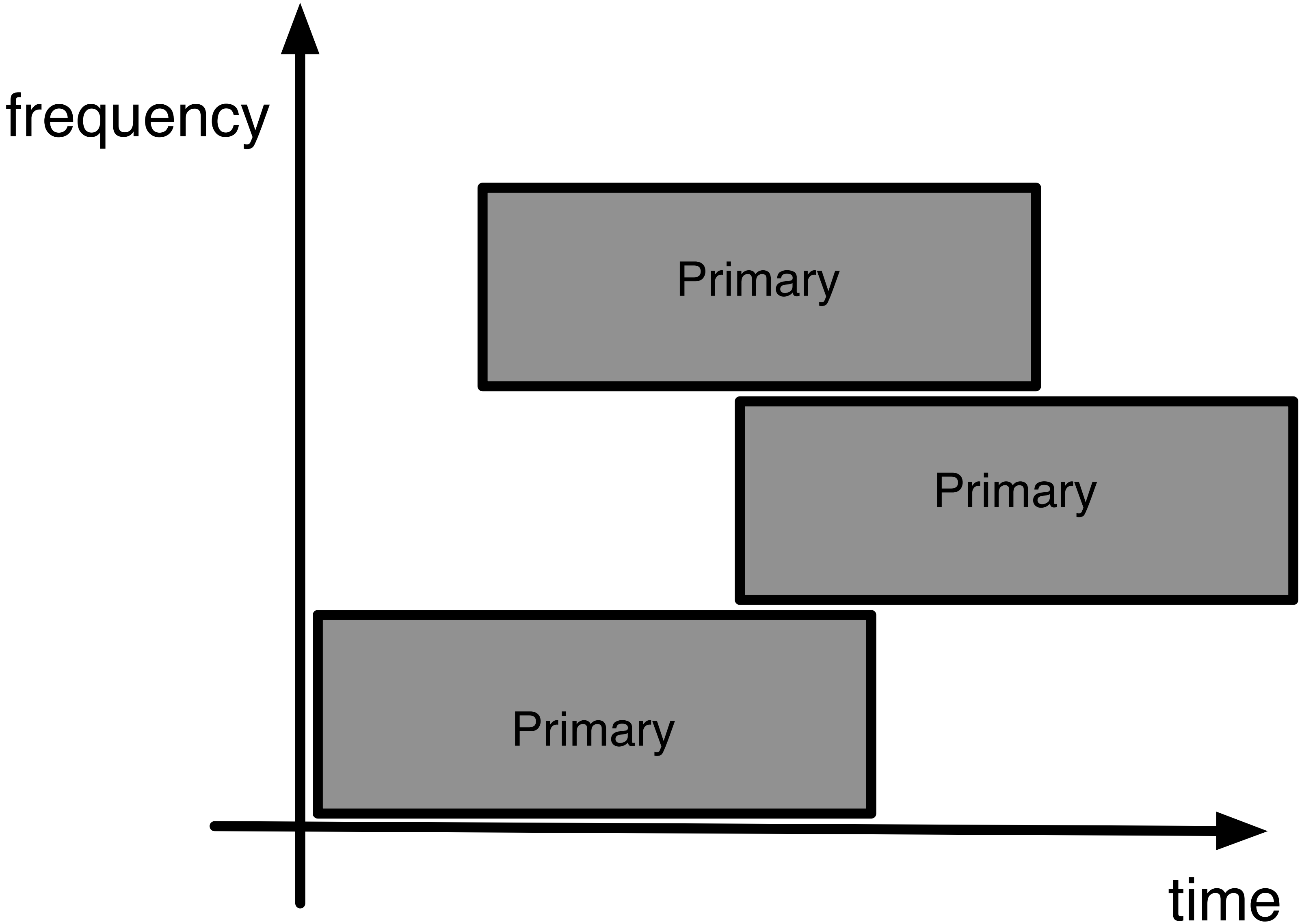


Secondary spectrum usage

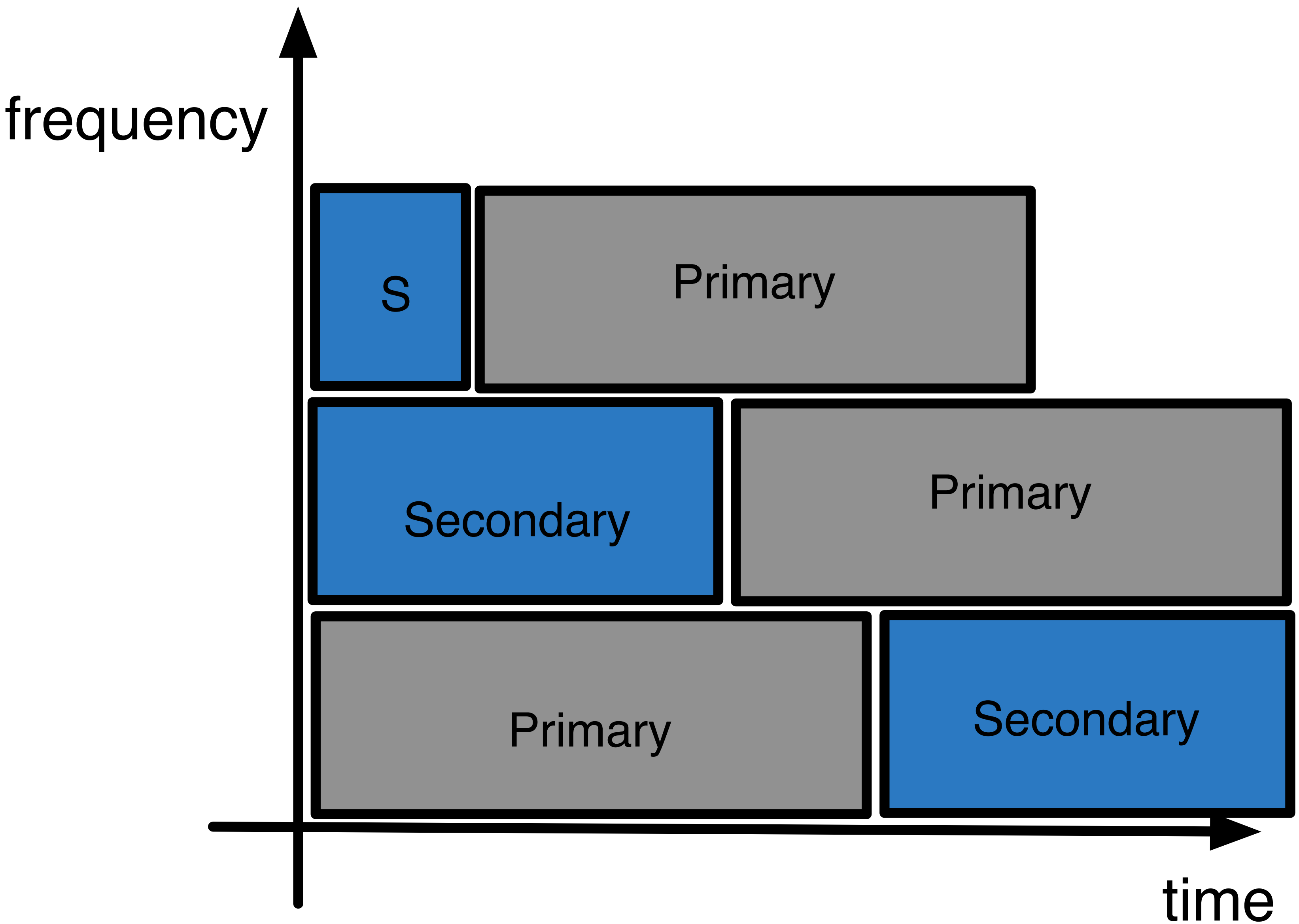


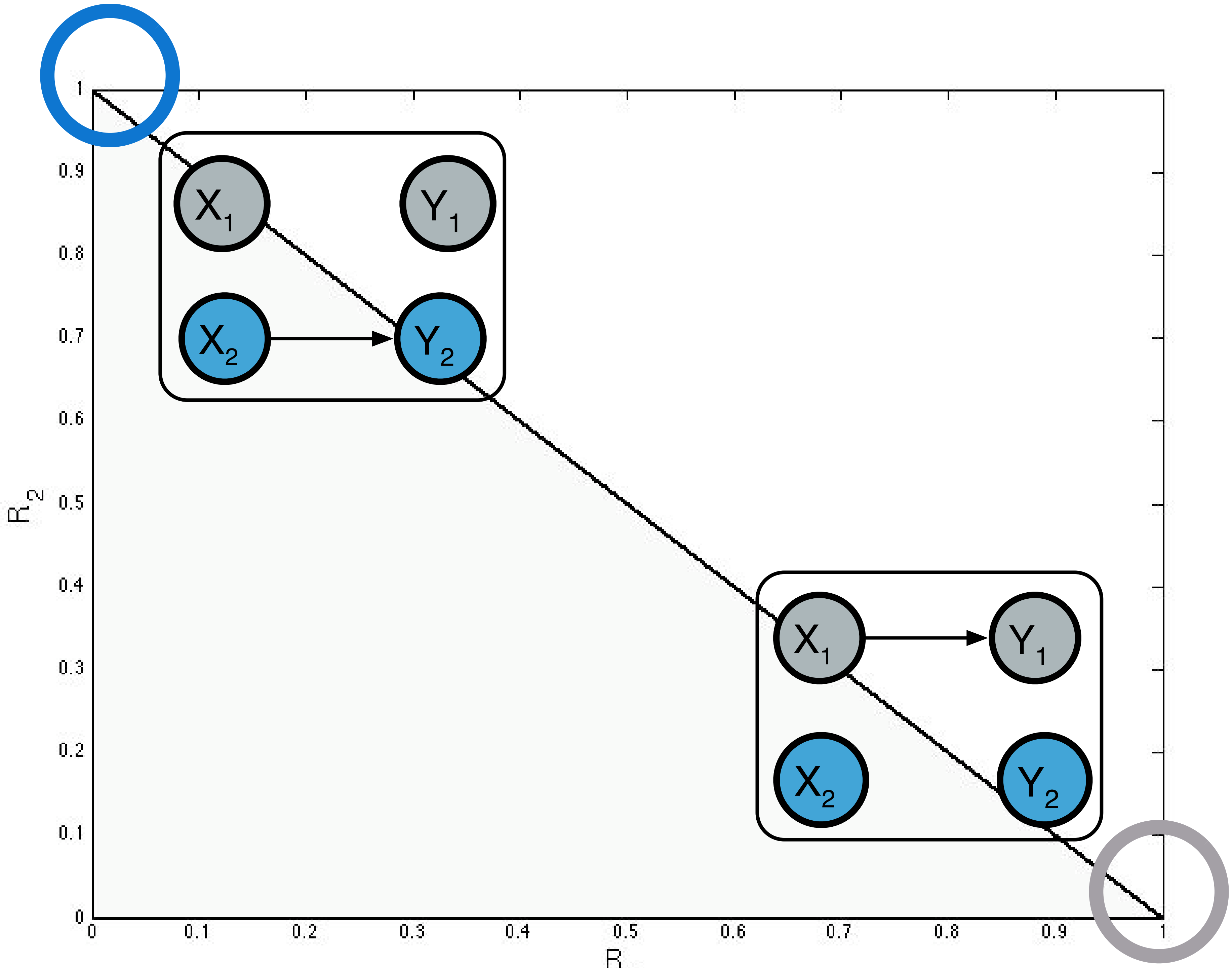
What can the cognitive link do?

1. White spaces

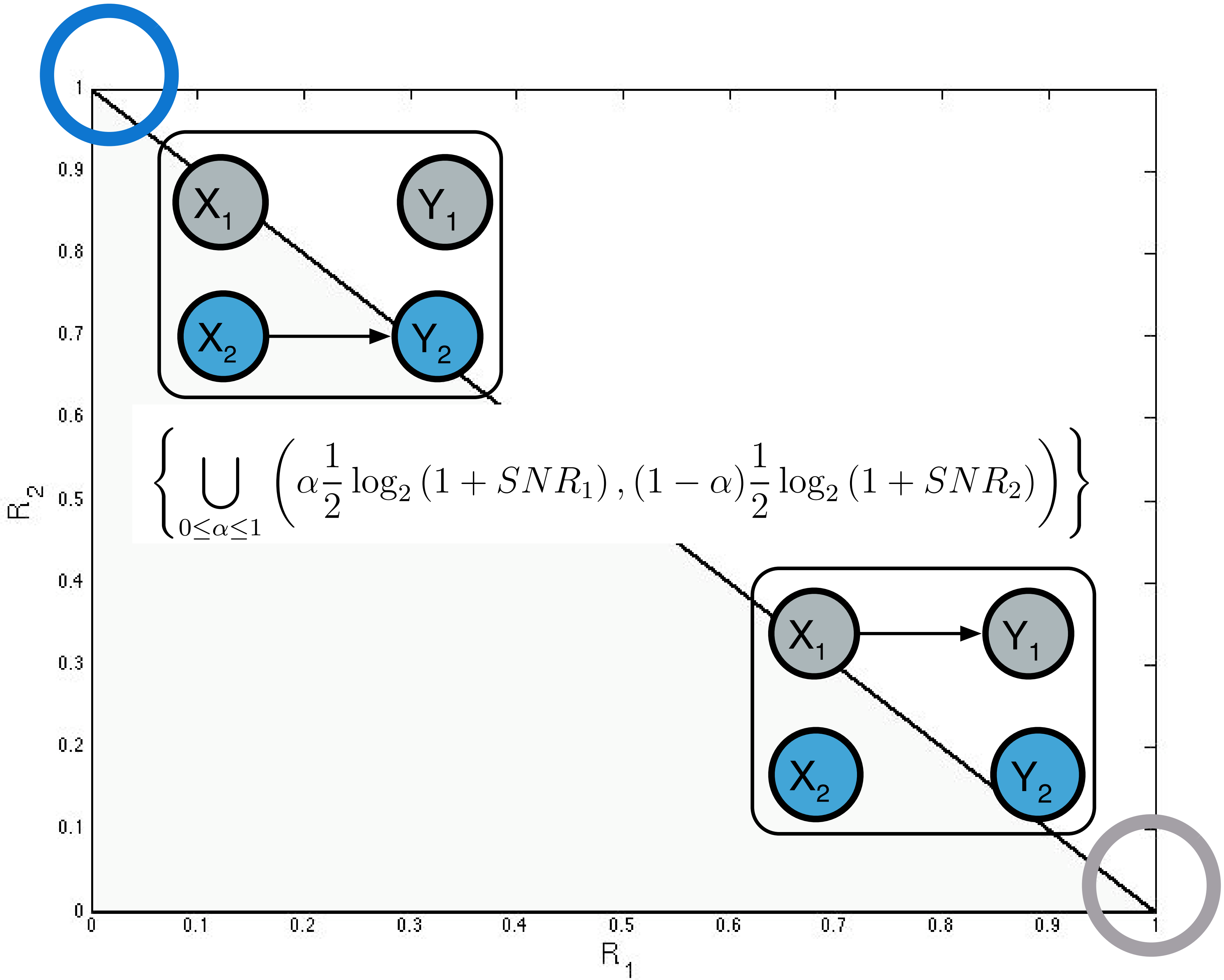


1. White spaces



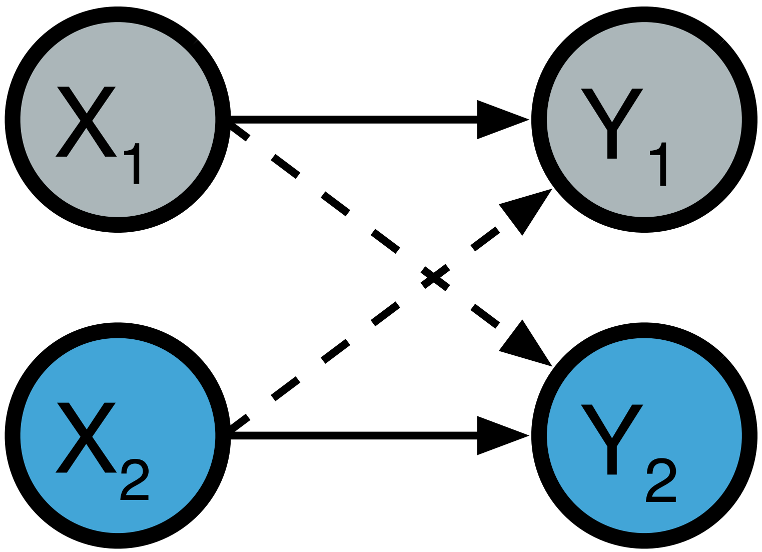


X1
Y1
X2
Y2
X1
Y1
X2
Y2
R2
R1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



$$\left\{ \bigcup_{0 \le \alpha \le 1} \left(\alpha \frac{1}{2} \log_2 (1 + SNR_1), (1-\alpha) \frac{1}{2} \log_2 (1 + SNR_2) \right) \right\}$$
X_1
Y_1
X_2
Y_2
R_1
R_2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

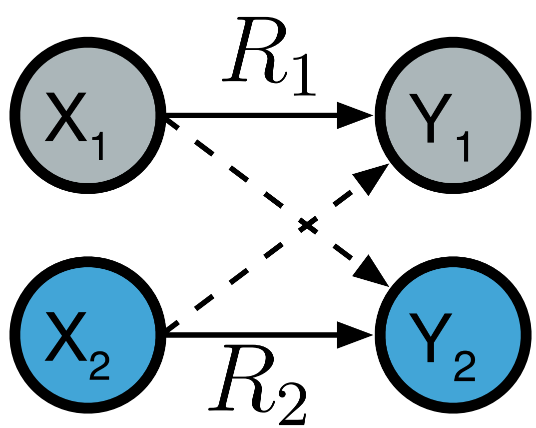
2. Just transmit



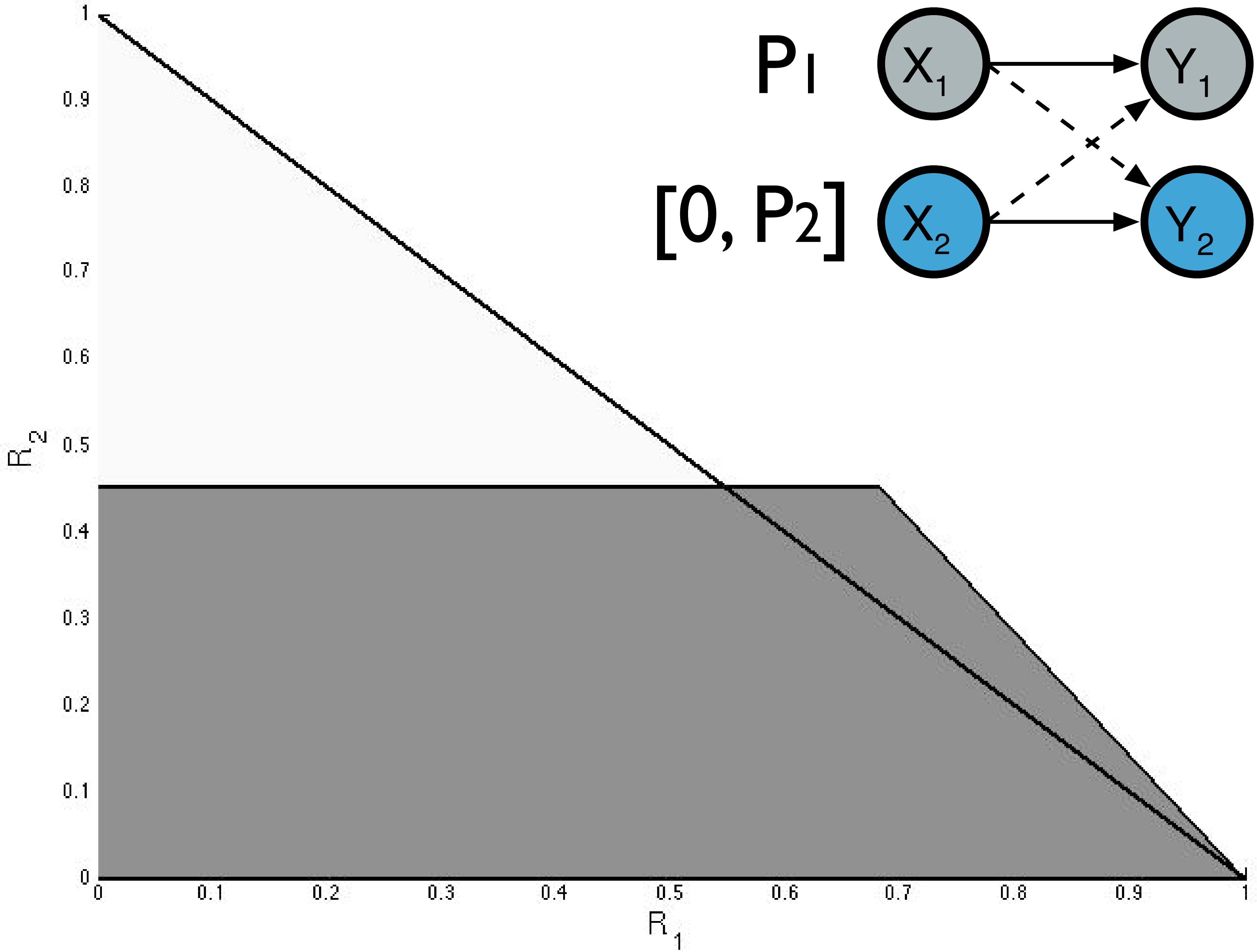
Interfere with each other!

2. Just transmit

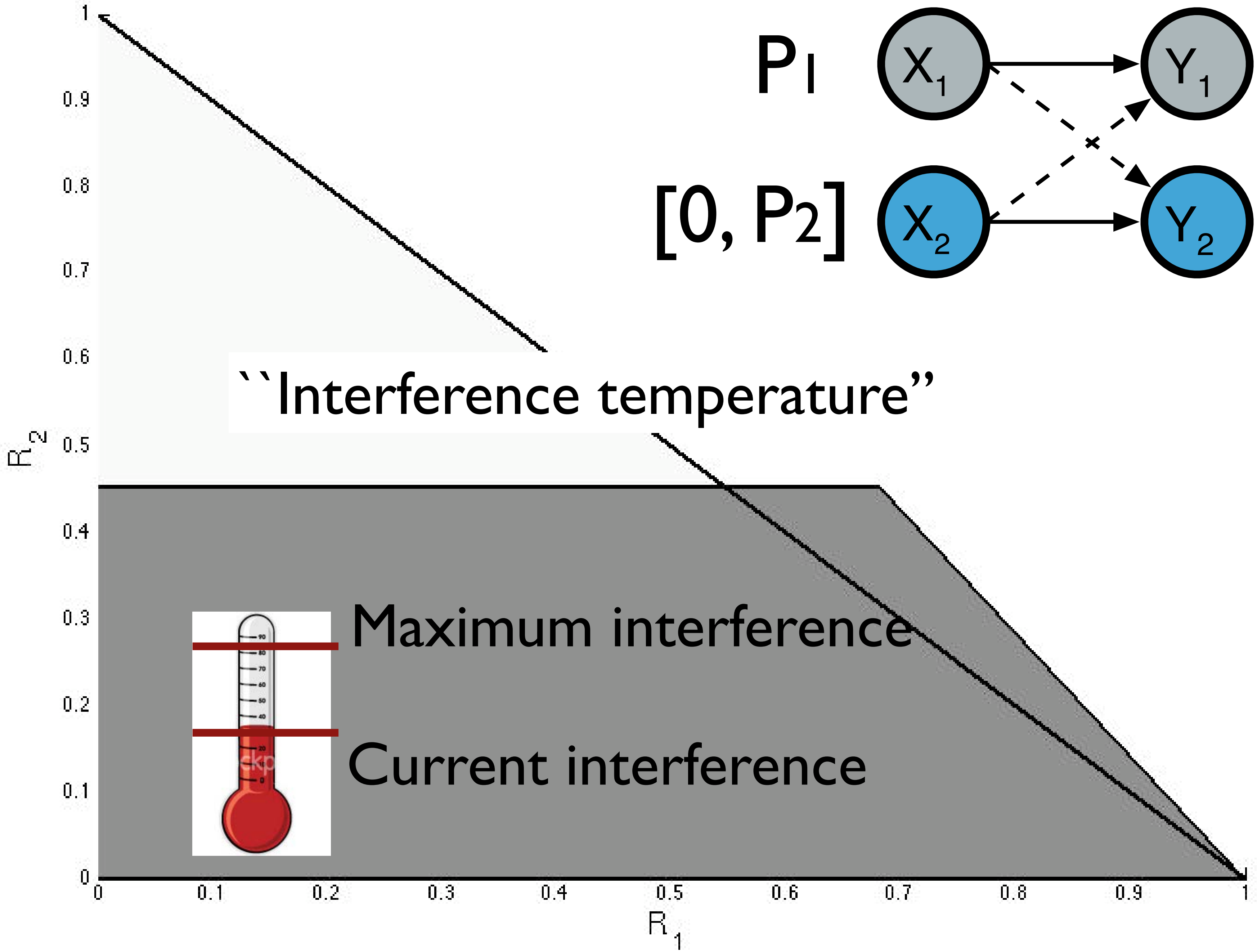
$$R_1 \leq \frac{1}{2}\log_2\left(1 + \frac{\text{Power of signal 1}}{\text{Interference from signal 2} + \text{ Noise}}\right)$$



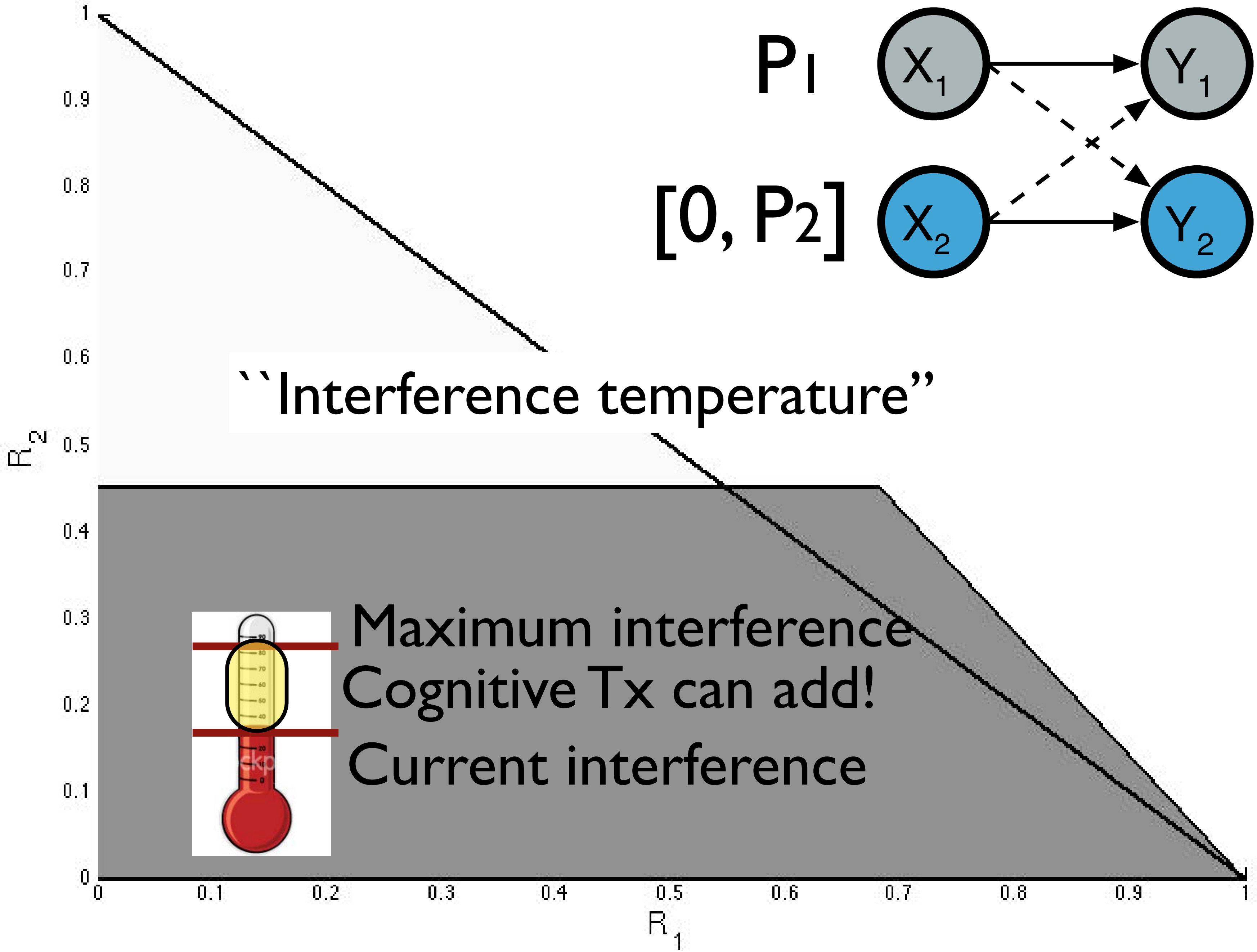
$$R_2 \leq \frac{1}{2}\log_2\left(1 + \frac{\text{Power of signal 2}}{\text{Interference from signal 1} + \text{ Noise}}\right)$$



P1
[0, P2]
X_1
X_2
Y_1
Y_2
R_2
R_1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

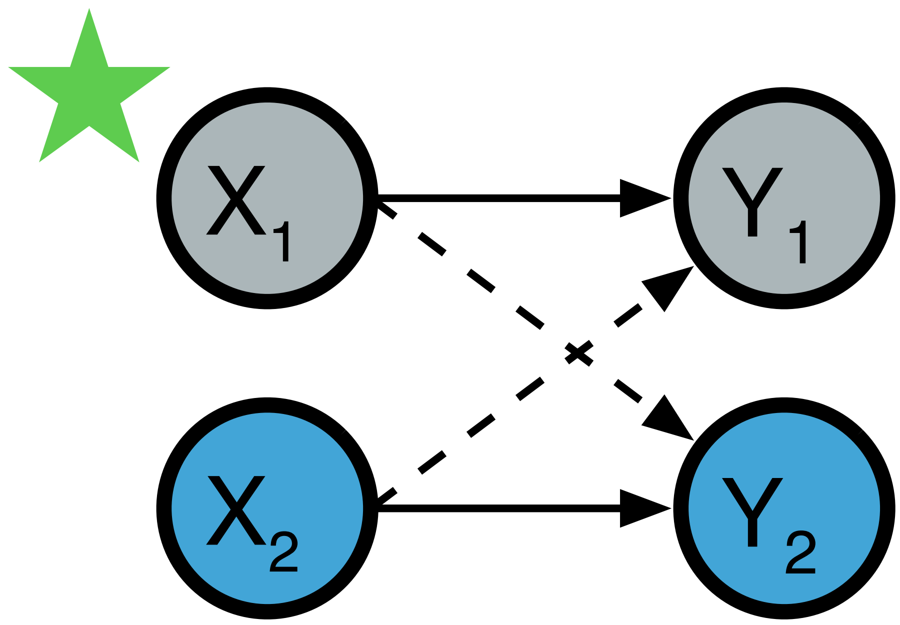


P1
[0, P2]
X1
X2
Y1
Y2
``Interference temperature"
Maximum interference
Current interference
R 2
R 1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



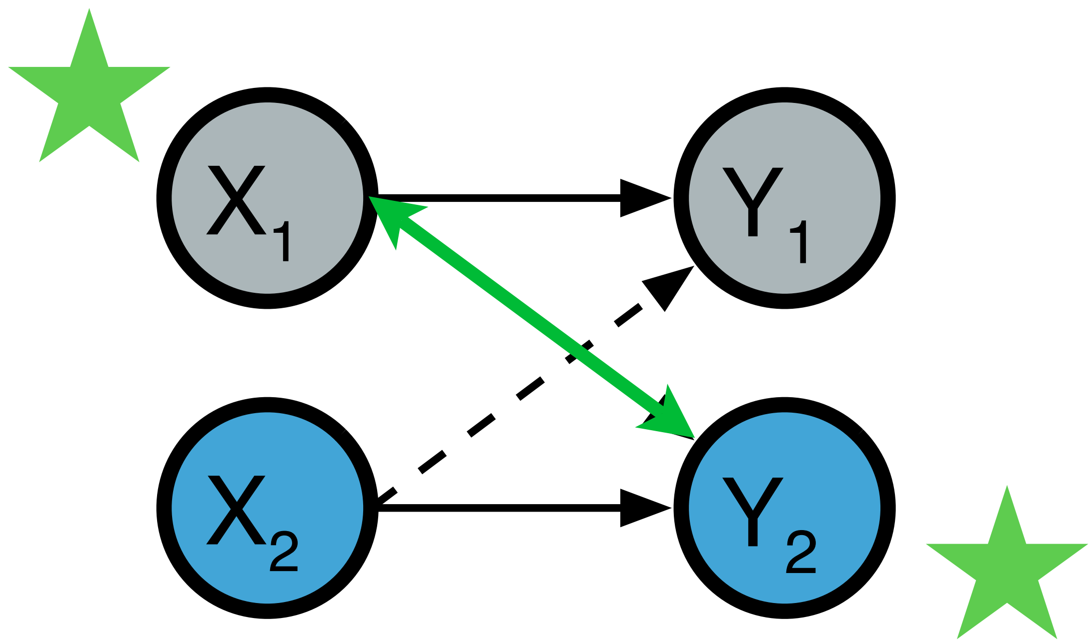
P1
[0, P2]
X_1
Y_1
X_2
Y_2
``Interference temperature"
Maximum interference
Cognitive Tx can add!
Current interference
R_2
R_1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

3. Opportunistic "cognitive" decoding

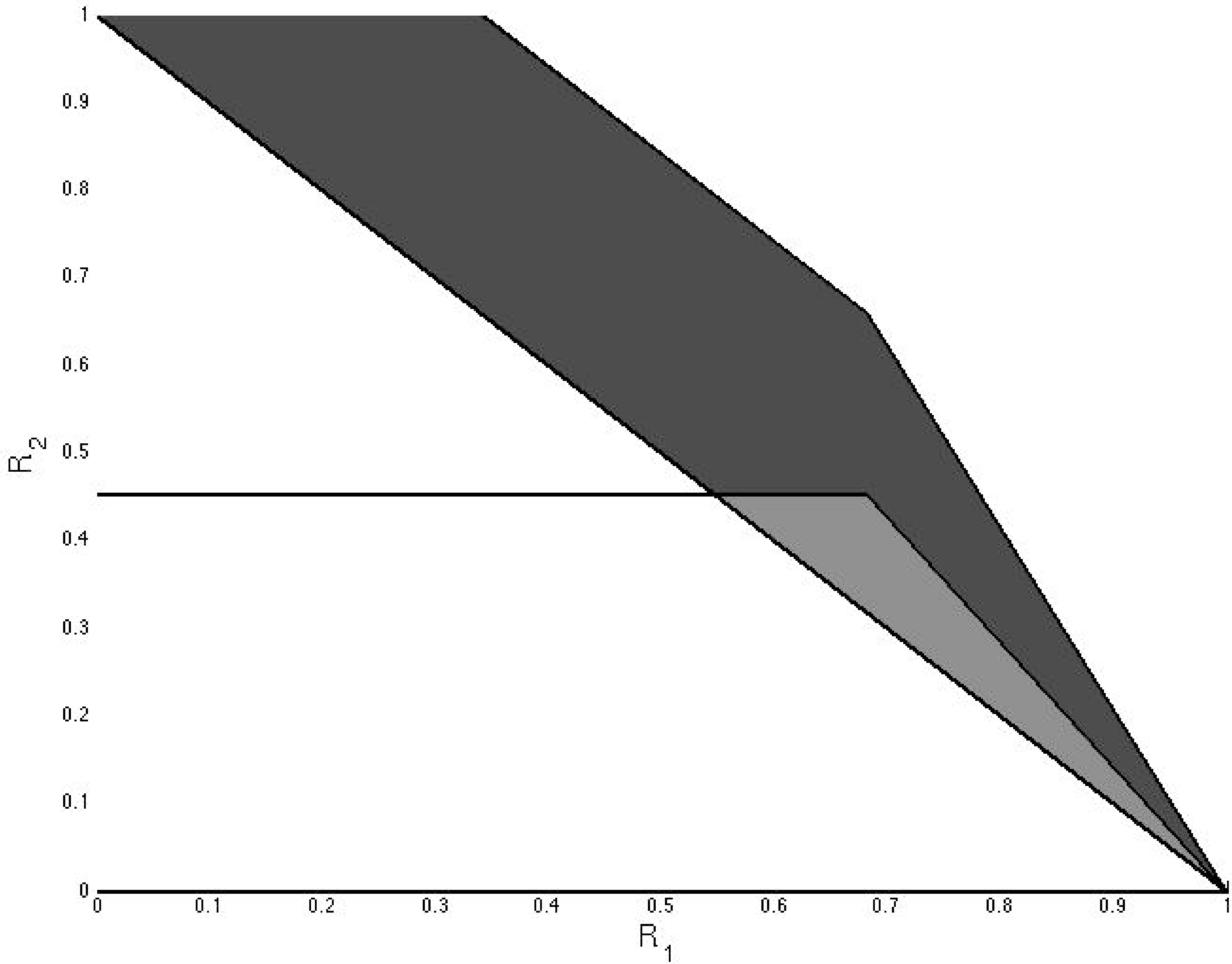


*[Popovski, Devroye 2008]**

3. Opportunistic "cognitive" decoding

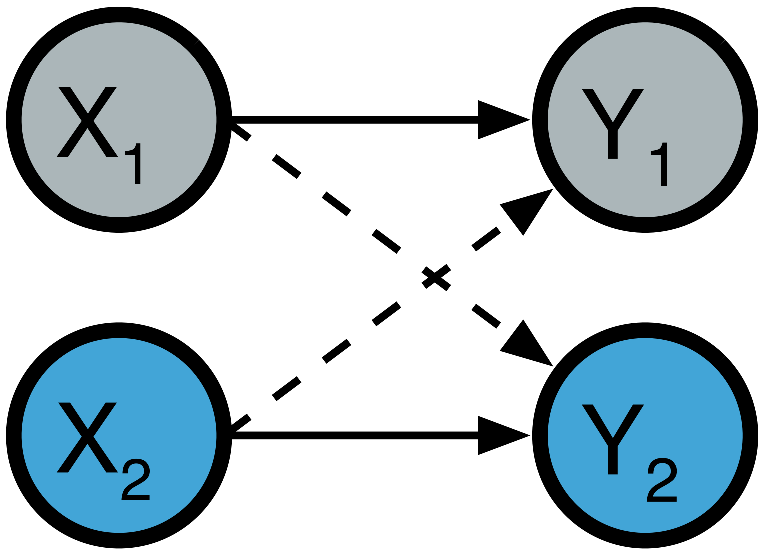


*[Popovski, Devroye 2008]**



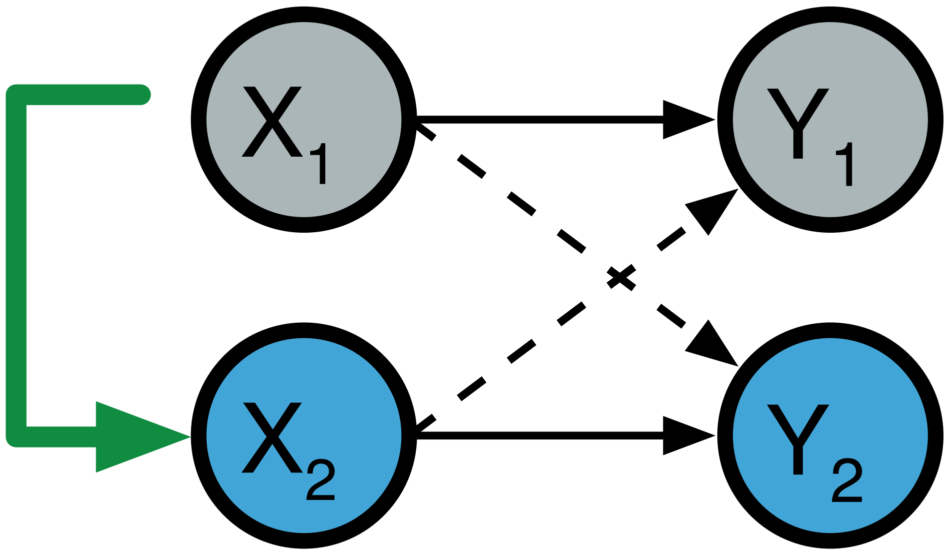
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
R_1

4. Cognitive transmission



Interference!

4. Cognitive transmission

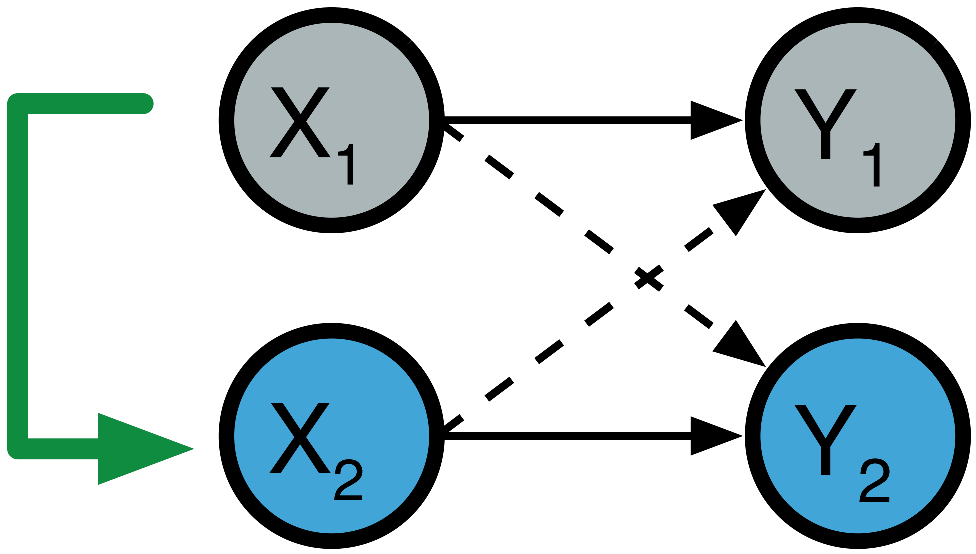


Interference can be reduced

[Devroye, Mitran, Tarokh 2006]

Our proposal:

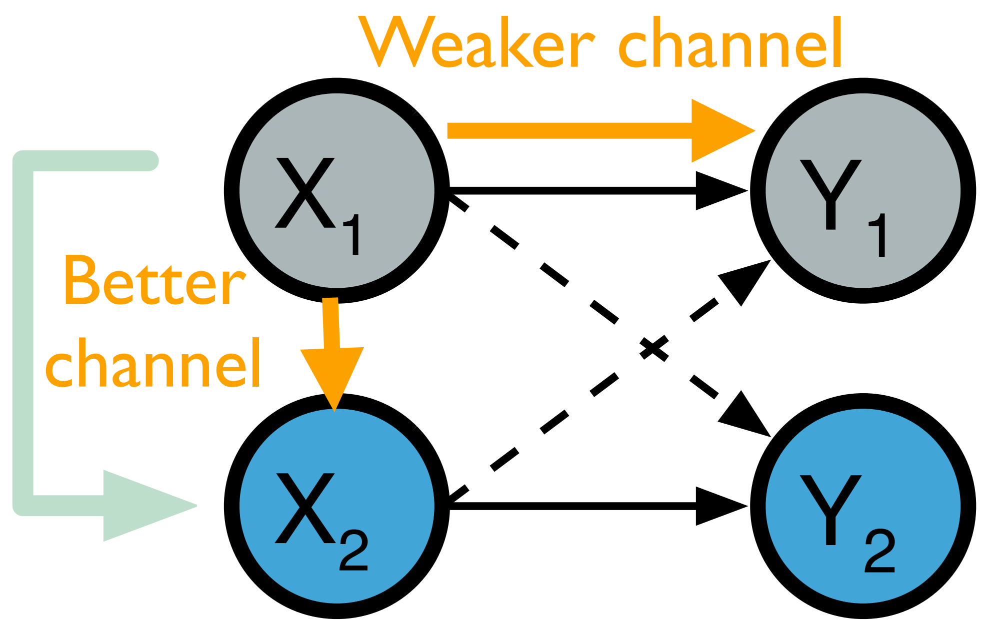
Simultaneous Cognitive Transmission



Assumption: Tx 2 knows message encoded by X_1 a-priori

Our proposal:

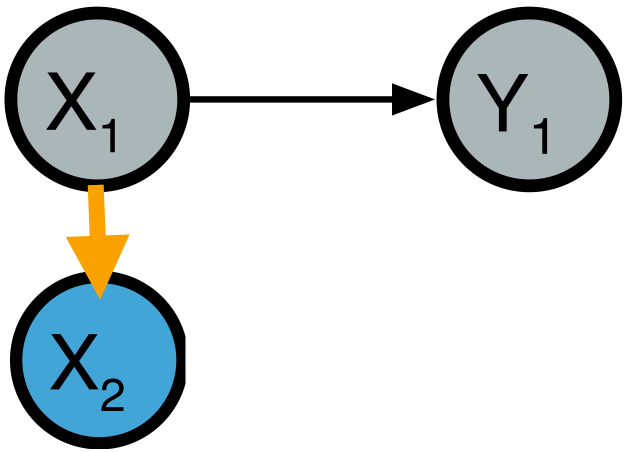
Simultaneous Cognitive Transmission



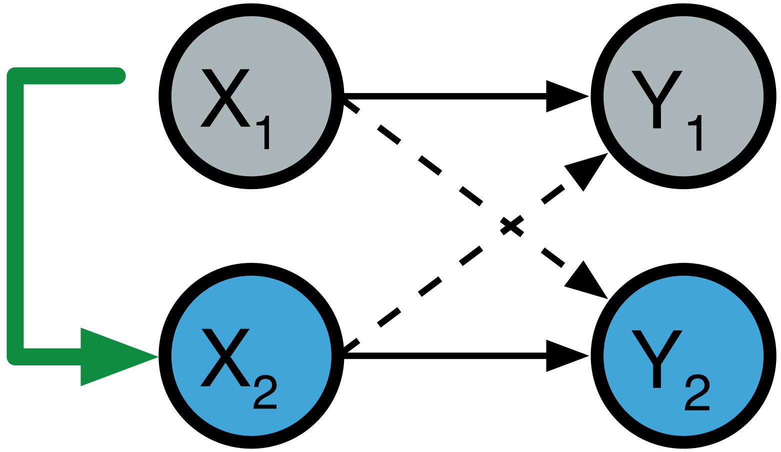
Cognitive Tx may obtain primary's message in a fraction of the time

Our proposal:

Simultaneous Cognitive Transmission

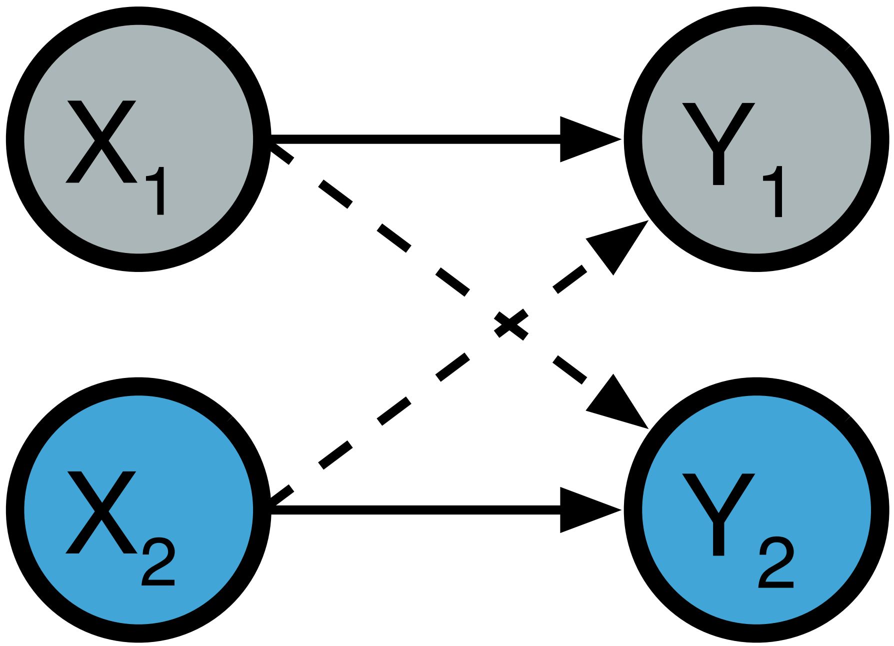


Primary transmission



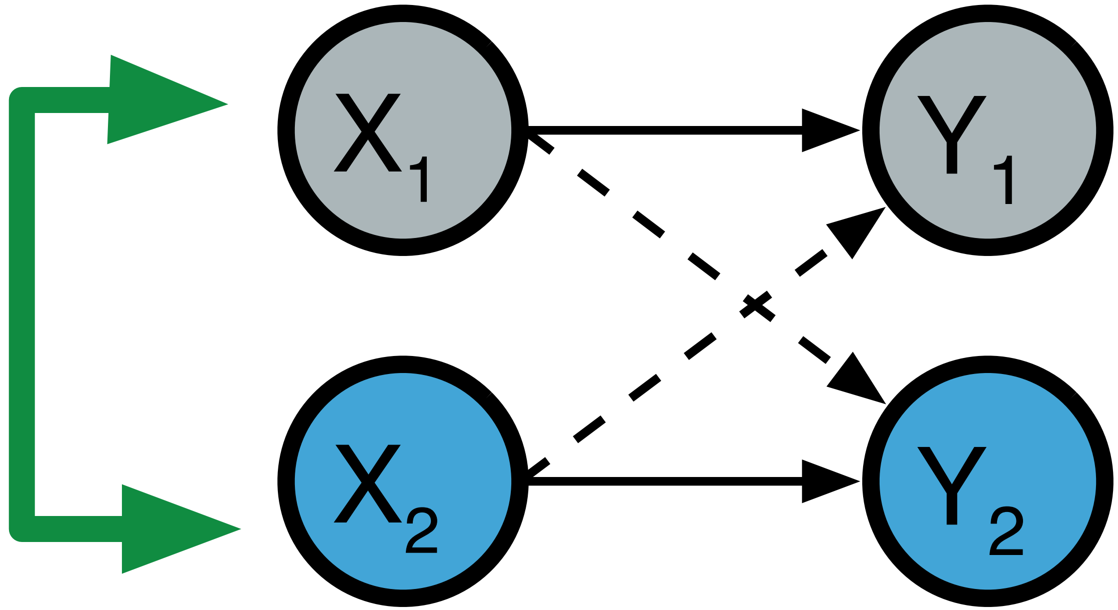
Re-transmission

Cognitive Tx may overhear primary's message



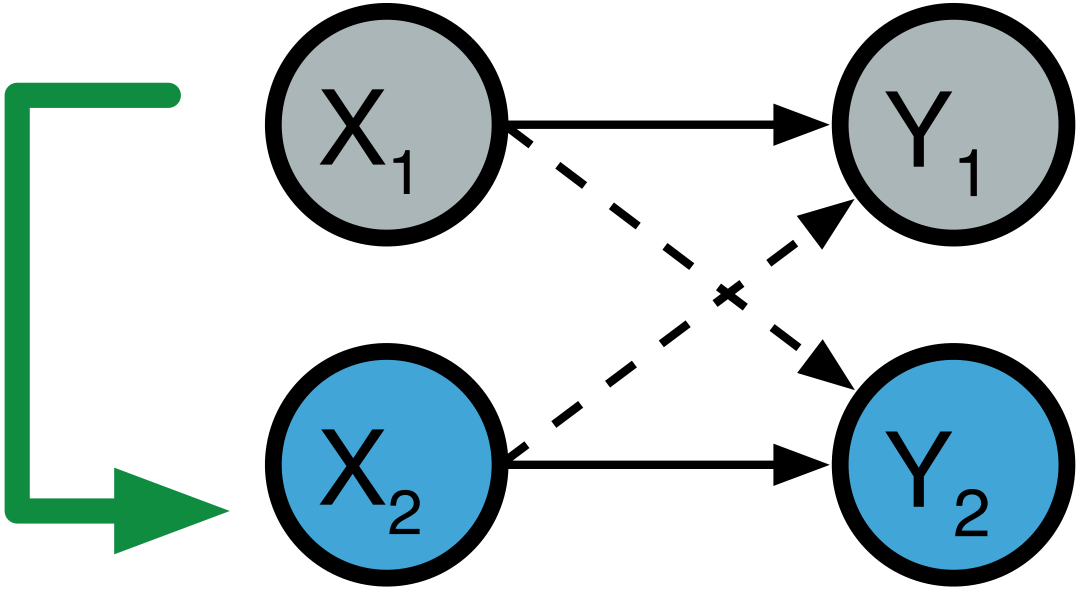
"Competitive"

Interference channel



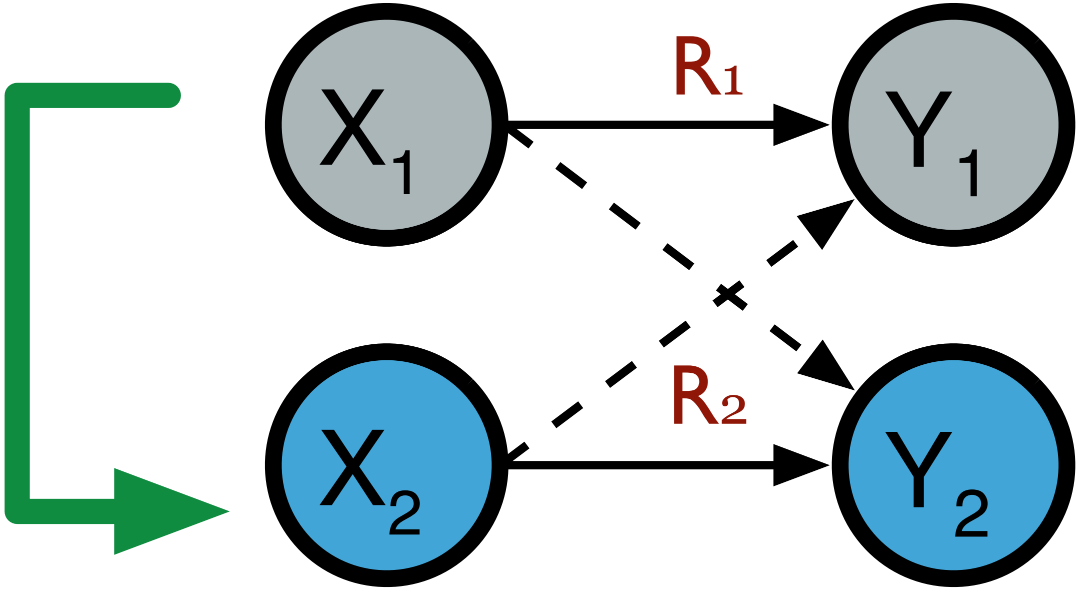
"Cooperative"

Broadcast channel



"Cognitive"

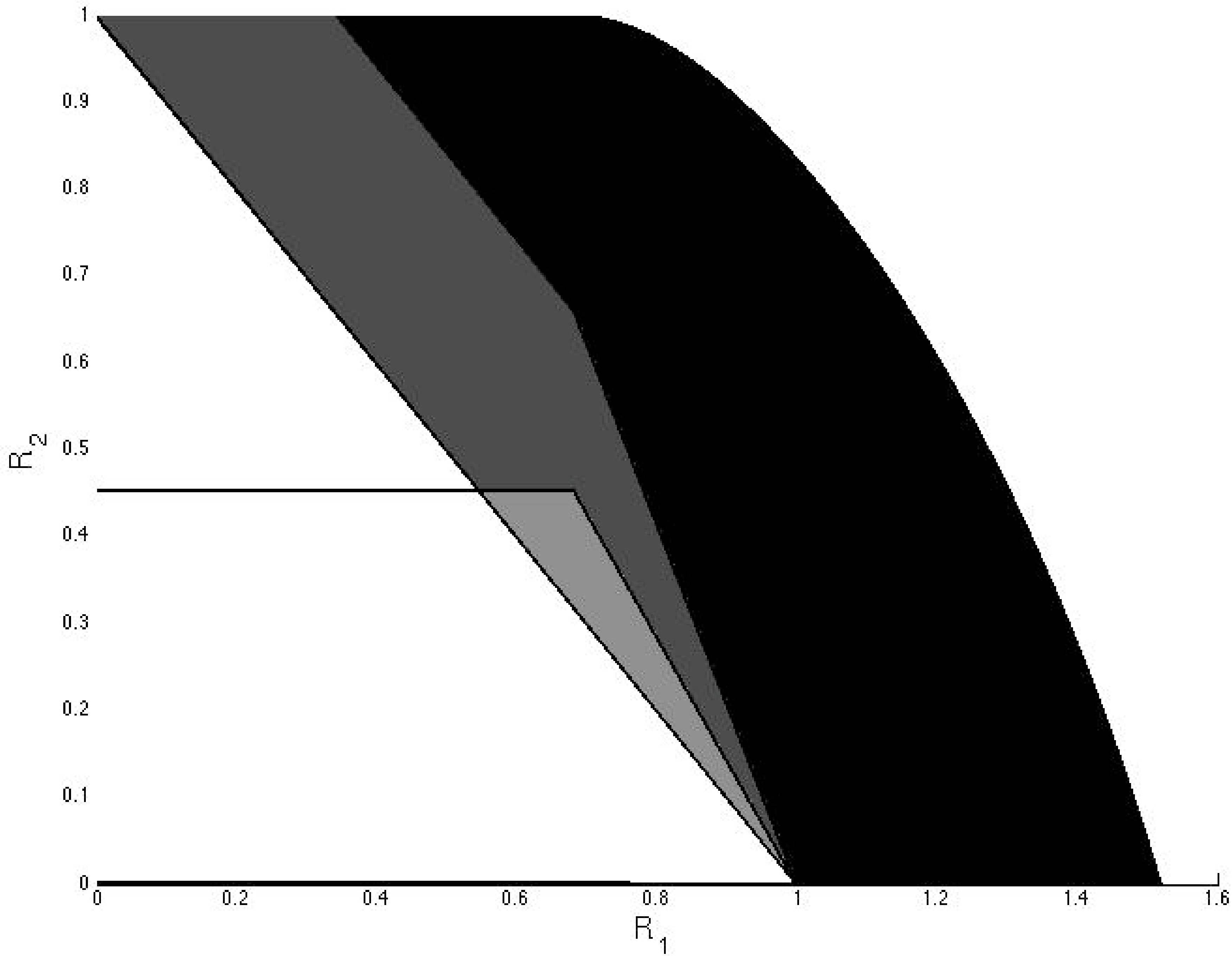
Cognitive channel



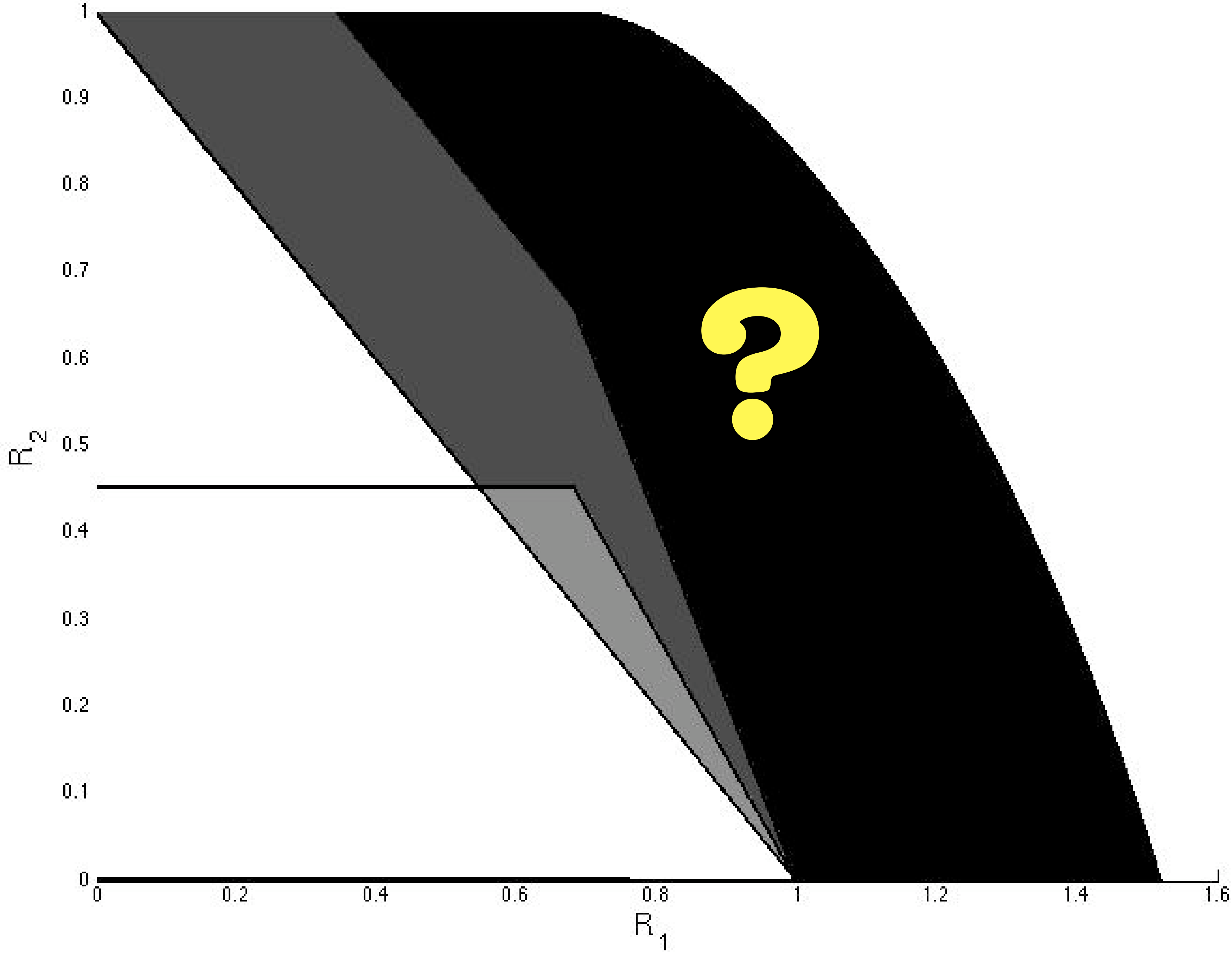
“Cognitive”

Cognitive channel

What rates (R_1, R_2) are achievable?

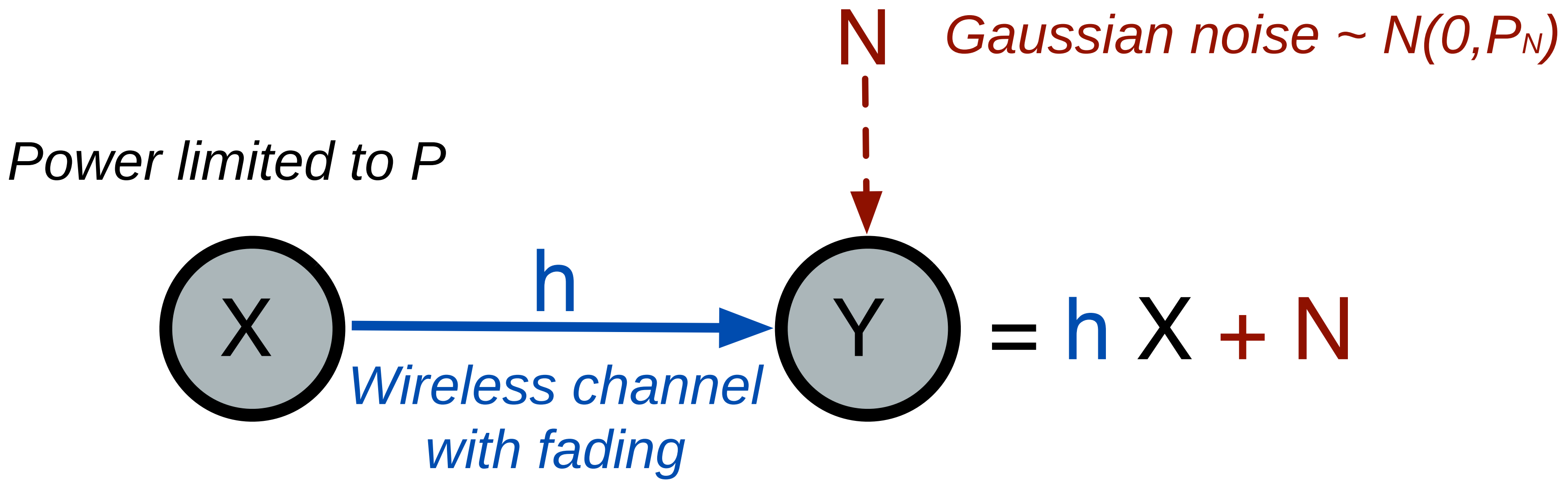


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1

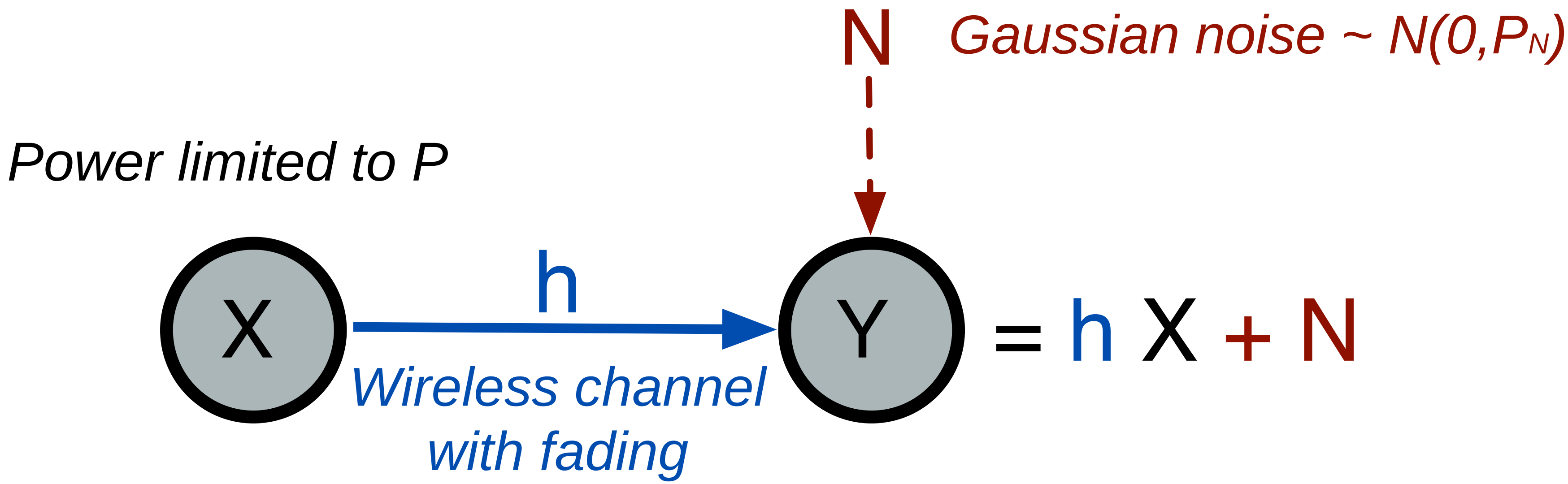


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1
?

Gaussian noise channel capacity



Gaussian noise channel capacity



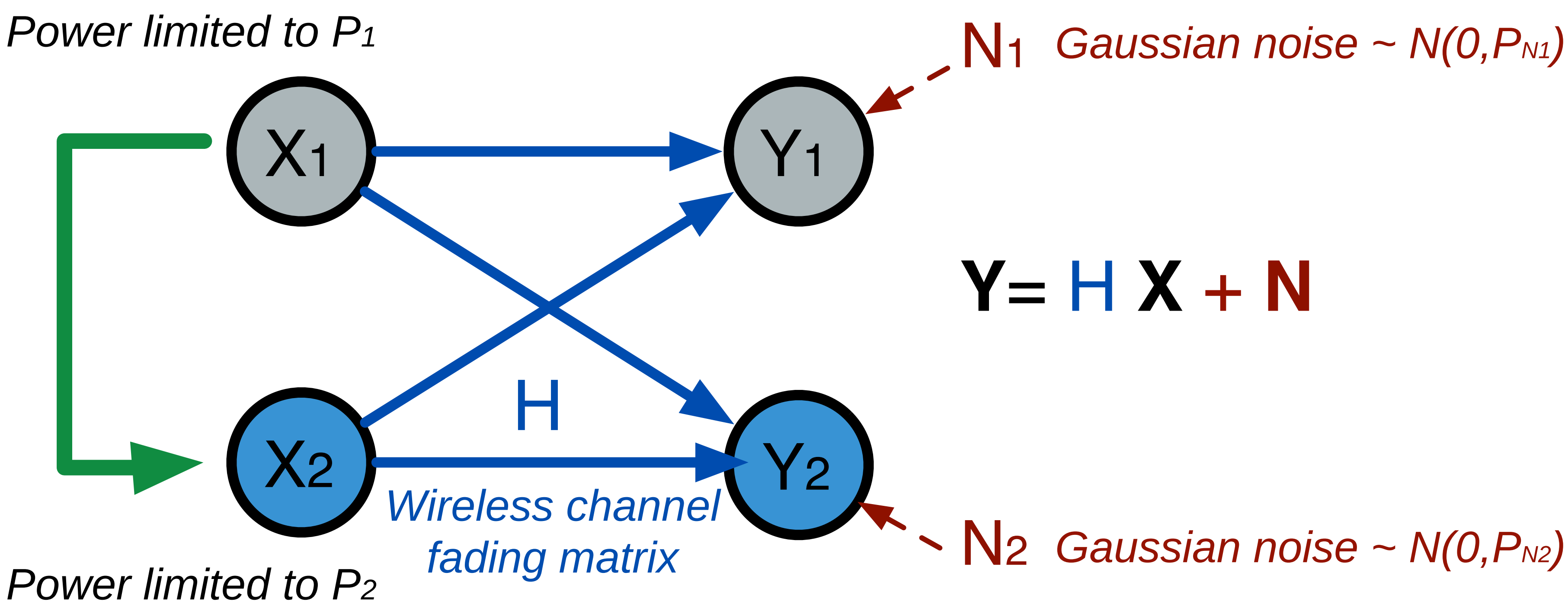
Capacity

$$C = \max_{p(x):E[|X|^2] \leq P} I(X;Y)$$

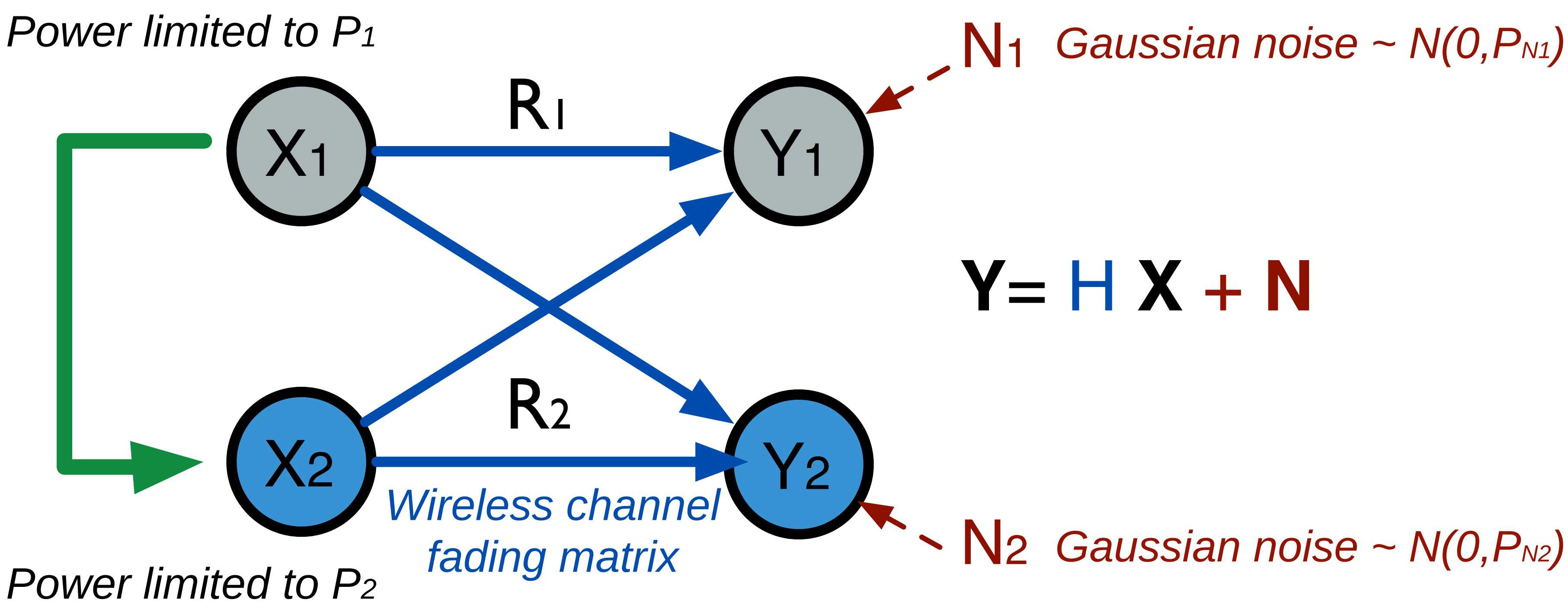
$$= \frac{1}{2} \log_2 \left(\frac{|h|^2 P + P_N}{P_N} \right)$$

$$= \frac{1}{2} \log_2 (1 + \text{SNR})$$ (bits/channel use)

Gaussian cognitive channel

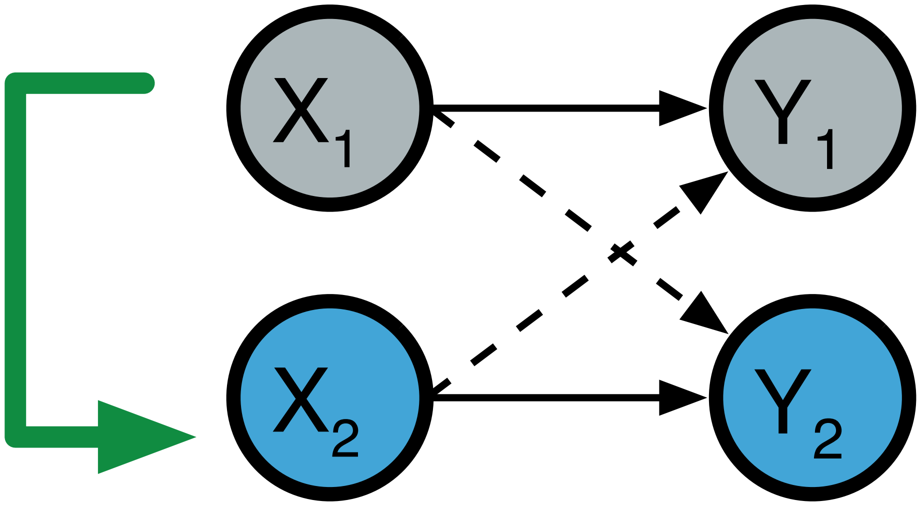


Gaussian cognitive channel



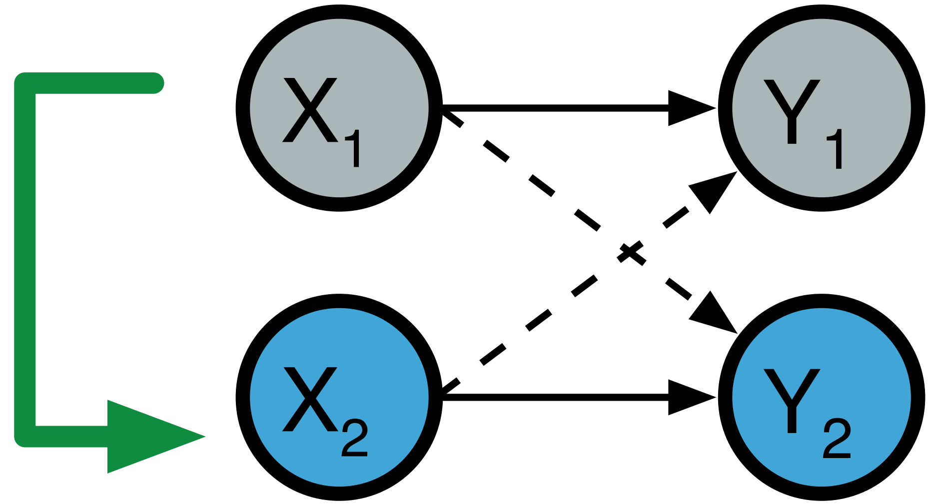
What rates are achievable?

Intuition



A priori message knowledge

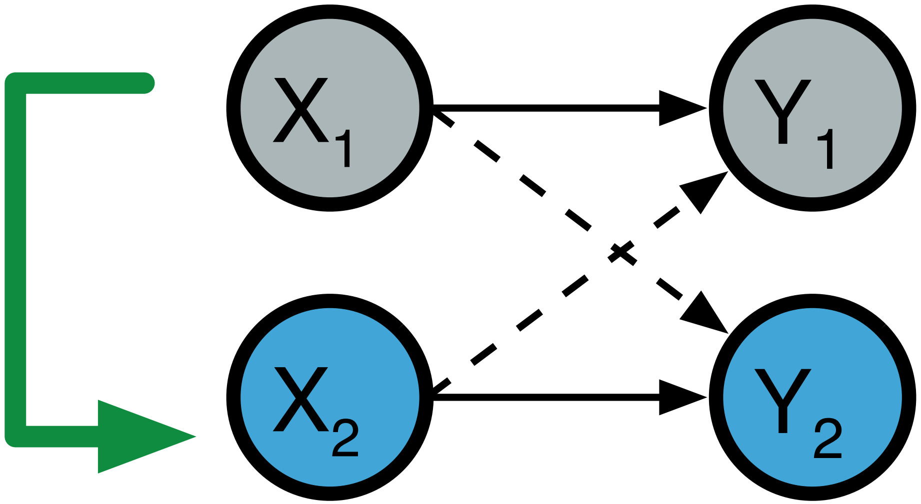
Intuition



A priori message knowledge

aid
transmission
SELFLESS

Intuition



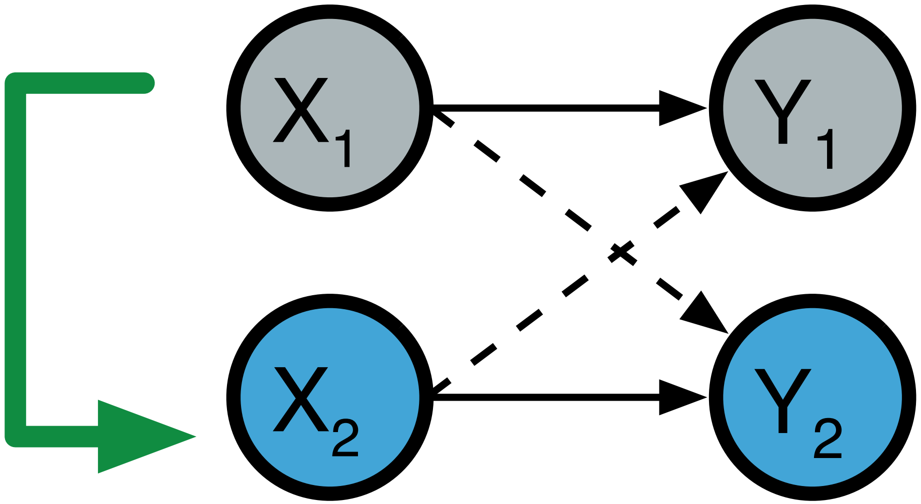
A priori message knowledge

aid transmission
SELFLESS

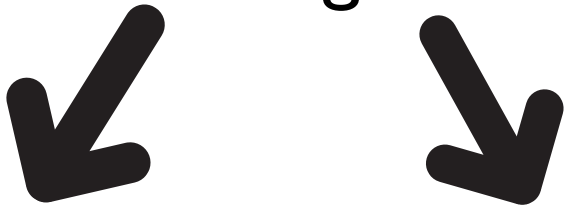
mitigate interference
SELFISH

Intuition

P_1'



A priori message knowledge



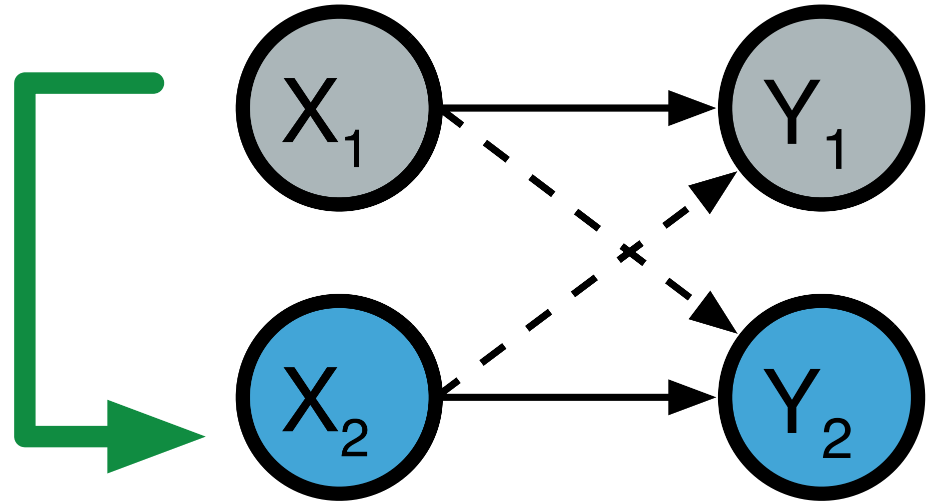
aid transmission
SELFLESS

mitigate interference
SELFISH

Intuition

P_1'

$$P_2 = \boxed{P_2'} + \boxed{P_2''}$$



A priori message knowledge

aid transmission
SELFLESS

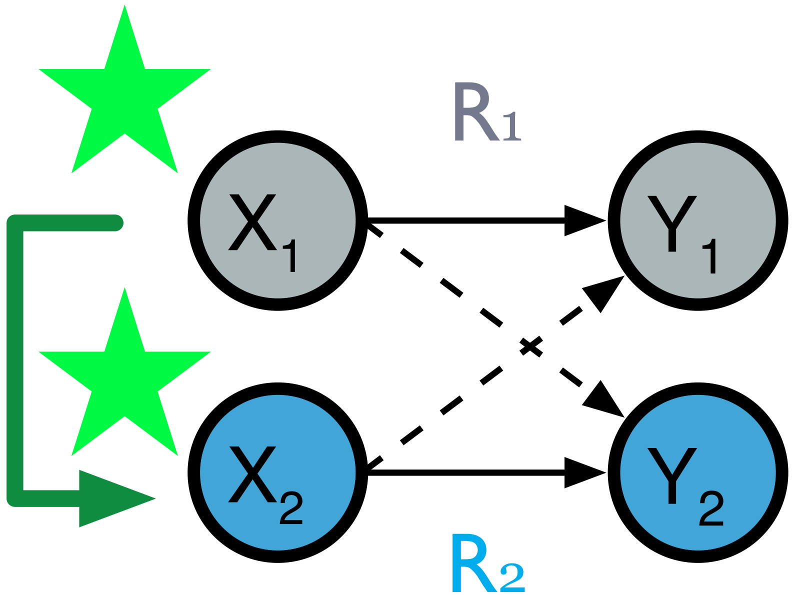
mitigate interference
SELFISH

Message 1

Message 1: encoded by a codeword which is generated jointly Gaussian according to $\mathcal{N}(0,B_1)$

$$\boxed{B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}}$$

message 1



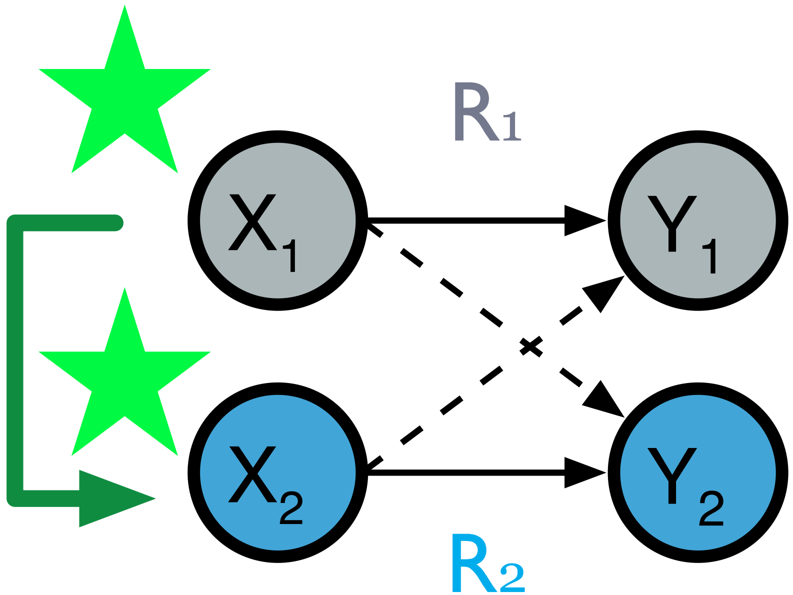
Message 1

Message 1: encoded by a codeword which is generated jointly Gaussian according to $\mathcal{N}(0,B_1)$

$$B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}$$

message 1

$$\begin{bmatrix} E[|X_1|^2] & E[X_1X_2] \\ E[X_1X_2] & E[|X_2|^2] \end{bmatrix}$$

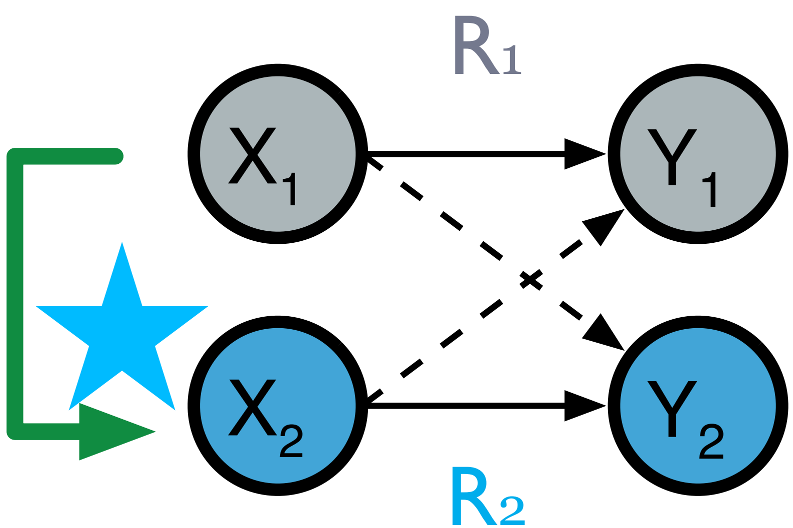


Message 2 ★

Message 2: encoded by a codeword which is generated as jointly Gaussian according to $\mathcal{N}(0, B_2)$

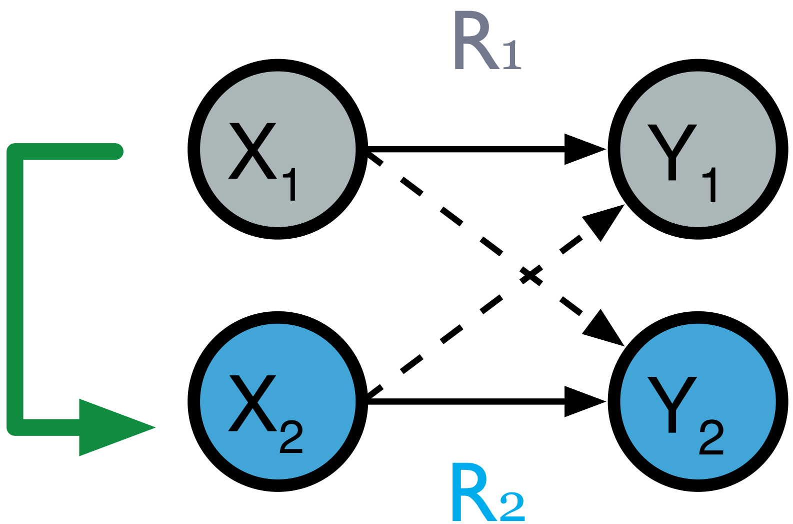
$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_2'' \end{bmatrix}$$

message 2



Send the superposition

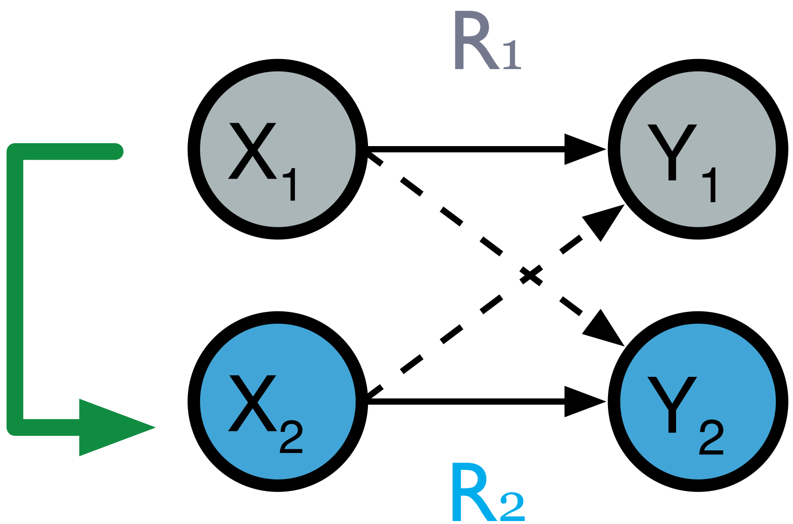
$\boxed{B_1 + B_2}$ Overall transmit covariance matrix



Send the superposition

$$B_1 + B_2 \preceq \begin{bmatrix} \boxed{P_1} & z \\ z & \boxed{P_2} \end{bmatrix}$$

Per antenna power constraints

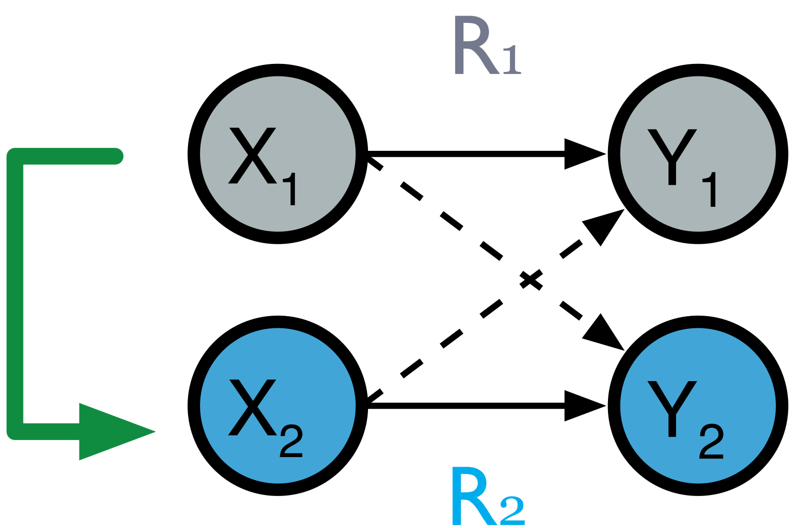


Send the superposition

$$B_1 + B_2 \preceq \begin{bmatrix} P_1 & \boxed{z} \\ \boxed{z} & P_2 \end{bmatrix}, \quad \boxed{z^2 \leq P_1 P_2}$$

Correlation between two antennas

Ensures Tx covariance matrix is positive semi-definite

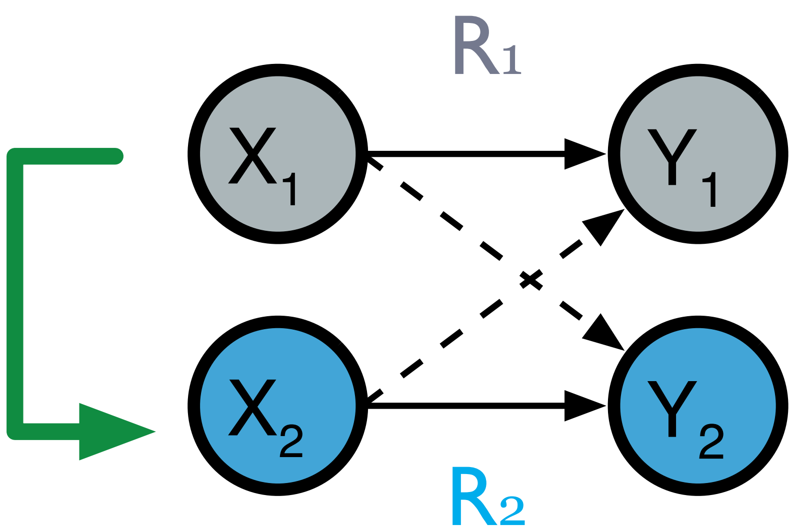


Send the superposition

$$B_1 + B_2 \preceq \begin{bmatrix} P_1 & \boxed{z} \\ \boxed{z} & P_2 \end{bmatrix}, \quad \boxed{z^2 \leq P_1 P_2}$$

Correlation between two antennas

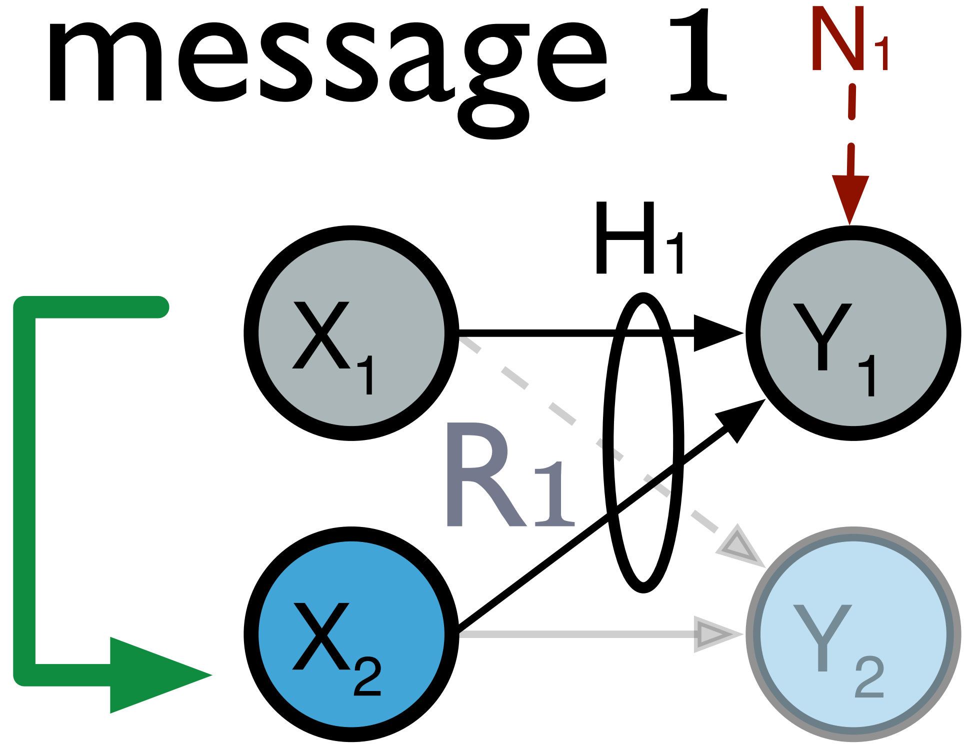
Ensures Tx covariance matrix is positive semi-definite



What rates R1, R2 are achievable?

R_1: Rate of message 1

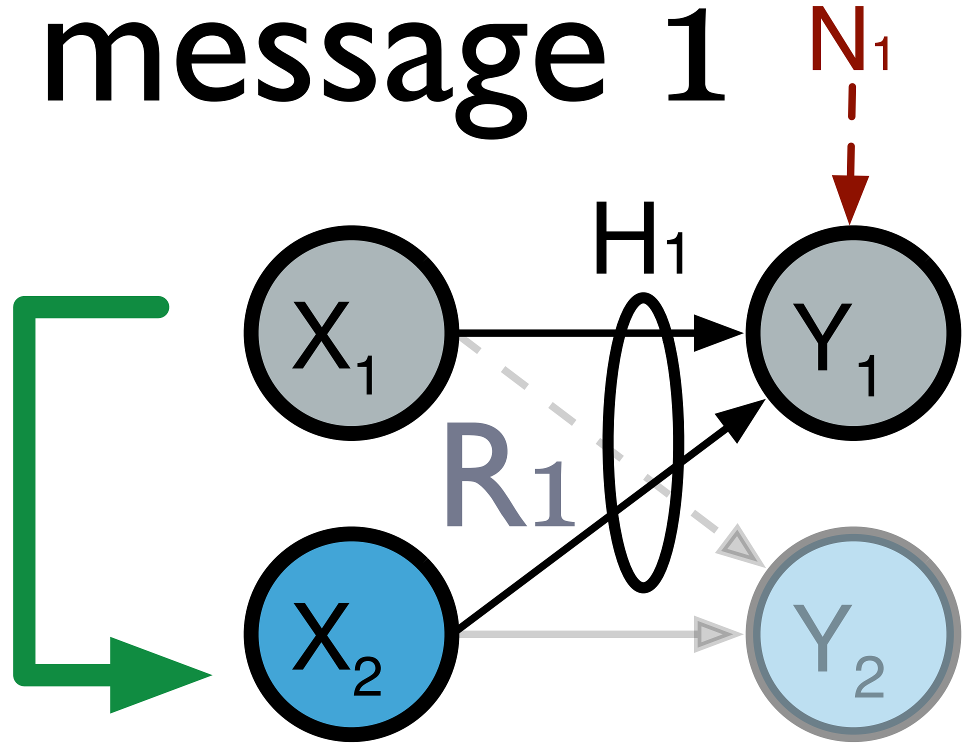
$$B = \boxed{B_1} + B_2$$



$$Y_1 = H_1 X + N_1$$

R_1 :Rate of message 1

$$B = \boxed{B_1} + B_2$$



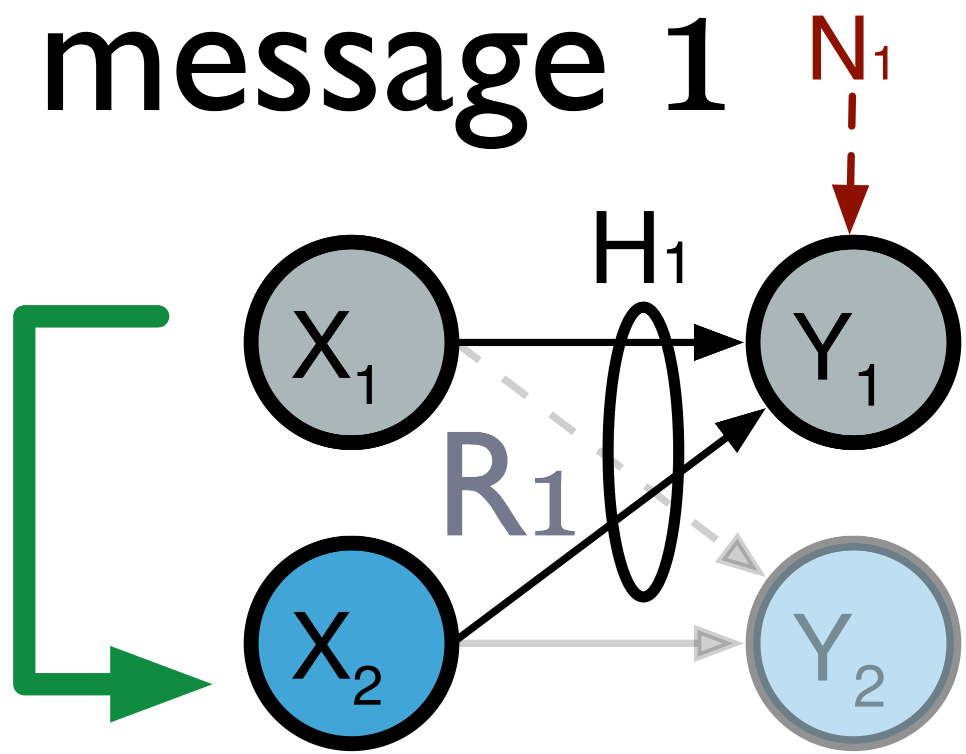
Signal power at Y1

$$R_1 \le \frac{1}{2}\log_2\left(\frac{\boxed{H_1(B_1+B_2)H_1^{\dagger} + P_{N_1}}}{\boxed{H_1(B_2)H_1^{\dagger} + P_{N_1}}}\right)$$

Interference + noise power

R_1 :Rate of message 1

$$B = \boxed{B_1} + B_2$$



Signal power at Y1

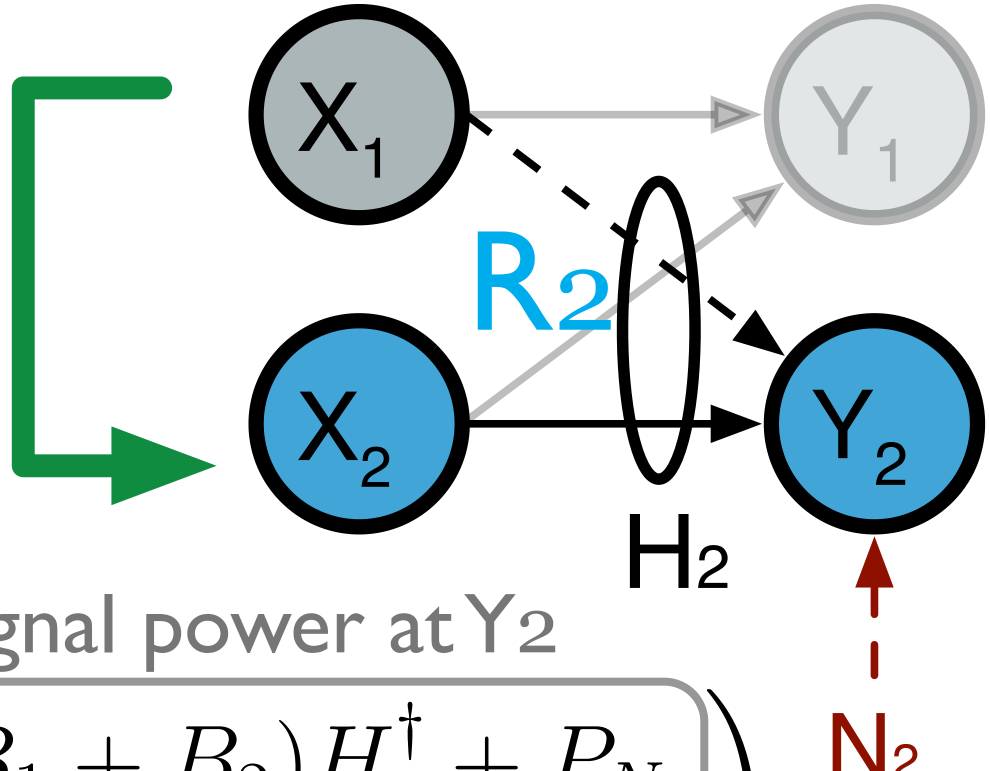
$$R_1 \leq \frac{1}{2}\log_2\left(\frac{\boxed{H_1(B_1 + B_2)H_1^\dagger + P_{N_1}}}{\boxed{H_1(B_2)H_1^\dagger + P_{N_1}}}\right)$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_2'' \end{bmatrix}$$

Interference + noise power

R_2 : Rate of message 2

$$B = B_1 + \boxed{B_2}$$



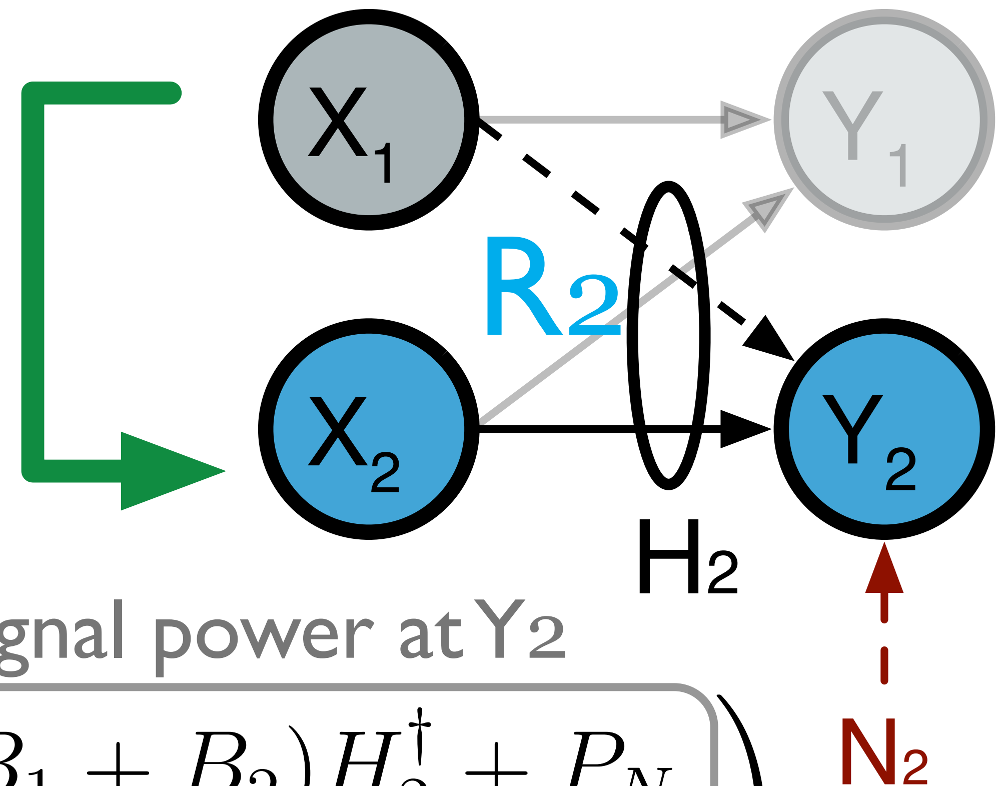
Signal power at Y2

$$R_2 \leq \frac{1}{2}\log_2\left(\frac{\boxed{H_2(B_1+B_2)H_2^\dagger + P_{N_2}}}{\boxed{H_2(B_1)H_2^\dagger + P_{N_2}}}\right)$$

Interference + noise power

R_2 : Rate of message 2

$$B = B_1 + \boxed{B_2}$$



$$R_2 \leq \frac{1}{2}\log_2\left(\frac{\overbrace{H_2(B_1 + B_2)H_2^\dagger + P_{N_2}}^{\text{Signal power at } Y_2}}{\underbrace{H_2(B_1)H_2^\dagger + P_{N_2}}_{\text{Interference + noise power}}}\right)$$

$$B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}$$

Since Tx 2 knows message 1,
we can do better!

Since Tx 2 knows message 1,
we can do better!

Dirty paper coding

Dirty-paper coding

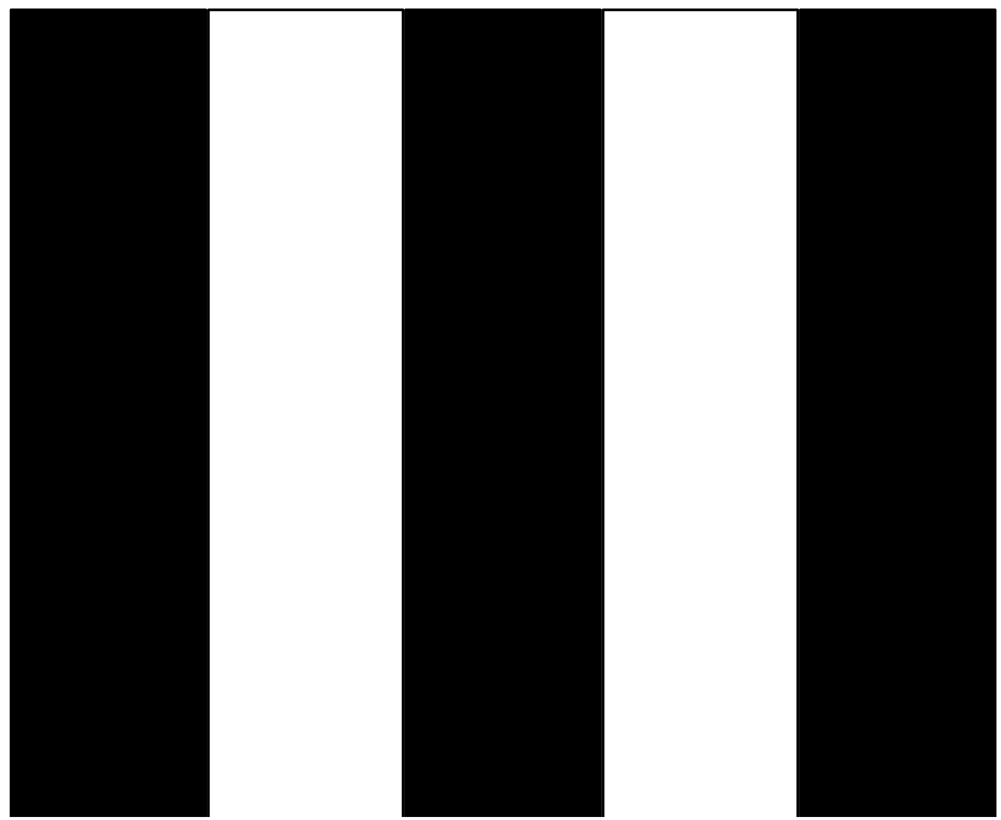
[Costa, 1983]

[Gel'fand, Pinsker, 1980]

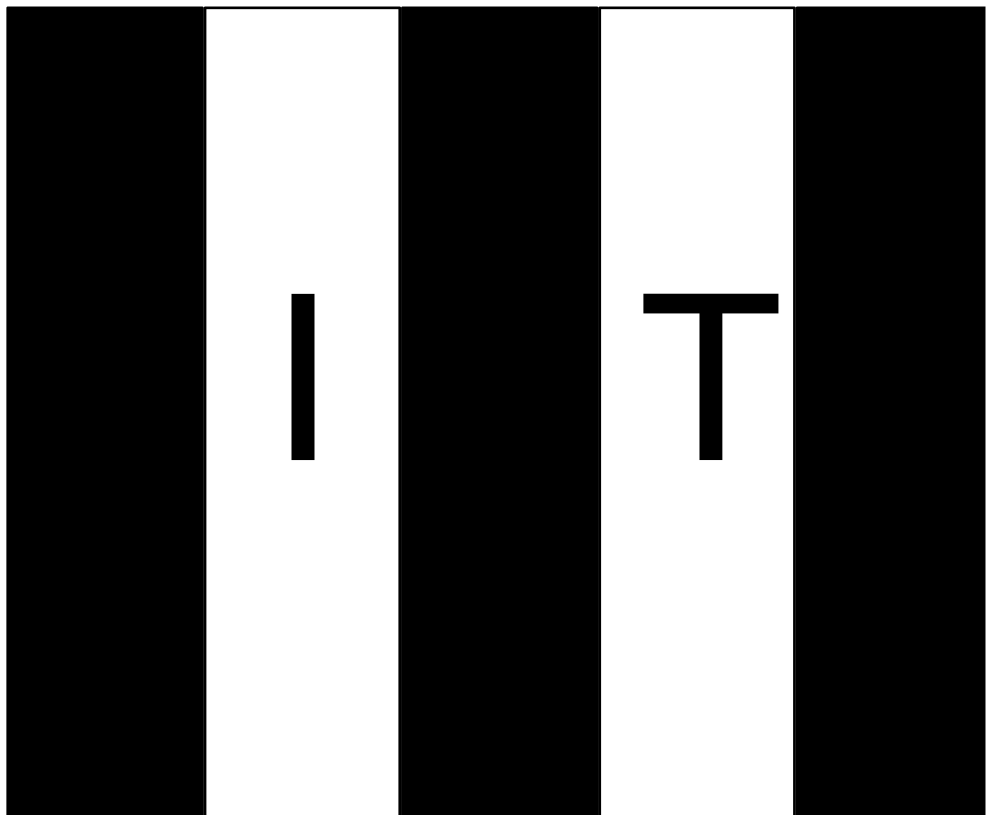
Dirty-paper coding

D I R T Y

Dirty-paper coding



Dirty-paper coding

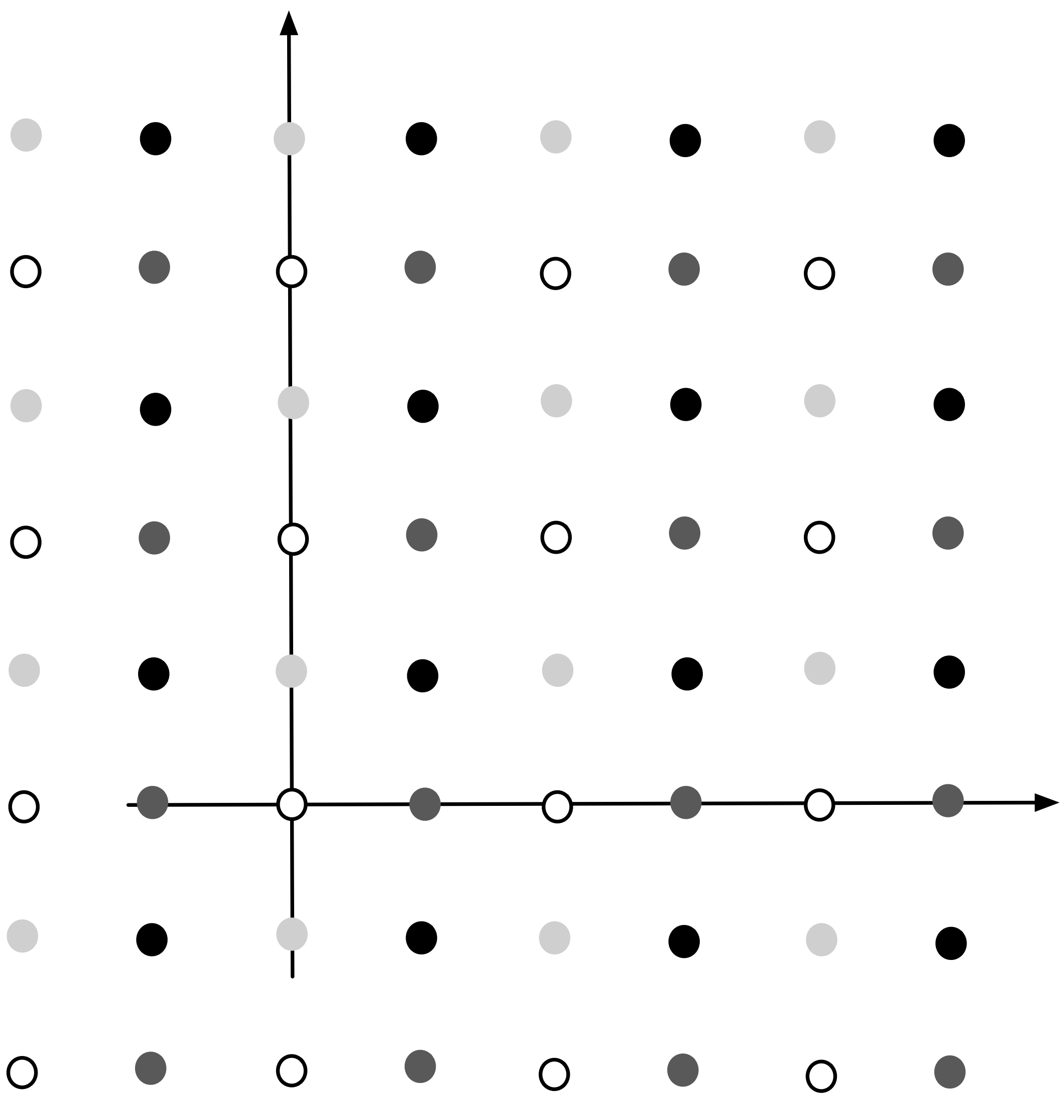


write in black ink?

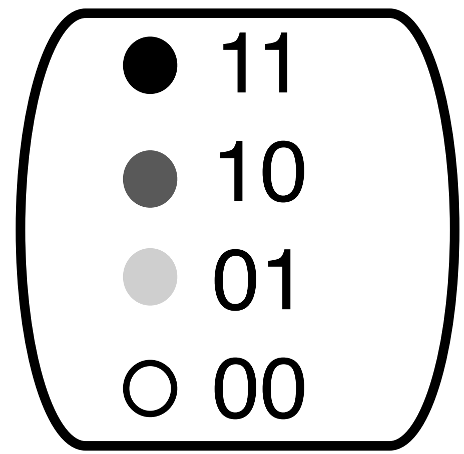
Dirty-paper coding

adjust your ink ✓

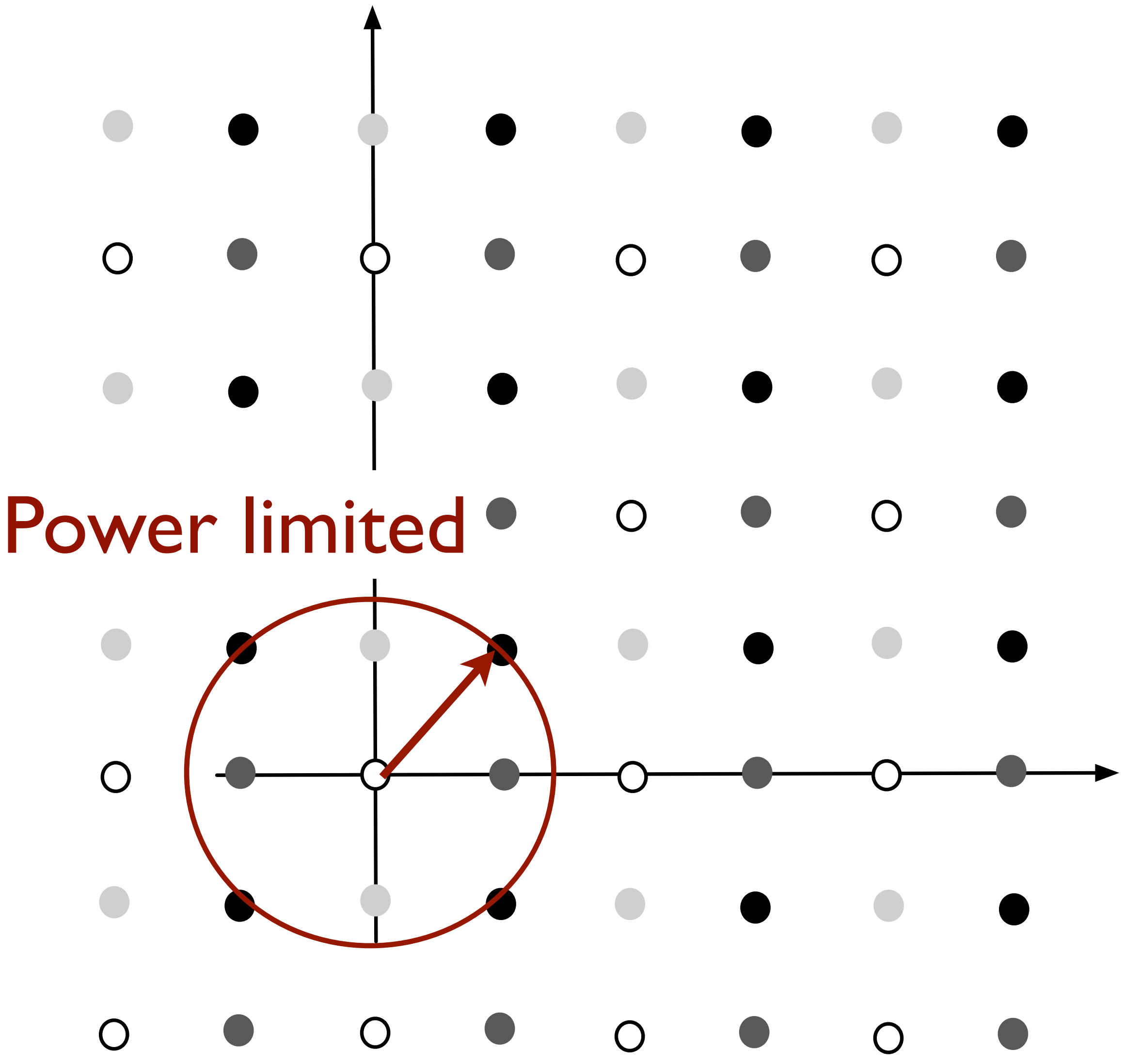
Example of dirty-paper coding



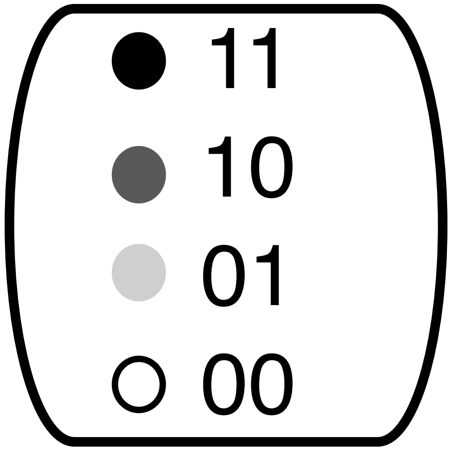
Send 2 bits:



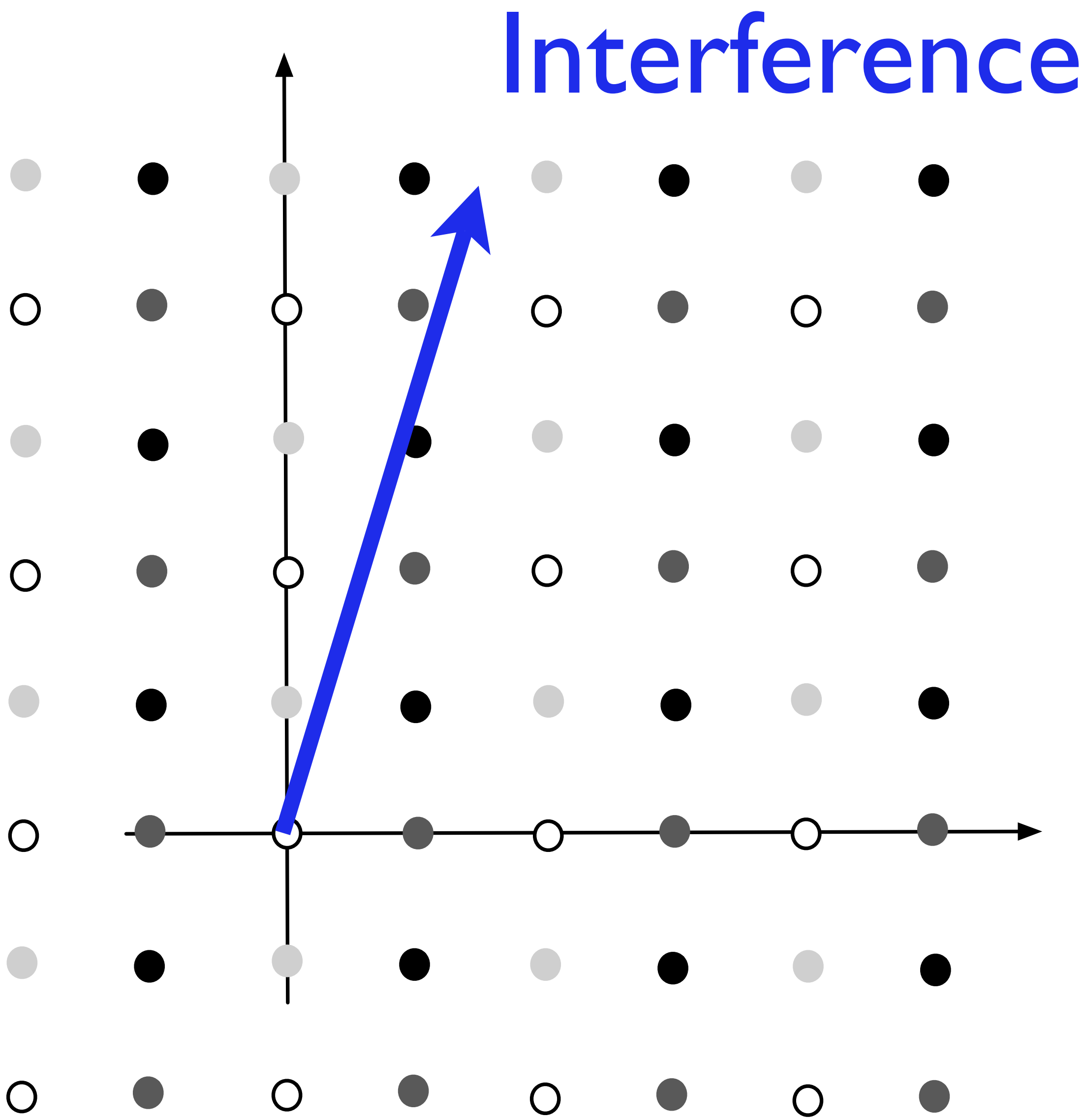
Example of dirty-paper coding



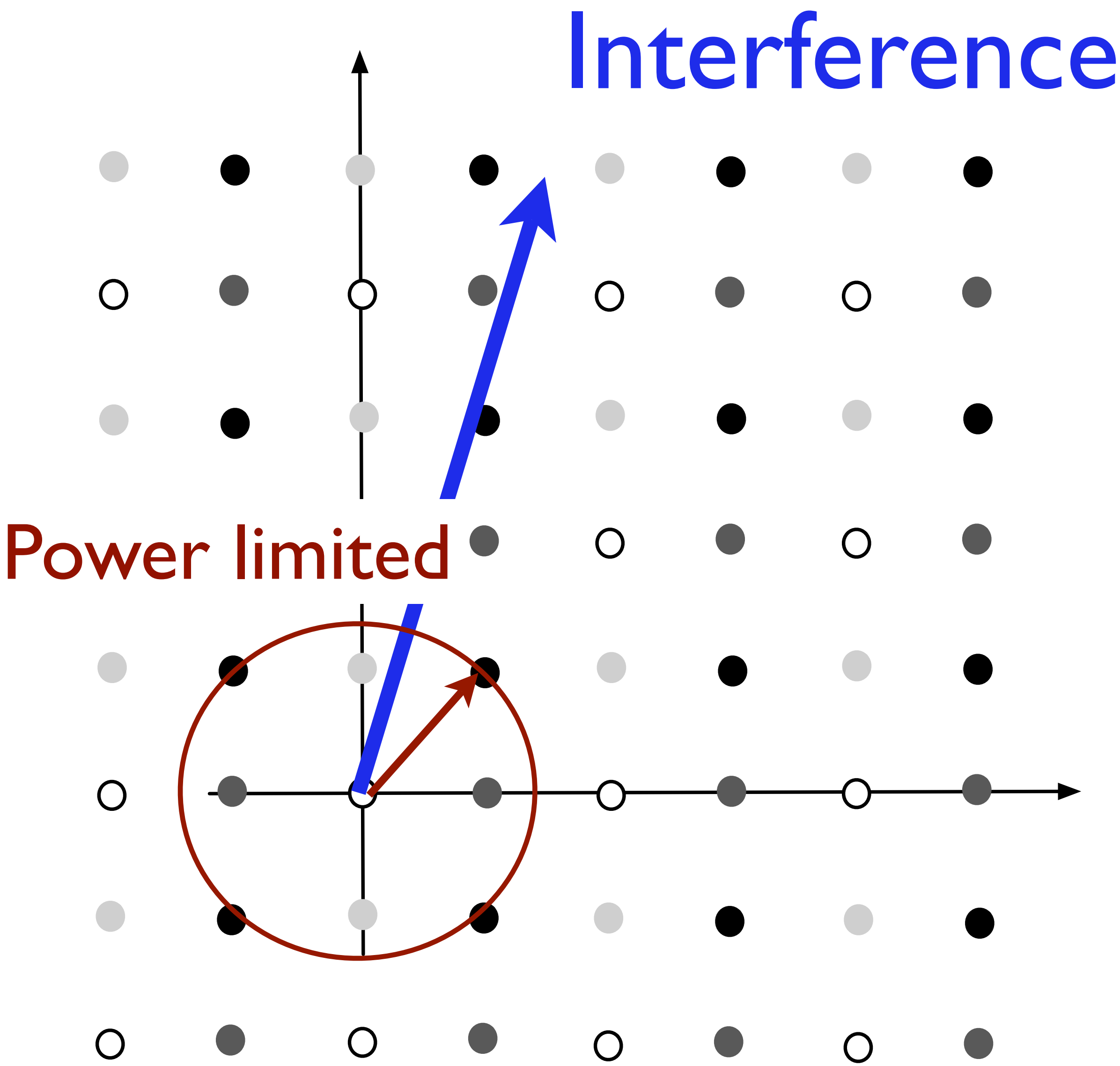
Send 2 bits:



Example of dirty-paper coding

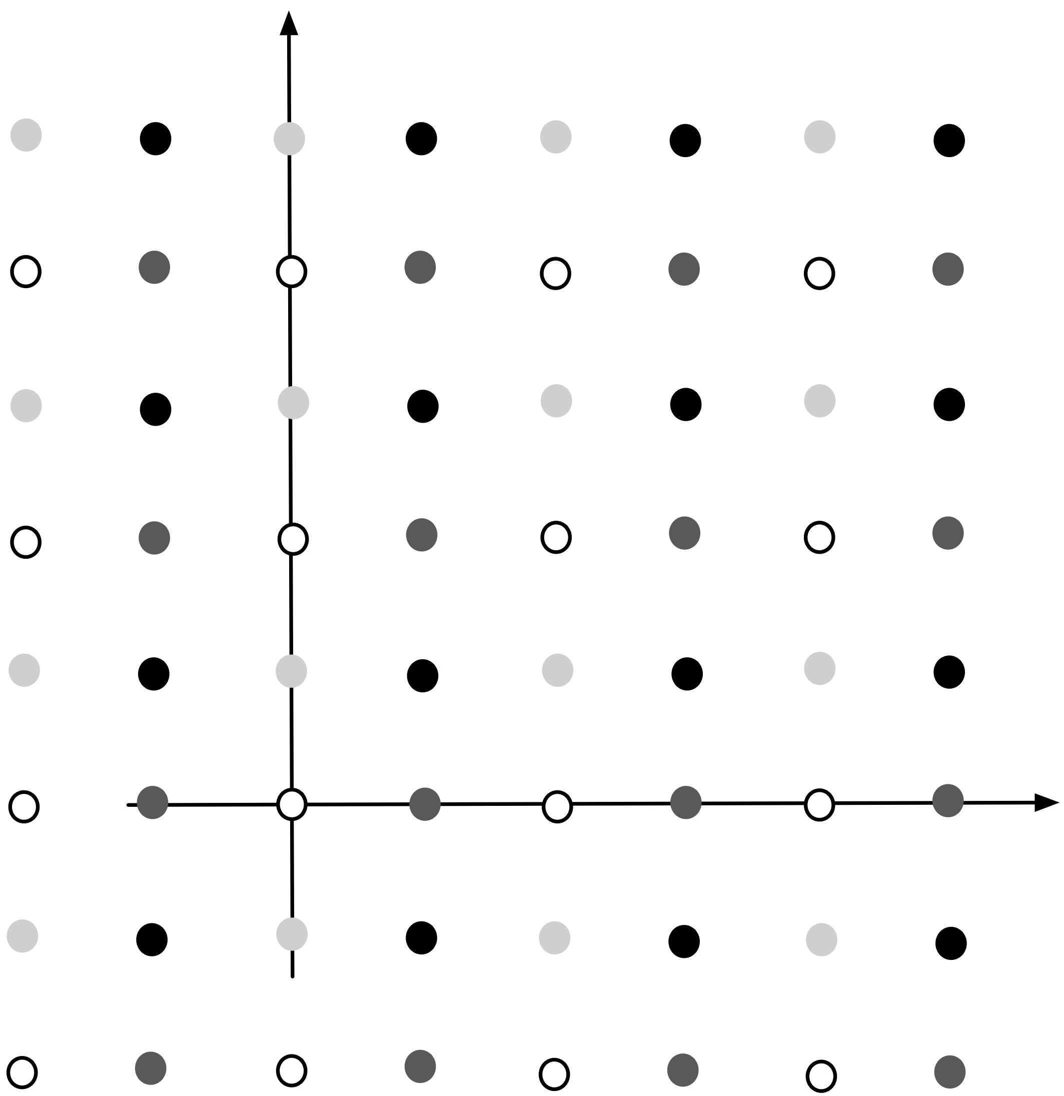


Example of dirty-paper coding

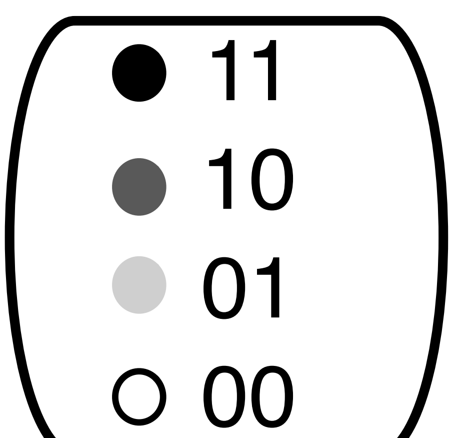


Do NOT have enough power to subtract off the interference!

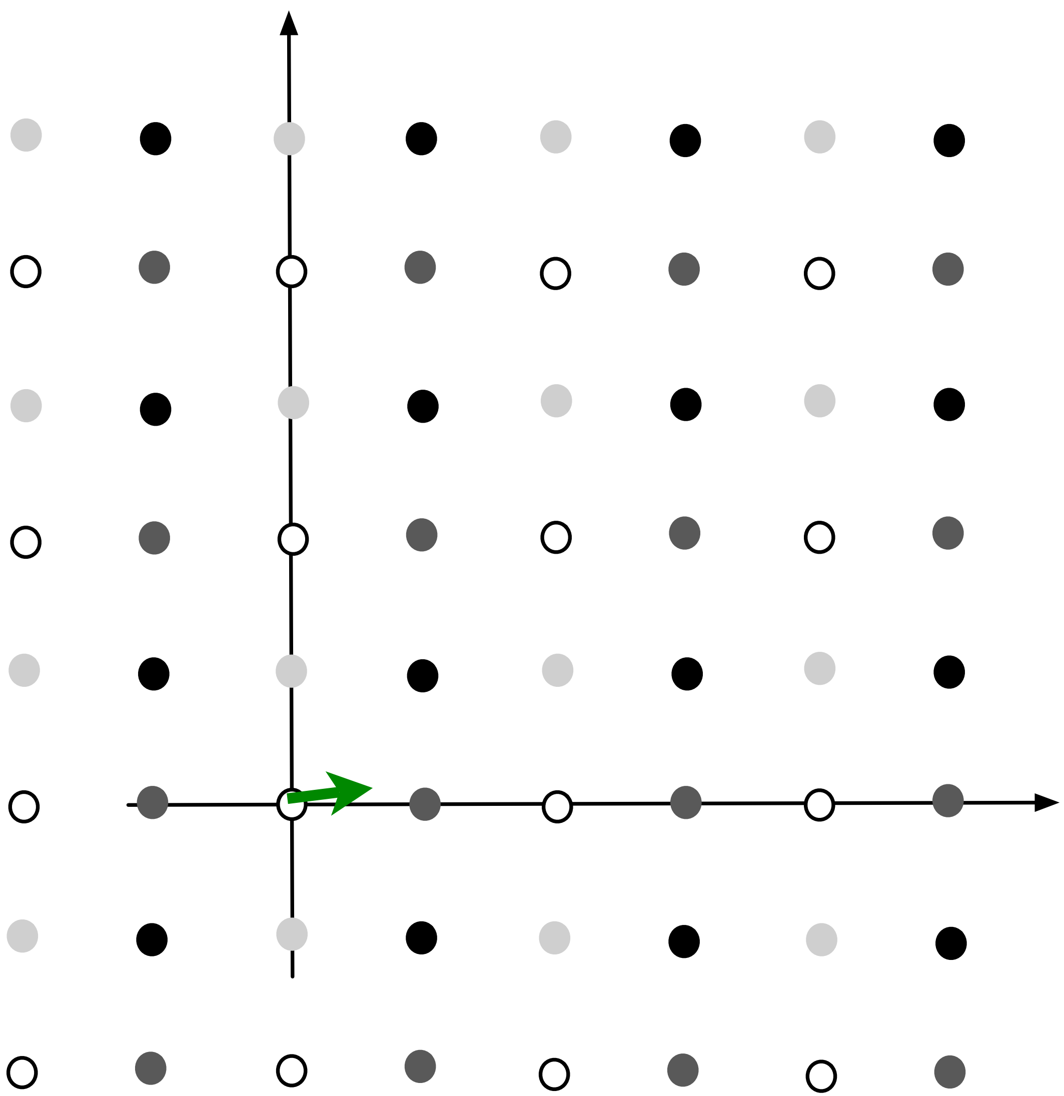
Example of dirty-paper coding



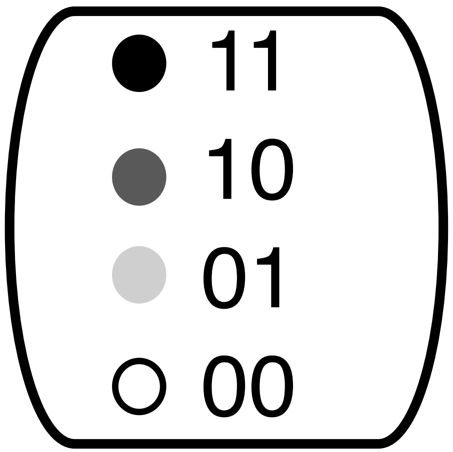
How to send 01?



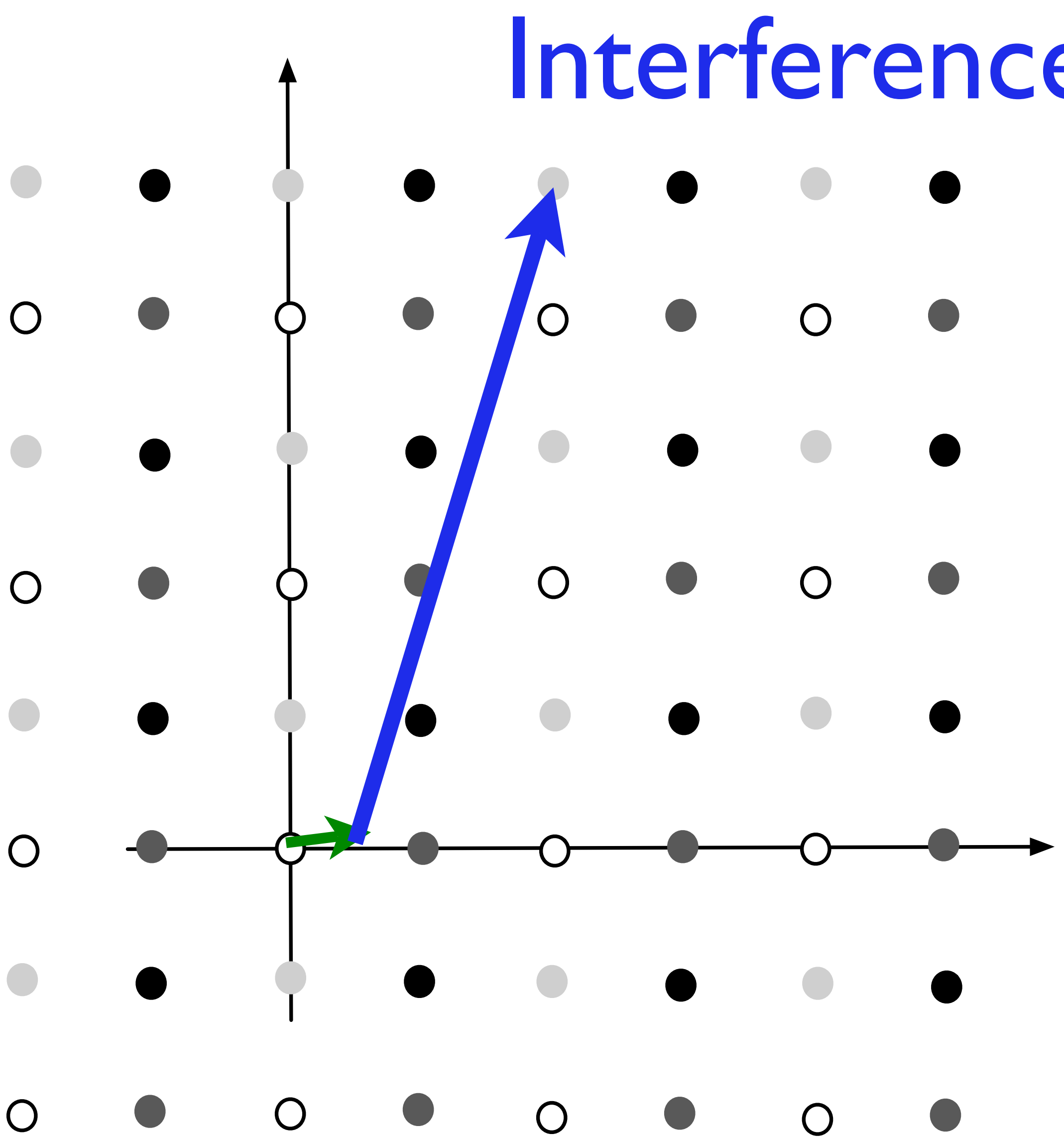
Example of dirty-paper coding



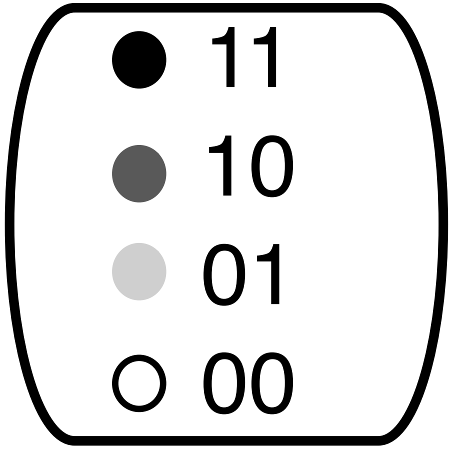
How to send 01?



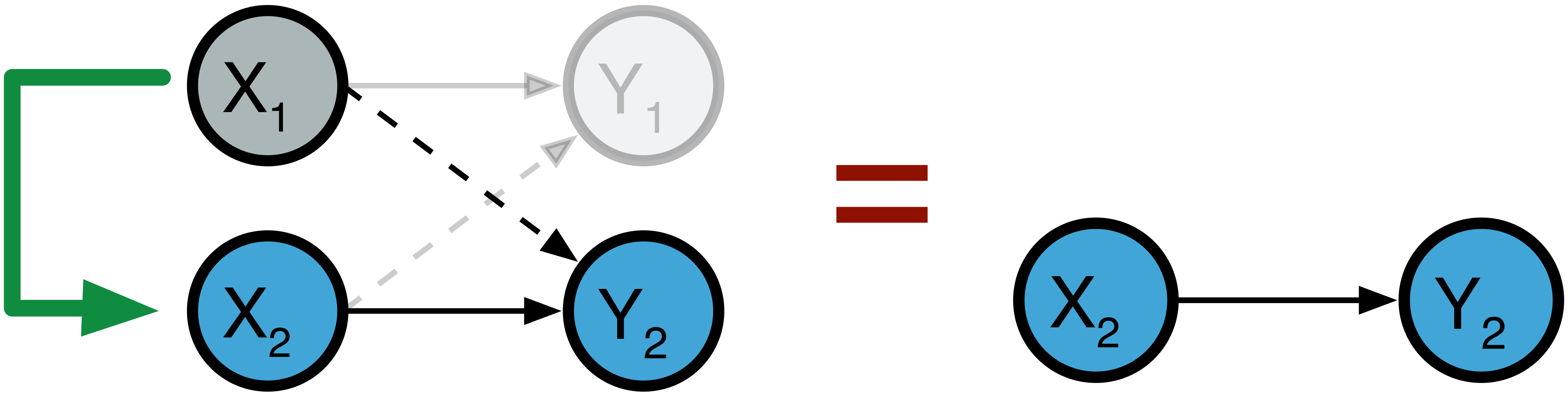
Example of dirty-paper coding



How to send 01?



Dirty-paper coding



NO power penalty!
NOT subtracting off interference!

Rate of message 2:
WITHOUT and **WITH** dirty-paper coding

WITHOUT

Signal power at Y2

$$R_2 \le \frac{1}{2}\log_2\left(\frac{\boxed{H_2(B_1+B_2)H_2^\dagger + P_{N_2}}}{\boxed{H_2(B_1)H_2^\dagger + P_{N_2}}}\right)$$

Interference + noise power

Rate of message 2:
WITHOUT and **WITH** dirty-paper coding

WITHOUT

$$R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_1 + B_2)H_2^\dagger + P_{N_2}}{H_2(B_1)H_2^\dagger + P_{N_2}}\right)$$

WITH

$$R_2 \leq \frac{1}{2}\log_2\left(\frac{\overbrace{\boxed{H_2(B_2)H_2^\dagger + P_{N_2}}}^{\text{Signal at Y2}}}{\underbrace{\boxed{P_{N_2}}}_{\text{No interference + noise}}}\right)$$

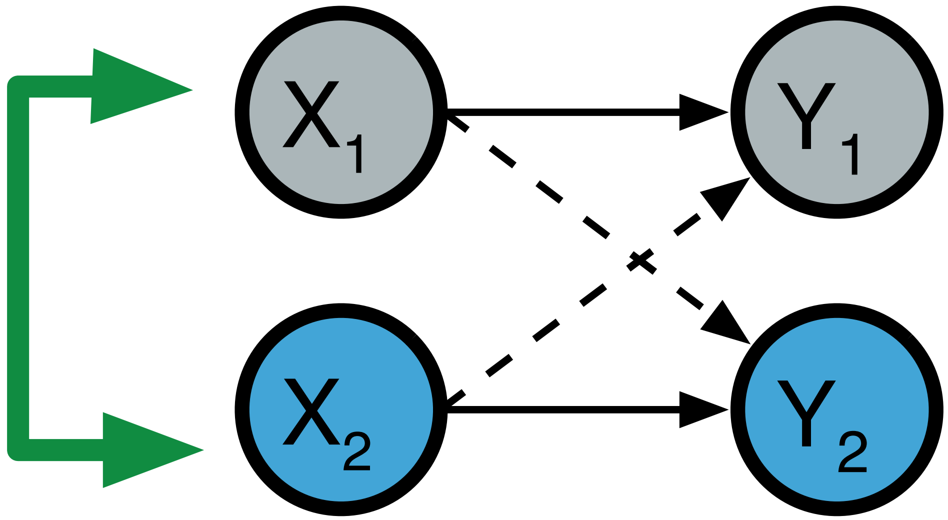
Gaussian cognitive channel

Cognitive region $=$ Convex hull of

$$\left\{ \begin{array}{c} (R_1, R_2): \\ \\ R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \\ \\ B_1, B_2 \succeq 0, \boxed{B_1 = \begin{bmatrix} P_1' & z \\ z & P_2' \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 & 0 \\ 0 & P_2'' \end{bmatrix}}, \;\; B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \;\; z^2 \leq P_1 P_2 \end{array} \right\}$$

Matrices with zeros

Gaussian MIMO broadcast channel region



Gaussian MIMO broadcast channel region

Cognitive permutation

MIMO BC region = Convex hull of

$$\left\{ \begin{array}{c} (R_1, R_2): \\ \boxed{\begin{array}{c} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \end{array}} \bigcup \begin{array}{c} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger+Q_1}{Q_1}\right) = R_1(\pi_{21}) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^\dagger+Q_2}{H_2(B_1)H_2^\dagger+Q_2}\right) = R_2(\pi_{21}) \end{array} \\ B_1, B_2 \succeq 0, \quad B_1 = \begin{bmatrix} P_1' & z' \\ z' & P_2' \end{bmatrix}, \; B_2 = \begin{bmatrix} P_1'' & z'' \\ z'' & P_2'' \end{bmatrix}, \; B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \; z^2 \le P_1 P_2 \end{array} \right\}$$

Gaussian MIMO broadcast channel region

Reverse permutation

MIMO BC region = Convex hull of

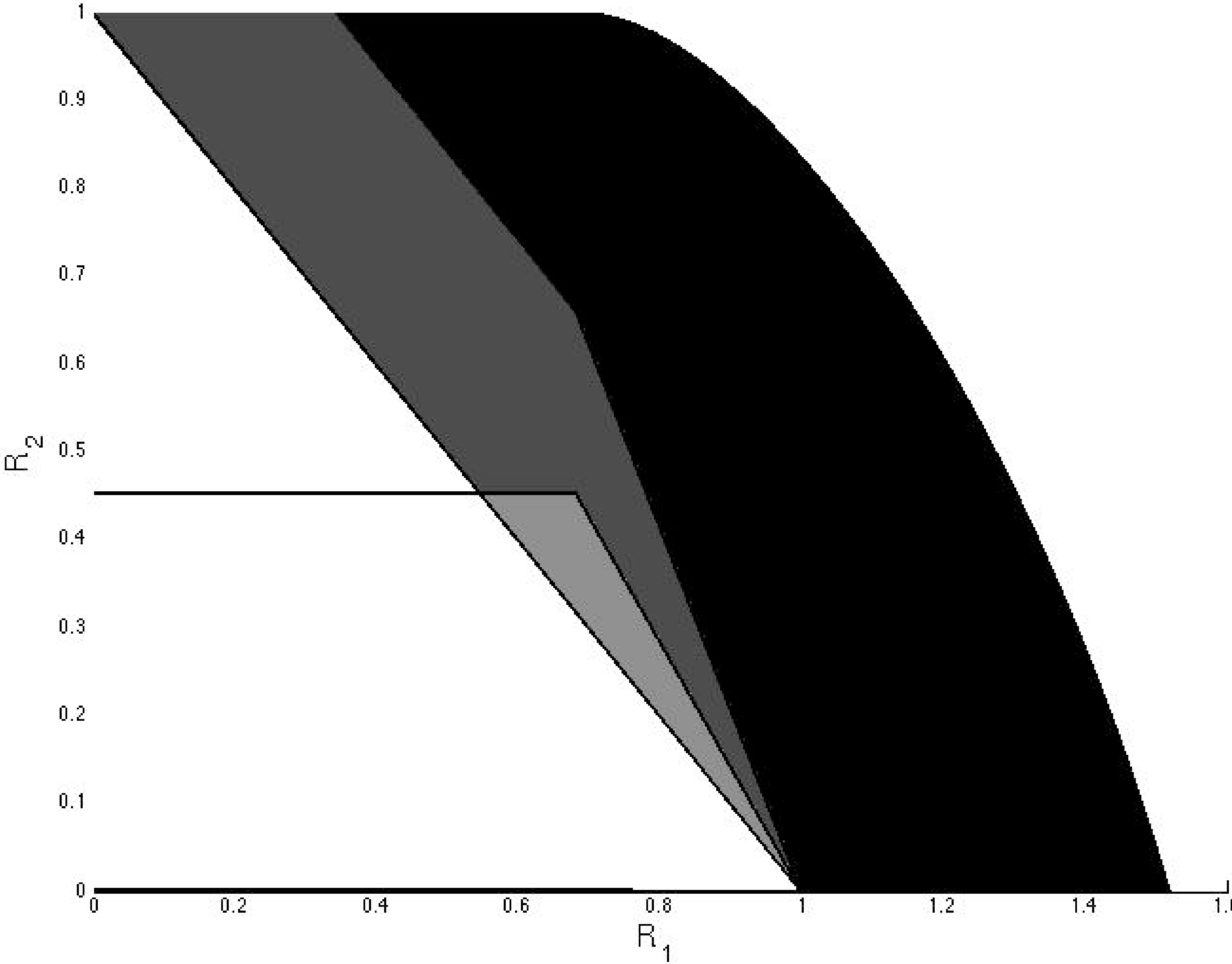
$$\left\{ \begin{array}{c} (R_1, R_2) : \\ \begin{array}{c} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \end{array} \bigcup \boxed{\begin{array}{c} R_1 \le \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger+Q_1}{Q_1}\right) = R_1(\pi_{21}) \\ R_2 \le \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^\dagger+Q_2}{H_2(B_1)H_2^\dagger+Q_2}\right) = R_2(\pi_{21}) \end{array}} \\ B_1, B_2 \succeq 0,\ \ B_1 = \begin{bmatrix} P_1' & z' \\ z' & P_2' \end{bmatrix},\ B_2 = \begin{bmatrix} P_1'' & z'' \\ z'' & P_2'' \end{bmatrix},\ \ B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix},\ \ z^2 \le P_1 P_2 \end{array} \right\}$$

Gaussian MIMO broadcast channel region

MIMO BC region $=$ Convex hull of

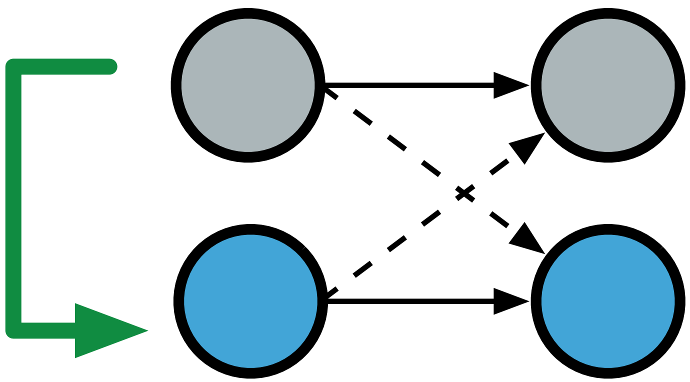
$$\left\{ \begin{array}{c} (R_1, R_2) : \\ \begin{array}{c} R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1+B_2)H_1^\dagger+Q_1}{H_1(B_2)H_1^\dagger+Q_1}\right) = R_1(\pi_{12}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_2)H_2^\dagger+Q_2}{Q_2}\right) = R_2(\pi_{12}) \end{array} \bigcup \begin{array}{c} R_1 \leq \frac{1}{2}\log_2\left(\frac{H_1(B_1)H_1^\dagger+Q_1}{Q_1}\right) = R_1(\pi_{21}) \\ R_2 \leq \frac{1}{2}\log_2\left(\frac{H_2(B_1+B_2)H_2^\dagger+Q_2}{H_2(B_1)H_2^\dagger+Q_2}\right) = R_2(\pi_{21}) \end{array} \\ B_1, B_2 \succeq 0, \boxed{B_1 = \begin{bmatrix} P_1' & z' \\ z' & P_2' \end{bmatrix}, \; B_2 = \begin{bmatrix} P_1'' & z'' \\ z'' & P_2'' \end{bmatrix}}, \; B_1 + B_2 \preceq \begin{bmatrix} P_1 & z \\ z & P_2 \end{bmatrix}, \; z^2 \leq P_1 P_2 \end{array} \right\}$$

Full matrices



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
R_2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
R_1

Strongest result



The discrete memoryless cognitive radio channel

Largest to date known general region

Cognitive channel:

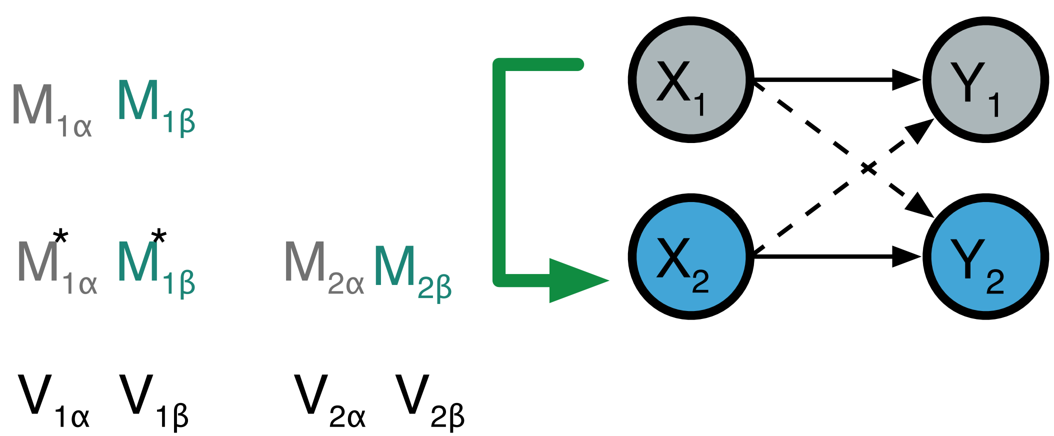
Achievable rate region

$(R_{1\alpha}, R_{1\beta}, R_{2\alpha}, R_{2\beta})$ **such that, for γ=α, β:**

$$
\begin{aligned}
R_{1\gamma} &= L_{1\gamma} \\
R_{2\gamma} &\leq L_{2\gamma} - I(V_{2\gamma}; V_{1\alpha}, V_{1\beta})
\end{aligned}
$$

$$
\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1, \mathbf{V}_{\overline{\mathbf{T}}}; \mathbf{V}_{\mathbf{T}}) + f(\mathbf{V}_{\mathbf{T}})
$$

$$
\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) \leq I(Y_2, \mathbf{V}_{\overline{\mathbf{T}}}; \mathbf{V}_{\mathbf{T}}) + f(\mathbf{V}_{\mathbf{T}})
$$



Cognitive channel:

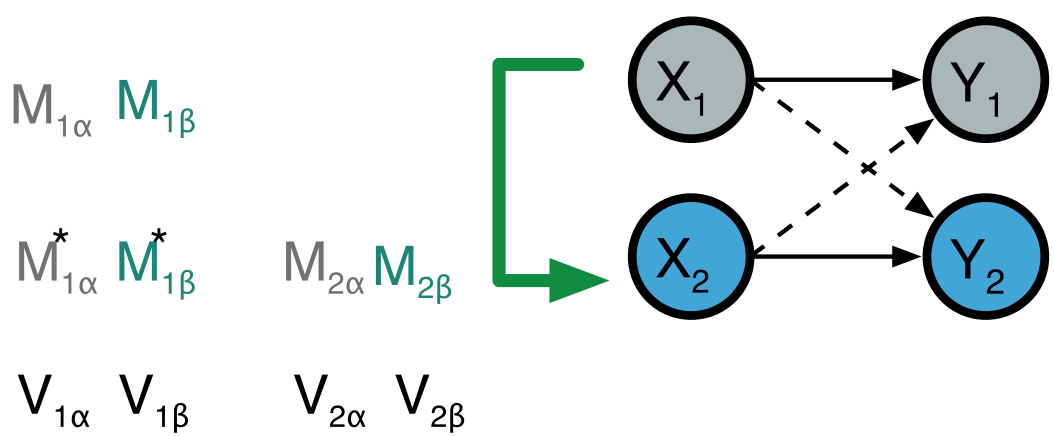
Achievable rate region

$(R_{1\alpha}, R_{1\beta}, R_{2\alpha}, R_{2\beta})$ **such that, for γ=α, β:**

$$\boxed{\begin{array}{rcl} R_{1\gamma} & = & L_{1\gamma} \\ R_{2\gamma} & \leq & L_{2\gamma} - I(V_{2\gamma}; V_{1\alpha}, V_{1\beta}) \end{array}}$$

interference mitigation

$$\begin{array}{rcl} \bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) & \leq & I(Y_1, \mathbf{V}_{\overline{\mathbf{T}}}; \mathbf{V}_{\mathbf{T}}) + f(\mathbf{V}_{\mathbf{T}}) \\ \bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) & \leq & I(Y_2, \mathbf{V}_{\overline{\mathbf{T}}}; \mathbf{V}_{\mathbf{T}}) + f(\mathbf{V}_{\mathbf{T}}) \end{array}$$



Cognitive channel:

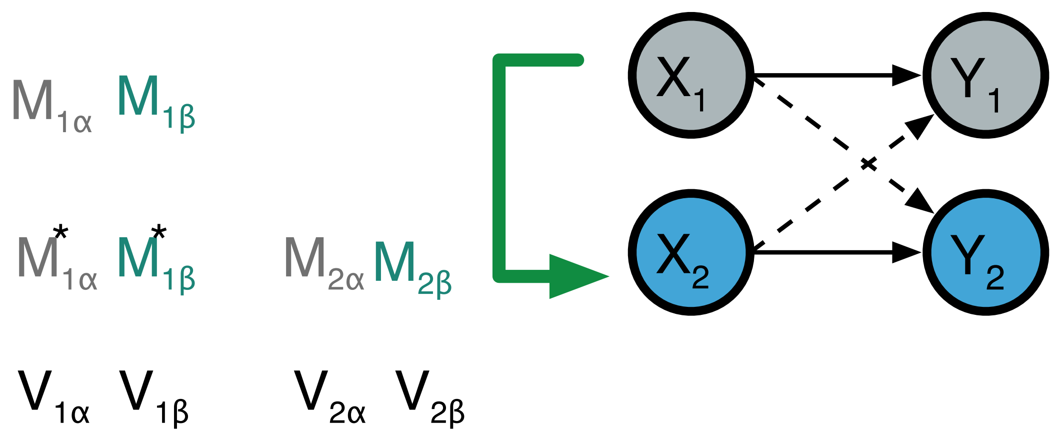
Achievable rate region

$(R_{1\alpha}, R_{1\beta}, R_{2\alpha}, R_{2\beta})$ such that, for γ=α, β:

$$
\begin{aligned}
R_{1\gamma} &= L_{1\gamma} \\
R_{2\gamma} &\leq L_{2\gamma} - I(V_{2\gamma}; V_{1\alpha}, V_{1\beta})
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) &\leq I(Y_1, \mathbf{V}_{\overline{\mathbf{T}}}; \mathbf{V}_{\mathbf{T}}) + f(\mathbf{V}_{\mathbf{T}}) \\
\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) &\leq I(Y_2, \mathbf{V}_{\overline{\mathbf{T}}}; \mathbf{V}_{\mathbf{T}}) + f(\mathbf{V}_{\mathbf{T}})
\end{aligned}
}
$$

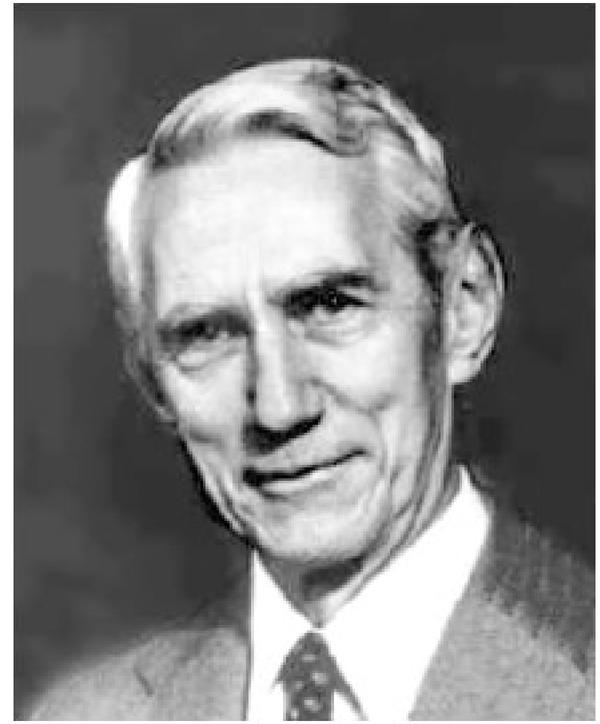
overlapping MAC channels



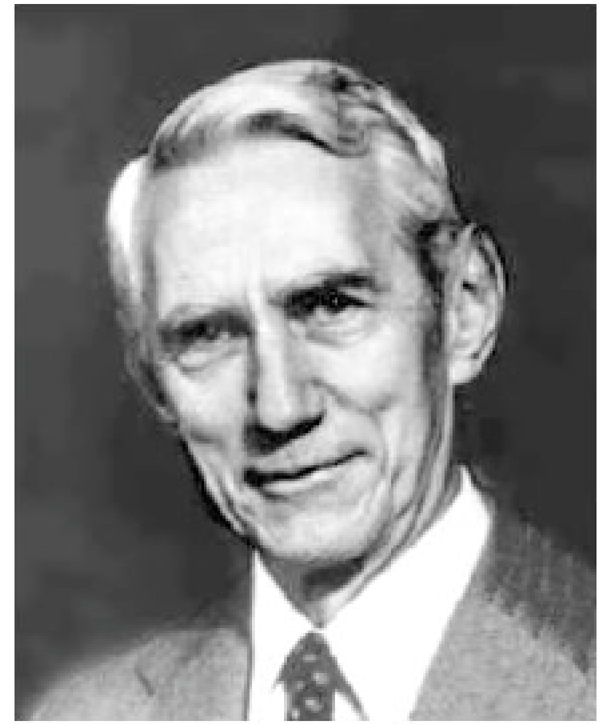
Impact?

meets

meets



meets

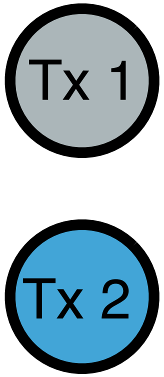


Quantitative, fundamental analysis in infant field

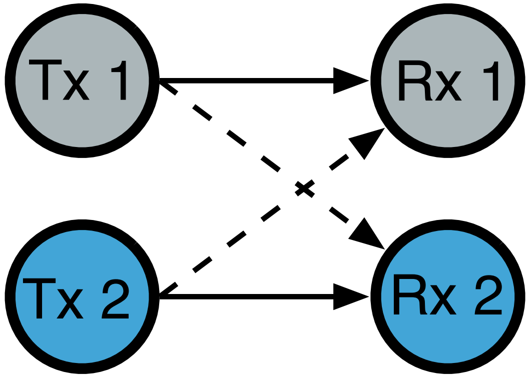
The Multiplexing Gain of MIMO X-Channels with Partial Transmit Side-Information

Natasha Devroye, Masoud Sharif

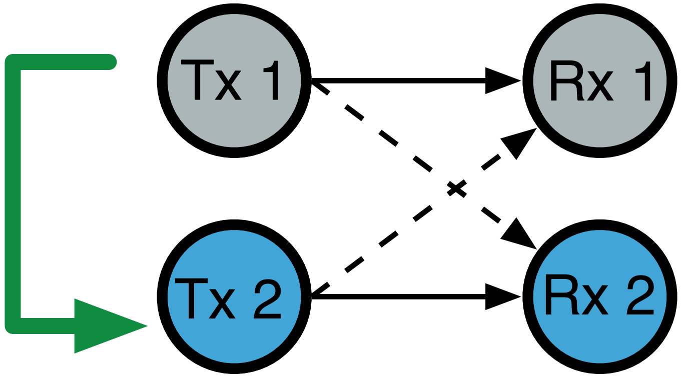
This talk



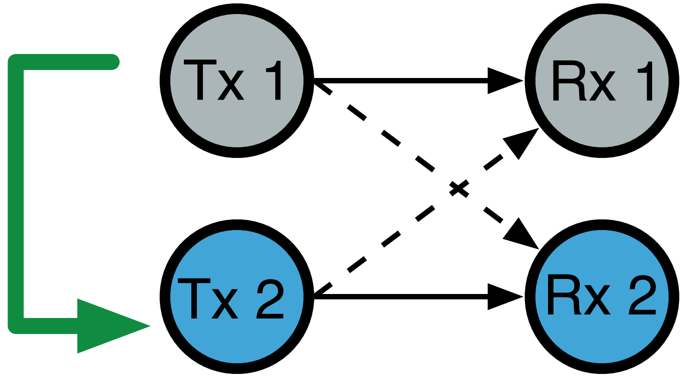
This talk



This talk

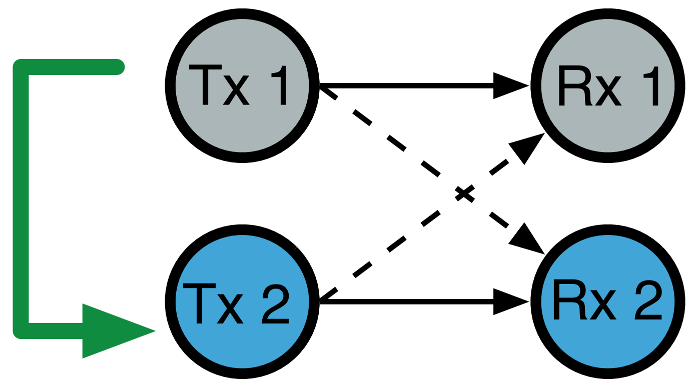


This talk



Side-information increases the achievable rates

This talk



Side-information increases the achievable rates

What about the *multiplexing gains (degrees of freedom)?*

Degrees of freedom (DOF)

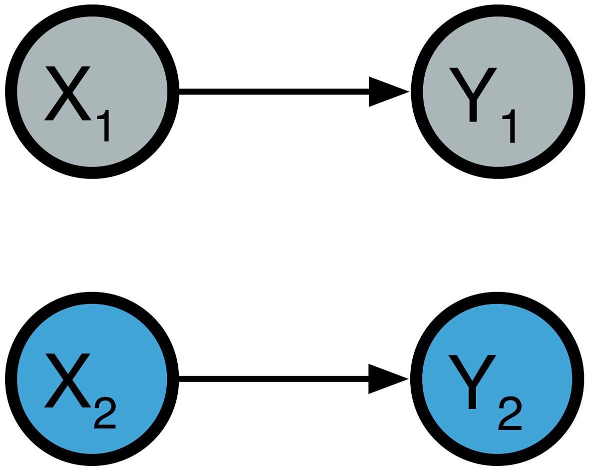
Capacity of a single input, single output Gaussian noise channel:

$$C = \frac{1}{2}\log_2\left(1 + \mathrm{SNR}\right)$$

Scales like log(SNR) as SNR $\rightarrow \infty$

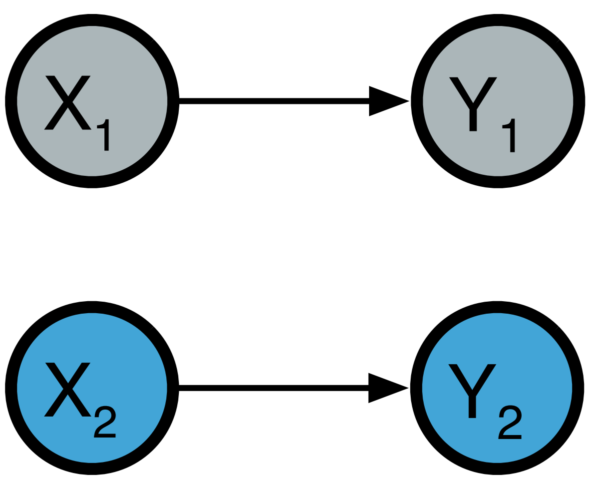
Degrees of freedom

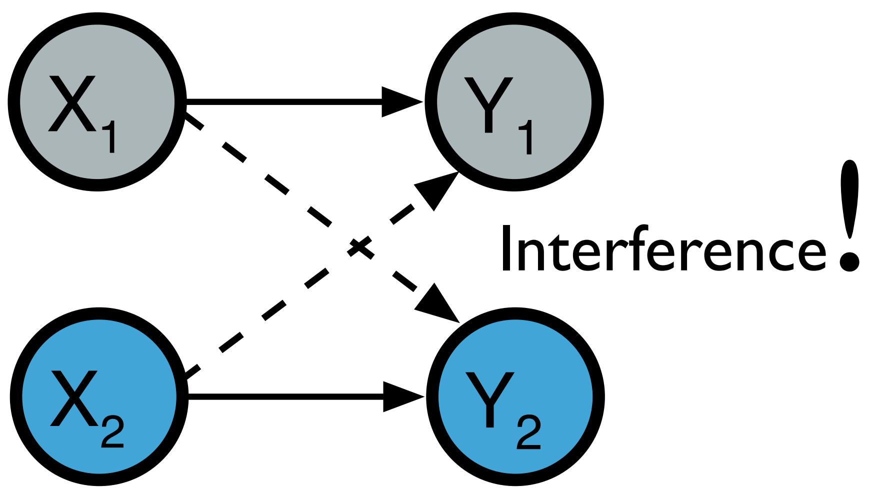
- Multiple-input multiple-output (MIMO) channels may have many information streams



Degrees of freedom

- Multiple-input multiple-output (MIMO) channels may have many information streams
- As SNR $\rightarrow \infty$ interference, rather than noise becomes the limiting factor





Degrees of freedom

Degrees of freedom (DOF) measures the number of point-to-point Gaussian channels contained in a MIMO channel as SNR $\rightarrow \infty$

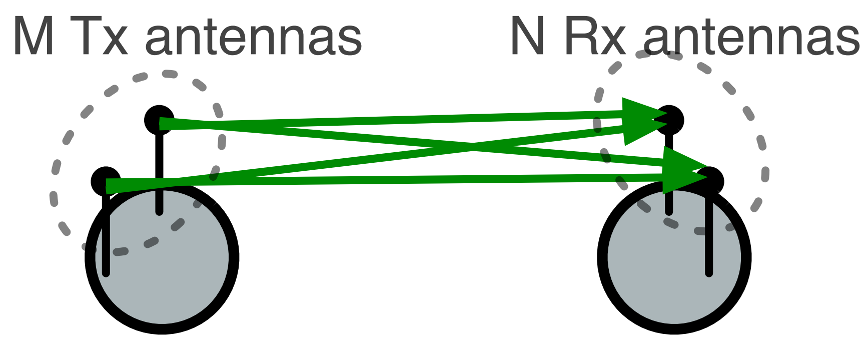
MIMO channel

$$\mathrm{DOF} = \lim_{\mathrm{SNR}\rightarrow\infty} \frac{\text{Sum capacity}(\mathrm{SNR})}{\log(\mathrm{SNR})}$$

point-to-point channel

Degrees of freedom

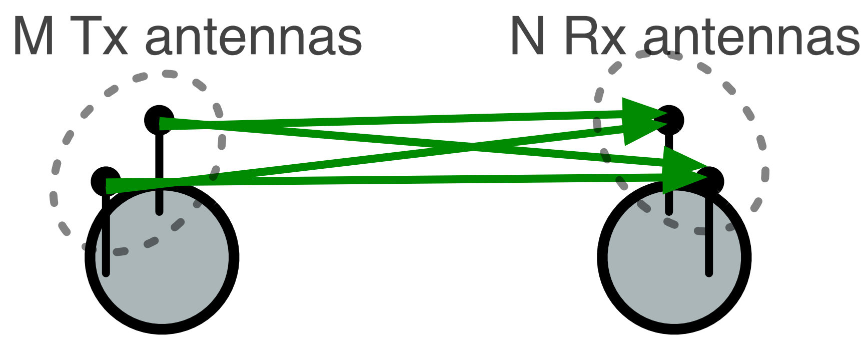
MIMO



DOF = min(M,N)=2

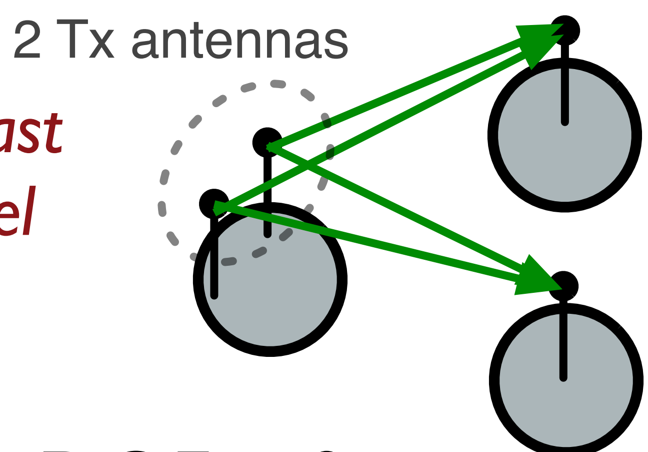
Degrees of freedom

MIMO



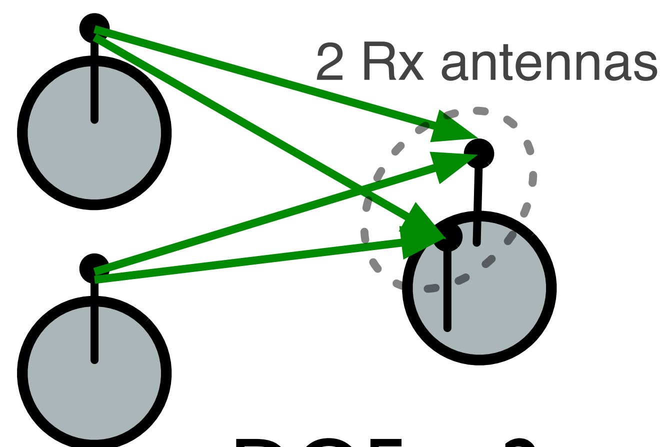
DOF = min(M,N)=2

Broadcast channel



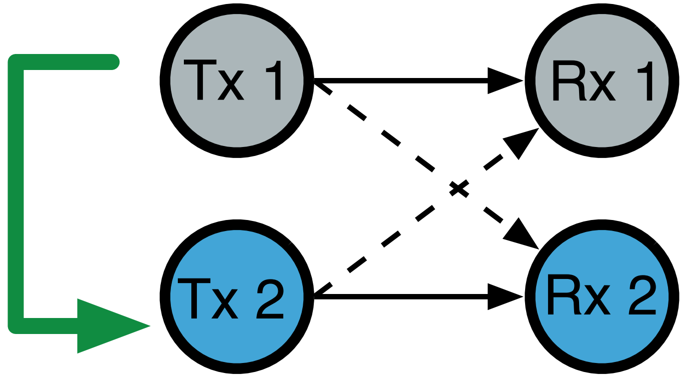
DOF = 2

Multiple-access channel



DOF = 2

This talk

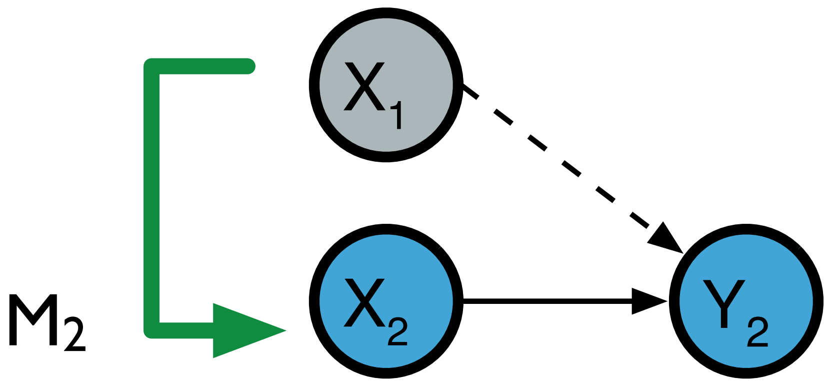


How does *asymmetric* side-information

affect the *degrees of freedom?*

Side-information:

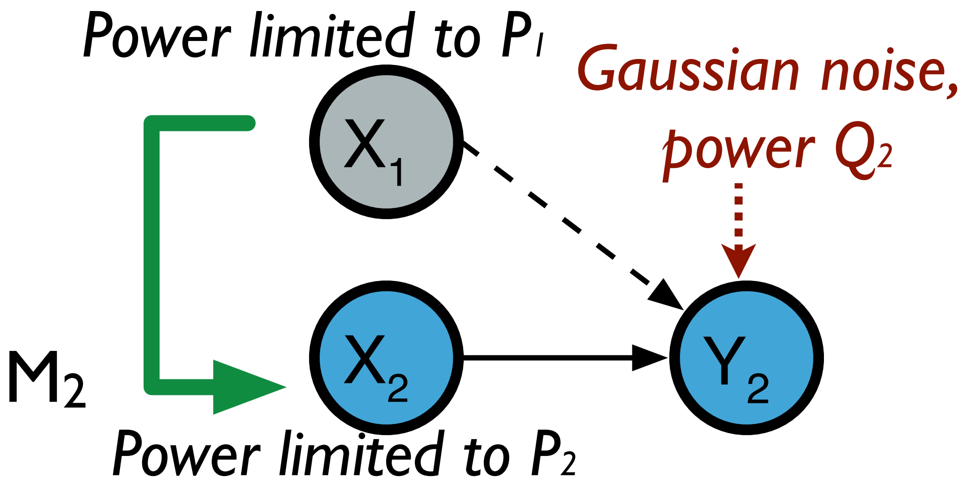
Gaussian noise channels



$$C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)$$

[Gel'fand, Pinsker 1980]

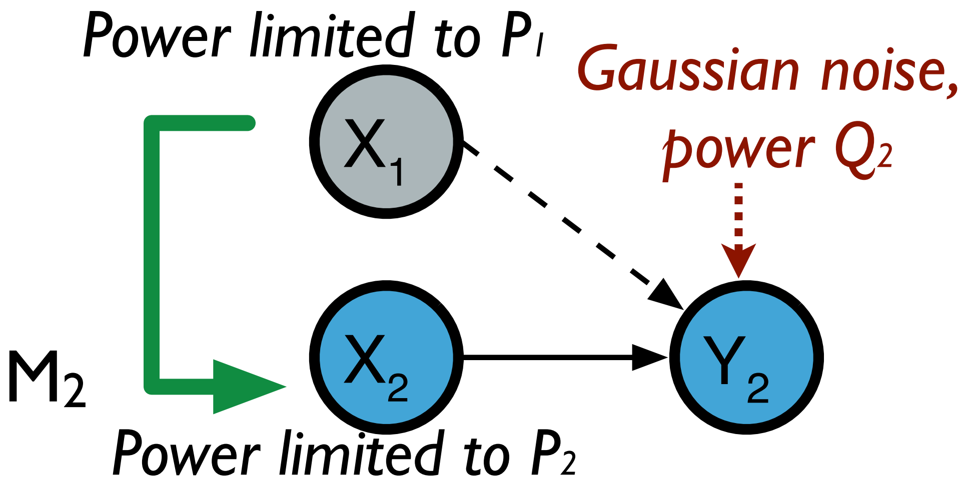
Side-information:
Gaussian noise channels



$$C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)$$

[Gel'fand, Pinsker 1980]

Side-information:
Gaussian noise channels



$$C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)$$

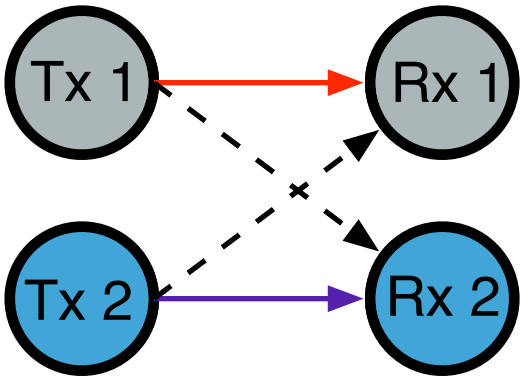
[Gel'fand, Pinsker 1980]

[Costa's "Writing on Dirty Paper" 1983]

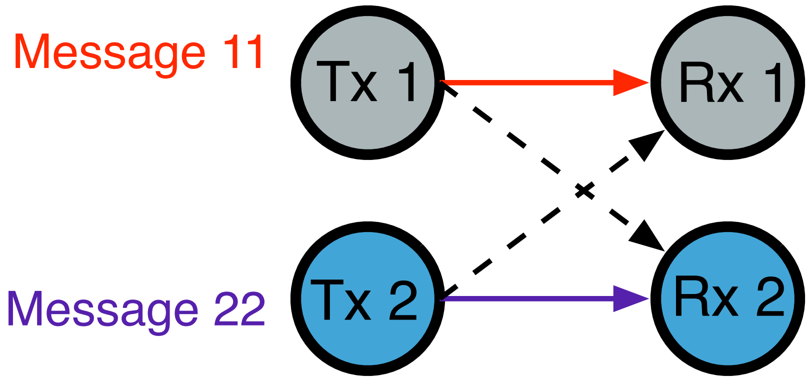
- Assume $M_2 = X_2 + \gamma X_1$
- Optimize ϒ to obtain $\gamma = \dfrac{P_2}{P_2 + Q_2}$
- Capacity is that of interference-free channel! $C = \dfrac{1}{2}\log_2\left(1 + \dfrac{P_2}{Q_2}\right)$

Degrees of freedom in the cognitive channel

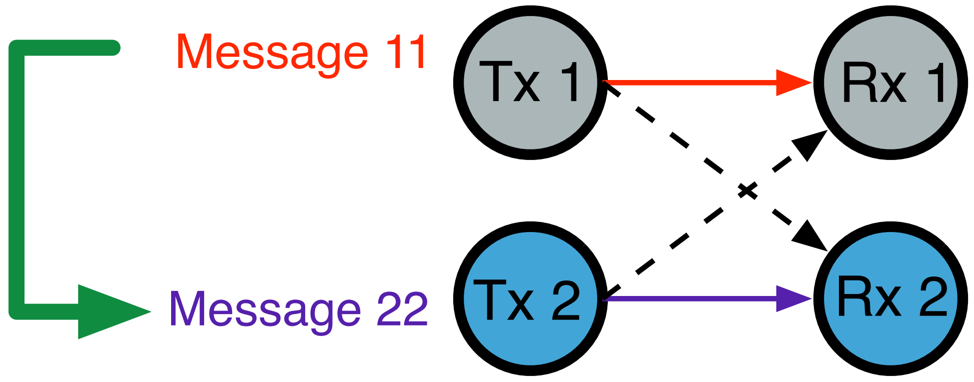
The cognitive channel



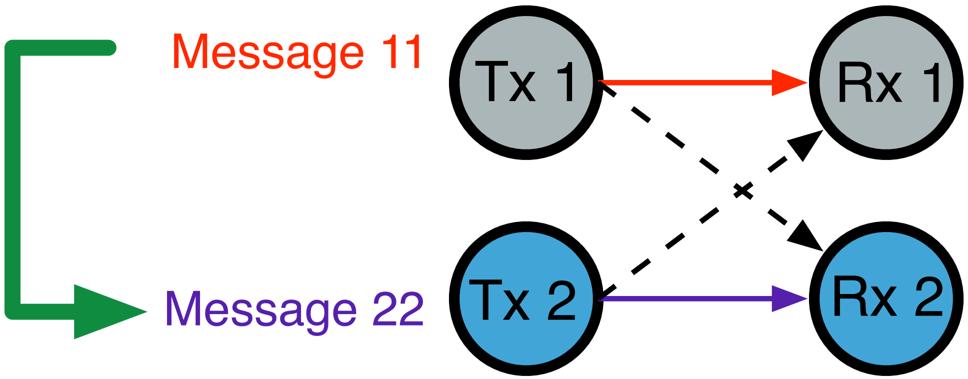
The cognitive channel



The cognitive channel

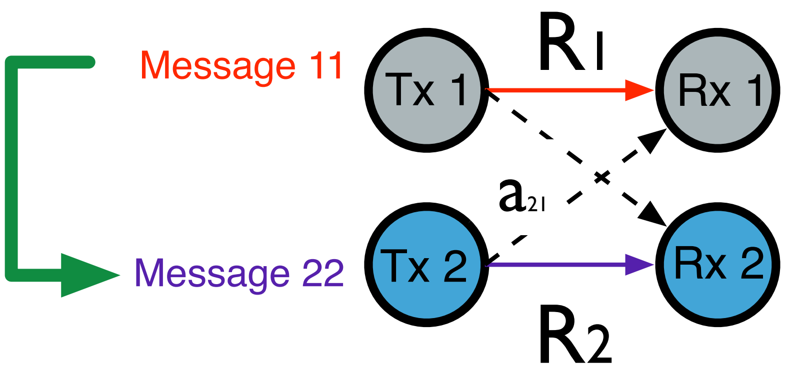


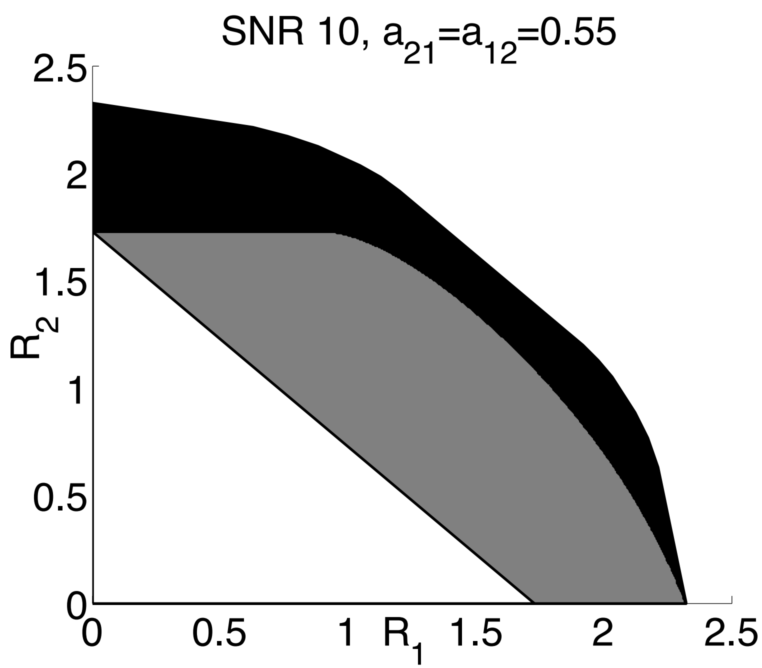
The cognitive channel



Interference channel with degraded message sets
Interference channel with partially-cognitive transmitter
Interference channel with unidirectional cooperation

The cognitive channel

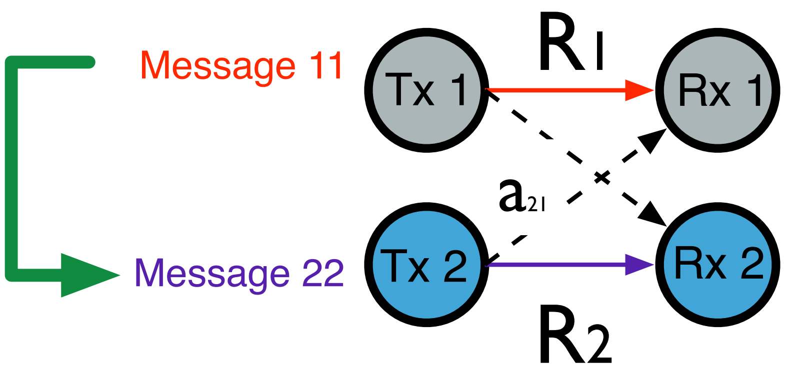


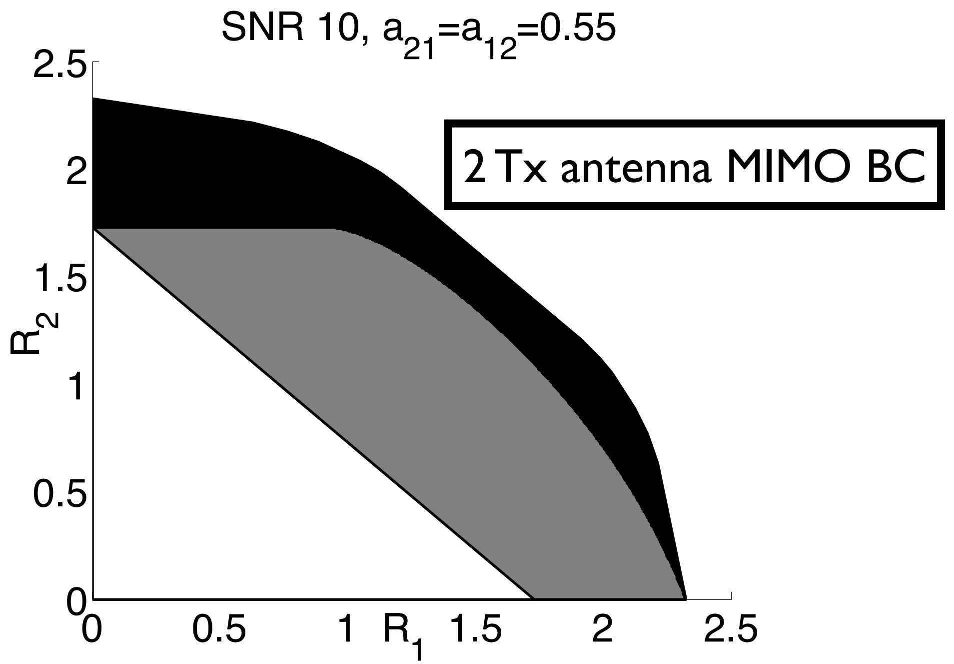


- Capacity region is known in Gaussian noise in the weak interference regime ($a_{21}<1$)
- Sum-rate capacity is known for $a_{21}>1$

[Jovicic, Viswanath 2007]

The cognitive channel

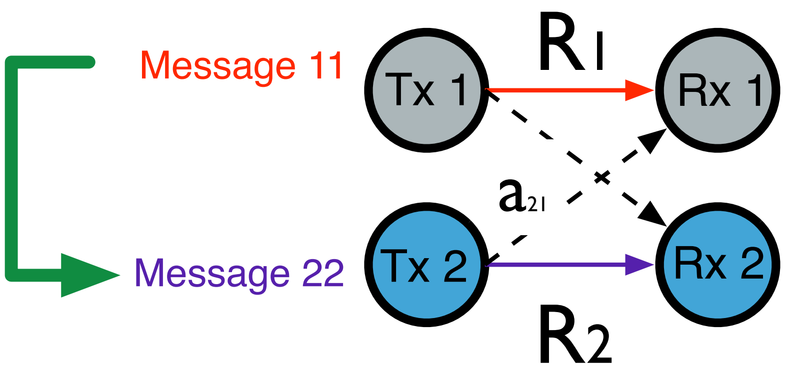


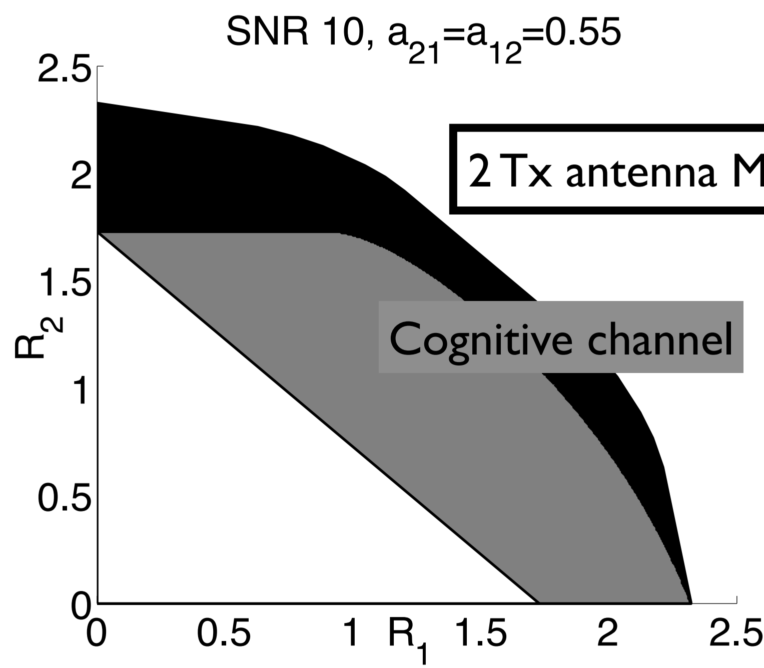


- Capacity region is known in Gaussian noise in the weak interference regime (a_{21}<1)
- Sum-rate capacity is known for a_{21}>1

[Jovicic, Viswanath 2007]

The cognitive channel

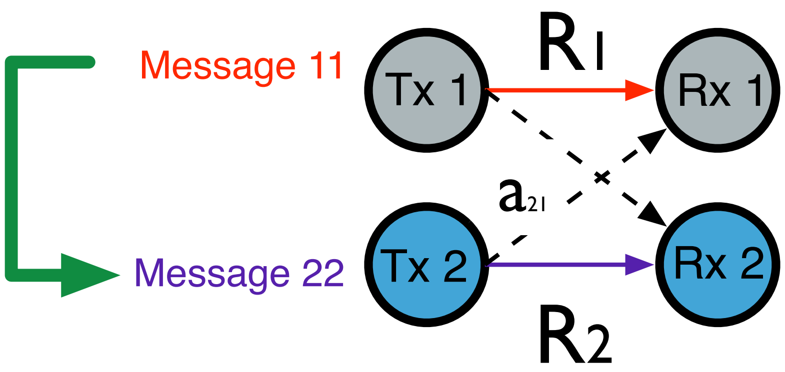


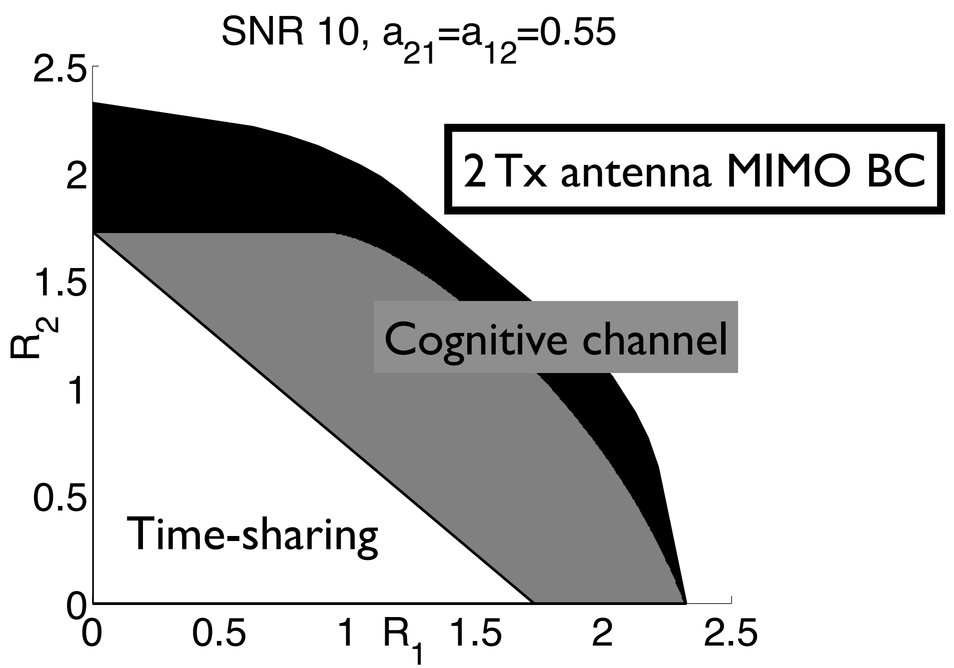


- Capacity region is known in Gaussian noise in the weak interference regime (a_{21}<1)
- Sum-rate capacity is known for a_{21}>1

[Jovicic, Viswanath 2007]

The cognitive channel

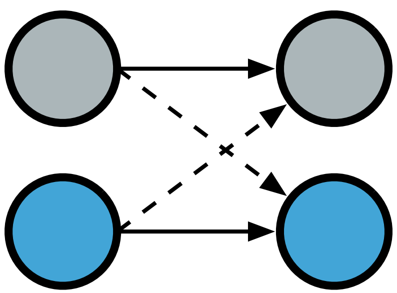




- Capacity region is known in Gaussian noise in the weak interference regime ($a_{21}<1$)
- Sum-rate capacity is known for $a_{21}>1$

[Jovicic, Viswanath 2007]

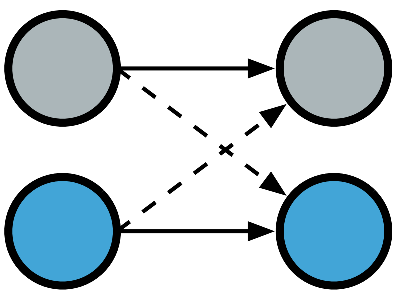
Degrees of freedom in *cognitive* channels



DOF = 1

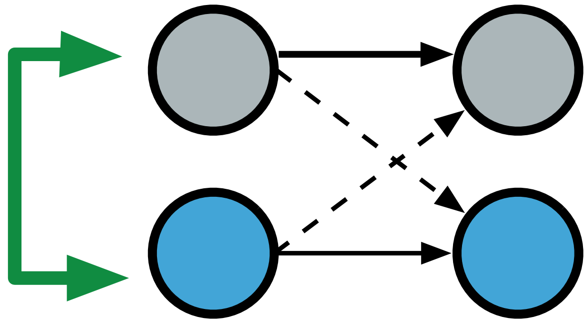
Interference channel

Degrees of freedom in *cognitive* channels



DOF = 1

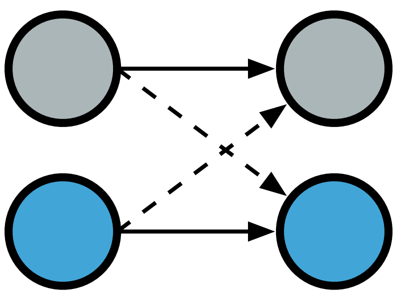
Interference channel



DOF = 2

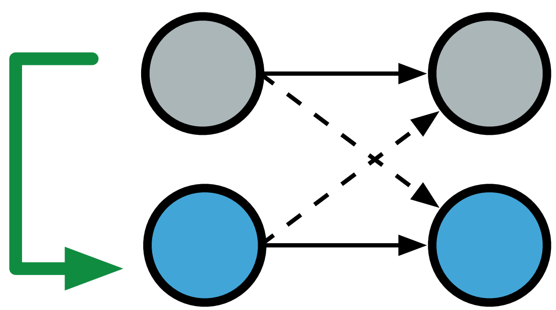
2 Tx antenna
broadcast channel

Degrees of freedom in *cognitive* channels



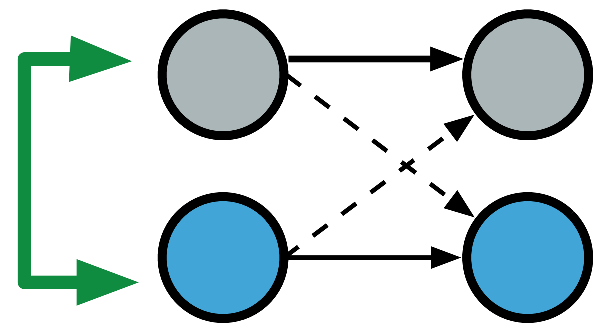
DOF = 1

Interference channel



$\boxed{\text{DOF} = 1}$

Cognitive channel

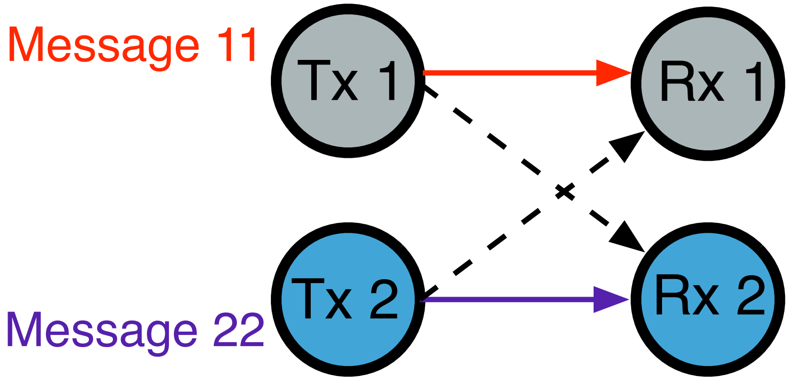


DOF = 2

2 Tx antenna
broadcast channel

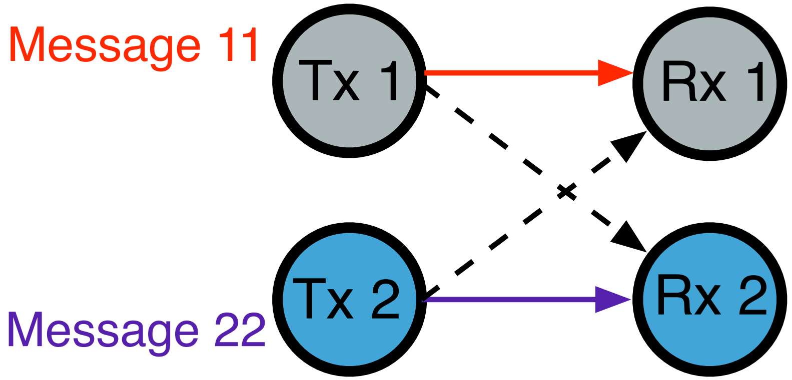
Degrees of freedom in the cognitive X channel

The X channel

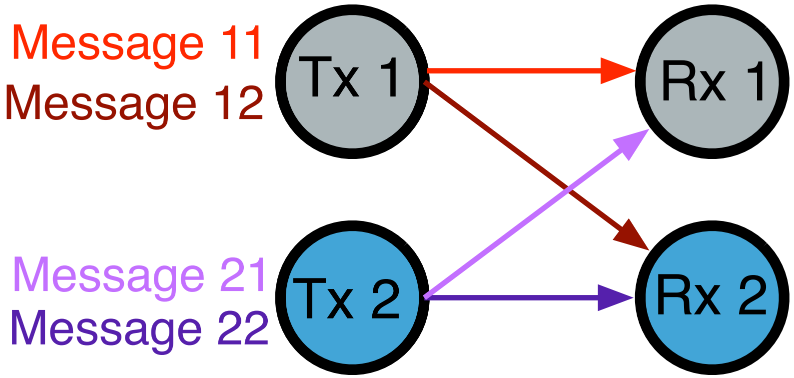


Interference channel
2 messages

The X channel

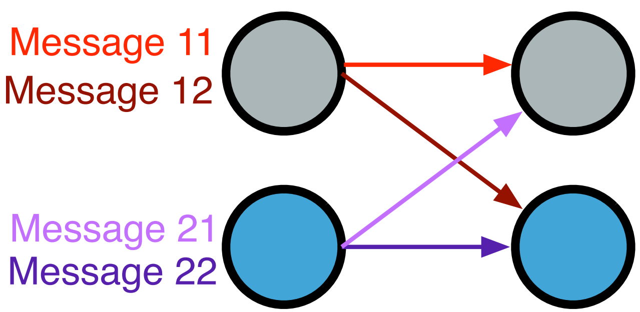


Interference channel
2 messages

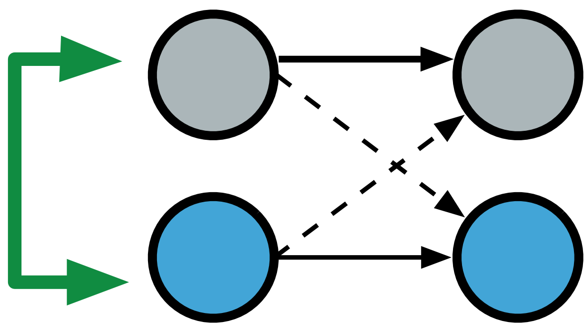


X channel
4 messages

Degrees of freedom in *cognitive* X channels



$1 \leq \text{DOF} \leq 4/3$

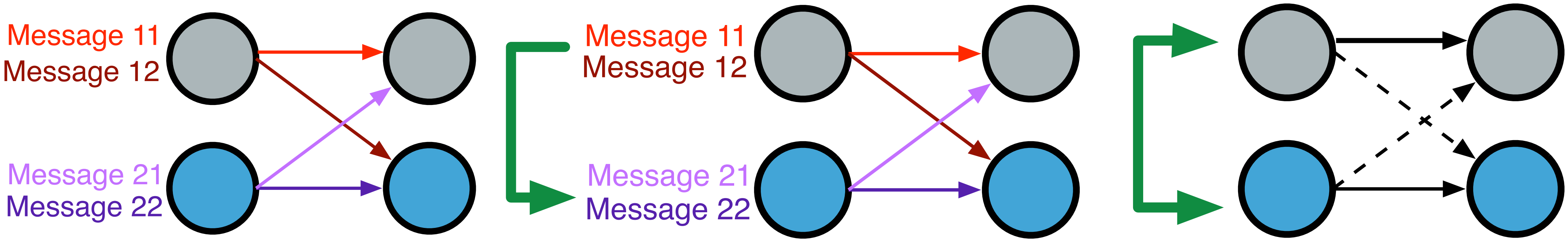


$\text{DOF} = 2$

[Maddah-Ali, Motahari, Khandani 2006]

[Jafar, Shamai 2007]

Degrees of freedom in *cognitive* X channels



1 ≤ DOF ≤ 4/3

DOF ?

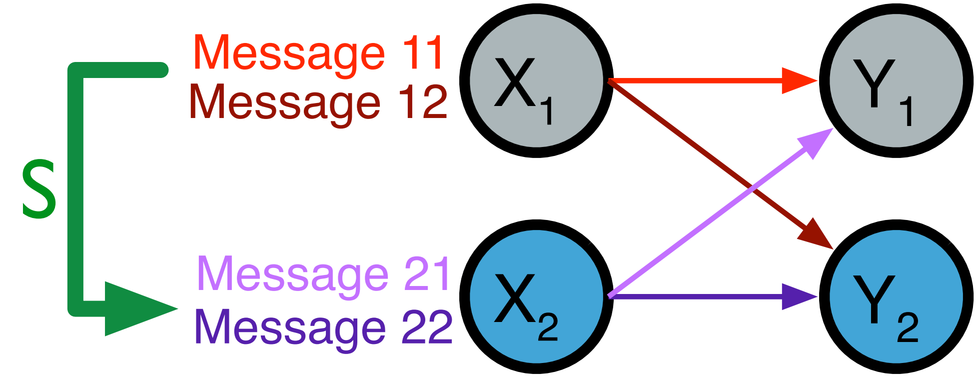
DOF = 2

[Maddah-Ali, Motahari, Khandani 2006]

[Jafar, Shamai 2007]

Cognitive X channel

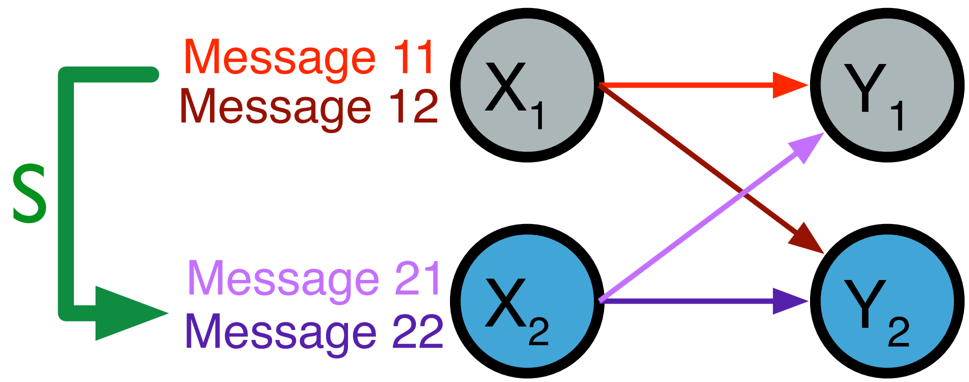
Several possibilities for the side-information S at the cognitive Tx 2:



1. Tx 2 knows message 11
2. Tx 2 knows message 11 and message 12
3. Tx 2 knows codeword which encodes message 11

Cognitive X channel

Several possibilities for the side-information S at the cognitive Tx 2:



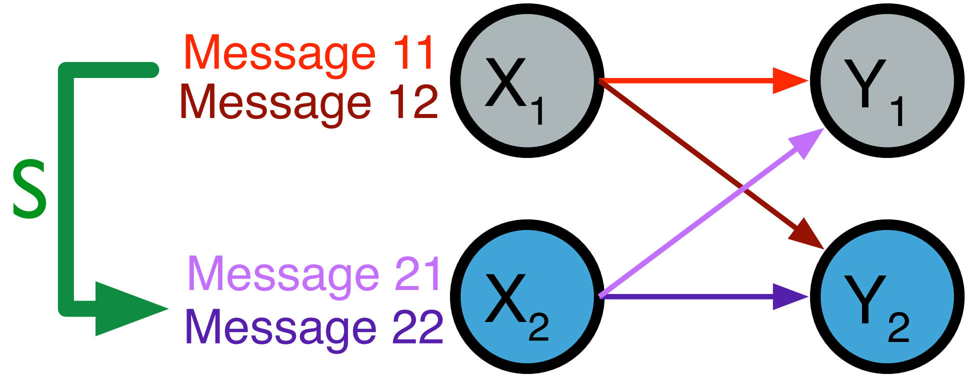
1. Tx 2 knows message 11
2. Tx 2 knows message 11 and message 12

This talk

3. Tx 2 knows codeword which encodes message 11

Cognitive X channel

Several possibilities for the side-information S at the cognitive Tx 2:



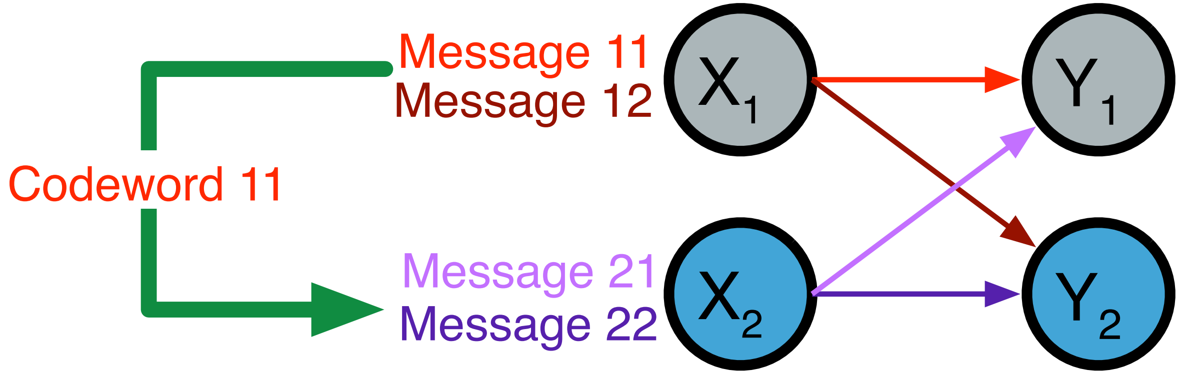
1. Tx 2 knows message 11
2. Tx 2 knows message 11 and message 12

[Jafar, Shamai 2007]

This talk

3. Tx 2 knows codeword which encodes message 11

Cognitive X Channel



Tx 1
- Encode message 12 as codeword M_{12}
- Encode message 11 as codeword M_{11} by dirty-paper coding against M_{12}

Tx 2
- Encode message 21 as codeword M_{21}
- Encode message 22 as codeword M_{22} by dirty-paper coding against M_{11} and M_{21}

Achievable rate region for a cognitive X channel

$(R_{11}, R_{12}, R_{21}, R_{22})$ such that:

$$R_{11} \leq I(M_{11}; Y_1 | M_{21}) - I(M_{11}; M_{12})$$
$$R_{21} \leq I(M_{21}; Y_1 | M_{11})$$
$$R_{11} + R_{21} \leq I(M_{11}, M_{21}; Y_1) + I(M_{11}; M_{21}) - I(M_{11}; M_{12})$$

$$R_{12} \leq I(M_{12}; Y_2 | M_{22})$$
$$R_{22} \leq I(M_{22}; Y_2 | M_{12}) - I(M_{22}; M_{11}, M_{21})$$
$$R_{12} + R_{22} \leq I(M_{12}, M_{22}; Y_2) + I(M_{12}; M_{22}) - I(M_{22}; M_{11}, M_{21})$$

Achievable rate region for a cognitive X channel

$(R_{11}, R_{12}, R_{21}, R_{22})$ **such that:**

MAC+Gel'fand-Pinsker to Rx 1

$$\left.\begin{aligned} R_{11} &\leq I(M_{11}; Y_1 | M_{21}) - I(M_{11}; M_{12}) \\ R_{21} &\leq I(M_{21}; Y_1 | M_{11}) \\ R_{11} + R_{21} &\leq I(M_{11}, M_{21}; Y_1) + I(M_{11}; M_{21}) - I(M_{11}; M_{12}) \end{aligned}\right\}$$

$$\begin{aligned} R_{12} &\leq I(M_{12}; Y_2 | M_{22}) \\ R_{22} &\leq I(M_{22}; Y_2 | M_{12}) - I(M_{22}; M_{11}, M_{21}) \\ R_{12} + R_{22} &\leq I(M_{12}, M_{22}; Y_2) + I(M_{12}; M_{22}) - I(M_{22}; M_{11}, M_{21}) \end{aligned}$$

Achievable rate region for a cognitive X channel

$(R_{11}, R_{12}, R_{21}, R_{22})$ **such that:**

MAC+Gel'fand-Pinsker to Rx 1

$$\left.\begin{aligned} R_{11} &\leq I(M_{11}; Y_1 | M_{21}) - I(M_{11}; M_{12}) \\ R_{21} &\leq I(M_{21}; Y_1 | M_{11}) \\ R_{11} + R_{21} &\leq I(M_{11}, M_{21}; Y_1) + I(M_{11}; M_{21}) - I(M_{11}; M_{12}) \end{aligned}\right\}$$

MAC+Gel'fand-Pinsker to Rx 2

$$\left.\begin{aligned} R_{12} &\leq I(M_{12}; Y_2 | M_{22}) \\ R_{22} &\leq I(M_{22}; Y_2 | M_{12}) - I(M_{22}; M_{11}, M_{21}) \\ R_{12} + R_{22} &\leq I(M_{12}, M_{22}; Y_2) + I(M_{12}; M_{22}) - I(M_{22}; M_{11}, M_{21}) \end{aligned}\right\}$$

Approach

[Costa's "Writing on Dirty Paper" 1980]

$$C = \max_{p(m_2,x_2|x_1)} I(M_2;Y_2) - I(M_2;X_1)$$

- Assume *M*2 to be $M_2 = X_2 + \gamma X_1$ (Gaussian), and one dirty-paper coding parameter ϒ
- Optimize ϒ
- Evaluate capacity

Approach

[Costa's "Writing on Dirty Paper" 1980]

$$C = \max_{p(m_2,x_2|x_1)} I(M_2;Y_2) - I(M_2;X_1)$$

- Assume M_2 to be $M_2 = X_2 + \gamma X_1$ (Gaussian), and one dirty-paper coding parameter γ
- Optimize γ
- Evaluate capacity

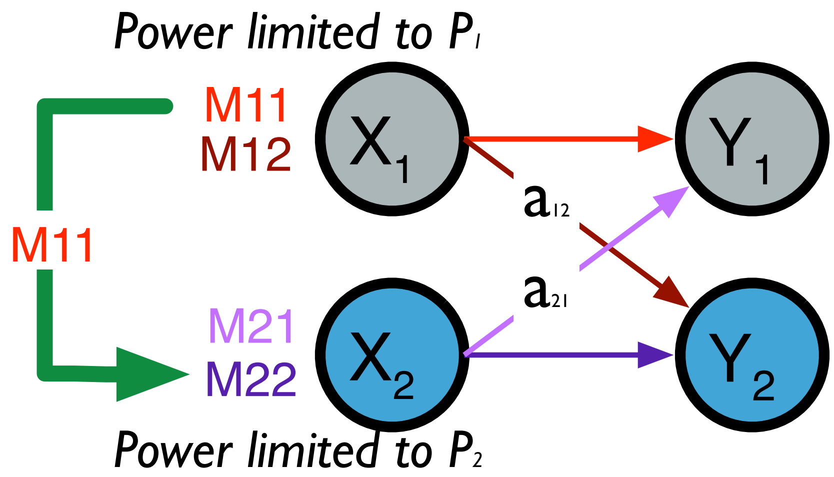
[This talk]

$$\begin{aligned} R_{11} &\leq I(M_{11};Y_1|M_{21}) - I(M_{11};M_{12}) \\ R_{21} &\leq I(M_{21};Y_1|M_{11}) \\ R_{11}+R_{21} &\leq I(M_{11},M_{21};Y_1) + I(M_{11};M_{21}) - I(M_{11};M_{12}) \end{aligned}$$

$$\begin{aligned} R_{12} &\leq I(M_{12};Y_2|M_{22}) \\ R_{22} &\leq I(M_{22};Y_2|M_{12}) - I(M_{22};M_{11},M_{21}) \\ R_{12}+R_{22} &\leq I(M_{12},M_{22};Y_2) + I(M_{12};M_{22}) - I(M_{22};M_{11},M_{21}) \end{aligned}$$

- Assume $M_{11}, M_{12}, M_{21}, M_{22}$ Gaussian and 2 dirty paper coding parameters γ_1, γ_2
- Optimize γ_1, γ_2
- Evaluate achievable sum-rate

Assumed input variables



$$
\begin{aligned}
M_{11} &= U_{11} + \gamma_1 U_{12} \\
M_{12} &= U_{12}, \\
M_{21} &= U_{21}, \\
M_{22} &= U_{22} + \gamma_2 (U_{21} + a_{12} U_{11})
\end{aligned}
\qquad
\begin{aligned}
U_{11} &\sim \mathcal{N}(0, P_{11}) \\
U_{12} &\sim \mathcal{N}(0, P_{12}) \\
U_{21} &\sim \mathcal{N}(0, P_{21}) \\
U_{22} &\sim \mathcal{N}(0, P_{22})
\end{aligned}
$$

$$
\begin{aligned}
P_1 &= P_{11} + P_{12} \\
\beta P_2 &= P_{21} + P_{22}
\end{aligned}
$$

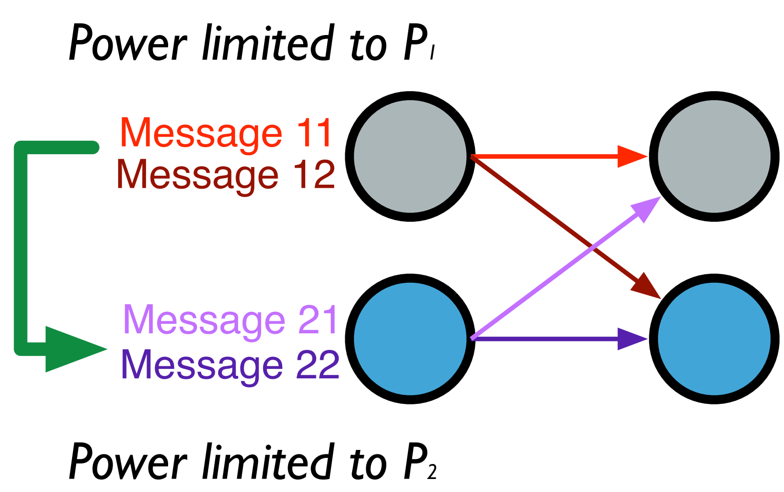
$$
\begin{aligned}
X_1 &= U_{11} + U_{12} \\
X_2 &= U_{21} + U_{22} + \sqrt{\tfrac{(1-\beta)P_2}{P_{11}}} U_{11} \\
Y_1 &= \left(1 + a_{21}\sqrt{\tfrac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + U_{12} + a_{21}(U_{21} + U_{22}) + N_1 \\
Y_2 &= \left(a_{12} + \sqrt{\tfrac{(1-\beta)P_2}{P_{11}}}\right) U_{11} + a_{12} U_{12} + (U_{21} + U_{22}) + N_2.
\end{aligned}
$$

Results

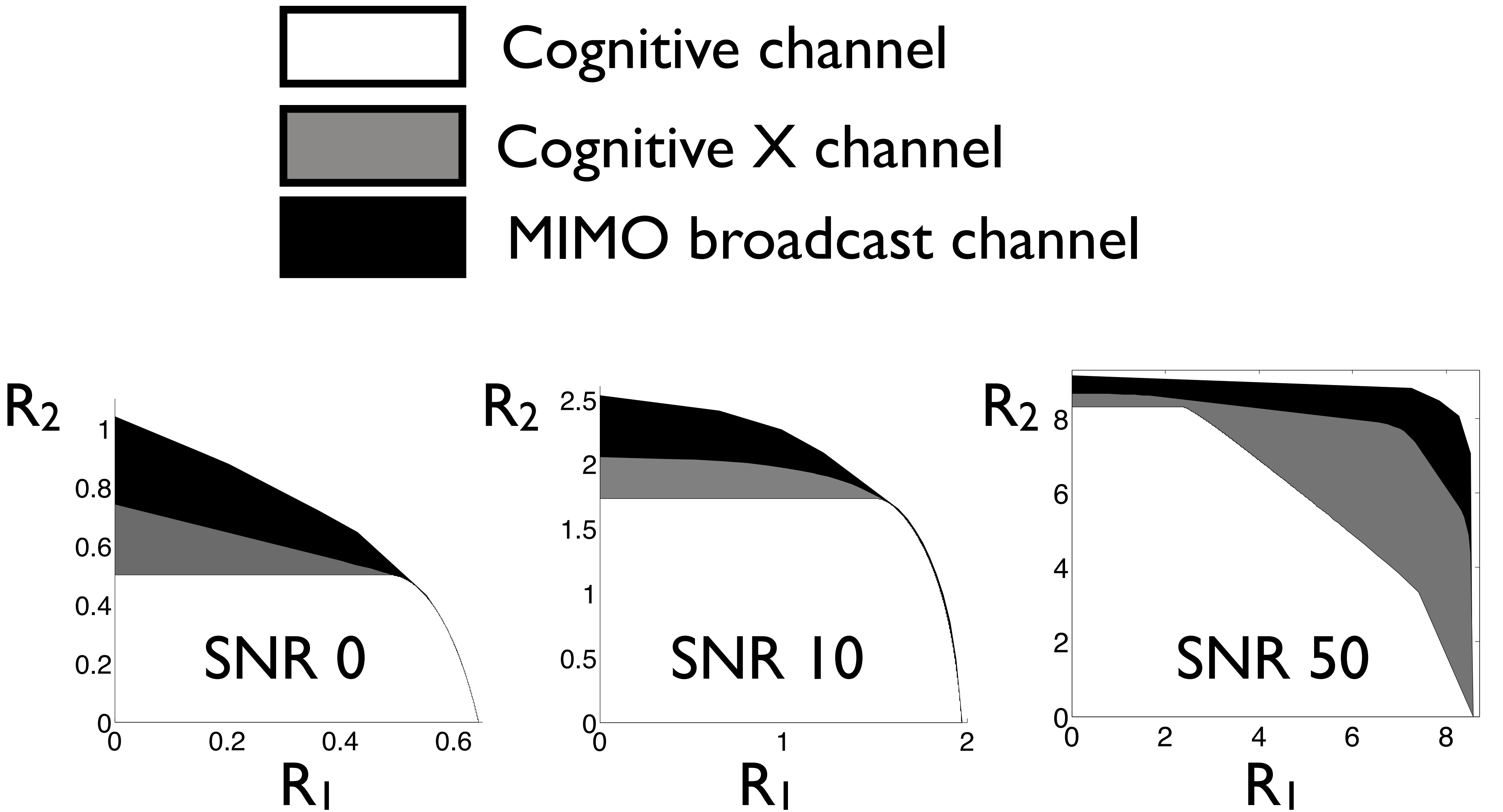
DOF of this cognitive X channel is **2**

- Power allocation that achieves this is

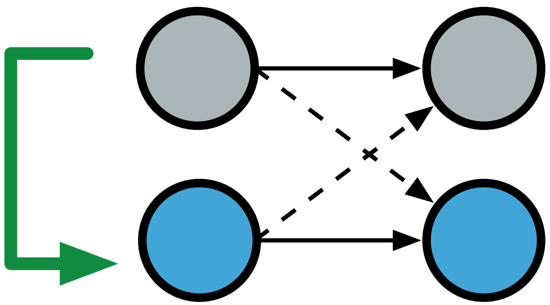
$$P_{11} = P_{12} = P_{21} = P \to \infty$$
$$P_{22} = \text{constant}$$



Rate regions

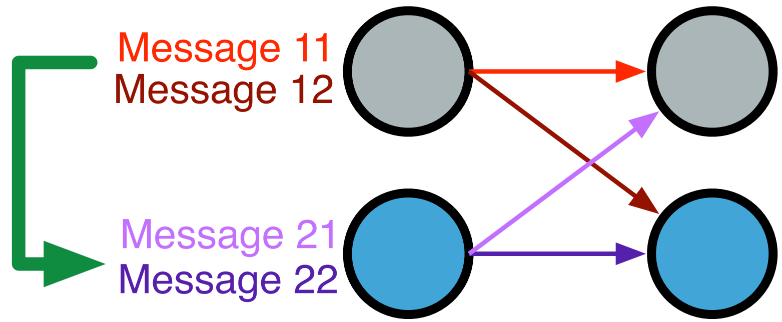


Conclusion



$\boxed{\text{DOF} = 1}$

Cognitive channel

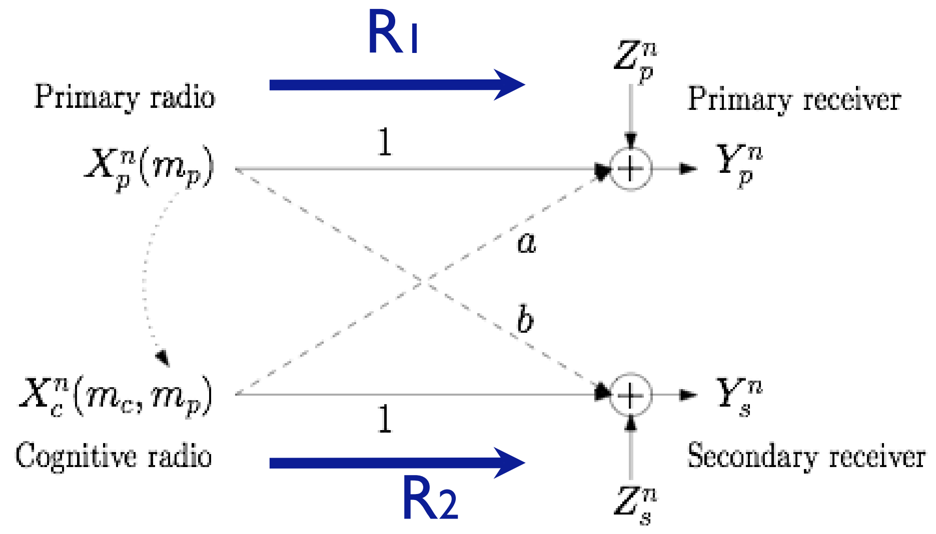


$\boxed{\text{DOF} = 2}$

Cognitive X channel
Side-information is one of Tx 1's codewords

Extra

DOF=1



- They prove that for a<1 *(weak interference)* the **capacity region** when both nodes are power limited to P is the set of all rate pairs (R_1, R_2) such that, for all $0 \le \alpha \le 1$,

$$R_1 \leq \log\left(1 + \frac{(\sqrt{P} + a\sqrt{\alpha P})^2}{1 + a^2(1-\alpha)P}\right)$$

$$R_2 \leq \log_2(1 + (1-\alpha)P)$$

DOF=1

- For any $0 \leq \alpha \leq 1$ the cognitive radio spends αP of its power amplifying the primary message, and $(1-\alpha)P$ of its power dirty-paper coding its own message

$$R_1 \leq \log\left(1 + \frac{(\sqrt{P} + a\sqrt{\alpha P})^2}{1 + \boxed{a^2(1-\alpha)P}}\right)$$

$$R_2 \leq \log_2(1 + (1-\alpha)P)$$

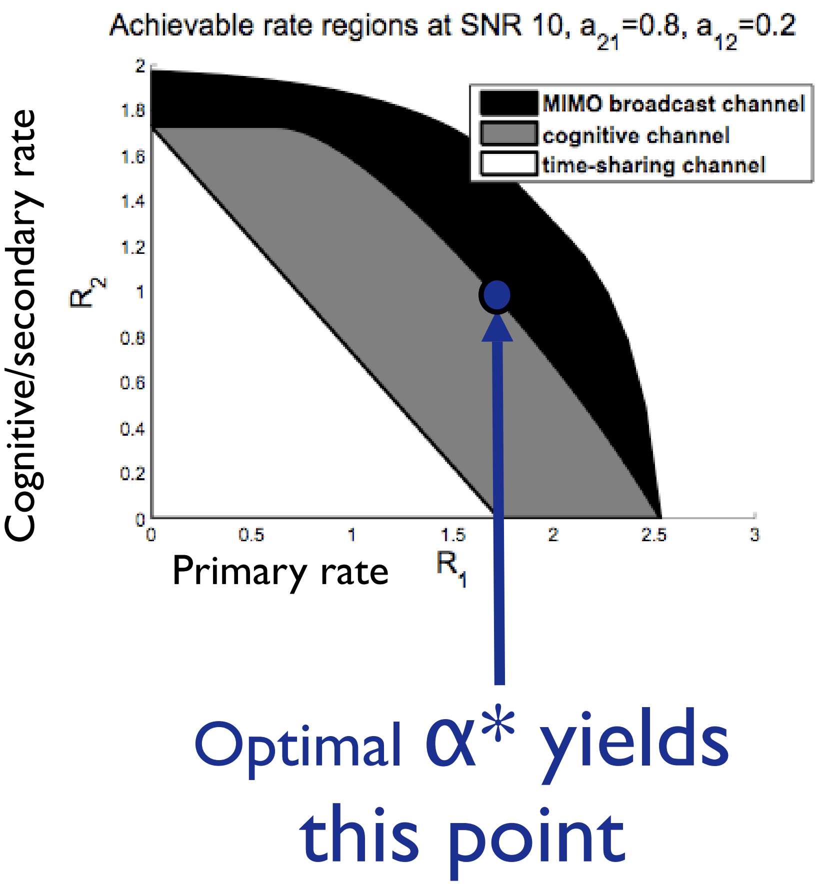
Interference at primary receiver due to cognitive transmission

DOF=1

- Moreover, they find the maximum rate the cognitive user may transmit at such that the primary user suffers no loss in rate (compared to cognitive-free transmission)

- This determines the optimal α* as:

$$\alpha^* = \left(\frac{\sqrt{P}\left(\sqrt{1+a^2P(1+P)}-1\right)}{a\sqrt{P}(1+P)} \right)^{\frac{1}{2}}$$



Selected DPC parameters

- γ_1 selected so as to maximize the sum-rate to Rx 1
- γ_2 selected so as to minimize the denominator of the sum-rate to Rx 2

$$\gamma_1 = \frac{P_{11}(1 + a_{21}\theta)}{P_{11}(1 + a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1}, \quad \gamma_2 = \frac{P_{22}}{P_{22} + N_2}$$

$$\theta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}.$$

Sum-rates

$$R_{11} + R_{21} \leq \frac{1}{2}\log_2\left(\frac{(P_{11})(P_{11}(1+a_{21}\theta)^2 + P_{12} + \alpha_{21}^2(P_{21}+P_{22}) + N_1)}{\gamma_1^2 P_{12}(P_{11}(1+a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1) - 2\gamma_1 P_{11}P_{12}(1+a_{21}\theta) + P_{11}(P_{12} + \alpha_{21}^2 P_{22} + N_1)}\right) \quad (5)$$

$$R_{12} + R_{22} \leq \frac{1}{2}\log_2\left(\frac{(P_{11}(a_{12}+\theta)^2 + a_{12}^2 P_{12} + P_{21} + P_{22} + N_2)(\gamma_1^2 P_{12}(P_{22} + \gamma_2^2(a_{12}+\theta)^2 P_{11}) + P_{11}P_{22})}{(P_{11} + \gamma_1^2 P_{12})(\gamma_2^2((P_{22}+N_2)(P_{21}+a_{12}^2) + P_{11}P_{21}\theta^2) + \gamma_2(-2P_{22}(P_{11}(a_{12}+\theta)^2 + P_{21})) + P_{22}(P_{11}(a_{12}+\theta)^2 + P_{21} + N_2))}\right) \quad (6)$$

$$R_{11} + R_{21} \leq \frac{1}{2}\log_2\left(\frac{(P_{11}(1+a_{21}\theta)^2 + P_{12} + \alpha_{21}^2(P_{21}+P_{22}) + N_1)(P_{11}(1+a_{21}\theta)^2 + \alpha_{21}^2 P_{22} + N_1)}{(\alpha_{21}^2 P_{22} + N_1)(P_{11}(1+a_{21}\theta)^2 + P_{12} + \alpha_{21}^2 P_{22} + N_1)}\right) \quad (7)$$

after substituting γ_1

$$\theta \triangleq \sqrt{\frac{(1-\beta)P_2}{P_{11}}}$$

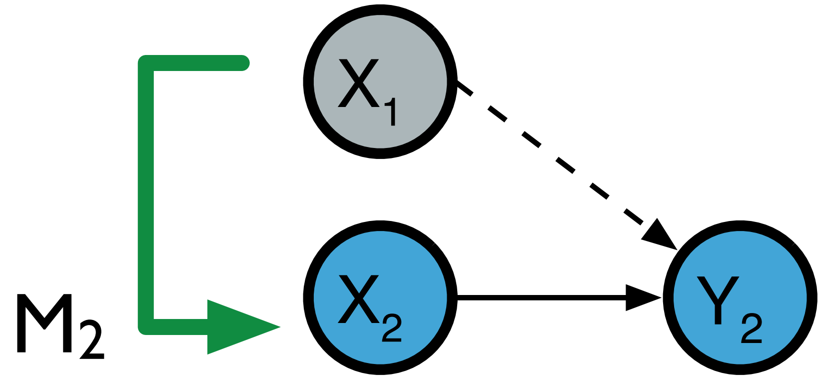
Marton's Region

- Used in *[Caire, Shamai 2003]*

$$\mathrm{co} \bigcup_{P_{U_1,U_2,X,Y_1,Y_2} \in \mathcal{P}} \left\{ \begin{array}{l} 0 \le R_1 \le I(U_1;Y_1) \\ 0 \le R_2 \le I(U_2;Y_2) \\ R_1 + R_2 \le I(U_1;Y_1) \\ \quad + I(U_2;Y_2) - I(U_1,U_2) \end{array} \right\}$$

Side-information:

discrete memoryless channels



$$C = \max_{p(m_2, x_2 | x_1)} \underbrace{I(M_2; Y_2)}_{\text{Point-to-point}} - \underbrace{I(M_2; X_1)}$$

Point-to-point

↓

Penalty for using non-causal side info

[Gel'fand, Pinsker 1980]

Formal Theorem 1

Theorem 1: Let $Z \triangleq (Y_1,\ Y_2,\ X_1,\ X_2,\ M_{11},\ M_{12},\ M_{21},\ M_{22})$, and let $\mathcal{P}$ be the set of distributions on Z that can be decomposed into the form

$$\begin{gathered} p(m_{11}|m_{12})p(m_{12})p(m_{21})p(m_{22}|m_{11},m_{21}) \\ p(x_1|m_{11},m_{12})p(x_2|m_{11},m_{21},m_{22}) \\ p(y_1|x_1,x_2)p(y_2|x_1,x_2), \end{gathered} \tag{3}$$

where we additionally require $p(m_{12},m_{22}) = p(m_{12})p(m_{22})$. For any $Z \in \mathcal{P}$, let $S(Z)$ be the set of all tuples $(R_{11},R_{12},R_{21},R_{22})$ of non-negative real numbers such that:

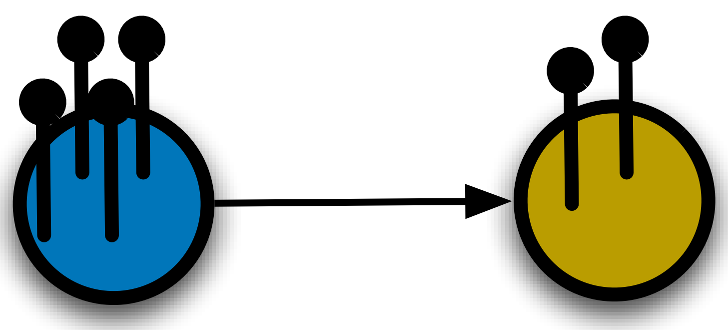
$$\begin{aligned} R_{11} &\leq I(M_{11};Y_1|M_{21}) - I(M_{11};M_{12}) \\ R_{21} &\leq I(M_{21};Y_1|M_{11}) \\ R_{11}+R_{21} &\leq I(M_{11},M_{21};Y_1) - I(M_{11};M_{12}) \end{aligned}$$

$$\begin{aligned} R_{12} &\leq I(M_{12};Y_2|M_{22}) \\ R_{22} &\leq I(M_{22};Y_2|M_{12}) - I(M_{22};M_{11},M_{21}) \\ R_{12}+R_{22} &\leq I(M_{12},M_{22};Y_2) - I(M_{22};M_{11},M_{21}) \end{aligned}$$

Any element in the closure of $\cup_{Z\in\mathcal{P}} S(Z)$ is achievable.

Degrees of freedom in *classical* channels

M_T
transmit antennas

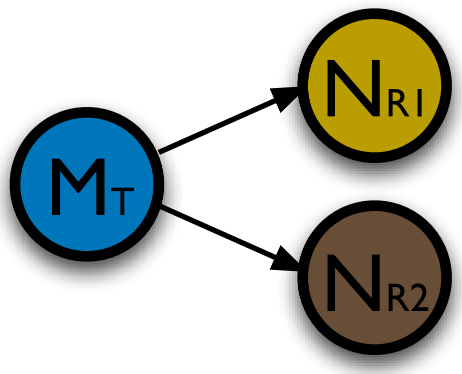


N_R
receive antennas

$$\mathrm{DOF} = \min(M_T, N_R)$$

Degrees of freedom in *classical* channels

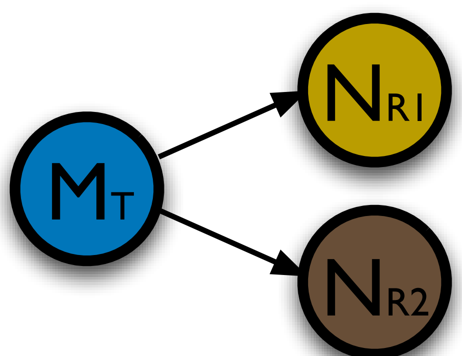
Broadcast channel



$\min(M_T, N_{R1} + N_{R2})$

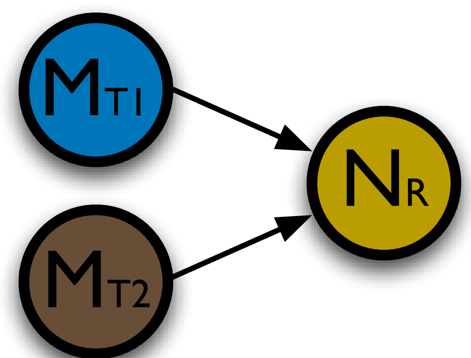
Degrees of freedom in *classical* channels

Broadcast channel



$\min(M_T, N_{R1} + N_{R2})$

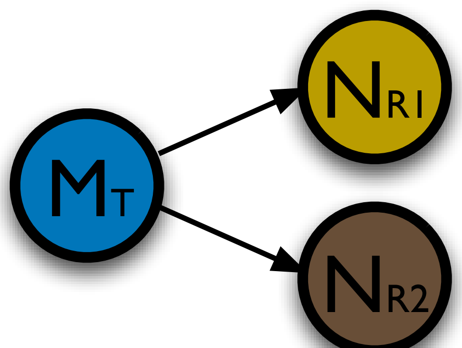
Multiple access channel



$\min(M_{T1} + M_{T2}, N_R)$

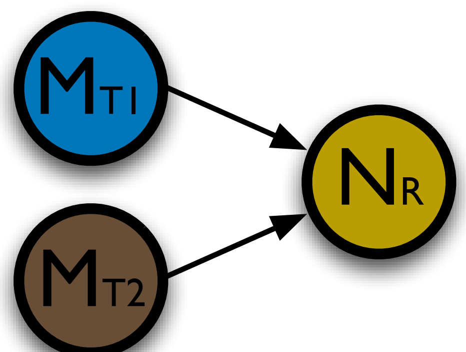
Degrees of freedom in *classical* channels

Broadcast channel



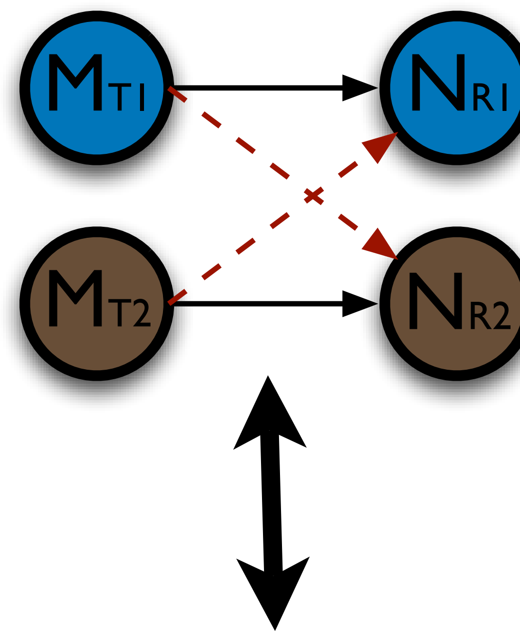
$\min(M_T, N_{R1} + N_{R2})$

Multiple access channel



$\min(M_{T1} + M_{T2}, N_R)$

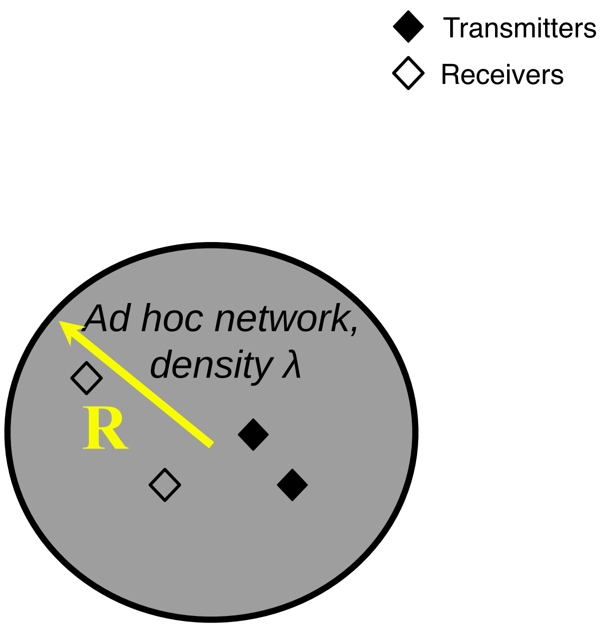
Interference channel



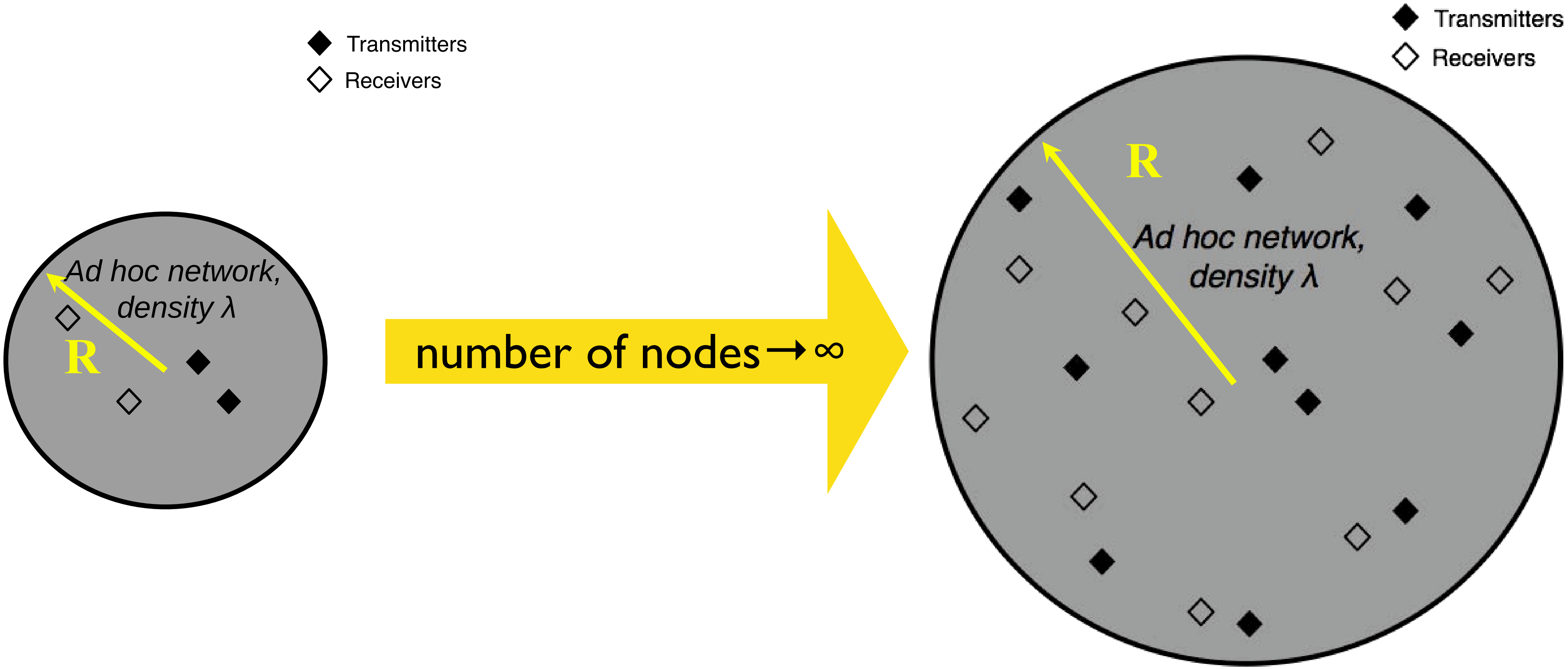
$\min(M_{T1} + M_{T2}, N_{R1} + N_{R2}, \max(M_{T1}, N_{R2}), \max(M_{T2}, N_{R1}))$

Cognitive networks

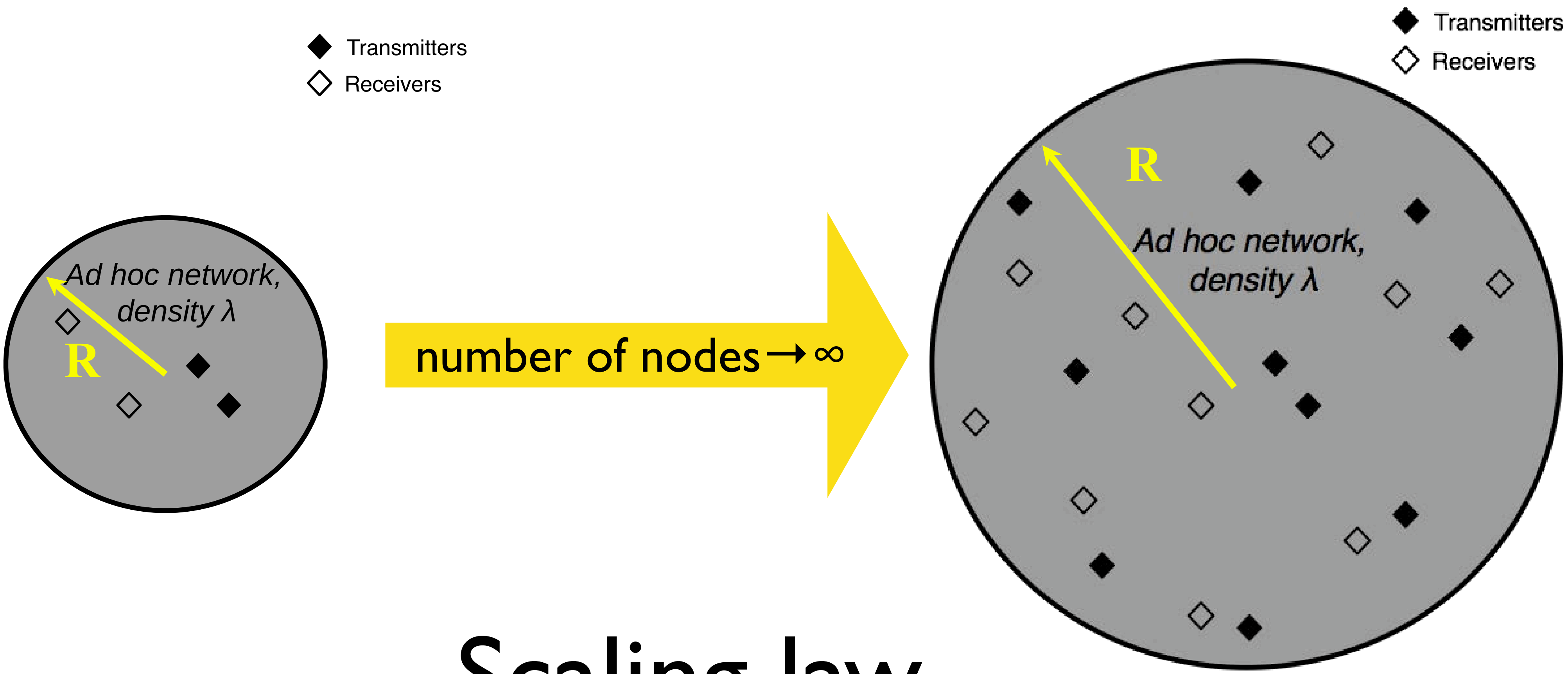
Wireless networks



Wireless networks



Wireless networks



Scaling law

Throughput(number of nodes)

Scaling laws

- **[Gupta+Kumar 2000]:** Non-cooperative ad hoc networks
  - per-node throughput ~ $O(1/\sqrt{n})$
  - Degradation is due to multi-hop and **interference** between nodes

Scaling laws

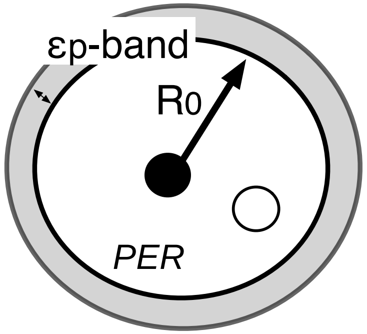
- **[Gupta+Kumar 2000]:** Non-cooperative ad hoc networks
  - per-node throughput ~ O(1/√n)
  - Degradation is due to multi-hop and **interference** between nodes
- **[Ozgur, Leveque, Tse 2007]:** Cooperative ad hoc networks
  - nodes may cooperate as in a MIMO system
  - per-node throughput ~ O(1) *(constant)*

Scaling laws

- **[Gupta+Kumar 2000]:** Non-cooperative ad hoc networks
  - per-node throughput ~ $O(1/\sqrt{n})$
  - Degradation is due to multi-hop and **interference** between nodes
- **[Ozgur, Leveque, Tse 2007]:** Cooperative ad hoc networks
  - nodes may cooperate as in a MIMO system
  - per-node throughput ~ $O(1)$ *(constant)*

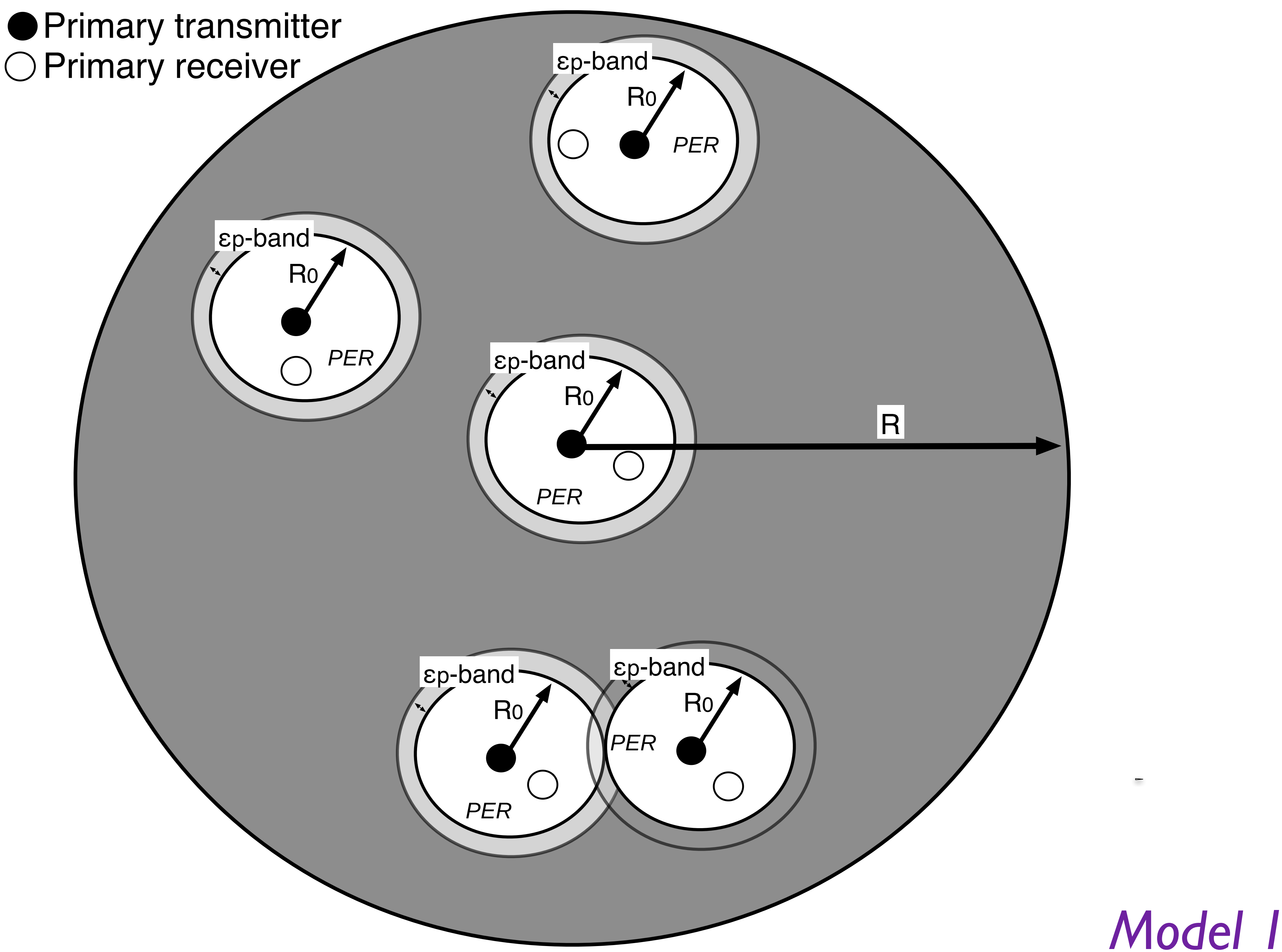
What about cognitive networks?

● Primary transmitter
○ Primary receiver

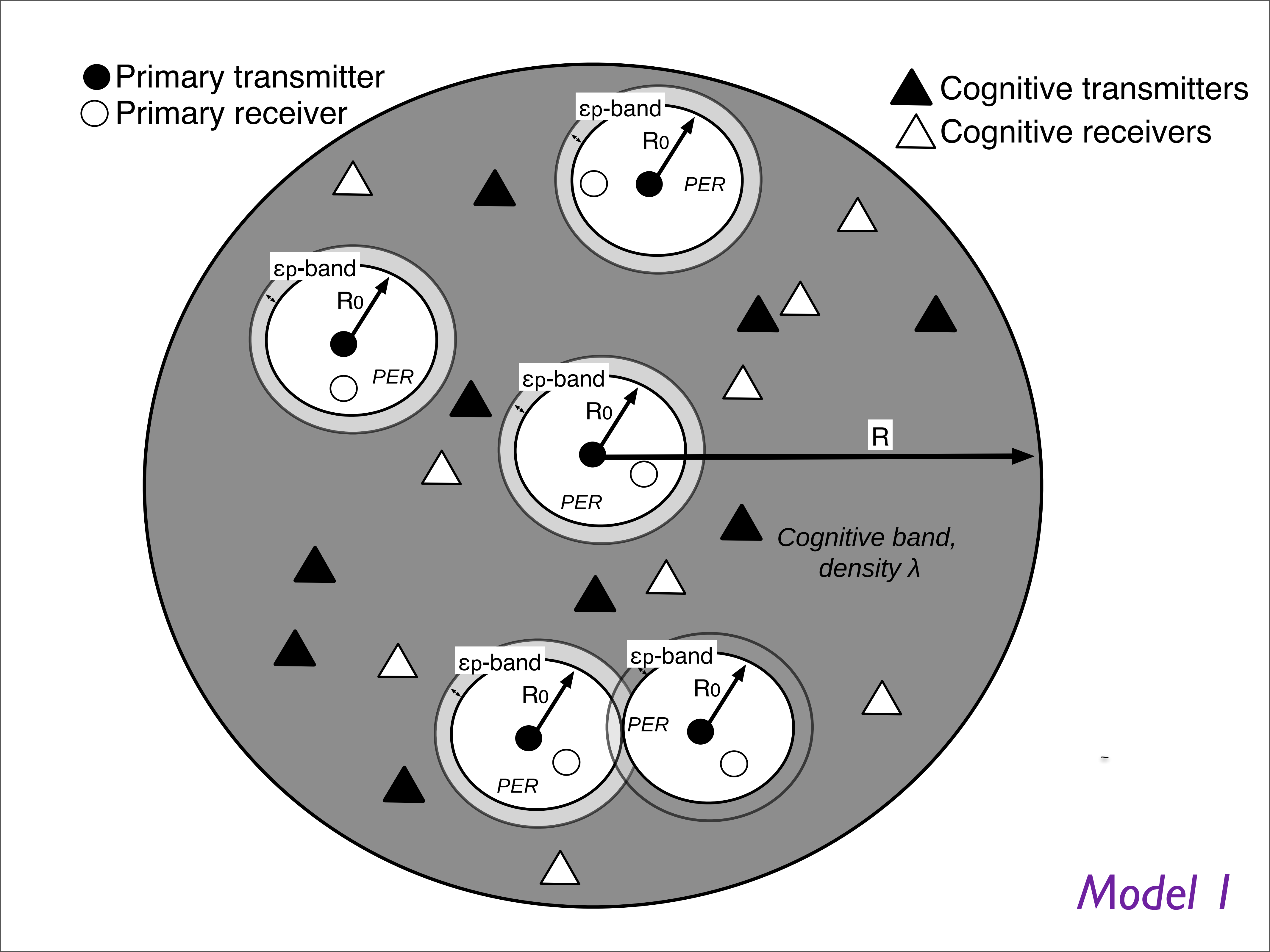


[Vu, Devroye, Tarokh 2007]

Model I



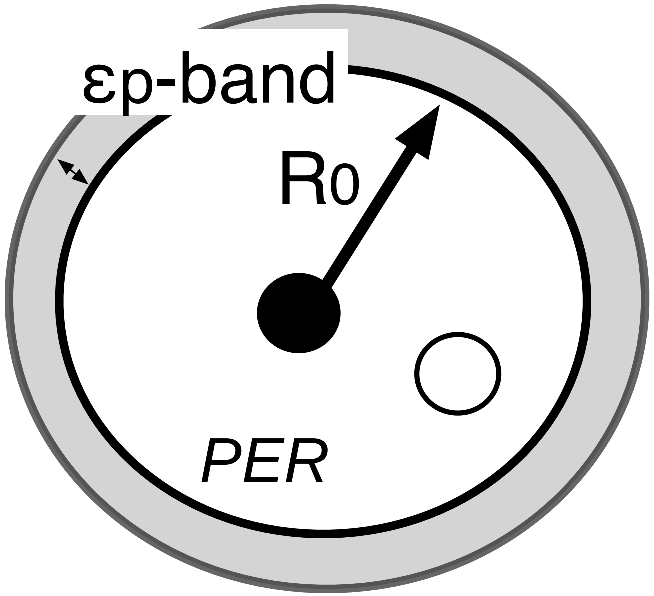
Primary transmitter
Primary receiver
εp-band
R0
PER
R
Model I



Primary transmitter
Primary receiver
Cognitive transmitters
Cognitive receivers
εp-band
R0
PER
εp-band
R0
PER
εp-band
R0
PER
R
Cognitive band,
density λ
εp-band
R0
PER
εp-band
R0
PER
Model I

Single hop

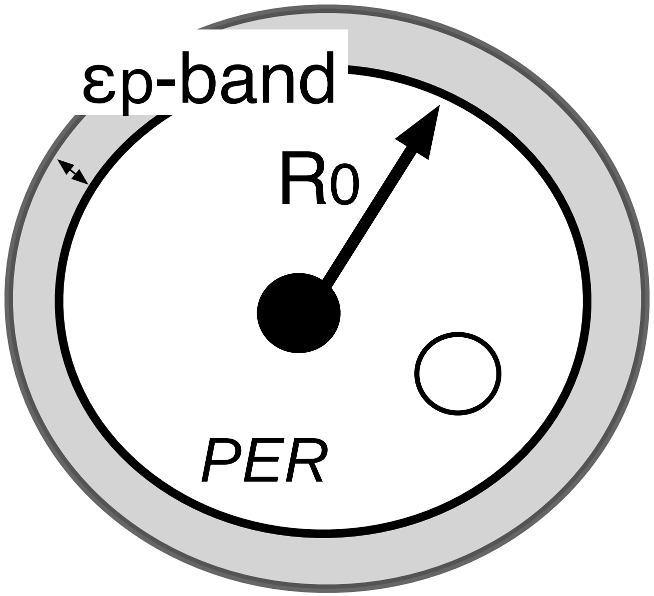
Primary nodes



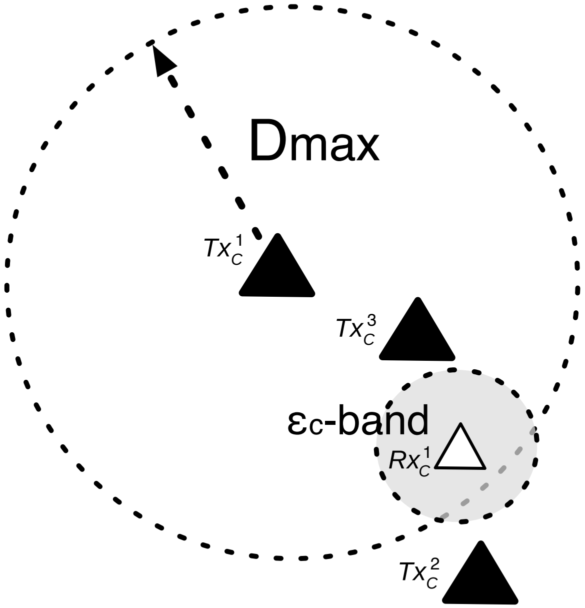
Model 1

Single hop

Primary nodes



Secondary nodes



Model 1

What we guarantee

[Vu, Devroye, Tarokh 2007]

Model 1

What we guarantee

Primary nodes don't suffer too much

$$\Pr\left[\text{primary user's rate} \leq C_0\right] \leq \beta$$

[Vu, Devroye, Tarokh 2007]

Model I

What we guarantee

Primary nodes don't suffer too much

$$\Pr\left[\text{primary user's rate} \le C_0\right] \le \beta$$

What we prove

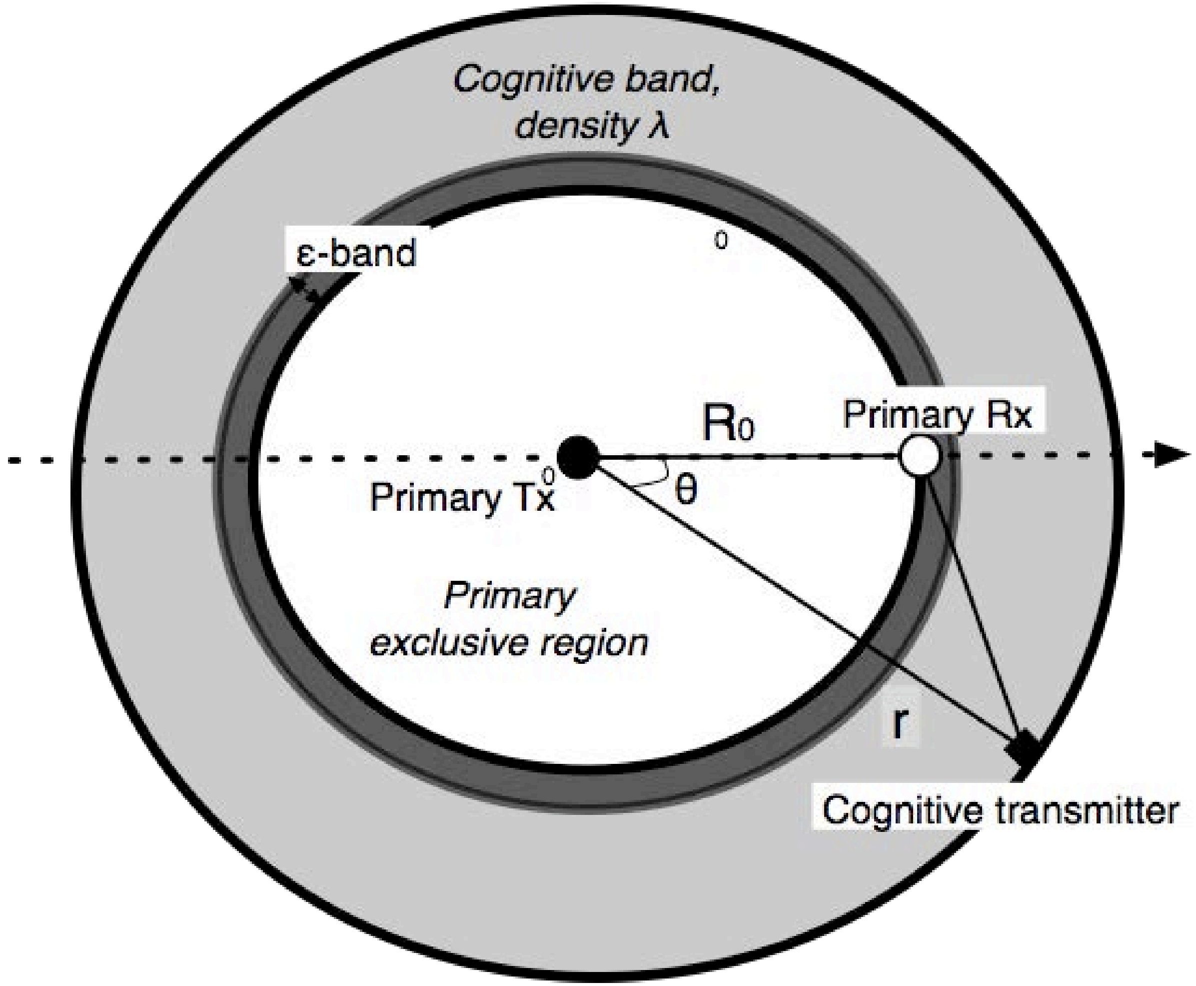
Sum-throughput per cognitive user scale as

O(1) as $n \rightarrow \infty$

while guaranteeing $\Pr\left[\text{primary user's rate} \le C_0\right] \le \beta$

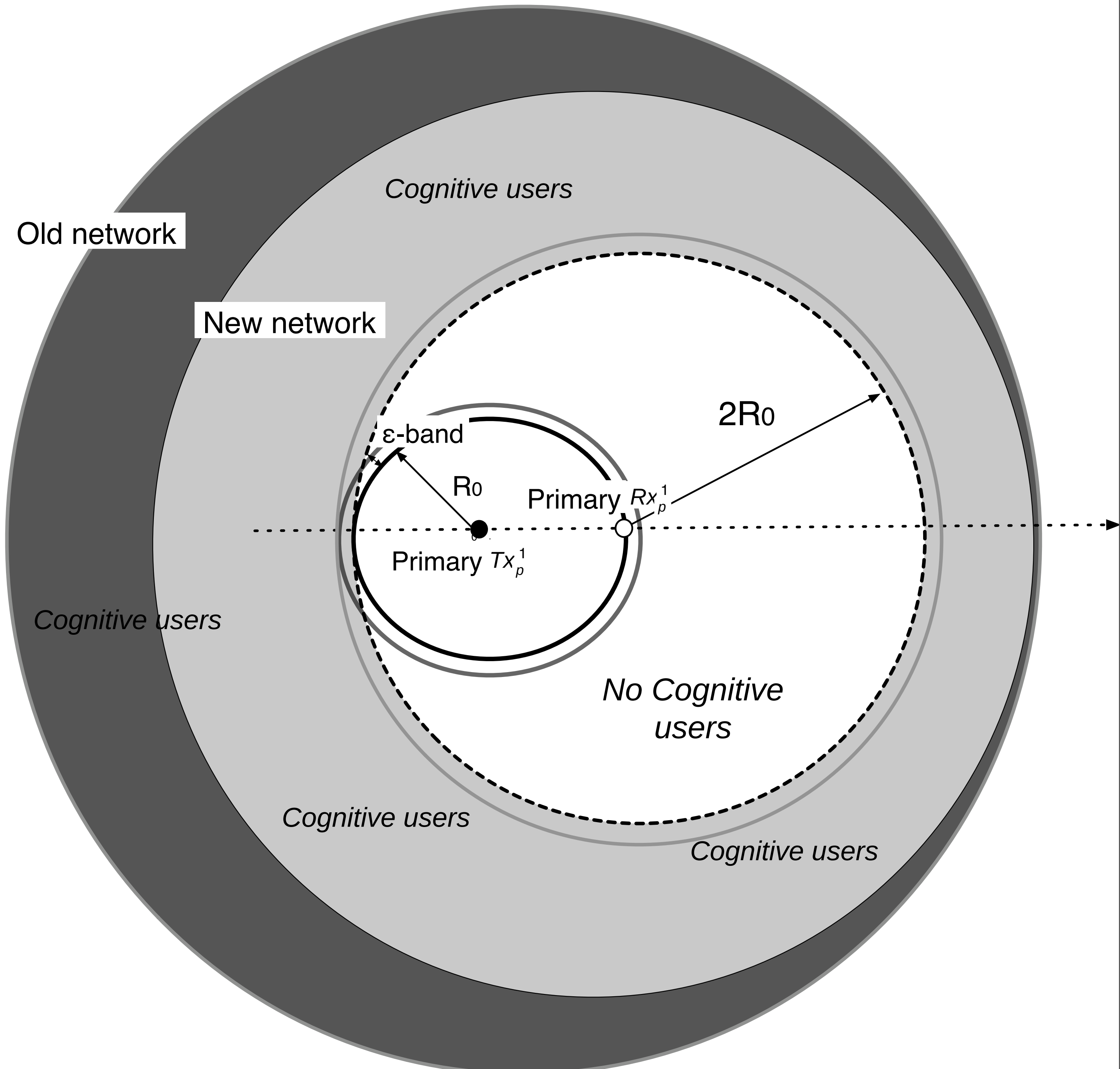
[Vu, Devroye, Tarokh 2007]

Model I

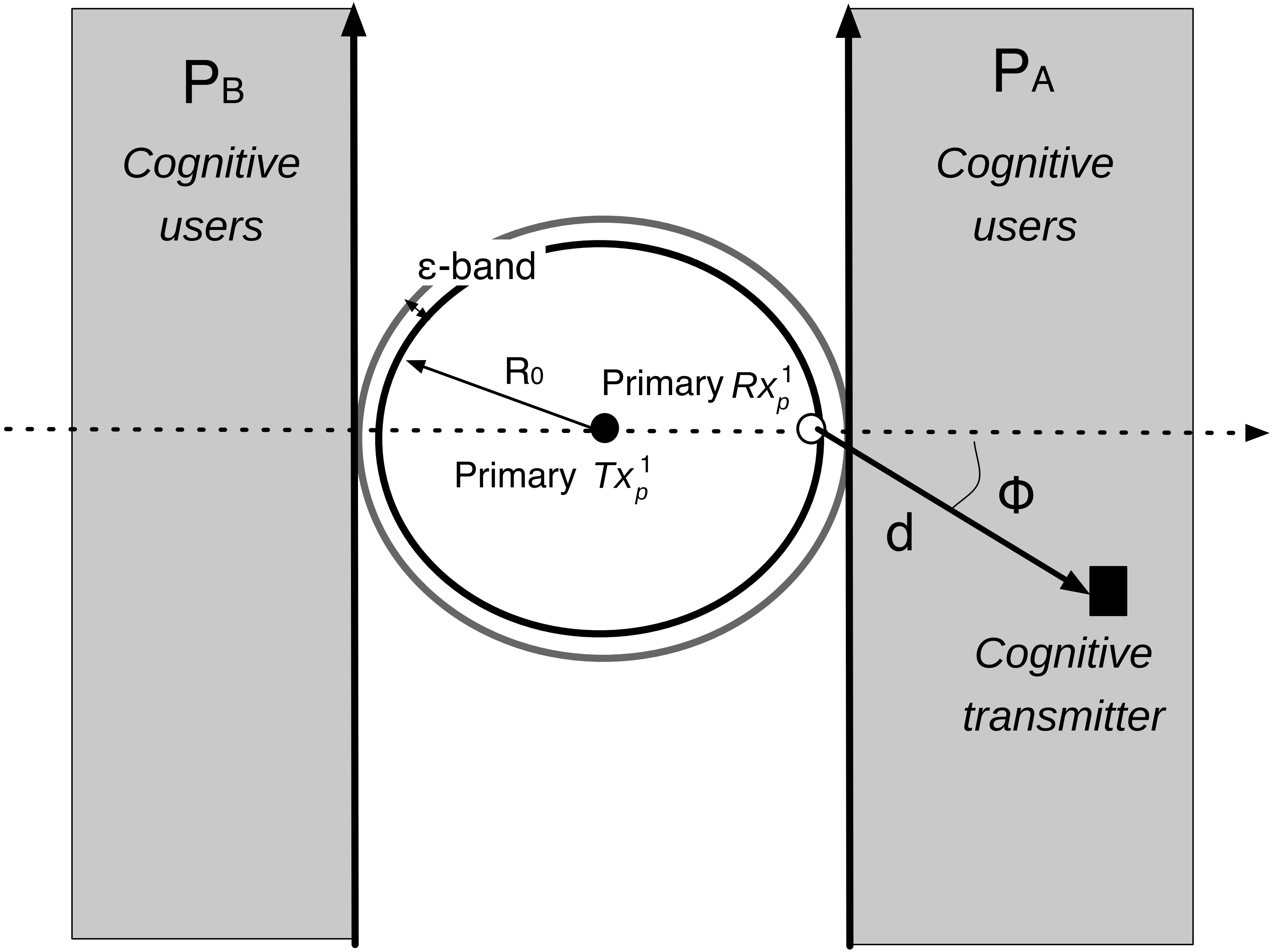


$$E[I_0]_{\alpha=4} = \lambda \pi P \left[-\frac{R^2}{(R^2 - R_0^2)^2} + \frac{(R_0 + \epsilon_p)^2}{\epsilon_p^2 (2R_0 + \epsilon_p)^2} \right]$$

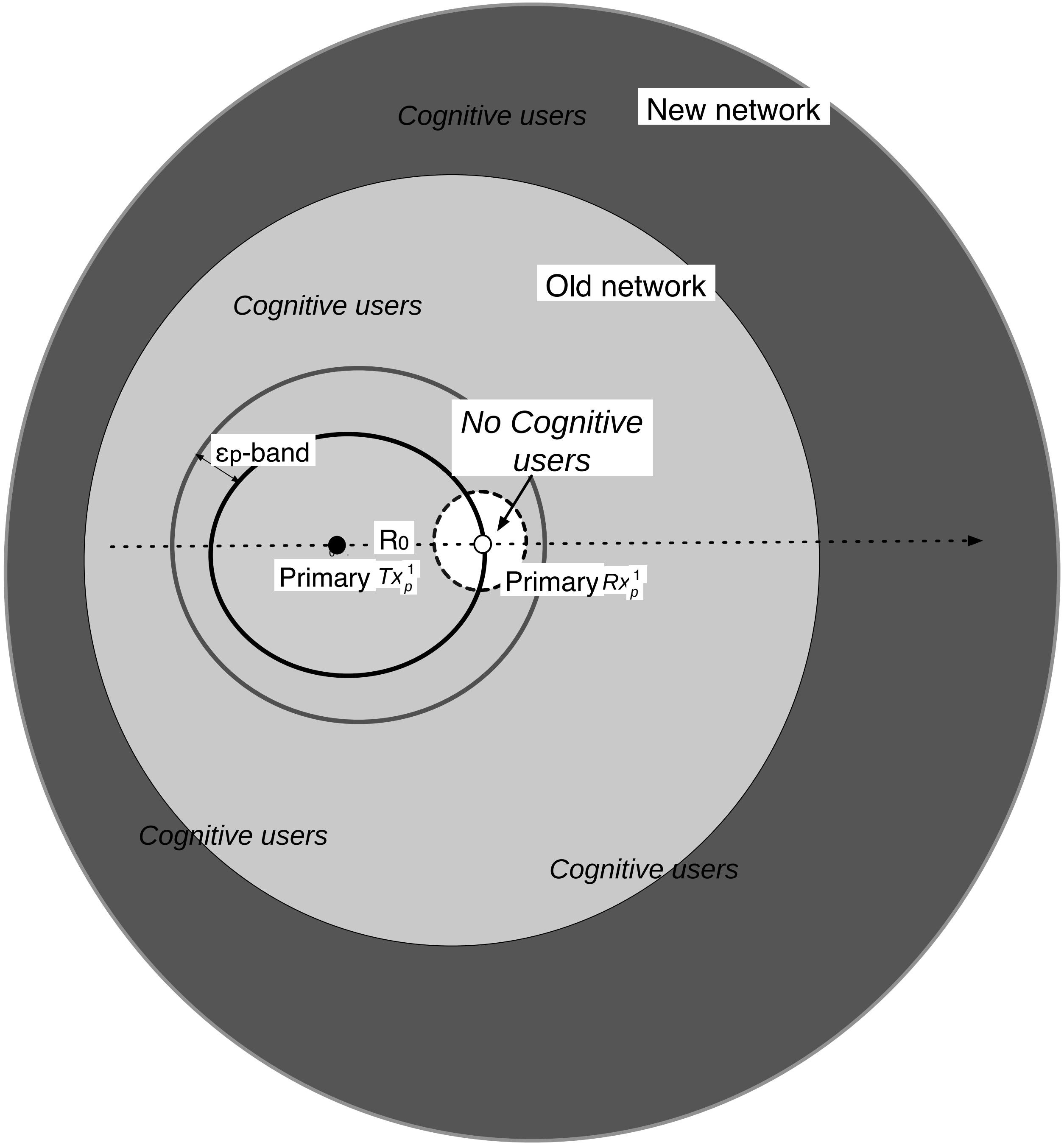
Exact calculation of interference



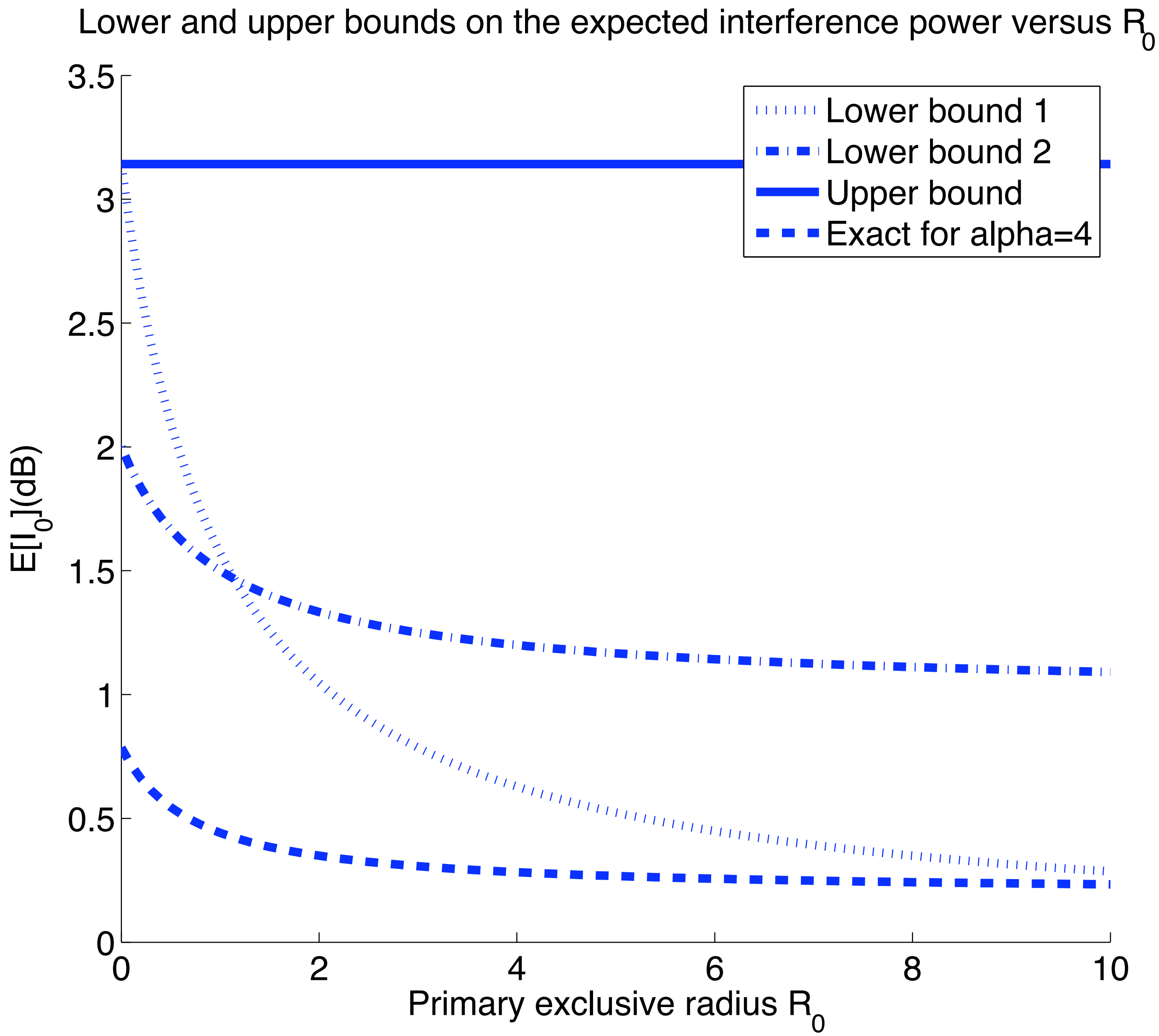
Lower bound on interference 1



Lower bound on interference 2

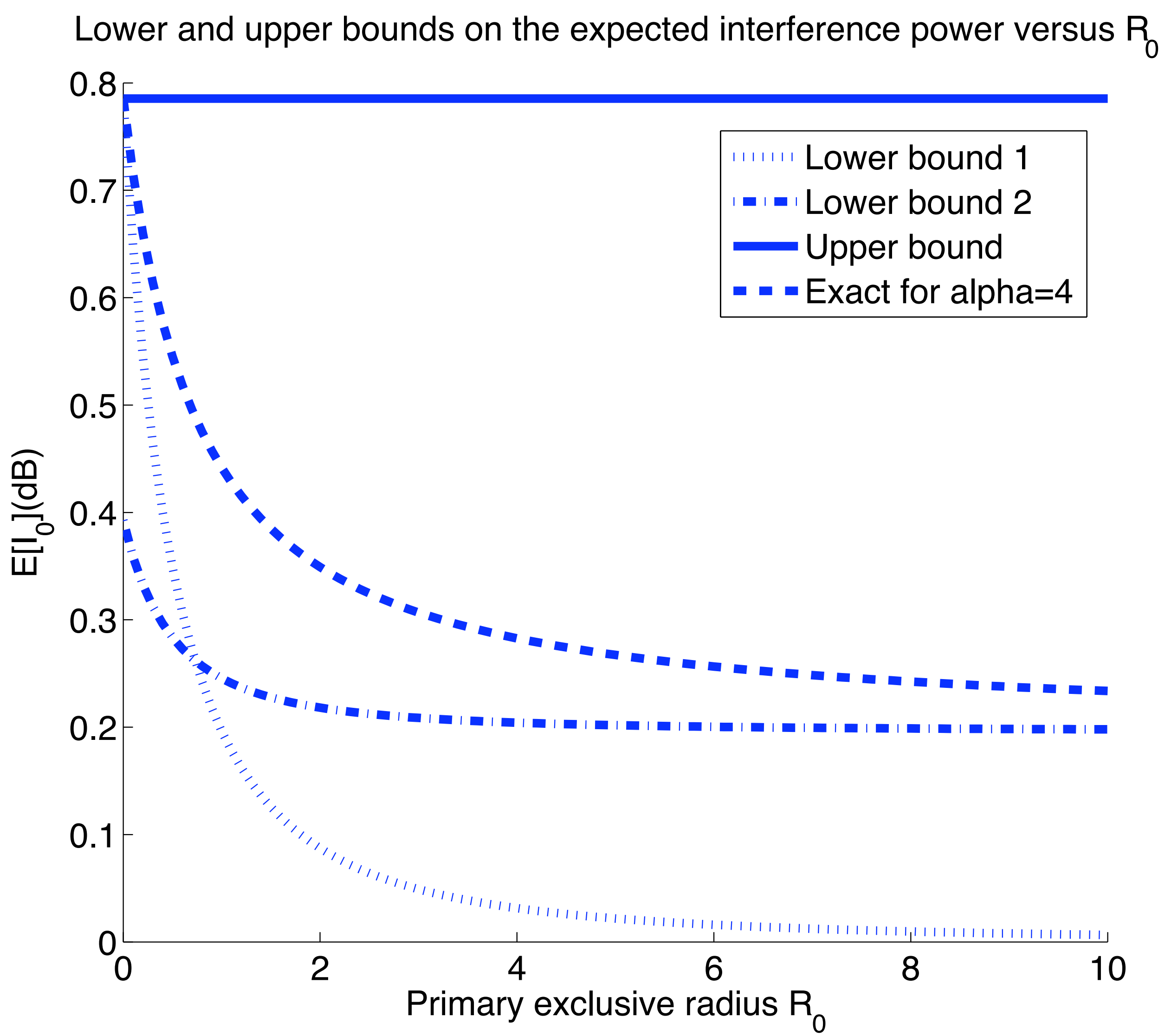


Outer bound on interference

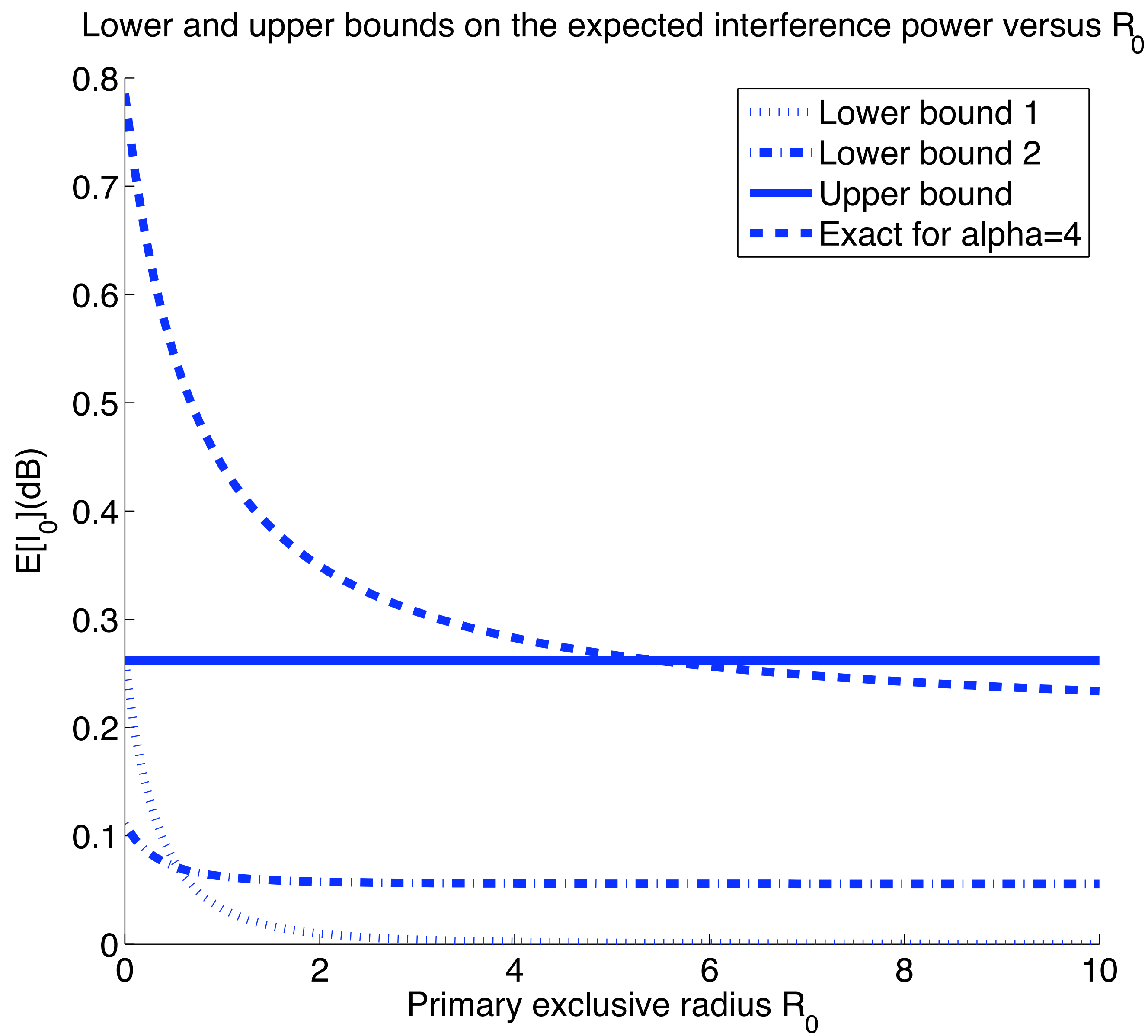


Lower and upper bounds on the expected interference power versus R0
Lower bound 1
Lower bound 2
Upper bound
Exact for alpha=4
E[I0](dB)
Primary exclusive radius R0
0
0.5
1
1.5
2
2.5
3
3.5
0
2
4
6
8
10

$\alpha = 3$

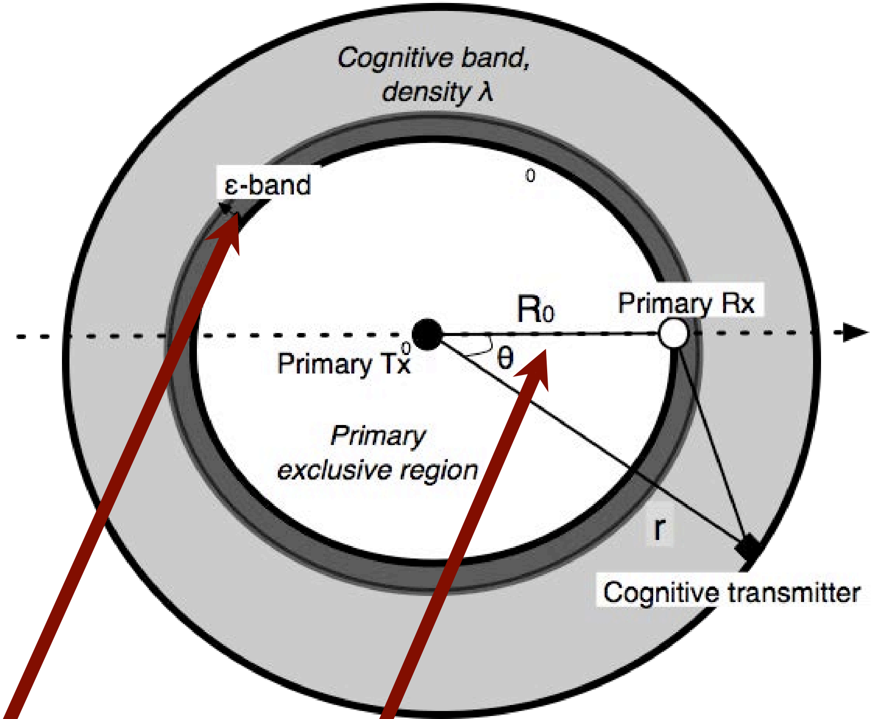

Lower and upper bounds on the expected interference power versus R_0
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$E[I_0]$(dB)
0
2
4
6
8
10
Primary exclusive radius R_0
Lower bound 1
Lower bound 2
Upper bound
Exact for alpha=4

α =4


Lower and upper bounds on the expected interference power versus R0
Lower bound 1
Lower bound 2
Upper bound
Exact for alpha=4
E[I0](dB)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
Primary exclusive radius R0

α =5

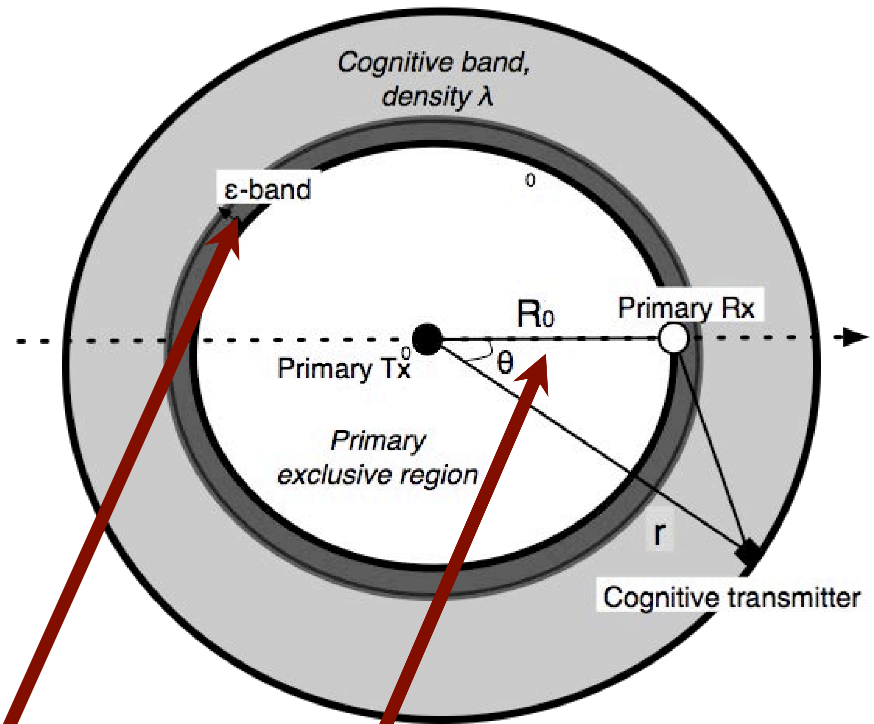
How to pick parameters



What R_0 and ϵ will guarantee $\Pr\left[\text{primary user's rate} \leq C_0\right] \leq \beta$?

Model I

How to pick parameters

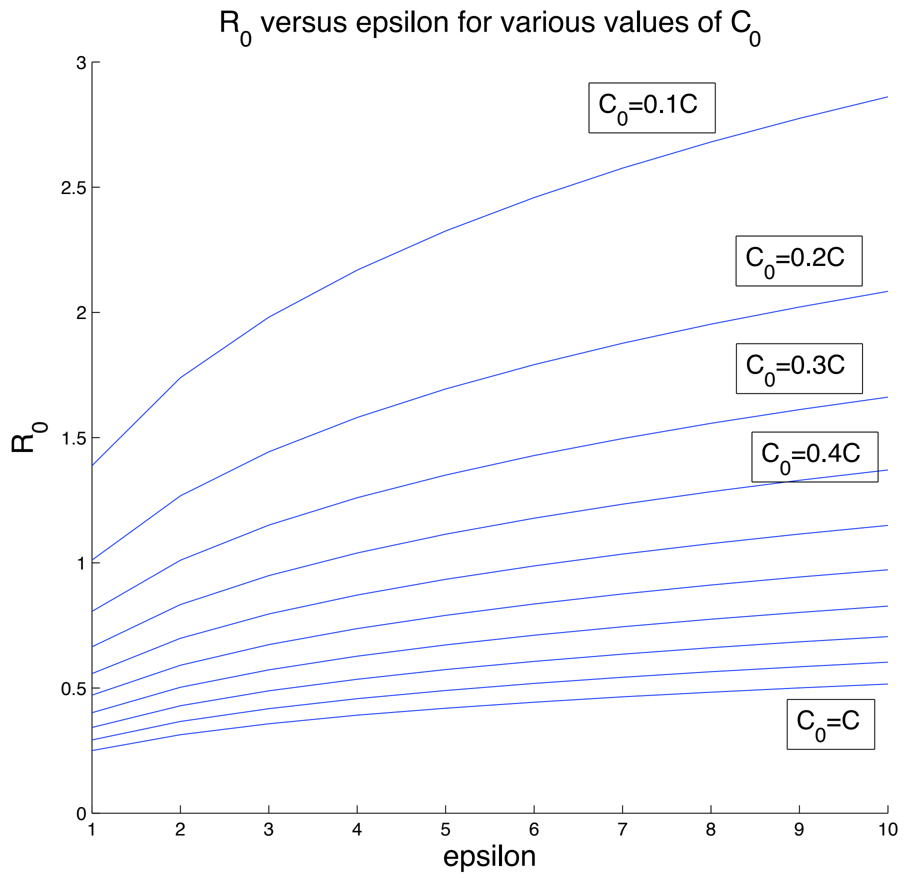


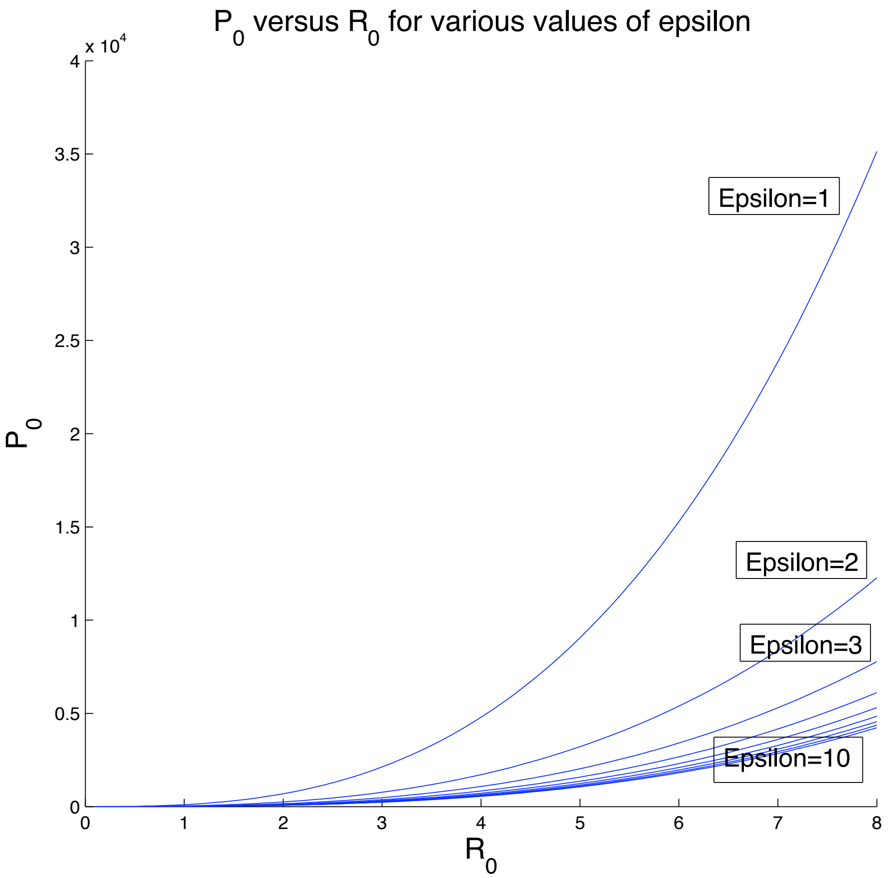
What R_0 and ϵ will guarantee $\Pr\left[\text{primary user's rate} \leq C_0\right] \leq \beta$?

Tradeoffs!

Model I

How to pick parameters

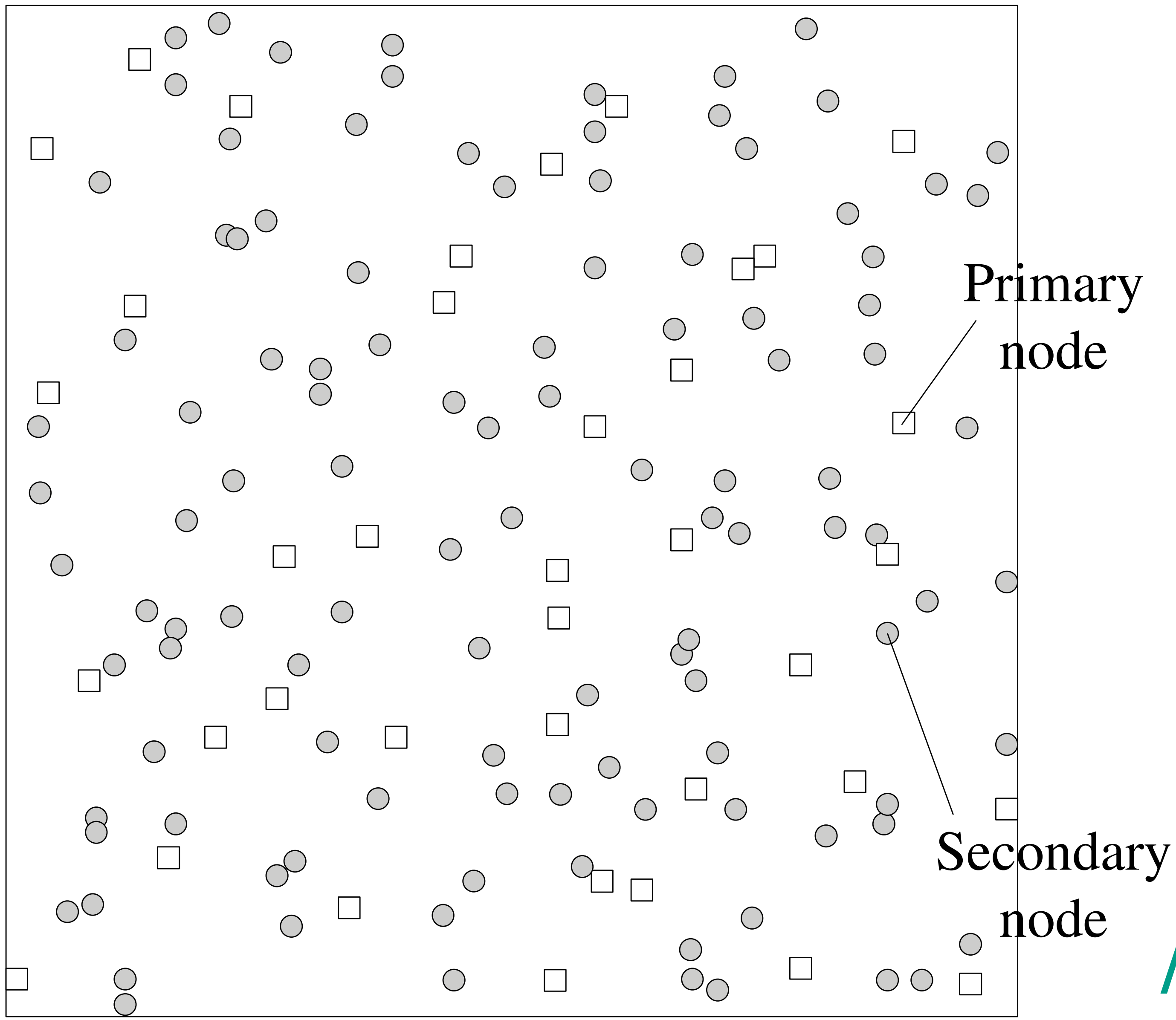




Model 1

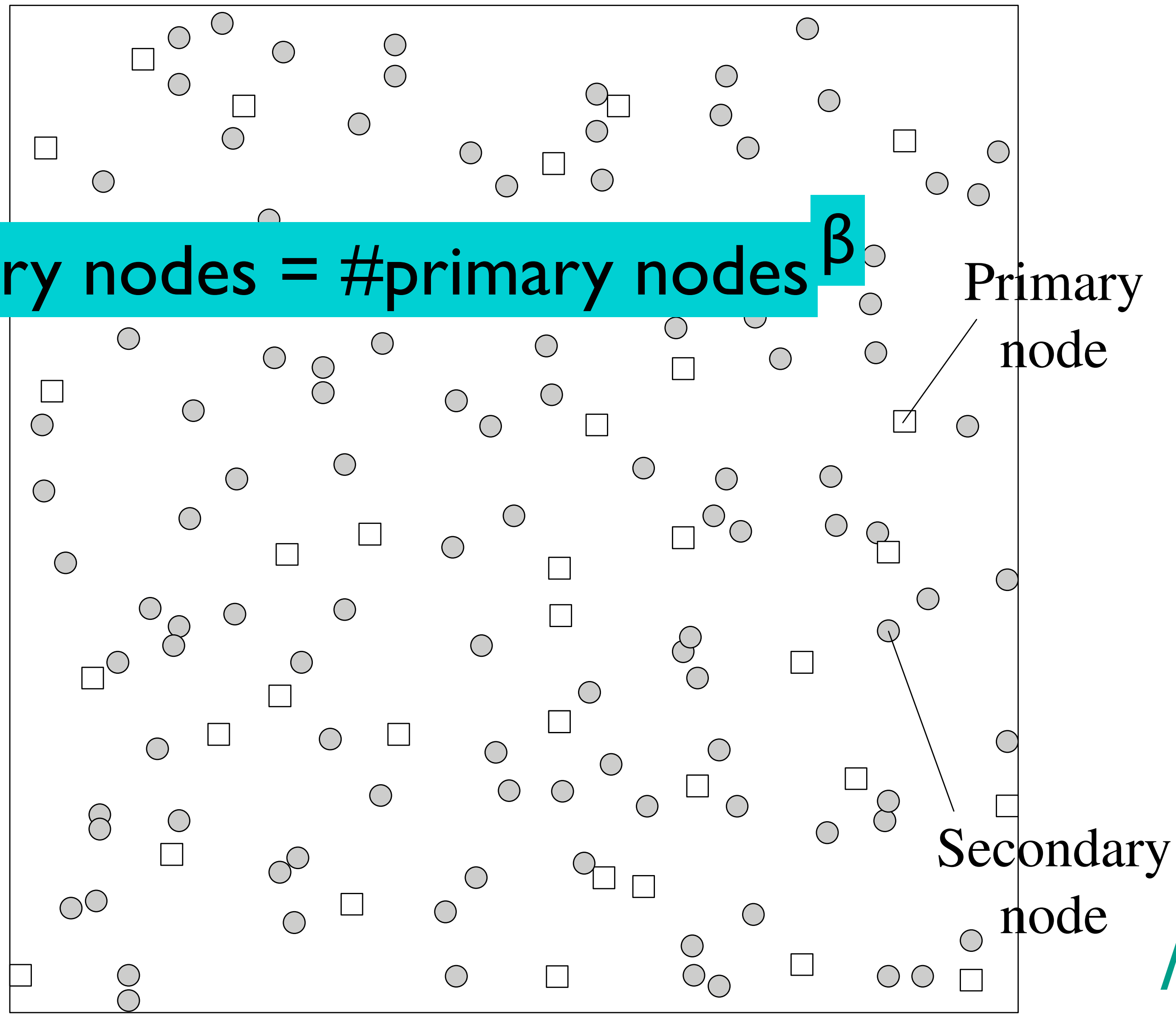
Model 2

Ad hoc cognitive networks



Model 2

Ad hoc cognitive networks



$$\#\text{secondary nodes} = \#\text{primary nodes}^{\beta}$$

Model 2

What we guarantee

Primary nodes act as if cognitive network does not exist

Primary nodes achieve same scaling law as if cognitive network does not exist

*[Jeon, Devroye, Vu, Chung, Tarokh 2008]**

Model 2

What we guarantee

Primary nodes act as if cognitive network does not exist

Primary nodes achieve same scaling law as if cognitive network does not exist

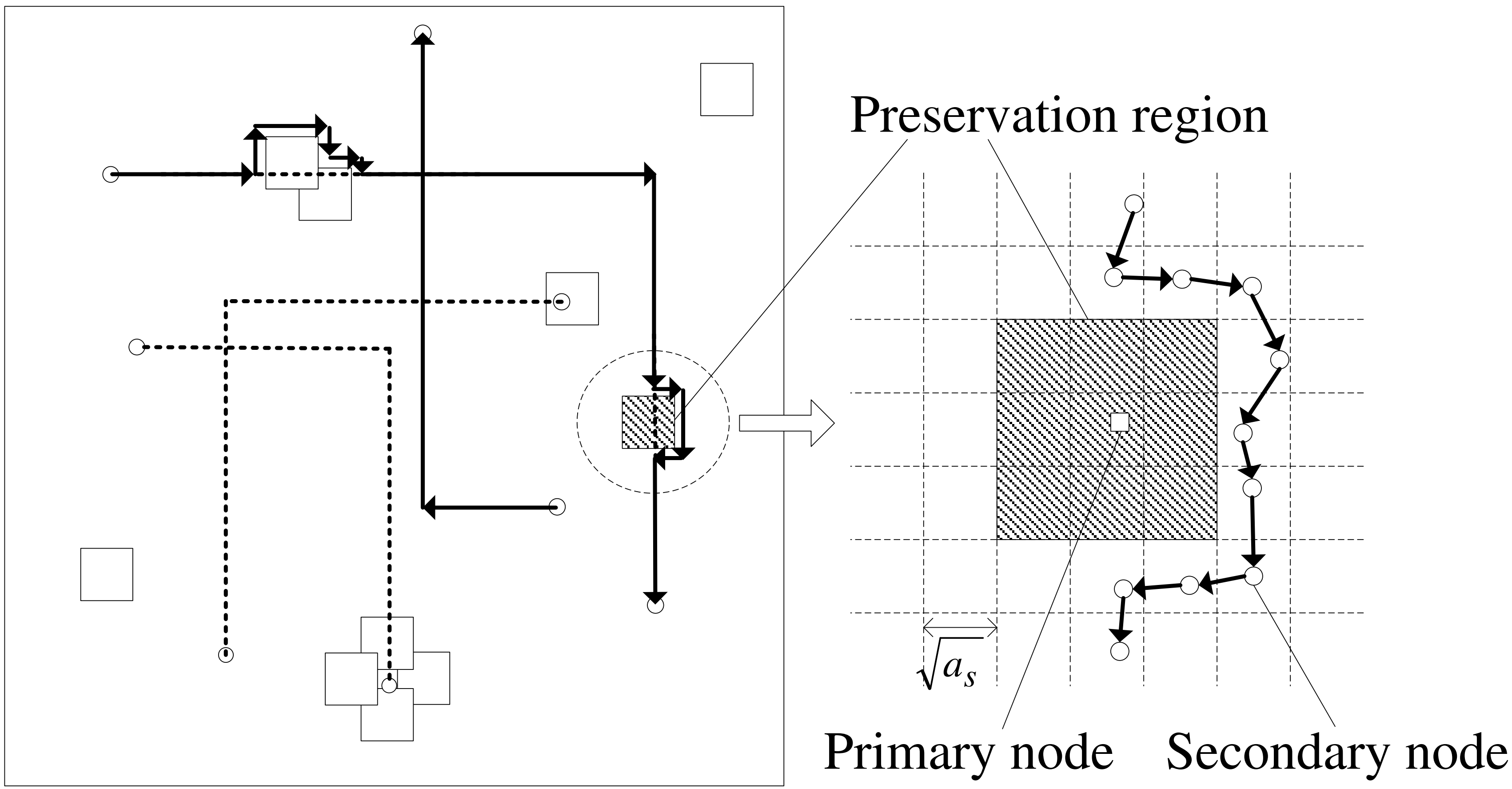
What we prove

$$T_p(n) = \Theta\left(\sqrt{\frac{1}{n \log n}}\right), \quad T_s(m) = \Theta\left(\sqrt{\frac{1}{m \log m}}\right)$$

*[Jeon, Devroye, Vu, Chung, Tarokh 2008]**

Model 2

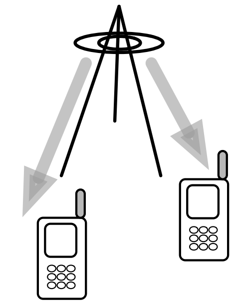
How



Improving Cellular Downlink Capacity

practical application of asymmetric cooperation

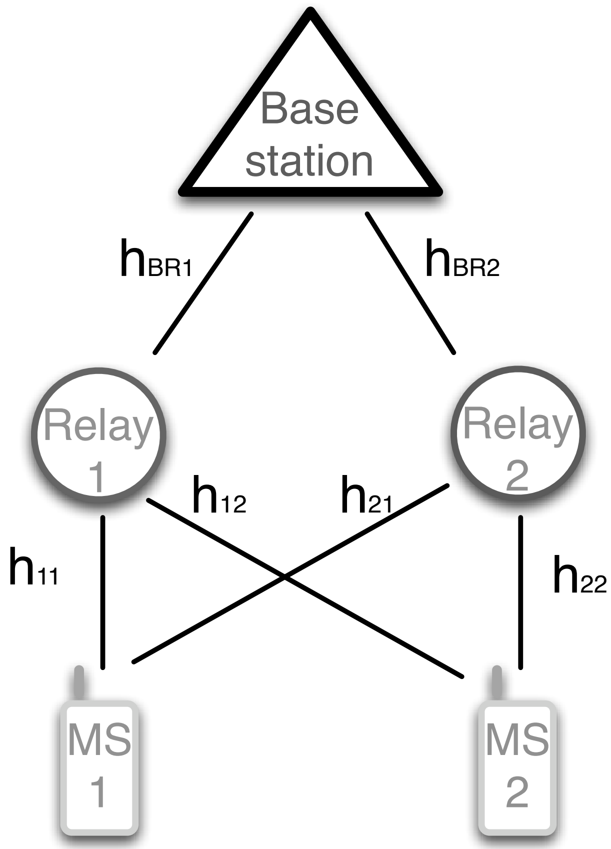
Motivation

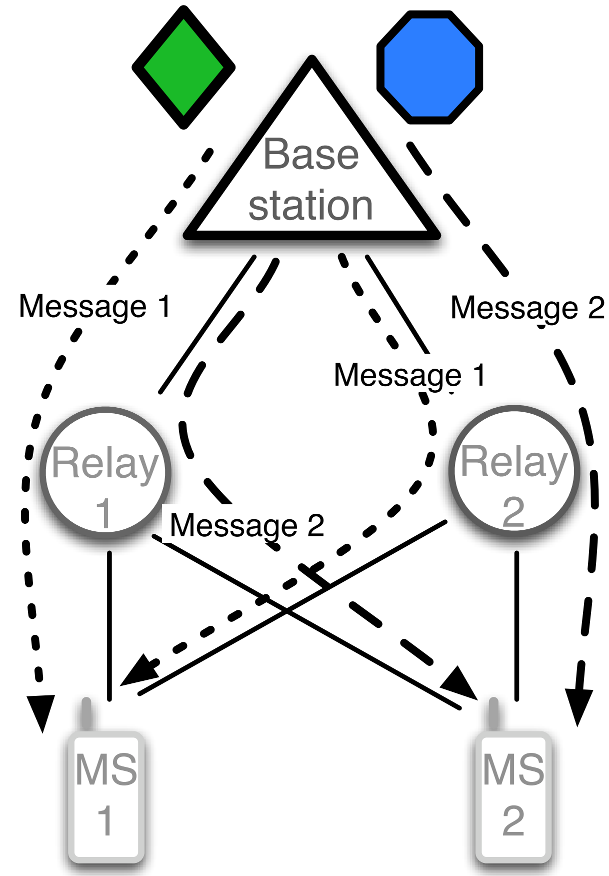


- Cellular providers are introducing *relays* to
  - extend cell coverage
  - boost transmission rates
  - improve spectral efficiency

All at lower costs than building new full-fledged base stations

Downlink cellular system

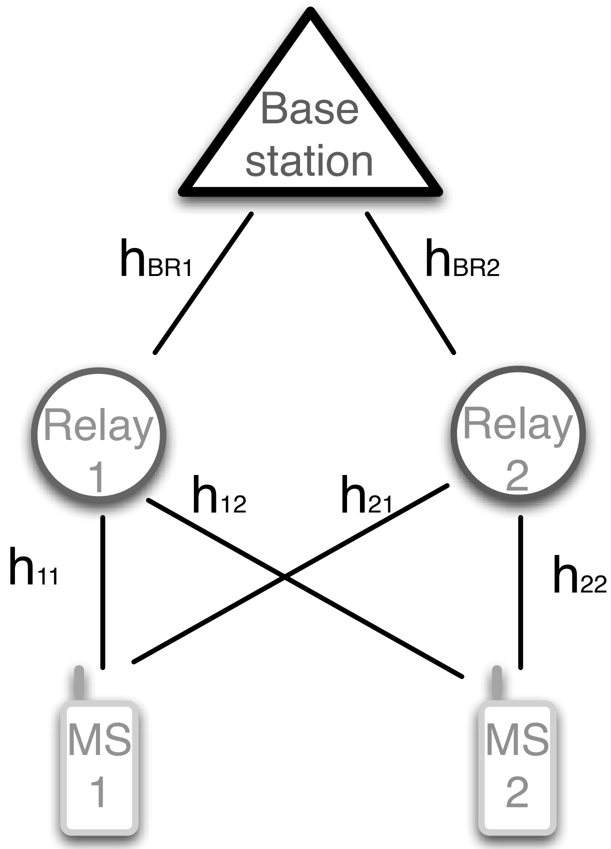


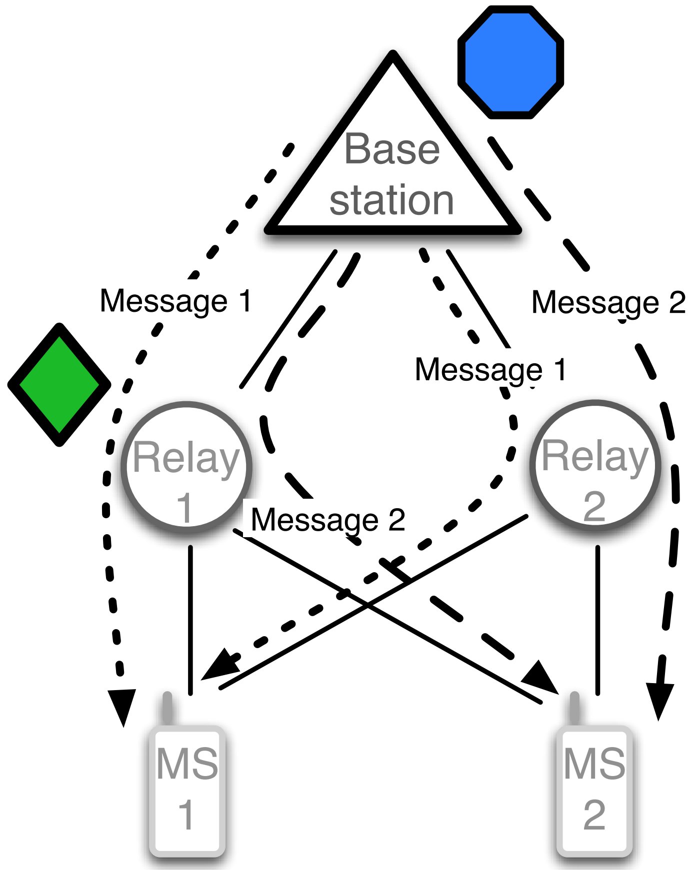


Downlink scheduling

1 base station, 2 relays, 2 mobiles, 2 messages

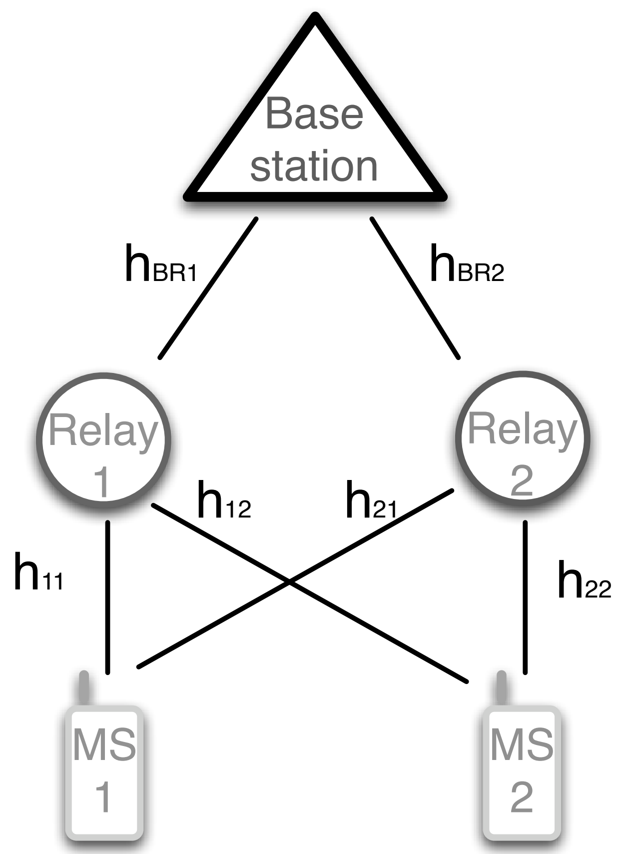
Downlink cellular system

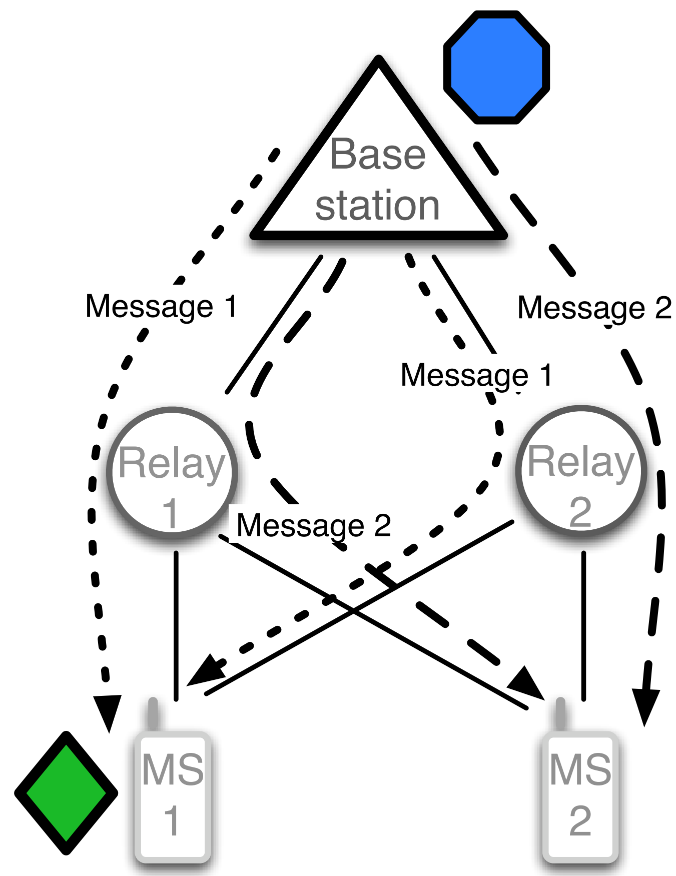




Downlink scheduling
1 base station, 2 relays, 2 mobiles, 2 messages

Downlink cellular system

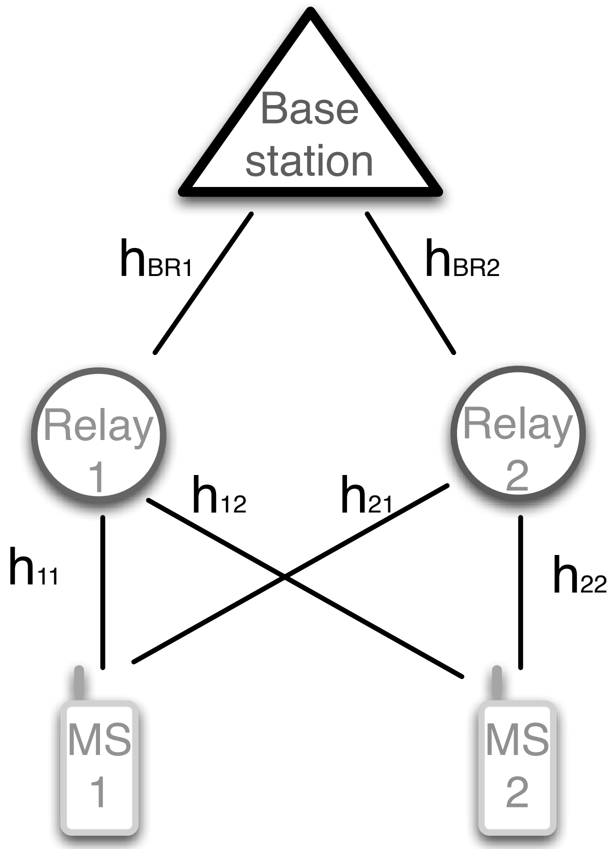


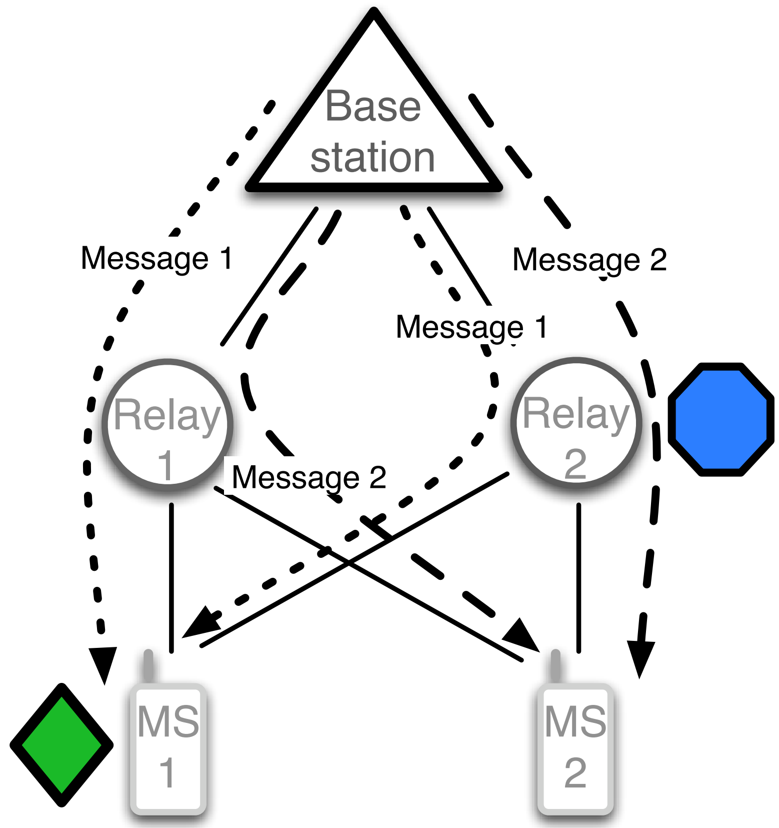


Downlink scheduling

1 base station, 2 relays, 2 mobiles, 2 messages

Downlink cellular system

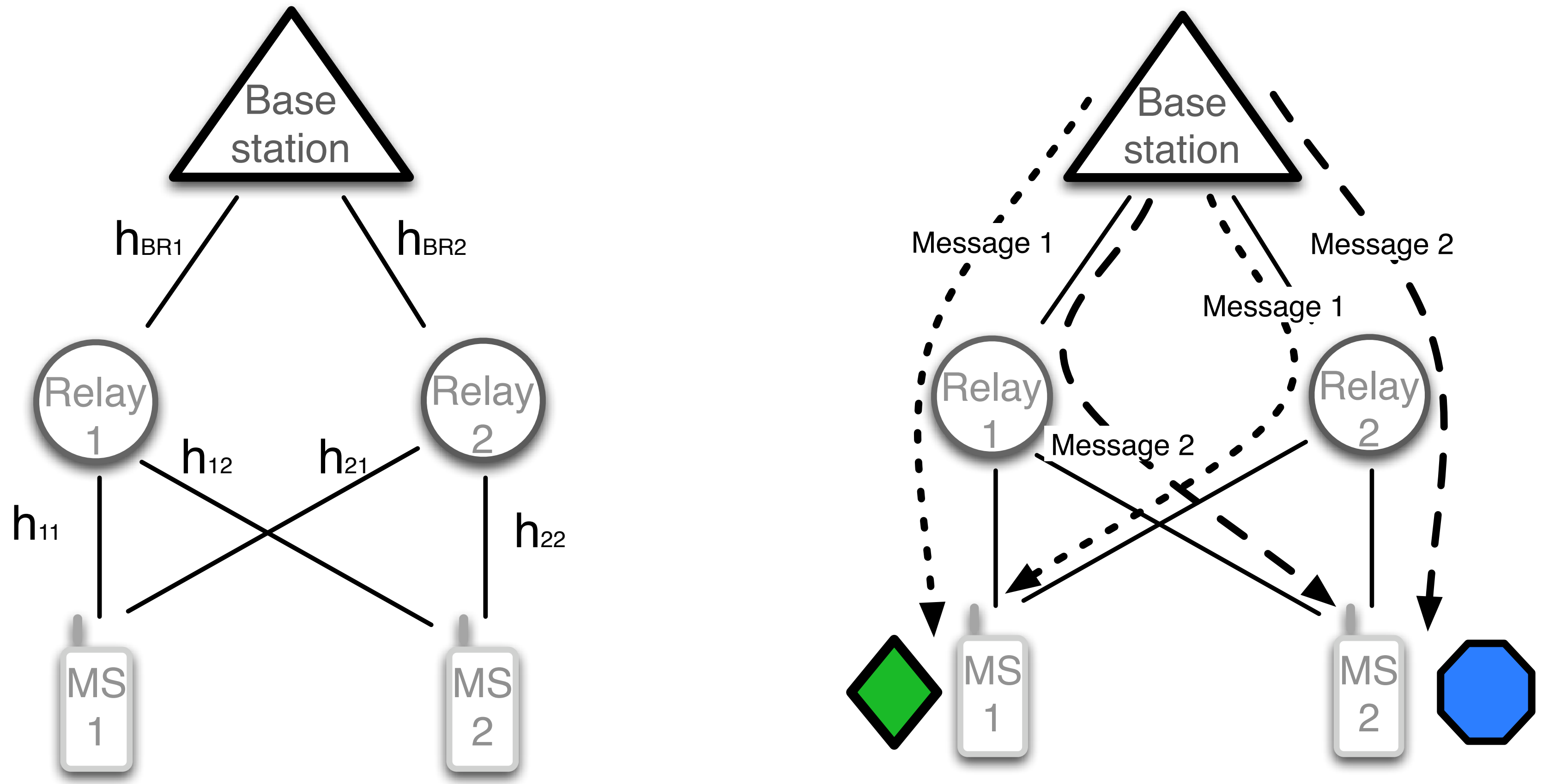




Downlink scheduling

1 base station, 2 relays, 2 mobiles, 2 messages

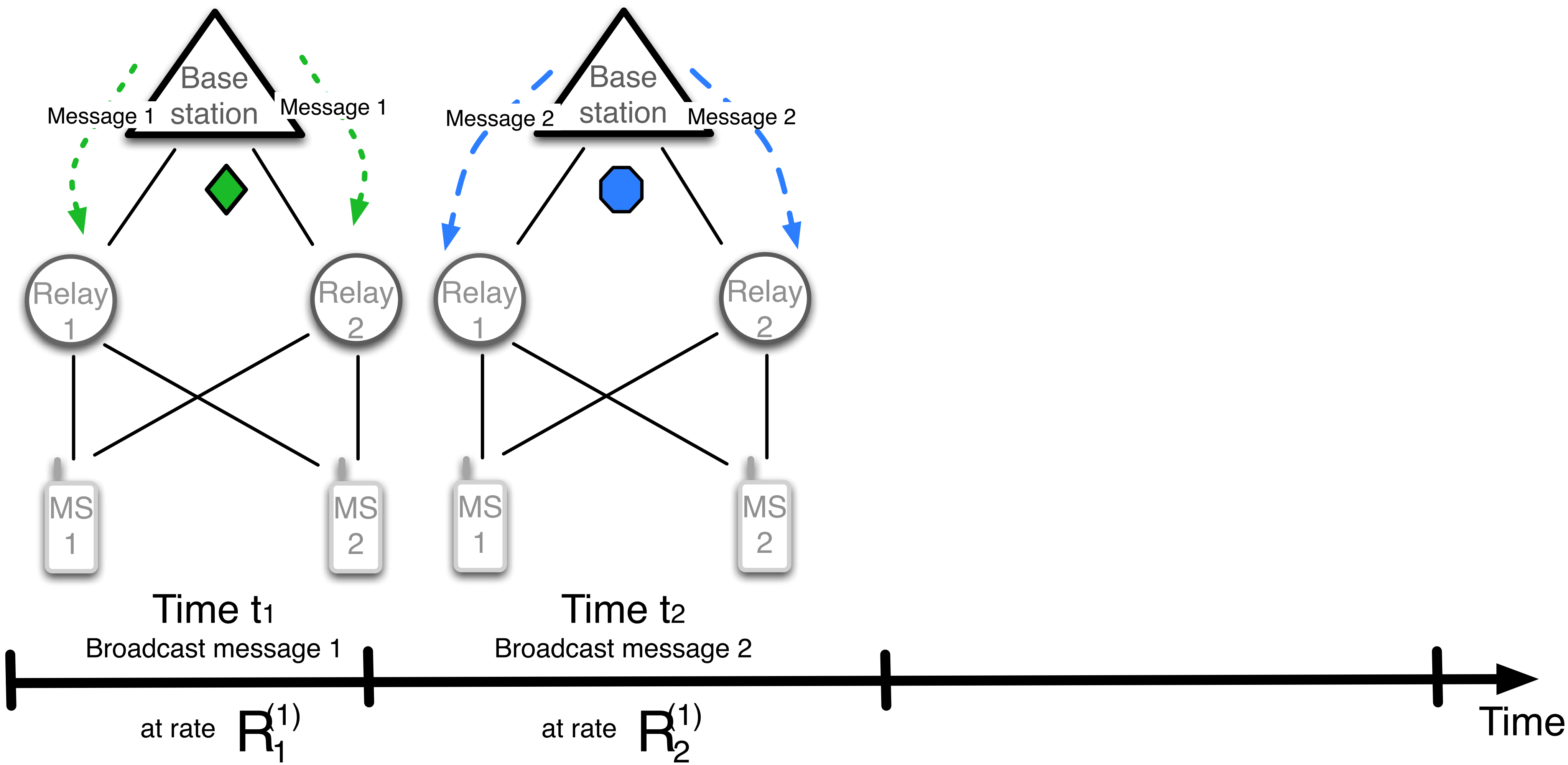
Downlink cellular system



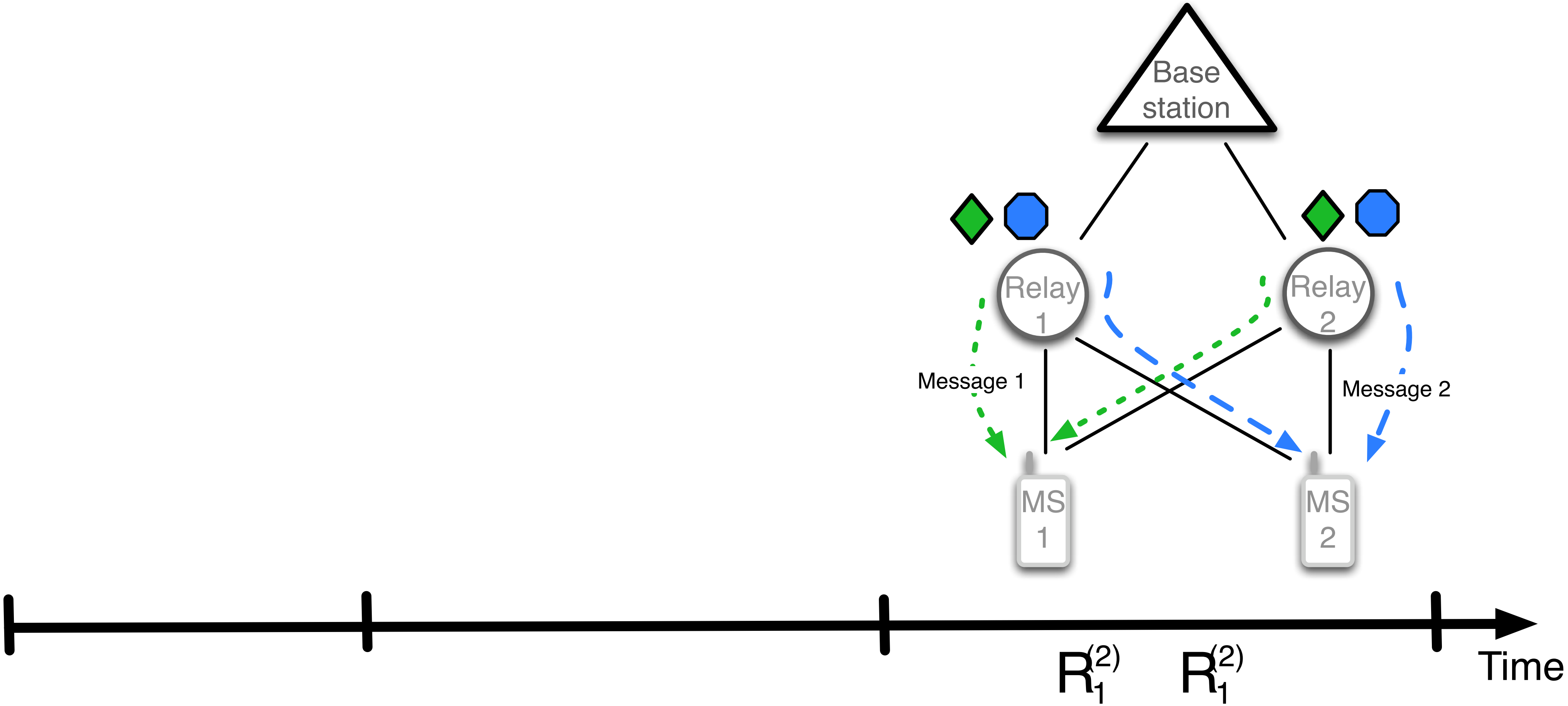
Downlink scheduling

1 base station, 2 relays, 2 mobiles, 2 messages

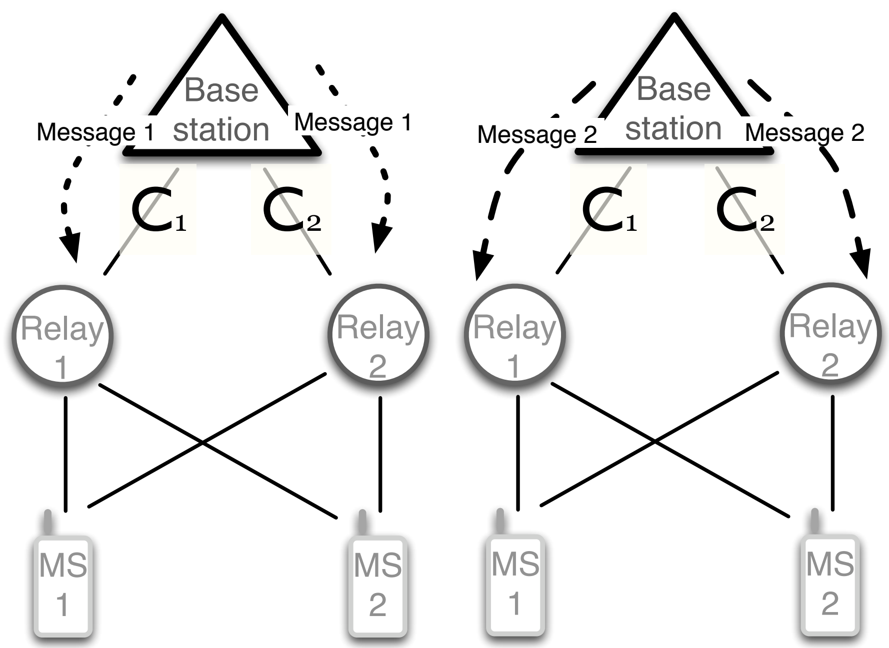
Problem setup: phase 1



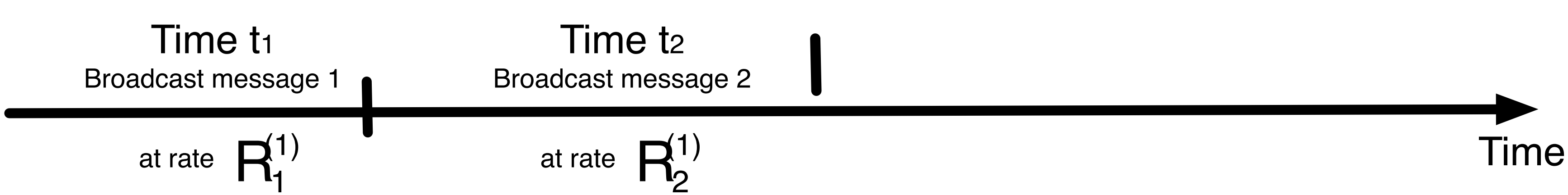
Problem setup: phase 2



Phase 1 may result in asymmetry



Phase 1: TDMA

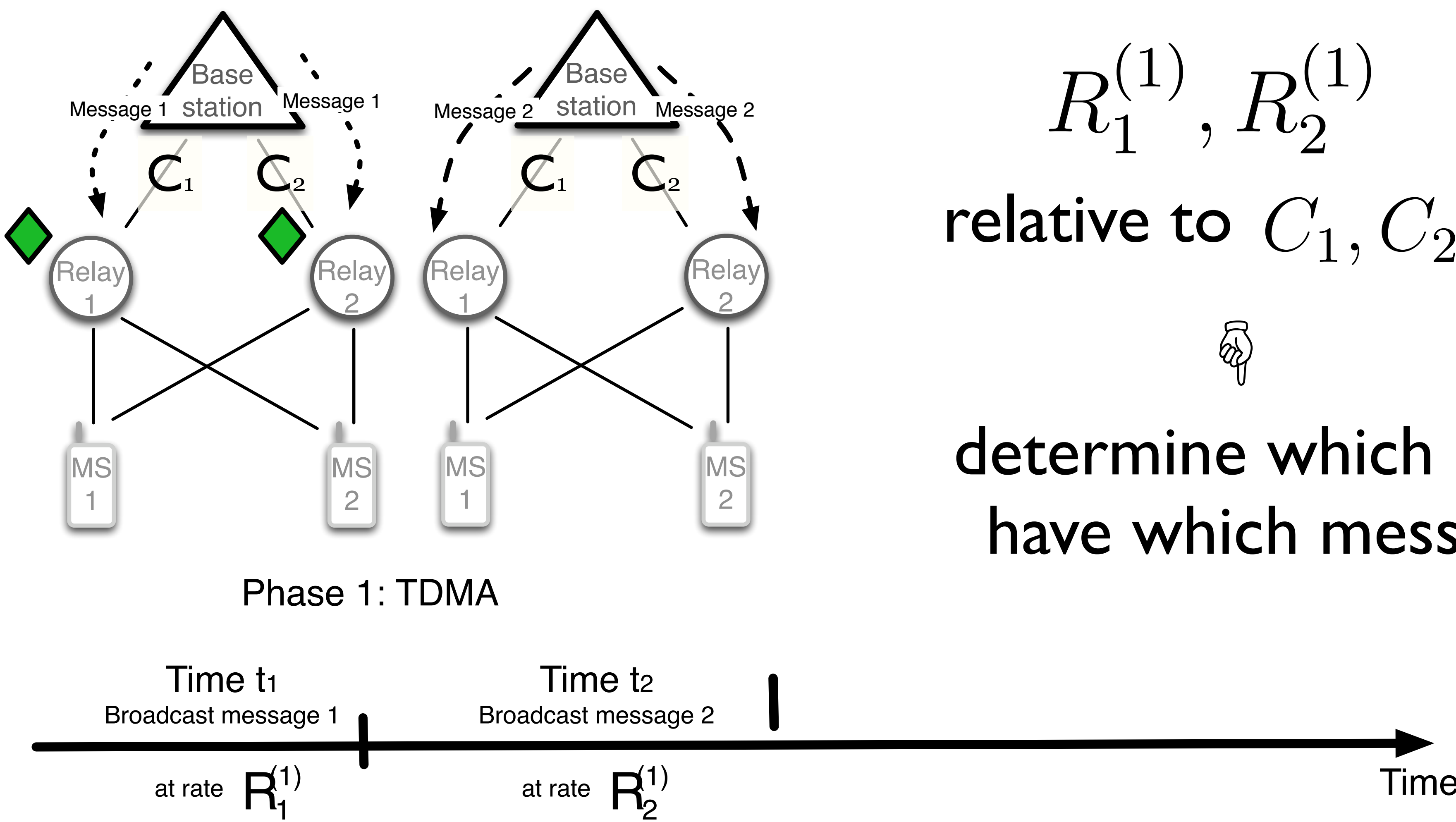


$R_1^{(1)}, R_2^{(1)}$

relative to C_1, C_2

determine which relays have which messages!

Phase 1 may result in asymmetry

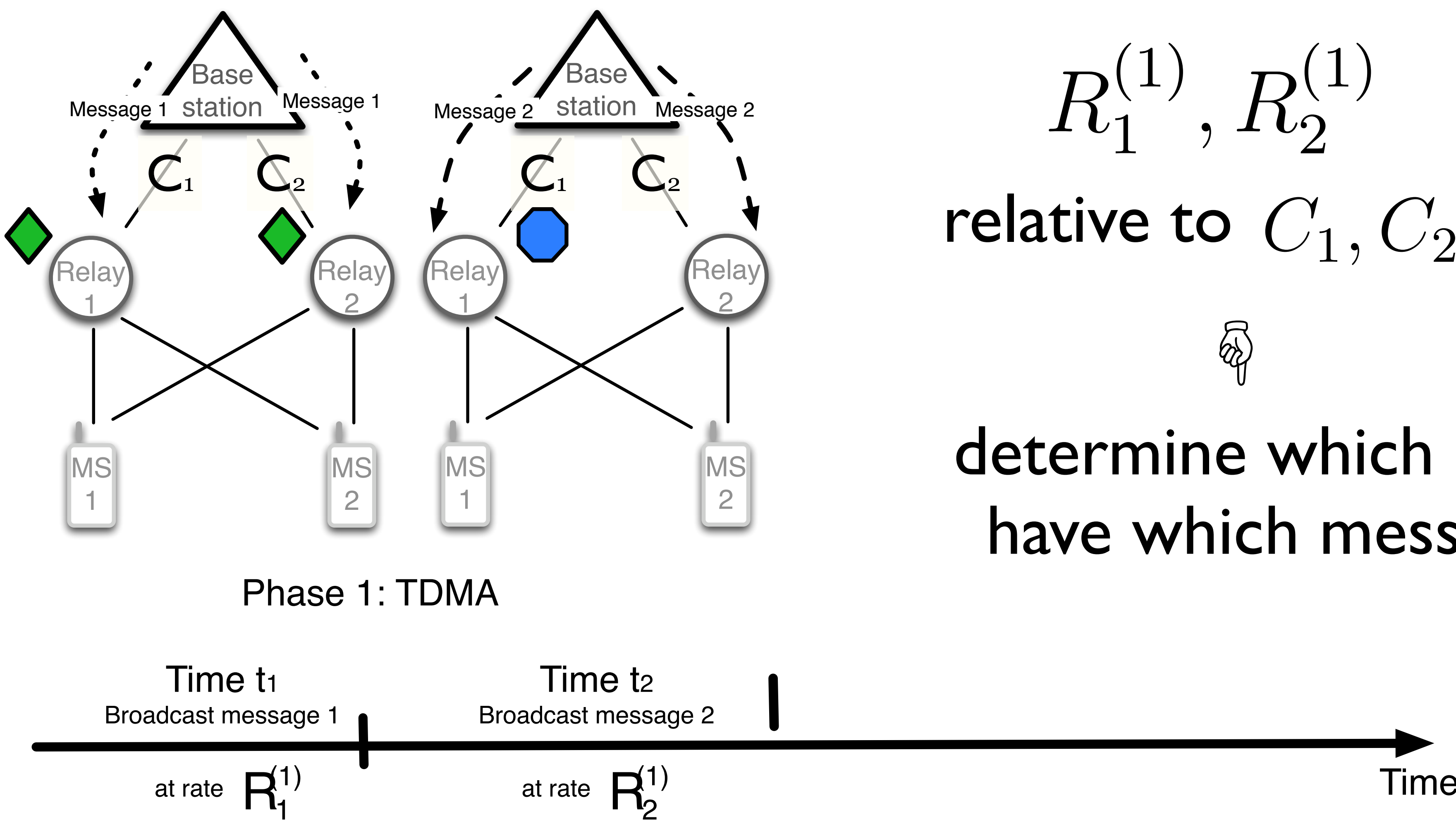


$R_1^{(1)}, R_2^{(1)}$

relative to C_1, C_2

determine which relays have which messages!

Phase 1 may result in asymmetry

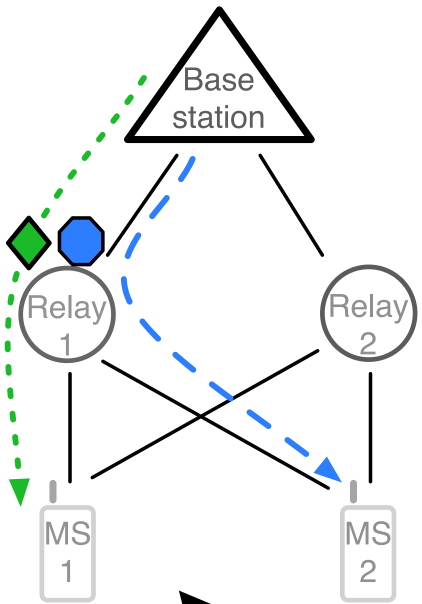


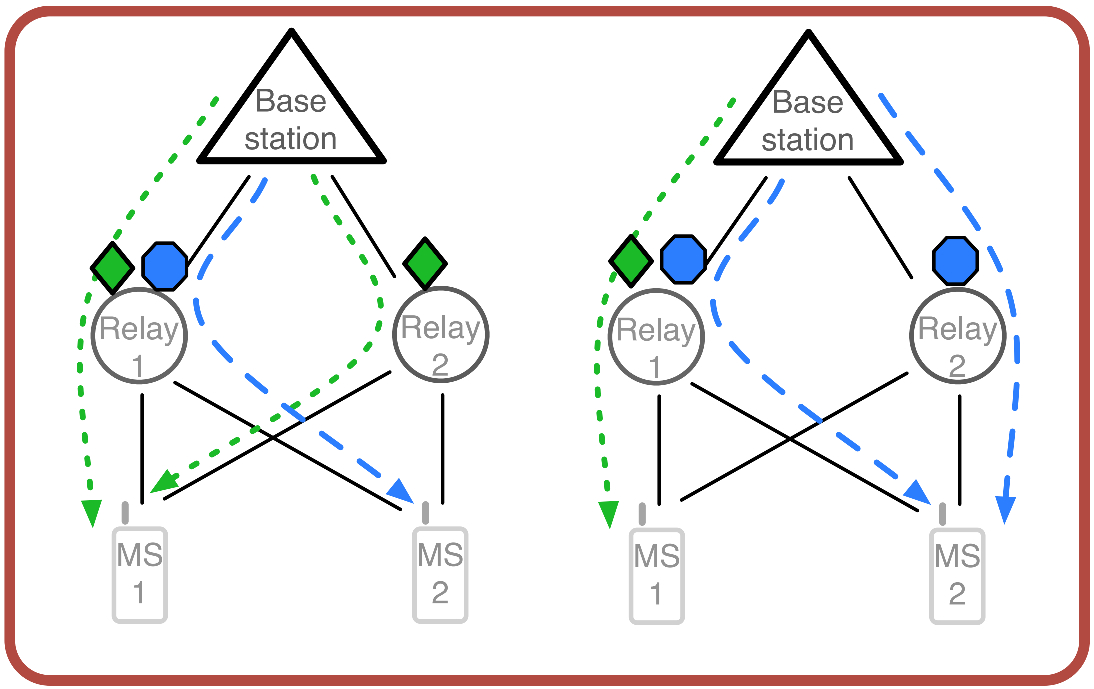
$$R_1^{(1)}, R_2^{(1)}$$

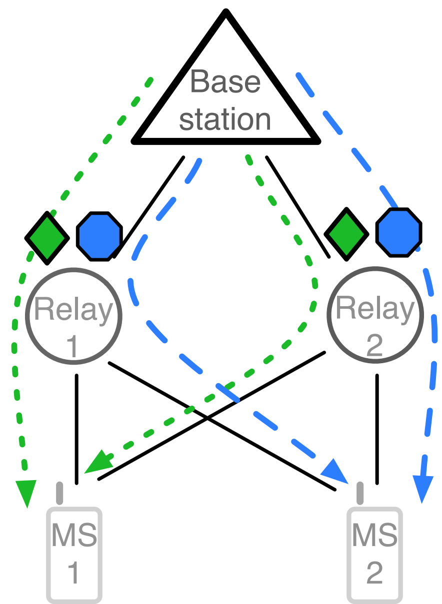
relative to C_1, C_2

determine which relays have which messages!

4 message knowledge cases





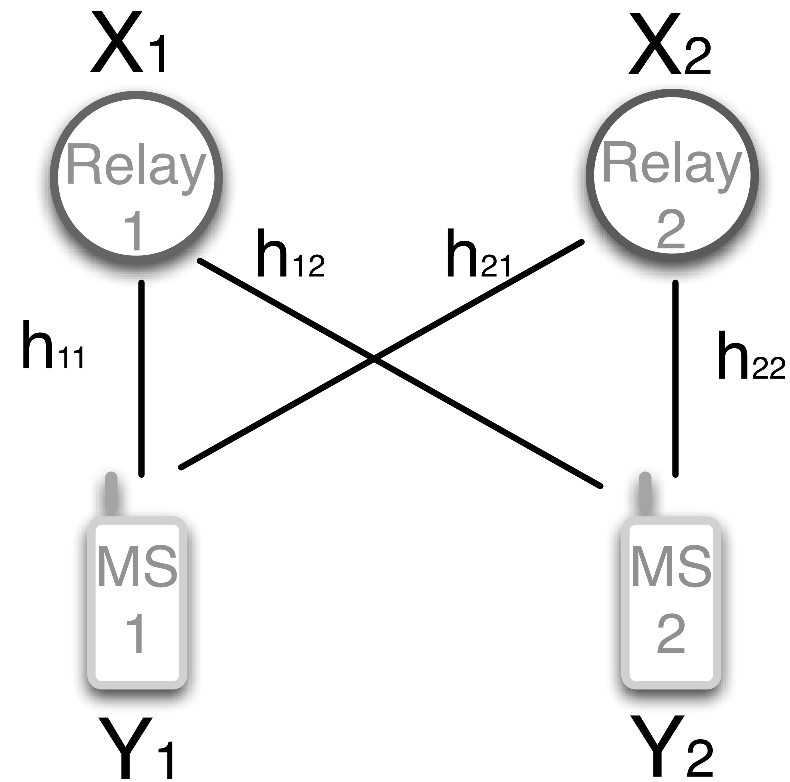


Asymmetric message knowledge

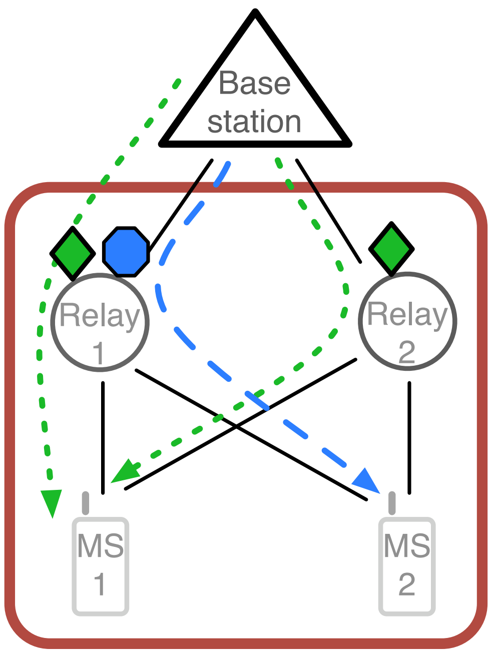
Symmetric message knowledge

Phase 2

$$\overbrace{\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}}^{\text{Output}} = \underbrace{\begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix}}_{\substack{\text{Channel} \\ \text{given, known}}} \underbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}_{\substack{\textbf{to be} \\ \textbf{determined,} \\ \textbf{depends on what} \\ \textbf{messages the} \\ \textbf{relays have!}}} + \overbrace{\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}}^{\text{Noise}}$$

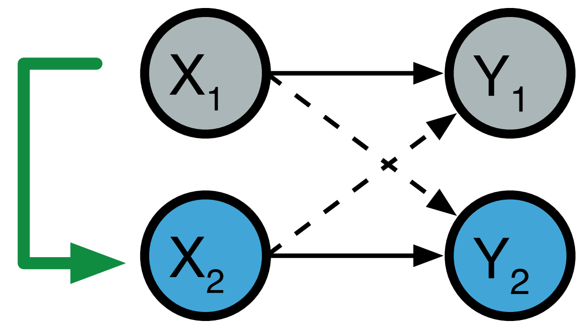


Equivalence to cognitive radio channel



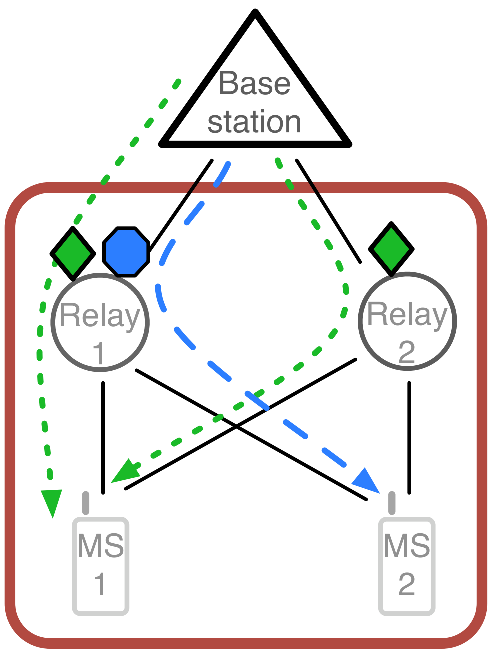
Asymmetric cases

$\equiv$

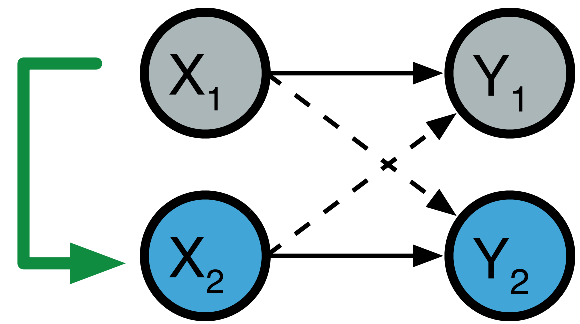


Cognitive radio channel

Equivalence to cognitive radio channel



≡



Asymmetric cases

Cognitive radio channel

☞ linear precoding

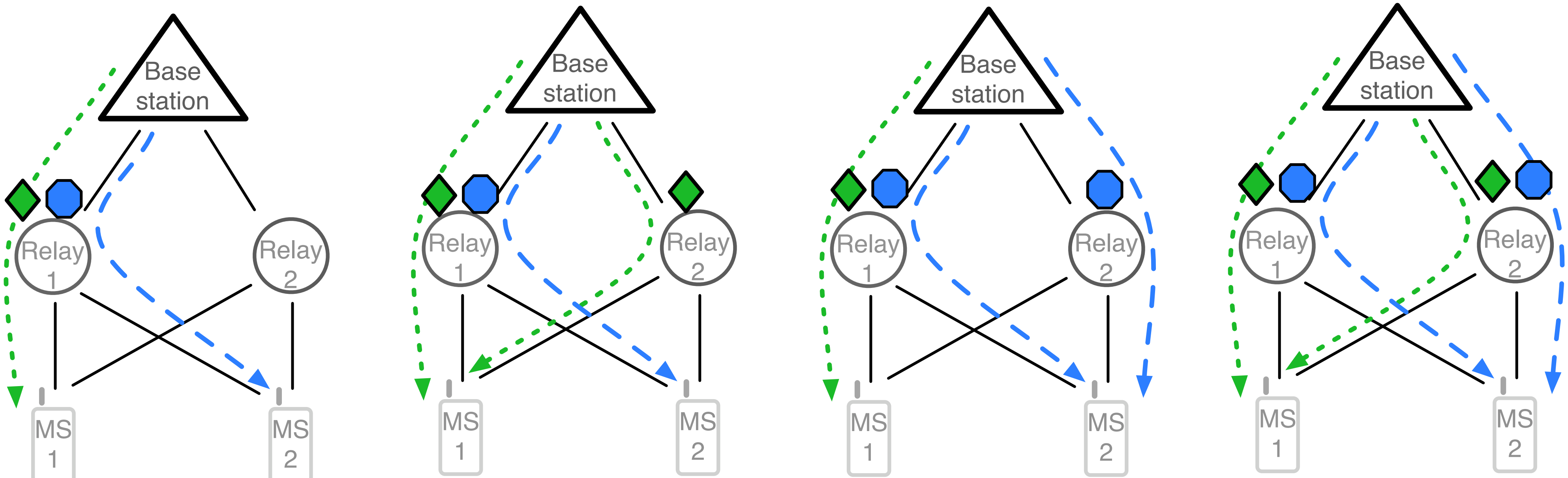
☞ dirty-paper coding

Linear precoding

$$\overbrace{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}^{\substack{\text{Transmitted signal} \\ \text{at the 2 relays, resp.}}} = \underbrace{\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}}_{\text{Precoding matrix B}} \overbrace{\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}}^{\text{Unit power messages}}$$

- Power constraints: $|b_{11}|^2 + |b_{12}|^2 + |b_{21}|^2 + |b_{22}|^2 \leq P_R$
- Message knowledge constraints: B has zeros

Linear precoding



$$B = \begin{bmatrix} b_{11} & b_{12} \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Constraints on the precoding matrices imposed by phase 1

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \overbrace{\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}}^{\mathrm{B}} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

Linear precoding

$$
\begin{aligned}
Y &= HX + N \\
&= HBU + N
\end{aligned}
$$

Linear precoding

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$$

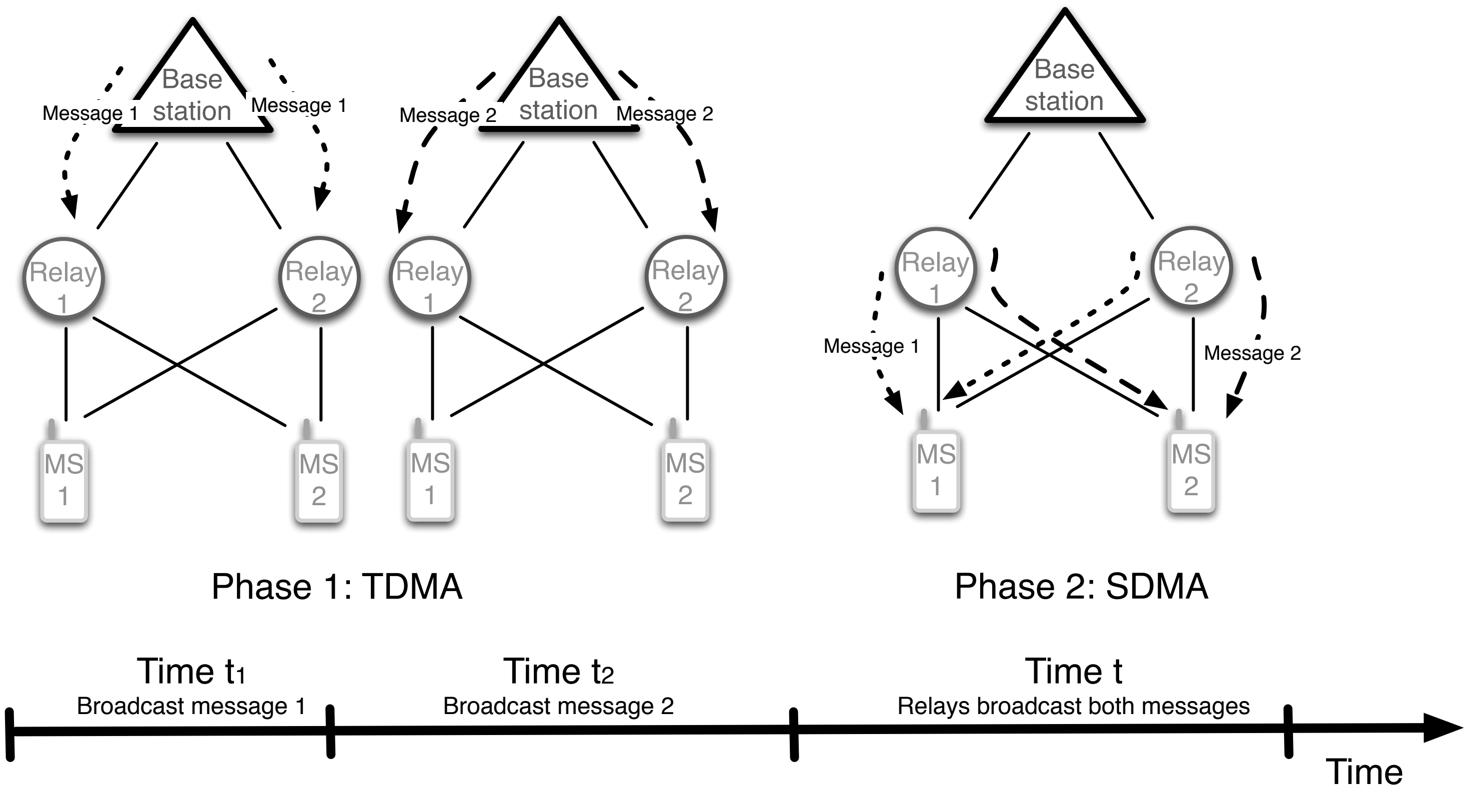
$$= \begin{bmatrix} (h_{11}b_{11} + h_{21}b_{21})U_1 + (h_{11}b_{12} + h_{21}b_{22})U_2 \\ (h_{12}b_{11} + h_{22}b_{21})U_1 + (h_{12}b_{12} + h_{22}b_{22})U_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}.$$

Receive SINR at Y1 $\quad \gamma_1 = \dfrac{(h_{11}b_{11} + h_{21}b_{21})^2}{(h_{11}b_{12} + h_{21}b_{22})^2 + N_1}$

Receive SINR at Y2 $\quad \gamma_2 = \dfrac{(h_{12}b_{12} + h_{22}b_{22})^2}{(h_{12}b_{11} + h_{22}b_{21})^2 + N_2}$

$$R_1^{(2)} = \frac{1}{2}\log_2(1+\gamma_1), \qquad R_2^{(2)} = \frac{1}{2}\log_2(1+\gamma_2)$$

Optimization



Need to select:

Phase 1: $t_1^{(1)},\ t_2^{(1)},\ R_1^{(1)},\ R_2^{(1)},\ n_1,\ n_2$

Phase 2: linear precoding B matrix

☞ We will do so according to two optimization criteria

Max throughput criterion

☞ Maximize the throughput over 2 phases

$$
\begin{aligned}
\max \quad & \frac{n_1 + n_2}{\frac{n_1}{R_1^{(1)}} + \frac{n_2}{R_2^{(1)}} + \max\left(\frac{n_1}{R_1^{(2)}}, \frac{n_2}{R_2^{(2)}}\right)} \\
\text{s.t.} \quad & n_1, n_2 \geq 0
\end{aligned}
$$

$$R_1^{(1)}, R_2^{(1)} \text{ are phase 1 rates}$$

$$R_1^{(2)}, R_2^{(2)} \text{ are phase 2 rates}$$

n_1, n_2 are number of bits sent to each mobile, variables to be optimized over

Extreme fairness criterion

☞ Maximize the throughput when forced to send one unit of information to each mobile

$$\max \quad \frac{n_1 + n_2}{\frac{n_1}{R_1^{(1)}} + \frac{n_2}{R_2^{(1)}} + \max\left(\frac{n_1}{R_1^{(2)}}, \frac{n_2}{R_2^{(2)}}\right)}$$

$$\text{s.t.} \quad n_1, n_2 \geq 0$$

$$R_1^{(1)}, R_2^{(1)} \text{ are phase 1 rates}$$

$$R_1^{(2)}, R_2^{(2)} \text{ are phase 2 rates}$$

$$\boxed{n_1 = n_2}$$

Optimization reduction

We reduce the non-linear optimization problem from one over 8 variables to one over 2 variables.

Max throughput optimization:

$$\max_{\substack{x, b_{11},\\ \alpha,\beta,\theta}} \frac{x+1}{x/R_1^{(1)} + 1/R_2^{(1)} + \max\left(x/\log_2\left(1+\frac{|h_{11}|^2}{|\alpha|^2+N_1/|b_{11}|^2}\right),\ 1/\log_2\left(1+\frac{|\beta|^2}{|h_{12}|^2+N_2/|b_{11}|^2}\right)\right)}$$

$$\text{s.t.} \quad x \ge 0 \quad |b_{11}|^2 \le P_R$$

$$g_1|\alpha|^2 + 2|g_{12}||\alpha||\beta|\cos(\theta_G+\theta) + g_2|\beta|^2 \le P_R/|b_{11}|^2 - 1$$

Simplified max throughput optimization:

$$\max_{t, |b_{11}|} \frac{x+1}{x/R_1^{(1)} + 1/R_2^{(1)} + \max\left(x/\log_2\left(1+\frac{|h_{11}|^2}{|\alpha(t,b_{11},\theta^*)|^2+N_1/|b_{11}|^2}\right),\ 1/\log_2\left(1+\frac{|\beta(t,b_{11},\theta^*)|^2}{|h_{12}|^2+N_2/|b_{11}|^2}\right)\right)}$$

$$\text{s.t.} \quad x \in \{0, x^*, \infty\}, \quad |b_{11}|^2 \le P_R, \quad t \in [0,\pi], \quad \theta^* \in \{-\theta_G, \pi-\theta_G\}$$

Optimization summary

Optimization

$$\max_{\substack{\mathbf{B}, n_1, n_2\\ R_1^{(1)}, R_2^{(1)}}} \frac{n_1+n_2}{\frac{n_1}{R_1^{(1)}}+\frac{n_2}{R_2^{(1)}}+\max\left(\frac{n_1}{\log_2(1+\gamma_1)}, \frac{n_2}{\log_2(1+\gamma_2)}\right)}$$

$$\text{s.t.} \quad n_1, n_2 \geq 0$$

$$|b_{11}|^2+|b_{12}|^2+|b_{21}|^2+|b_{22}|^2 \leq P_R$$

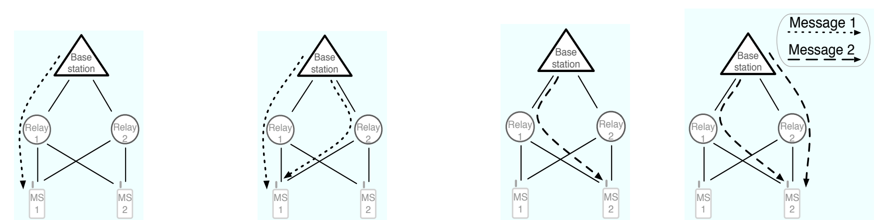
$$\text{If } R_1^{(1)} \geq \log_2\left(1+|h_{BR_1}|^2 P_B\right) \text{ then } b_{11}=0$$

$$\text{If } R_2^{(1)} \geq \log_2\left(1+|h_{BR_1}|^2 P_B\right) \text{ then } b_{12}=0$$

$$\text{If } R_1^{(1)} \geq \log_2\left(1+|h_{BR_2}|^2 P_B\right) \text{ then } b_{21}=0$$

$$\text{If } R_2^{(1)} \geq \log_2\left(1+|h_{BR_2}|^2 P_B\right) \text{ then } b_{22}=0$$

Step 1: Single message



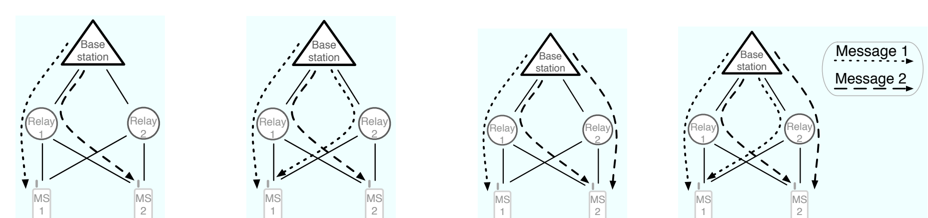
$R_1^{(1)} = \log_2(1+|h_{BR_1}|^2 P_B)$, $R_1^{(2)} = \log_2(1+|h_{11}|^2 P_R)$

$R_1^{(1)} = \log_2(1+|h_{BR_2}|^2 P_B)$, $R_1^{(2)} = \log_2(1+(|h_{11}|^2+|h_{21}|^2) P_R)$

$R_2^{(1)} = \log_2(1+|h_{BR_1}|^2 P_B)$, $R_2^{(2)} = \log_2(1+|h_{12}|^2 P_R)$

$R_2^{(1)} = \log_2(1+|h_{BR_2}|^2 P_B)$, $R_2^{(2)} = \log_2(1+(|h_{12}|^2+|h_{22}|^2) P_R)$

Step 2: Dual message

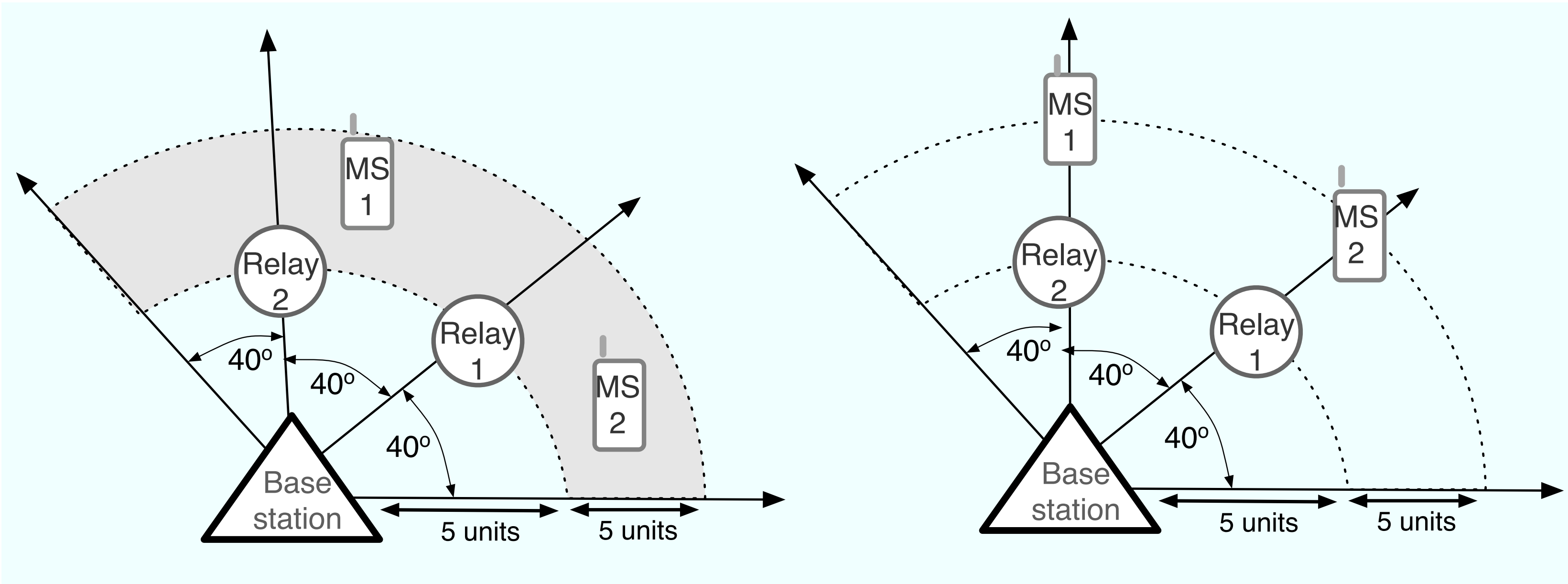


Step 3: Select best

Max throughput

Simulation Setup

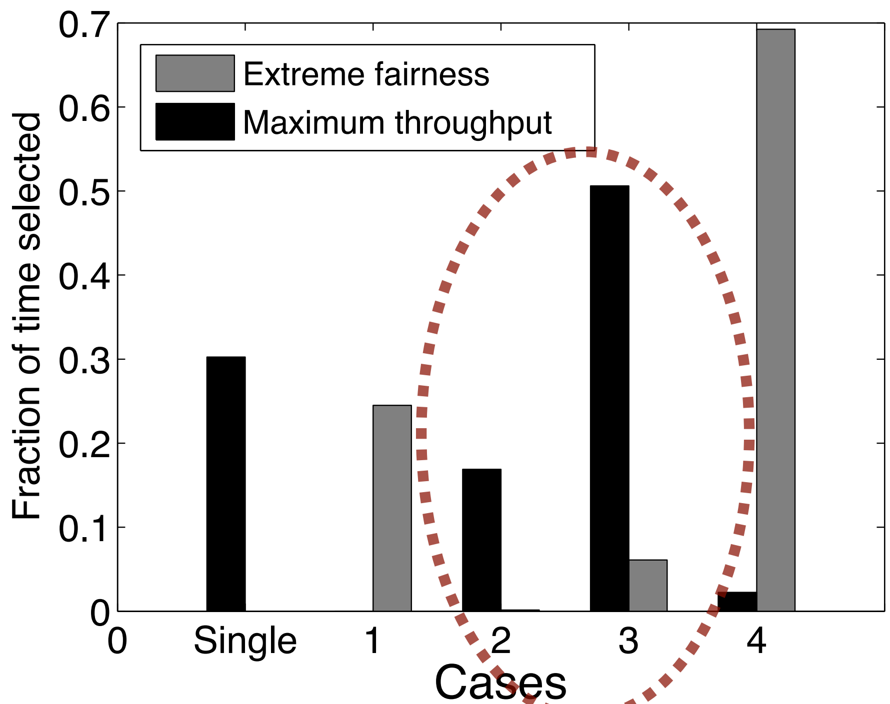
Rayleigh fading, pathloss



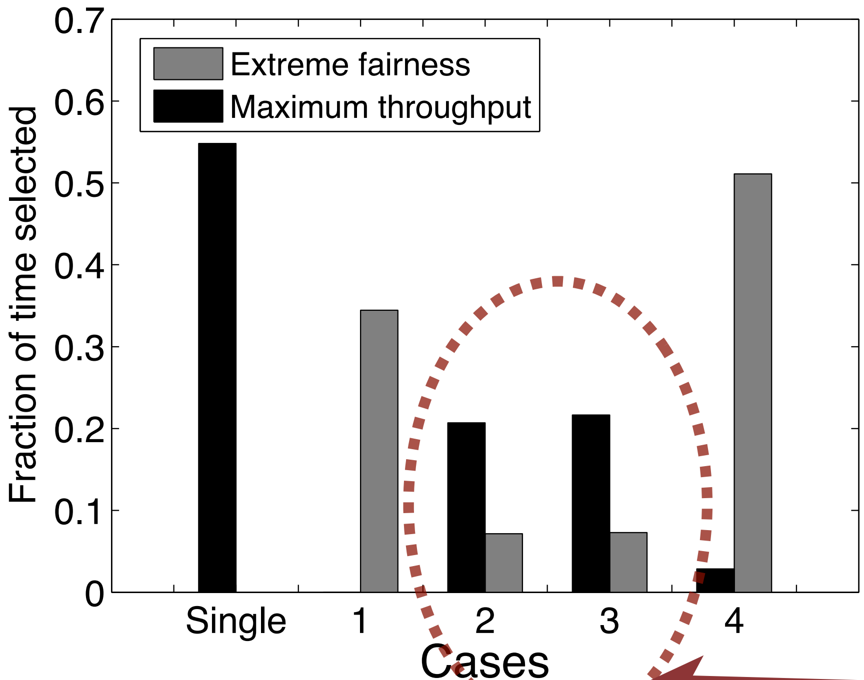
Random MS placement

Fixed MS placement

Simulation Results



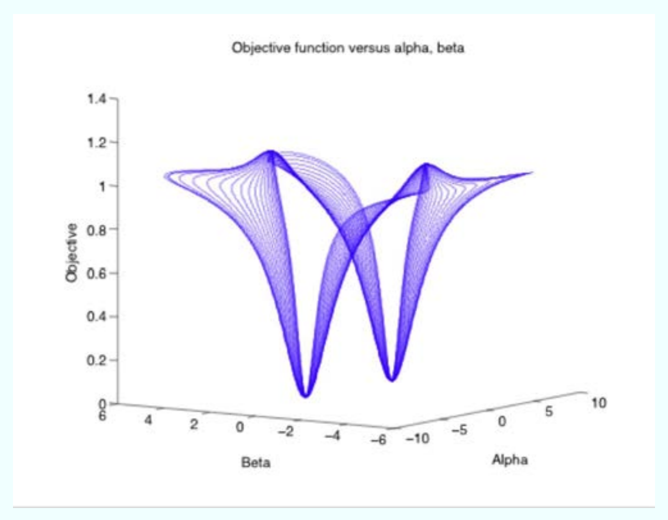
Fixed MS

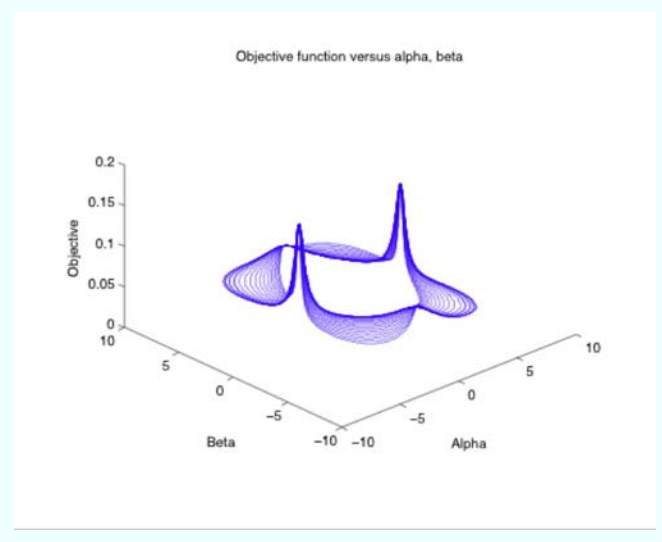


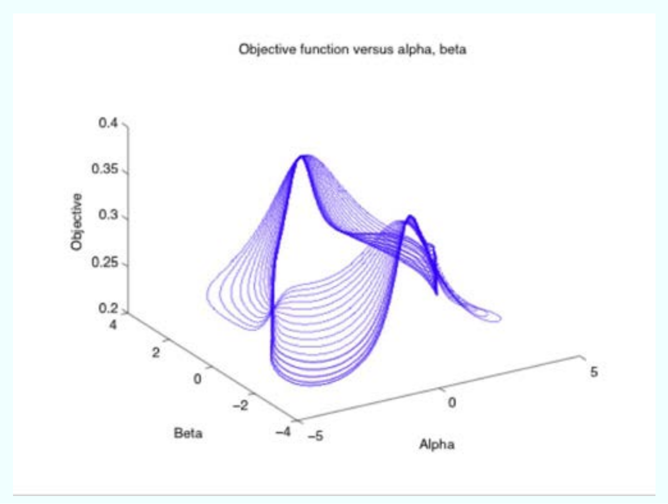
Random MS

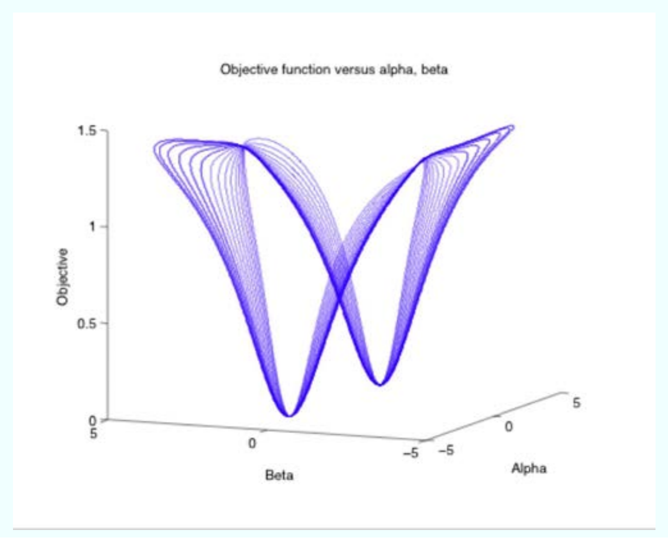
Asymmetric cases are throughput optimal often!

Optimization





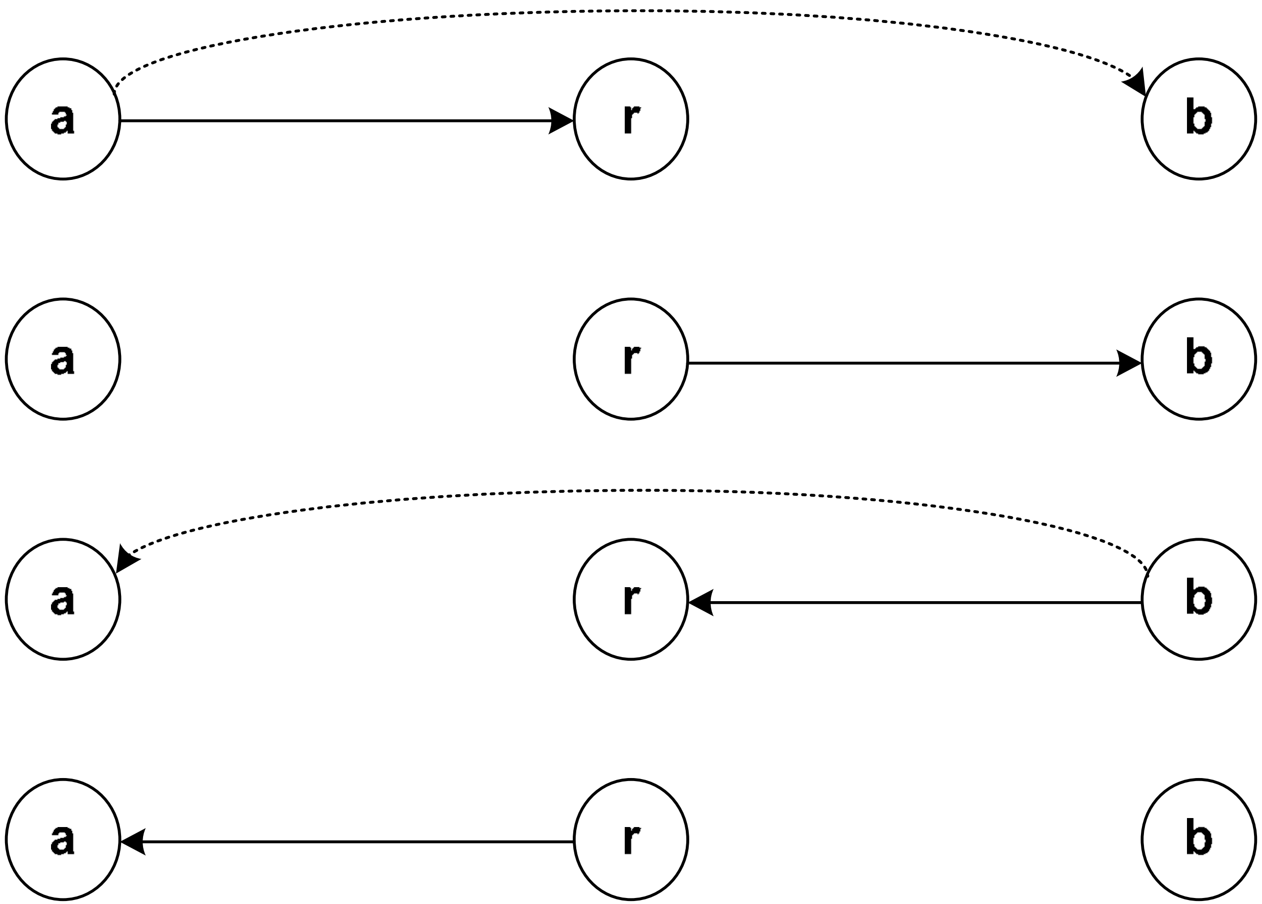




Bi-directional relaying

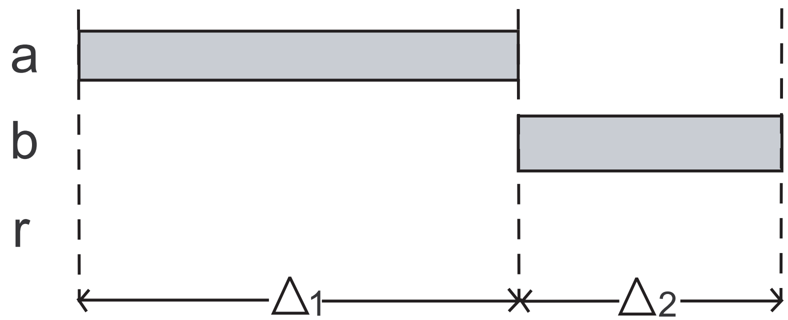
Sang Joon Kim, Patrick Mitran,
Natasha Devroye, Vahid Tarokh

Nodes a and b want to exchange messages over a *shared half-duplex memoryless* channel with the help of a relay

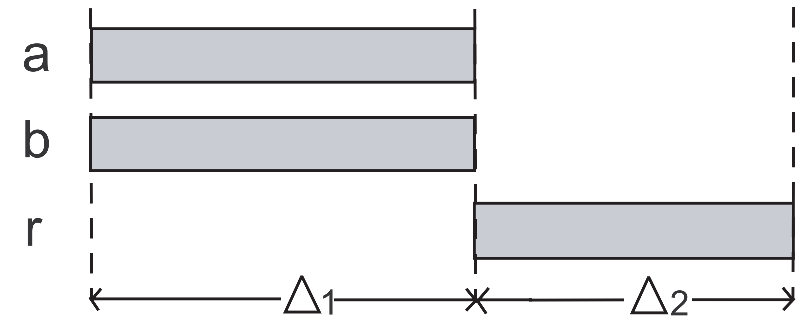


[Naïve four phase bi-directional cooperation]

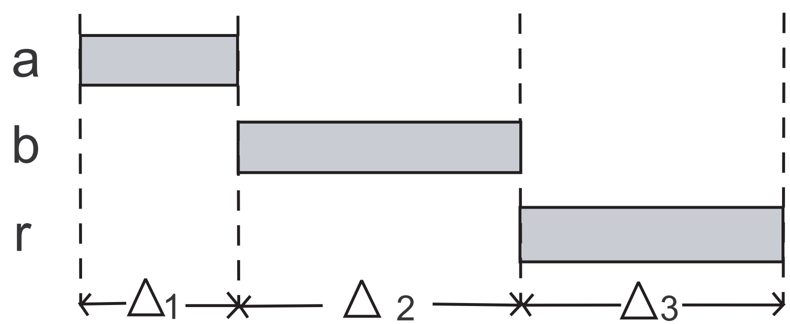
Protocols in time



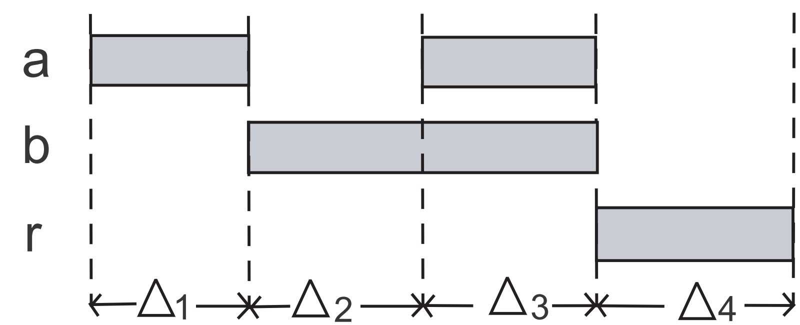
(i) DT protocol



(ii) MABC protocol



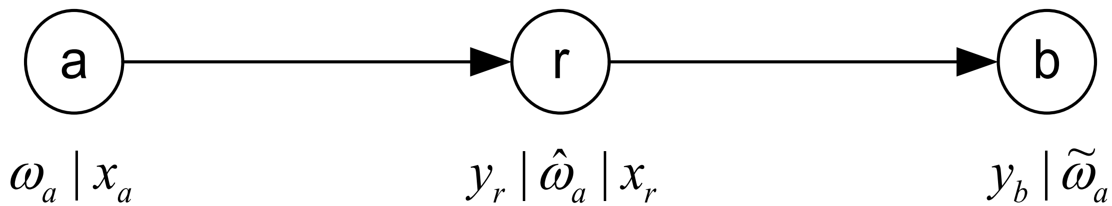
(iii) TDBC protocol



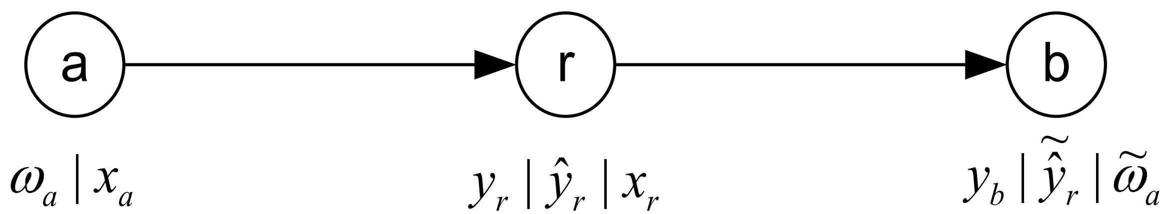
(iv) HBC protocol

Protocols in relaying

- Decode and Forward (D&F)



- Compress and Forward (C&F)



- Mixed Forward : One way uses D&F and the other used C&F

Results

Find inner and outer bounds of capacity region

- Inner bound (achievable region)

$$R \le C_{in} \Rightarrow \lim_{n \to \infty} P_e = 0$$

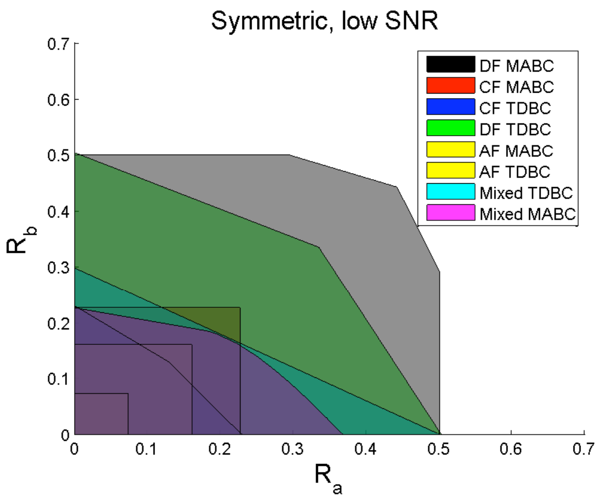
- Outer bound

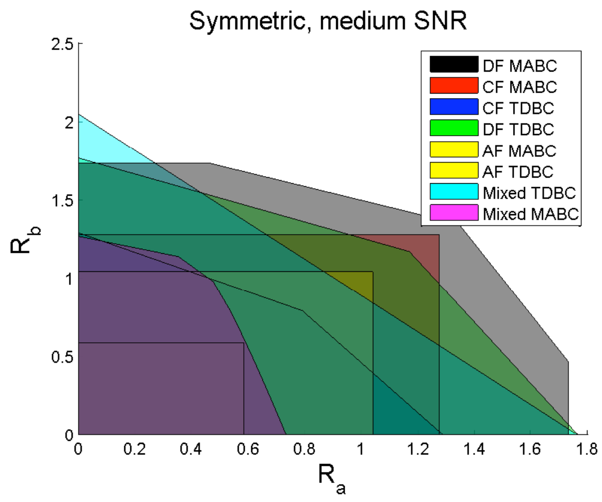
$$R \le C_{out} \Leftarrow \lim_{n \to \infty} P_e = 0$$

- Capacity C lies in

$$C_{in} \le C \le C_{out}$$

Plots





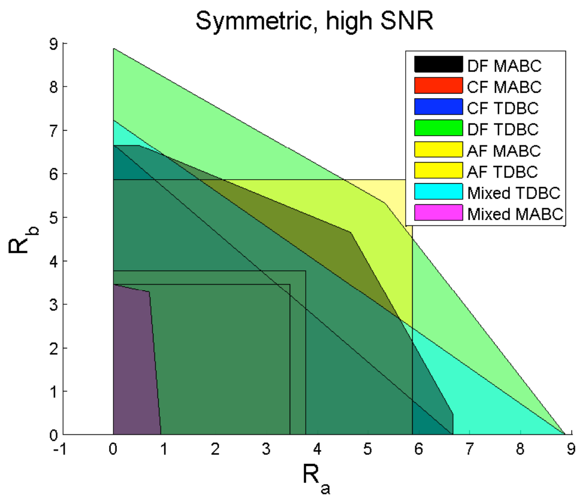


Fig. 3. Comparison of bi-directional regions with $h_{ar} = h_{br} = 1$, $h_{ab} = 0.2$, $P_a = P_b = P_r = 1$ and $N_a = N_b = N_r = 1$.

Fig. 4. Comparison of bi-directional regions with $h_{ar} = h_{br} = 1$, $h_{ab} = 0.2$, $P_a = P_b = P_r = 10$ and $N_a = N_b = N_r = 1$.

Fig. 5. Comparison of bi-directional regions with $h_{ar} = h_{br} = 1$, $h_{ab} = 0.2$, $P_a = P_b = P_r = 10^5$ and $N_a = N_b = N_r = 1$.

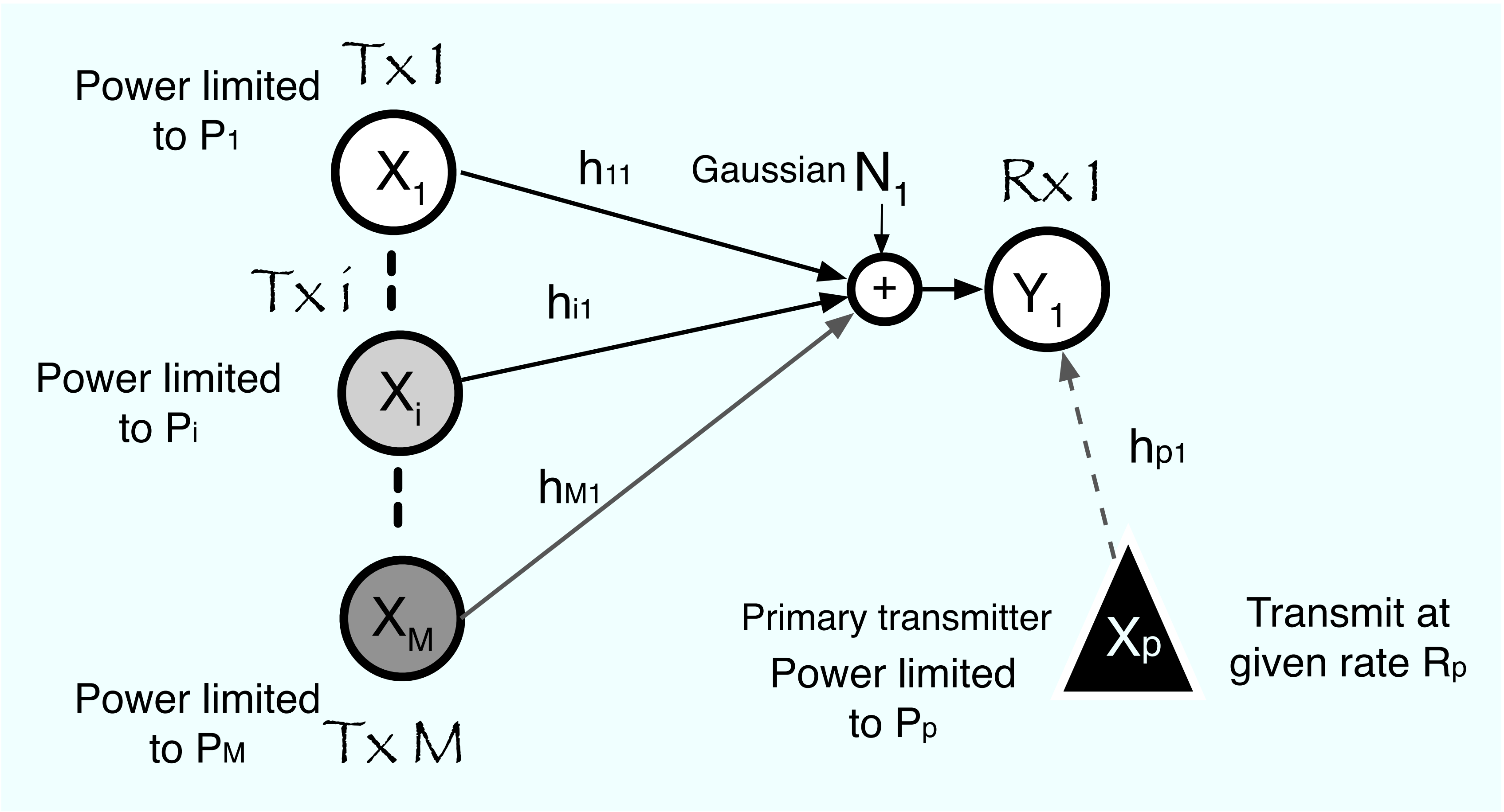
Opportunistic interference cancellation

Petar Popovski, Natasha Devroye

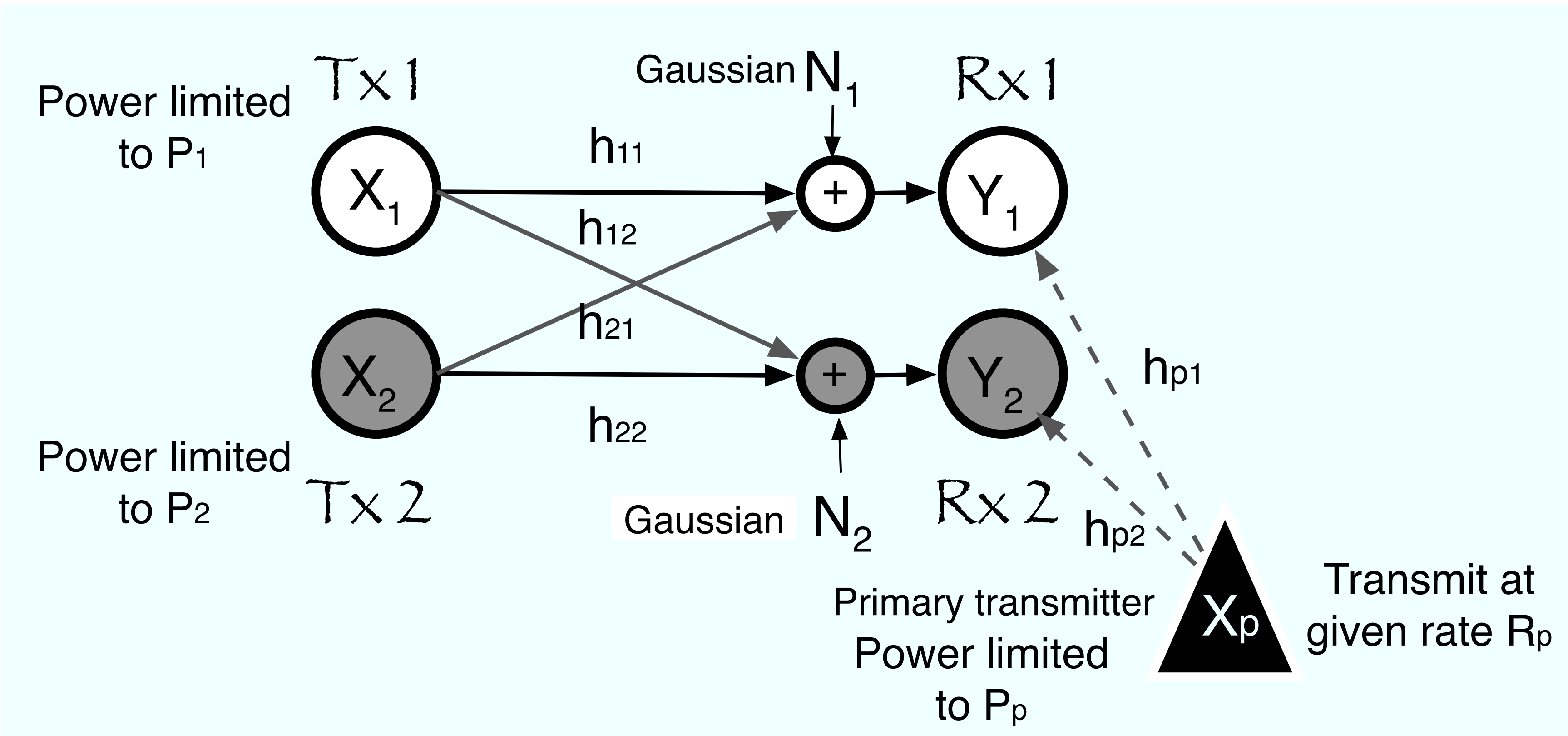
Motivation

- Rather than Tx-side cognition, can the Rxs behave in a cognitive fashion?
- Assume the cognitive Rx knows the primary's codebook
- Assume the cognitive Tx knows at what power it may transmit so as not to harm the primary Rx
- Assume primary system does not change

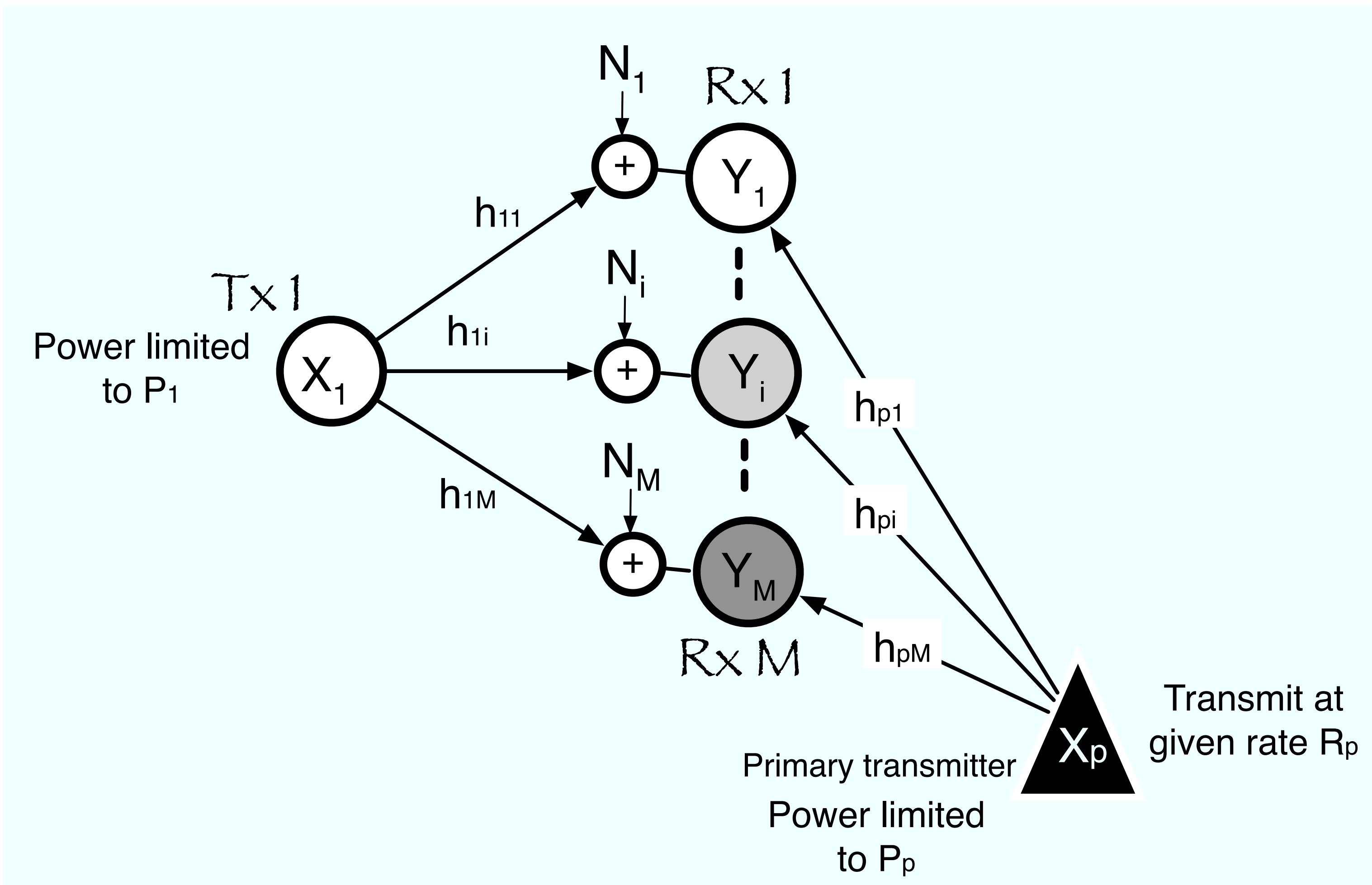
Scenario 1: MAC



Scenario 2: Interference



Scenario 3: Broadcast



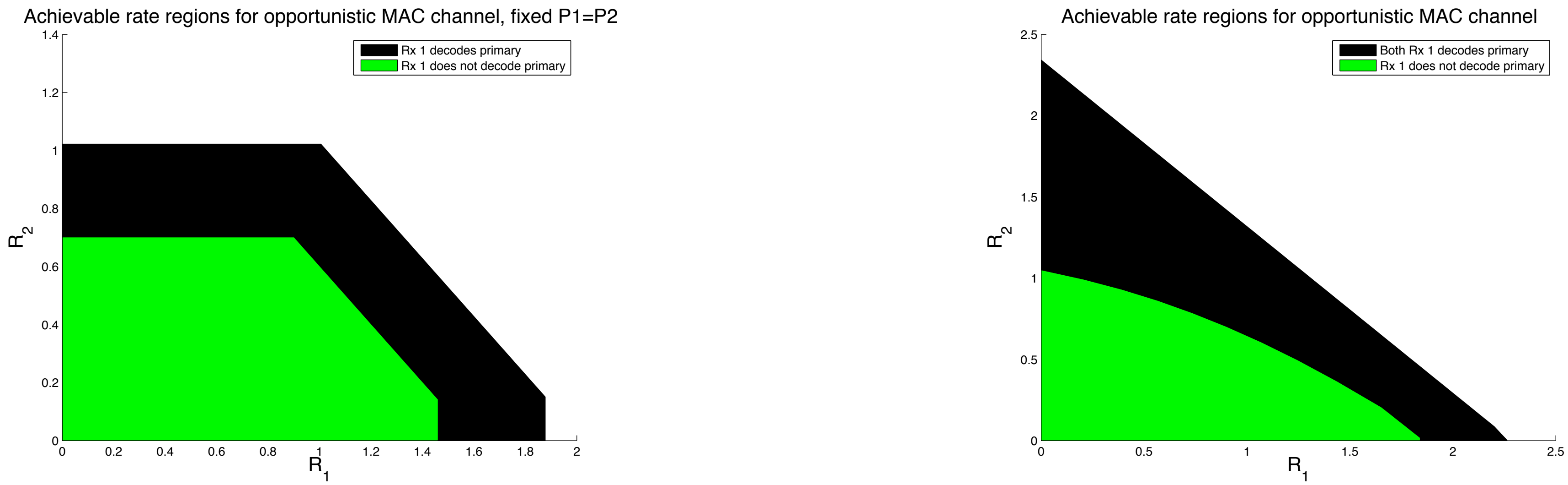
Key idea

- The channel to the cognitive Rx is a multiple access channel: opportunistically decode the primary signal if its rate is below the capacity from the primary to secondary.
- Obtain the rate region described by the MAC but with the MAC rate set to the primary rate. Project down 1 dimension.

Nice technical result

- Can achieve all points on the boundary without time sharing with the primary
- Achieved by:
  - MAC: single message split at cognitive Tx
  - Int: cannot in general achieve
  - BC: cannot in general achieve

Gains for opportunistic MAC

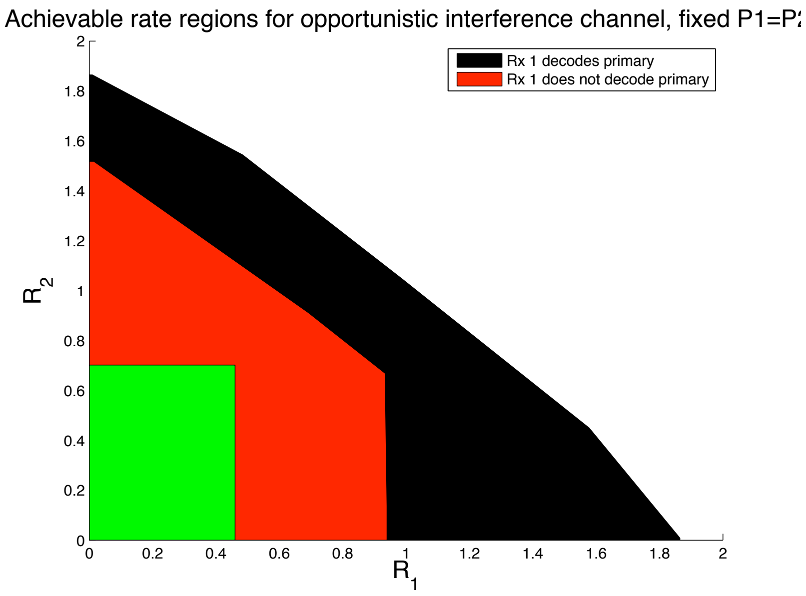


(a) Rate regions for opportunistic MAC channel with equal, fixed powers $P_1 = P_2$.

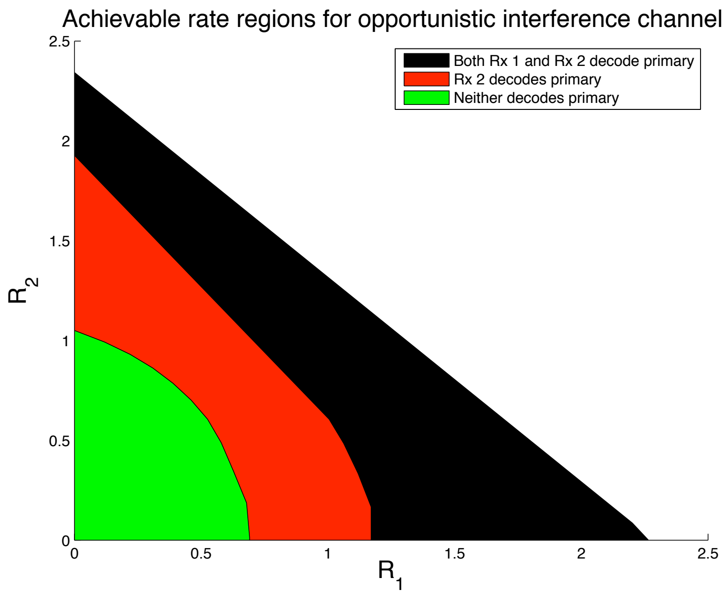
(b) Rate regions for opportunistic MAC channel with $(P_1, P_2) \in \mathcal{P}_{MAC}$.

Fig. 5. Opportunistic MAC channel regions.

Gains for opportunistic interference



Rate regions for opportunistic interference channel with equal, fixed powers $P_1 = P_2$.



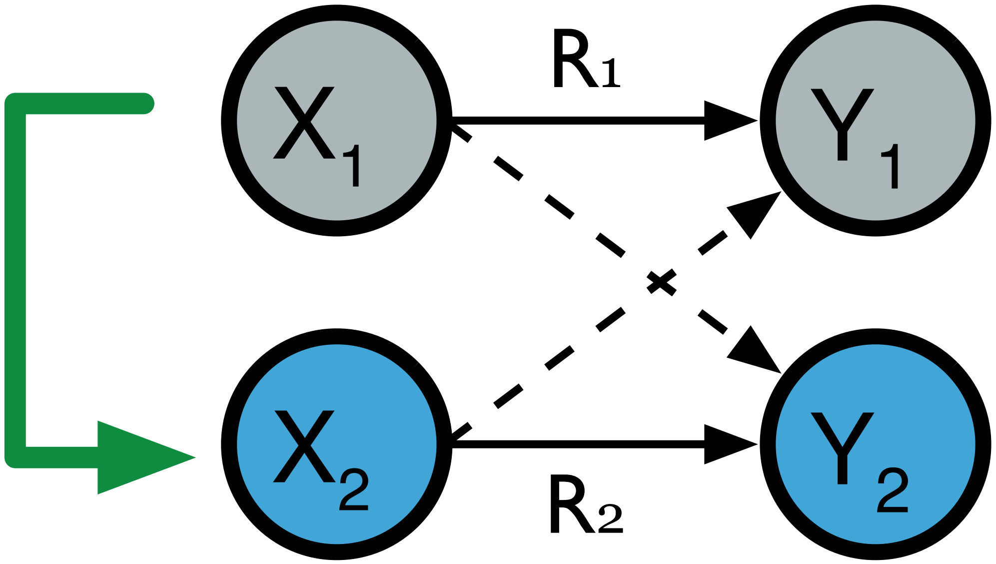
Rate regions for opportunistic interference channel with $(P_1, P_2) \in \mathcal{P}_{INT}$.

Fig. 6. Opportunistic interference channel regions.

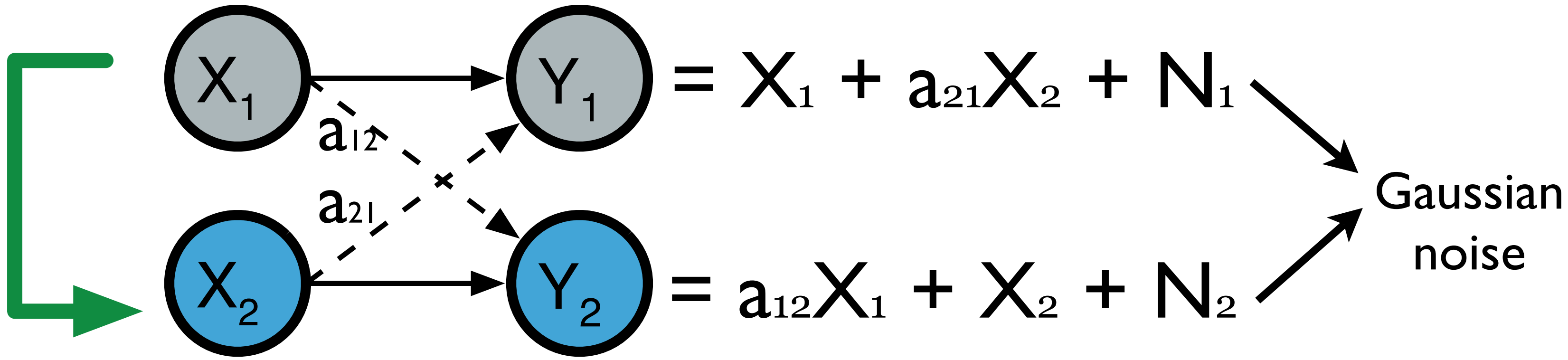
Causal cognitive radio channel

The question

What is the capacity of the cognitive radio channel?

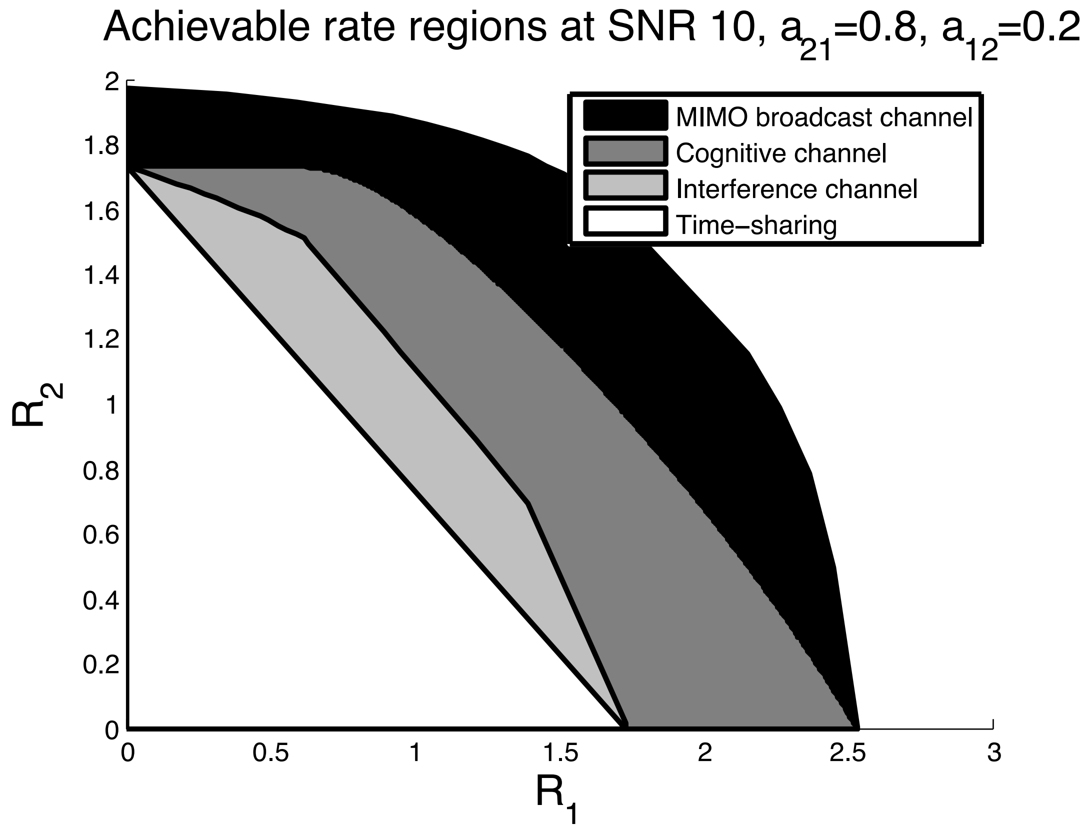


Achievable region: graphical



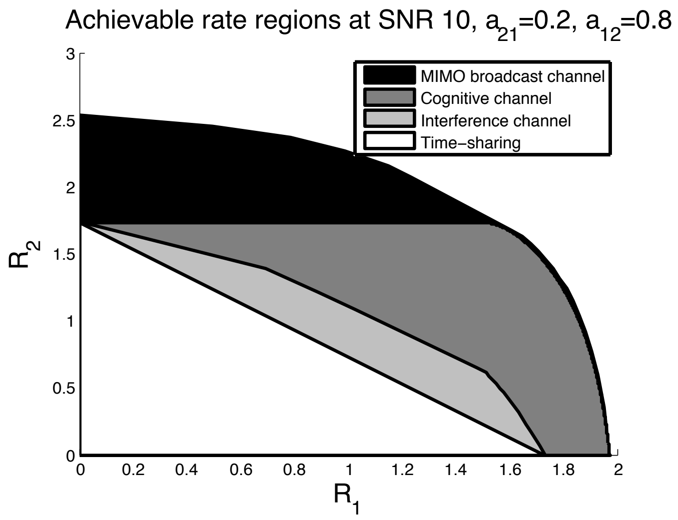
$a_{21}=0.8$

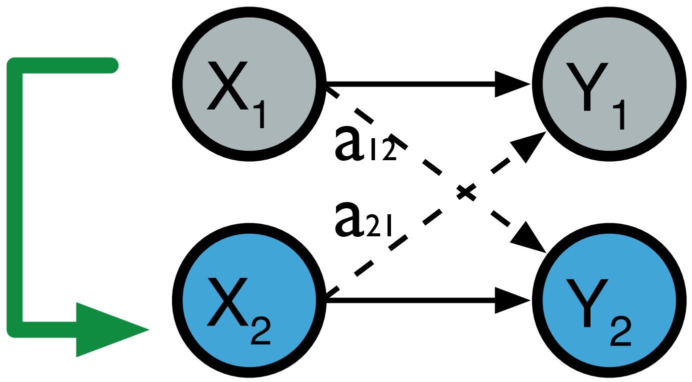
$a_{12}=0.2$



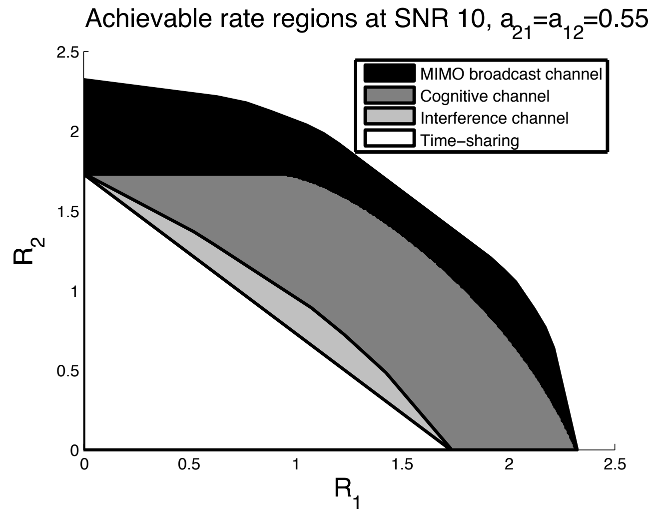
Different cross-over parameters

a21=0.2
a12=0.8



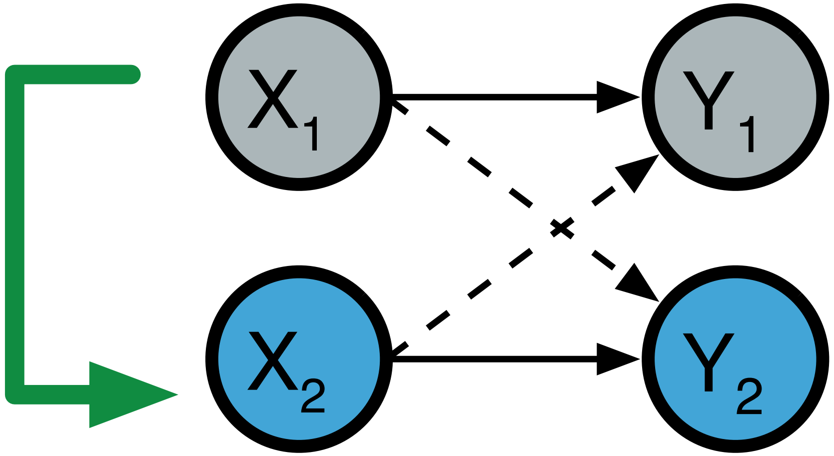


a21=0.55
a12=0.55



Causal message knowledge?

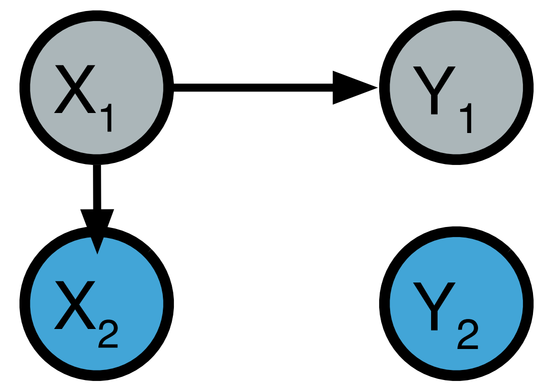
What if this link
is not free?



Protocol 1

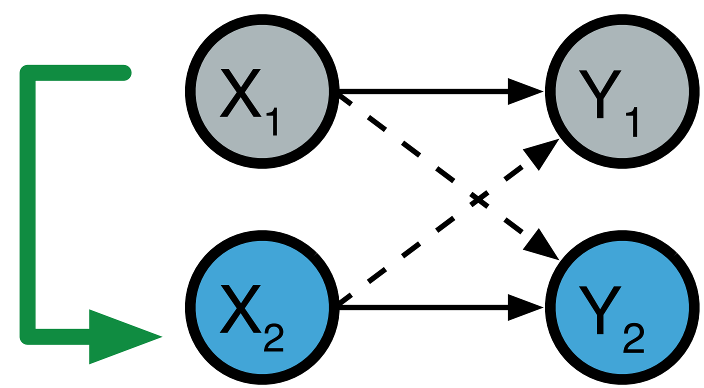
Phase 1:

broadcast channel



Phase 2:

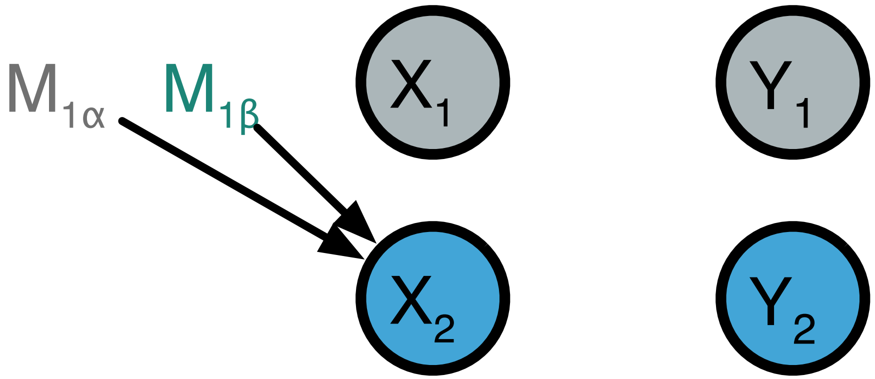
cognitive radio channel



Protocol 2

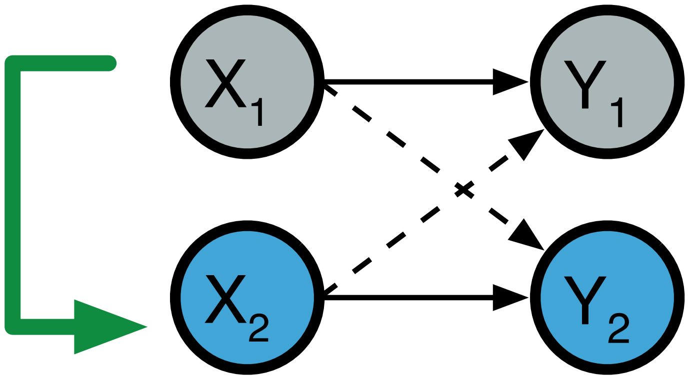
Phase 1:

multiple access channel



Phase 2:

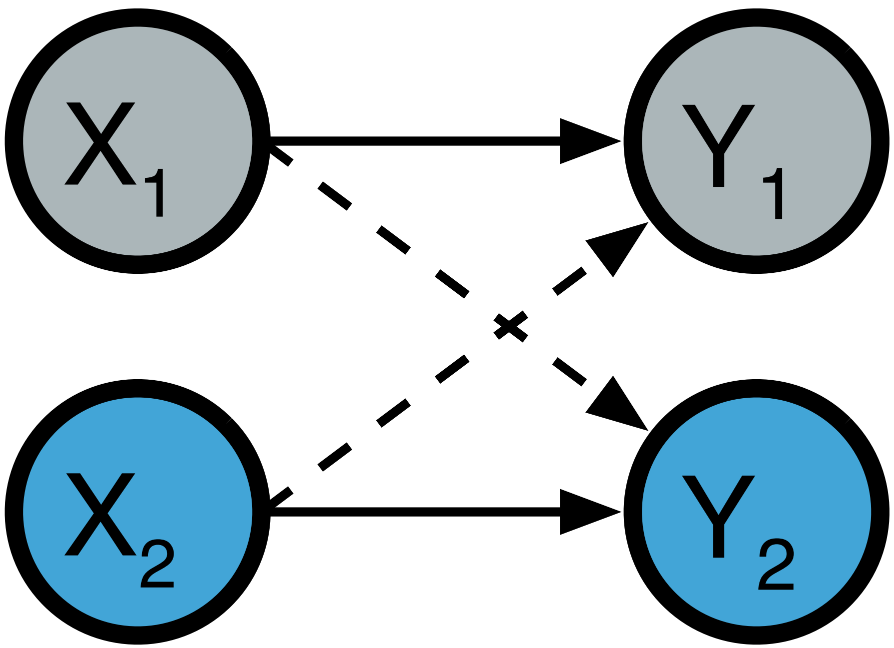
cognitive radio channel



Protocol 3

Only one phase: interference channel.

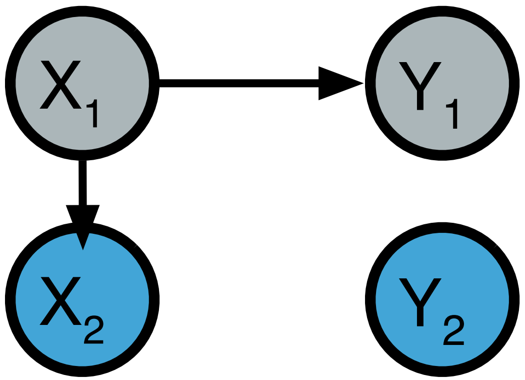
X_2 does not know X_1 and does not dirty paper code against it.



Protocol 4

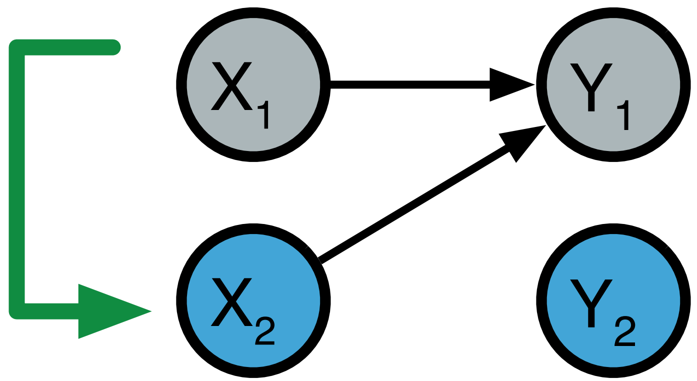
Phase 1:

broadcast channel



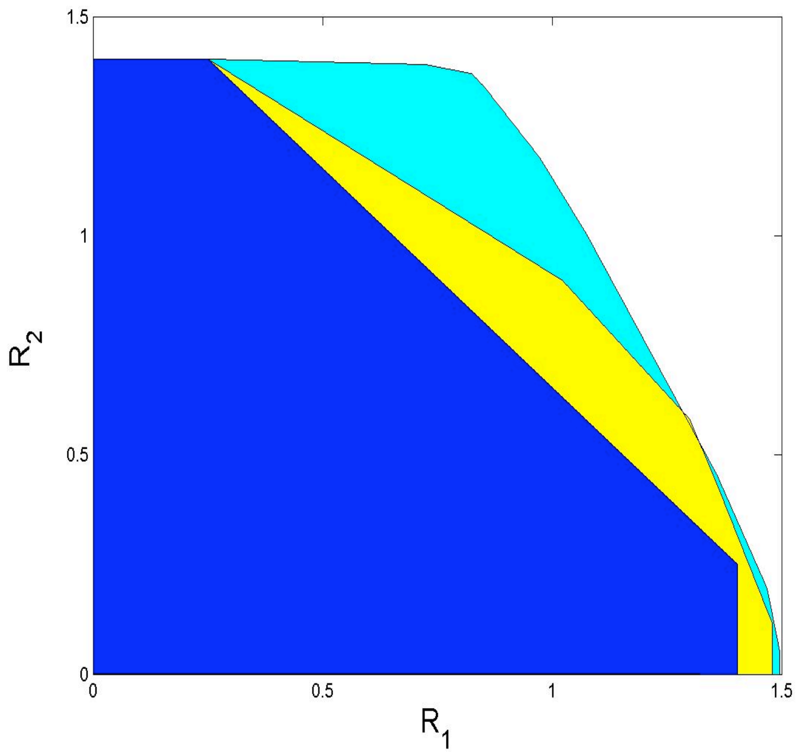
Phase 2:

X_2 aids X_1 in sending it message. X_2 does not send any information of its own.

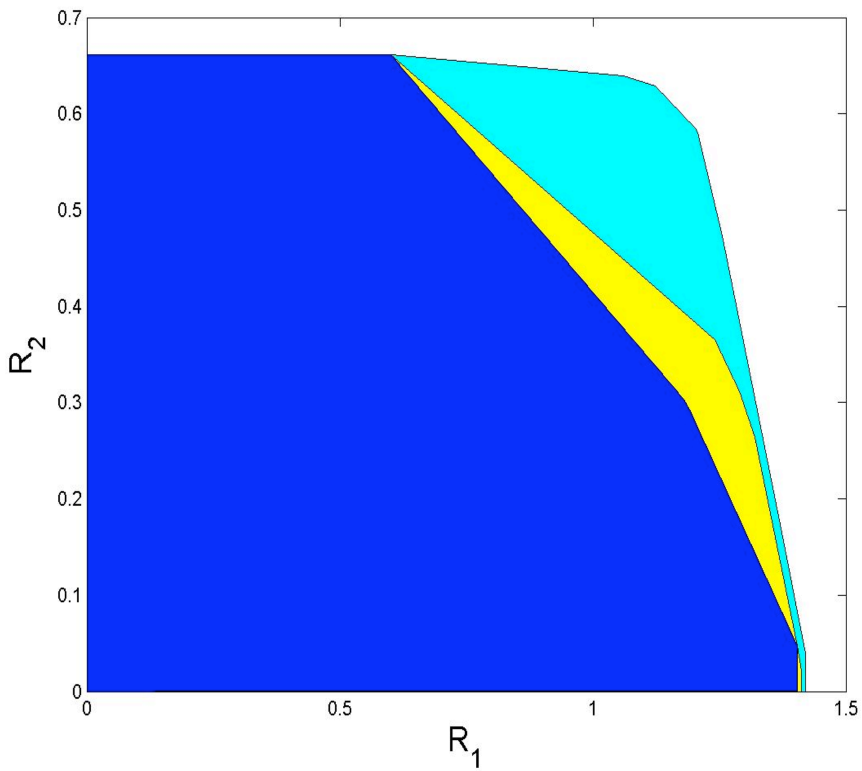


Causal achievable regions

Blue: G=1 Yellow: G=10 Cyan: Genie-Aided



$P_1=P_2=6$



$P_1=6$, $P_2=1.5$

Extra

Cognitive radio channel

Unused spectrum

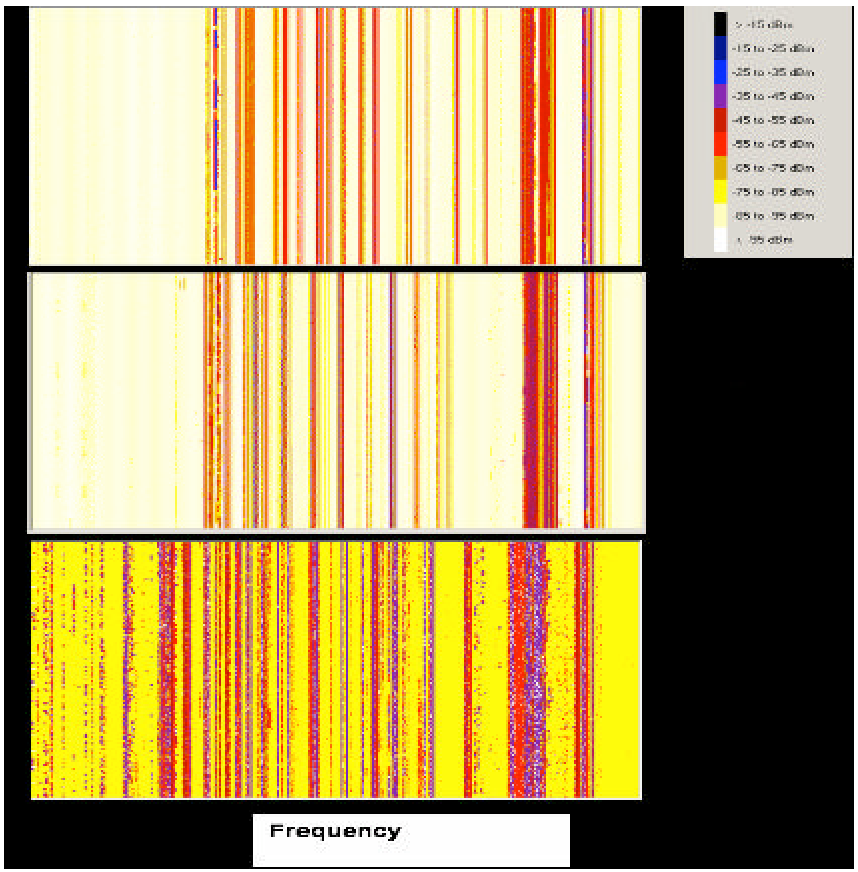


Figure 1: Occupancy of approximately 700 megahertz of spectrum below 1 GHz

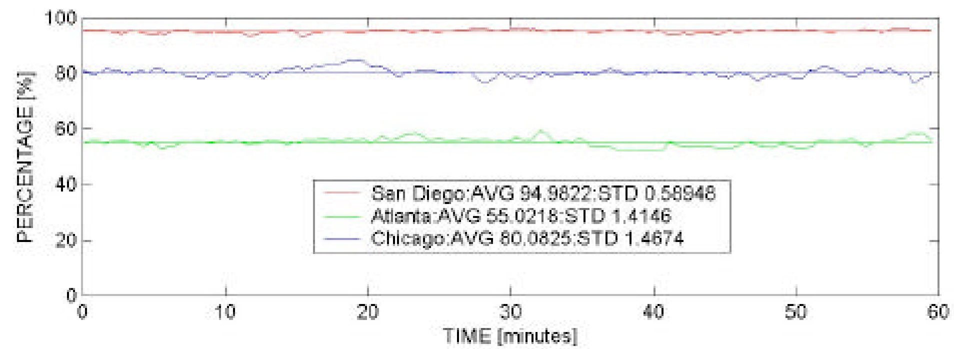


Figure 3: Use of a 7 megahertz band below 1 GHz (percentage of a 30-second window by 7 megahertz block that was idle)

Federal Communications Commission
Spectrum Policy Task Force
Report of the Spectrum
Efficiency Working Group
November 15, 2002
http://www.fcc.gov/sptf/files/SEWGFinalReport_1.pdf

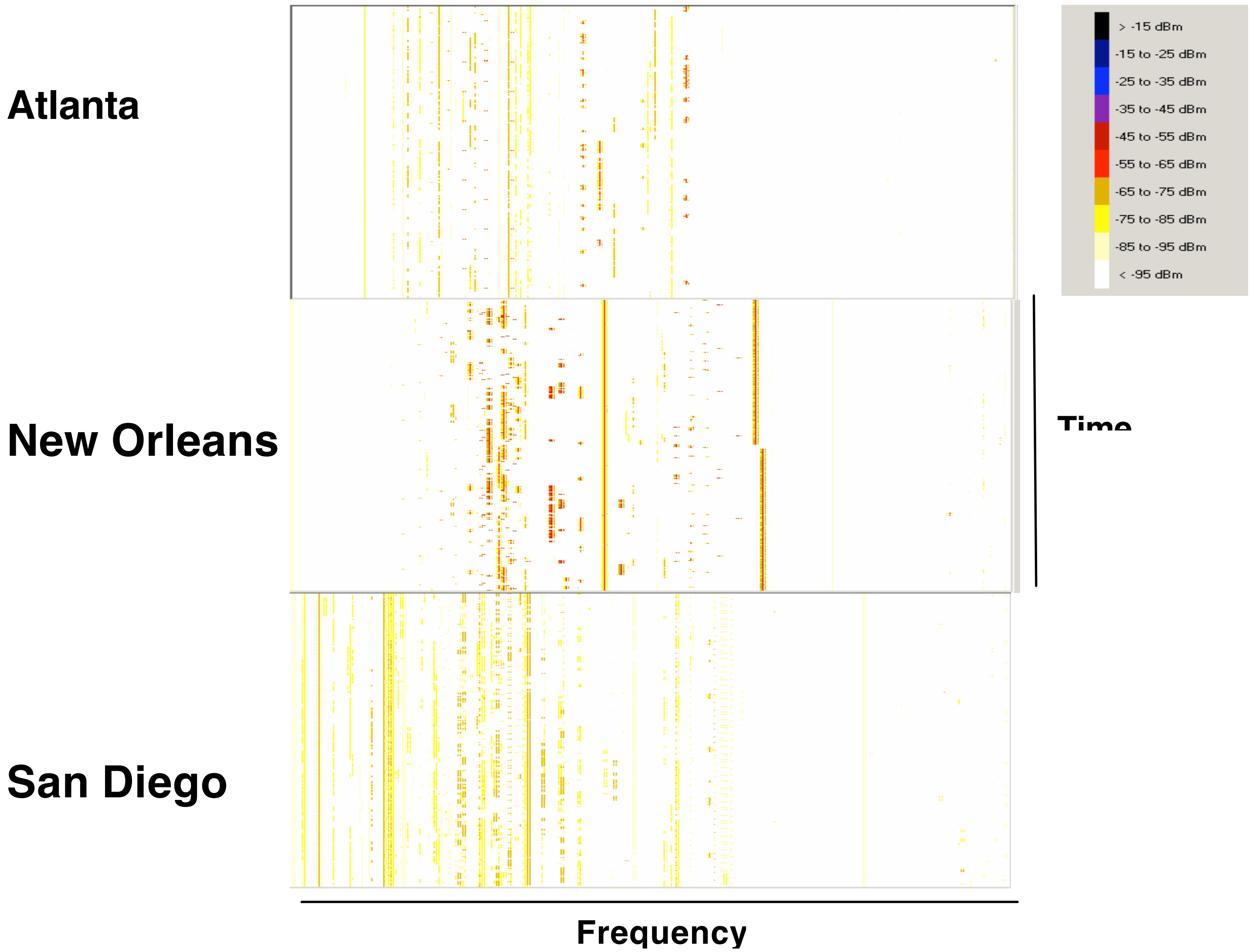


Figure 2: Occupancy of a 7.5 megahertz UHF Land Mobile band

Formal definitions

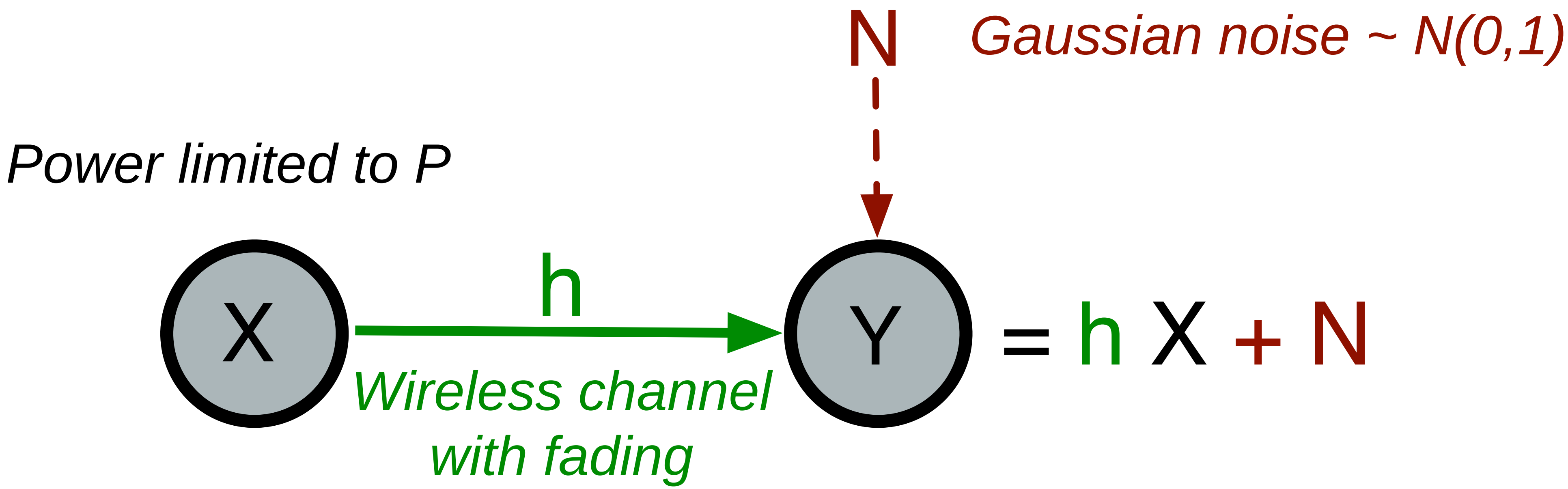
A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there exists an $(2^{\lceil nR_1 \rceil}, 2^{\lceil nR_2 \rceil}, n, P_e)$ code such that $P_e \leq \epsilon$.

An $(2^{\lceil nR_1 \rceil}, 2^{\lceil nR_2 \rceil}, n, P_e)$ code consists of encoding functions that map messages $W_1 \in \{1, 2, \cdots 2^{\lceil nR_1 \rceil}$ and $W_2 \in \{1, 2, \cdots 2^{\lceil nR_2 \rceil}$ and decoding functions that recover these messages such that the average error probability is less than P_e.

Achievable rate region: set of achievable rate pairs (R_1, R_2).

Capacity region: the closure of the set of all achievable rate pairs (R_1, R_2).

Why Cooperate to Communicate?

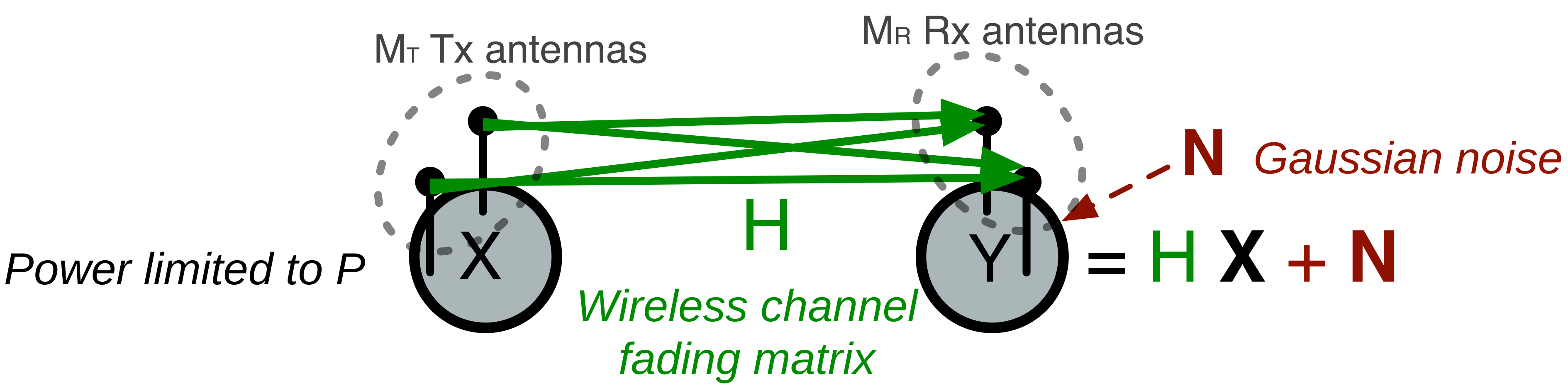


Channel capacity
(bits/channel use):

$$C = \begin{cases} \frac{1}{2}\log_2(1 + |h|^2 P/P_N) & \text{Instantaneous} \\ E_h\left[\frac{1}{2}\log_2(1 + |h|^2 P/P_N)\right] & \text{Ergodic} \end{cases}$$

Why Cooperate to Communicate?

2 Tx antenna, 2 Rx antenna Multiple Input Multiple Output (MIMO) fading channel with Gaussian noise

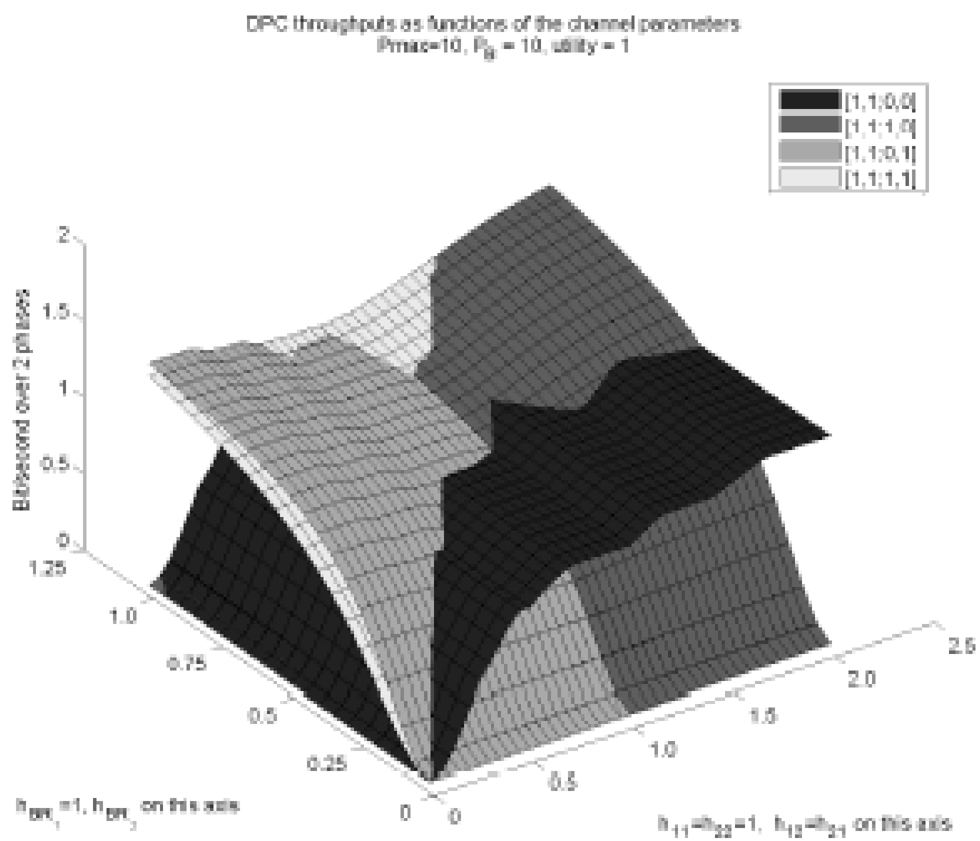


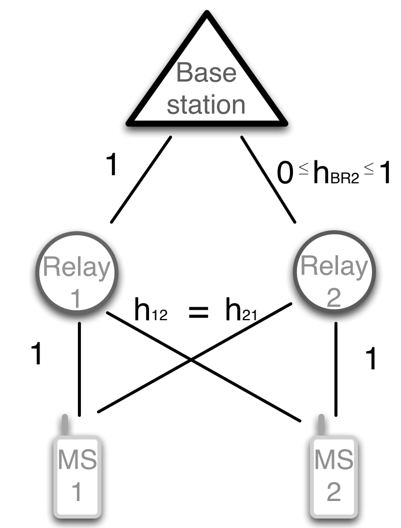
Channel capacity: $C = \begin{cases} \max_{\mathbf{Q}:Tr(\mathbf{Q})=P} \frac{1}{2}\log_2 \left|\mathbf{I}_{\mathbf{M_R}} + \mathbf{HQH}^\dagger\right| & \text{Instantaneous} \\ \max_{\mathbf{Q}:Tr(\mathbf{Q})=P} E_{\mathbf{H}}\left[\frac{1}{2}\log_2 \left|\mathbf{I}_{\mathbf{M_R}} + \mathbf{HQH}^\dagger\right|\right] & \text{Ergodic} \end{cases}$

Extra

Asymmetric cooperation downlink

Which of 4 schemes is optimal





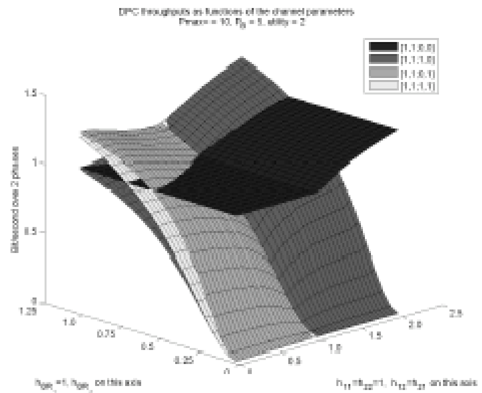


Fig. 15. DPC, Max throughput, $P_{max} = 10, P_B = 5$

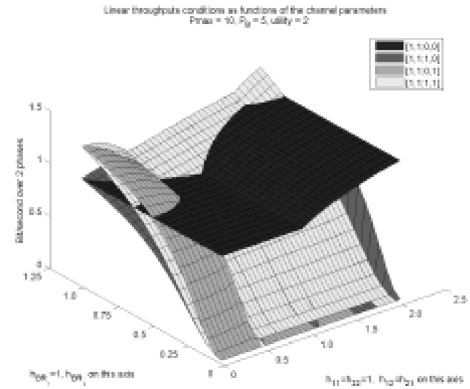


Fig. 16. Linear, Max throughput, $P_{max} = 10, P_B = 5$

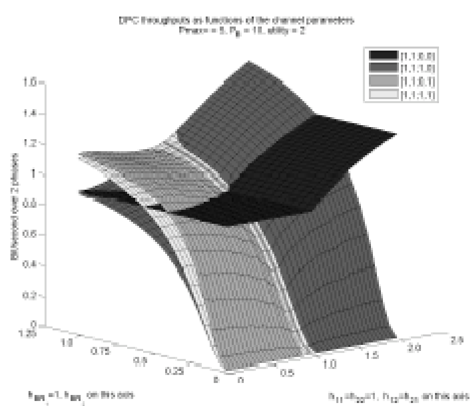


Fig. 17. DPC, Max throughput, $P_{max} = 5, P_B = 10$

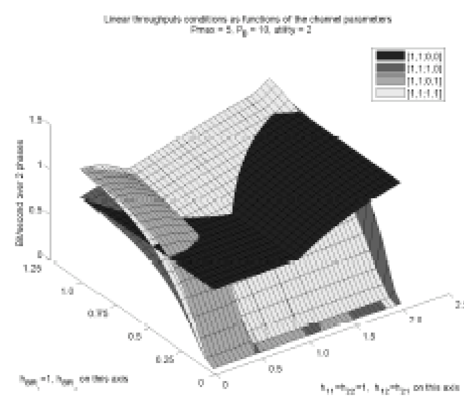


Fig. 18. Linear, Max throughput, $P_{max} = 5, P_B = 10$

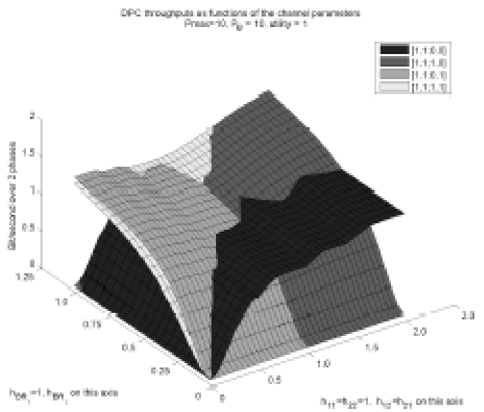


Fig. 7. DPC, Extreme Fairness, $P_{max} = 10, P_B = 10$

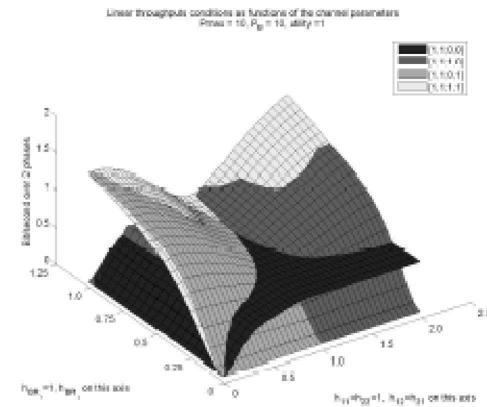


Fig. 8. Linear, Extreme Fairness, $P_{max} = 10, P_B = 10$

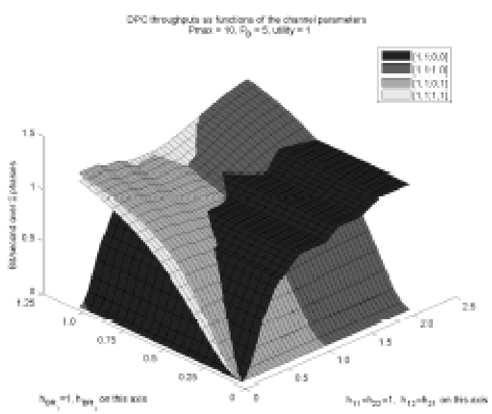


Fig. 9. DPC, Extreme Fairness, $P_{max} = 10, P_B = 5$

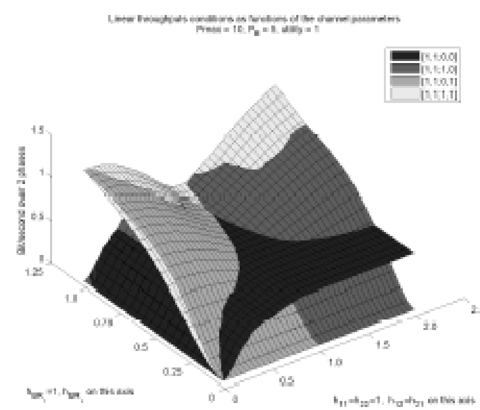
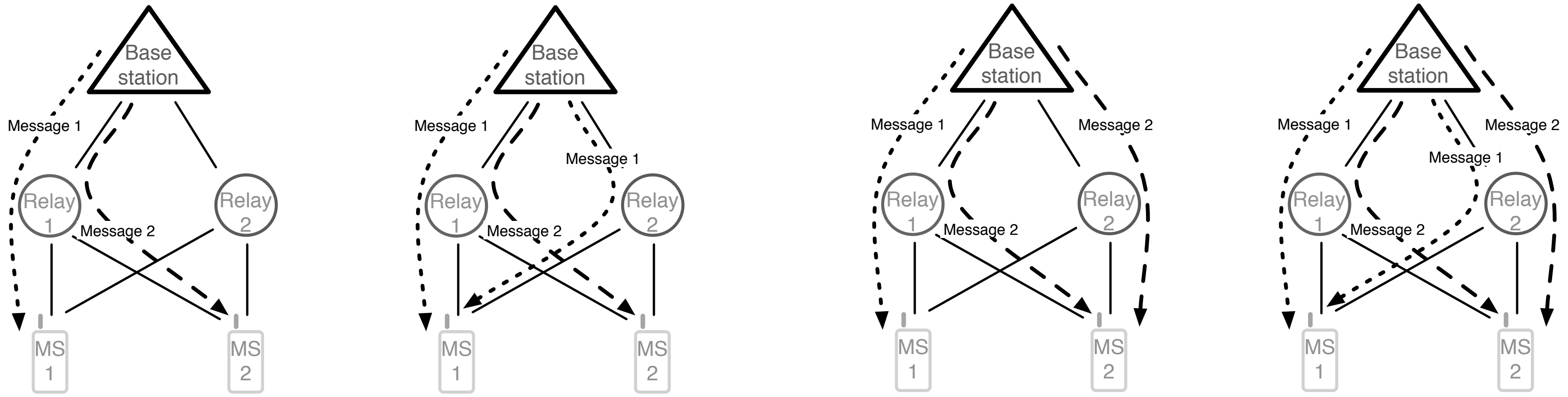
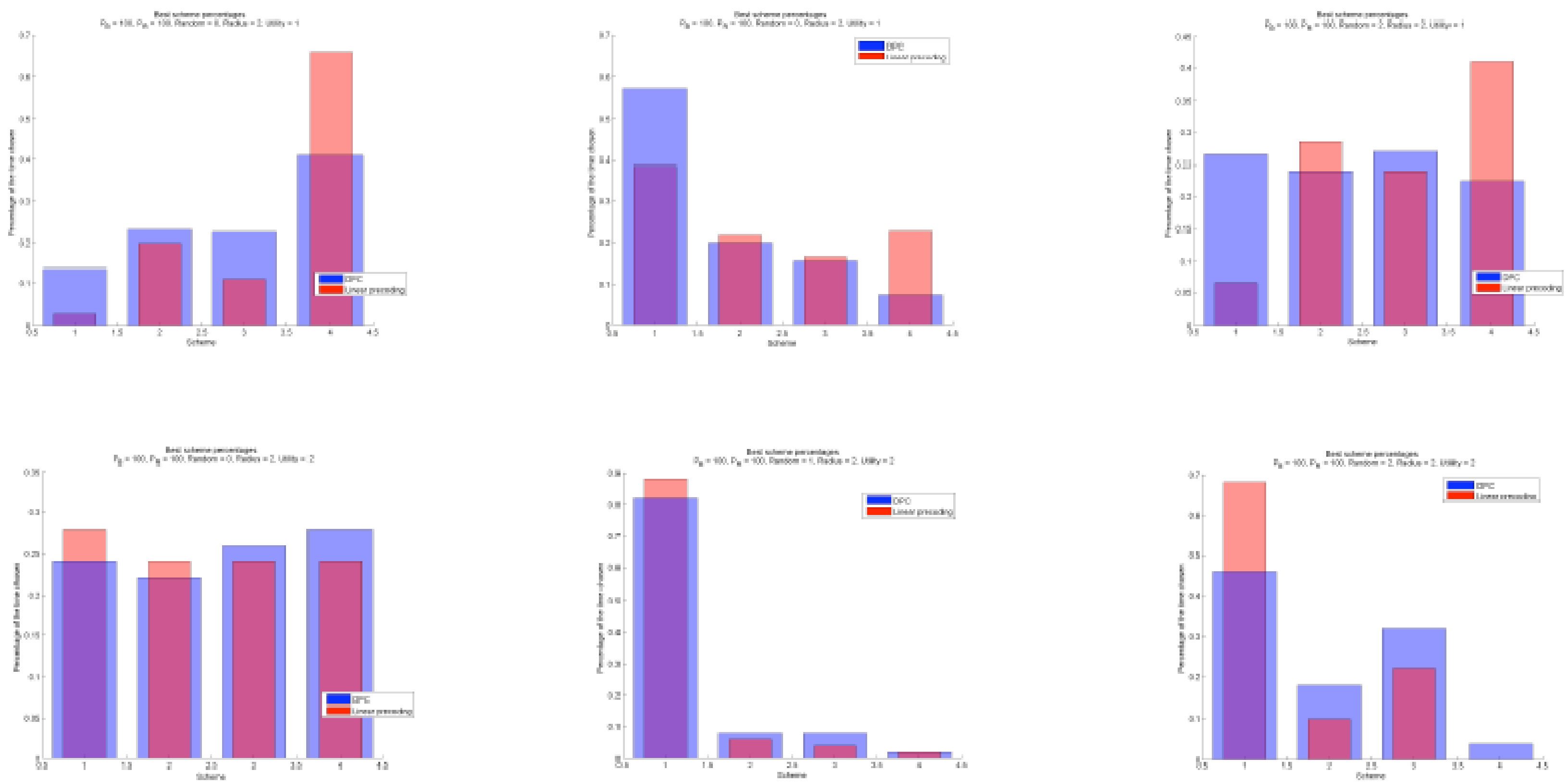


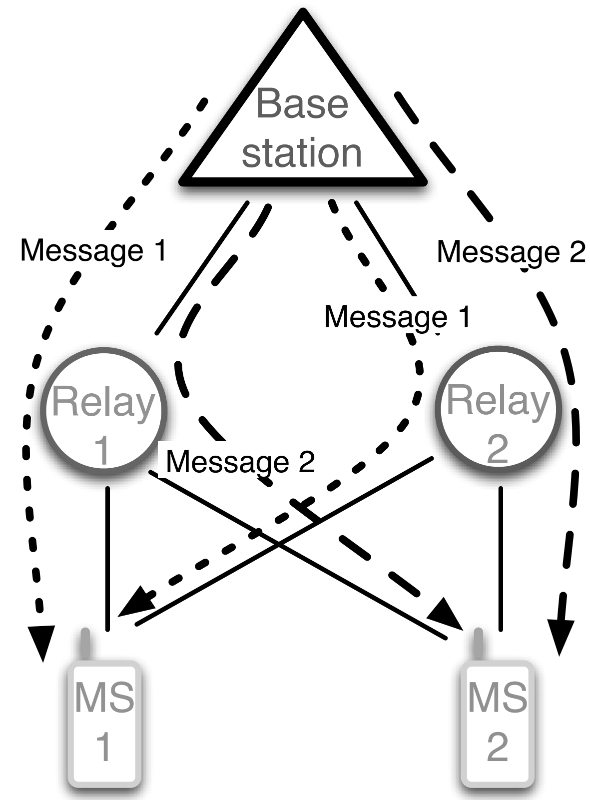
Fig. 10. Linear, Extreme Fairness, $P_{max} = 10, P_B = 5$


Base station
Message 1
Message 2
Relay 1
Relay 2
MS 1
MS 2



Baselines for comparison

- Depends on criterion
- Round robin with relay
- Best 2 hop overall



Gains of relay cooperation over non-cooperative schemes

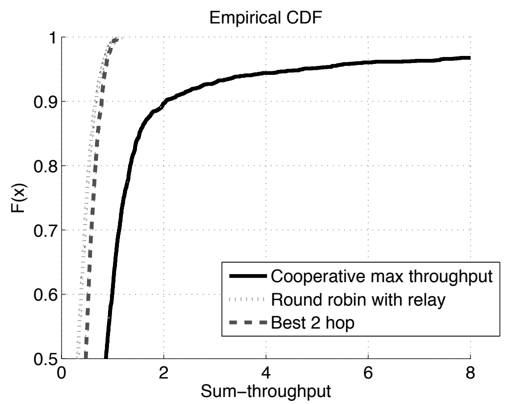


Fig. 5. CDF of sum throughput under the max throughput criterion, *random MS* placement.

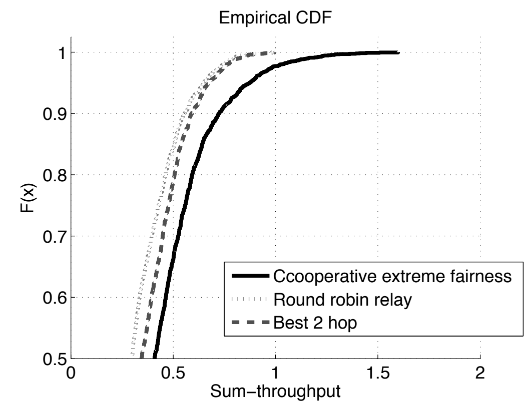


Fig. 6. CDF of sum throughput under the extreme fairness criterion, *random MS* placement.

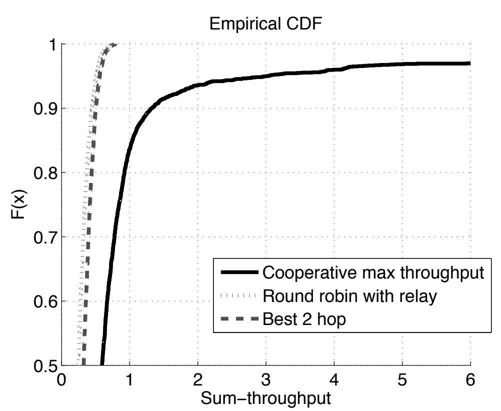


Fig. 8. CDF of sum throughput under the max throughput criterion, *fixed MS* placement.

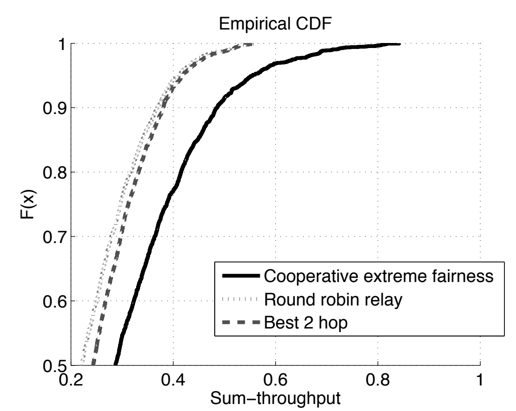
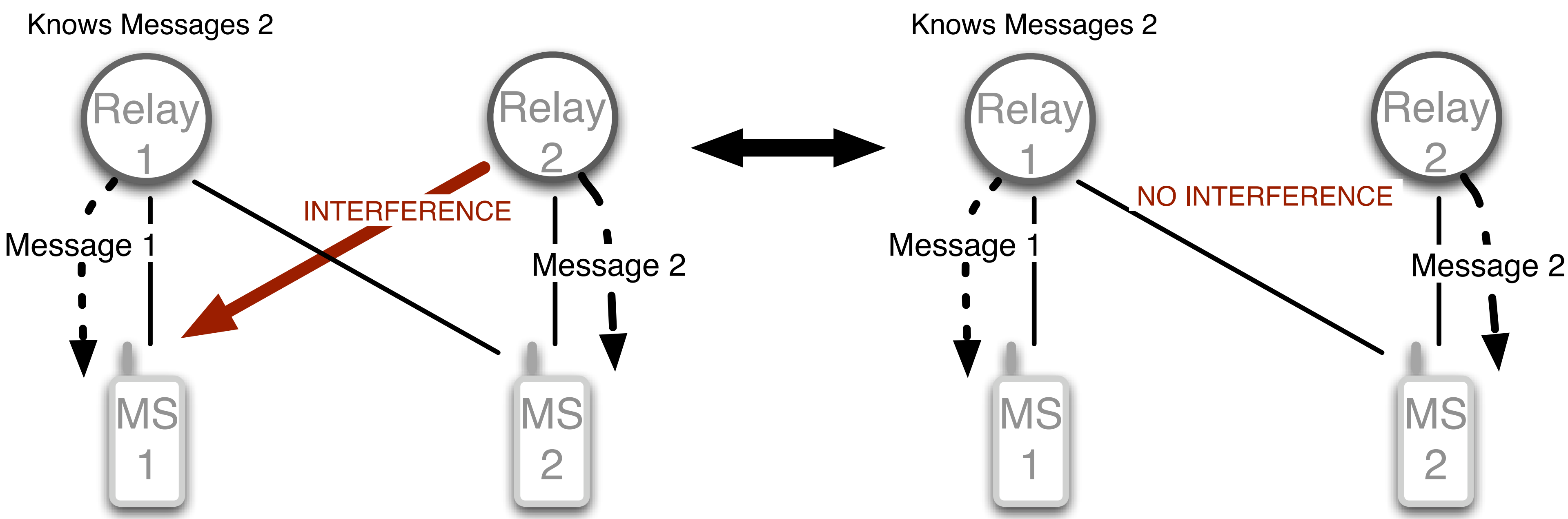


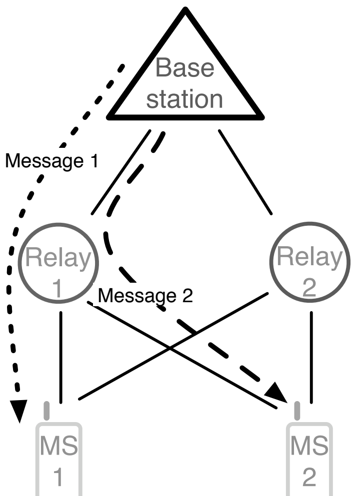
Fig. 9. CDF of sum throughput under the extreme fairness criterion, *fixed MS* placement.

Dirty-paper coding

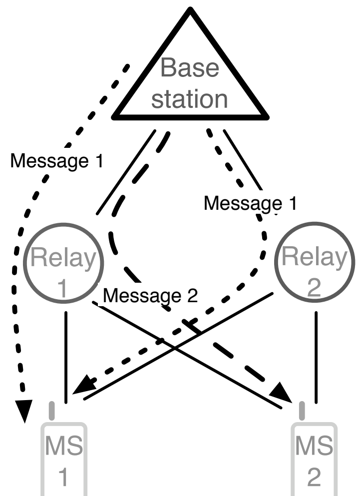
Optimal encoding scheme for a broadcast channel



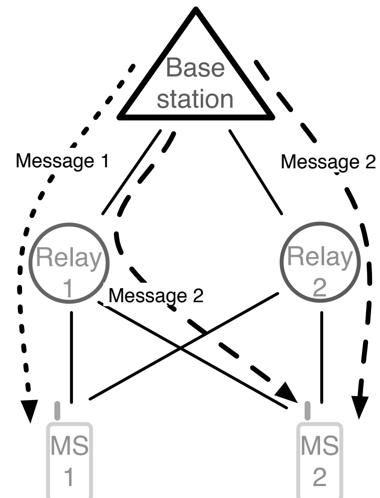
Dirty-paper coding



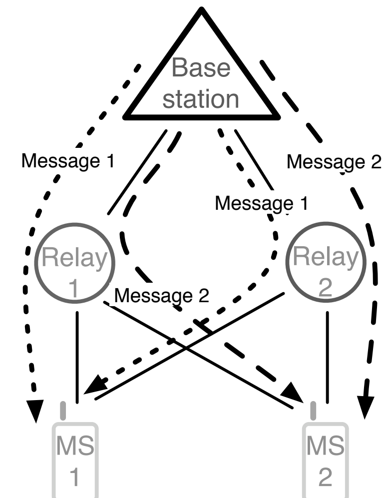
1) Case 1: relay 1 knows both messages



2) Case 2: relay 1 knows both messages, relay 2 knows message 1



3) Case 2: relay 1 knows both messages, relay 2 knows message 2



4) Case 4: both relays know both messages

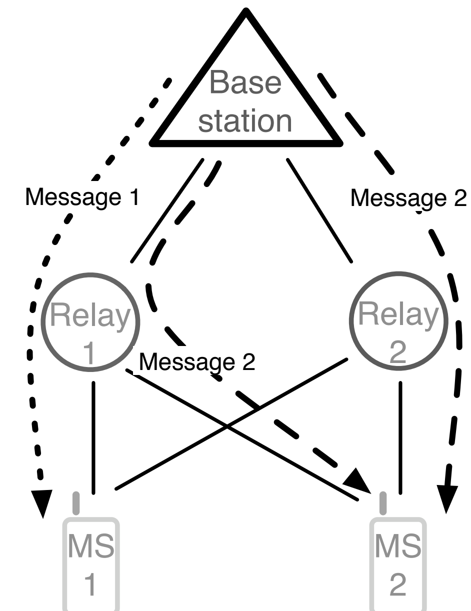
$$B_1 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \quad B_1 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$$

$$B_2 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} x & x \\ x & x \end{bmatrix} \quad B_2 = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$$

Power constraint: $trace(B_1 + B_2) \leq P_R$

Dirty-paper coding

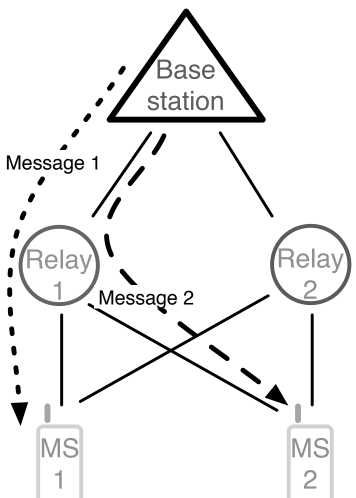
- The achievable rates can be written in terms of the transmit covariance matrices B_1 and B_2 as:



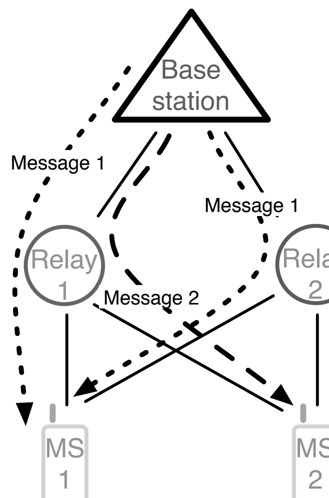
3) Case 3: relay 1 knows both messages, relay 2 knows message 2

$$R_1 \leq \frac{1}{2}\log_2\left(\frac{|H_1(B_1)H_1^T + N_1|}{|N_1|}\right), \qquad R_2 \leq \frac{1}{2}\log_2\left(\frac{|H_2(B_1+B_2)H_2^T|}{|H_2 B_1 H_2^T + N_2|}\right)$$

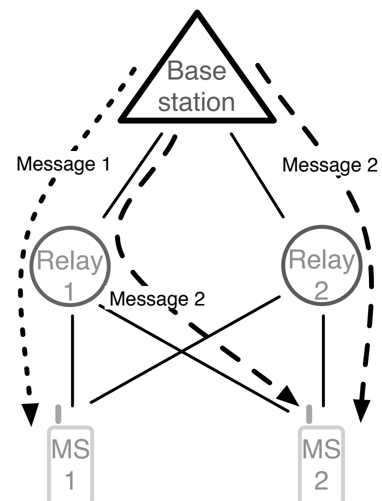
Optimization



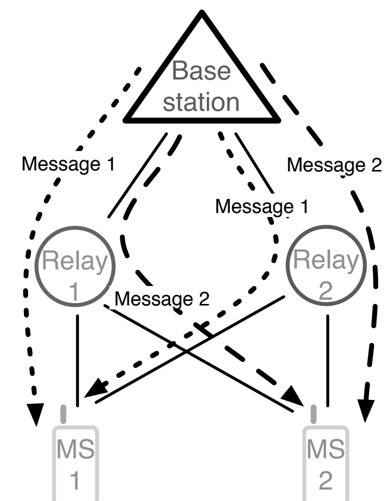
1) Case 1: relay 1 knows both messages



2) Case 2: relay 1 knows both messages, relay 2 knows message 1



3) Case 2: relay 1 knows both messages, relay 2 knows message 2

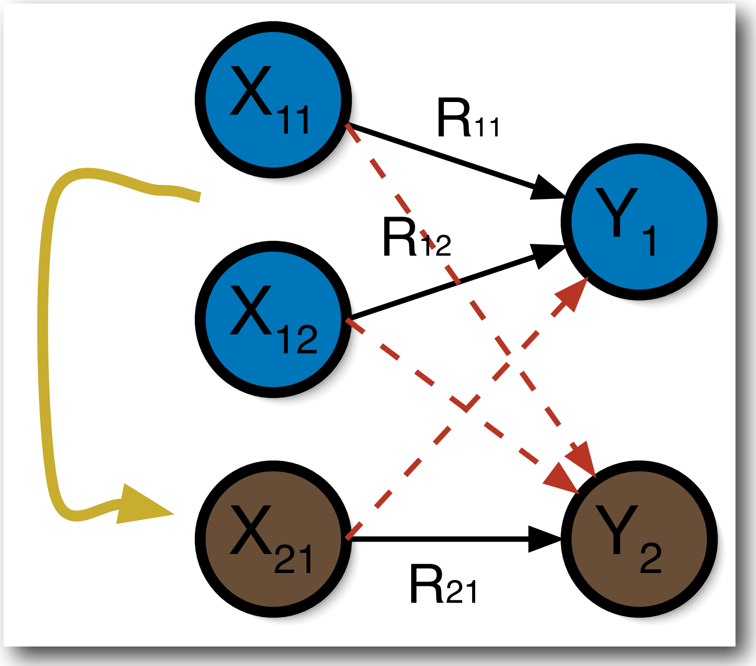


4) Case 4: both relays know both messages

For **each** of the 4 cases, find the parameters that maximize throughput or extreme fairness criteria

Cognitive multiple access networks

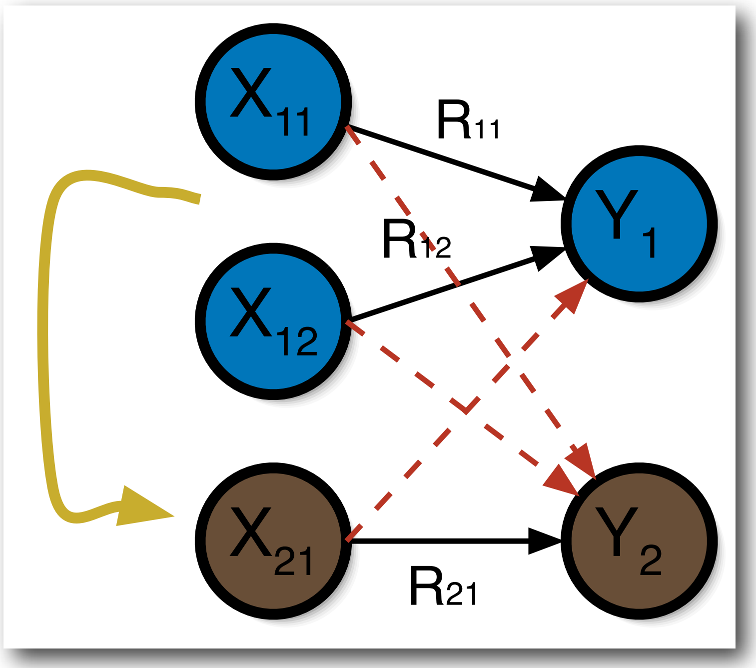
Natasha Devroye, Patrick Mitran, Vahid Tarokh



A priori message knowledge

aid transmission

mitigate interference

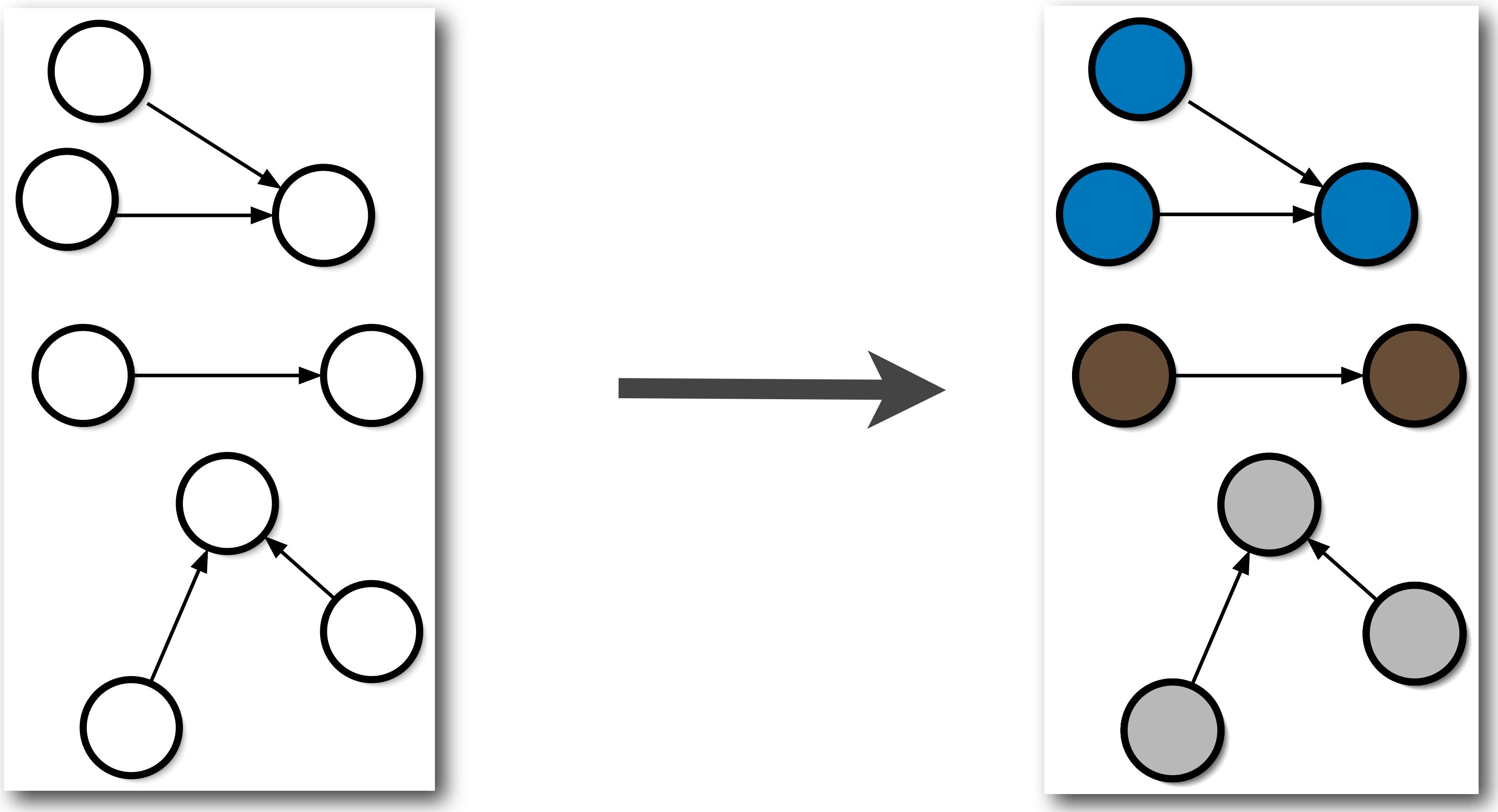


A priori message knowledge

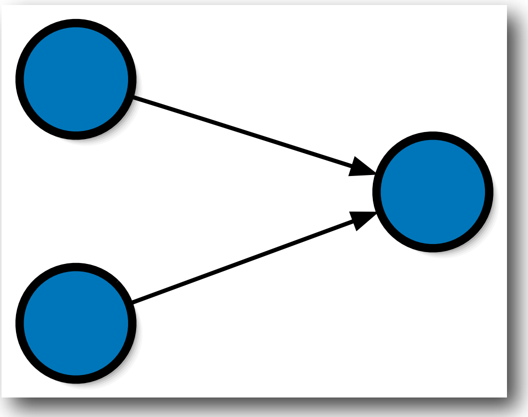
aid transmission

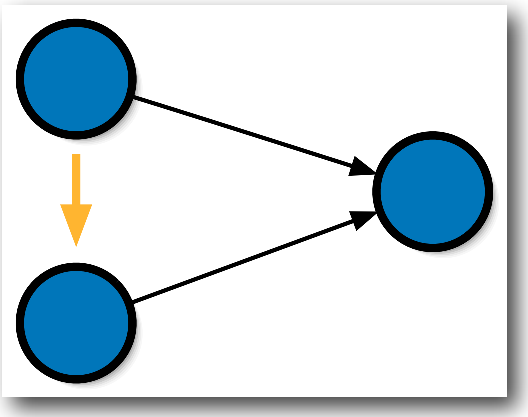
mitigate interference

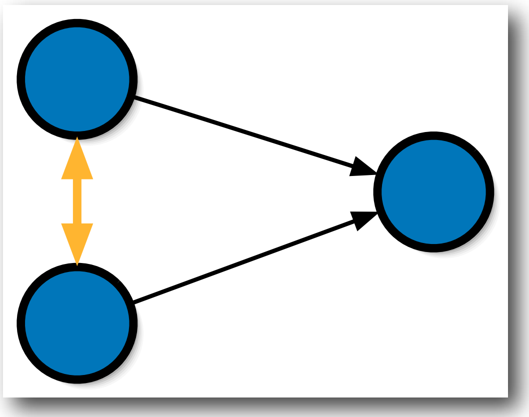
OR BOTH



Disjoint clusters interfere (inter-cluster)
Nodes within a cluster interfere (intra-cluster)



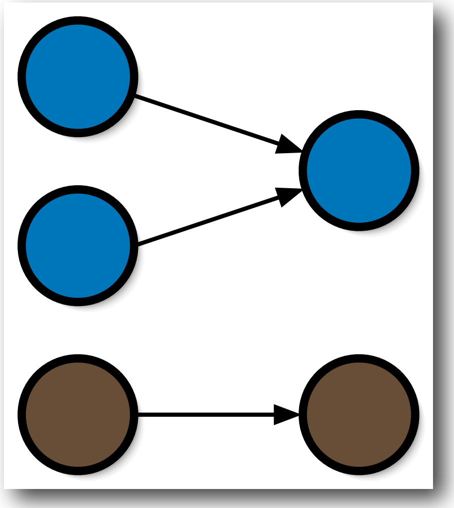


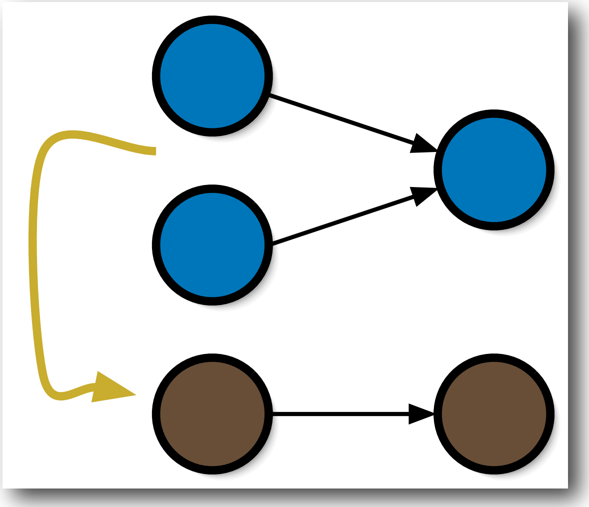


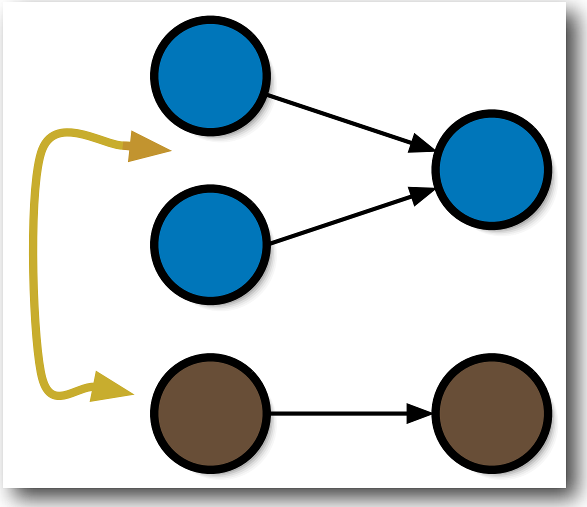
Competitive:
MAC

Cognitive:
Van der Meulen
Prelov

Cooperative:
2x1 vector
MISO







Competitive:
generalized
interference

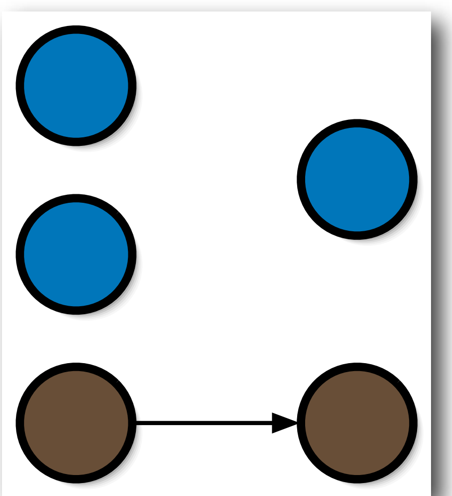
Cognitive:
THIS TALK

Cooperative:
almost, but not
quite broadcast!

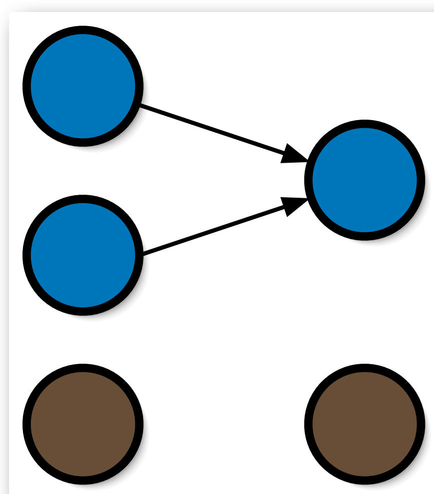
- Motivation and definition
- Relation to previous work
- Theorem intuition
- Achievable region in Gaussian case

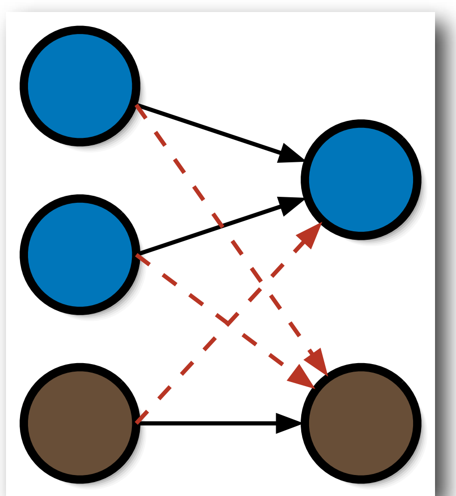
Traditionally

Proposed: simultaneous

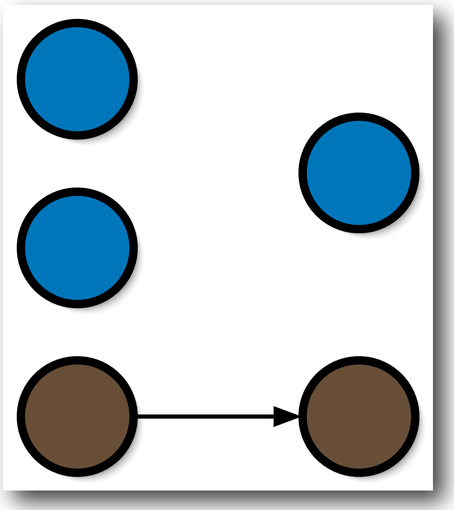


OR

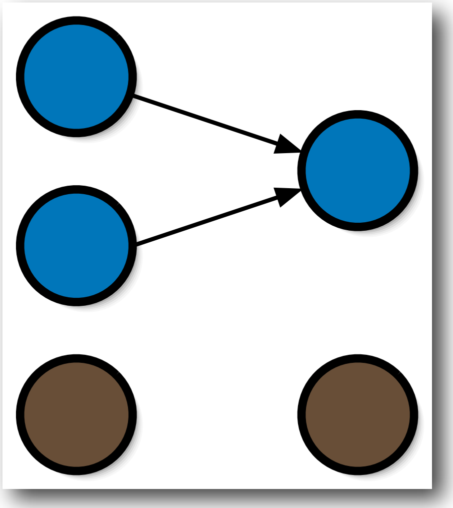




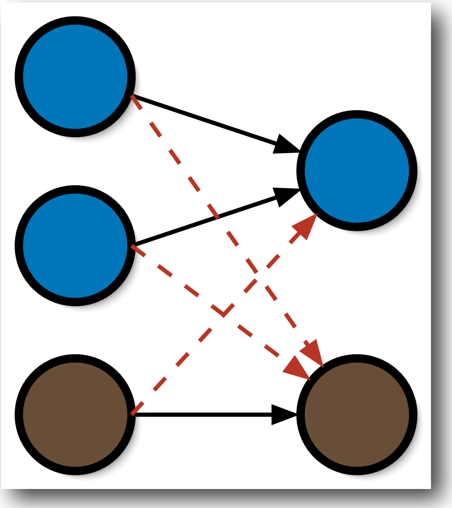
Traditionally



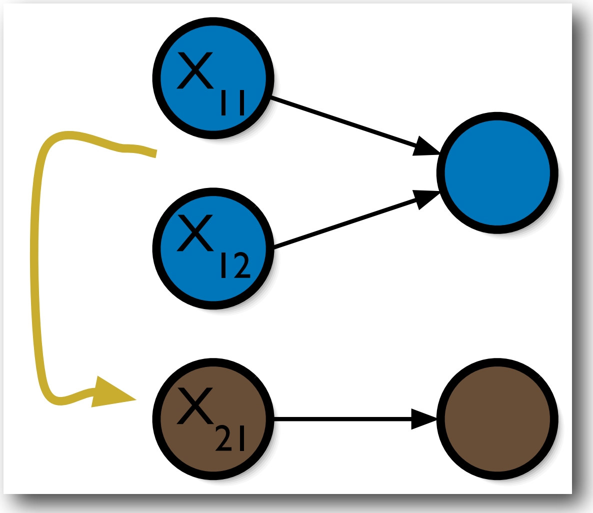
OR



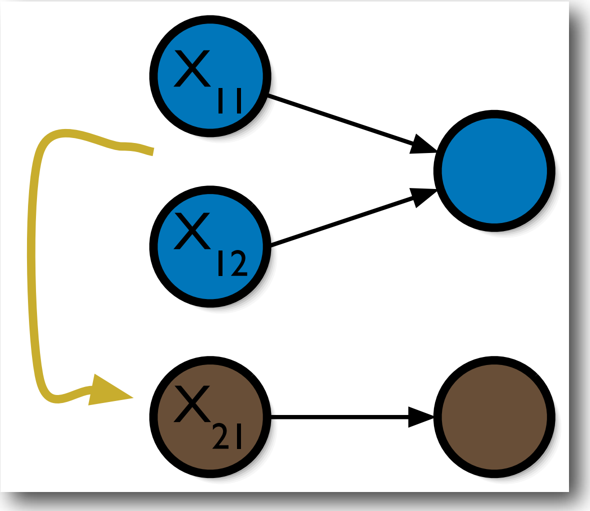
Proposed: simultaneous



The catch: interference



If X_{21} is close to X_{11}, X_{12}
it can obtain their messages

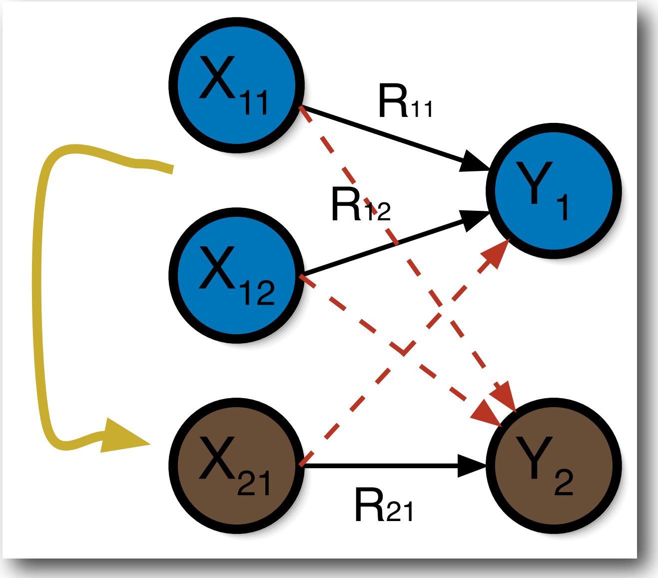


If X_{21} is close to X_{11}, X_{12}
it can obtain their messages

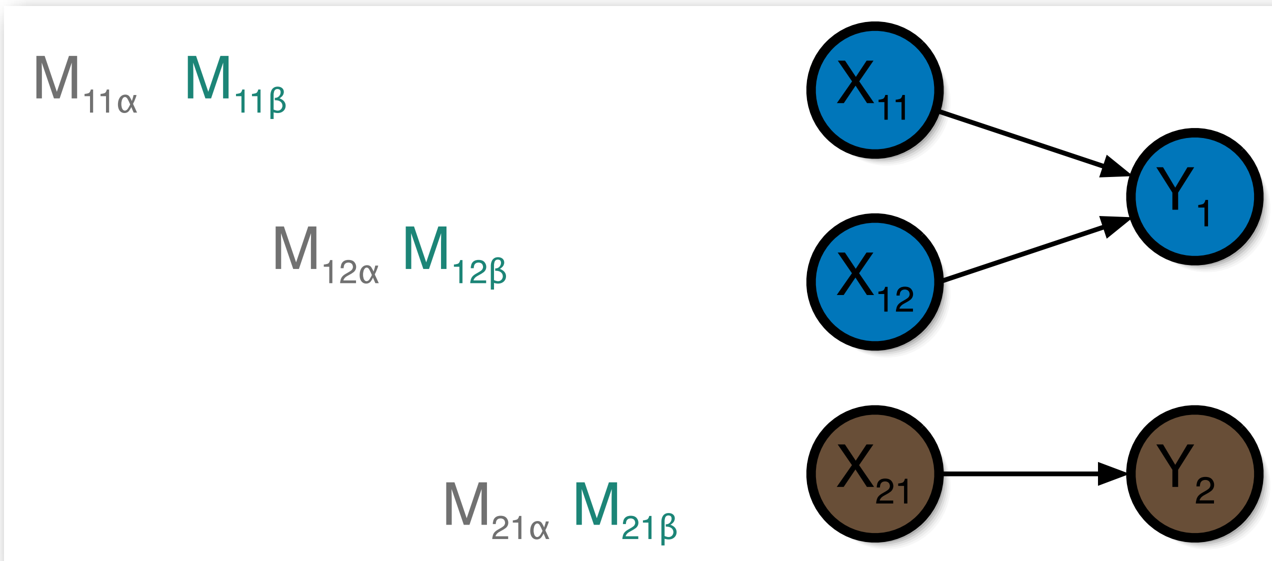
Genie assumption: X_{21} knows X_{11}, X_{12} *a priori*

X_{21} knows the messages of X_{11} and X_{12} a priori.

Asymmetric problem.



During simultaneous transmission, what rates R_{11}, R_{12}, R_{21} are achievable?

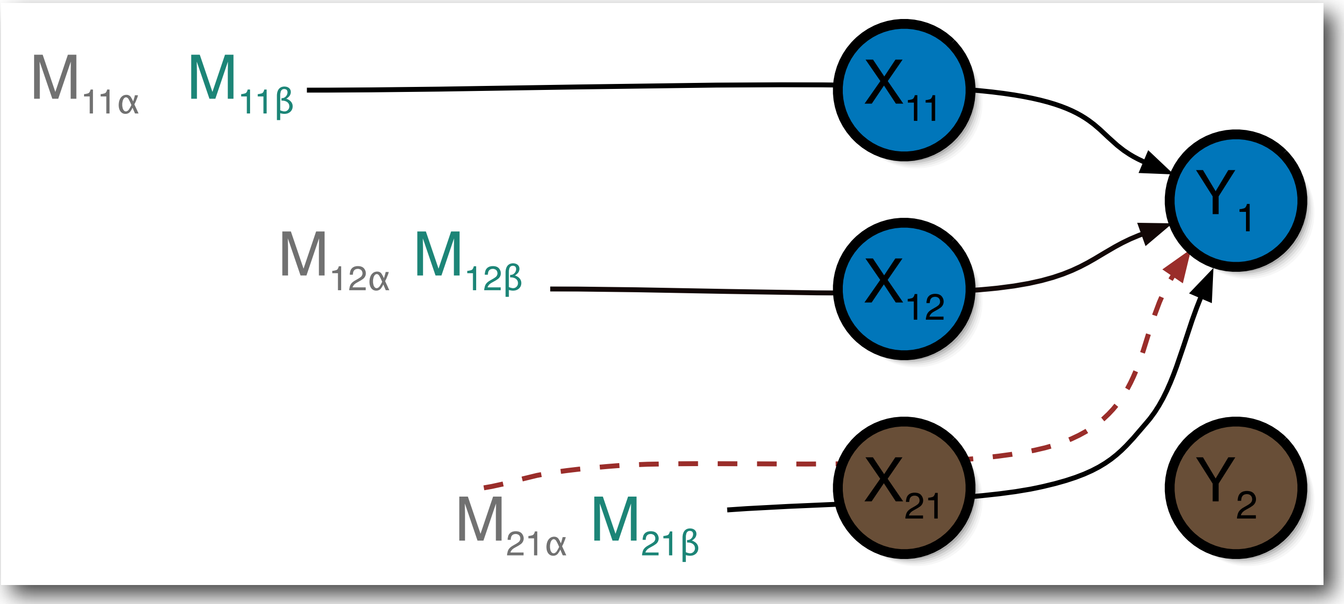


Private variables $M_{11\alpha}$, $M_{12\alpha}$, $M_{21\alpha}$

(intended for one receiver only)

Public variables $M_{11\beta}$, $M_{12\beta}$, $M_{21\beta}$

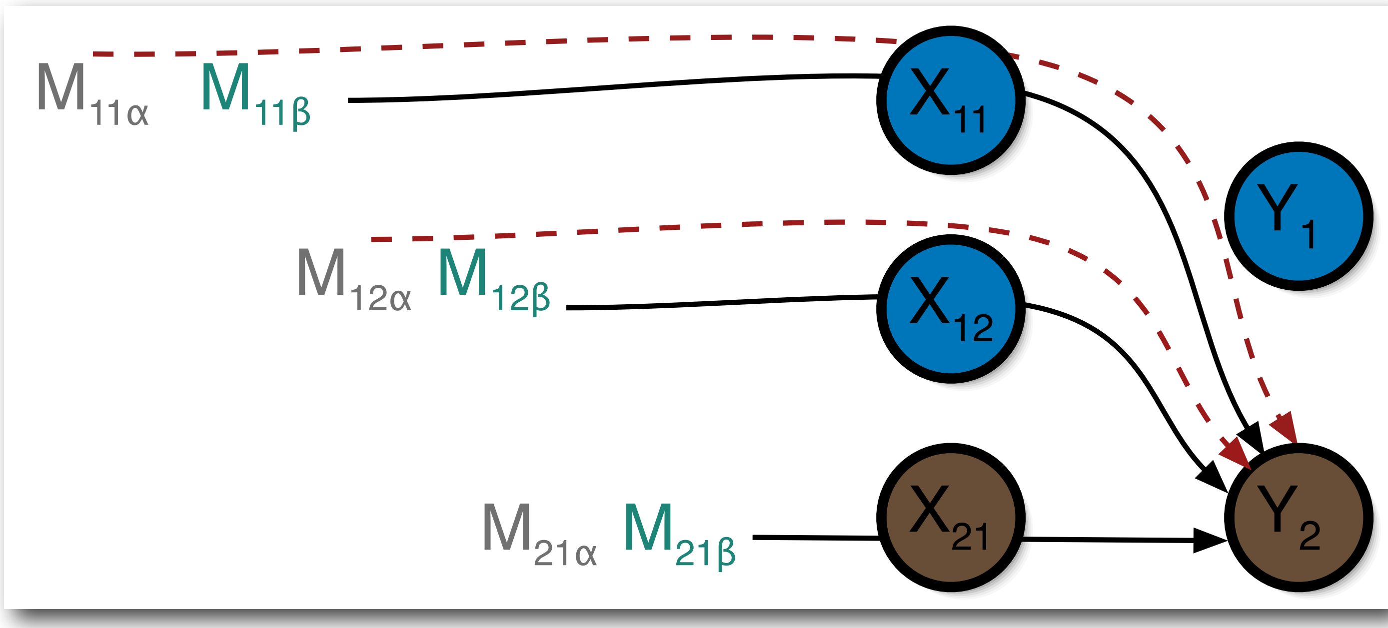
(intended for all receivers)



$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1; \mathbf{M}_T | \mathbf{M}_{\overline{T}})$$

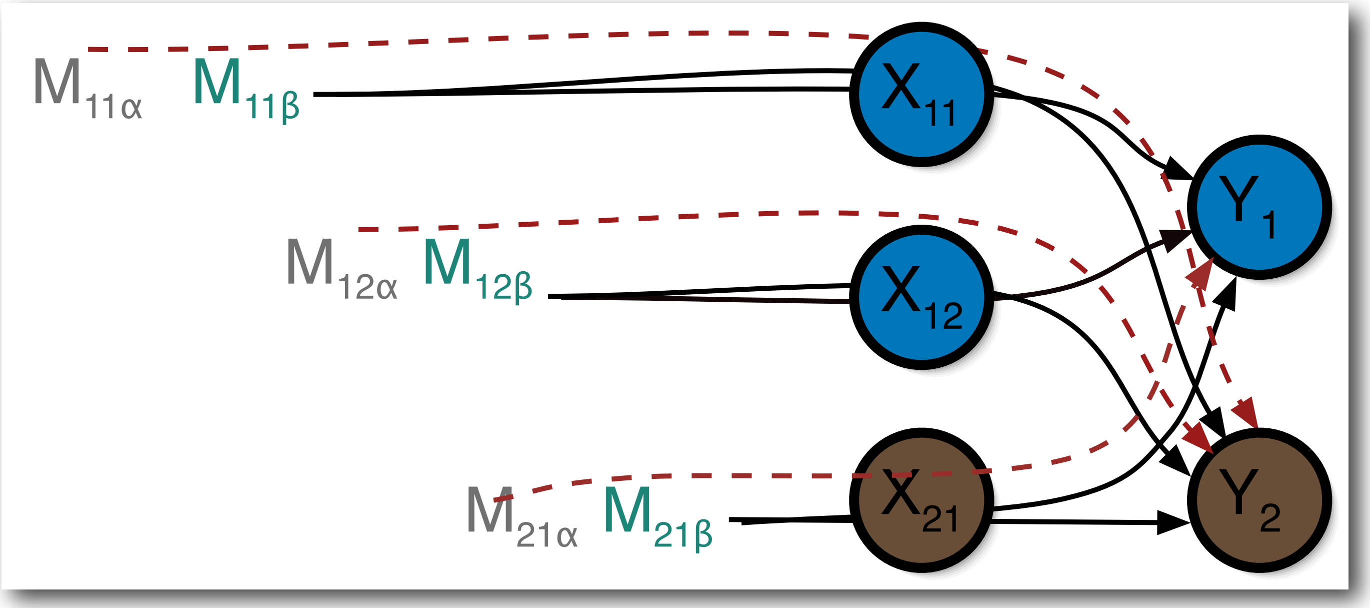
$$T_1 = \{11\alpha, 11\beta, 12\alpha, 12\beta, 21\beta\}$$

L_t is the rate of M_t



$$\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) \leq I(Y_2; \mathbf{M}_T | \mathbf{M}_{\overline{T}})$$

$$T_2 = \{11\beta, 12\beta, 21\alpha, 21\beta\}$$

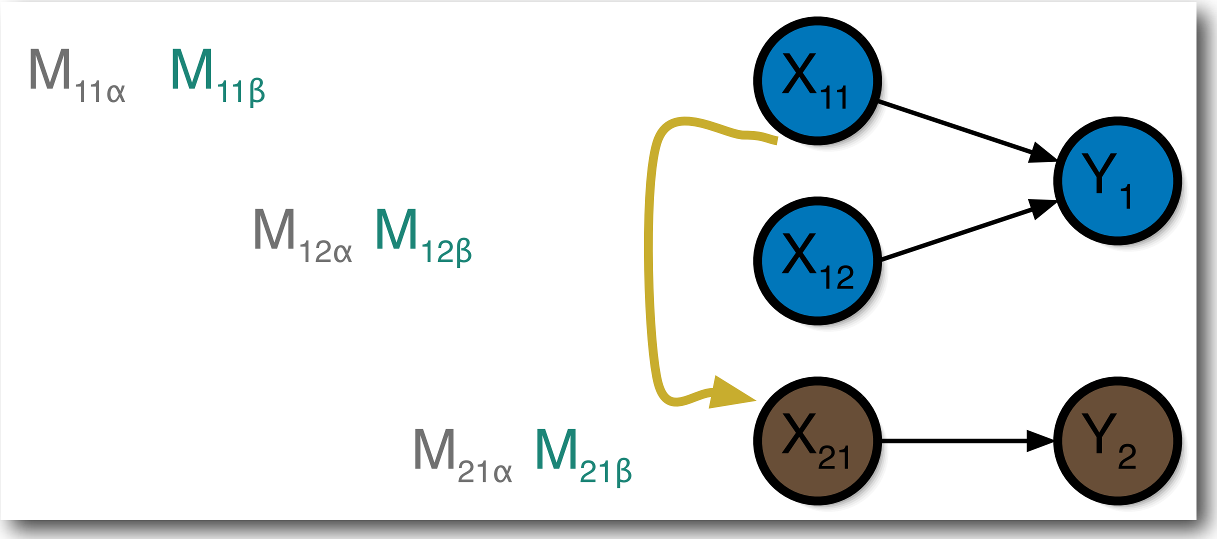


$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1; \mathbf{M}_T | \mathbf{M}_{\overline{T}})$$

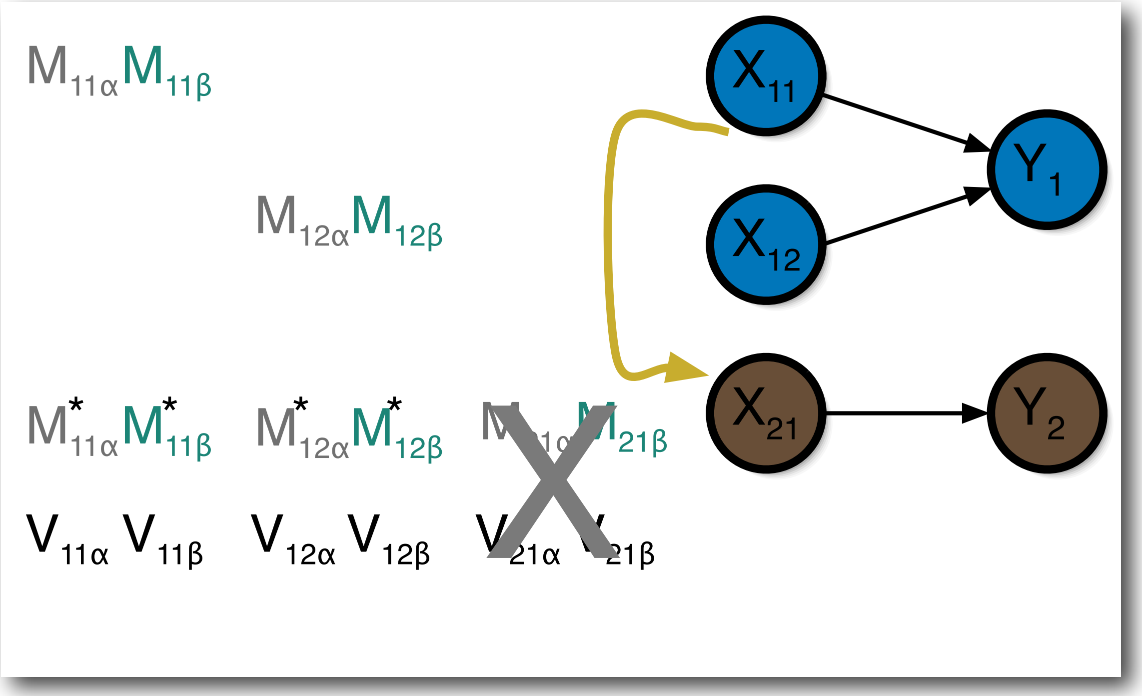
$$\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) \leq I(Y_2; \mathbf{M}_T | \mathbf{M}_{\overline{T}})$$

Secondary transmitter has prior knowledge of the primary's message and can:

1. Aid the primary transmitters
2. Use knowledge of interference to mitigate it

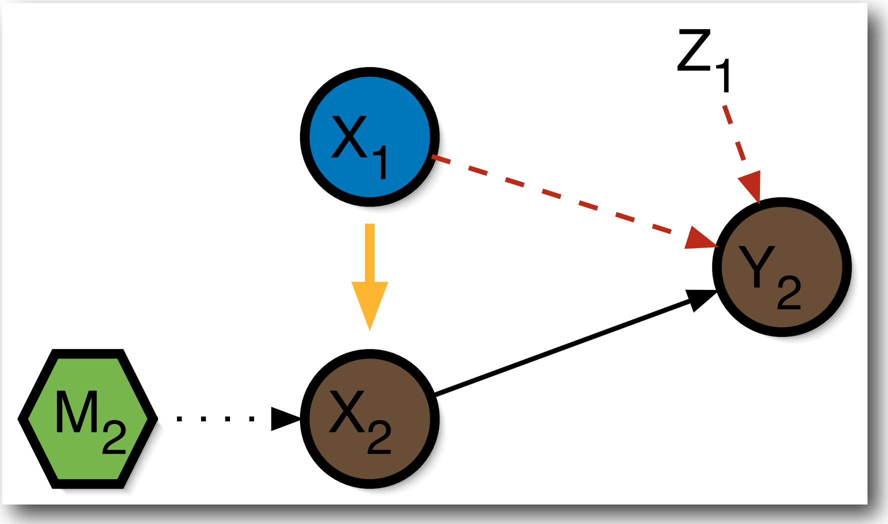


a priori message knowledge allows secondary transmitter to act as relay

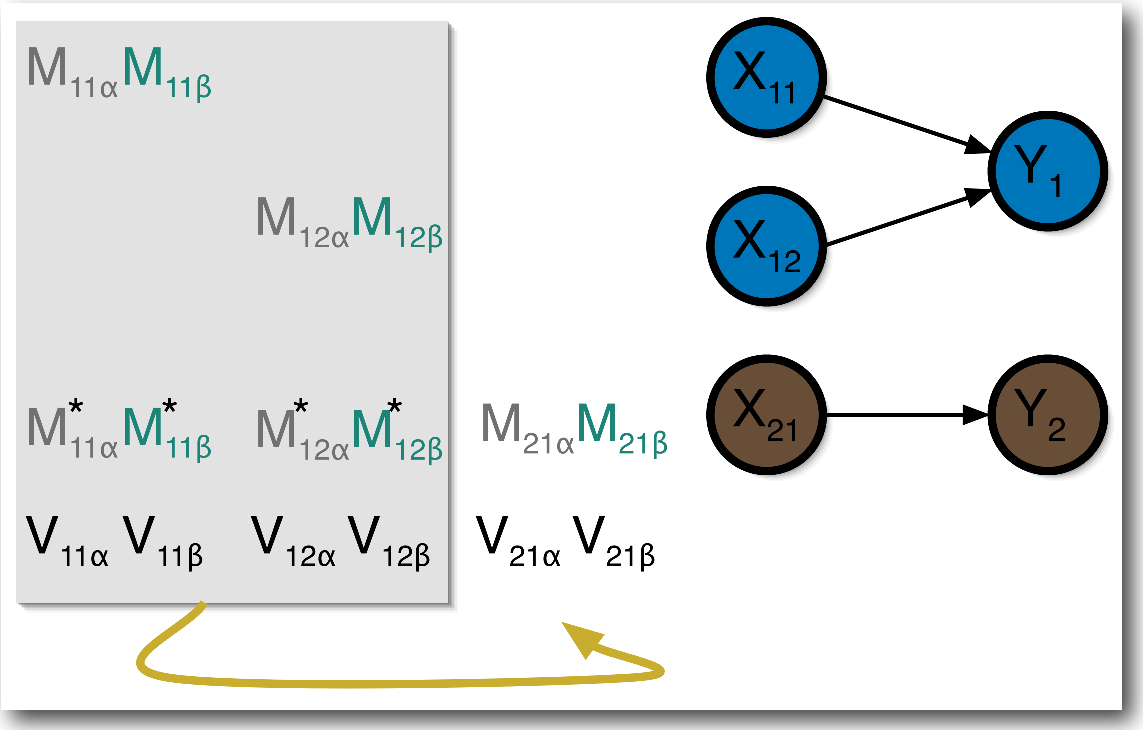


$M^*_{11\alpha} M^*_{11\beta}$ $M^*_{12\alpha} M^*_{12\beta}$ ``aid'' random variables

$V_{11\alpha}$ $V_{11\beta}$ $V_{12\alpha}$ $V_{12\beta}$ (M,M*) combined for notational clarity



$$C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)$$



R_t is the rate of V_t

L_t is the rate of M_t

$$R_{1ik} = L_{1ik}$$

$$R_{2jk} \leq L_{2jk} - I(V_{2jk}; \mathbf{V_1})$$

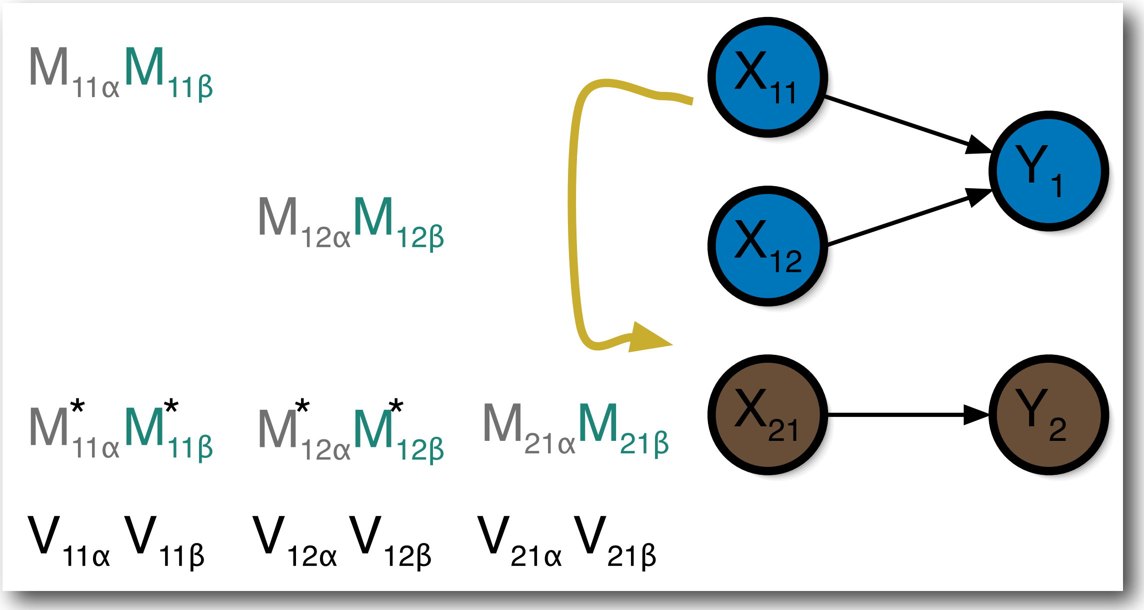
Achievable rate region:

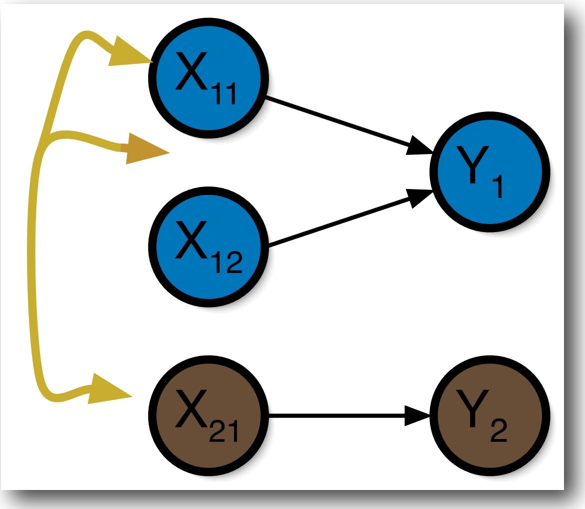
$$\left.\begin{aligned} R_{1ik} &= L_{1ik} \\ R_{2jk} &\leq L_{2jk} - I(V_{2jk};\mathbf{V_1}) \end{aligned}\right\}$$

Gel'fand-Pinsker coding

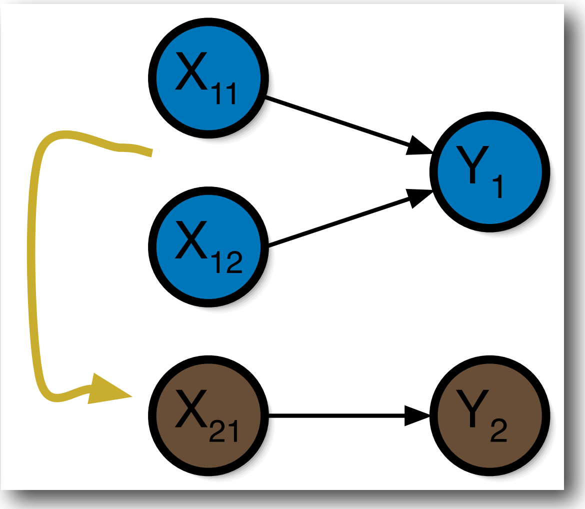
$$\left.\begin{aligned} \bigcap_{T\subset T_1}\left(\sum_{t_1\in T} L_{t_1}\right) &\leq I(Y_1;\mathbf{V}_T|\mathbf{V}_{\overline{T}}) \\ \bigcap_{T\subset T_2}\left(\sum_{t_2\in T} L_{t_2}\right) &\leq I(Y_2;\mathbf{V}_T|\mathbf{V}_{\overline{T}}) \end{aligned}\right\}$$

Overlapping MACs





is an outer bound for

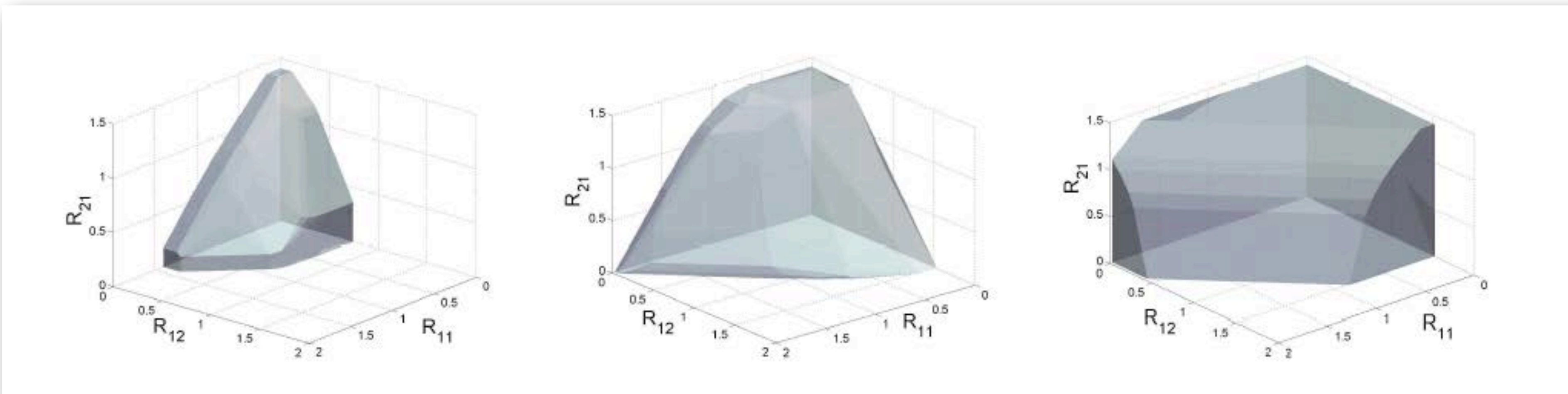


along with the bounds

$$R_{11} \leq I(Y_1; X_{11}, X_{21} | X_{12})$$

$$R_{12} \leq I(Y_1; X_{12}, X_{21} | X_{11})$$

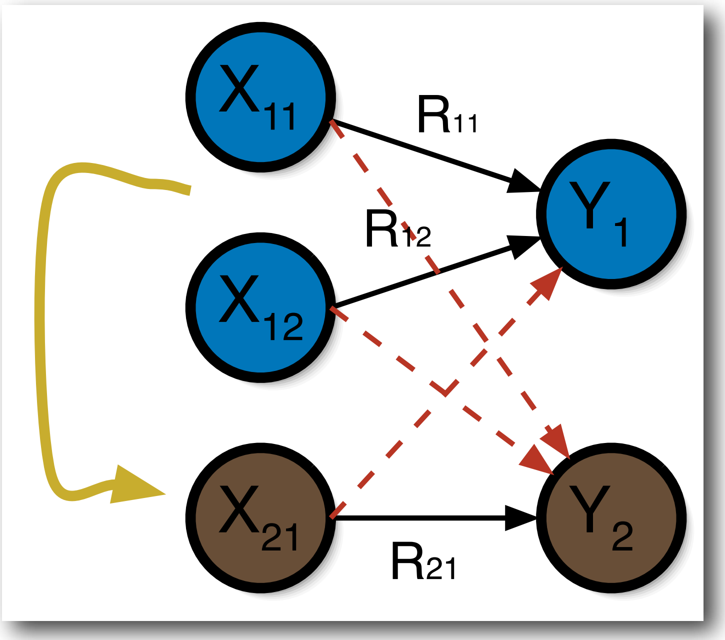
$$R_{21} \leq I(Y_2; X_{21} | X_{11}, X_{12})$$



Competitive
MAC channel

Cognitive
MAC channel

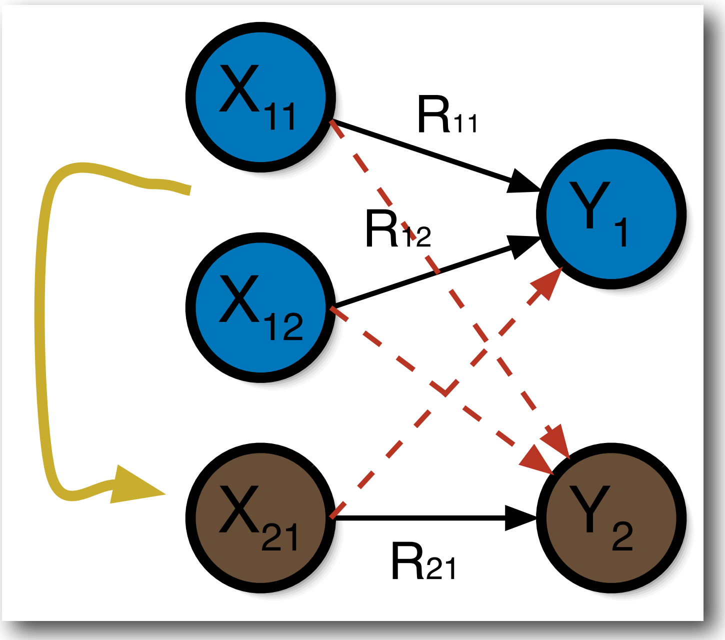
Outer bound
3x2 MIMO
+ inequalities



A priori message knowledge

aid transmission

mitigate interference

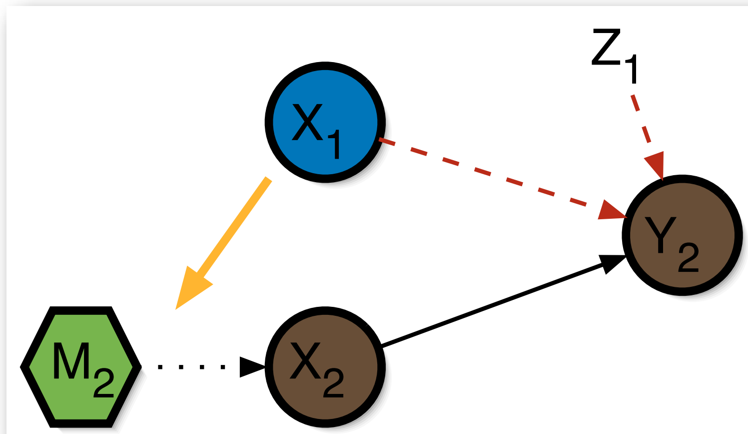


A priori message knowledge

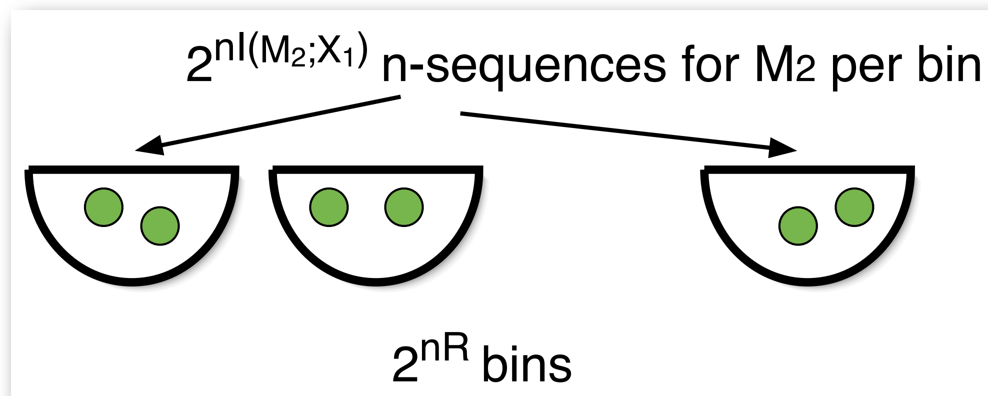
aid transmission

mitigate interference

OR BOTH



To send *bin number i*, given interference X_1 look in bin *i* for M_2 such that (M_2, X_1) are jointly typical.



$$C = \max_{p(m_2, x_2 | x_1)} I(M_2; Y_2) - I(M_2; X_1)$$

$$\bigcap_{T \subset T_G} \left(\sum_{t \in T} R_t \right) \leq I(g(\mathbf{X_1}); \mathbf{M}_T | \mathbf{M}_{\overline{T}})$$

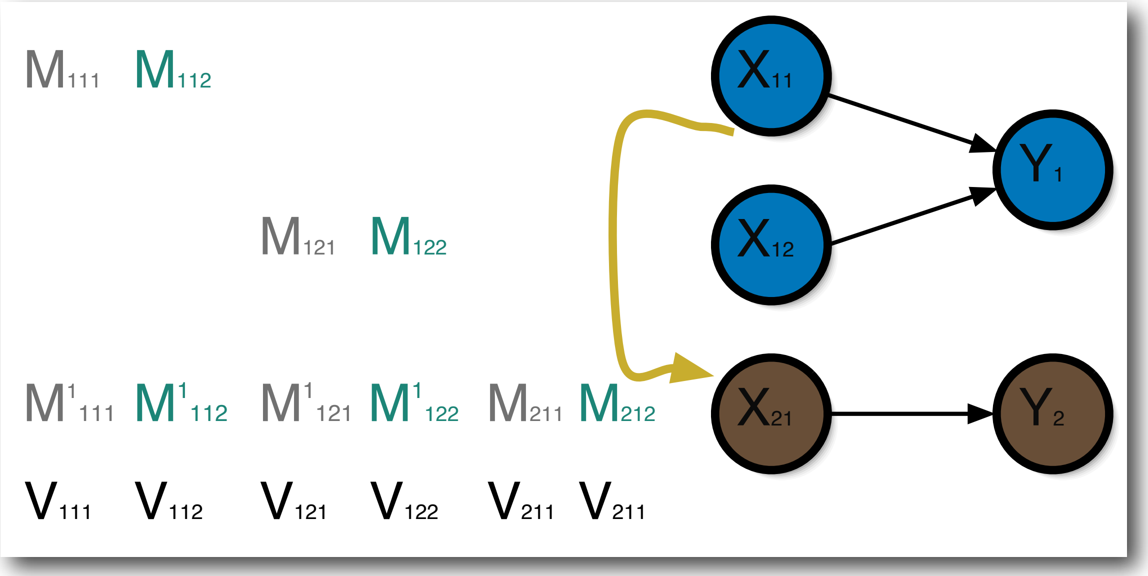
MAC equations for
S_2 to get X_1's messages

$$R_{1ik} = L_{1ik}$$
$$R_{2jk} \leq L_{2jk} - I(V_{2jk}; \mathbf{V_1})$$

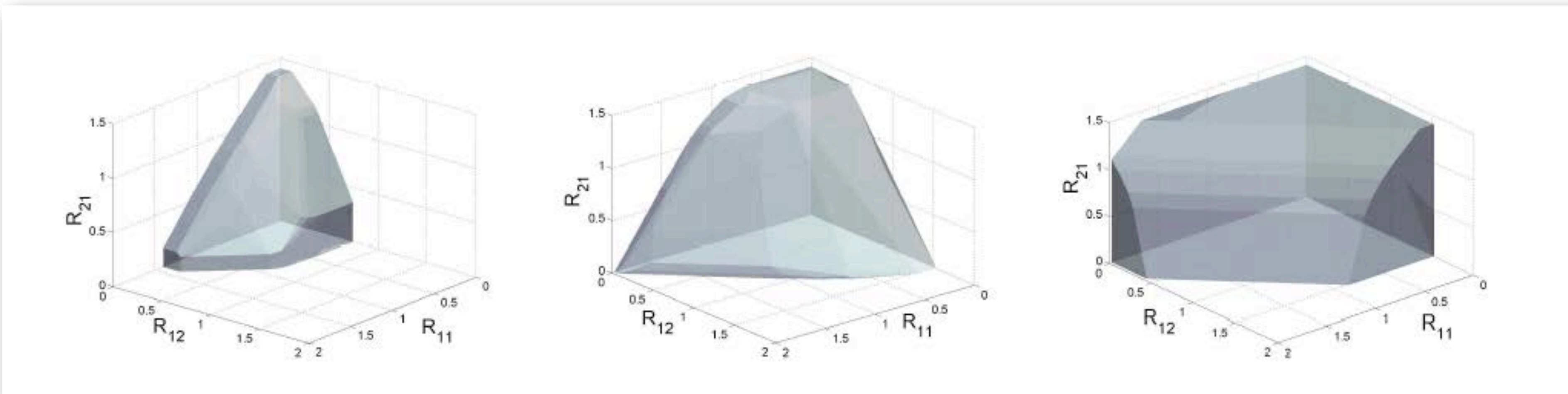
Gel'fand-Pinsker coding

$$\bigcap_{T \subset T_1} \left(\sum_{t_1 \in T} L_{t_1} \right) \leq I(Y_1, \mathbf{V}_{\overline{T}}; \mathbf{V}_T | W)$$
$$\bigcap_{T \subset T_2} \left(\sum_{t_2 \in T} L_{t_2} \right) \leq I(Y_2, \mathbf{V}_{\overline{T}}; \mathbf{V}_T | W),$$

Overlapping MACs



$$
\begin{aligned}
&\left[P(m_{11\alpha})P(m_{11\beta})P(x_{11}|m_{11\alpha},m_{11\beta})\right]\\
&\times\left[P(m_{12\alpha})P(m_{12\beta})P(x_{12}|m_{12\alpha},m_{12\beta})\right]\\
&\quad\times\left[P(m^*_{11\alpha}|m_{11\alpha})P(m^*_{11\beta}|m_{11\beta})\right]\\
&\qquad\times\left[P(m^*_{12\alpha}|m_{12\alpha})P(m^*_{12\beta}|m_{12\beta})\right]\\
&\qquad\quad\times\left[P(m_{21\alpha}|\mathbf{v_1})P(m_{21\beta}|\mathbf{v_1})\right]\\
&\qquad\qquad\times\left[P(x_{21}|m_{21\alpha},m_{21\beta},\mathbf{m}^*)\right]P(y_1|\mathbf{x_1},\mathbf{x_2})P(y_2|\mathbf{x_1},\mathbf{x_2})
\end{aligned}
$$



Competitive
MAC channel

Cognitive
MAC channel

Outer bound
3x2 MIMO
+ inequalities

Volume 0.6536

Volume 1.5064

Volume 2.9127

2-Dimensional Generalized Processor Sharing (2D-GPS)

Raymond Yim

Natasha Devroye

Background: 1D-GPS

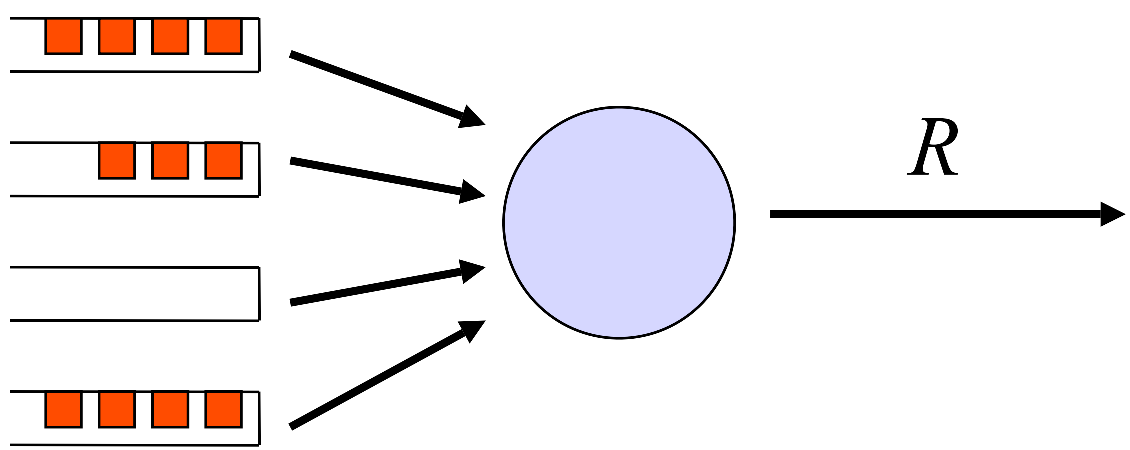
Rate Guarantees

$\phi_1 = 0.2$

$\phi_2 = 0.3$

$\phi_3 = 0.4$

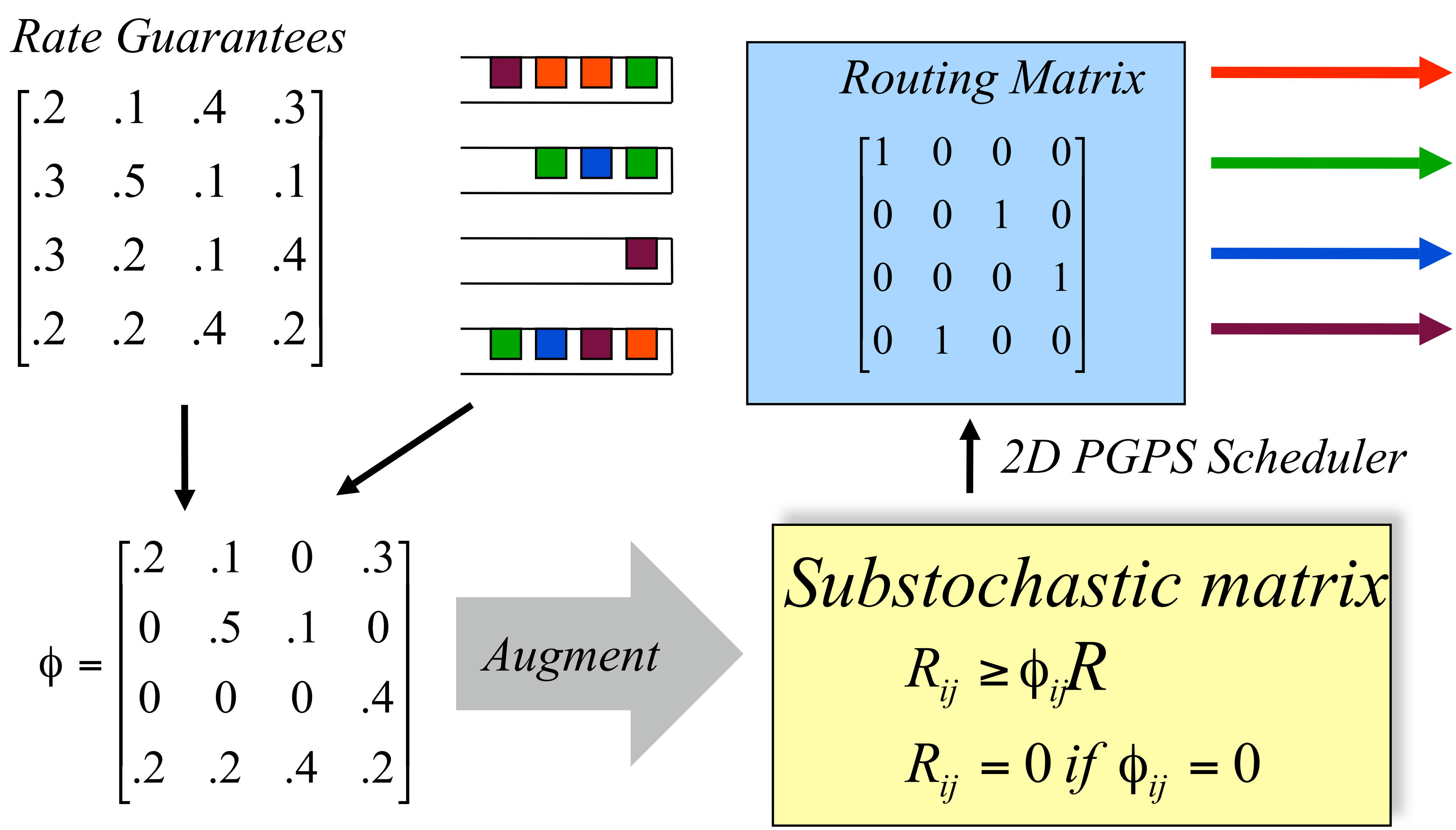
$\phi_4 = 0.1$



$$R_i = \frac{\phi_i R}{\sum_{j:b_j \neq 0} \phi_j} \; if \; b_i \neq 0$$

The goal of Packet-by-Packet GPS (PGPS) server is to mimic the result of GPS

2D-GPS: Overview



$$\begin{bmatrix} .2 & .1 & .4 & .3 \\ .3 & .5 & .1 & .1 \\ .3 & .2 & .1 & .4 \\ .2 & .2 & .4 & .2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\phi = \begin{bmatrix} .2 & .1 & 0 & .3 \\ 0 & .5 & .1 & 0 \\ 0 & 0 & 0 & .4 \\ .2 & .2 & .4 & .2 \end{bmatrix}$$

$$R_{ij} \geq \phi_{ij} R$$

$$R_{ij} = 0 \; if \; \phi_{ij} = 0$$

Fairness in Augmentation

- In 1D-GPS, solutions are both
  - Max-Min Fair
  - Proportionally Fair
- In 2D-GPS, the two fairness metrics will give rise to two different results in general.

Max-Min Fairness

$$\begin{bmatrix} 1/5 & 0 & 0 \\ 0 & 1/2 & 1/10 \\ 2/5 & 0 & 2/5 \end{bmatrix} \xrightarrow{\times \frac{5}{4}} \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 5/8 & 1/8 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

$$\xrightarrow{\times \frac{4}{3}} \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 5/6 & 5/6 \\ 1/2 & 0 & 1/2 \end{bmatrix} \xrightarrow{\times \frac{3}{2}} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 5/6 & 5/6 \\ 1/2 & 0 & 1/2 \end{bmatrix}$$

Proportional Fairness

$$\min \sum_{i,j} \left| R_{ij} - k\phi_{ij} \right| \quad \text{subjected to}$$

$$R_{ij} \geq \phi_{ij}$$

$$R_{ij} = 0 \; if \; \phi_{ij} = 0$$

$$\sum_i R_{iq} \leq 1 \; \forall \; q$$

$$\sum_j R_{pj} \leq 1 \; \forall \; p$$

$$k \geq 1$$

$$\left(\sum_j R_{pj} - 1 \right) \left(\sum_i R_{iq} - 1 \right) = 0 \; if \; \phi_{ij} \neq 0$$

The non-augmentability constraints are non-linear

Birkhoff Decomposition

- Any NxN doubly stochastic matrix can be represented by a convex sum of at most $(N\text{-}1)^2+1$ permutation matrices.

$$A = \sum_i \lambda_i P^i, \quad \sum_i \lambda_i = 1$$

Lemma

- Any non-augmentable matrix R from ϕ can be represented by a convex sum of non-absorbable zero-enforced permutation matrices of ϕ.

$$A = \sum_i \lambda_i P^i, \quad \sum_i \lambda_i = 1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & X & 0 \end{bmatrix}, \begin{bmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & X & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & X \\ X & 0 & 0 \\ 0 & X & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & X \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Transform problem

Lemma 3: Let $P = \{P^1, \cdots, P^z\}$ be the set of all non-absorbable zero-enforced permutation matrices of Φ, and let p_{ij}^n be the entries of matrix P^n for $n \in \{1, \cdots z\}$. Let $\lambda = (\lambda_1, \cdots, \lambda_z)$ be a vector of dimension $z = |P|$. Suppose λ and k are the solution to the optimization problem:

$$\min \sum_{i,j} \left| \sum_{n:p_{ij}^n=1} \lambda_n - k\phi_{ij} \right|, \tag{7}$$

subjected to

$$\sum_{n:p_{ij}^n=1} \lambda_n \geq \phi_{ij} \quad \forall \quad i, j, \tag{8}$$

$$\sum_n \lambda_n = 1, \tag{9}$$

$$\lambda_n \geq 0 \quad \forall \quad n, \tag{10}$$

$$k \geq 0, \tag{11}$$

and the service rate matrix $R = \sum_{n=1}^{z} \lambda_n P^n$ satisfies the non-augmentability constraint. Then, we say R is the proportional fair solution to Φ, and k is the proportional increase of Φ.

Example 2: Using Φ from Example 1. We first write the service rate matrix R as a convex combination of non-absorbable zero-enforced permutation matrices. That is, $R = \lambda_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \lambda_4 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. The non-augmentability constraint is $\lambda_2\lambda_3 = 0$. Solve the two linear optimization problems, one with $\lambda_2 = 0$ and one with $\lambda_3 = 0$, the minimum cost result from $\lambda_2 = 0$. The resulting $R = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.85 & 0.15 \\ 0.6 & 0 & 0.4 \end{bmatrix}$.

Example

$$\Phi = \begin{bmatrix} \frac{1}{5} & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{10} \\ \frac{2}{5} & 0 & \frac{2}{5} \end{bmatrix}$$

Max-min augmentation

$$\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{5}{6} & \frac{1}{6} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

PF augmentation

$$\begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.85 & 0.15 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

Compound Dirty Paper Channel

Patrick Mitran, Natasha Devroye, Vahid Tarokh
Harvard University

- Results on the capacities of channels with side information known at the transmitter assume *full channel knowledge*
- In realistic cognitive radio channels, we should not assume full channel knowledge. Results for cognitive radio channels use results from channels with known side-info at the transmitter

- Channel with side information at the transmitter
- Fading channels, where the channel state is unknown to the transmitter, but known to the receiver
- What are bounds on the capacity?

Theorem 2: Let $p^{\beta}_{Y|X,S}$ be a compound channel continuously parameterized in $\beta \in \mathcal{C}$ ($\mathcal{C}$ compact) where the input and output alphabets are standard. Then, the capacity of the compound channel with side-information at the transmitter, C, is bounded by $C_L \leq C \leq C_U$ where

$$C_L = \sup_{p_{U|X,q(S),W}, p_{X|q(S),W}, P_W} \left[\inf_{\beta \in \mathcal{C}} \left[I^{\beta}(U;Y|W) - I(U;q(S)|W) \right] \right] \tag{17}$$

$$C_U = \sup_{p^{\beta}_{U|X,S,W}, p_{X|S,W}, P_W} \left[\inf_{\beta \in \mathcal{C}} \left[I^{\beta}(U;Y|W) - I(U;S|W) \right] \right], \tag{18}$$

and the suprema are over all distributions on standard alphabets U, all distributions on finite alphabet random variables W and all quantizers $q(.)$ for S. $\square$

Returning to the cognitive radio scenario, we consider the problem of encoding a message V with knowledge of a Gaussian interfering signal S of power Q. The encoder output X is also power constrained to $P = Q$ and the signal received at the decoder is $Y = \beta_1 X + \beta_2 S + Z$ where Z is independent Gaussian noise and the compound parameter is $\beta := (\beta_1, \beta_2)$.

Similar to Costa's scheme, we suggest $U = X + \alpha S$, where α is now chosen as a function of the second order statistics of β_1 and β_2. The scheme proposed in Section V selects

$$\alpha = \frac{\mu_1^* \mu_2 SNR}{(|\mu_1|^2 + \sigma_1^2) SNR + 1}. \tag{5}$$

We note the following three facts about this choice for Ricean fading channels where β_1 and β_2 have K-factors K_1 and K_2 respectively:

1) If $K_1, K_2 \to \infty$, then the scheme is identical to Costa's with $\alpha = P/(P+N)$ and the interference is perfectly mitigated.
2) If either $K_1 \to 0$ or $K_2 \to 0$, the scheme treats the interferer as noise.
3) The performance does not depend on the phase difference between μ_1 and μ_2 as this choice of α rotates the mean channels so that their phases are aligned.

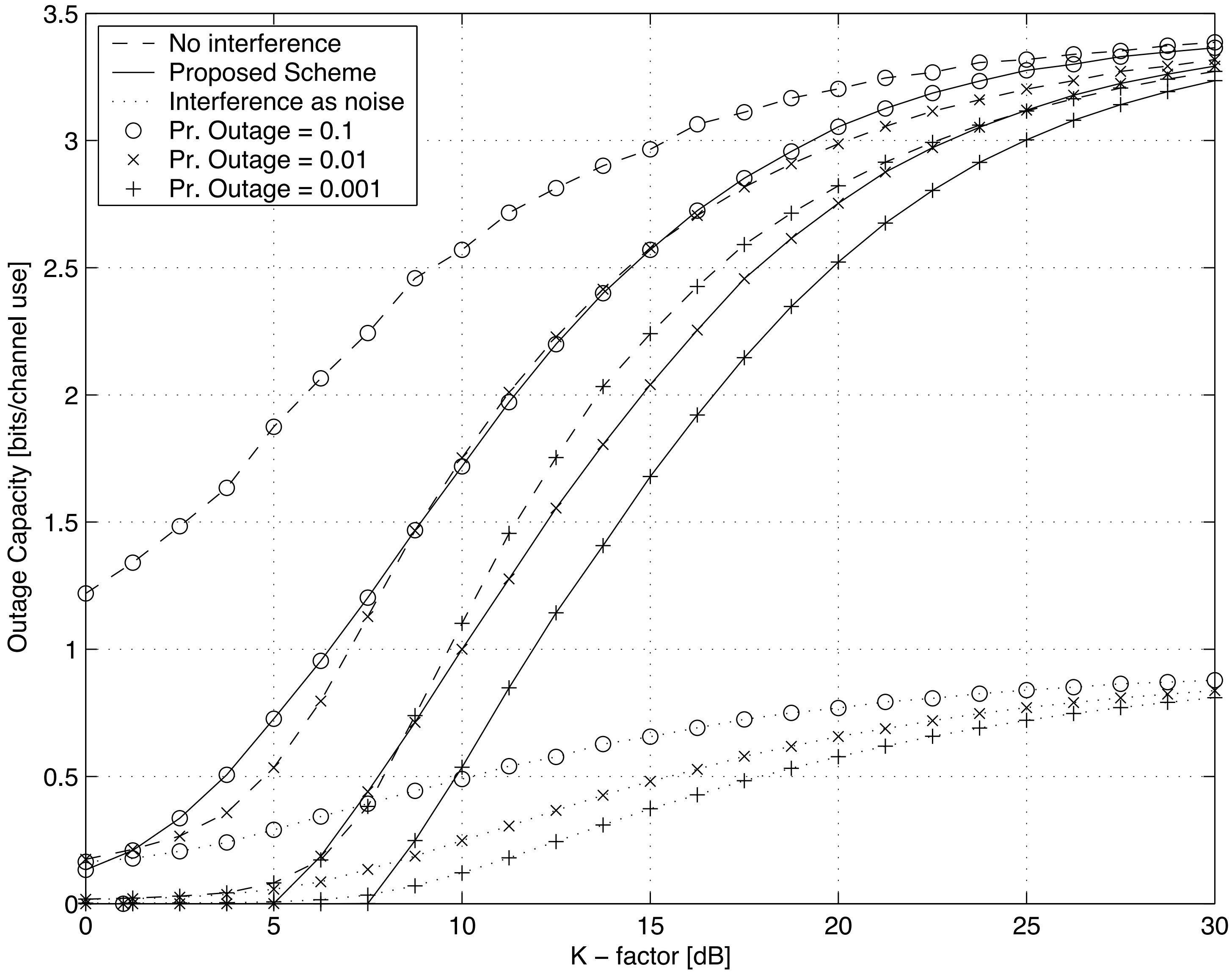


Fig. 6. Communications over a fading channel with a fading interferer whose signal, but not fading coefficient, is known at the transmitter for $SNR = 10$ dB with $P = Q = 1$.

SDR Implementation of Collaborative Communications

Oh-Soon Shin, H.T. Kung, Vahid Tarokh
Harvard University

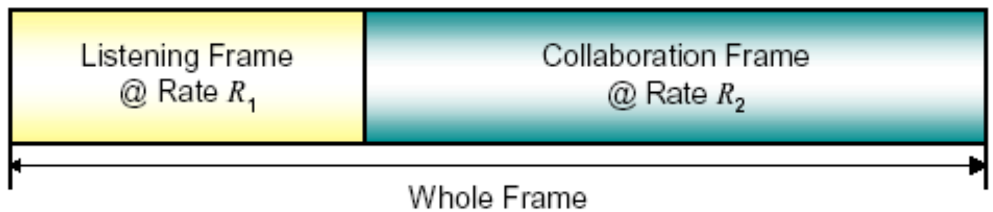
John Chapin's Team
Vanu Inc.

- Design and implementation of an OFDM-based space-time collaborative system
- Our lab completed the simulations in Matlab/C, and Vanu Inc. will do the software radio platform design and implementation.

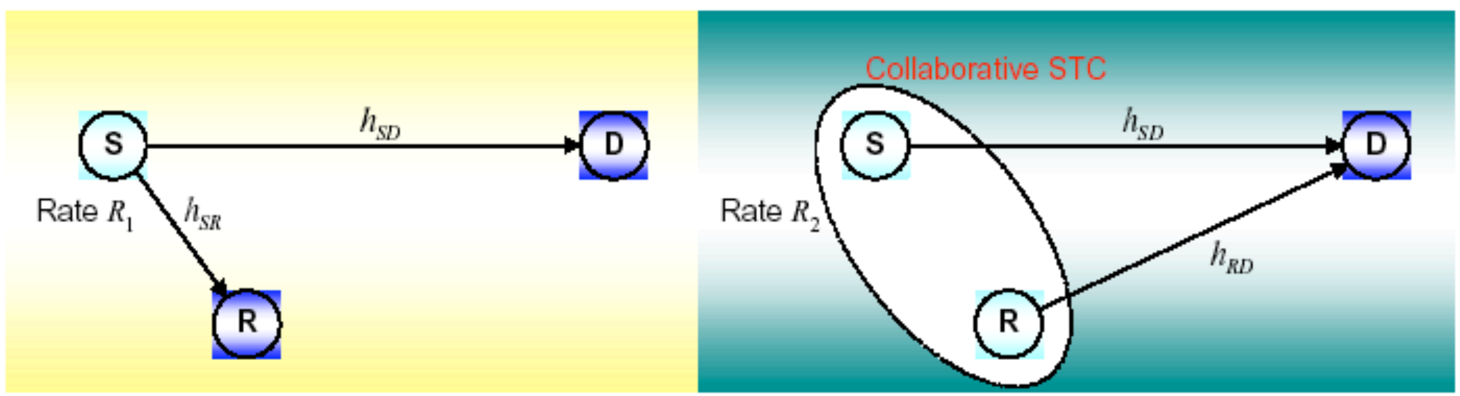
- Carrier frequencies: 902-928 MHz
- Bandwidth: < 5 MHz signal
- 64 subcarriers, spaced at 72kHz

Parameter	Value
Total bandwidth (W)	4.625 MHz
Total number of subcarriers (N_T)	64
Number data subcarriers (N_D)	48
Number pilot subcarriers (N_P)	4
Number of guard or null subcarriers (N_G)	12
Subcarrier frequency spacing (Δ_F)	72.27 kHz
IFFT/FFT period (T_{FFT})	13.838 μsec (64 samples)
Guard interval duration (T_{GI})	2.162 μsec (10 samples)*
OFDM symbol duration (T_{SYM})	16.0 μsec ($T_{FFT} + T_{GI}$)

- Each PHY frame consists of two subsequent phases



- T-R configuration during each phase

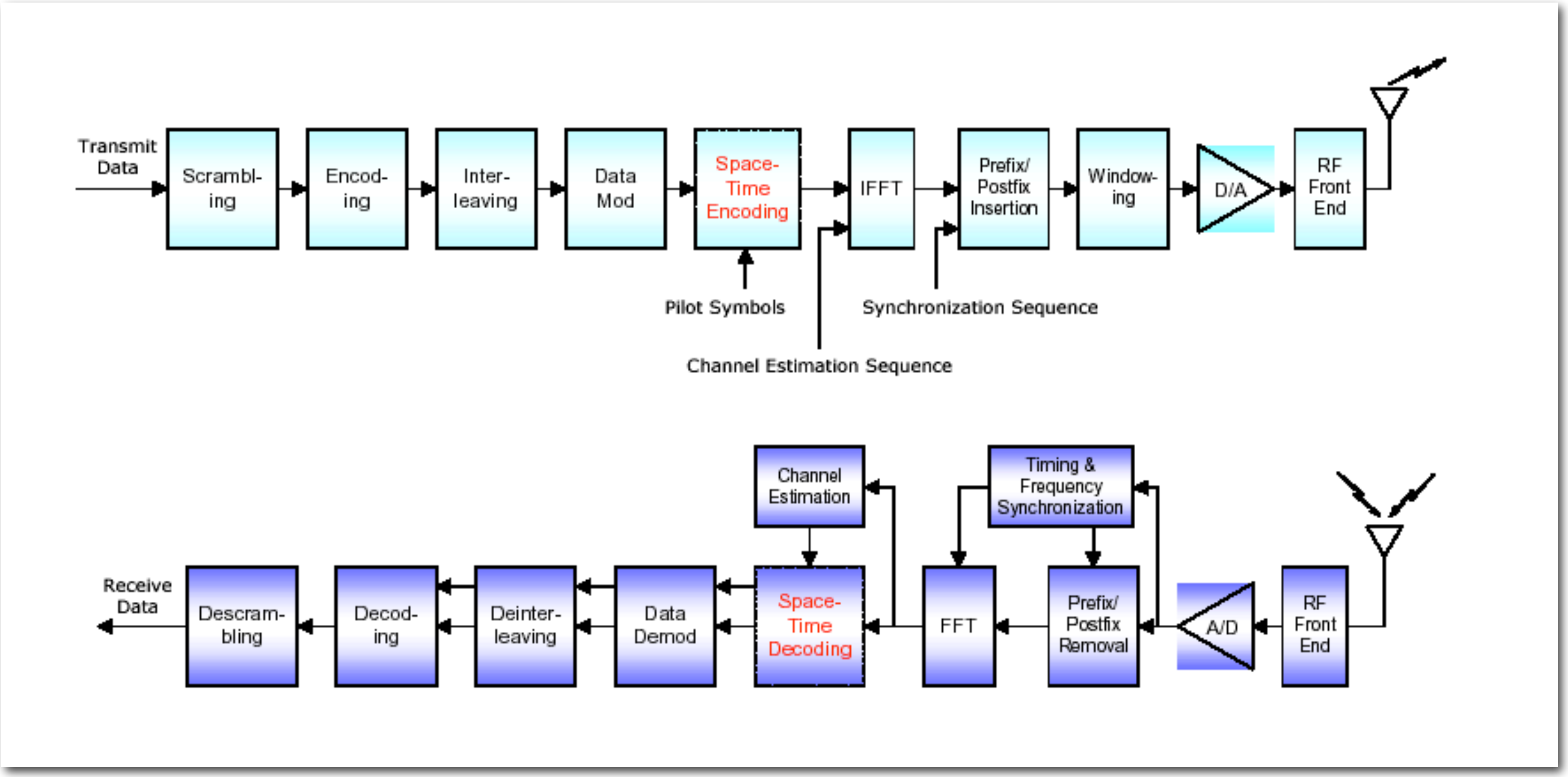


Listening Phase

Collaboration Phase

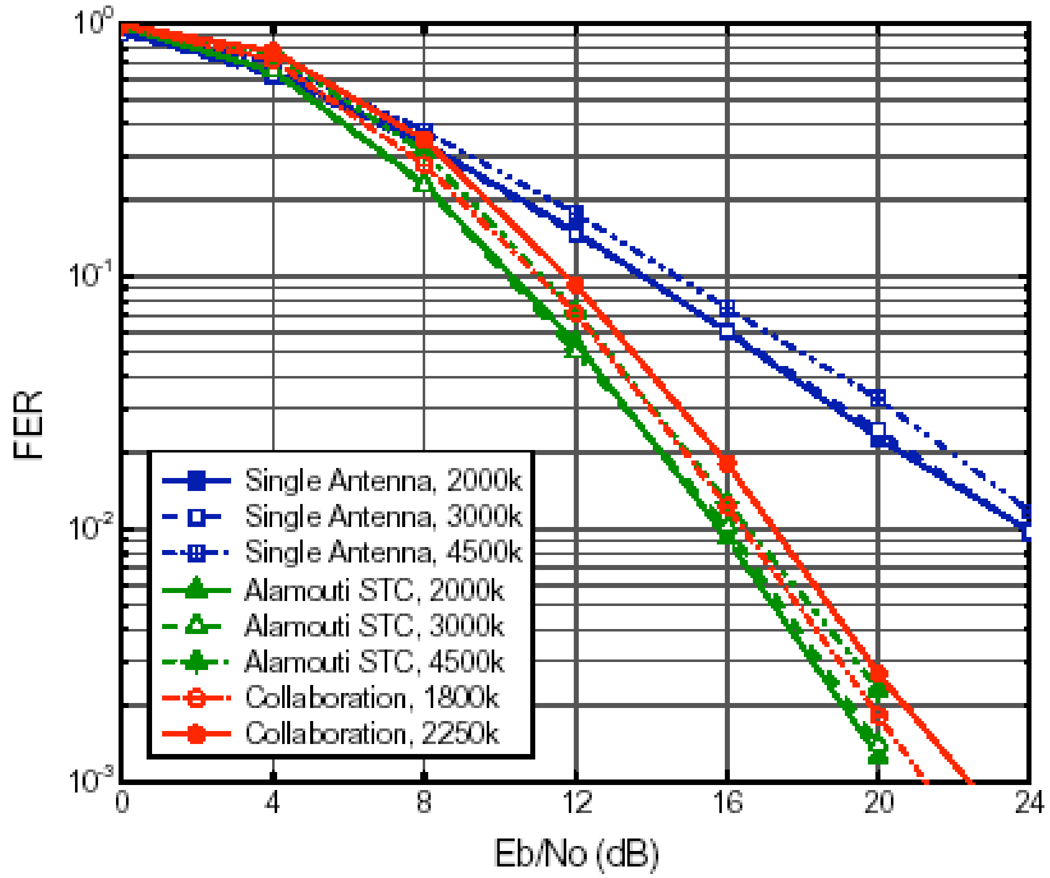
* S: Source, D: Destination, R: Relay

- Listening phase
  - The source broadcasts an information frame
  - The destination and relay try to decode the frame
  - If the destination can decode it correctly, it will ignore the following collaboration frame
- Collaboration phase
  - If the relay succeeded in decoding of the listening frame, the source and relay will transmit space-time coded signal
  - Otherwise, the relay should be silent and only the source transmits as in the listening phase
  - For more reliable decision, the destination can combine the signals from the listening and collaboration phases



Transmit Data
Scrambl-ing
Encod-ing
Inter-leaving
Data Mod
Space-Time Encoding
IFFT
Prefix/ Postfix Insertion
Window-ing
D/A
RF Front End
Pilot Symbols
Synchronization Sequence
Channel Estimation Sequence
Channel Estimation
Timing & Frequency Synchronization
Receive Data
Descram-bling
Decod-ing
Deinter-leaving
Data Demod
Space-Time Decoding
FFT
Prefix/ Postfix Removal
A/D
RF Front End

Behaves more like a 2x1 system than a 1x1 system



- Develop an equivalent system for cognitive transmission
- Toughest to tackle will be developing codes that layer relaying and dirty-paper coding schemes in the same transmission
- Same synchronization issues
- 2 phase protocols seem practical

To cooperate or to select?

Natasha Devroye, Sumeet Sandhu
Intel Research

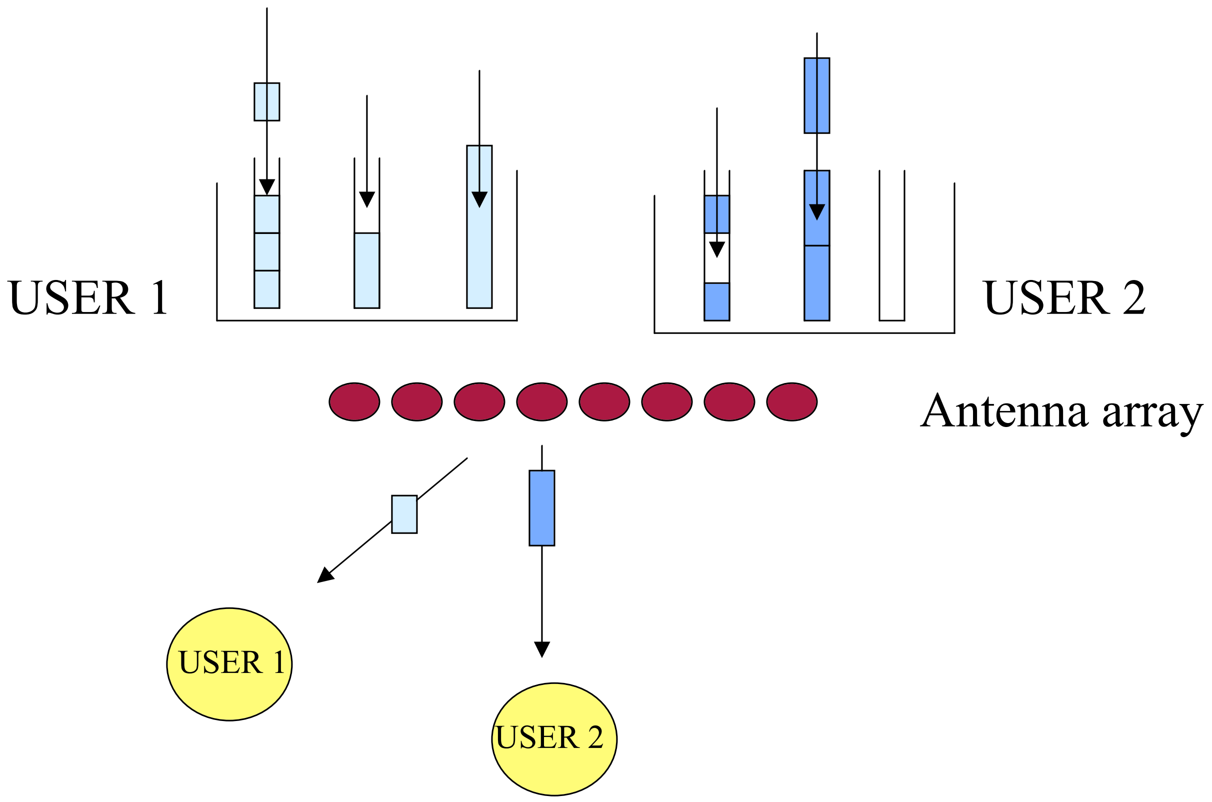
- Evaluate gains of cooperation between base stations in an 802.16d wireless network over no-cooperation, as well as selection.
- System level simulations that take shadowing, fading, sectorization, pathloss, realistic channel models, and interference from other base-stations/users into account.

- Cooperation performs only slightly better than selection.
- Shadowing drastically alters all simulations.
- Shadowing diversity exists.

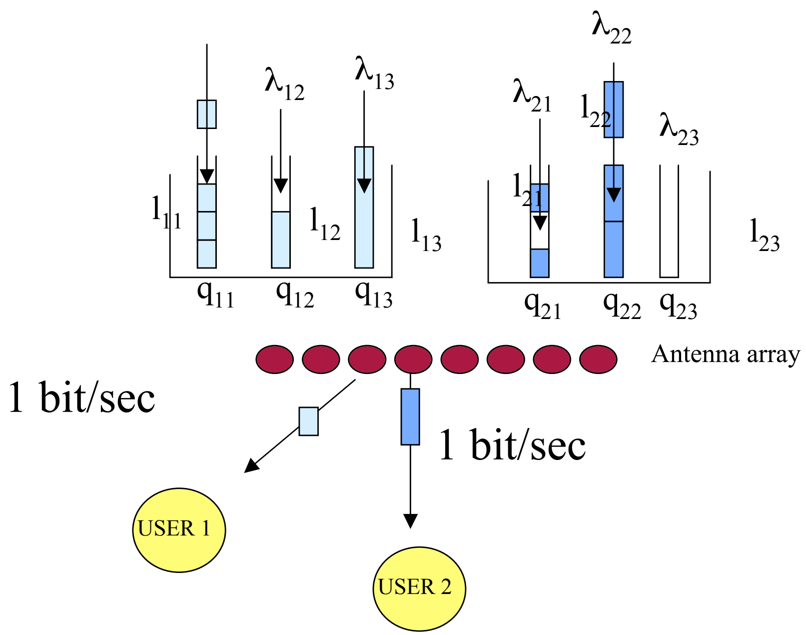
Scheduling for SDMA

Natasha Devroye, Vahid Tarokh
Harvard University

Goal: Produce a stable system of queues that is stochastically smaller that any queue length process produced by another stable policy.



- Found necessary conditions on Poisson arrival rates of various length packets that ensure the stability the queue length process.
- Simulated various scheduling algorithms, and saw their throughput.



The system of queues with arrival rates $\lambda_{11}, \lambda_{12}, \ldots, \lambda_{1K}, \lambda_{21}, \lambda_{22}, \ldots, \lambda_{2K}$ is stable if $\exists \sigma_{ij}$ for $i, j \in \{0, 1, \ldots, K\}$ such that

$$\Updownarrow$$

$$\sigma_{ij} = 0 \text{ if } C_{ij} = 0$$

$$\lambda_{1i} = \sum_{j=0}^{K} \sigma_{ij}$$

$$\lambda_{2j} = \sum_{i=0}^{K} \sigma_{ij}$$

$$\sum_{i=0}^{K} \sum_{j=0}^{K} \sigma_{ij} \max\{l_{1i}, l_{2j}\} < 1$$

- Base the decision of who to schedule on the current parameters and queue lengths only.
- If there exists a pair (i*, j*) with $C_{i^*j^*}=1$, and non-empty queues, send the pair such that:

$$\text{Criteria 1: } (i^*, j^*) = \arg\max_{i,j}(q_{1i}, q_{2j})$$
$$\text{Criteria 2: } (i^*, j^*) = \arg\max_{i,j}(q_{1i}q_{2j})\left(\frac{l_{1i}+l_{2j}}{\max\{l_{1i},l_{2j}\}}\right)$$
$$\text{Criteria 3: } (i^*, j^*) = \arg\max_{i,j}\left(\frac{q_{1i}q_{2j}}{q_{1i}+q_{2j}}\right)\left(\frac{l_{1i}+l_{2j}}{\max\{l_{1i},l_{2j}\}}\right)$$
$$\text{Criteria 4: } (i^*, j^*) = \arg\max_{i,j}\left(\frac{q_{1i}q_{2j}}{q_{1i}+q_{2j}}\right)\left(\frac{l_{1i}+l_{2j}}{\max\{l_{1i},l_{2j}\}}\right)^2$$

EBCOT Image Compression

Natasha Devroye, Fabrice Labeau
McGill University

- Year long project which consisted of reading D. Taubman's ``High performance scalable image compression with EBCOT" *IEEE Trans. on Image Processing*, vol.9, 2000.
- Learned all background material on image compression techniques, and coded the described algorithm, which forms the basis of JPEG2000 in C++.

- 2-D Wavelet Transform
- Bit-plane encoding
- Adaptive Arithmetic Coding
- On each bit plane, uses Quad-tree, and 4 more coding passes
- Post-compression rate distortion optimization